Out-of-equilibrium phases of monitored circuits

From measurement-only to measurement-free

Matteo Ippoliti (Stanford University)

9/20/2021, KITP, “INFOVERSALITY 21”

Phase structure out of equilibrium

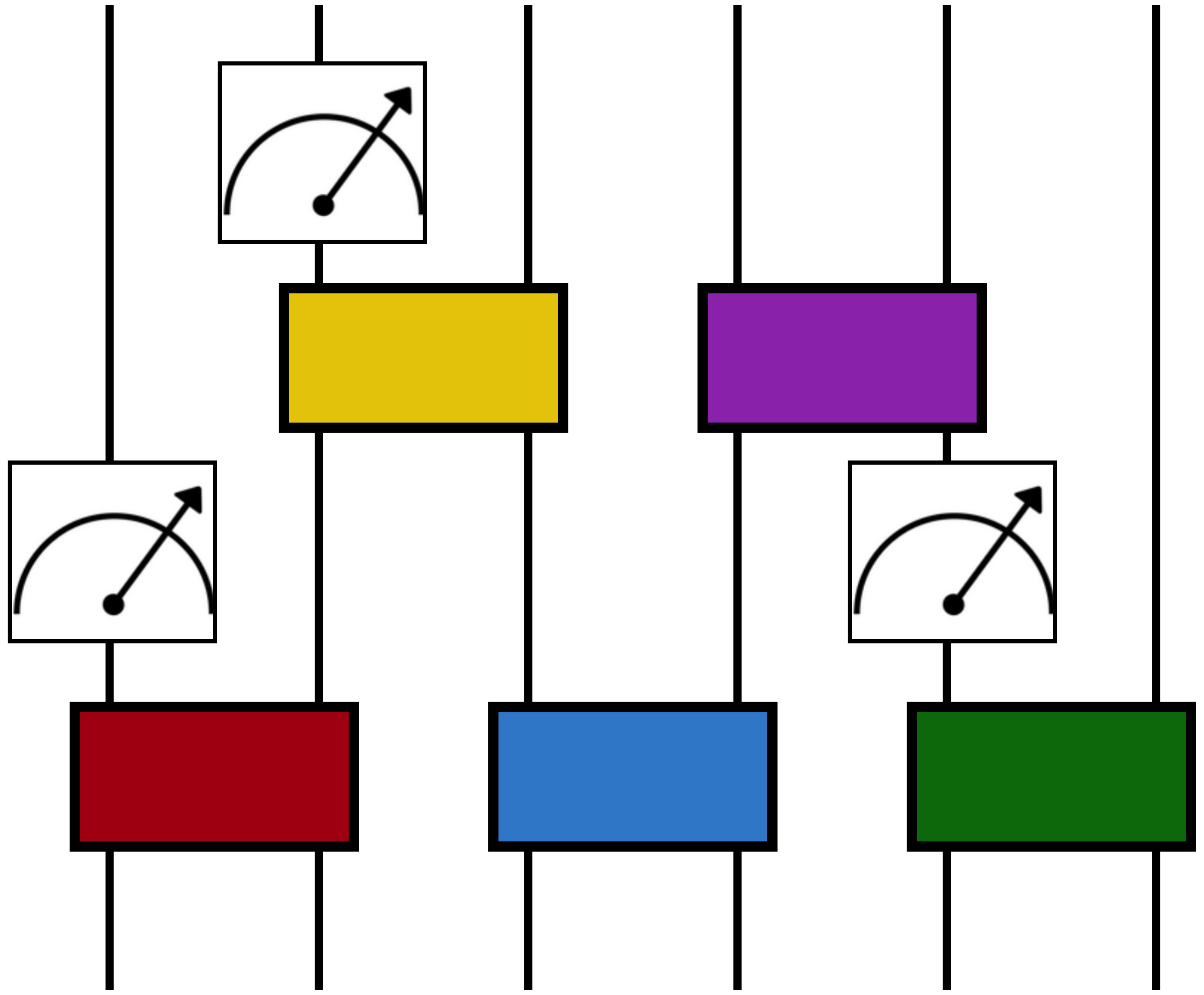
- Expanding ideas of **order**, **phases**, & **universality** to new domains
- Non-equilibrium systems like to return to equilibrium
  - **Quantum thermalization** — foundations of stat-mech, chaos, hydrodynamics…

- Some ways around thermalization:
  - Driven/dissipative systems
  - Many-body localization (MBL)
  - **Monitored circuits**

Monitored circuits

Two fundamental processes in quantum mechanics:

- **unitary** time evolution $-i\partial_t |\psi\rangle = H|\psi\rangle$
- measurement — "wavefunction **collapse**" $|\psi\rangle \mapsto \dfrac{P|\psi\rangle}{\|P|\psi\rangle\|}$



"**Monitored circuits**":

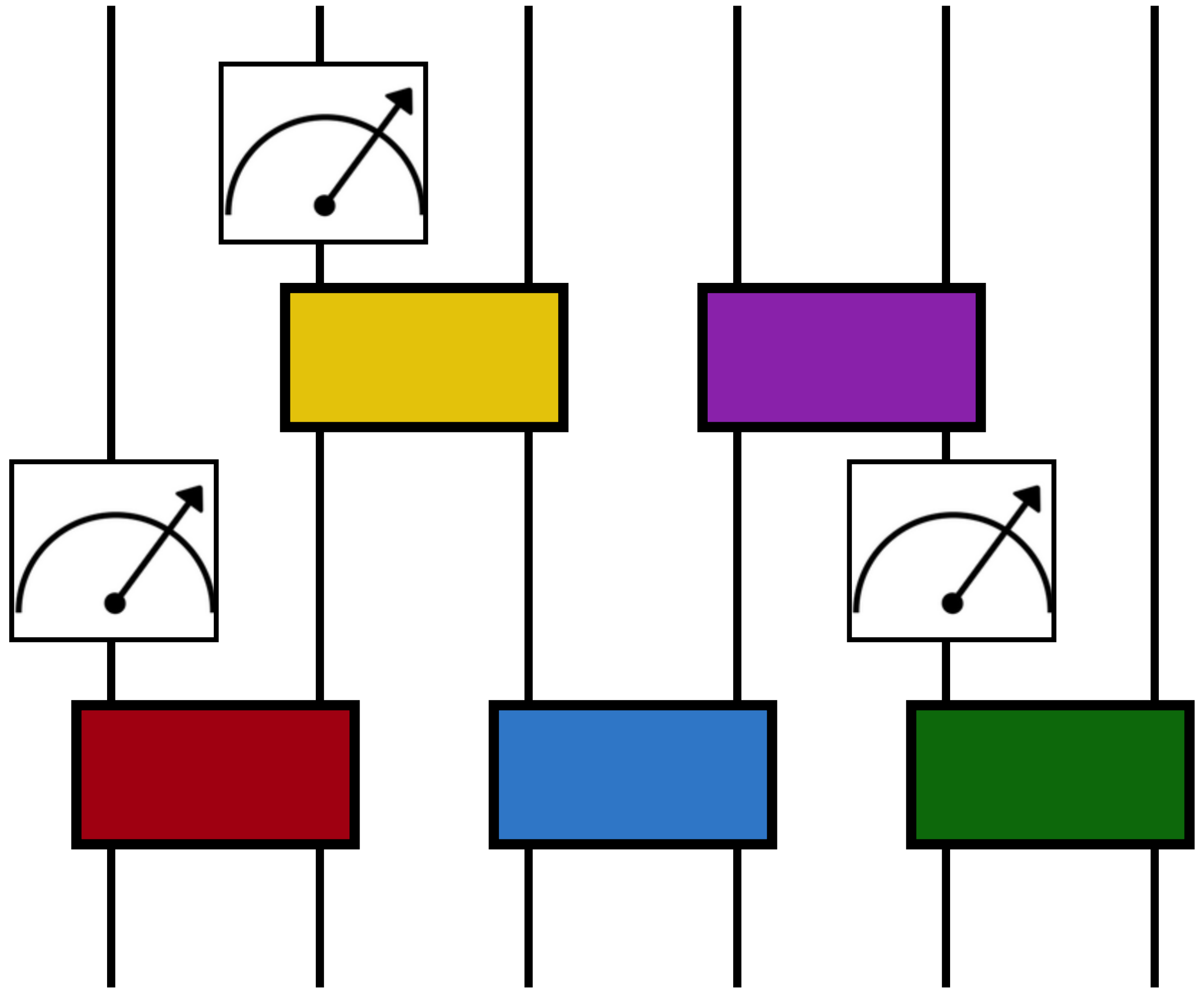
unitary dynamics + measurements*

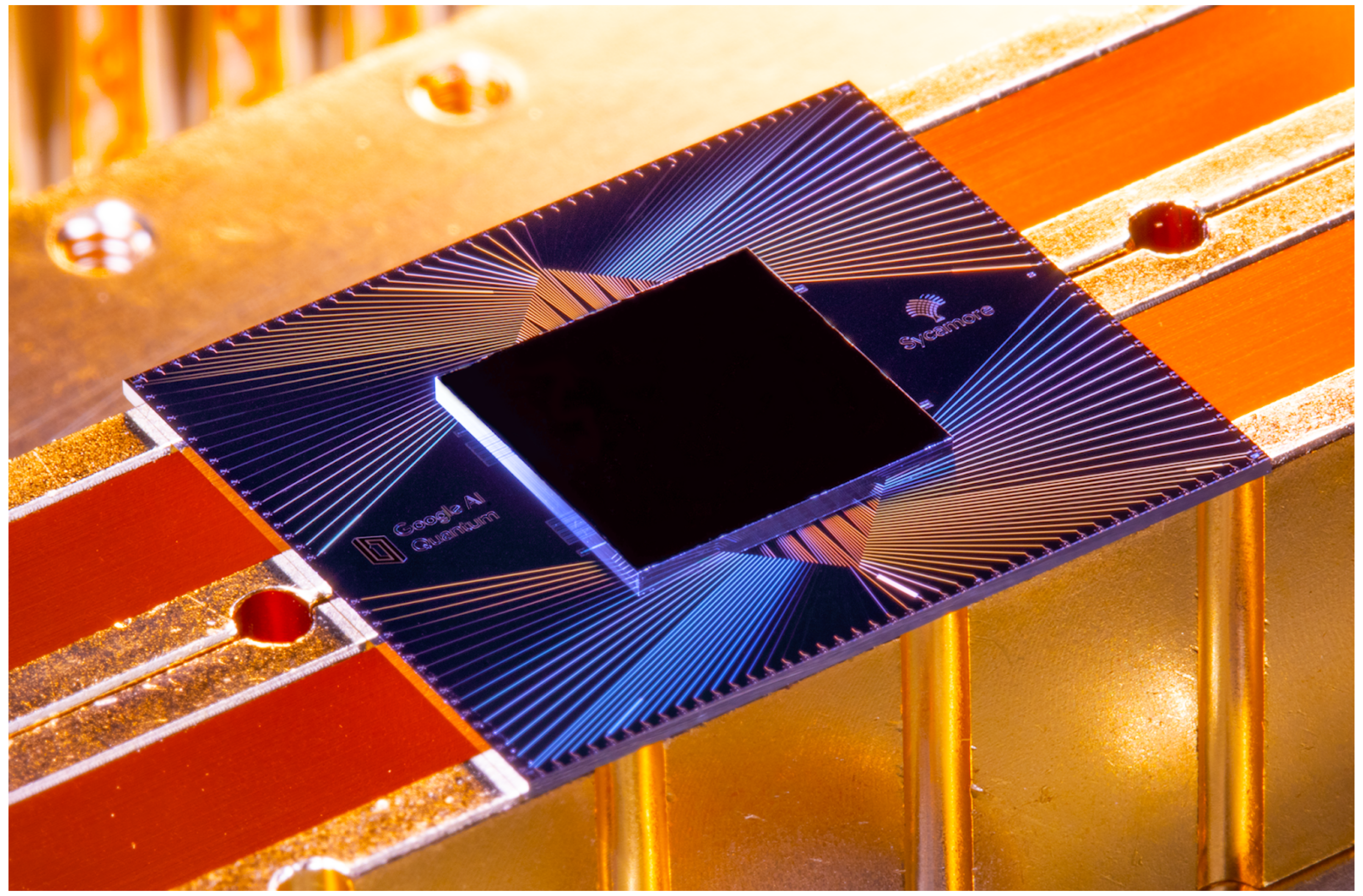
(*postselected: **pure** quantum trajectories)

[Li, Chen, Fisher PRB 2018; Nahum, Ruhman, Skinner PRX 2019]

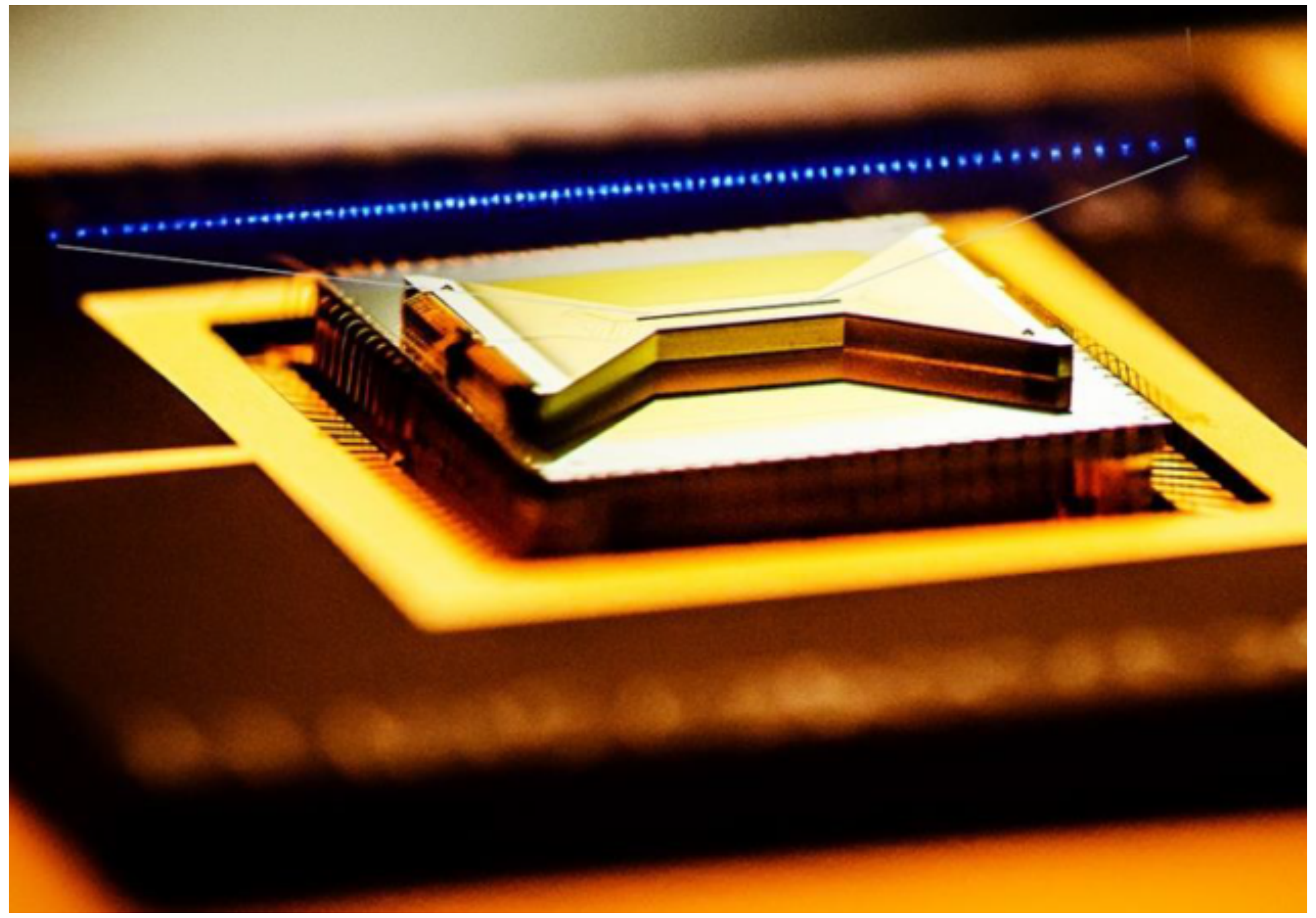
Monitored circuits

- Experimentally motivated by advent of NISQ devices
  - New capabilities for **control** and **measurement**
  - Ideal for exploring **nonequilibrium phenomena** in **many-body** setting

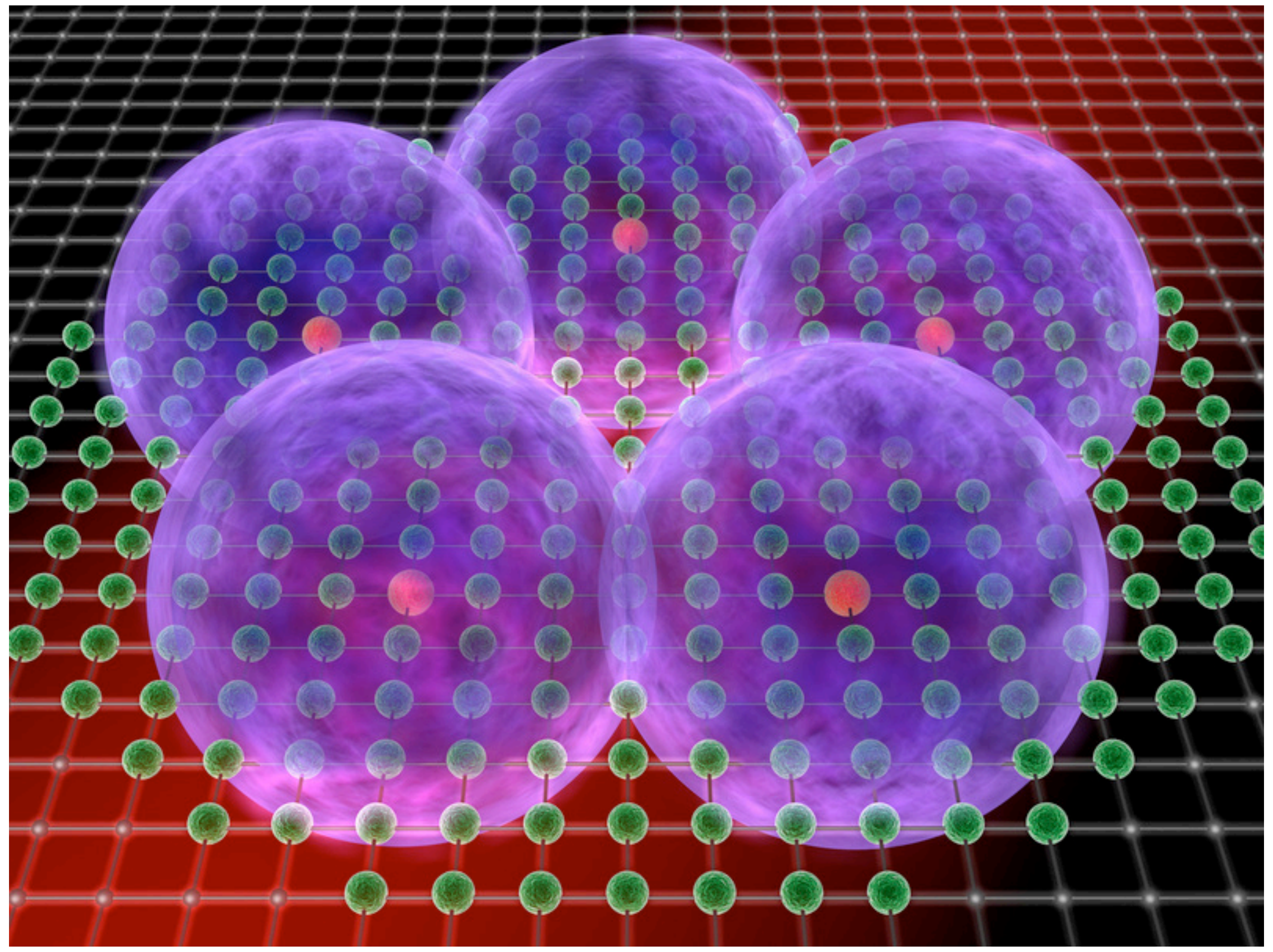




Superconducting qubits (Google AI Quantum)

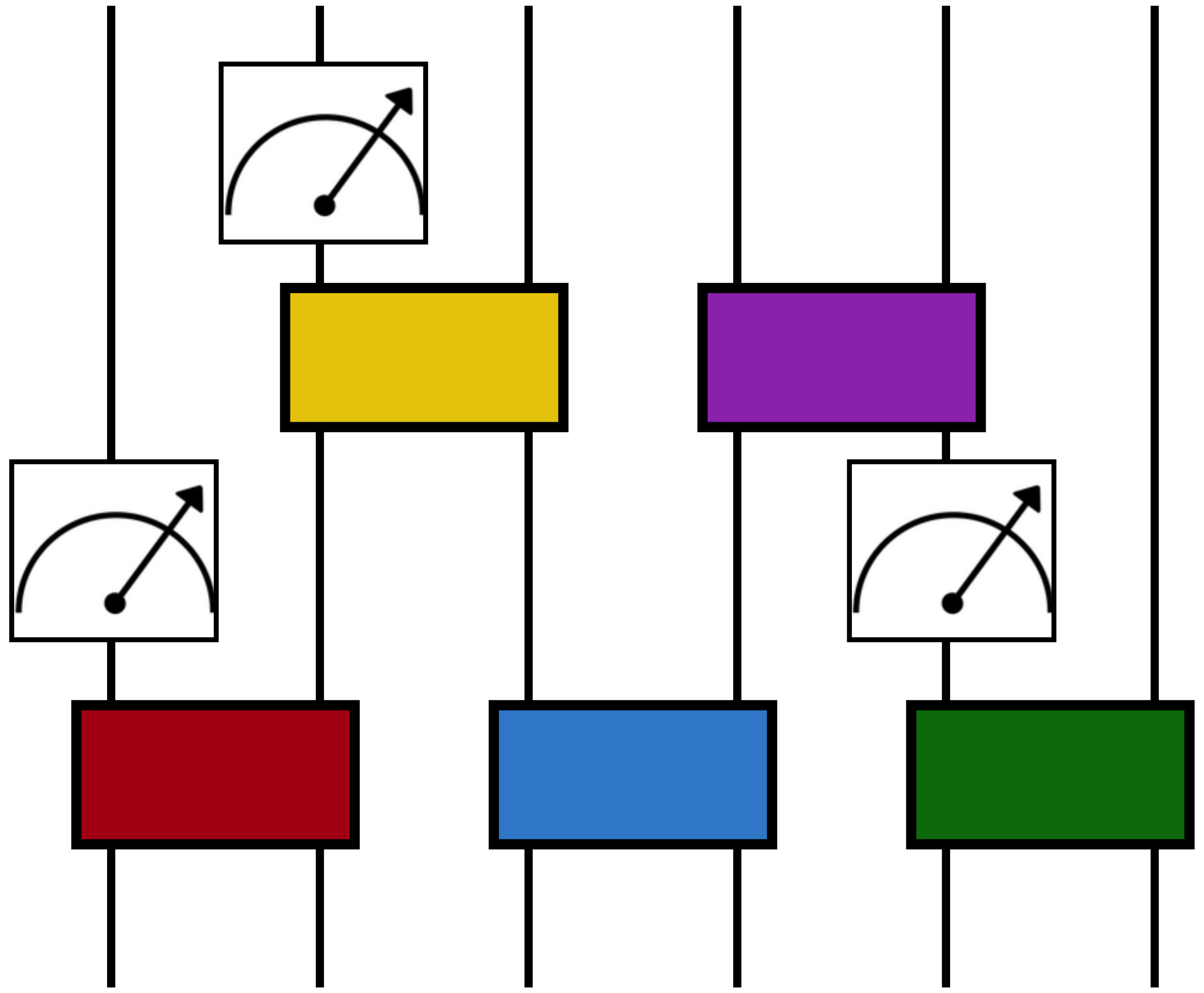


Trapped ions (IonQ)

