Strong Modes and Almost Strong Modes of Floquet Spin Chains

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Daniel J. Yates and Aditi Mitra, *Strong and almost strong modes of Floquet spin chains in Krylov subspaces*, arXiv:2105.13246.

Daniel J. Yates, Alexander G. Abanov, and Aditi Mitra, *Long-lived π edge modes of interacting and disorder-free Floquet spin chains*, arXiv:2105.13766.

Daniel J. Yates, Alexander G. Abanov, and Aditi Mitra, *Dynamics of almost strong edge modes in spin chains away from integrability*, Phys. Rev. B **102**, 195419 (2020).

Daniel J. Yates, Alexander G. Abanov, and Aditi Mitra, *Lifetime of almost strong edge-mode operators in one dimensional, interacting, symmetry protected topological phases*, Phys. Rev. Lett. **124**, 206803 (2020).

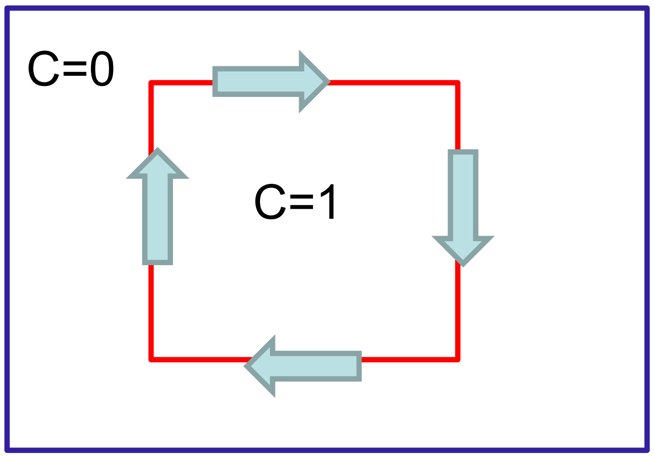
Daniel J. Yates, Fabian H. L. Essler, and Aditi Mitra *Almost strong $0, \pi$ edge modes in clean, interacting 1D Floquet systems*, Phys. Rev. B **99**, 205419 (2019).

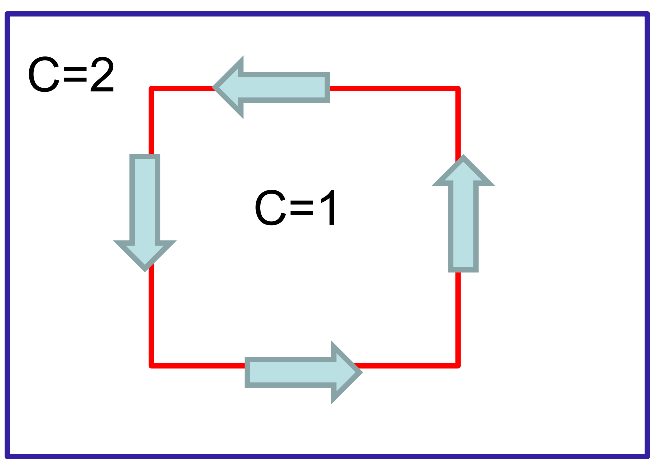
Strong modes are edge modes of certain spin chains.

They can be related to localized zero modes of topological phases.

Wiki definition of symmetry protected topological (SPT) phase:

(a) *distinct SPT states with a given symmetry cannot be smoothly deformed into each other without a phase transition, if the deformation preserves the symmetry*.
(b) *however, they all can be smoothly deformed into the same trivial product state without a phase transition, if the symmetry is broken during the deformation*.





Symmetry Protected Topological Phases for static (free) Fermions:

The notion of SPTs is powerful because the symmetries are difficult to remove (unlike crystal symmetries).

$TR : T^2 = 0, \pm,$
$PH : C^2 = 0, \pm,$ — Anti-Unitary

$Chiral : S = 0,1$ — Unitary

$$HS = -SH$$

IQHE (T=0,C=0,S=0)

complex case:

Cartan\d	0	1	2	3	4	5	6	7	8	9	10	11	⋯
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	⋯
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	⋯

SSH

real case:

Cartan\d	0	1	2	3	4	5	6	7	8	9	10	11	⋯
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	⋯
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	⋯
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	⋯
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	⋯
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	⋯
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	⋯
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	⋯
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	⋯

$T^2 = +,$
$C^2 = +,$
$S = 1$

This Talk:
Kitaev-type Chains: Floquet driving and interactions

Topological insulators and superconductors: ten-fold way and dimensional hierarchy
Shinsei Ryu, Andreas Schnyder, Akira Furusaki, Andreas Ludwig
New J. Phys. 12, 065010 (2010)

Periodic table for topological insulators and superconductors,
Alexei Kitaev (2009)

Kitaev chain:

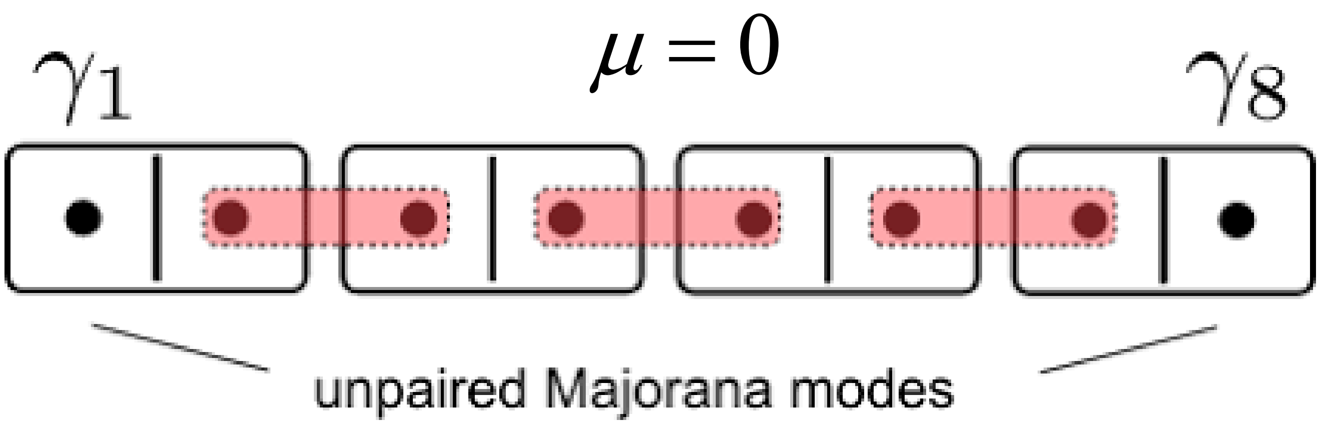
A. Y. Kitaev, Physics-Uspekhi **44**, 131 (2001).

$$\gamma_{j,1} = c_j + c_j^\dagger, \quad \gamma_{j,2} = i\left(c_j^\dagger + c_j\right),$$

$$H = -\mu \sum_j c_j^\dagger c_j + \sum_{j=0}^{N-1} \Big[-t\left(c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1}\right) - |\Delta| \left(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger\right) \Big].$$

Equivalent to the transverse field Ising model

$$H = J \sum_i \sigma_i^x \sigma_{i+1}^x - \mu \sum_i \sigma_i^z$$



Strong zero modes Ψ_0

P. Fendley, Journal of Physics A: Mathematical and Theoretical **49**, 30LT01 (2016).

$\mathcal{D}$ A discrete Z2 symmetry of the Hamiltonian (for example, Fermion parity)

Ψ_0 Commutes with the Hamiltonian in the thermodynamic limit

$$[H, D] = 0,$$
$$[H, \Psi_0] \approx 0,$$
$$H|n\rangle = \varepsilon_n |n\rangle$$

$\{\Psi_0, \mathcal{D}\} = 0$ Anti-commutes with the discrete symmetry

$\Psi_0^2 = O(1)$ Normalizable

Above implies an eigenspectrum phase which is doubly-degenerate

$$\{|n\rangle, \Psi_0|n\rangle\}$$

Edge mode exists at all energies

$|n\rangle$: EVEN; $\Psi_0|n\rangle$: ODD

Example: Kitaev chain (transverse field Ising) with open boundary conditions

P. Fendley, Journal of Physics A: Mathematical and Theoretical **49**, 30LT01 (2016).

$$H = J\sum_i \sigma_i^x \sigma_{i+1}^x - \mu \sum_i \sigma_i^z$$

$$\mathcal{D} = \sigma_1^z \sigma_2^z \ldots \sigma_L^z$$

Or Fermion Parity

$$\Psi_0 = \sum_{s=0}^{\infty} (-\mu/J)^s \, \sigma_{s+1}^x \prod_{j=1}^{s} \sigma_j^z$$

⇒

$$\Psi_0(\mu = 0) = \sigma_1^x$$

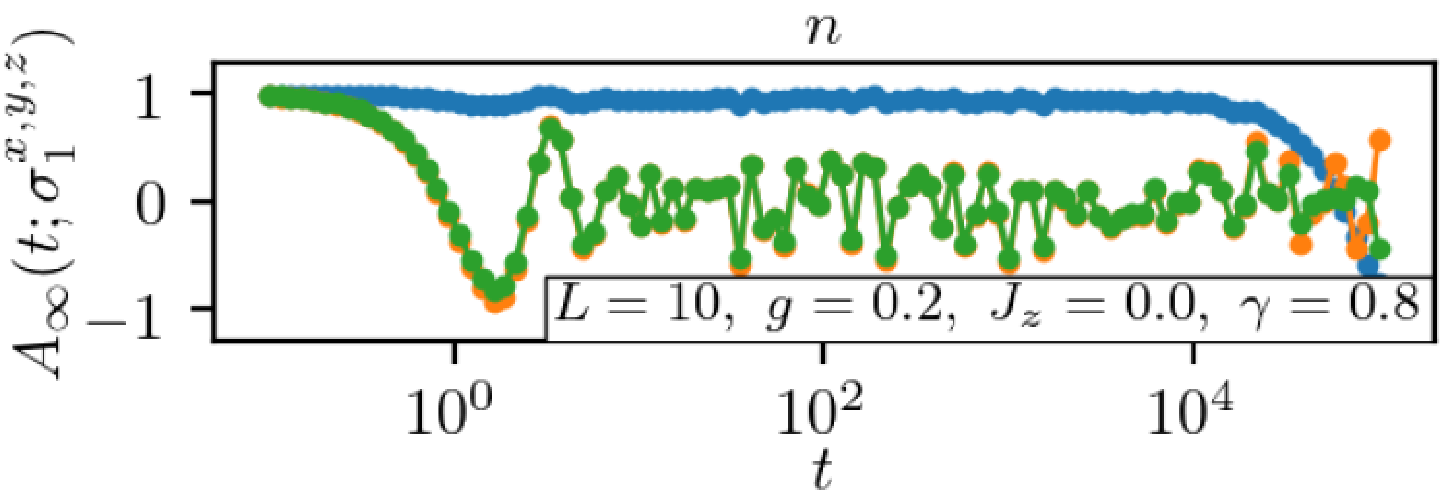
Normalizable in the topologically non-trivial phase

$$\Psi_0^2 = \frac{1}{1-(\mu/J)^2}$$

Topological phase transition when operator is non-normalizable

Identification of strong modes: Infinite temperature correlation function of the boundary spin.

