Integrable Domain Walls in ABJM theory

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Based on:

- T. Gombor & C.K., arXiv: 2207.06866[hept-th], to appear in Phys. Lett. B.
- C.K, D.L. Vu & K. Zarembo, arXiv:2112.10438[hept-th], JHEP 02 (2022) 070
- C.K., D. Müller & K. Zarembo, arXiv:2106.08116[hep-th], JHEP 09 (2021) 004, arXiv:2011.12192, JHEP 03 (2021) 100

Talking Integrability: Spins, Fields and Strings

Aug. 29^{th}, 2022

AdS/CFT

QFT in lower D* $\longleftrightarrow$ String theory in 10D

- Conformal symmetry
- Supersymmetry
- Planar integrability

AdS/dCFT

Domain wall $\longleftrightarrow$ Probe D-brane

- Conformal symmetry partially broken
- Supersymmetry partially or completely broken

In the planar limit, $N \to \infty$:

* $\mathcal{N} = 4$ SYM in 4D, ABJM-theory in 3D ← Focus of this talk

Motivation

- Gain insight on the interplay between conformal symmetry, supersymmetry and integrability
- Test the AdS/CFT dictionary for set-ups with supersymmetry partially or completely broken (all tests positive)
- Exact results for novel types of observables such as one-point functions
- Produce input data for the boundary conformal bootstrap program.
- Interesting connections to statistical physics and QI: matrix product states and quantum quenches
- Novel characterization of integrable boundary states at the discrete level,
  Novel examples of integrable boundary states
- Novel "microscopic duality relations" for correlation functions
  Strong predictive/constraining power.