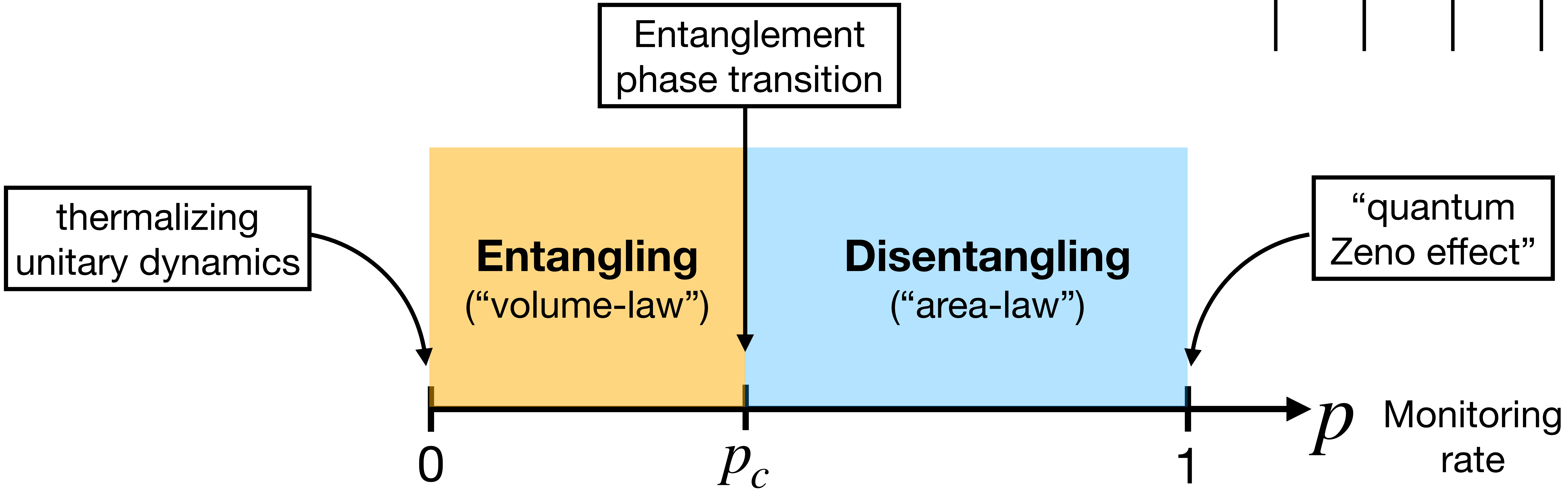


Rydberg atoms (Bloch lab, Munich)

Monitored circuits

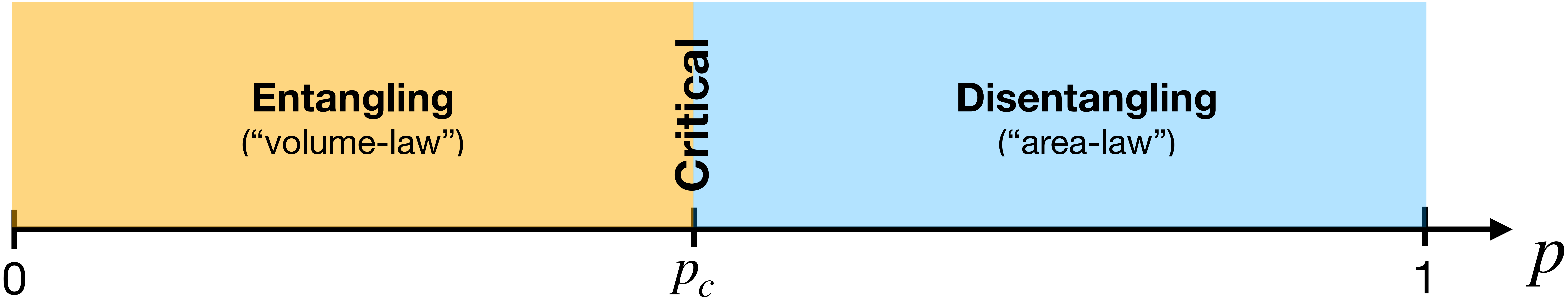


- New **entanglement phase transition**
- New arena for **out-of-equilibrium universality** & phase structure



[Li, Chen, Fisher PRB 2018; Nahum, Ruhman, Skinner PRX 2019]

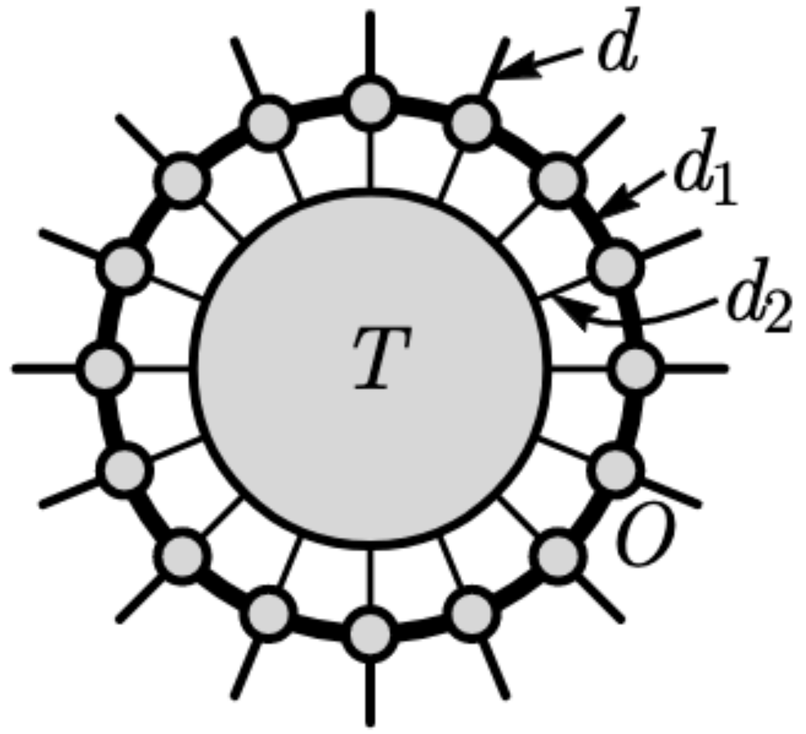
Entanglement phases & transitions



$S(\ell) \sim s\ell + f(\ell)$

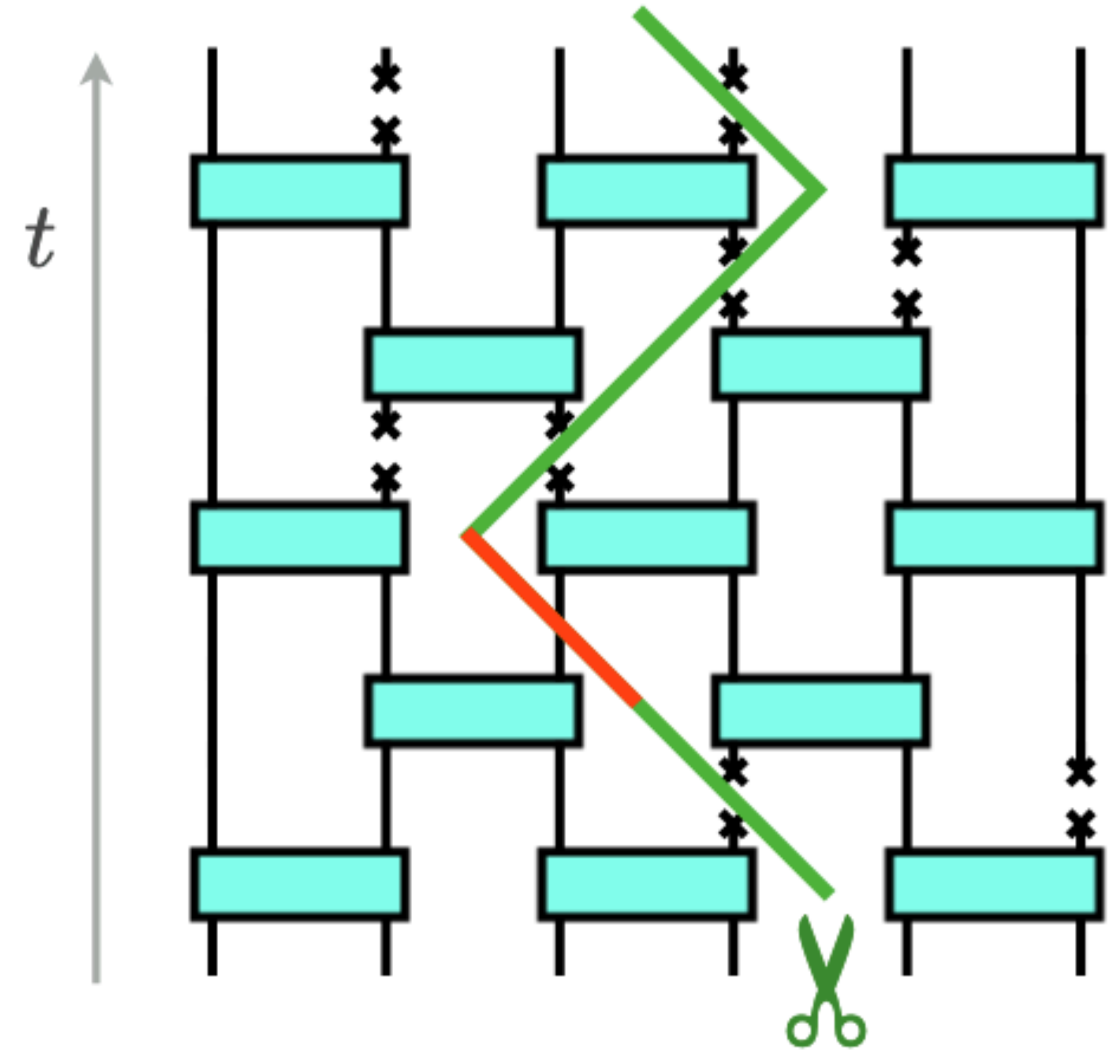
$S(\ell) \sim \log \ell$

$S(\ell) \sim \text{const}\,.$



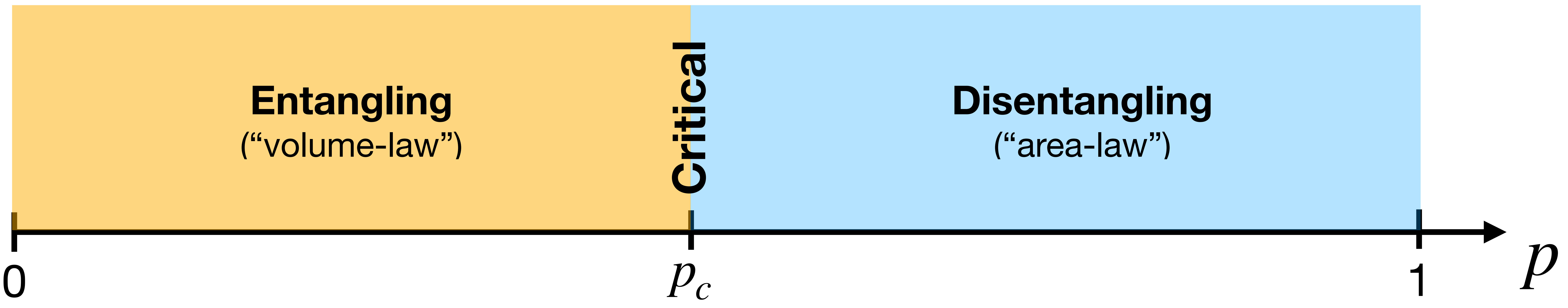
[Fan, Vijay, Vishwanath, You, PRB 2021]

Scrambling unitaries form **quantum code**, hide info from measurements



[Nahum, Ruhman, Skinner PRX 2019]

Entanglement phases & transitions



Questions:

- Nature of the phase?
- Connections to active QEC?

- What is the CFT?

- Interplay with quantum order?

- **Experimental access?** Limiting "postselection overhead" / "decoding complexity"?

Part 1: Measurement-Only

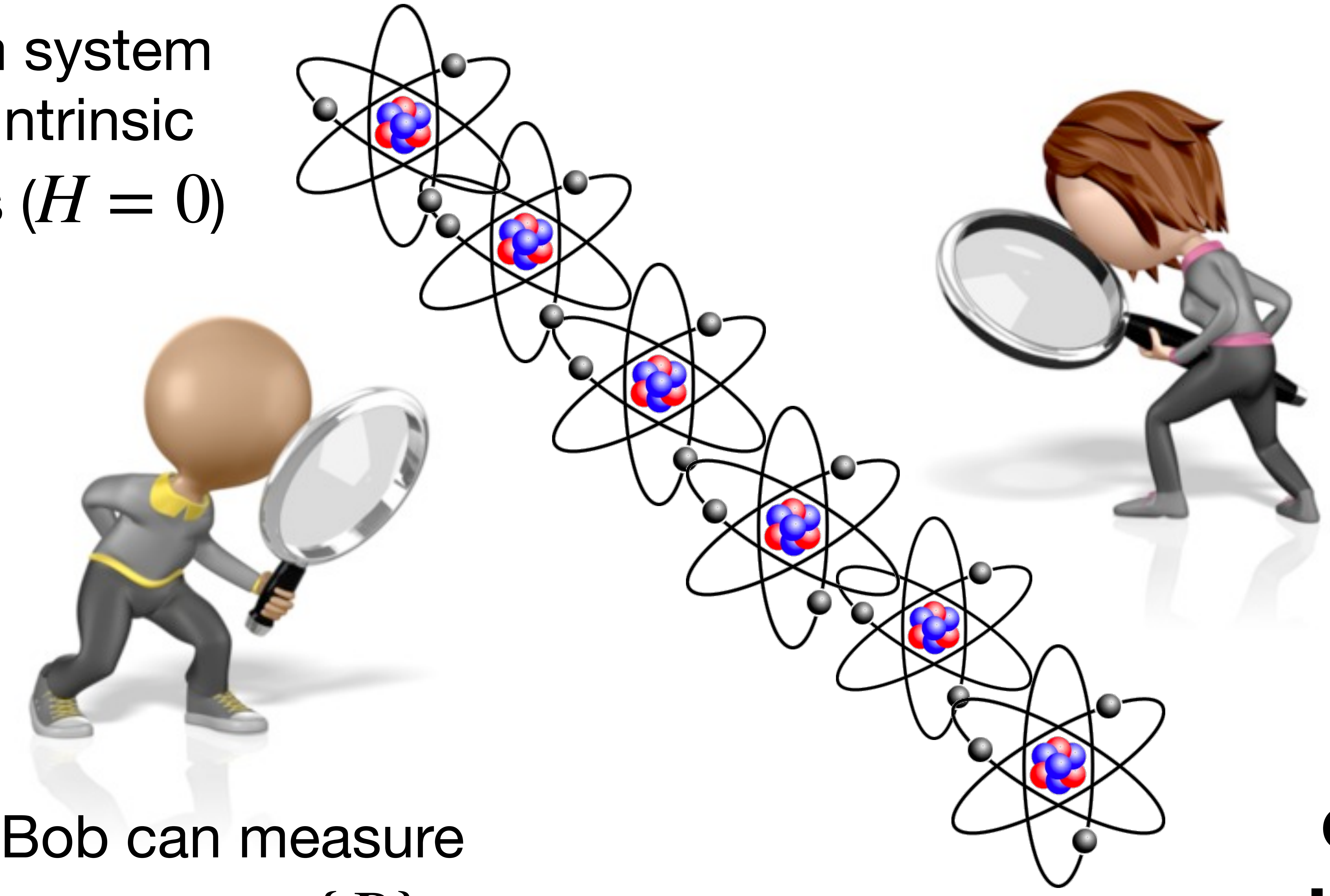
PHYSICAL REVIEW X **11,** 011030 (2021)

Entanglement Phase Transitions in Measurement-Only Dynamics

Matteo Ippoliti,[1] Michael J. Gullans,[2] Sarang Gopalakrishnan,[3,4,5] David A. Huse,[2,6] and Vedika Khemani[1]

Measurement-only dynamics

Quantum system with no intrinsic dynamics ($H = 0$)