$$A_\infty(t) = \frac{1}{2^L}\mathrm{Tr}\left[\sigma_1^x(t)\sigma_1^x(0)\right].$$



$$H = \sum_i \left[J \left(\frac{1+\gamma}{2} \right) \sigma_i^x \sigma_{i+1}^x + J \left(\frac{1-\gamma}{2} \right) \sigma_i^y \sigma_{i+1}^y + J_z \sigma_i^z \sigma_{i+1}^z + g \sigma_i^z \right].$$

Strong modes have been constructed for

a). Free fermions: Jz=0, g/J<1. Kitaev 2006 and
Yates, Abanov, Mitra, arxiv:2002.00098 for general anisotropy.

b). XYZ chain, g=0. Fendley, arxiv:1512.03441

c). Parafermionic chains. Fendley: arxiv:1209.0472

Adding interactions:

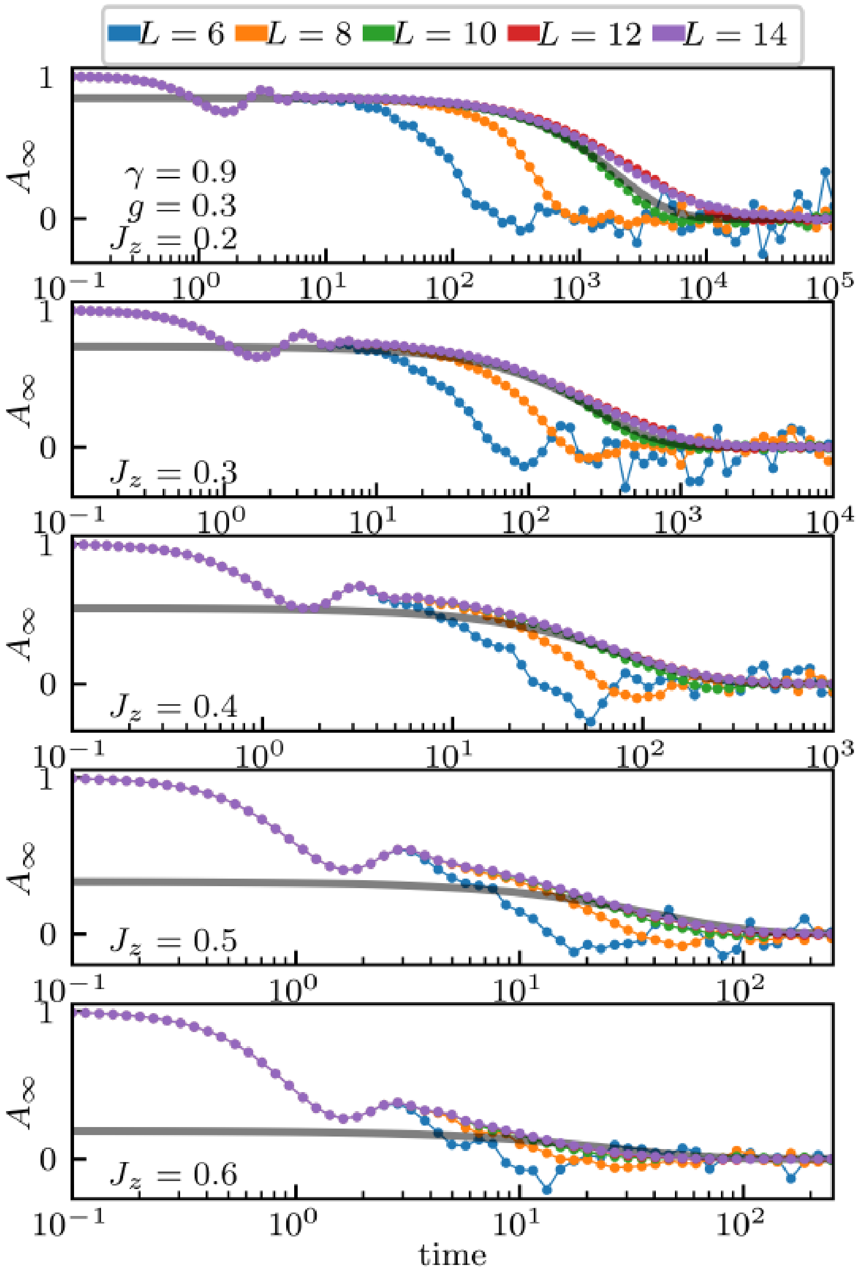
Almost strong zero modes:

Here the edge mode lives for times that grows exponentially with system size, eventually saturating at large enough system sizes. Yet these times are very long (>> 1/temperature)

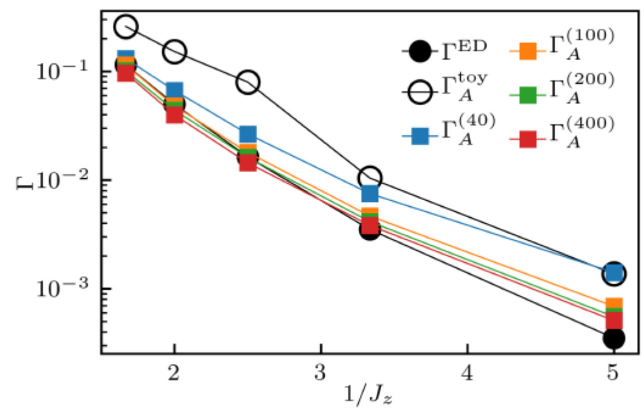
J. Kemp, N. Y. Yao, C. R. Laumann, and P. Fendley, Journal of Statistical Mechanics: Theory and Experiment **2017**, 063105 (2017).
D. V. Else, P. Fendley, J. Kemp, and C. Nayak, Phys. Rev. X **7**, 041062 (2017).

$$H = J\sum_i \sigma_i^x \sigma_{i+1}^x - \mu \sum_i \sigma_i^z + J_z \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$A_\infty(t) \sim e^{-\Gamma t} \qquad \Gamma \sim e^{-cJ/J_z}, c = O(1)$$



$$H = \sum_i \Bigg[J\left(\frac{1+\gamma}{2}\right)\sigma_i^x\sigma_{i+1}^x + J\left(\frac{1-\gamma}{2}\right)\sigma_i^y\sigma_{i+1}^y + J_z\sigma_i^z\sigma_{i+1}^z + g\sigma_i^z \Bigg].$$



$$\Gamma \sim e^{-cJ/J_z}, c = O(1)$$

Daniel J. Yates, Alexander G. Abanov, and Aditi Mitra, *Dynamics of almost strong edge modes in spin chains away from integrability*, Phys. Rev. B **102**, 195419 (2020).

Daniel J. Yates, Alexander G. Abanov, and Aditi Mitra, *Lifetime of almost strong edge-mode operators in one dimensional, interacting, symmetry protected topological phases*, Phys. Rev. Lett. **124**, 206803 (2020).

Domain Wall Counting Argument:

D. V. Else, P. Fendley, J. Kemp, and C. Nayak, Prethermal strong zero modes and topological qubits, Phys. Rev. X **7**, 041062 (2017).

J. Kemp, N. Y. Yao, and C. R. Laumann, Symmetry-enhanced boundary qubits at infinite temperature, Phys. Rev. Lett. **125**, 200506 (2020).

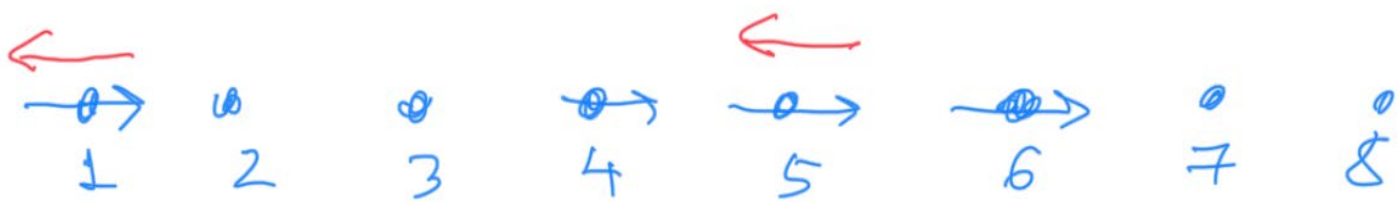
D. V. Else, B. Bauer, and C. Nayak, Prethermal phases of matter protected by time-translation symmetry, Phys. Rev. X **7**, 011026 (2017).