Plan of the talk

I. Correlators in AdS/dCFT = spin chain overlaps

II. Domain walls in ABJM theory

III. Novel exact overlap formulas for ABJM theory

IV. Predicting overlaps from duality relations

V. Future directions

AdS/CFT and Overlaps

Conformal operators $\longleftrightarrow$ String states

$\updownarrow$ $\updownarrow$

Eigenstates of integrable super spin chain: $|\mathbf{u}\rangle$

Minahan. Zarembo ‘02

Beisert, Staudacher ‘03

Karch. Randall ‘01

Co-dimension one defect $\longleftrightarrow$ Karch-Randall probe brane

$|\Psi_0\rangle$ (integrable) boundary state describing defect / probe brane

$\langle\Psi_0|\mathbf{u}\rangle$ is a one-point function

De Leeuw, C.K. Zarembo ‘15

Pair of determinant operators $\longleftrightarrow$ Giant graviton

Similar idea: $|\Psi_0\rangle \sim$ determinant operators/giant graviton

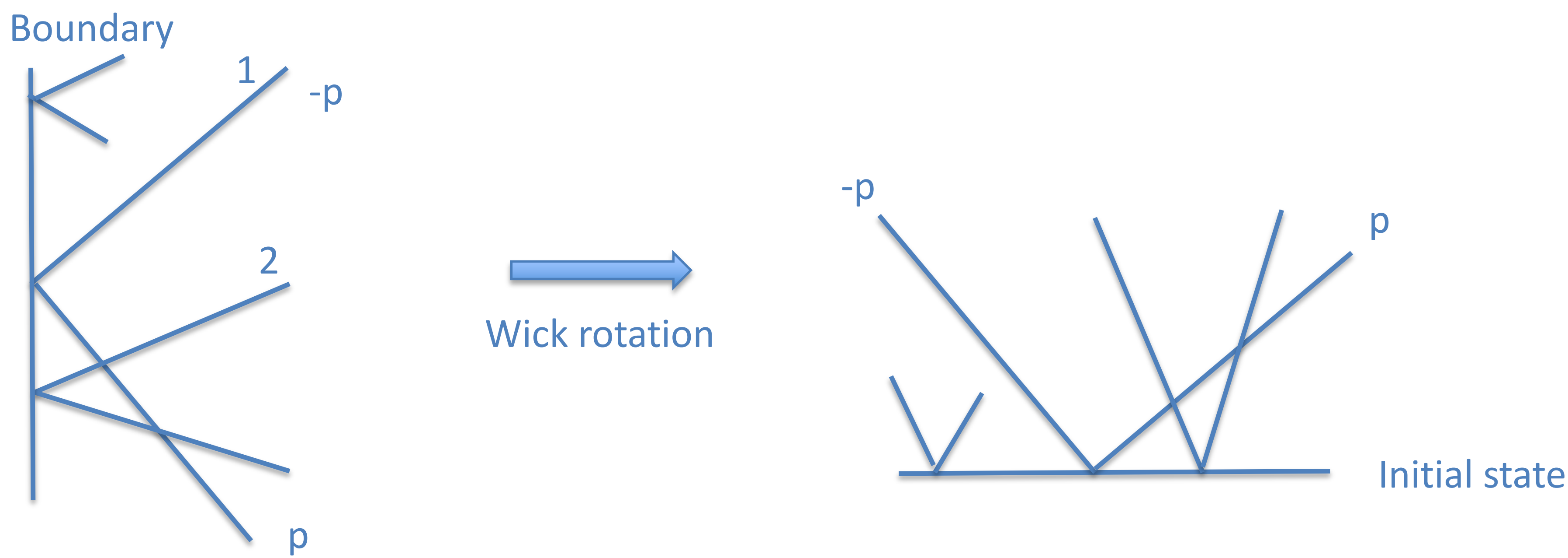
Jiang, Komatsu Vescovi ‘19

$\langle\Psi_0|\mathbf{u}\rangle$ is a three-point function

Integrable boundaries in integrable QFTs

Ghoshal, Zamolodchikov '93

- No particle production or annihilation
- Pure reflection, possibly change of internal quantum numbers
- Yang-Baxter relations fulfilled (order of reflection does not matter)



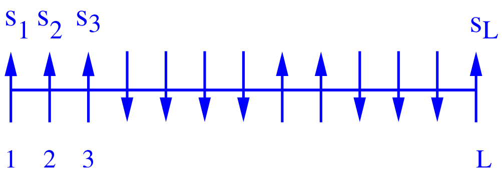
Pure reflection
+BYB for reflection matrix

Entangled (p,-p) pairs
+BYB for initial state

Piroli, Pozsgay, Vernier '17

Spin chain language: $Q_{2m+1}|\Psi_0\rangle = 0$

Integrable boundary states



$s_{L+m} = s_m$ $\quad |\Psi\rangle = |s_1 s_2 s_3 \ldots s_L\rangle$

Eigenstates: $H_0|\mathbf{u}\rangle = E_0|\mathbf{u}\rangle$

Integrable boundary state $\langle\Psi_0|$: $Q_{2n+1}|\Psi_0\rangle = 0$

Expect $\langle\Psi_0|\mathbf{u}\rangle$ computable in closed form

Types of boundary states of relevance for AdS/dCFT:

Matrix product states: $|B\rangle = |\mathrm{MPS}\rangle = \sum_{\{s_i\}} \mathrm{Tr}(t_{s_1} \ldots t_{s_L})|s_1 \ldots s_L\rangle$

De Leeuw, C.K., Zarembo '15

Valence Bond States: $|\mathrm{VBS}\rangle = |K\rangle^{\otimes \frac{L}{2}}, \qquad K = \sum_{s_1, s_2} K_{s_1, s_2}|s_1 s_2\rangle$

C.K., Müller, Zarembo '20

Cross cap states: $|C\rangle = |c\rangle\rangle^{\otimes L/2}$, where $|c\rangle\rangle = |\uparrow\rangle_j|\uparrow\rangle_{\frac{L}{2}+j} + |\downarrow\rangle_j|\downarrow\rangle_{\frac{L}{2}+j}$