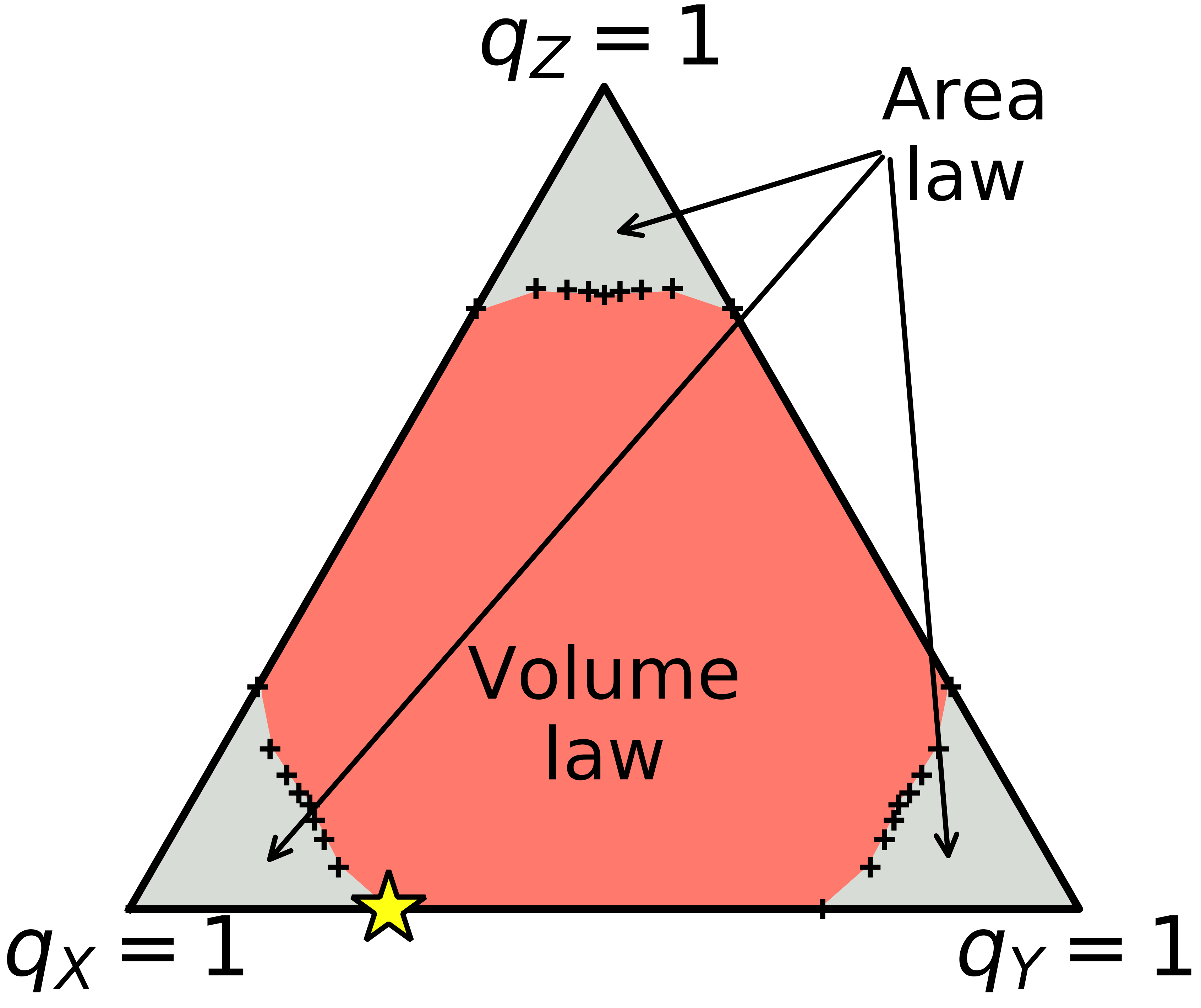
Alice can measure operators $\{A\}$

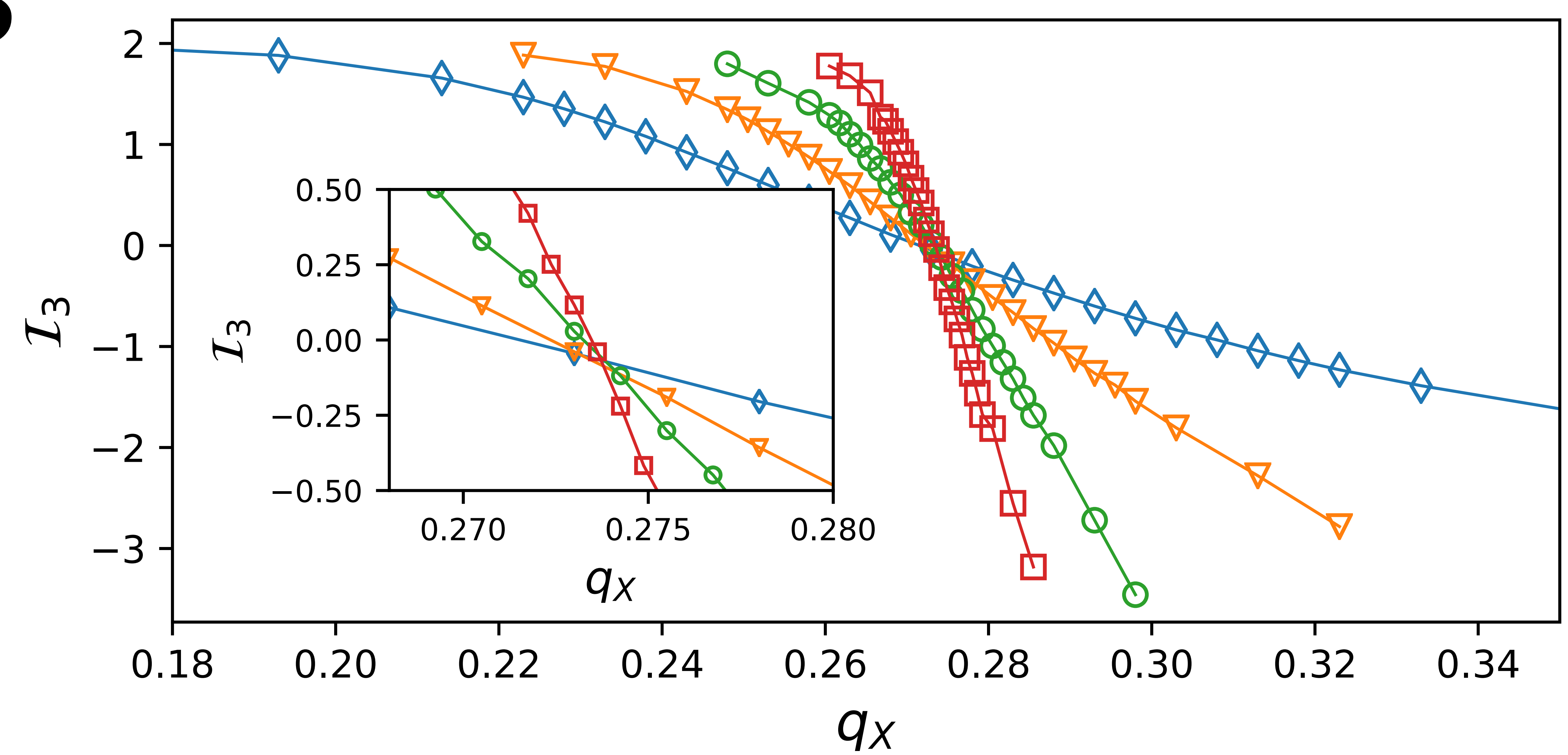
Bob can measure operators $\{B\}$

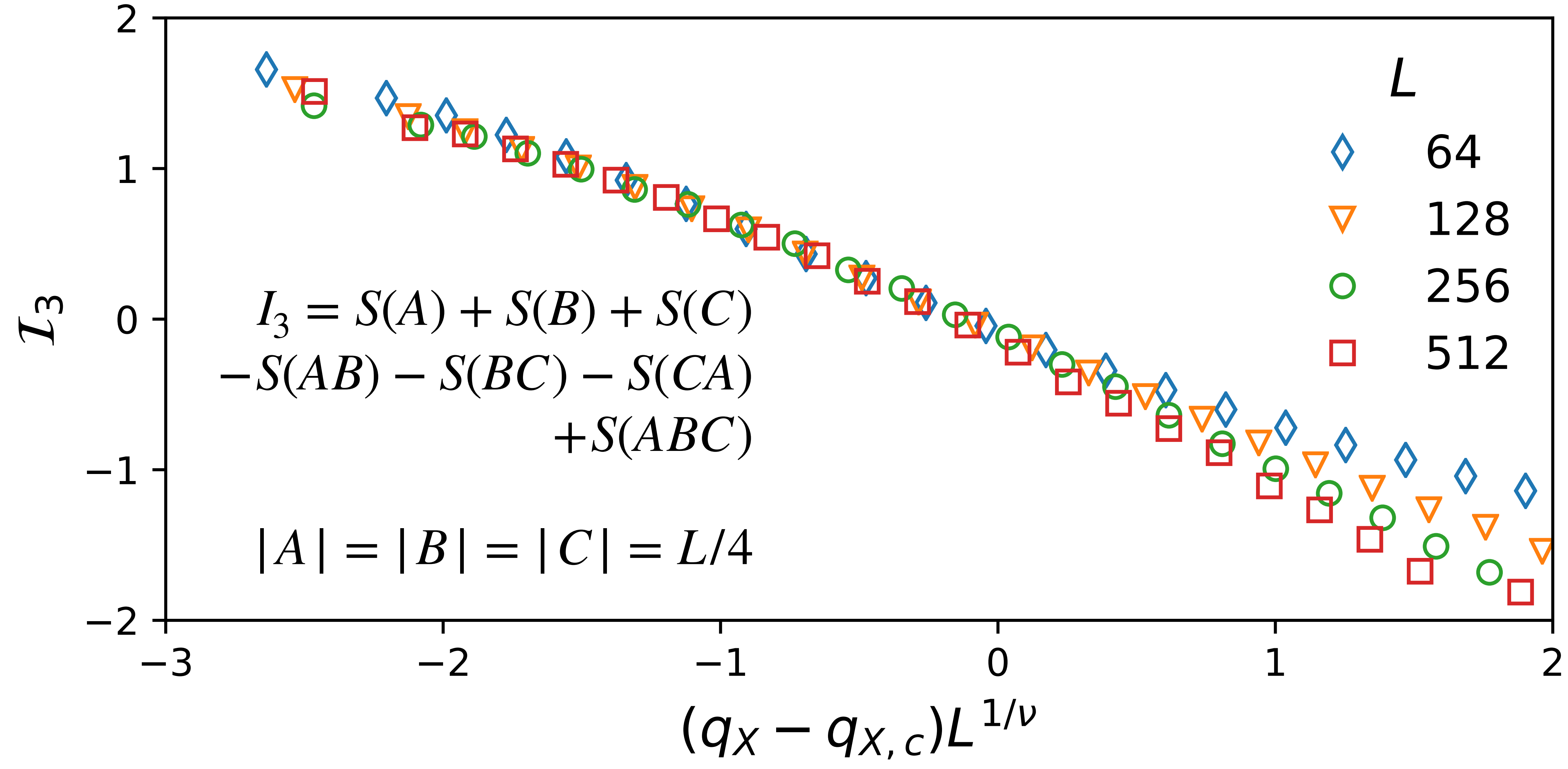
Steady-state entanglement in the system?

Entanglement phases

- Example: measure {XXX, XXY, … ZZZ} (27 operators: generic model)
- $\mathrm{Prob}(\sigma^a \sigma^b \sigma^c) = q_a q_b q_c$
- Entanglement phase diagram vs probabilities q_X, q_Y, q_Z:

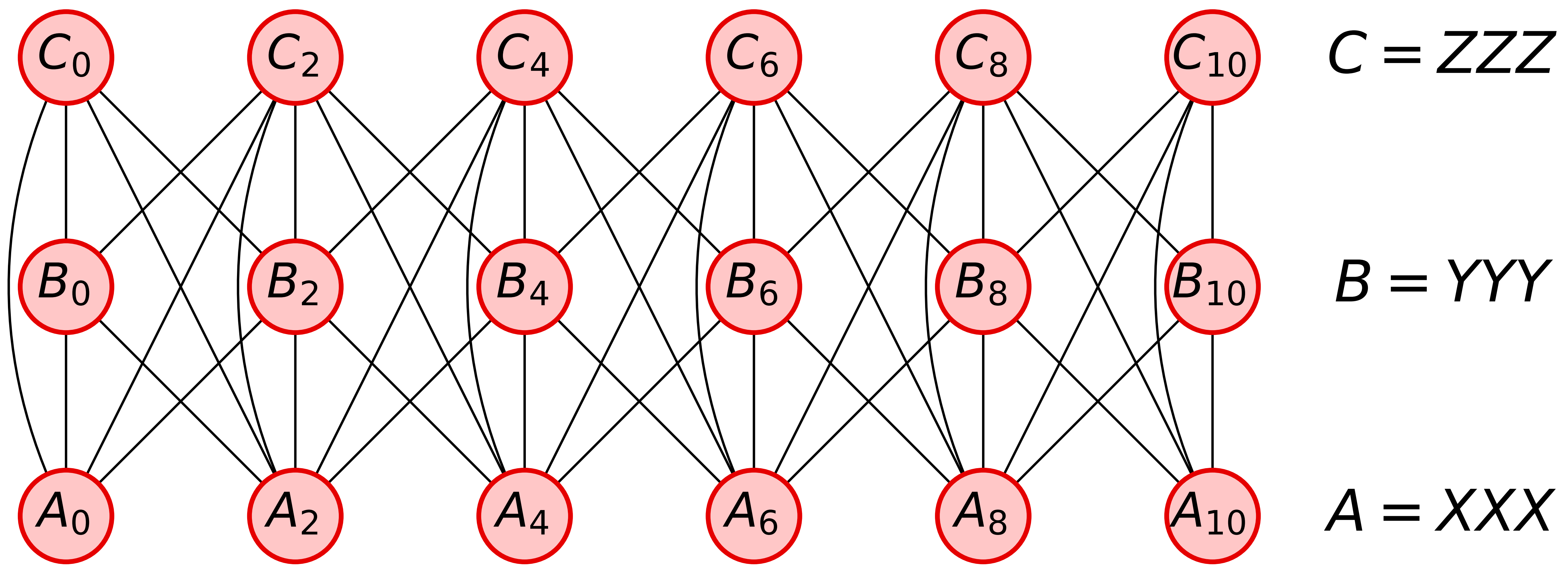






A different mechanism for the EPT

- No scrambling unitaries: entangling phase is surprising
- What drives the entanglement phase transition?
- "**Measurement frustration**" — competition between incompatible measurements



"Frustration graph"

node = operator

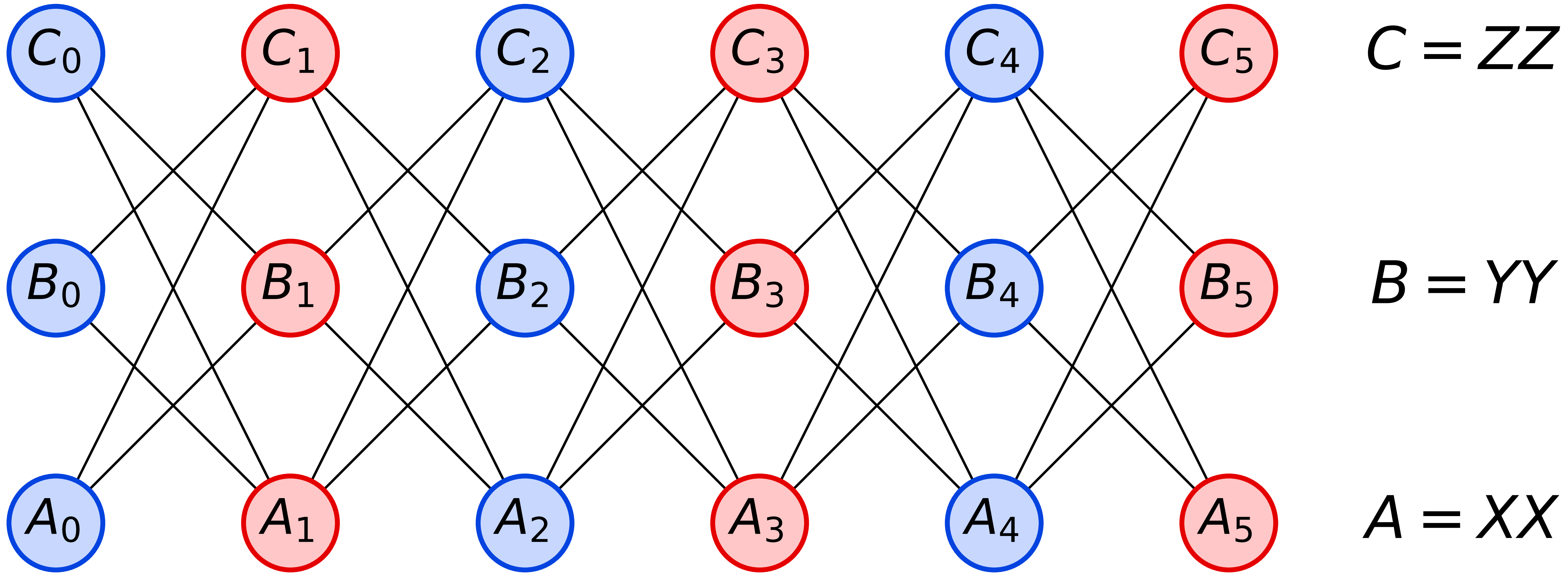
edge = anticommutation

Can reveal interesting hidden structure, e.g. free fermions

[Chapman, Flammia, Quantum 4, 278 (2020)]

A different mechanism for the EPT

- No scrambling unitaries: entangling phase is surprising
- What drives the entanglement phase transition?
- "**Measurement frustration**" — competition between incompatible measurements



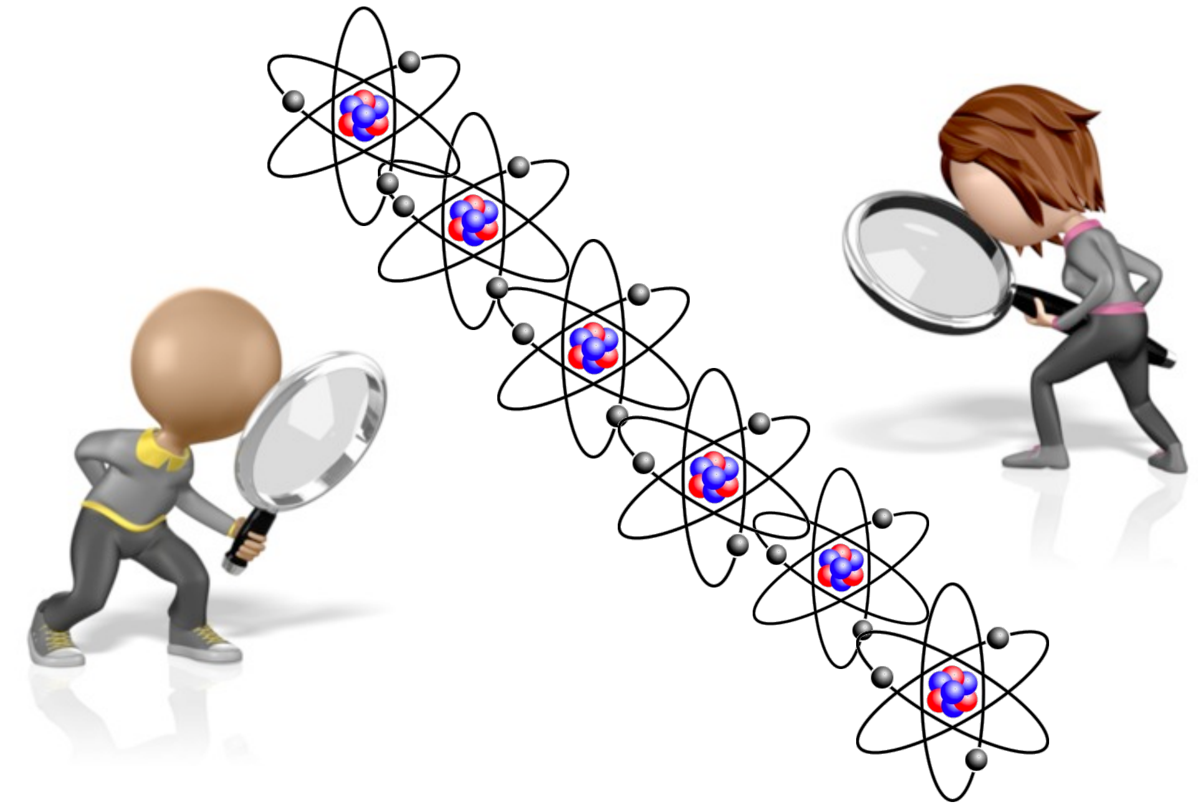
"Frustration graph"
node = operator
edge = anticommutation

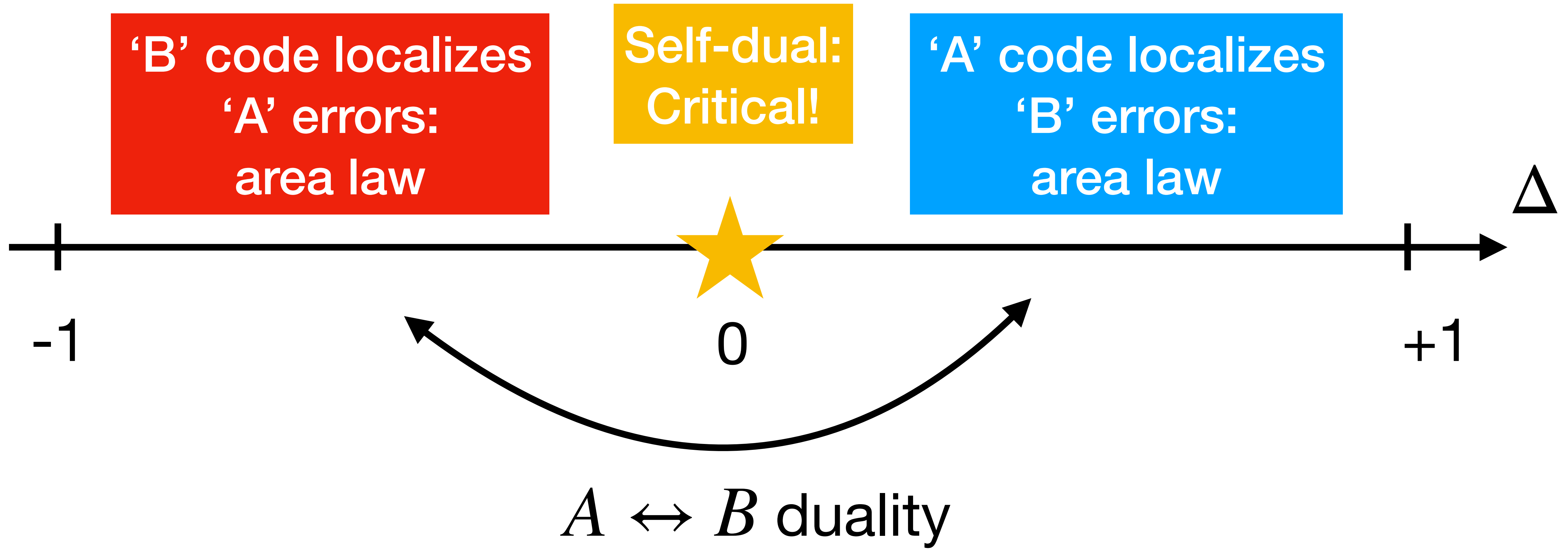
Bipartition of the frustration graph obstructs volume-law phase

Can reveal interesting hidden structure, e.g. free fermions
[Chapman, Flammia, Quantum 4, 278 (2020)]

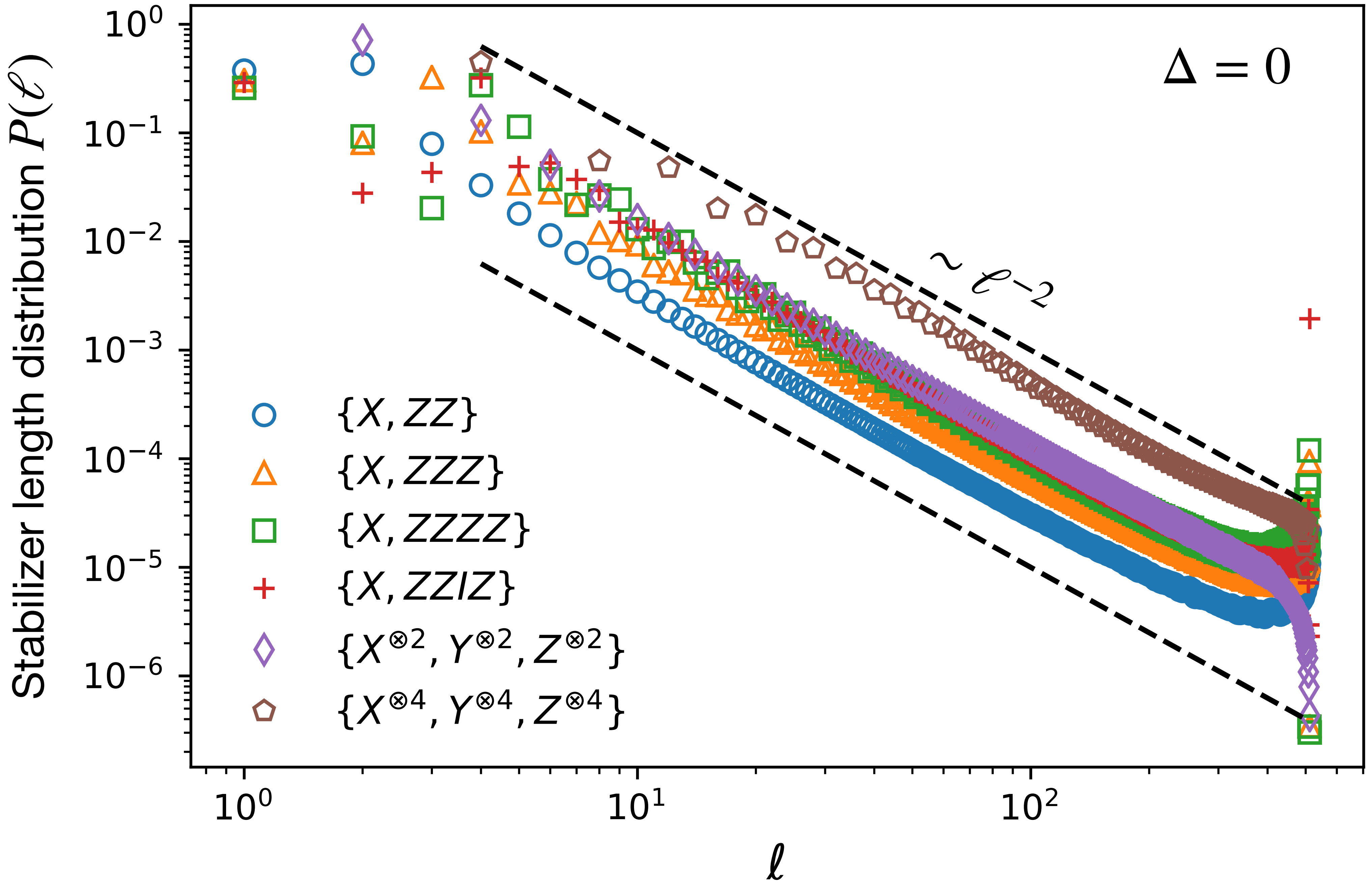
Ordered phases & dualities

- Consider two sets of **identical commuting operators**, e.g. *{A=XX, B=ZZ}* or *{A=XXX, B=ZZZ}*
- Phase diagram parametrized by $\Delta = P_A - P_B$
  - Symmetric around $\Delta = 0$ (duality)
  - Area-law at $\Delta = \pm 1$ (fully un-frustrated)
  - In between?