D. J. Yates, A. G. Abanov, and A. Mitra, Lifetime of almost strong edge-mode operators in one-dimensional, interacting, symmetry protected topological phases, Phys. Rev. Lett. **124**, 206803 (2020).

$$H = J_x \sum_i \sigma_i^x \sigma_{i+1}^x + O(g) + O(J_z)$$

$$N = \sum_i \sigma_i^x \sigma_{i+1}^x$$

$$H = J_x \sum_i \sigma_i^x \sigma_{i+1}^x = J_x N$$



Energy cost $= J_x$

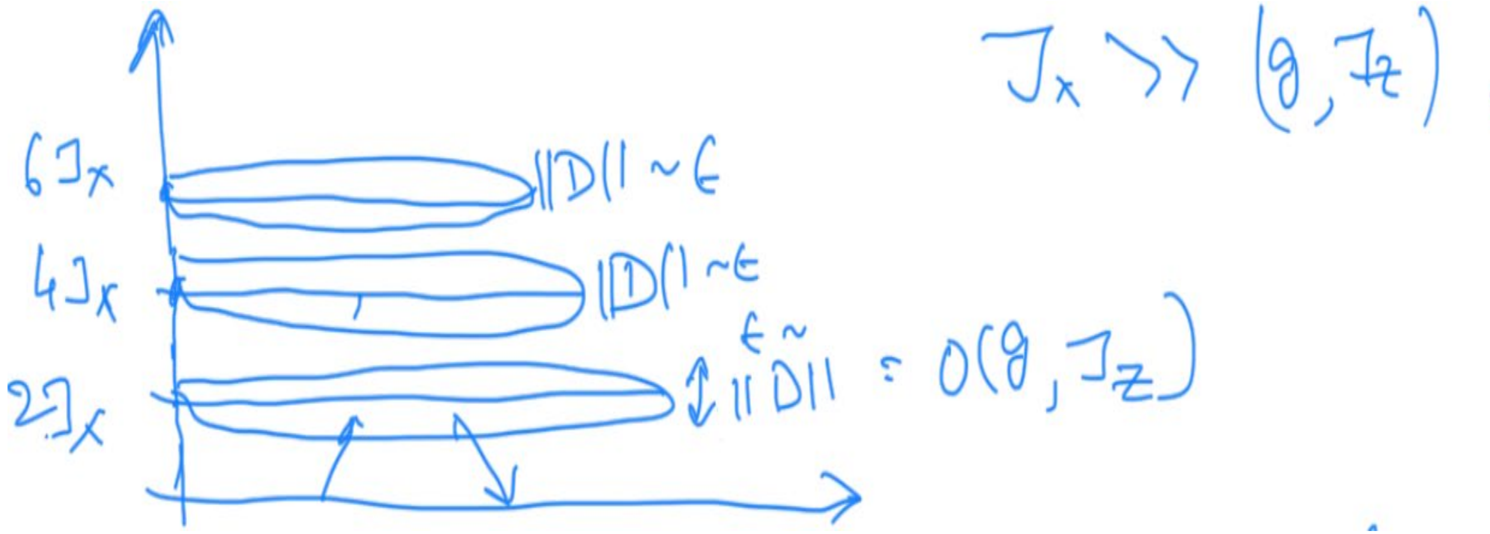
Energy cost $= 2J_x$

$$H = J_x N + D \quad \text{where} \quad [D, N] = 0$$

$$D = J_z \sum_i P_j \sigma_j^z \sigma_{j+1}^z, P_j = \frac{1}{2}\left(1 - \sigma_{j-1}^x \sigma_j^x \sigma_{j+1}^x \sigma_{j+2}^x\right).$$

$$H = J_x N + D + V$$

where $[D, N] = 0$, and $[N, V] \neq 0$. Example,



Creation and annihilation of a pair of domain walls costs energy of the order of the bandwidth $\epsilon \ll \hat{J}_x$. Thus, we need J_x/ϵ number of applications of V to offset the energy Jx needed to flip the edge spin.

Thus, the lifetime goes as $[||V||]^{\hat{J}_x/\epsilon}$. Numerics indicate that,

$$1/\epsilon = O(1/J_z, 1/g)$$

WHAT ABOUT INTERACTING FLOQUET CHAINS?

GENERAL VIEW: CLOSED, DISORDER-FREE FLOQUET SYSTEMS HEAT TO INFINITE TEMPERATURE.

A. Lazarides, A. Das, and R. Moessner, Phys. Rev. E **90**, 012110 (2014).

H. Kim, T. N. Ikeda, and D. A. Huse, Phys. Rev. E **90**, 052105 (2014).

L. D'Alessio and M. Rigol, Phys. Rev. X **4**, 041048 (2014).

P. Ponte, A. Chandran, Z. Papi, and D. A. Abanin, Annals of Physics **353**, 196 (2015).

A. Haldar, R. Moessner, and A. Das, Phys. Rev. B **97**, 245122 (2018).

SO NAÏVE EXPECTATION: NO PROTECTED EDGE MODES.

LONG LIVED EDGE MODES IN INTERACTING FLOQUET SPT PHASES ARE KNOWN TO PRIMARILY OCCUR WITH DISORDER (MBL PREVENTS HEATING).

Y. Bahri, R. Ronen, E. Altman, and A. Vishwanath, Nature Communications **6**, 7341 (2015).

V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

G. J. Sreejith, A. Lazarides, and R. Moessner, Phys. Rev. B **94**, 045127 (2016).

I.-D. Potirniche, A. C. Potter, M. Schleier-Smith, A. Vishwanath, and N. Y. Yao, Phys. Rev. Lett. **119**, 123601 (2017).

A. Kumar, P. T. Dumitrescu, and A. C. Potter, Phys. Rev. B **97**, 224302 (2018).

QUESTION WE POSED: STRONG/ALMOST STRONG MODES IN CLEAN FLOQUET SYSTEMS?

YES!

NO NEED TO BE IN THE HIGH-FREQUENCY DRIVING REGIME

Floquet System: Almost Strong zero/pi modes: Two different eigenspectrum phases

$$\Psi_{0,\pi}$$

$$U(T)$$: Time-evolution operator over one period.

$$\Rightarrow \quad \Psi_{0,\pi}^2 = O(1)$$

$$\Rightarrow \quad \{\Psi_{0,\pi}, \mathcal{D}\} = 0$$

See also:

M. Thakurathi, A. A. Patel, D. Sen, and A. Dutta, Phys. Rev. B **88**, 155133 (2013).

G. J. Sreejith, A. Lazarides, and R. Moessner, Phys. Rev. B **94**, 045127 (2016).

Eigenspectrum Phase-I

$$[\Psi_0, U(T)] \approx 0$$

$$\{|n\rangle, \Psi_0|n\rangle\}$$

Degenerate pairs of states

Eigenspectrum Phase-II

$$\{\Psi_\pi, U(T)\} \approx 0$$

$$\{|n\rangle, \Psi_\pi|n\rangle\}$$

Pairs with quasi-energy separated by π/T

Boundary time-crystal

1D Kitaev chain with nn and nnn couplings
+ periodic driving

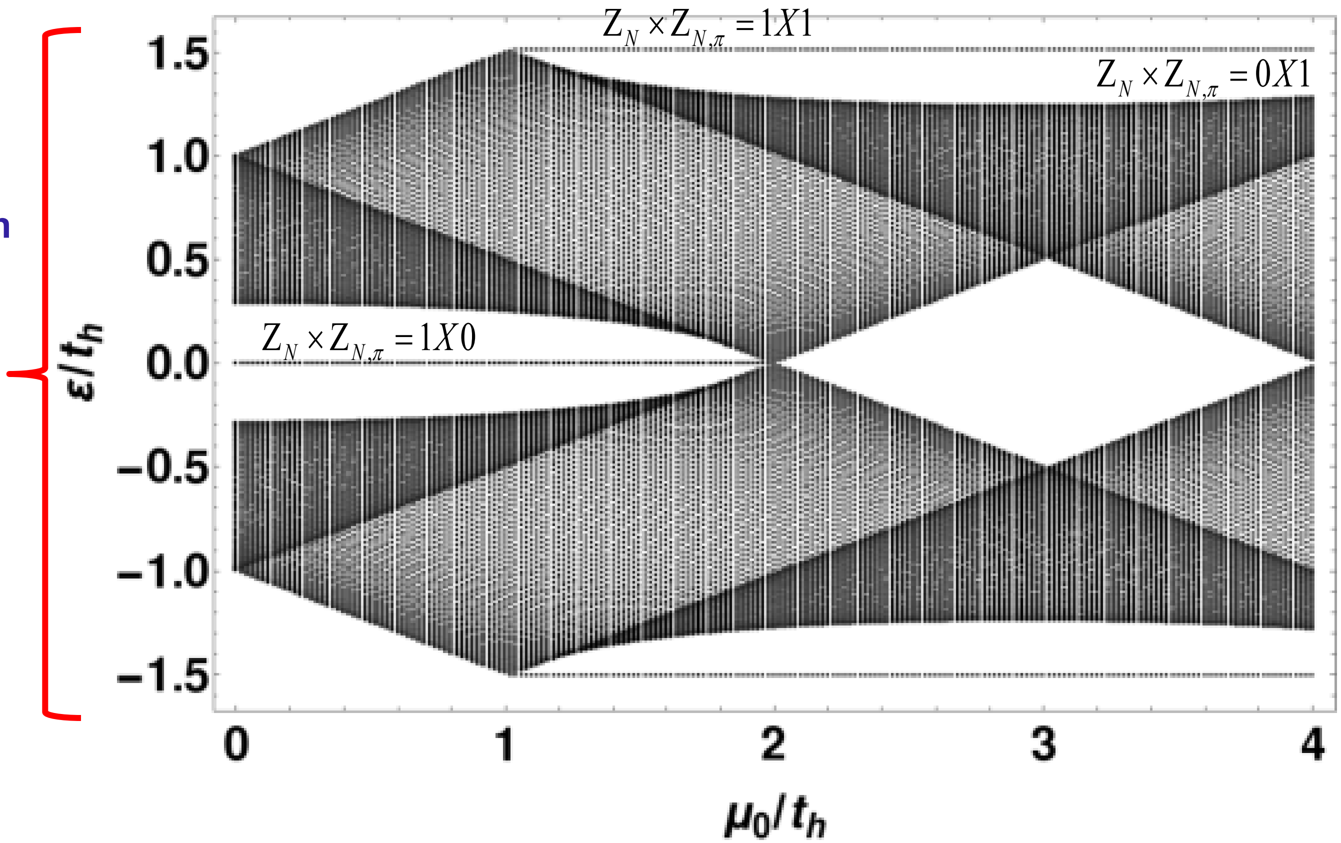
$$
\begin{aligned}
H &= \sum_i \Big[-t_h c_i^\dagger c_{i+1} - \Delta(t) c_i^\dagger c_{i+1}^\dagger - \mu(t)\left(c_i^\dagger c_i - \frac{1}{2}\right) \\
&\qquad - t_h' c_i^\dagger c_{i+2} - \Delta'(t) c_i^\dagger c_{i+2}^\dagger + h.c. \Big] \\
&= \sum_k \left(c_k^\dagger \;\; c_{-k}\right) H_{\mathrm{BdG}}(k,t) \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}. \qquad (1)
\end{aligned}
$$