Caetano, Komatsu '21, Gombor '22, Ekman '22

Overlap Formulas

Selection rule

$\langle \Psi_0 | \mathbf{u} \rangle \neq \mathbf{0} \iff \{\mathbf{u_j}\} = \{-\mathbf{u_i}, \mathbf{u_i}\}$ $\mathbb{Z}_2$ parity invariance

Ingredients:

de Leeuw, C.K. & Zarembo '15 de Leeuw, C.K. & Mori '16 de Leeuw, C.K. & Linardopoulos '18

For $|\mathrm{MPS}_k\rangle$:

- Superdeterminant of Gaudin matrix: $\mathbb{D} = S\det G = \frac{\det(G_+)}{\det(G_-)}$

  $\langle \mathbf{u} | \mathbf{u} \rangle = \det G = \det G_+ \det G_-$

- Fused transfer matrices: Sums of ratios of Baxter polynomials: $\sum_{a=-\frac{k}{2}}^{a=\frac{k}{2}} \dots$

  de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

For $|\mathrm{VBS}\rangle$:

- No sums involved Poszgay '18 Gombor '21

For cross cap states:

- No Baxter polynomials involved Caetano, Komatsu '21 Gombor '22 Ekman '22

ABJM theory

ABJM theory in 3D $\longleftrightarrow$ Type IIA strings on $AdS_4 \times CP^3$

$\mathcal{N} = 6$ susy

Field content: $A_\mu, \hat{A}_\mu, \Psi_A, Y^A, \;\; A = 1, 2, 3, 4$

Gauge symmetry: $U(N)_k \times \hat{U}(N)_{-k}$

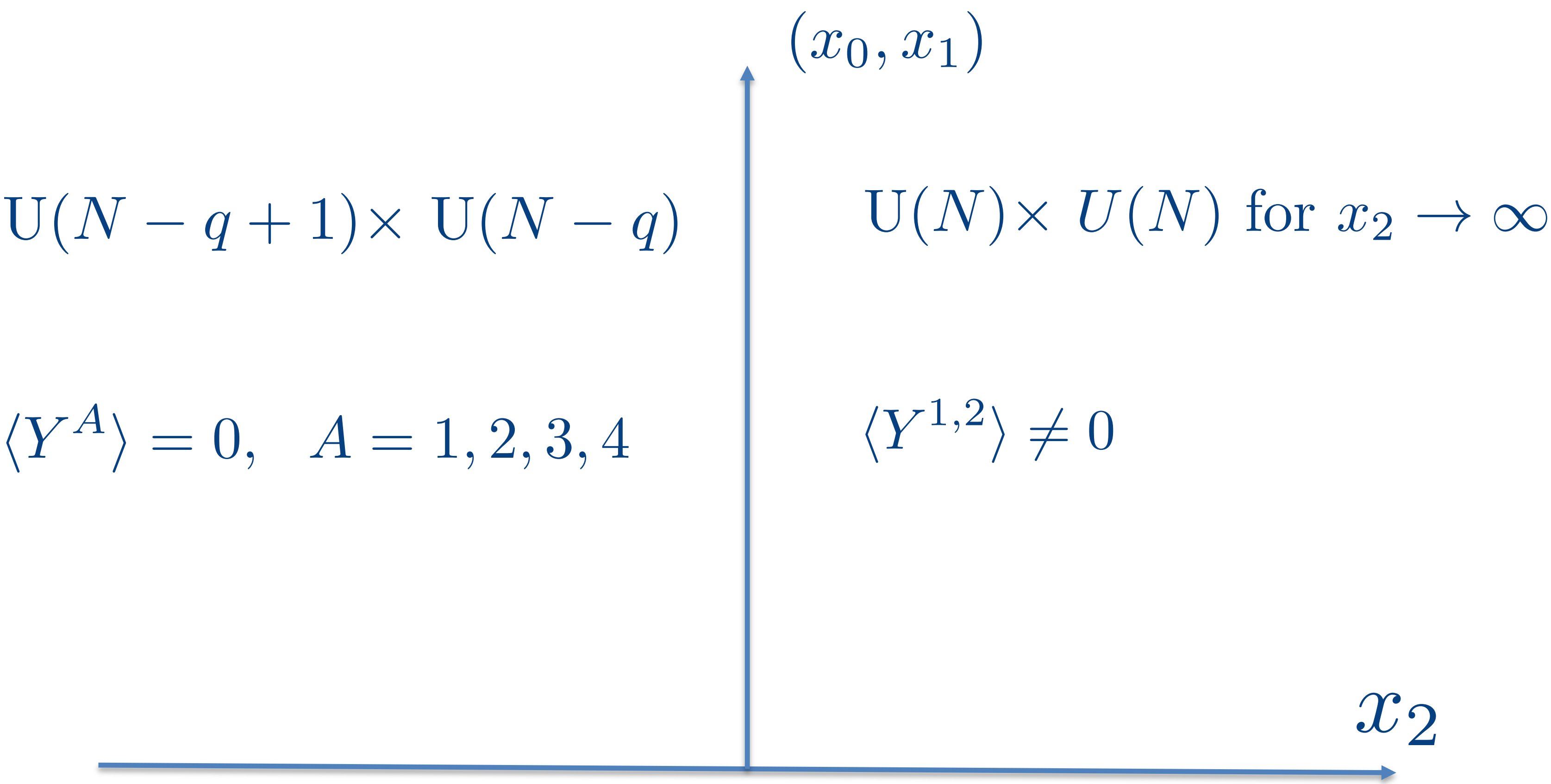
Planar 't Hooft limit: $N, k \to \infty$, $\lambda = \frac{N}{k}$ fixed