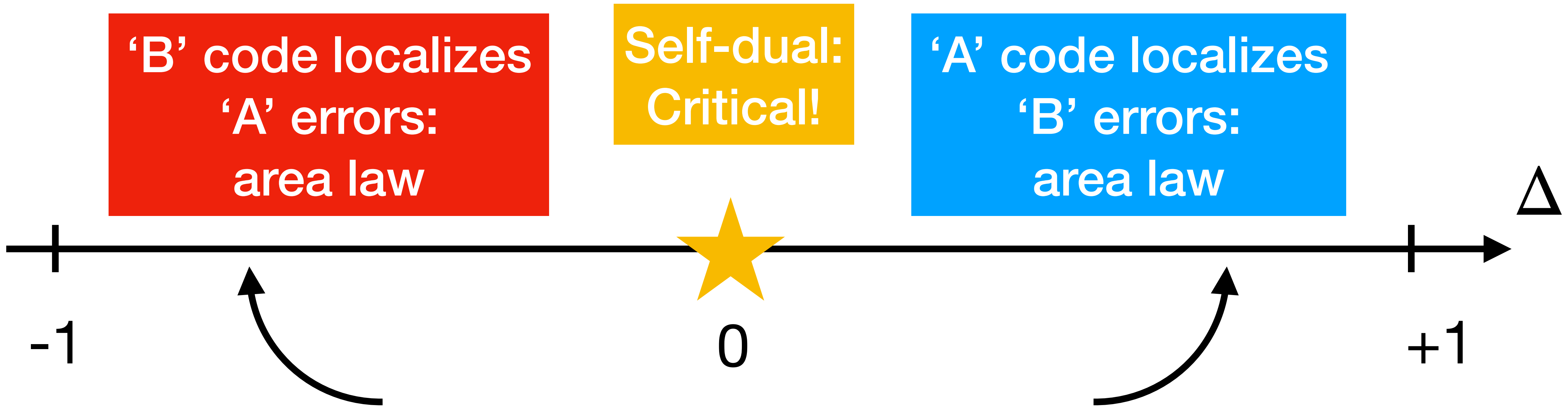




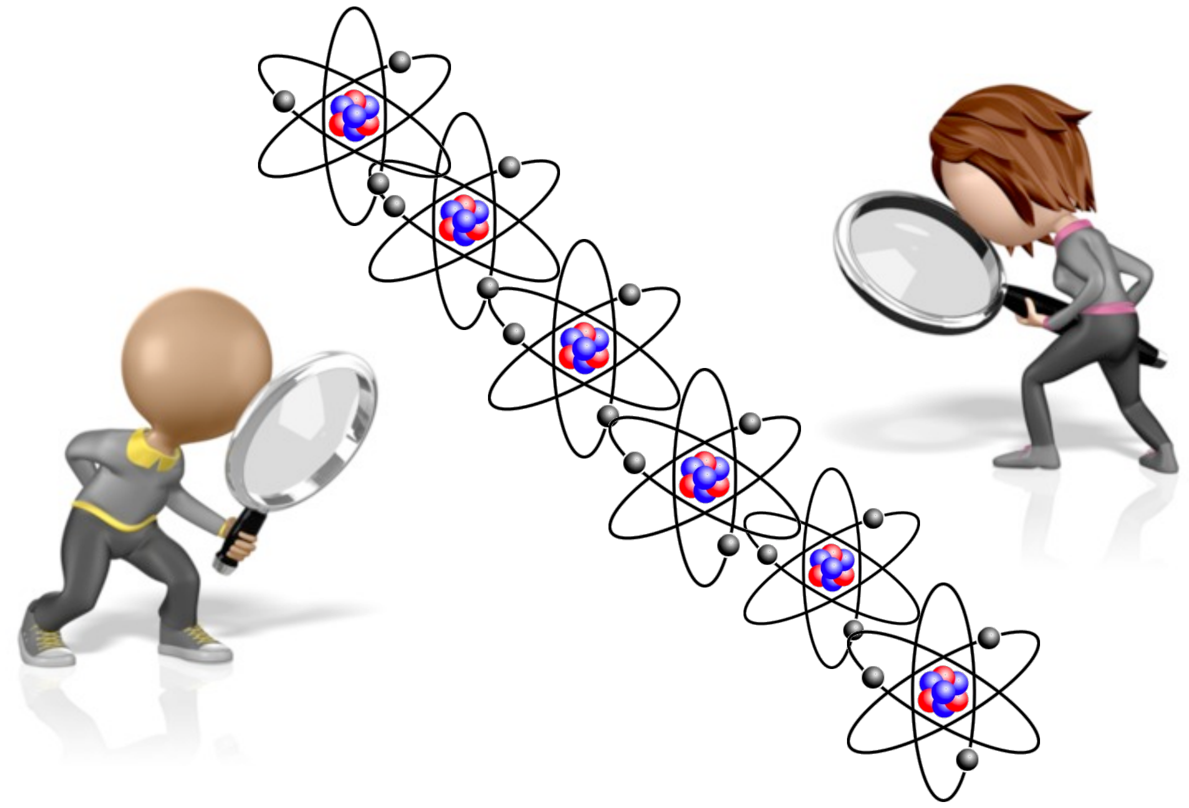
Same phenomenology for all symmetric, **bipartite** frustration graphs:



Ordered phases & dualities



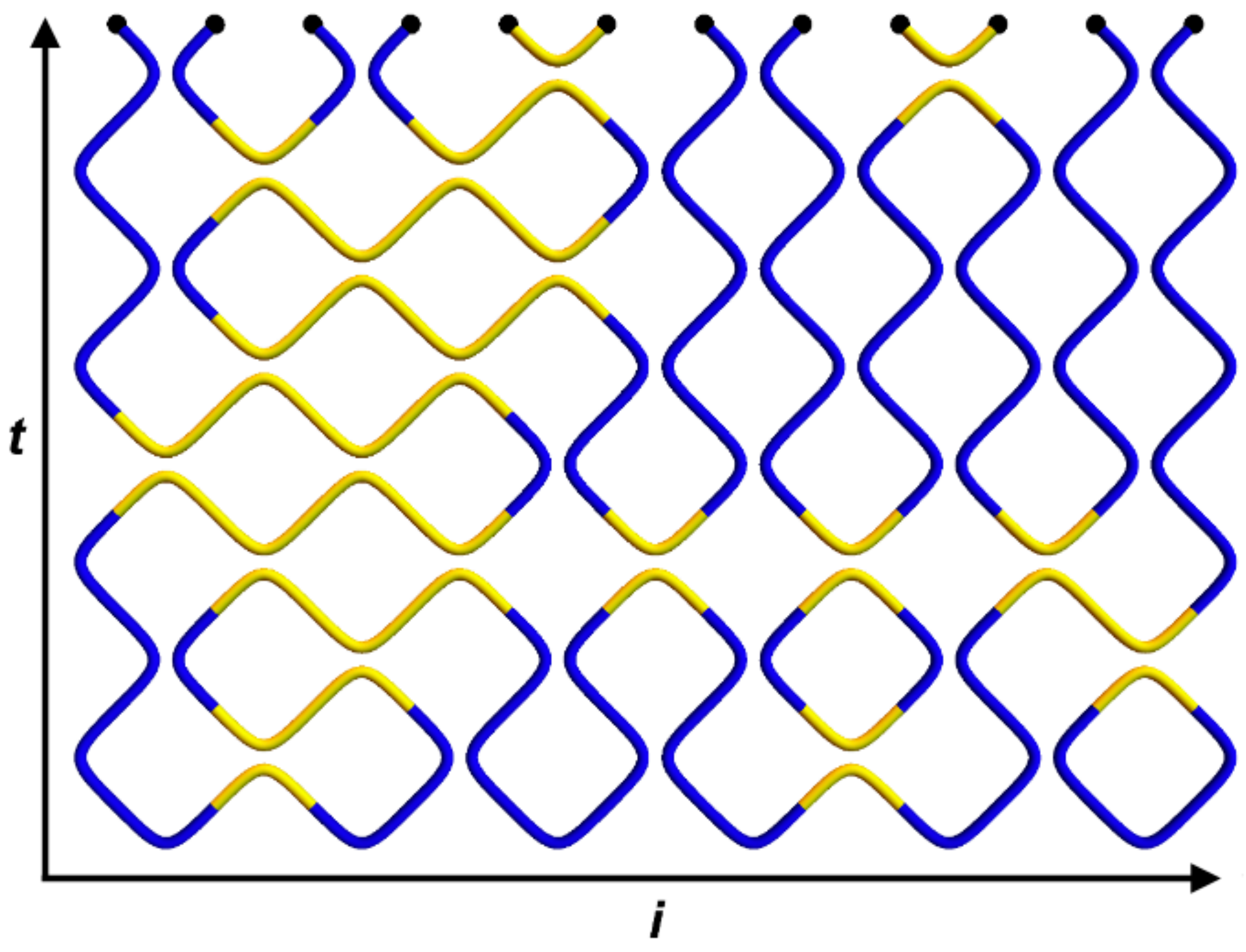
Protected "logical qubits":
(topological) **order parameters**

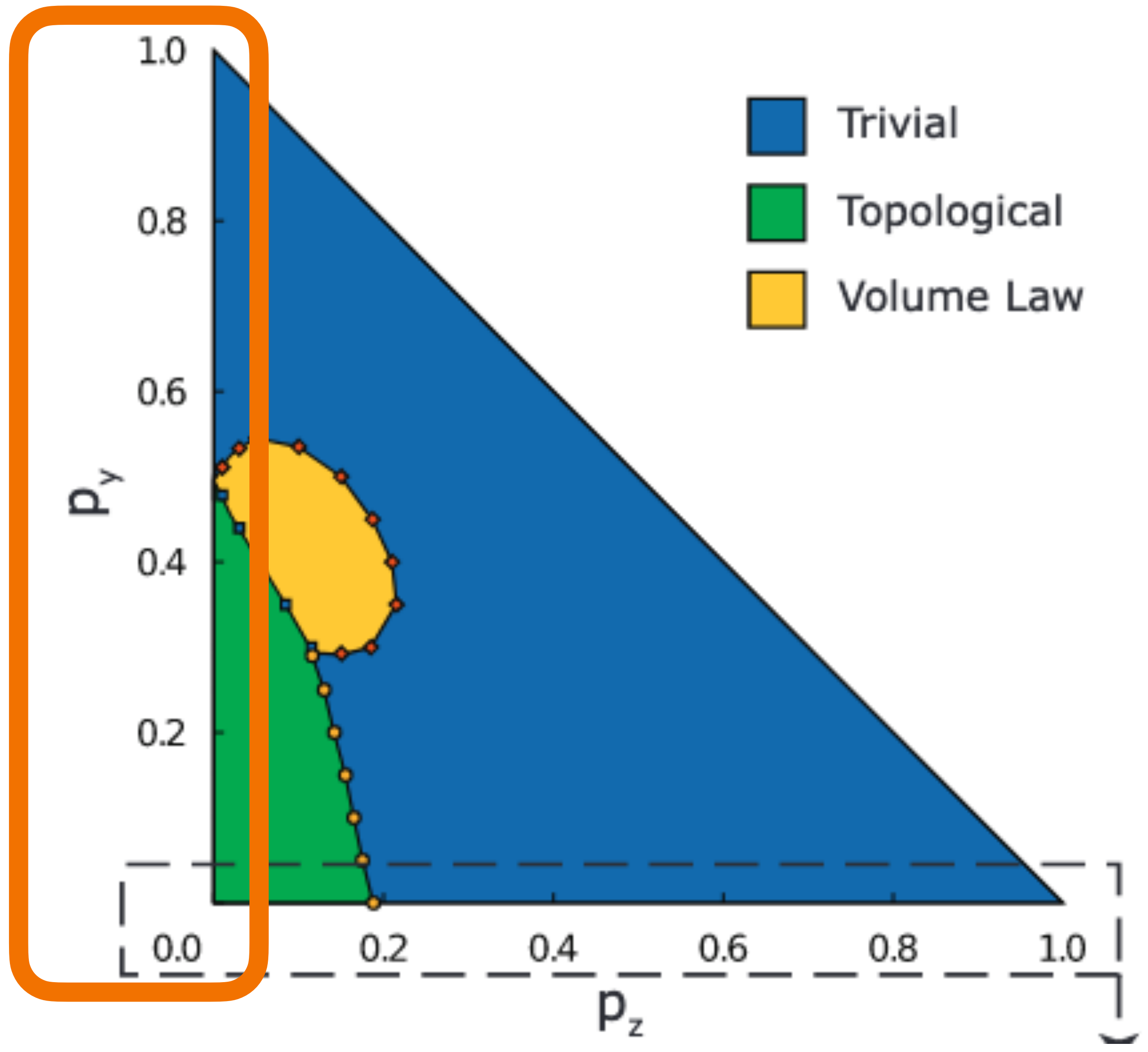


Duality: A = {toric code}, B = {Y errors}
{Z errors} break bipartition → vol. law phase

Ising case {X, ZZ}:
free Majoranas, maps
to loop percolation

[Nahum, Skinner, PRR (2020);
Lavasani, Alavirad, Barkeshli,
Nat Phys (2021);
Sang, Hsieh, PRR (2021);
Bao, Choi, Altman, arXiv:2102;
Li, Fisher, arXiv:2108]





[Lavasani, Alavirad, Barkeshli,
arXiv:2011.06595]

Part 2: Measurement-Free

PHYSICAL REVIEW LETTERS **126,** 060501 (2021)

Editors' Suggestion

Postselection-Free Entanglement Dynamics via Spacetime Duality

Matteo Ippoliti and Vedika Khemani
Department of Physics, Stanford University, Stanford, California 94305, USA

arXiv.org > quant-ph > arXiv:2103.06873

Quantum Physics

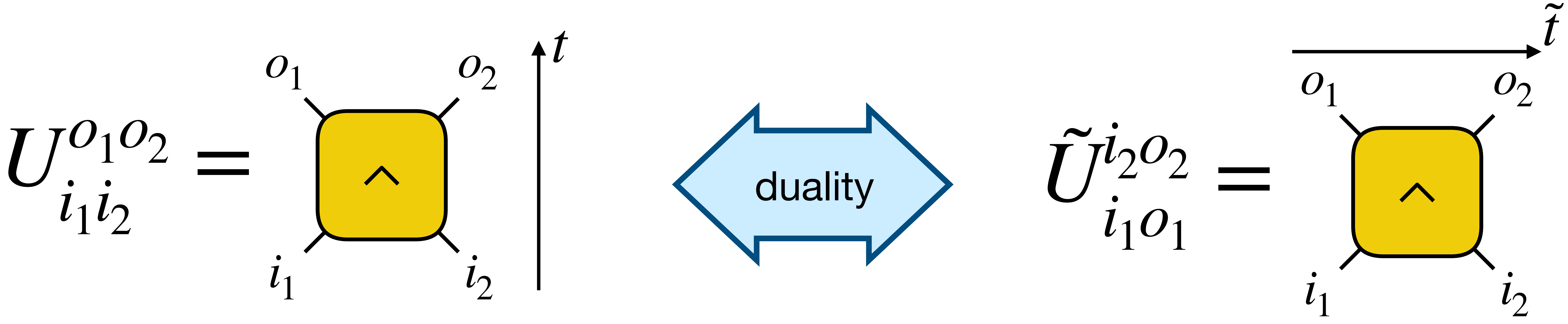
[Submitted on 11 Mar 2021 (v1), last revised 24 May 2021 (this version, v2)]

Fractal, logarithmic and volume-law entangled non-thermal steady states via spacetime duality

Matteo Ippoliti, Tibor Rakovszky, Vedika Khemani

Spacetime duality

- Exchange the roles of **space** and **time**
  - Map every unitary gate U to a dual matrix $\tilde{U}$

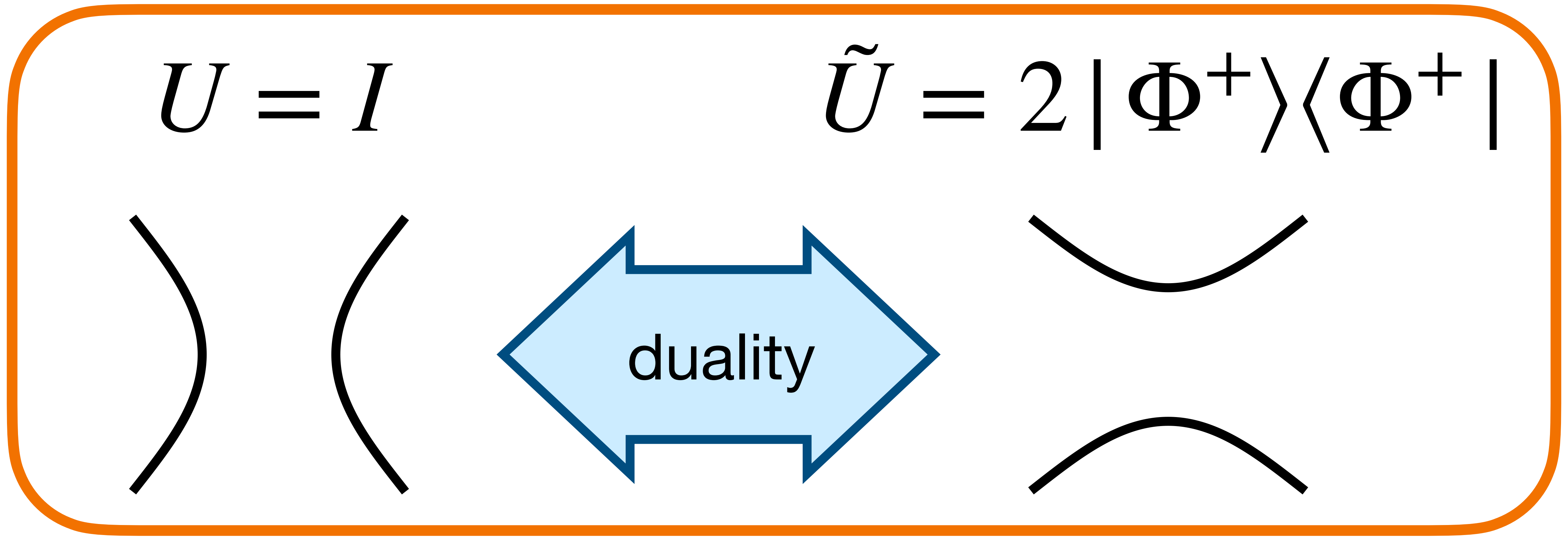


[Bertini, Kos, Prosen 2018-21; Gopalakrishnan, Lamacraft 2019; …]

- **Dual-unitary** gate: U & $\tilde{U}$ both unitary
- Fine-tuned condition: generically, $\tilde{U}$ is **not unitary**
  - $\tilde{U} = 2VH$, V unitary, H Hermitian → deterministic (postselected) measurement
- Use this to **simulate monitoring?** Avoid **postselection?**

Spacetime duality

Example:

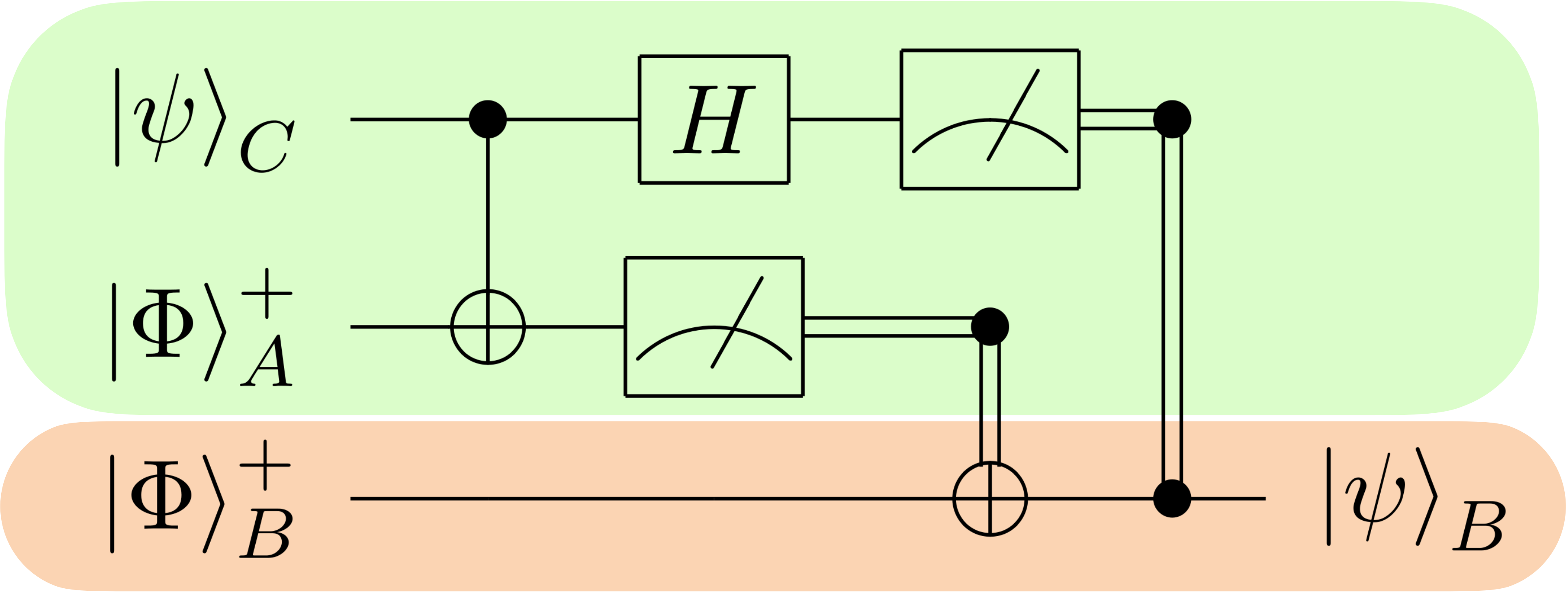


Bell pair state

$|\Phi^+\rangle \propto |00\rangle + |11\rangle$

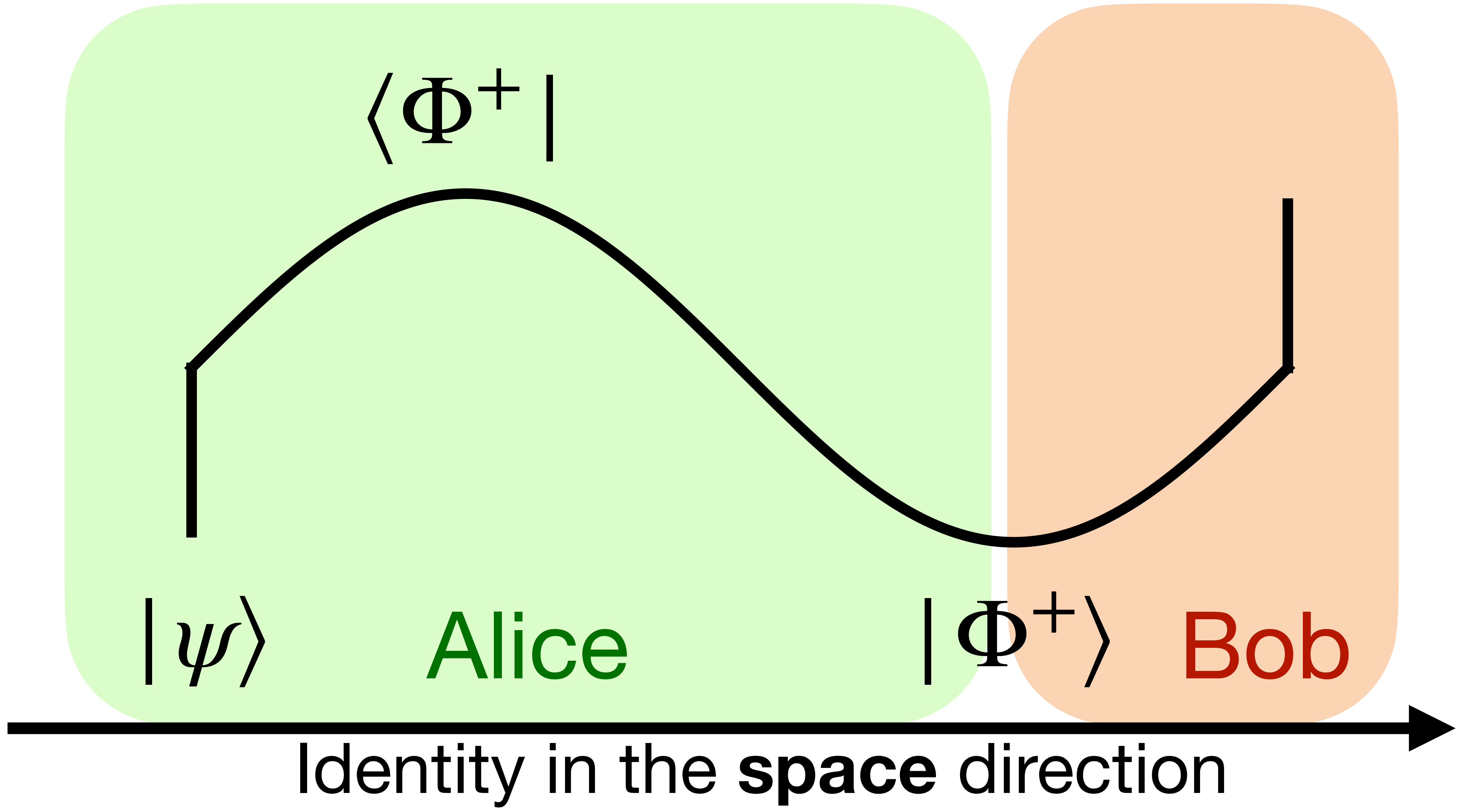
$\Phi^+_{ij} \propto \delta_{ij}$

Aside: **quantum teleportation**

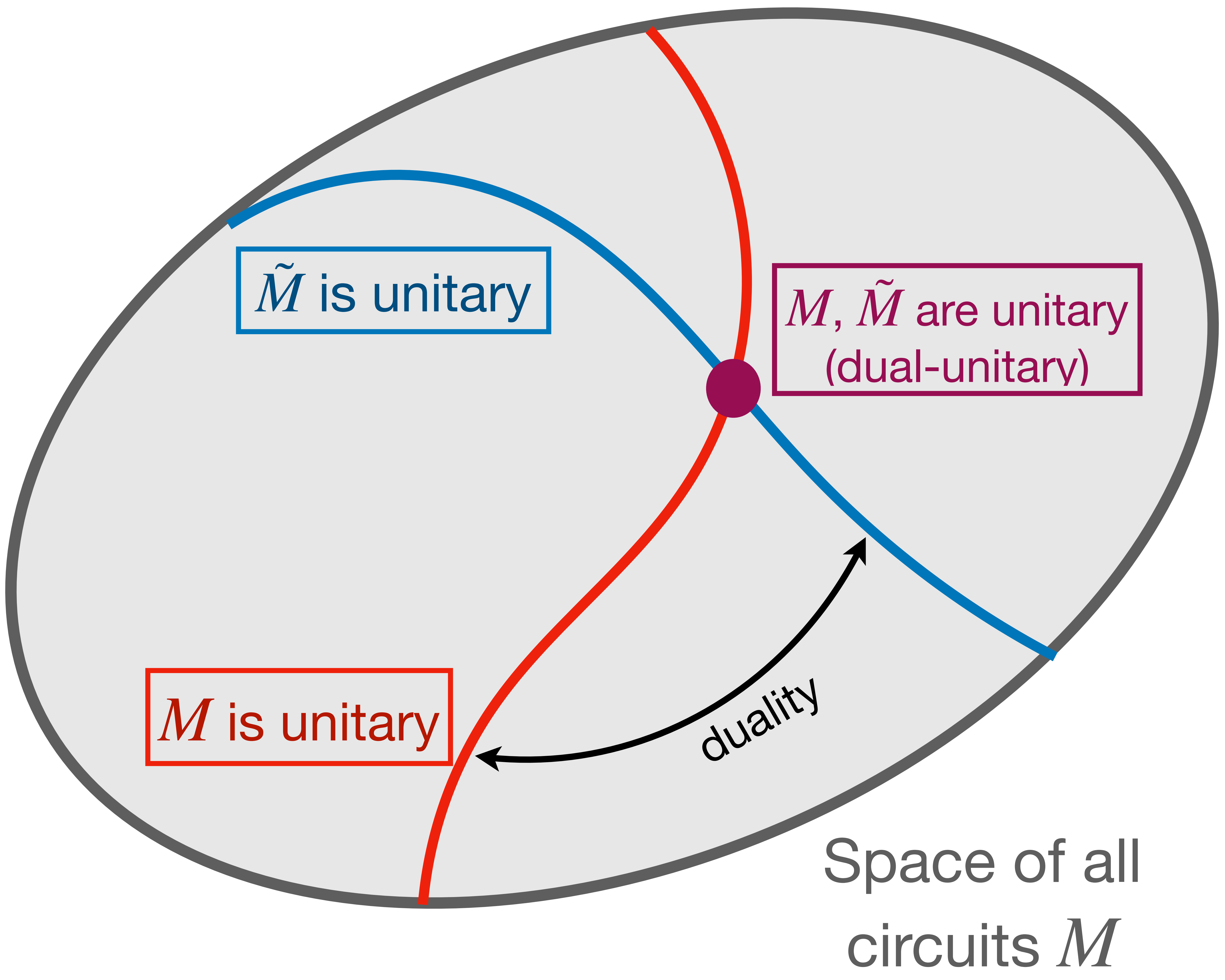


[from Wikipedia]