Topological phase transitions
via gap-opening and closing at two
quasi-energies 0 and half-drive-frequency.

$$\mu(t) = \mu_0 + \xi \sin(\Omega t).$$

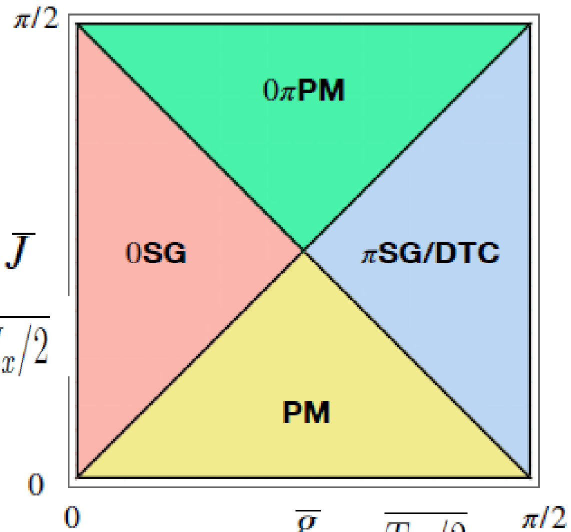
Quasi-energy-spectrum for open boundary conditions and nn only

Floquet Brillouin Zone (FBZ)



A consequence of Pi Strong Modes: Stable Discrete or Floquet Time Crystals (FTC)

V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, Phase structure of driven quantum systems, Phys. Rev. Lett. **116**, 250401 (2016).
C. W. von Keyserlingk, V. Khemani, and S. L. Sondhi, Absolute stability and spatiotemporal long-range order in floquet systems, Phys. Rev. B **94**, 085112 (2016).



V. Khemani, R. Moessner, and S. Sondhi, A brief history of time crystals, arXiv:1910.10745 (2019).

Spectral pairing at pi leads to:

$$\langle \epsilon_\pm, p=\pm 1|\sigma_i^x(nT)\sigma_j^x(0)|\epsilon_\pm, p=\pm 1|\rangle$$
$$\approx \langle \epsilon_\pm, p=\pm 1|\left[U^\dagger\right]^n \sigma_i^x U^n|\epsilon_\mp, p=\mp 1|\rangle$$
$$= (-1)^n$$

$J = \overline{TJ_x/2}$, $g = \overline{T\mu/2}$

$$U(T) = e^{\frac{-iTJ_x}{2}H_{xx}} e^{\frac{-iT\mu}{2}H_z},$$

$$H_{xx} = \sum_{i=1}^{L} \sigma_i^x \sigma_{i+1}^x, \quad H_z = \sum_{i=1}^{L} \sigma_i^z.$$

Critical properties of Floquet or Discrete Time Crystals in 1D

N. Y. Yao, A. C. Potter, I.-D. Potirniche, and A. Vishwanath, Discrete time crystals: Rigidity, criticality, and realizations, Phys. Rev. Lett. **118**, 030401 (2017).
W. Berdanier, M. Kolodrubetz, S. A. Parameswaran, and R. Vasseur, Floquet quantum criticality, Proceedings of the National Academy of Sciences **115**, 9491 (2018).
D. Yates, Y. Lemonik, and A. Mitra, Central charge of periodically driven critical kitaev chains, Phys. Rev. Lett. **121**, 076802 (2018).
W. Berdanier, M. Kolodrubetz, S. A. Parameswaran, and R. Vasseur, Strong-disorder renormalization group for periodically driven systems, Phys. Rev. B **98**, 174203 (2018).

Critical properties of prethermal Floquet or Discrete Time Crystal in d>2

Muath Natsheh, Andrea Gambassi, and Aditi Mitra, *Critical properties of the prethermal Floquet time crystal*, Phys. Rev. B **103**, 224311 (2021).

Muath Natsheh, Andrea Gambassi, and Aditi Mitra, *Critical properties of the Floquet time crystal within the Gaussian approximation*, Phys. Rev. B **103**, 014305 (2021).

Edge mode diagnostic: A measure of edge decoherence

$$A(nT) = \frac{1}{2^L}\mathrm{Tr}\left[\sigma_1^x(nT)\sigma_1^x\right] = \frac{1}{2^L}\mathrm{Tr}\left[a_1(nT)a_1\right]$$

$$A_\psi(nT) = \langle\psi|\sigma_1^x(nT)\sigma_1^x|\psi\rangle$$

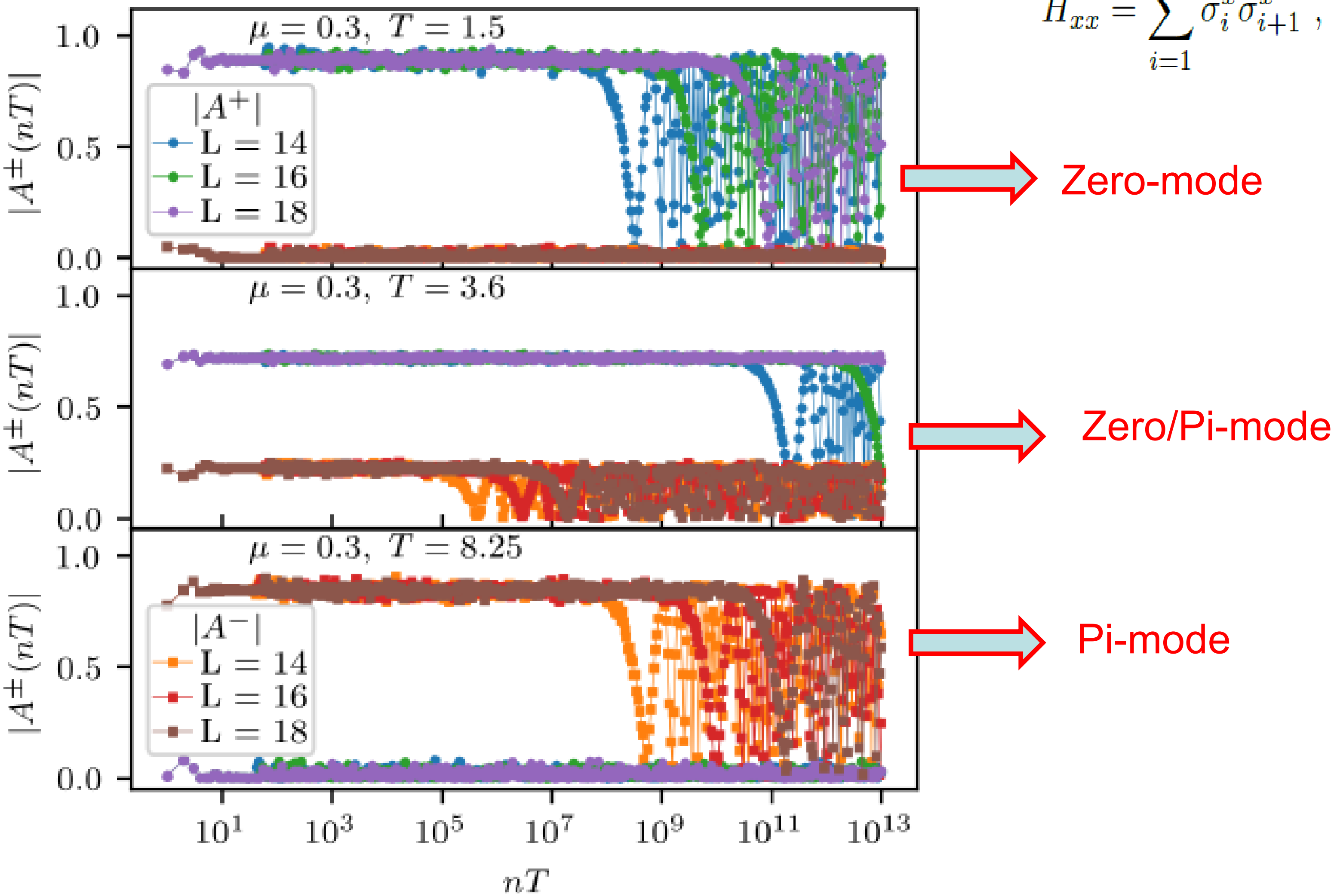
For detecting Pi mode: $A^-(nT) = \left[A(nT) - A((n+1)T)\right]/2$