Integrable in the planar limit

$$
\begin{aligned}
\mathcal{L} = \quad & \frac{k}{4\pi} \operatorname{tr} \Big[\varepsilon^{\mu\nu\lambda} \Big(A_\mu \partial_\nu A_\lambda + \frac{2}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \Big) \\
& + D_\mu Y_A^\dagger D^\mu Y^A + \frac{1}{12} Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + \frac{1}{12} Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C Y_A^\dagger \\
& - \frac{1}{2} Y^A Y_A^\dagger Y^B Y_C^\dagger Y^C Y_B^\dagger + \frac{1}{3} Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger + \text{fermions} \Big].
\end{aligned}
$$

The defect set-up of $|\mathrm{MPS}\rangle$

ABJM theory



Classical fields

Classical e.o.m.: $\frac{d^2 Y^\alpha}{dx_2^2} = \text{“}Y\,Y\,Y\,Y\,Y\text{”}$

$\langle Y^3 \rangle = \langle Y^4 \rangle = 0$

$A_\mu = \hat{A}_\mu = 0, \;\; \Psi^A = 0$

$\Uparrow$

BPS eqns: $\frac{dY^\alpha}{dx_2} = \frac{1}{2} Y^\alpha Y^\dagger_\beta Y^\beta - \frac{1}{2} Y^\beta Y^\dagger_\beta Y^\alpha, \qquad \alpha, \beta = 1, 2$

Basu-Harvey eqns.

Basu & Harvey ‘04

Analogy with $\mathcal{N} = 4$ SYM:

Nahm eqns (1st order) $\Longrightarrow$ classical eqns of motion (2nd order)

Nahm ‘80

Classical fields

Terashima '08

$$\langle Y^\alpha \rangle = \frac{1}{\sqrt{x_2}} \begin{pmatrix} S^\alpha_{(q-1)\times q} & 0 \\ 0 & 0_{(N-q+1)\times(N-q)} \end{pmatrix}, \quad \alpha = 1, 2$$

$$\langle Y^3 \rangle = \langle Y^4 \rangle = 0$$

$$S^1_{ij} = \delta_{i,j-1}\sqrt{i}, \qquad S^2_{ij} = \delta_{ij}\sqrt{q-i}, \qquad i = 1, \ldots, q-1, \;\; j = 1, \ldots, q$$

$$S^1_{(q-1)\times q} = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 & \sqrt{q-1} \end{pmatrix} \quad S^2_{(q-1)\times q} = \begin{pmatrix} \sqrt{q-1} & 0 & 0 & \ldots & 0 & 0 \\ 0 & \sqrt{q-2} & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 1 & 0 \end{pmatrix}$$