Via spacetime duality:



Monitored circuits & spacetime duality

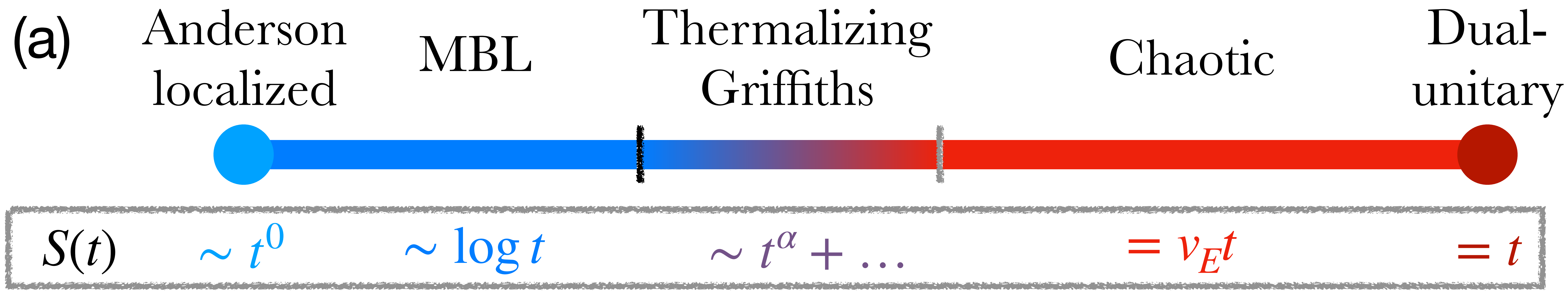


Submanifold of
analytically tractable,
experimentally feasible
monitored circuits

Entanglement in spacetime-dual circuits

Relationship between

- **growth** of entanglement in **unitary** dynamics, $S(t)$
- **spatial scaling** of entanglement in **non-unitary** steady states, $\tilde{S}_\infty(\tilde{\ell})$



Example: “kicked Ising”,

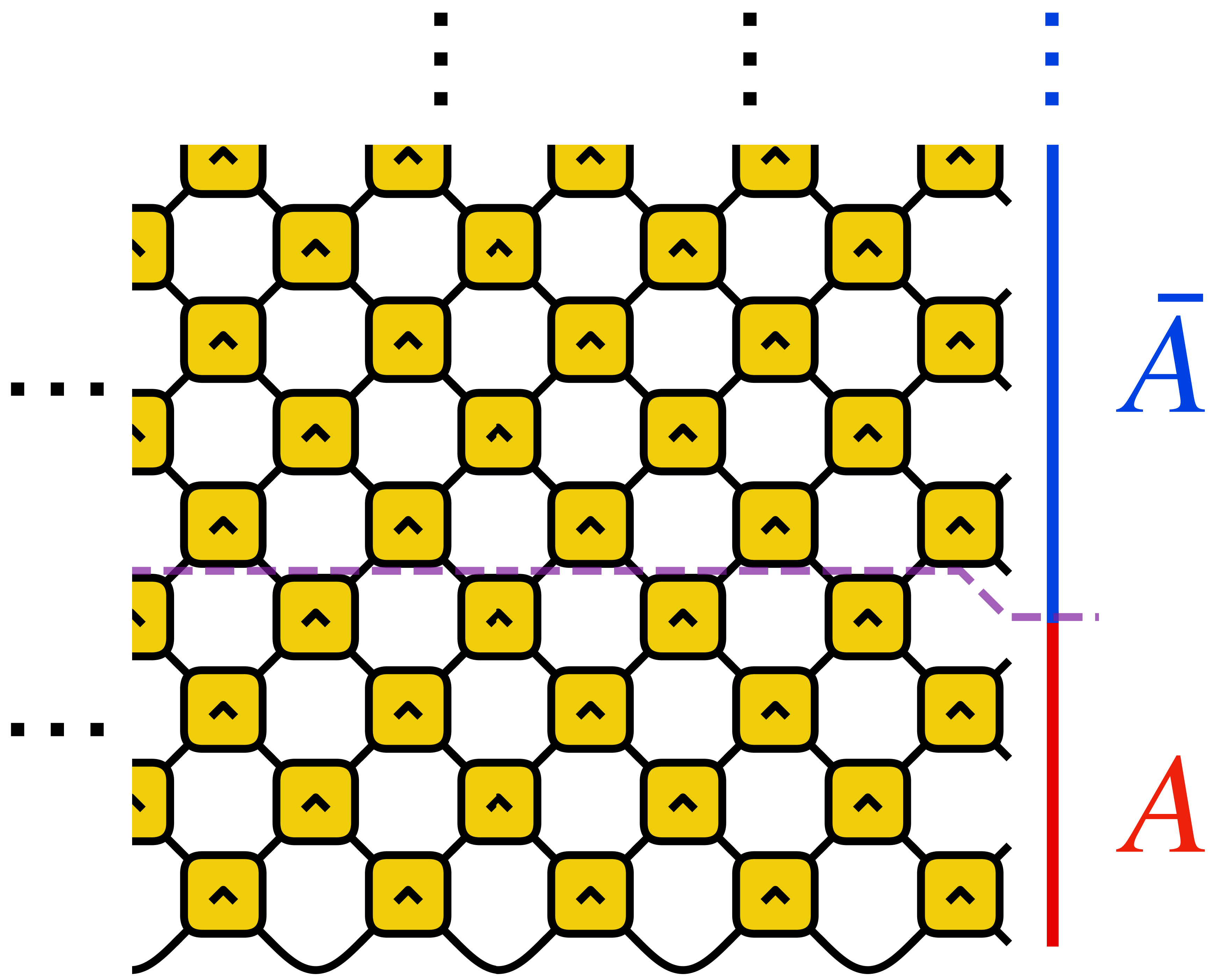
$$U = e^{-i\sum_n g_n X_n} e^{-i\sum_n J_n Z_n Z_{n+1} + h_n Z_n}$$

Entanglement in spacetime-dual circuits

spacetime-dual circuit $\tilde{U}$**,**
late-time state,
subsystem of size $|A| \equiv \tilde{\ell}$

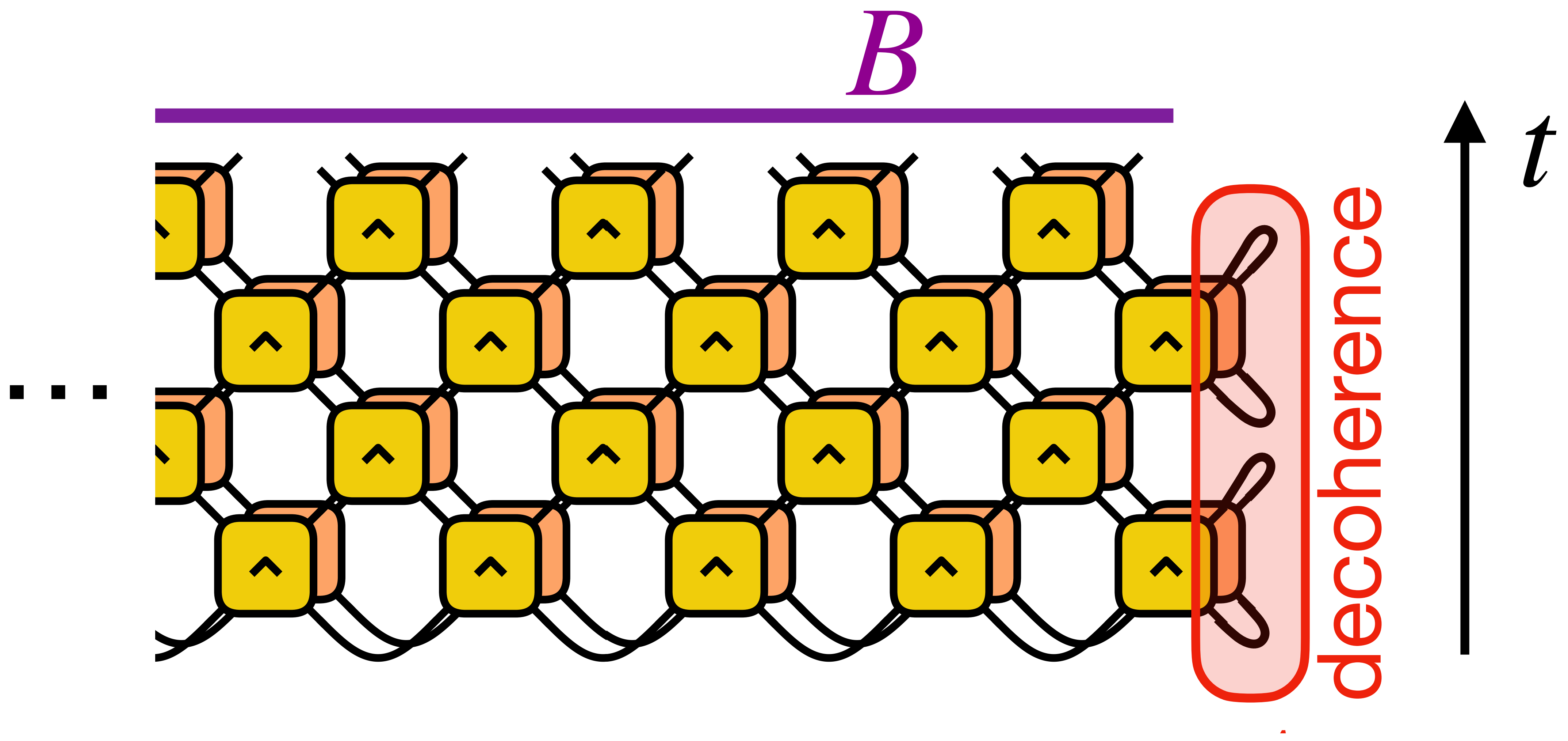
Key technical result:
$$\tilde{S}_\infty(\tilde{\ell}) = S_{\mathrm{dec}}(t = \tilde{\ell})$$

unitary circuit U **+**
edge decoherence,
depth $\tilde{\ell}$,
semi-infinite chain



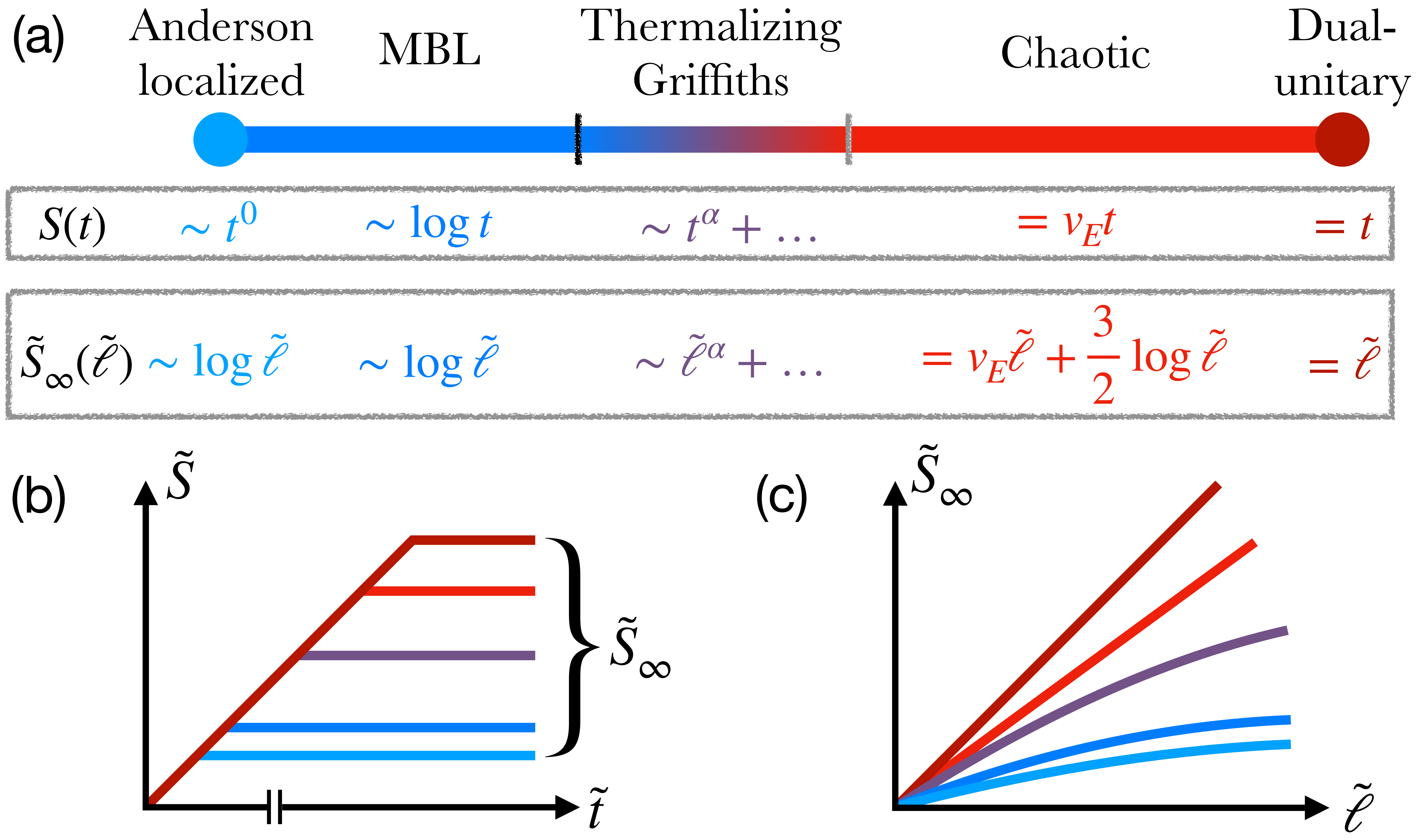
infinite depth (steady state)

$S_{\mathrm{dec}}(t) = S[\rho_B(t)]$



Entanglement in spacetime-dual circuits

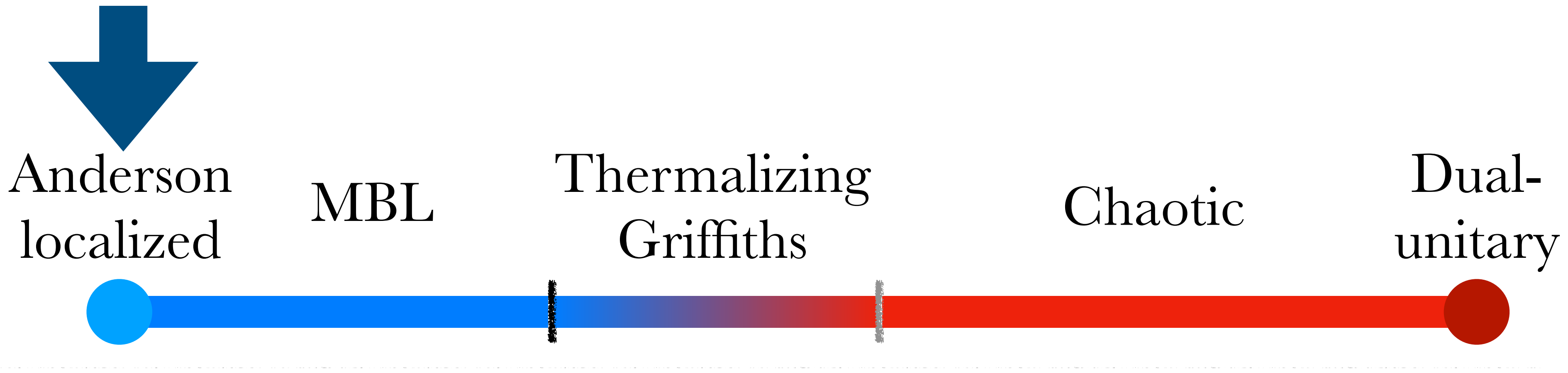
Idea: relate **growth** of entanglement in **unitary** dynamics, $S(t)$, to **spatial scaling** of entanglement in **non-unitary** steady states, $\tilde{S}_\infty(\tilde{\ell})$



Results:

- New steady states with **fractal entanglement** $\sim \tilde{\ell}^\alpha, 0 < \alpha < 1$
- Absence of **area-law** steady states
- Log **corrections** from edge decoherence (leading or subleading)

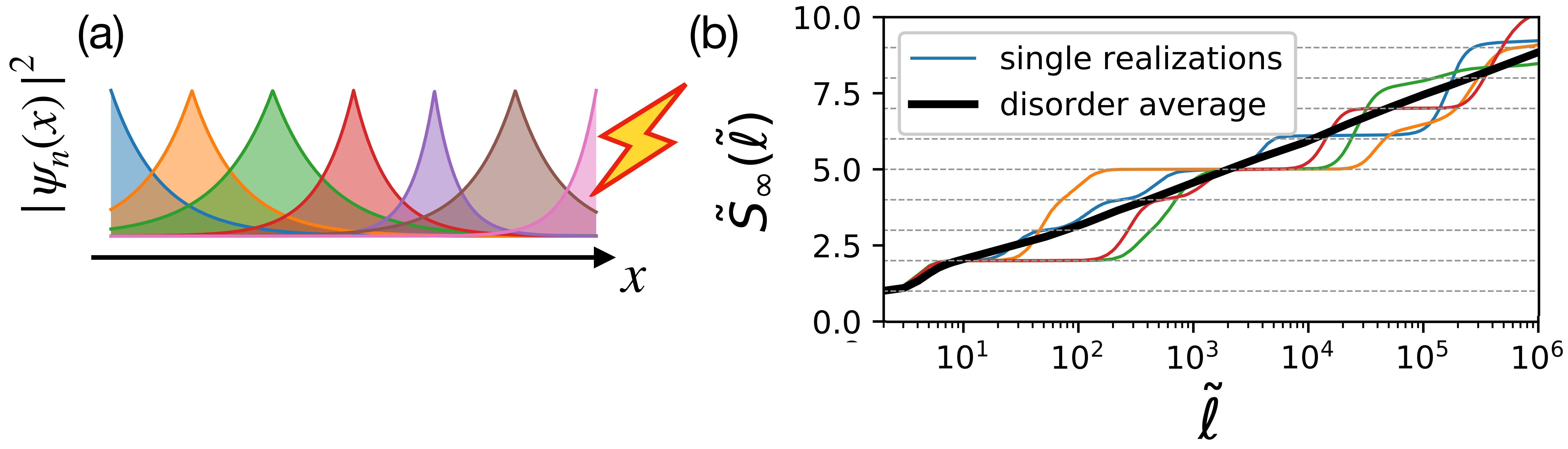
Duals of localized circuits



Disordered free-fermion circuits: **Floquet-Anderson localization**, $\psi_n(x) \sim e^{-|x-n|/\xi}$

$S(t)$ saturates to an **area-law.** Effect of edge decoherence?