For detecting 0 mode: $A^+(nT) = \left[A(nT) + A((n+1)T)\right]/2$

Free Floquet System: Strong zero/pi modes

$$U(T) = e^{\frac{-iTJ_x}{2}H_{xx}} e^{\frac{-iT\mu}{2}H_z} ,$$

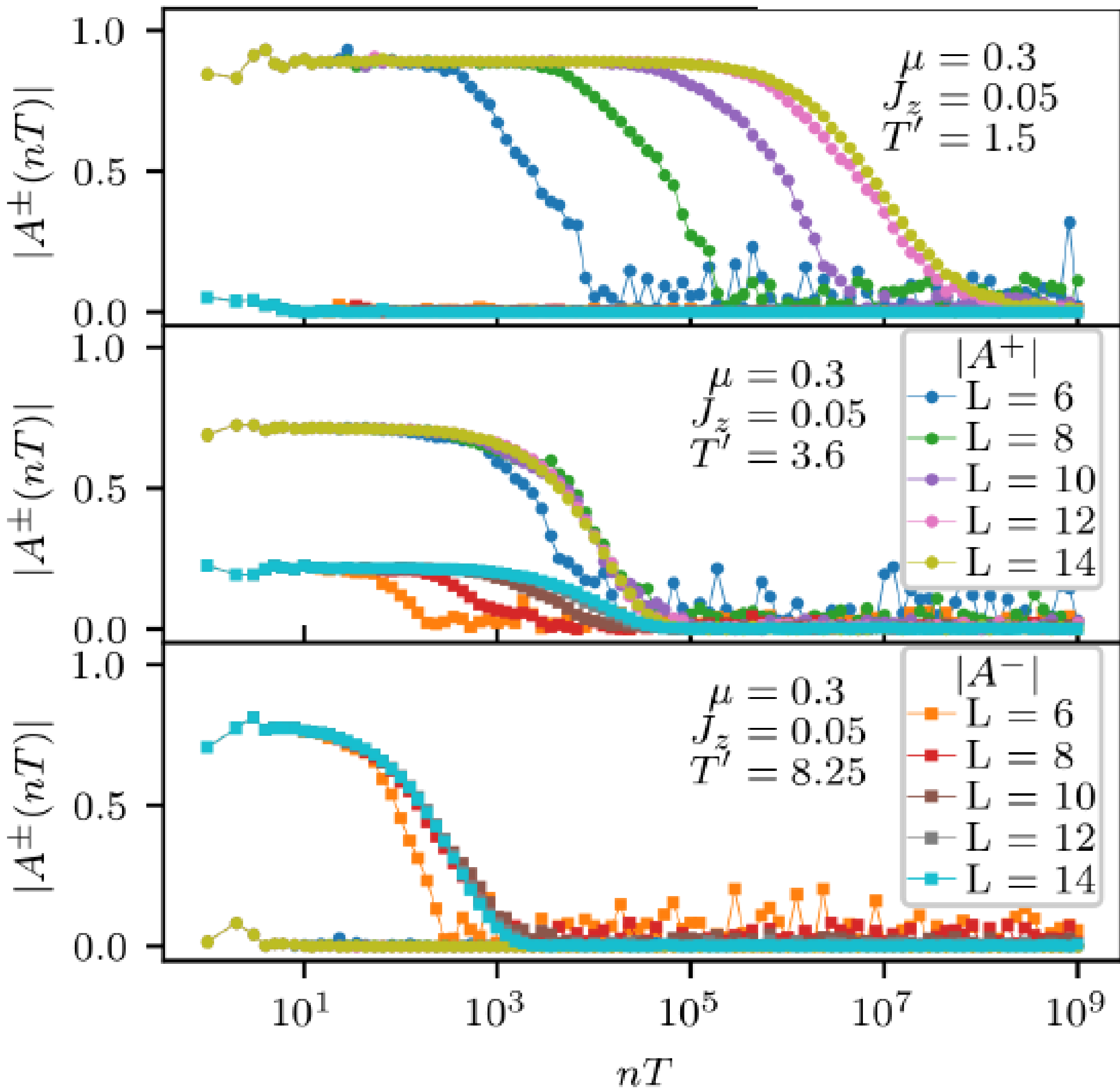
$$H_{xx} = \sum_{i=1}^{L} \sigma_i^x \sigma_{i+1}^x , \quad H_z = \sum_{i=1}^{L} \sigma_i^z .$$



Interacting Floquet System: Almost-Strong zero/pi modes

$$U(T) = e^{-i\frac{TJ_z}{3}H_{zz}} e^{-i\frac{TJ_x}{3}H_{xx}} e^{-i\frac{T\mu}{3}H_z}$$

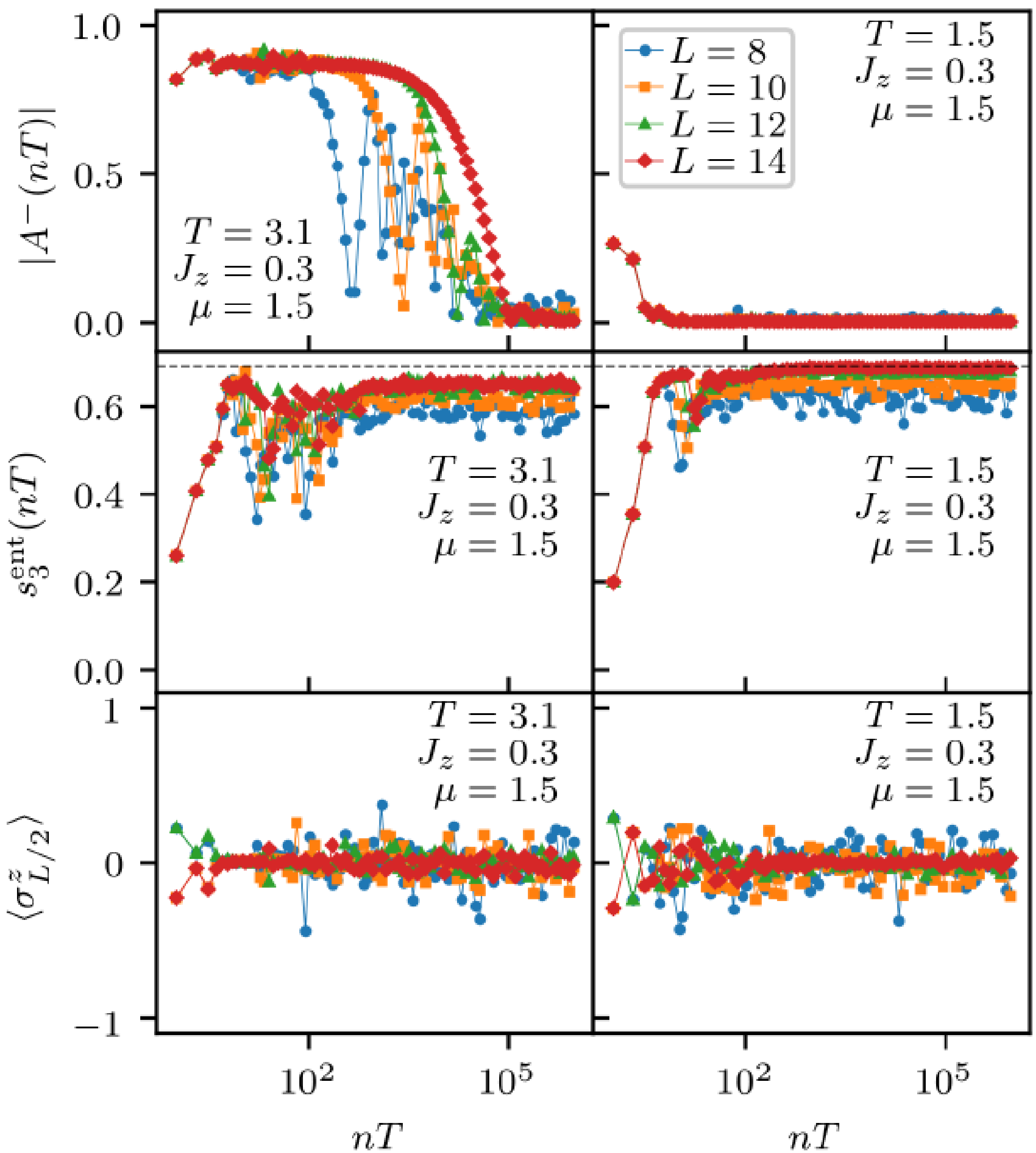
$$H_{zz} = \sum_{i=1}^{L-1} \sigma_i^z \sigma_{i+1}^z$$



Almost strong zero-mode

Almost strong Zero/pi-mode

Almost strong pi-mode



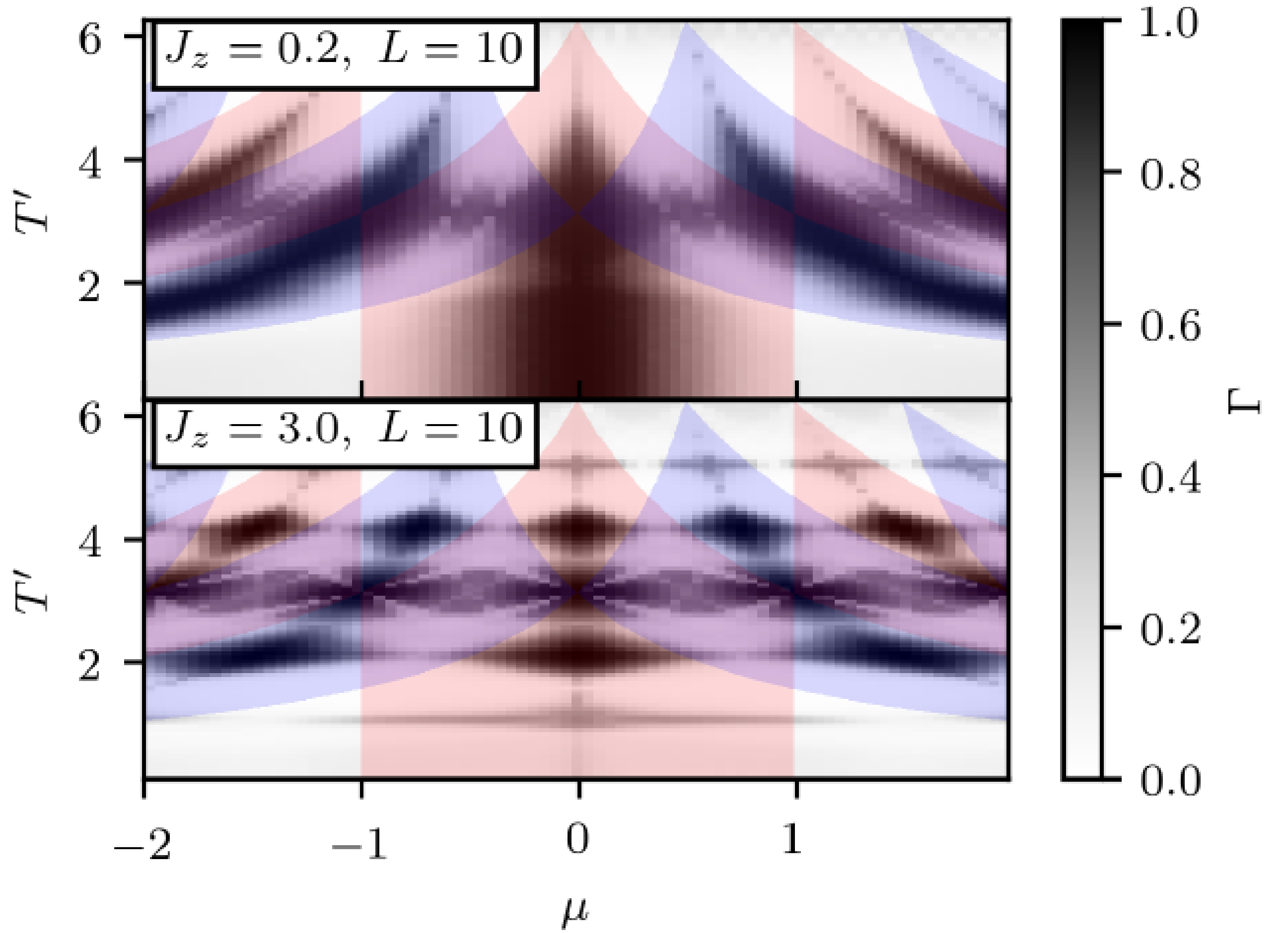
Lifetime of almost strong modes exceeds thermalization times by many orders.