One-point functions and MPS

$$\langle \mathcal{O}_\Delta(x) \rangle = \frac{C}{|x_2|^\Delta}$$

Cardy ´84

McAvity & Osborn '95

Due to vevs scalar operators can have non-zero 1-pt fcts at tree-level

$$\langle \mathcal{O}_\Delta(x) \rangle = \Big(\mathrm{Tr}(Y^{\alpha_1} Y^\dagger_{\beta_1} \ldots Y^{\alpha_L} Y^\dagger_{\beta_L}) + \ldots \Big) |_{Y^{\alpha_i} \to \langle Y^{\alpha_i} \rangle}$$

$\mathcal{O}_\Delta(x) \sim$ eigenstate of integrable alternating $SU(4)$ spin chain

$$\mathrm{Tr}(Y^{\alpha_1} Y^\dagger_{\beta_2} \ldots Y^{\alpha_L} Y^\dagger_{\beta_L}) \sim |s^{\alpha_1} \bar{s}_{\beta_2} \ldots s^{\alpha_L} \bar{s}_{\beta_L} \rangle$$

Spin chain Hamiltonian

$$H = \lambda^2 \sum_{l=1}^{2L} \left(1 - P_{l,l+2} + \frac{1}{2} P_{l,l+2} K_{l,l+1} + \frac{1}{2} K_{l,l+1} P_{l,l+2} \right),$$

Minahan & Zarembo '08

One-point functions and $|\mathrm{MPS}\rangle$

$$\langle \mathcal{O}_\Delta(x)\rangle = \left(\mathrm{Tr}(Y^{\alpha_1}Y^\dagger_{\beta_1}\ldots Y^{\alpha_1}Y^\dagger_{\beta_1}+\ldots\right)|_{Y^{\alpha_i}\to\frac{S^{\alpha_i}}{\sqrt{x_2}}}$$

C.K., Vu & Zarembo '21,

Two Matrix Product States associated with the defect:

$$|\mathrm{MPS}_{\mathrm{q}-1}\rangle = \sum_{\vec{\alpha},\vec{\beta}} \mathrm{Tr}[S^{\alpha_1}S^\dagger_{\beta_1}\ldots S^{\alpha_L}S^\dagger_{\beta_L}]|s^{\alpha_1}\bar{s}_{\beta_1}\ldots s^{\alpha_L}\bar{s}_{\beta_L}\rangle,$$

Object to calculate: $C_q(\mathbf{u}) = \dfrac{\langle \mathrm{MPS}_{q-1}|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{\frac{1}{2}}}$ ← Bethe eigenstate

Alternatively

$$|\widehat{\mathrm{MPS}}_q\rangle = \sum_{\vec{\alpha},\vec{\beta}} \mathrm{Tr}[S^\dagger_{\alpha_1}S^{\beta_1}\ldots S^\dagger_{\alpha_L}S^{\beta_L}]|\bar{s}_{\alpha_1}s^{\beta_1}\ldots \bar{s}_{\alpha_L}s^{\beta_L}\rangle,$$

Object to calculate: $C_q(\mathbf{u}) = \dfrac{\langle \widehat{\mathrm{MPS}}_q|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{\frac{1}{2}}}$

Connection to SU(2) reps.