- Orbital ψ_n decoheres in time $\tau_n \sim e^{n/\xi}$ [overlap w/ edge], contributes 1 bit to S_{dec}



Result: $\tilde{S}_\infty(\tilde{\ell}) \sim \xi \log(\tilde{\ell})$

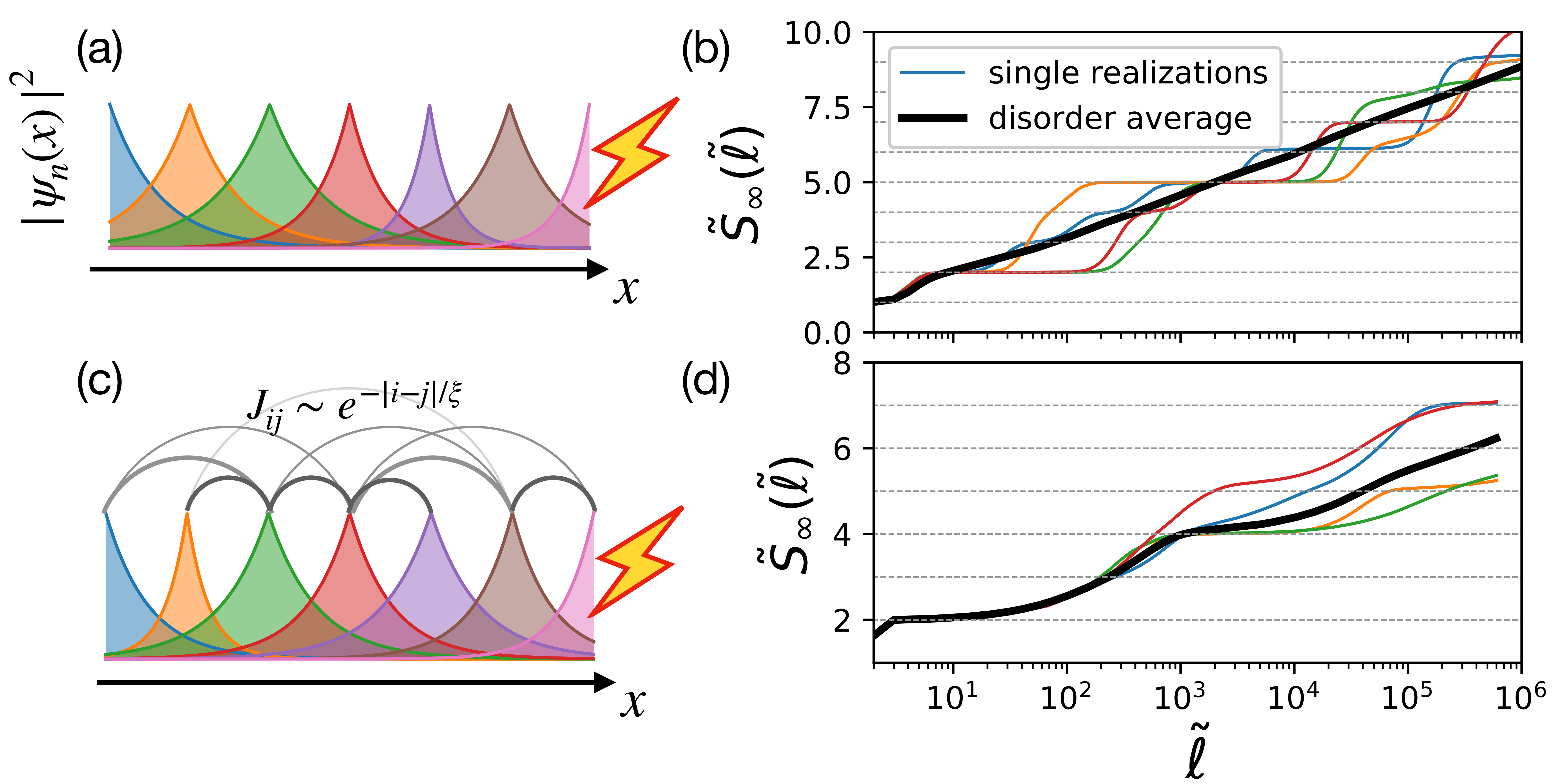
← Decoherence of Anderson orbitals visible in each realization

Duals of localized circuits

Disordered free-fermion circuits: **Floquet-Anderson localization**, $\psi_n(x) \sim e^{-|x-n|/\xi}$

$S(t)$ saturates to an **area-law.** Effect of edge decoherence?

- Orbital ψ_n decoheres in time $\tau_n \sim e^{n/\xi}$ [overlap w/ edge], contributes 1 bit to S_{dec}

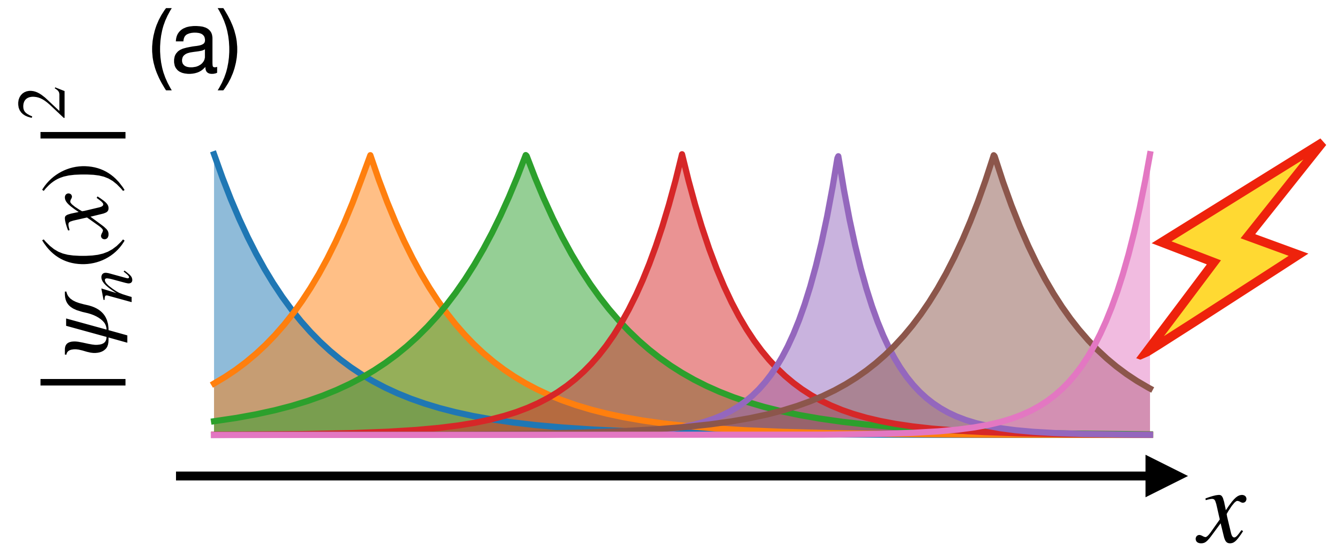


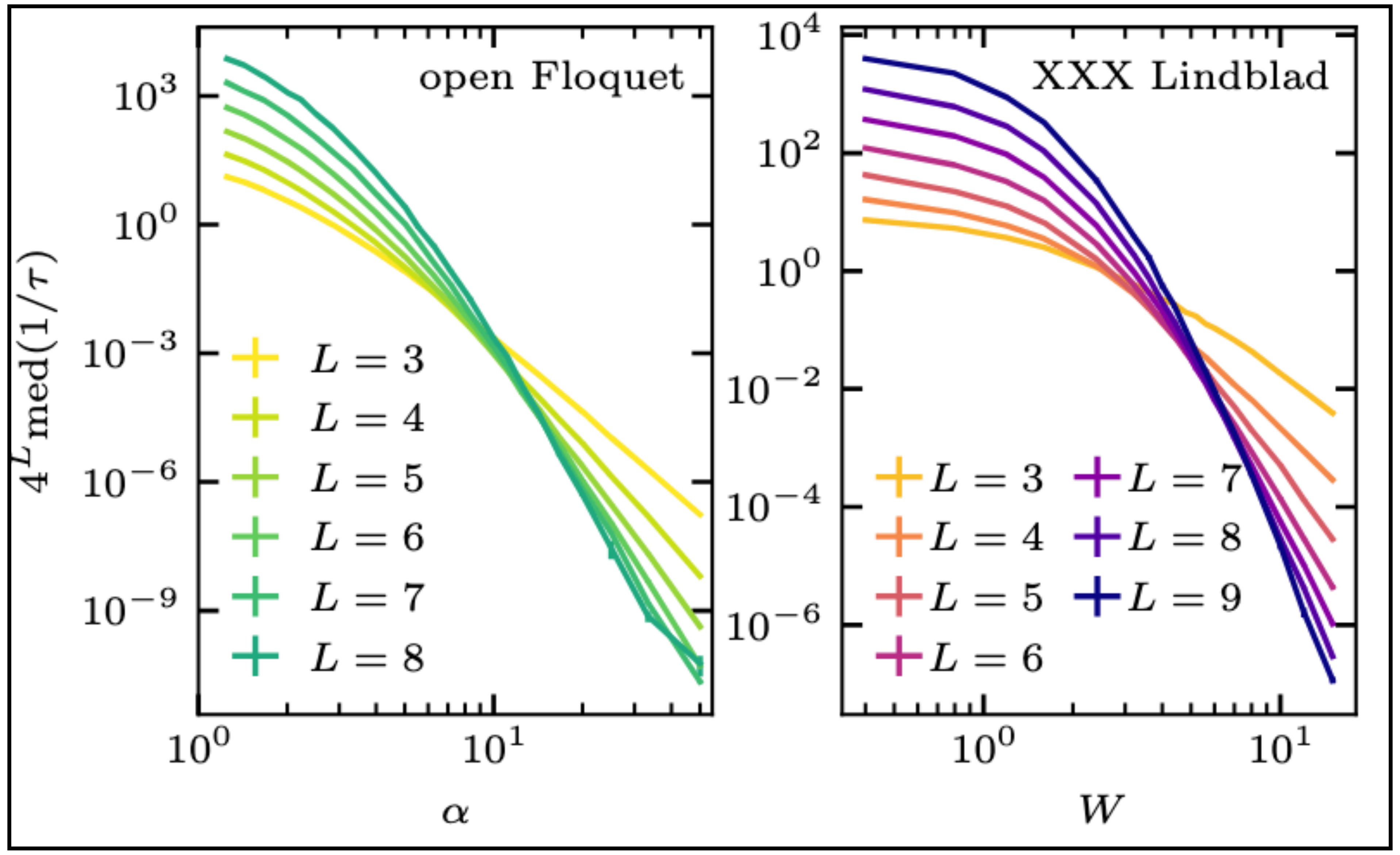
Result: $\tilde{S}_\infty(\tilde{\ell}) \sim \xi \log(\tilde{\ell})$

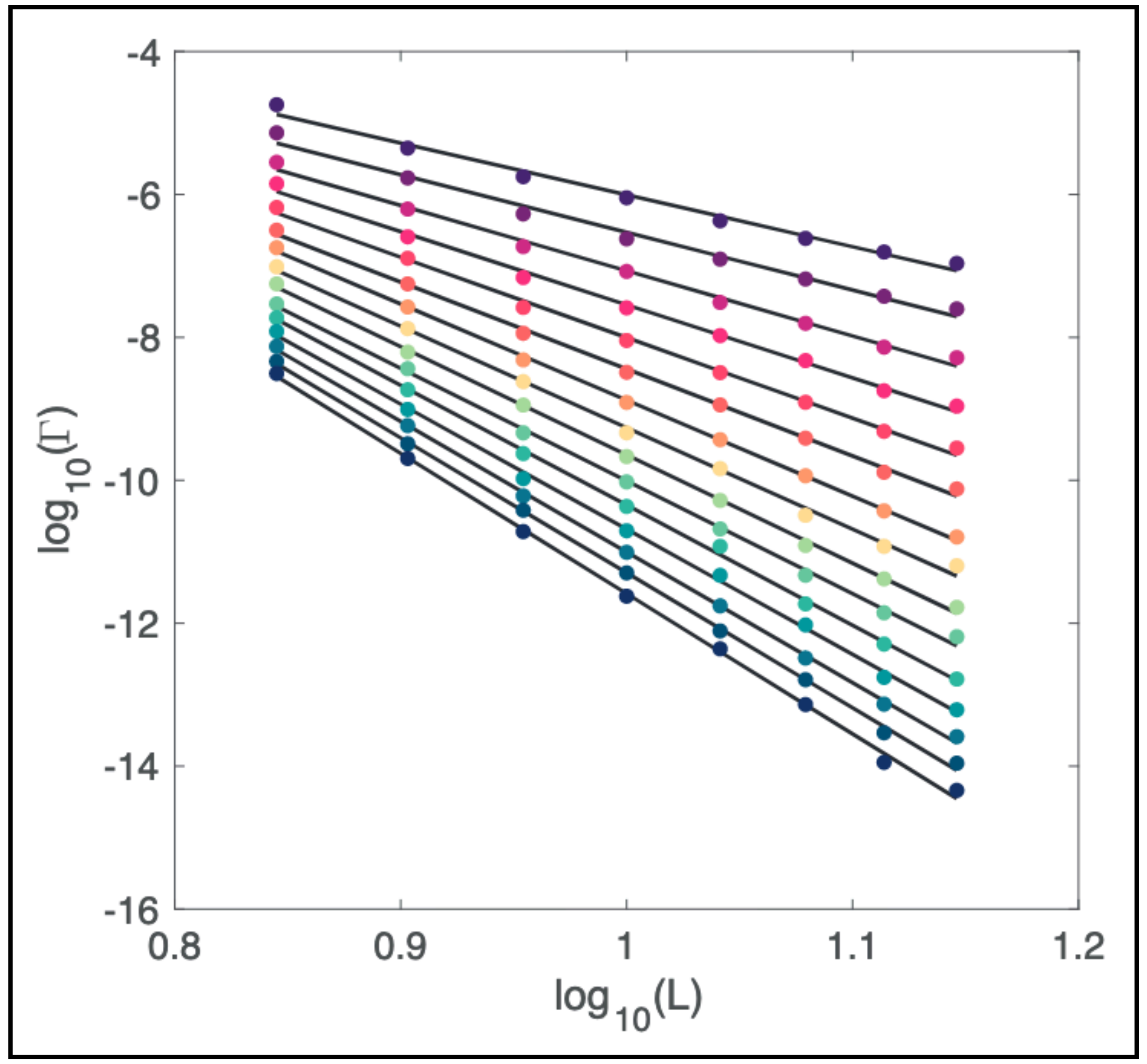
← Decoherence of Anderson orbitals visible in each realization

MBL: Same picture, Anderson orbitals ↦ 'Local integrals of motion' [LIOMS / lbits]

Duals of localized circuits

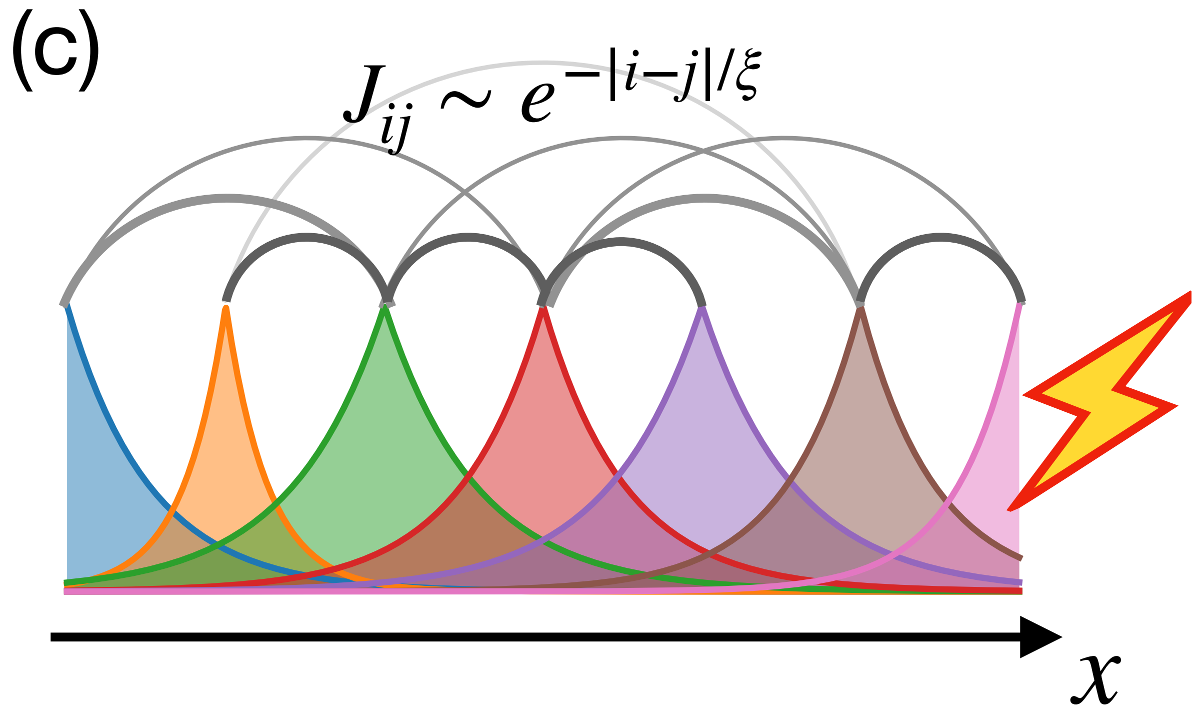






[Morningstar, … Huse, arXiv:2107.05642]

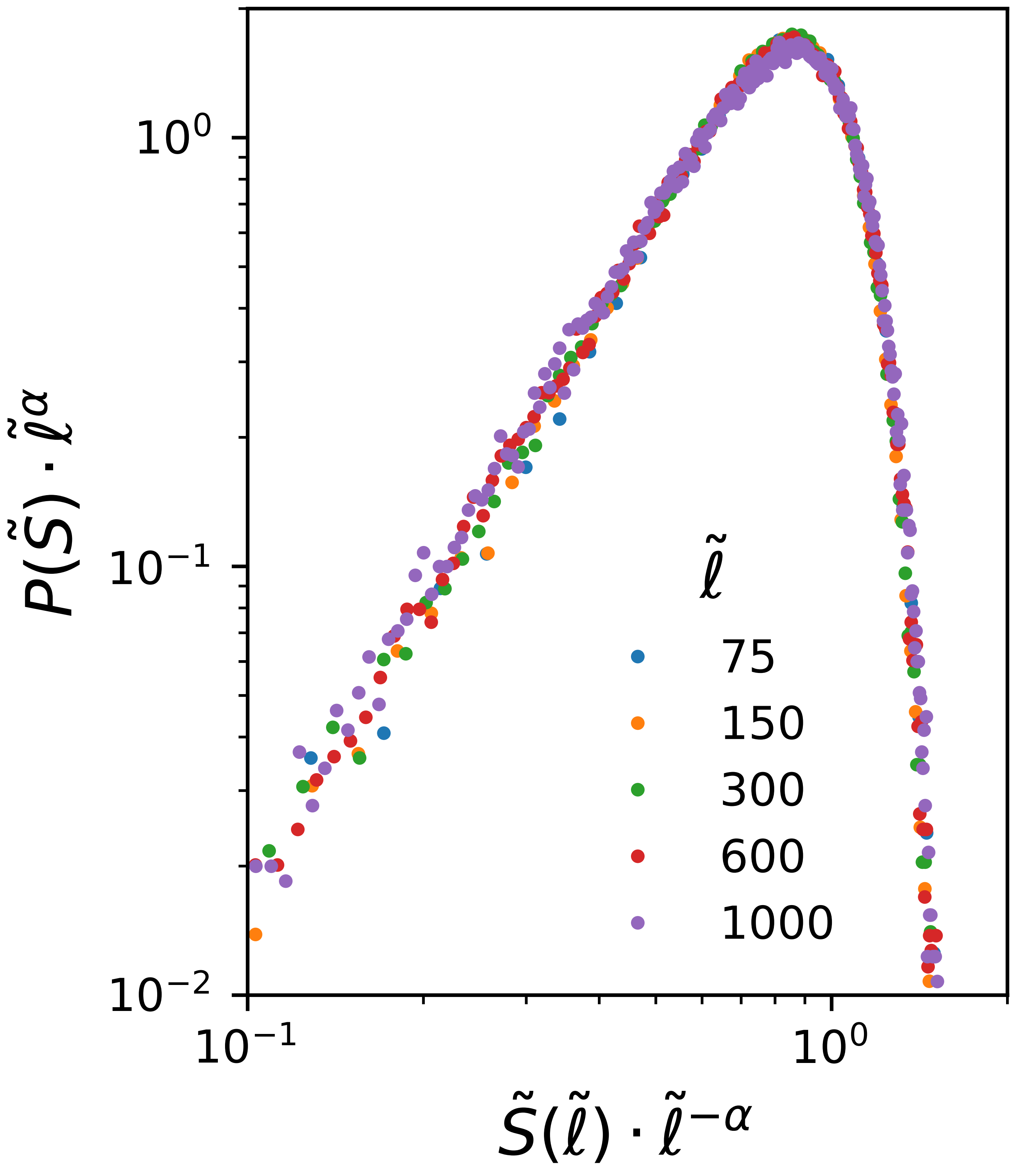
[Sels, arXiv:2108.10796]

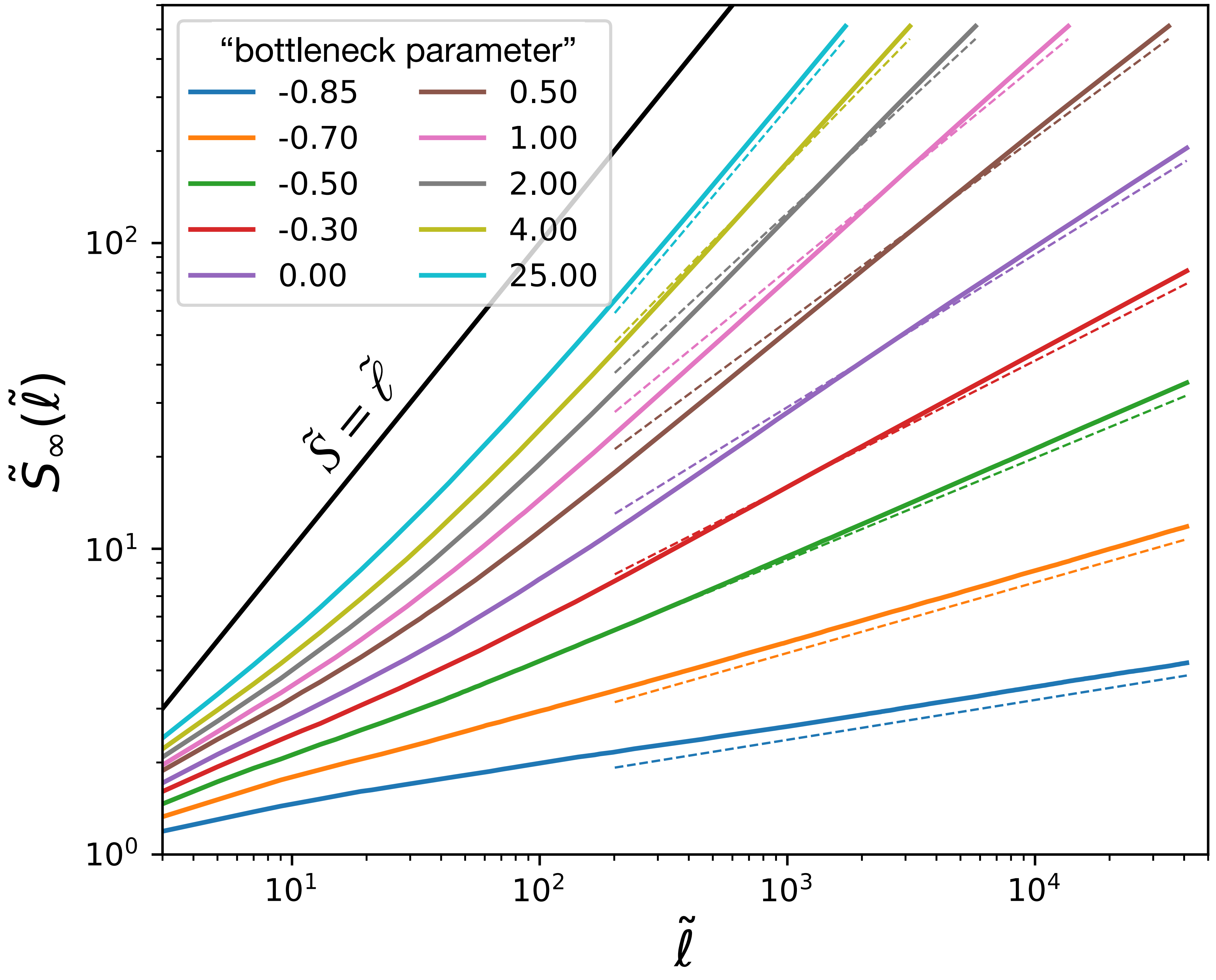


Edge decoherence as a probe of **MBL (in)stability**

Duals of “Griffiths” circuits

- Circuit model of “rare region effects” [Nahum, Ruhman, Huse, PRB **98**, 035118 (2018)]
- Entire **distribution** of $\tilde{S}_\infty$ (not just mean!) collapses onto power-law ansatz
- Statistically self-similar over all length scales: “**fractal**” **entanglement**