Initial state: classical Neel state

Large regions of parameters support almost strong modes:

$$U(T) = e^{-i\frac{TJ_z}{3}H_{zz}} e^{-i\frac{TJ_x}{3}H_{xx}} e^{-i\frac{T\mu}{3}H_z}$$

$$\Gamma = \frac{1}{2^L}\sum_s \max_{s'} |\langle s|\sigma_1^x|s'\rangle|^2$$



Blue/Red regions: Strong zero/pi mode of the free Floquet system

T' Effective drive period introduced in order to compare the binary and ternary drive results

Long lifetimes imply tunneling processes.

A convenient way to arrive at a tunneling picture in an interacting and driven problem is to map the operator dynamics to single-particle quantum mechanics.

This is possible to do using Krylov subspace techniques.

Operator spreading in Krylov subspace: Mapping to a single particle-problem

$$e^{iHt}Oe^{-iHt} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \mathcal{L}^n O, \qquad \mathcal{L}O = [H, O].$$

Heisenberg time-evolution can be mapped to motion of a free particle in a Krylov basis

$$|O_0) = |\sigma_1^x) \longrightarrow |A_1) = \mathcal{L}|O_0) \longrightarrow b_1 = \sqrt{(A_1|A_1)} \longrightarrow |O_1) = |A_1)/b_1$$

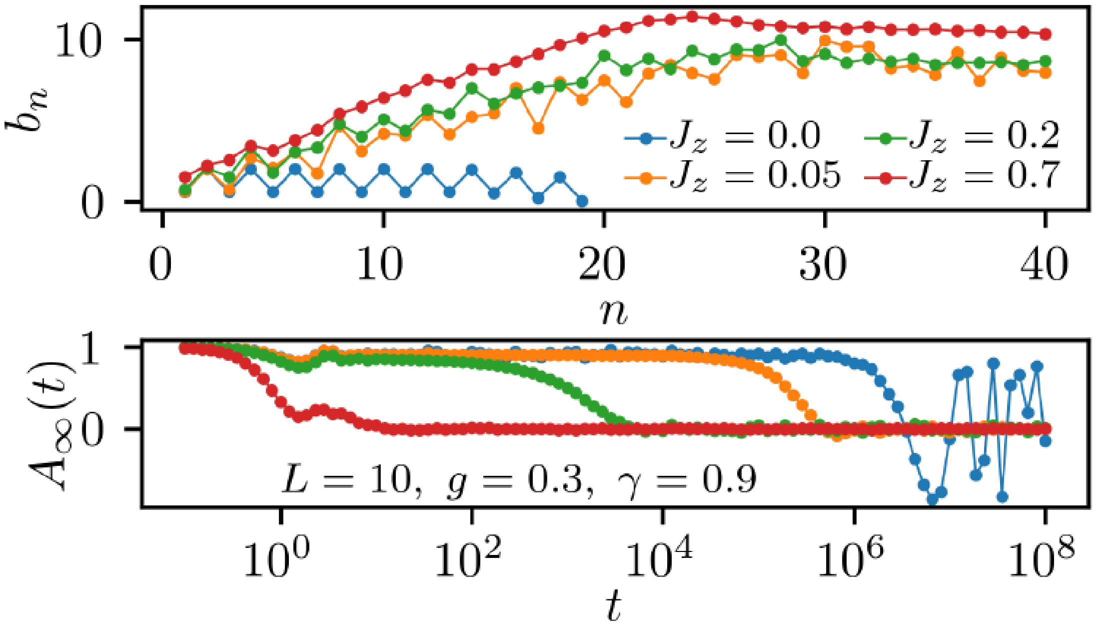
$$\mathcal{L}|O_n) = b_n|O_{n+1}) + b_{n-1}|O_{n-1}),$$

bn: Lanczos coefficients

$$\mathcal{L} = \begin{pmatrix} & b_1 & & \\ b_1 & & b_2 & \\ & b_2 & & \ddots \\ & & \ddots & \end{pmatrix}.$$

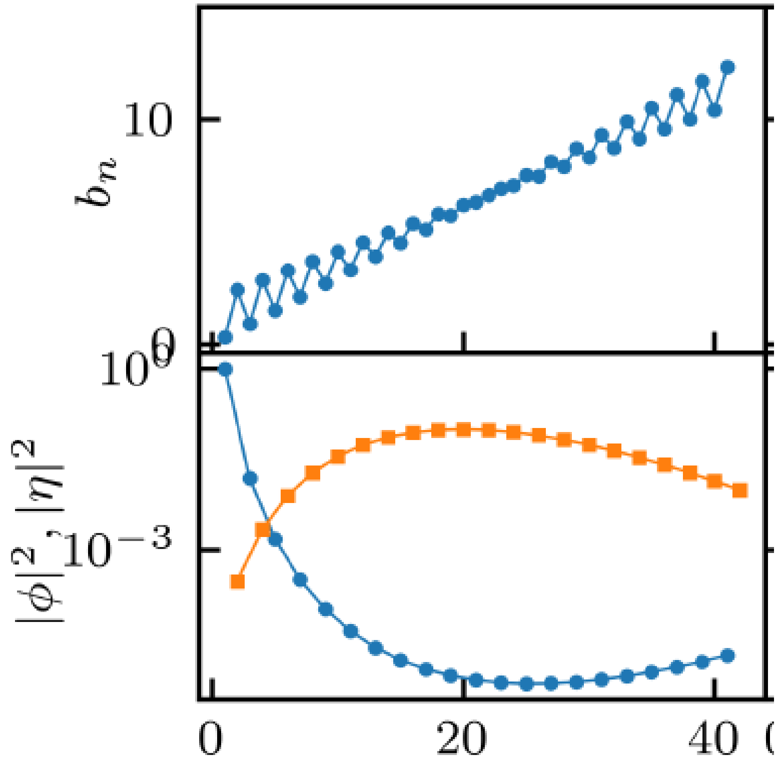
V. Vishwanath and G. Müller, *The Recursion Method: Applications to Many-Body Dynamics*, Springer, New York (2008).
D. E. Parker, X. Cao, A. Avdoshkin, T. Scaffidi, and E. Altman, Phys. Rev. X **9**, 041017 (2019).

$$A_\infty(t) = (e^{i\mathcal{L}t})_{1,1}.$$



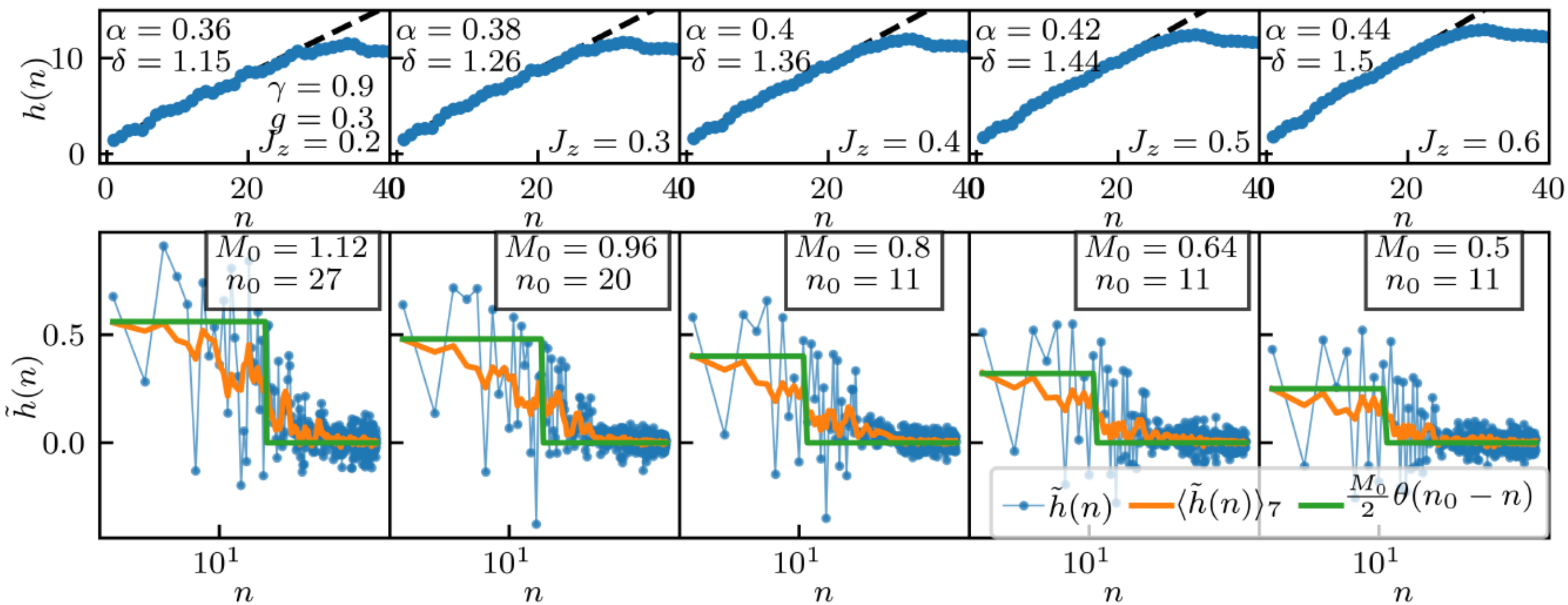
Krylov basis: Generalized SSH-type model, with increasing hopping decreasing dimerization.