C.K., Vu, Zarembo '20

Nastase '09

For $\alpha = 1, 2$:

$$\Phi^{\alpha}_{\beta} = Y^{\alpha} Y^{\dagger}_{\beta} \equiv \Phi^{i} (\sigma_i)^{\alpha}_{\beta} + \Phi \delta^{\alpha}_{\beta}, \qquad (q-1) \times (q-1) \text{ matrix}$$

$$\frac{d\Phi^i}{dx} = \frac{i}{2} \epsilon^{ijk} \left[\Phi^j, \Phi^k \right]$$

Nahm's equation

$$\frac{d\Phi}{dx} = \Phi^i \Phi^i - \Phi^2$$

Solution: $\Phi^i = \frac{t^i}{x}, \qquad (\{t^i\} = (q-1)\text{-dim. irrep. of SU(2)})$

$$\Phi = \frac{q}{2x} I_{q-1}$$

For $q = 2$: $\Phi^{\alpha}_{\beta} = \frac{1}{x} \delta^{\alpha}_{\beta}$, i.e. $|\text{MPS}_1\rangle = |\text{VBS}\rangle$

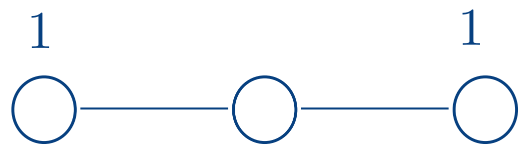
Similarly for $\widehat{\Phi}^{\alpha}_{\beta} = Y^{\dagger}_{\beta} Y^{\alpha}$, with q-dimensional rep. of $SU(2)$

Overlaps with $|\mathrm{VBS}\rangle$, i.e. $q = 2$

Vacuum state: $\mathrm{Tr}(Y^1 Y_2^\dagger)^L$

Excited states described via Bethe roots $\{u_1^{(i)}\}_{i=1}^{K_1}$, $\{u_2^{(j)}\}_{j=1}^{K_2}$, $\{u_3^{(k)}\}_{k=1}^{K_3}$

Baxter polynomials



$Q_1(u)$ $\quad Q_2(u)$ $\quad Q_3(u)$

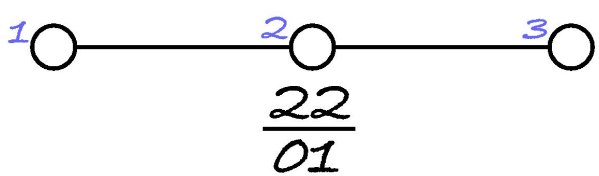
Selection Rules: $K_1 = K_2 = K_3 = L$

Z_2 symmetry: $\Omega : \{u_1^{(k)}\} \leftrightarrow \{-u_3^{(k)}\}$, $\{u_2^{(k)}\} \leftrightarrow \{-u_2^{(k)}\}$

Result for $|\mathrm{VBS}\rangle$: $$C = 2^{-L} Q_2(i) \sqrt{\frac{S\det G}{Q_2(0) Q_2(\frac{i}{2})}}$$

Gombor '21,

C.K., Vu & Zarembo '21,



Higher bond dimension $|\text{MPS}_q\rangle$

$$\frac{\langle \text{MPS}_{q-1}|\mathbf{u}\rangle}{\sqrt{\langle \mathbf{u}|\mathbf{u}\rangle}} = \sum_{k=1}^{q-1} \frac{Q_1(-\frac{i}{2})Q_1(-i(q-\frac{1}{2}))}{Q_1(-i(k-\frac{1}{2}))Q_1(-i(k+\frac{1}{2}))} \times$$

$$\left(\frac{k}{2}\right)^L \frac{Q_2(ik)}{\sqrt{\bar{Q}_2(0)\bar{Q}_2(i/2)}} \sqrt{\frac{\det G_+}{\det G_-}}.$$