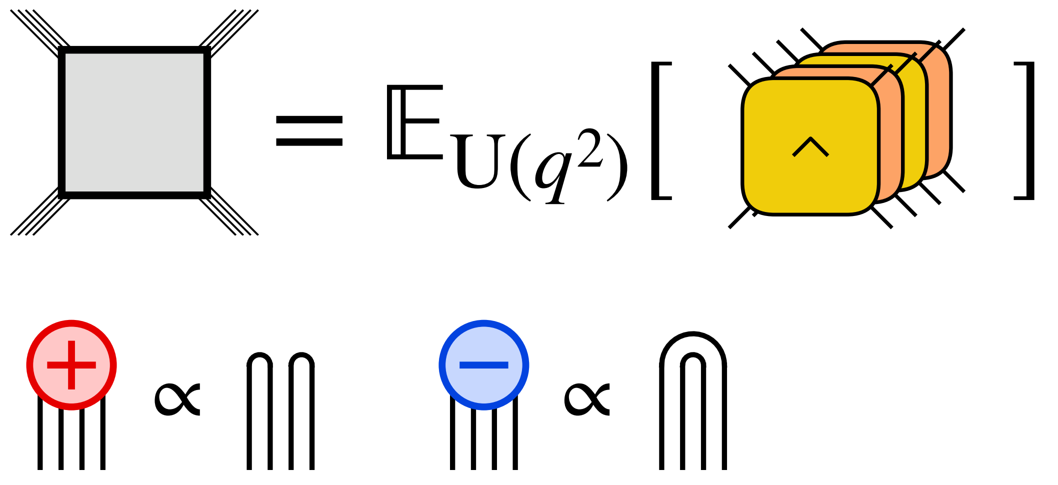
Beyond the **area/log/volume**-“laws” of (generic) many-body unitary dynamics

Examples in unitary dynamics (free or fine-tuned):

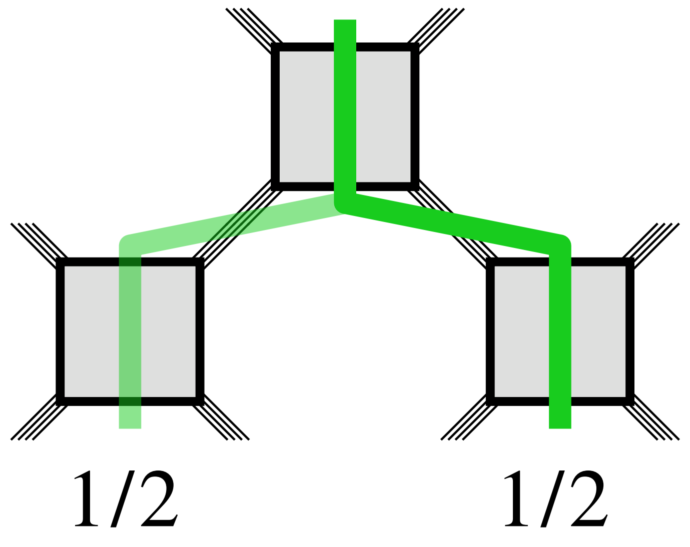
- Marginally Anderson-localized eigenstates [Nandkishore Potter 14]
- Motzkin chains [Movassagh Shor 16]
- Free fermions w/ long-range hopping & fractal Fermi sea [Gori…Trombettoni 14]

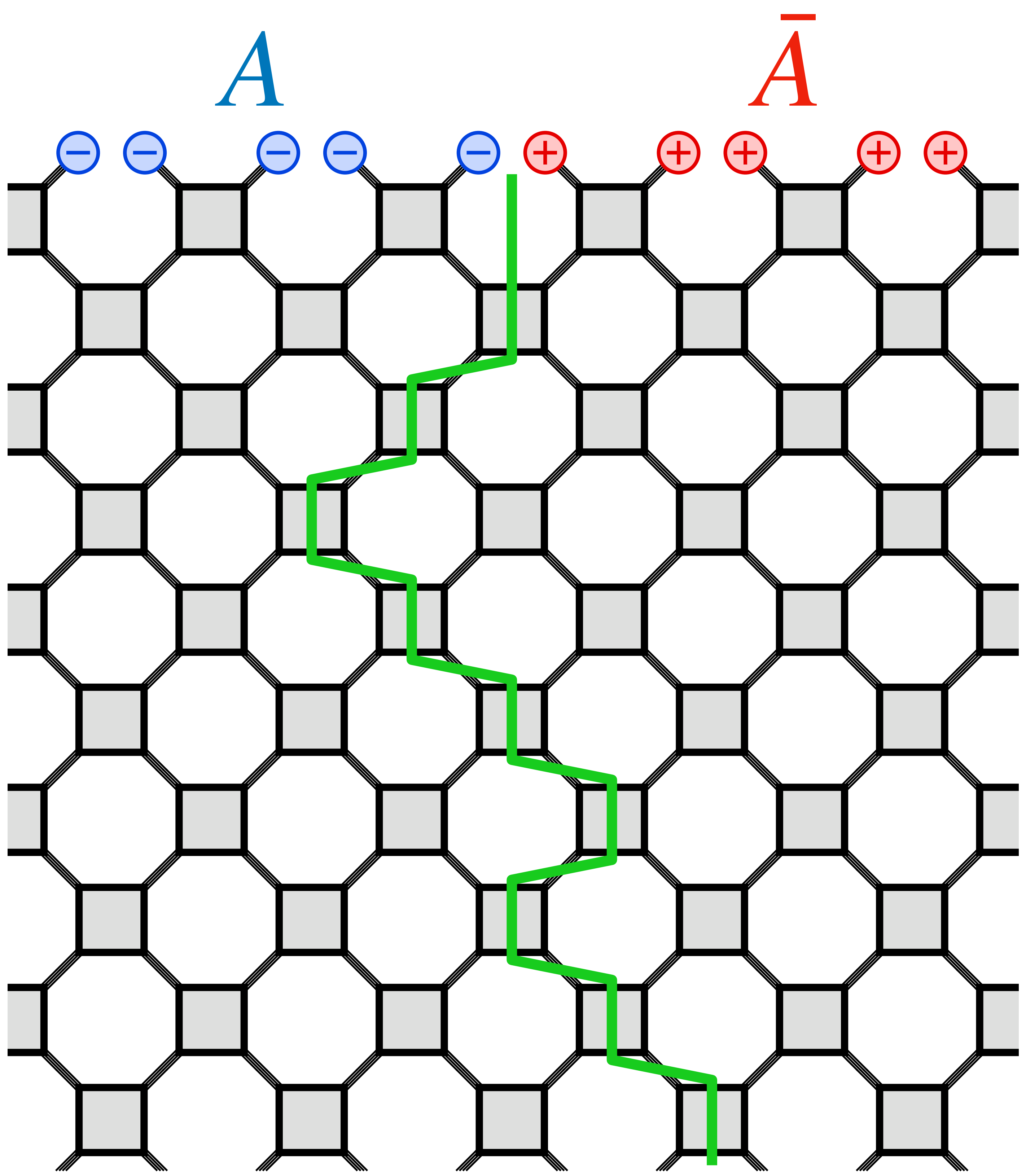
Duals of chaotic circuits

- Helpful toy model: **Haar-random circuits**
  - All gates I.I.D. with Haar measure in unitary group $U(q)$ [q: on-site qudit dimension]
  - Analytical techniques to evaluate **purity** $\mathbb{E}[\mathrm{Tr}\rho^2] \equiv q^{-S_2^{(a)}}$ ("annealed average" of entropy)



Random walk step





Entanglement growth in the unitary circuit: mapping to **random walk**,

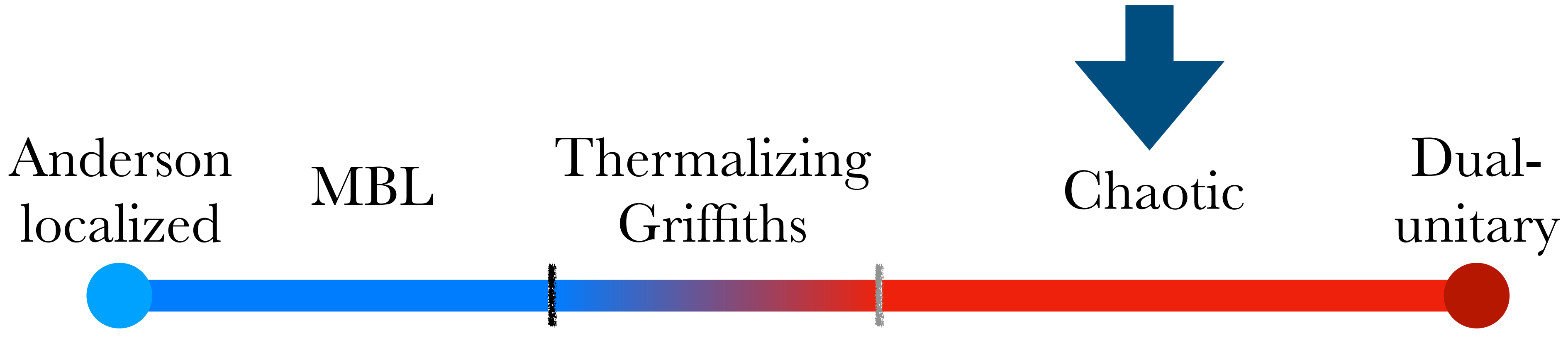
$$\mathbb{E}[\mathrm{Tr}(\rho^2)] = q^{-v_E t}$$

$$S_2^{(a)} = v_E t$$

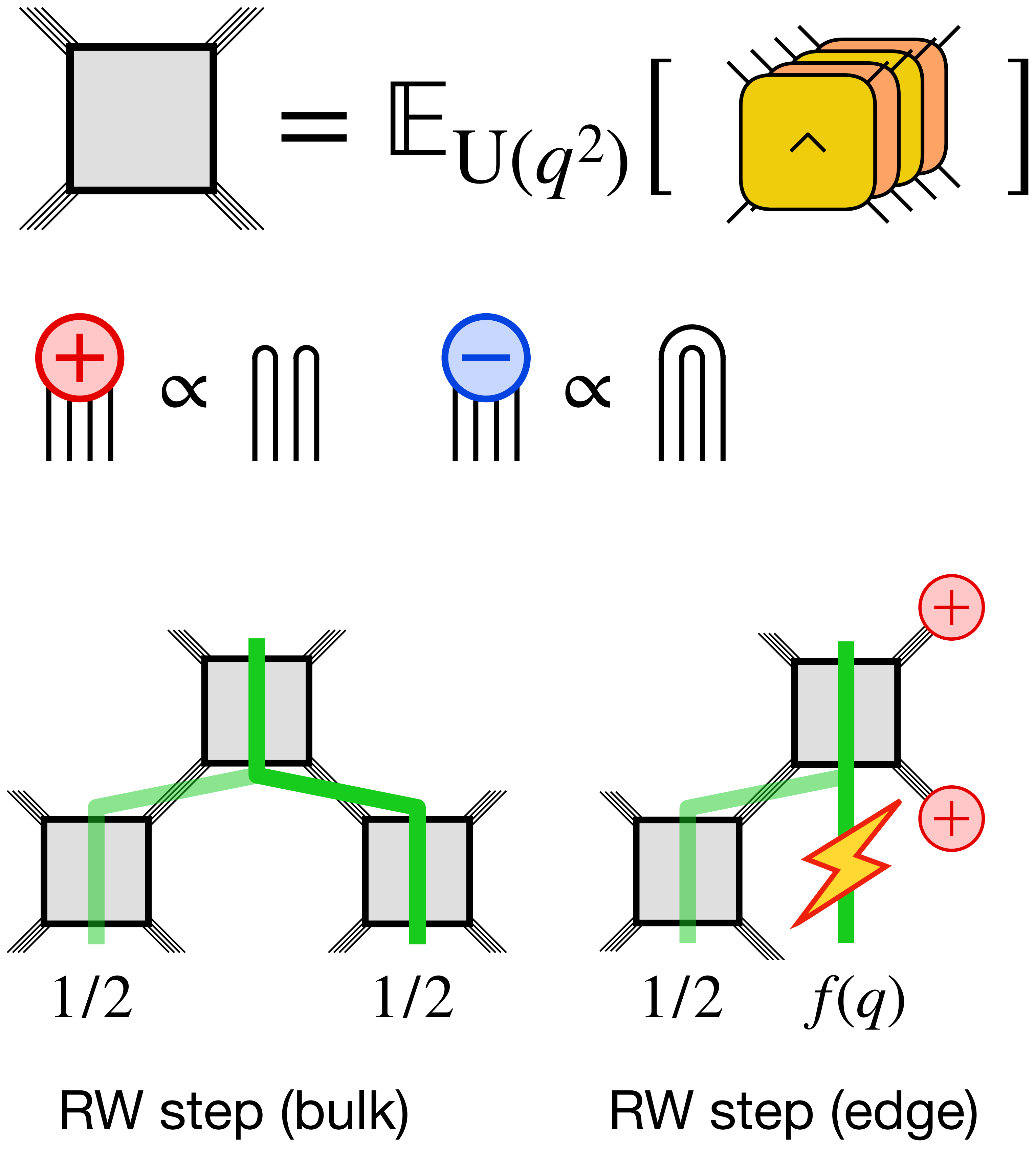
(entanglement velocity $v_E = \log_q[(q + q^{-1})/2]$)

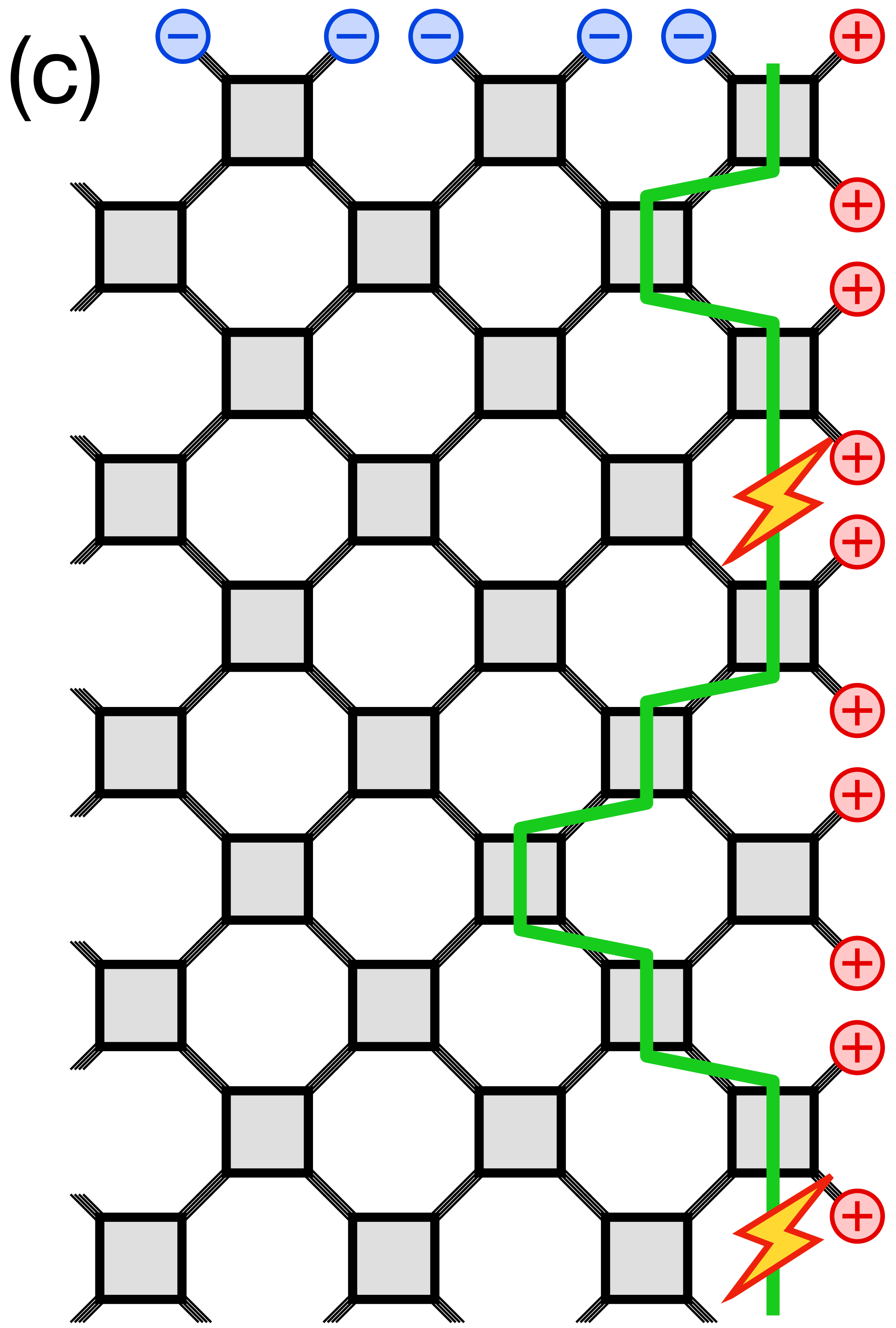
[Nahum, Vijay, Haah 17; von Keyserlingk, Rakovszky, Pollmann, Sondhi 17]

Duals of chaotic circuits



- Spacetime duality → **edge decoherence**
- Modifies random walk problem: **partially absorbing boundary**





RW survival probability: universal correction to purity,

$$\mathbb{E}[\mathrm{Tr}(\rho^2)] \sim q^{-v_E t}/\sqrt{t}$$

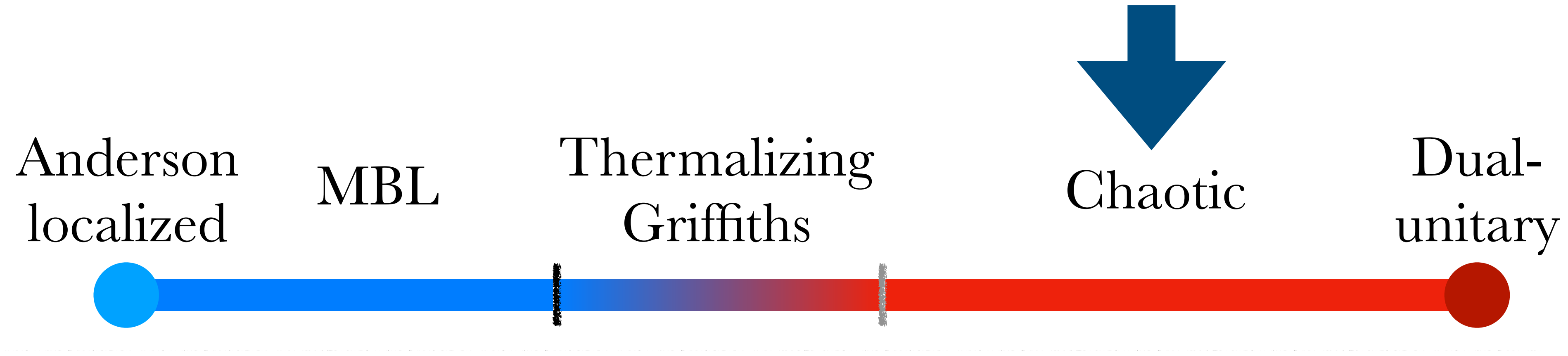
Entropy:

$$\tilde{S}_2^{(a)} = v_E \tilde{\ell} + (1/2)\log \tilde{\ell}$$

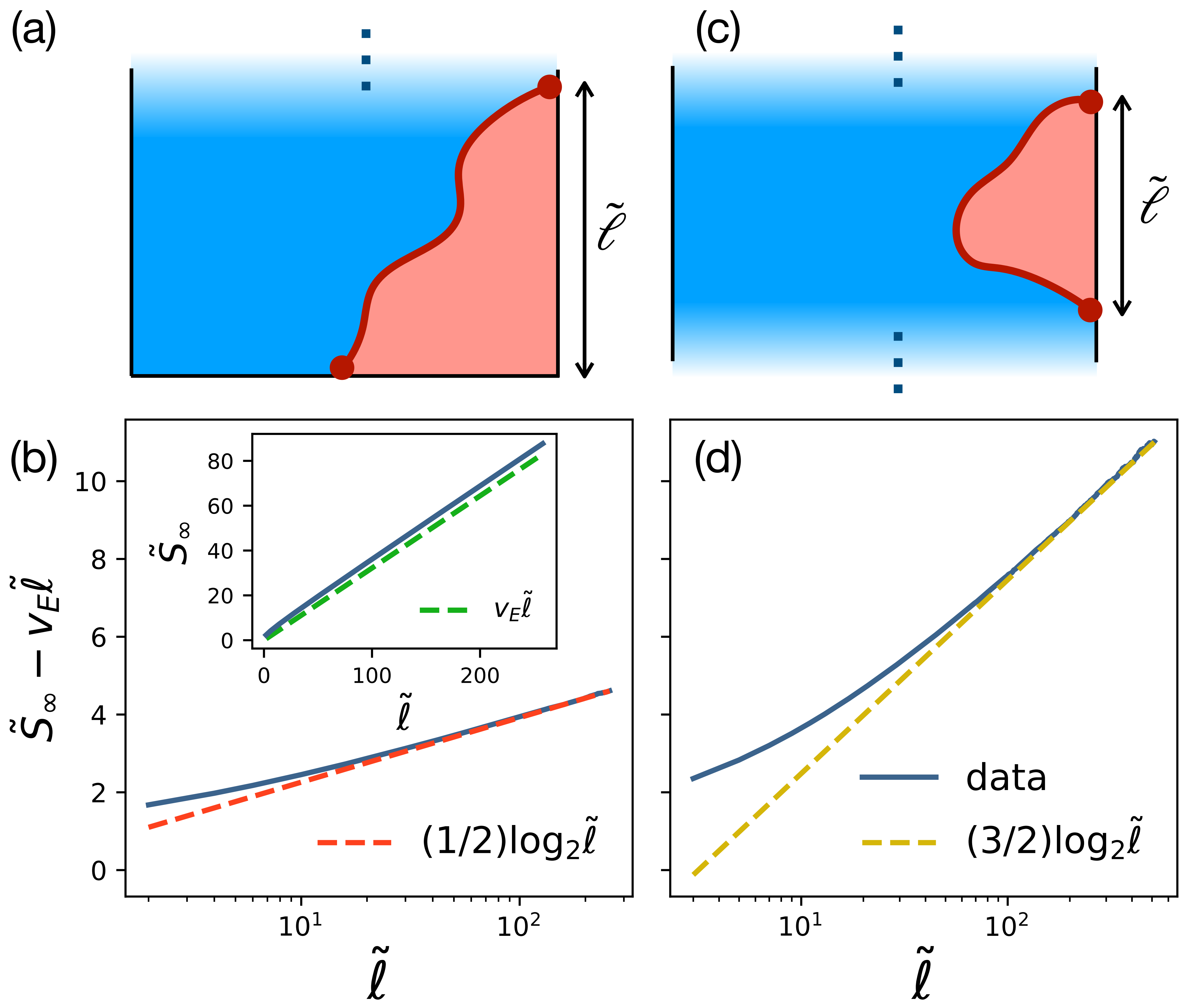
volume-law term (velocity ↔ density)

non-thermal correction

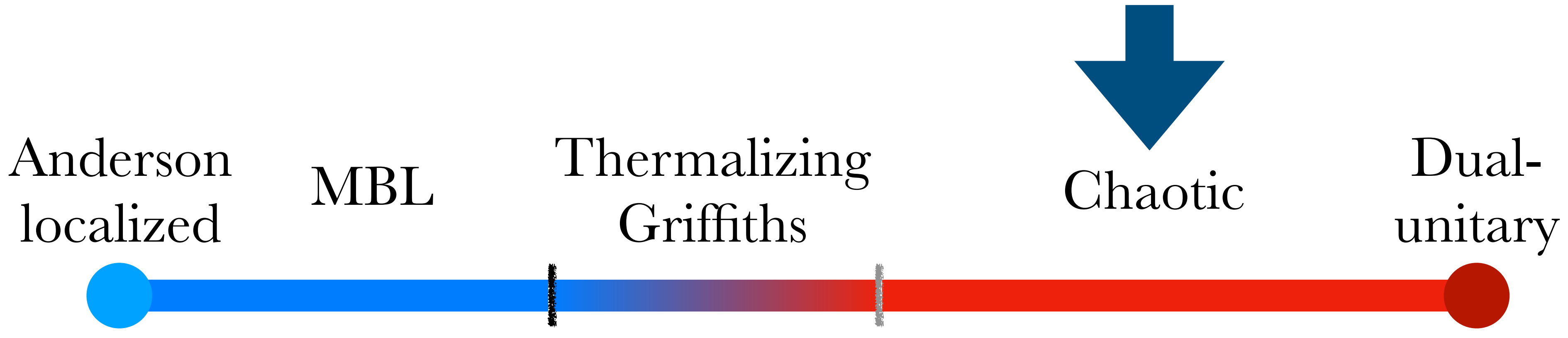
Duals of chaotic circuits



- Subsystem near the edge of semi-infinite system:
  $\tilde{S}(\tilde{\ell}) = v_E\tilde{\ell} + (1/2)\log\tilde{\ell}$
- In the bulk of infinite system:
  $\tilde{S}(\tilde{\ell}) = v_E\tilde{\ell} + (3/2)\log\tilde{\ell}$
- **Same subleading correction** as in generic monitored circuits [Fan, Vijay, Vishwanath, You PRB 2021; Li, Fisher PRB 2021]