A linear slope alone is not sufficient to remove edge modes:



$$b_n = \begin{cases} \alpha_1 n + \delta_1 & n \text{ odd} \\ \alpha_2 n + \delta_2 & n \text{ even.} \end{cases}$$

Map the model to a continuum model: Dirac model on half-line



$$
\begin{aligned}
b_n &= h_n + (-1)^n \tilde{h}_n, \\
h_n &= \alpha n + \delta, \\
\tilde{h}_n &= \frac{M_0}{2\left[\left(\frac{n}{n_0}\right)^{\beta} + 1\right]}.
\end{aligned}
$$

$$
i\partial_t \chi = \left[\sigma^z i\partial_X + \sigma^y m(X)\right]\chi\,,
$$

$$
m(X) = 2\tilde{h}(X) - \frac{\partial_X \tilde{h}(X)}{2h(X)} \approx 2\tilde{h}(X)\,.
$$

$$
\Gamma_A \sim 4M_0 e^{-\frac{M_0}{\alpha}\log\left(\frac{\alpha x_0}{\delta}\right)}\,.
$$

M_0 is dimerization at the left end of the wire, and $\alpha \propto J_z$ is the linear slope.

Strong and almost strong modes of Floquet spin chains in Krylov sub-spaces.

Two kinds of Krylov subspaces are possible:

1. From the Floquet Hamiltonian.

$$[U^\dagger]^m O U^m = e^{iH_F mT} O e^{-iH_F mT} = \sum_{n=0}^{\infty} \frac{(imT)^n}{n!} \mathcal{L}^n O,$$

2. From the Floquet Unitary: Arnoldi procedure.

Daniel J. Yates and Aditi Mitra, *Strong and almost strong modes of Floquet spin chains in Krylov subspaces*, arXiv:2105.13246.

Daniel J. Yates, Alexander G. Abanov, and Aditi Mitra, *Long-lived π edge modes of interacting and disorder-free Floquet spin chains*, arXiv:2105.13766.

Hopping parameters of the Krylov Hamiltonian: Binary-drive

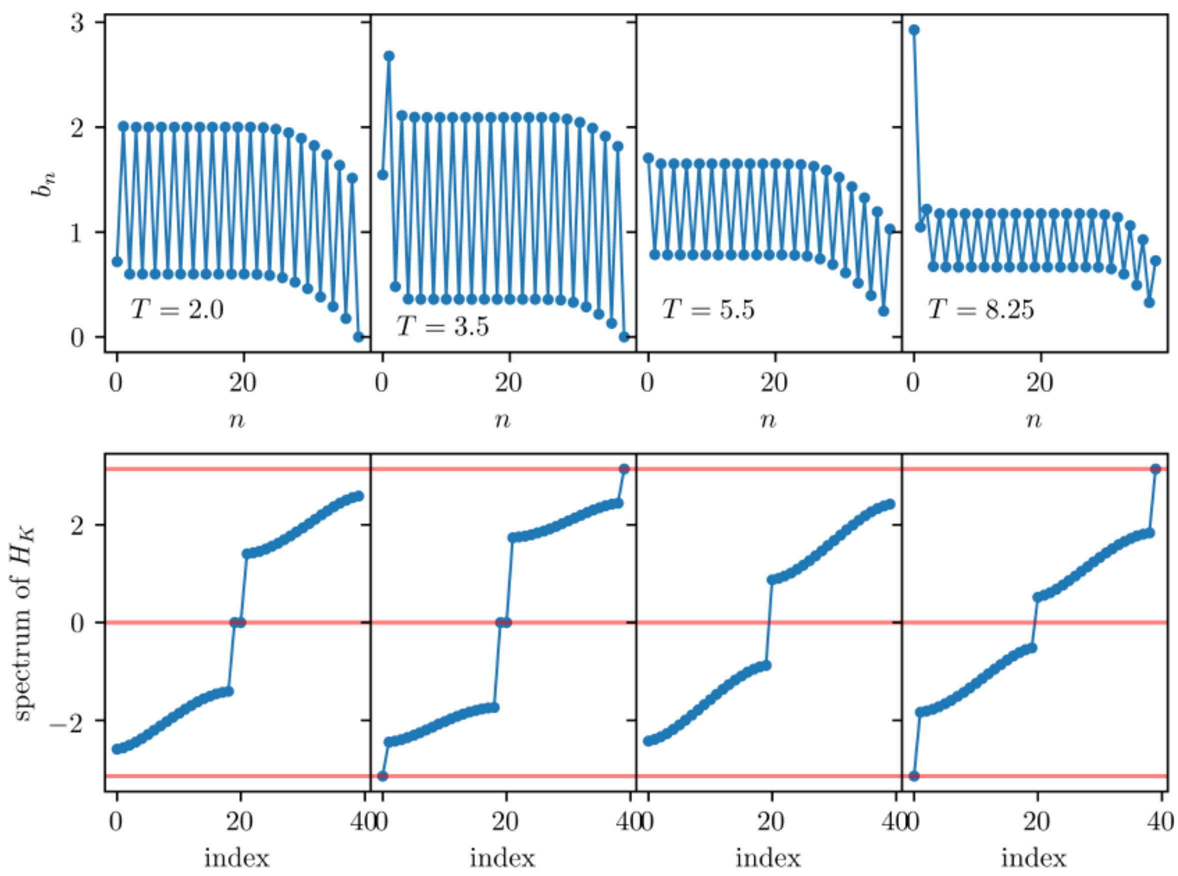
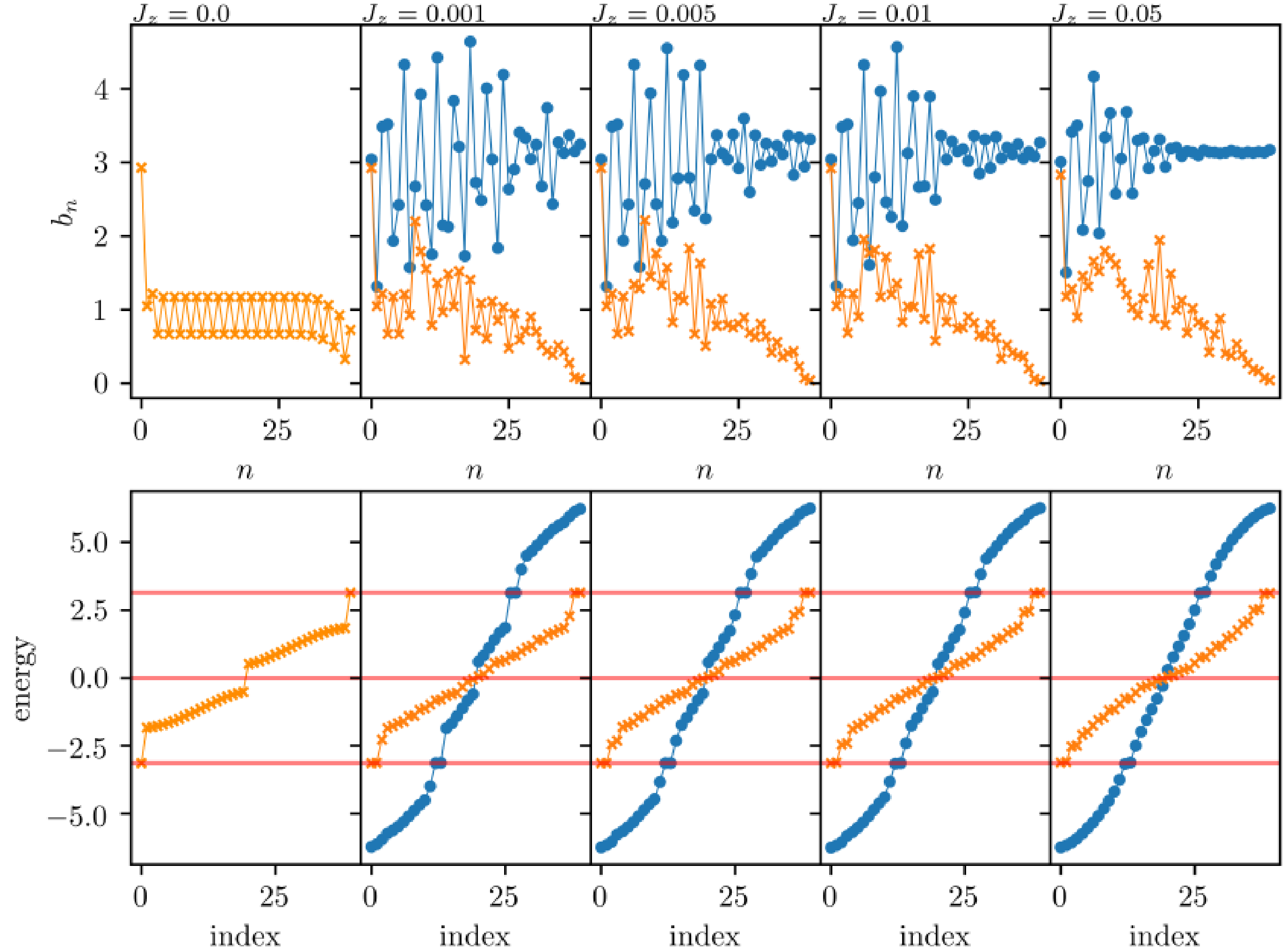