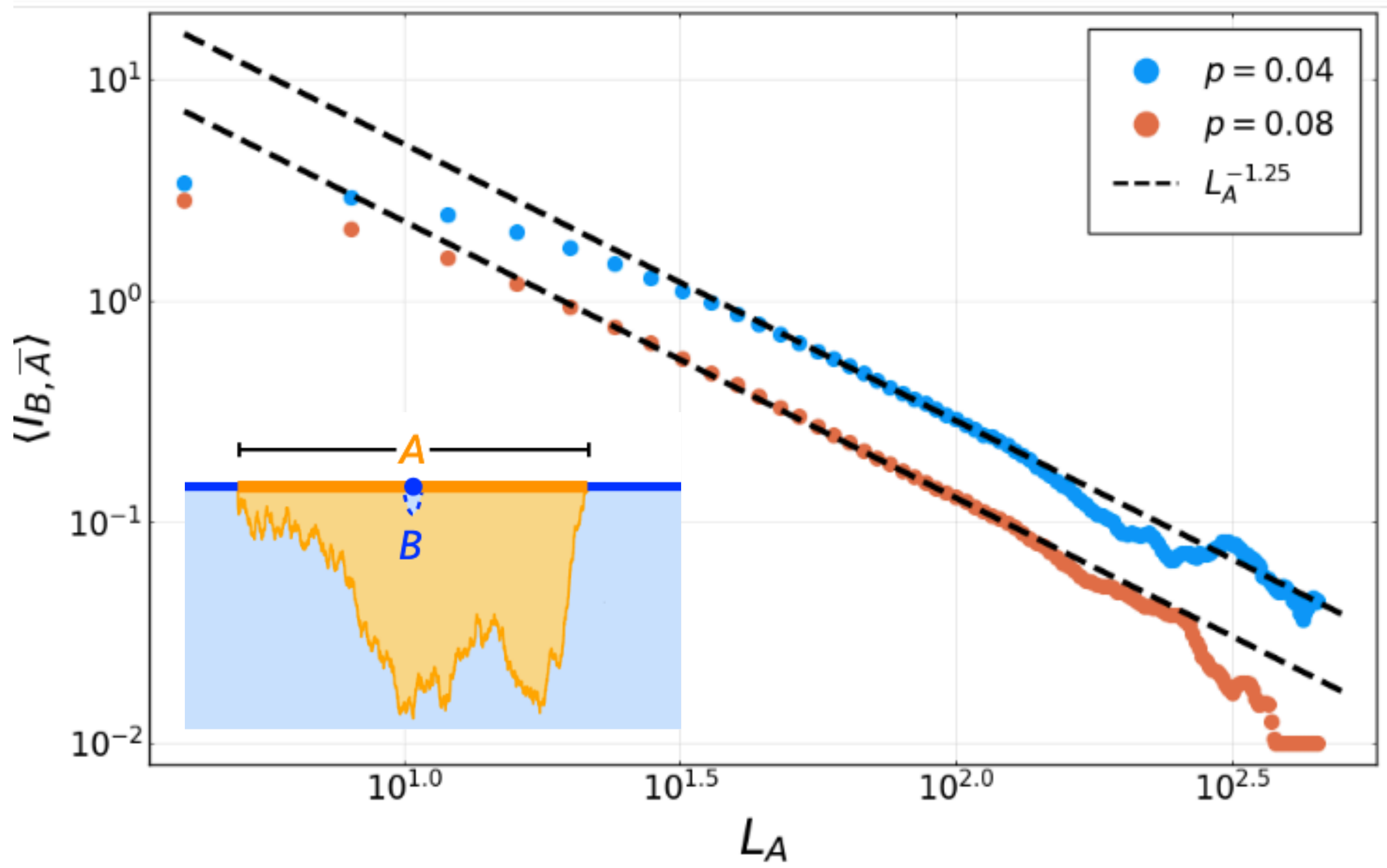


Duals of chaotic circuits



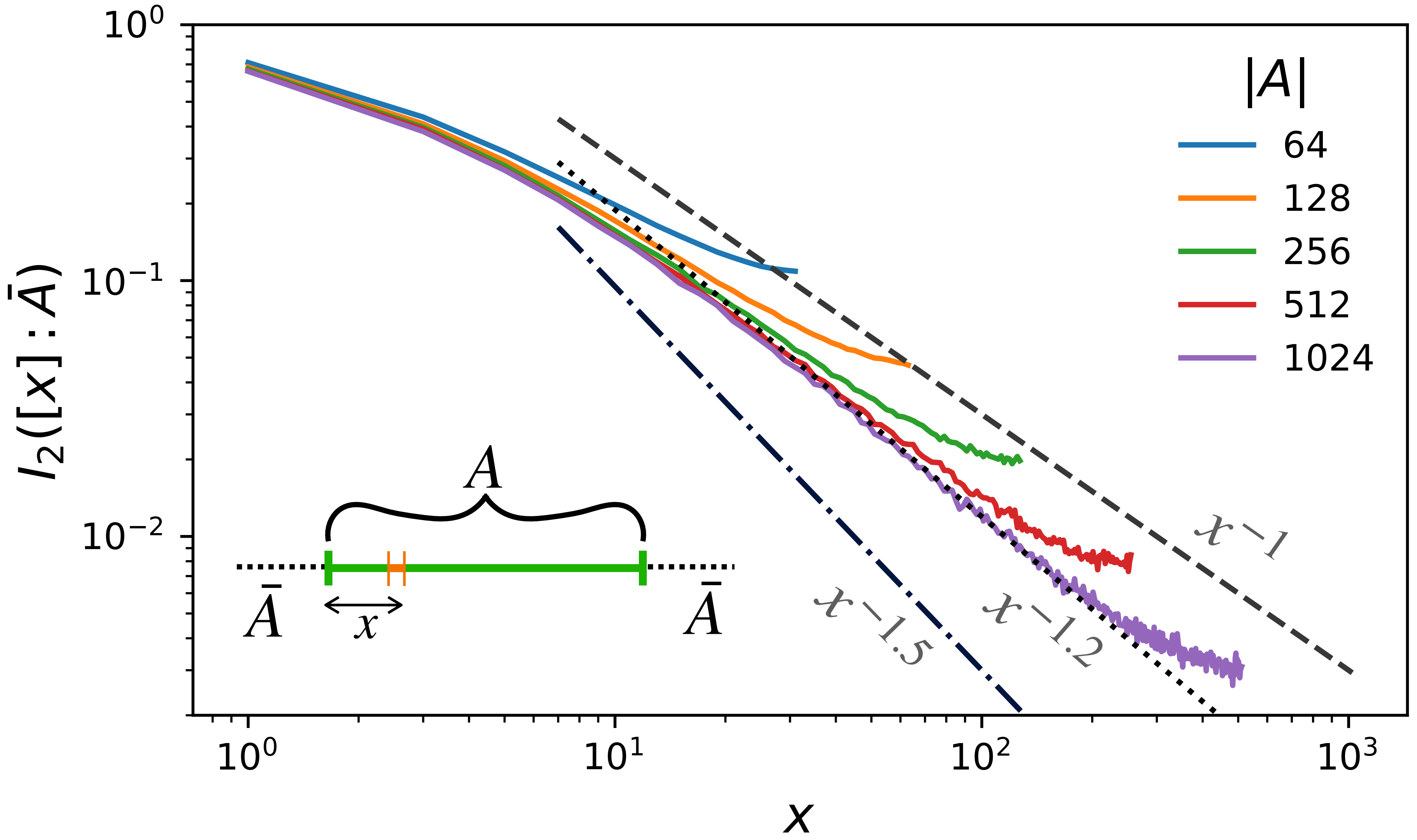
- Subsystem near the edge of semi-infinite system:
  $\tilde{S}(\tilde{\ell}) = v_E \tilde{\ell} + (1/2) \log \tilde{\ell}$
- In the bulk of infinite system:
  $\tilde{S}(\tilde{\ell}) = v_E \tilde{\ell} + (3/2) \log \tilde{\ell}$

[Li, Vijay, Fisher, arXiv:2105.13352]



- **Same subleading correction** as in generic monitored circuits [Fan, Vijay, Vishwanath, You PRB 2021; Li, Fisher PRB 2021]
- **Same scaling** of "qubit-environment" mutual information ($\sim x^{-1.25}$)

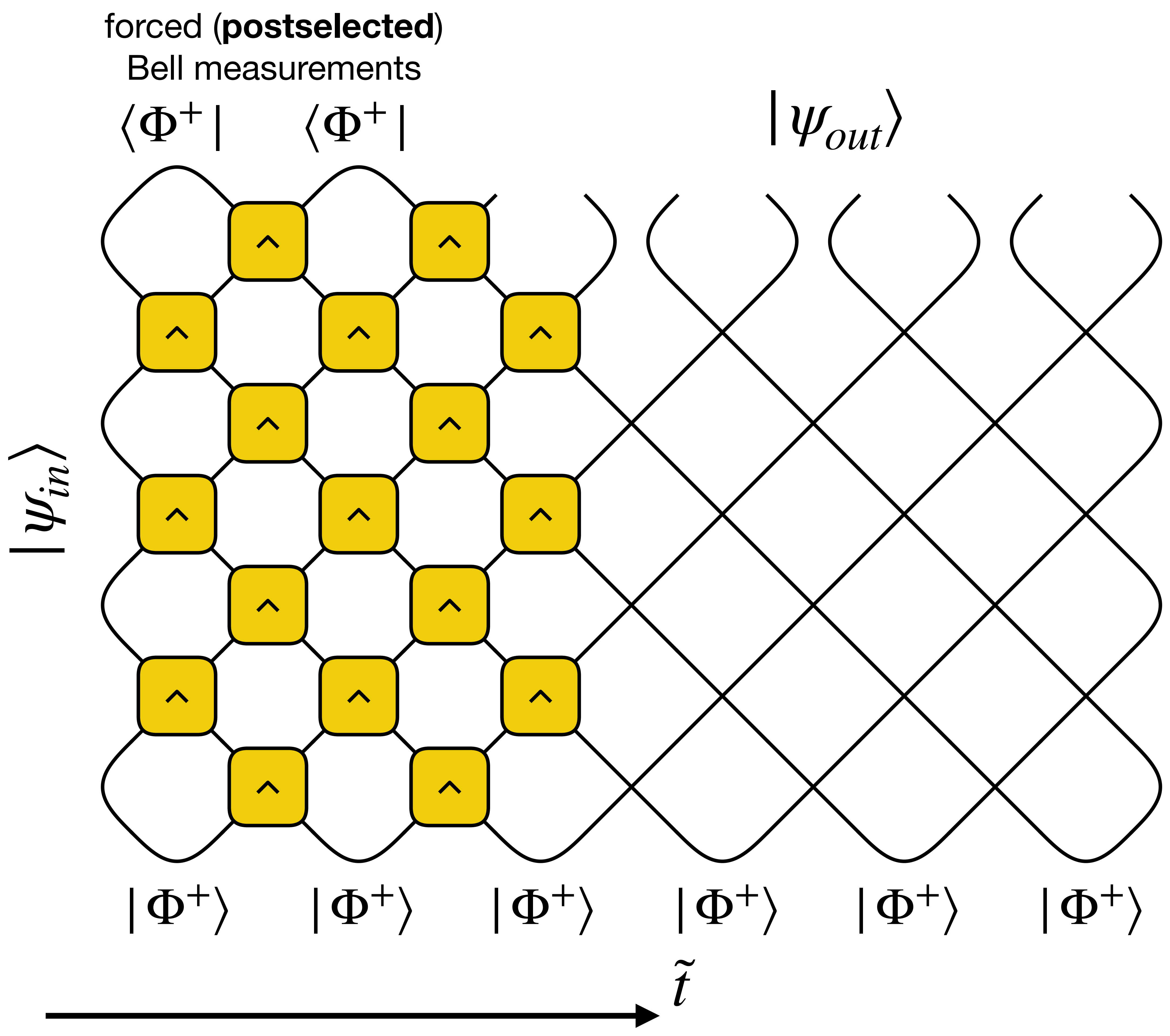
[MI, Rakovszky, Khemani, arXiv:2103.06873]



Conjecture:

Same non thermal volume-law phase

Spacetime duality in the lab



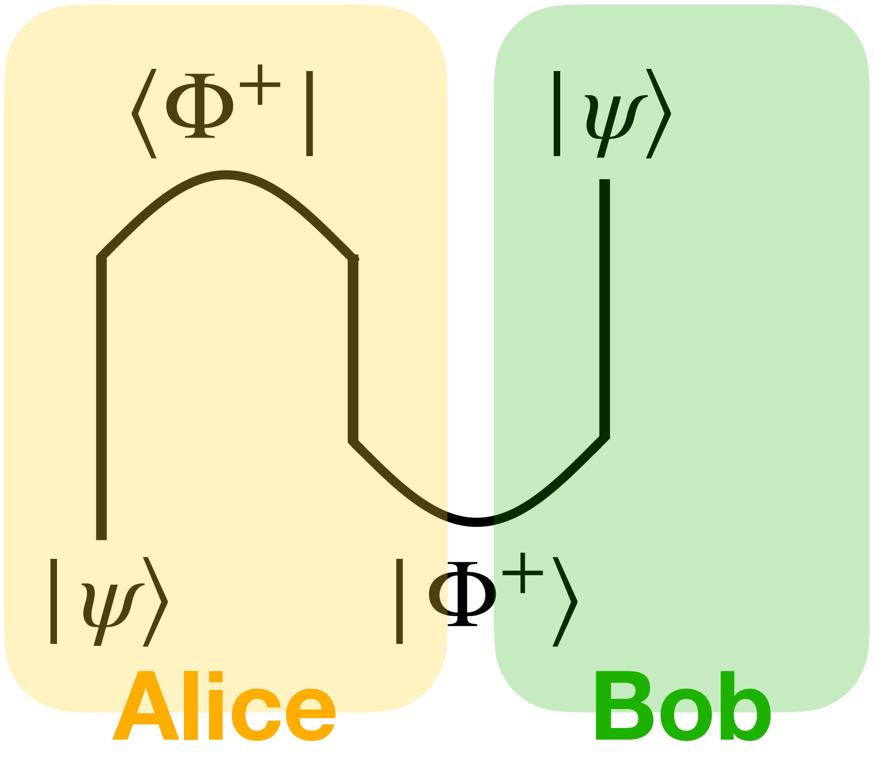
Implements $\tilde{U}$ with:

- 2-qubit unitary gates
- ancilla qubits
- $O(L)$ postselected measurements

[Cf. generic unitary-measurement circuits: $O(LT)$ measurements]

Many-body quantum teleportation

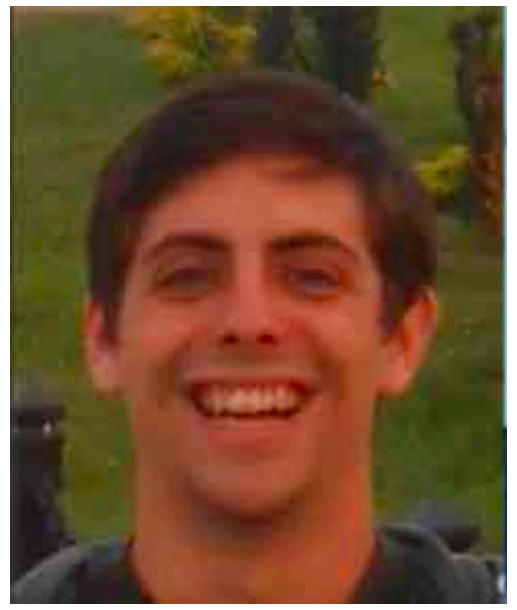
cf. single-qubit protocol:



Conclusion

- **Monitored circuits**
  - New arena for universality out of equilibrium
- **Measurement-only circuits**
  - Beyond the “unitary vs measurement” paradigm
  - Interplay of entanglement phases & ordered phases
- **Spacetime-dual circuits**
  - Analytically tractable, postselection-free
  - Universal phases (non-thermal volume-law)
  - New phases (fractally entangled)

Thank you!

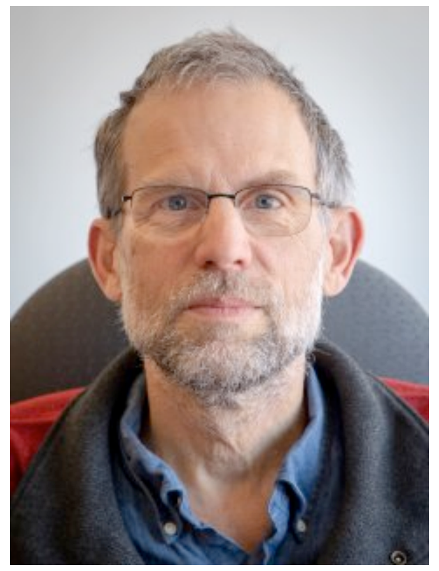


Tibor Rakovszky
(Stanford)

Vedika Khemani
(Stanford)

Sarang
Gopalakrishnan
(Penn State)

Michael Gullans
(UMD, NIST)



David Huse
(Princeton)

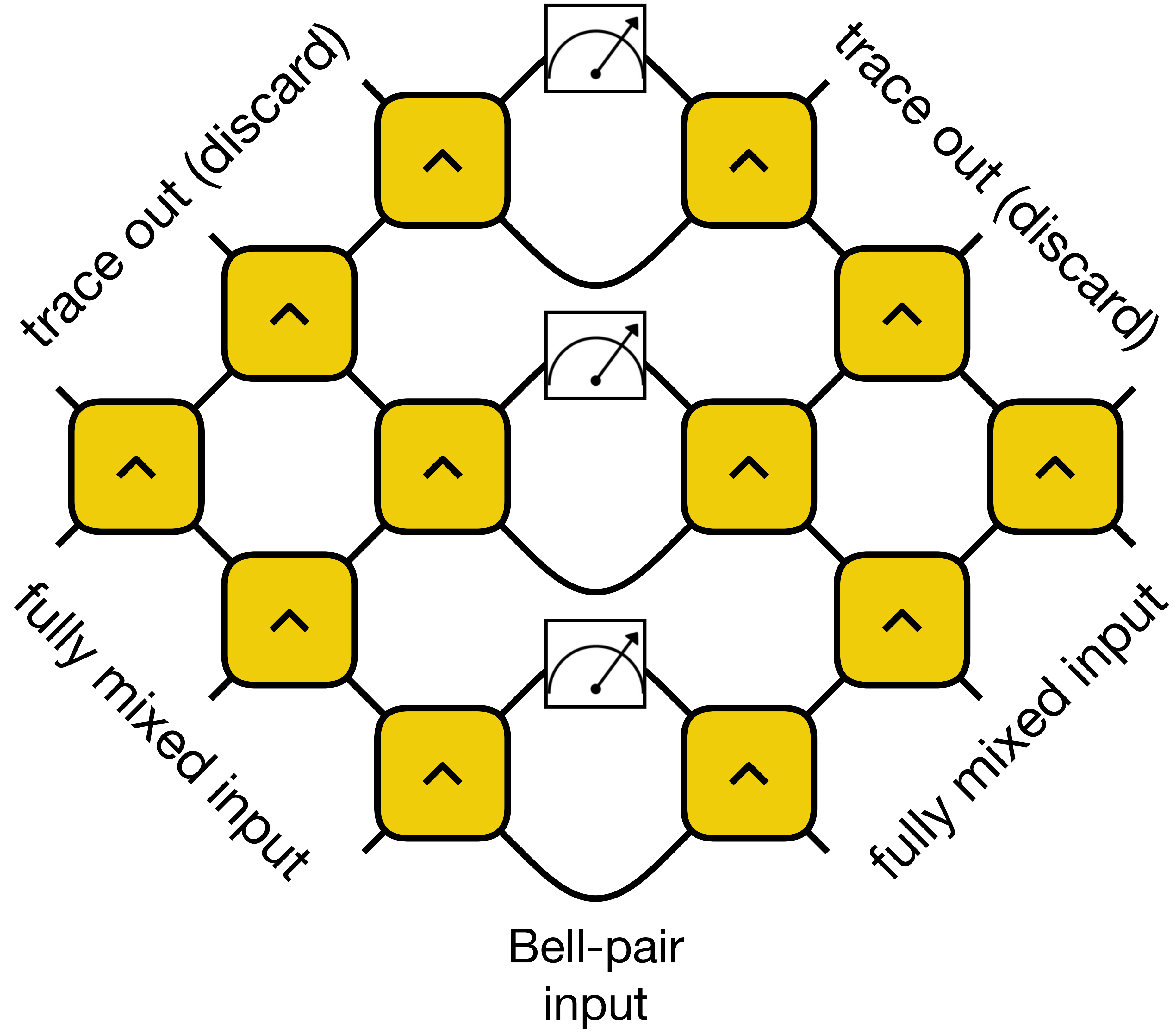
MI, M. Gullans, S. Gopalakrishnan, D. Huse, V. Khemani, PRX **11**, 011030 (2021)
MI, V. Khemani, PRL **126**, 060501 (2021)
MI, T. Rakovszky, V. Khemani, arXiv:2103.06873

Funding:
DARPA DRINQS program, Gordon and Betty Moore Foundation's EPiQS Initiative Grant GBMF8686

Postselection-free purification dynamics

Operational protocol

- **Initialize** $\mathbb{1} \otimes |B^+\rangle\langle B^+| \otimes \mathbb{1}$
- **Evolve**:
  - *Bell measurement* on middle bond
  - *Unitary gates* everywhere else
- If Bell measurement yields "**wrong**" outcome ($\neq |B^+\rangle$), **stop** and record a **failure**; else continue
- When T consecutive Bell measurements yield "**right**" outcome ($= |B^+\rangle$), record a **success**



If N_{tot} trials, of which $N_+(T)$ successful:

$$N_+(T)/N_{\text{tot}} = \text{Tr}(\rho^2) = e^{-S_2}$$

If $N_+(T) \sim N_{\text{tot}} e^{-T/\tau}$, **entropy density** measurable as decay time constant: $s = (2\tau)^{-1}$