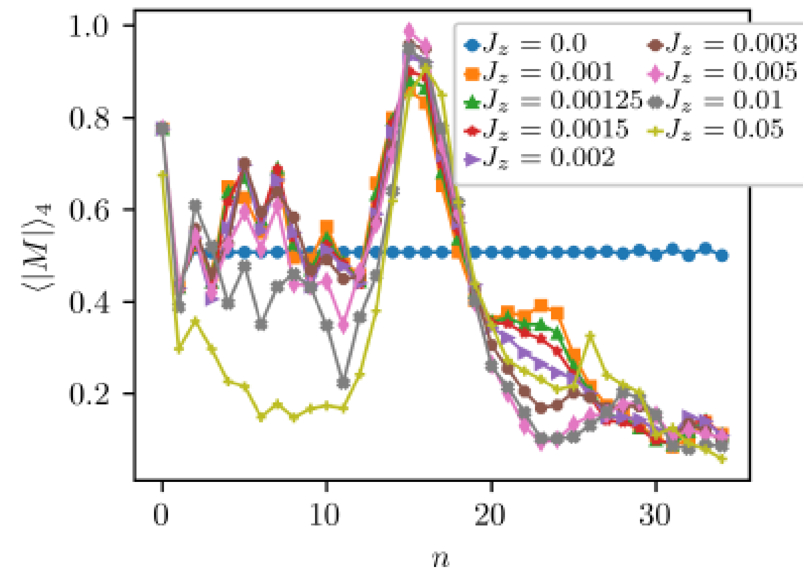
FIG. 2. Upper panel: The b_ns for the binary drive with $g = 0.3$ and system size $L = 20$. Lower panel: The corresponding spectra. For both upper and lower panels, from left to right $T = 2.0, 3.5, 5.5, 8.25$. These parameters correspond to SZM, SZM-SPM, trivial, and SPM phases respectively. Horizontal red lines in lower columns correspond to energies 0 and $\pm\pi$ in units of T^{-1}.

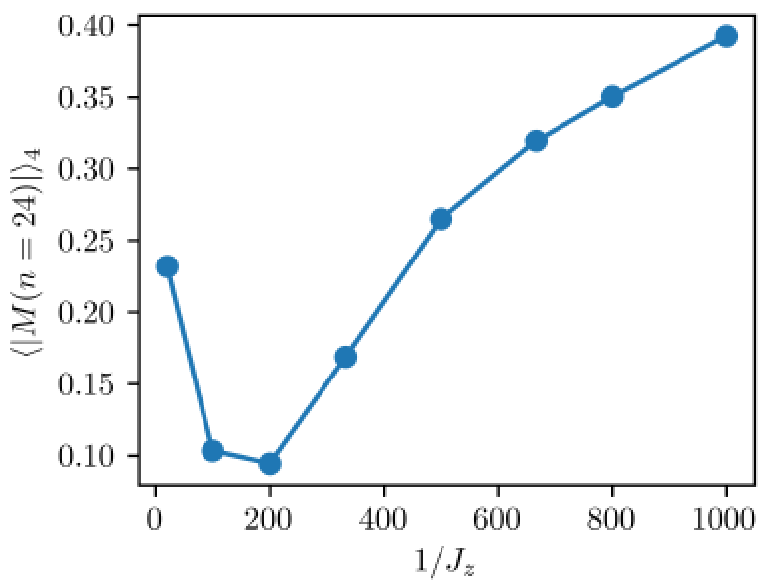
$$U = e^{-i\frac{T}{2}J_x H_{xx}} e^{-i\frac{T}{2}gH_z},$$

Hopping parameters of the Krylov Hamiltonian: Ternary-drive and pi-mode



$$U = e^{-i\frac{T}{2}J_z H_{zz}} e^{-i\frac{T}{2}J_x H_{xx}} e^{-i\frac{T}{2}g H_z}$$





$$i\partial_t \bar{\Psi} = \left[m(x)\sigma^y + \sigma^x i\partial_x \right] \bar{\Psi}$$

$$M(2n) = -M(2n+1) = m$$

$$m(x) = M_0 \theta(x - X_0)$$

$$\Gamma \approx 4M_0 e^{-2M_0 X_0}$$

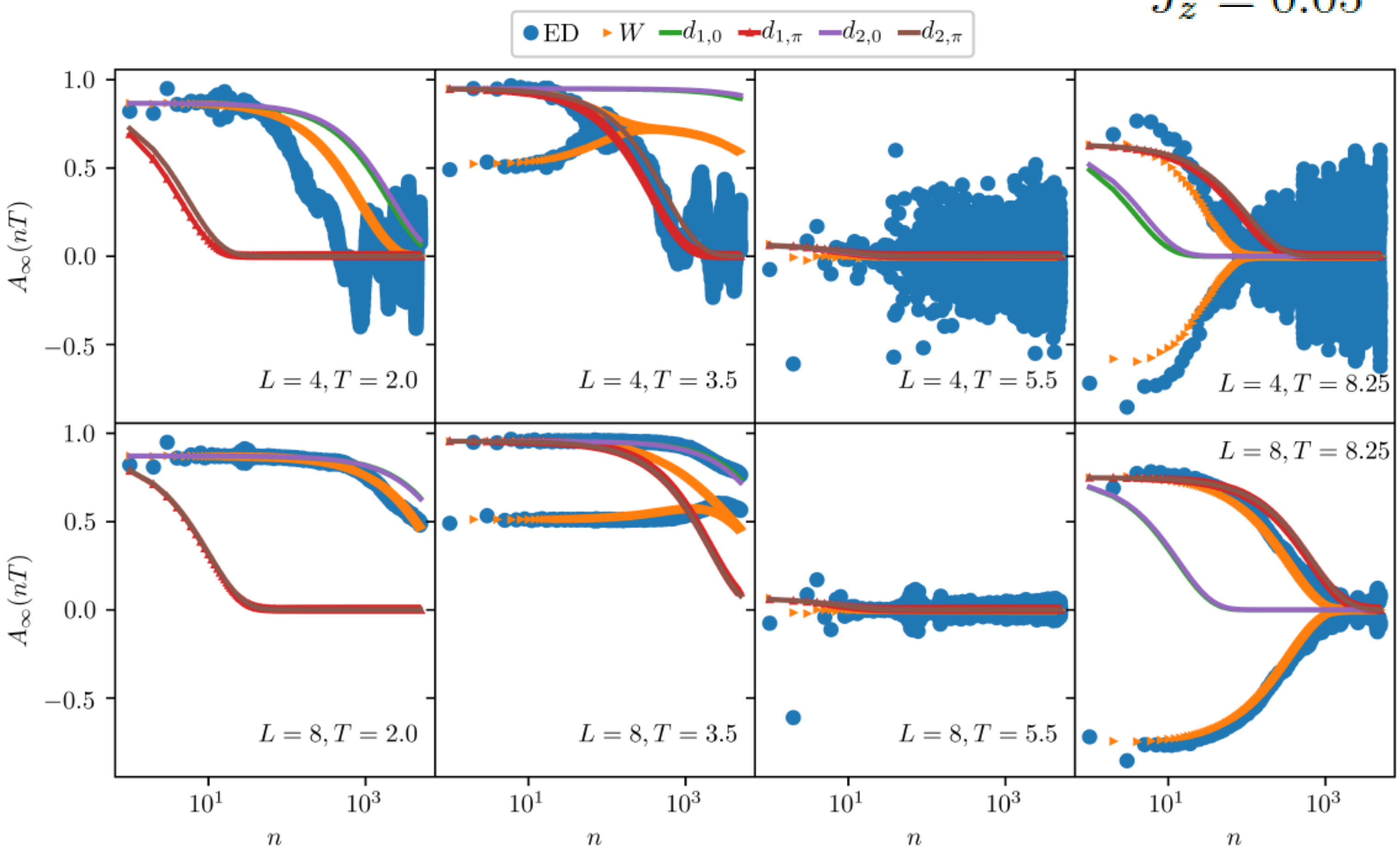
Arnoldi method: Mapping to a unitary matrix

$$W|O) = U^\dagger \hat{O} U$$
$$W^n|O) = U(n)^\dagger \hat{O} U(n).$$

$$W|n) = w_{n+1,n}|n+1) + \sum_{l=1}^{n} w_{l,n}|l).$$

$$W = \begin{pmatrix} w_{1,1} & w_{1,2} & \cdots \\ w_{2,1} & w_{2,2} & \cdots \\ 0 & w_{3,2} & \cdots \\ 0 & 0 & \ddots \end{pmatrix}.$$

$J_z = 0.05$



ED
W
$d_{1,0}$
$d_{1,\pi}$
$d_{2,0}$
$d_{2,\pi}$
$A_\infty(nT)$
$L = 4, T = 2.0$
$L = 4, T = 3.5$
$L = 4, T = 5.5$
$L = 4, T = 8.25$
$L = 8, T = 2.0$
$L = 8, T = 3.5$
$L = 8, T = 5.5$
$L = 8, T = 8.25$
n

Conclusions

Although clean interacting Floquet systems heat to infinite temperature, if an almost strong mode is present, edge modes are quasi-stable. They live for a time that exceeds bulk thermalization times by many orders of magnitude.

Good news for realizing non-abelian edge modes by Floquet driving.

Developed an analytic and computational scheme to extract the non-perturbatively long lifetimes of almost strong modes. This method involves mapping the operator dynamics to single particle dynamics in Krylov subspaces. The method can be applied to any dimension, and to both static and Floquet systems.

The Krylov Hamiltonian of almost strong modes has some universal features such as topologically non-trivial dimerization whose finite spatial extent gives an estimate for the tunneling time of the almost strong mode.