Gombor, C.K. '22

Recursive strategy based on nested coordinate Bethe ansatz

Bajnok, Gombor '21

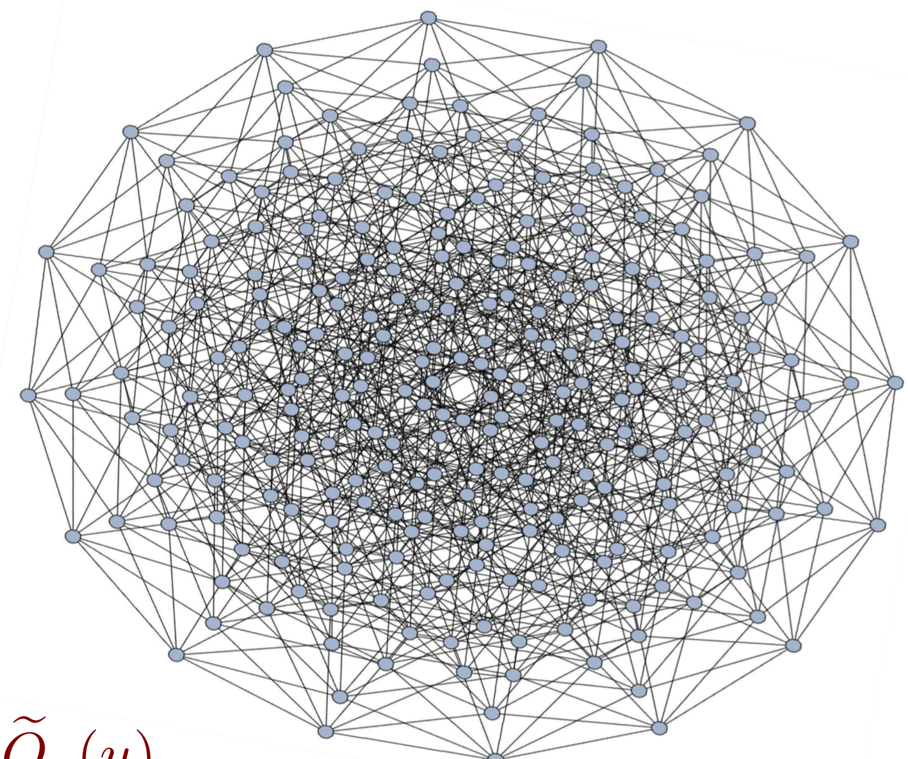
- Cut open MPS to $|\text{MPS}_{q-1}^{j,k}\rangle$
- Assume $\langle \text{MPS}_{q-1}^{k,k}|\mathbf{u}\rangle$ factorizes into reflection factors
- Calculate reflection matrix $K_{ab}(p,-p)$
- Relate reflection matrix recursively via nesting to $SU(2)$ reflection factor
- Check numerically

Other sectors via fermionic duality

Full $|\mathrm{VBS}\rangle$ overlap for $\mathcal{N}=4$ SYM singled out by transforming covariantly under fermionic spin chain duality

Many equivalent ways of writing the Bethe equations — QQ-system

For $\mathcal{N}=4$ SYM, # different choices of Q-functions $=2^8$



Connected via dualities

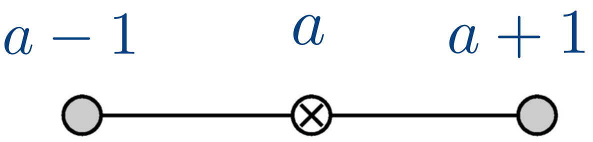
- Fermionic (Change of Dynkin diagram)
- Bosonic

Dualities = Change of variables in the Bethe equations: $Q_a(u) \to \widetilde{Q}_a(u)$

Fermionic dualities in general

- Allow one to move between the Bethe equations corresponding to any two Dynkin diagrams of a super Lie algebra (of type $SU(N|M)$)

- Involve a fermionic node and its neighbours only



$$Q_a \to \widetilde{Q}_a \;:\quad Q_a \widetilde{Q}_a = Q^-_{a-1} Q^+_{a+1} - Q^+_{a-1} Q^-_{a+1}$$

- Changes the nature of neighbouring nodes $\otimes \longleftrightarrow \bigcirc$ and the signs of off-diagonal elements in the Cartan matrix

- Dualized node non-momentum carrying $\Longrightarrow$ Dynkin labels unchanged

- Dualized node momentum carrying $\Longrightarrow$ Dynkin labels change on neighbouring nodes

Transformation of super Gaudin determinant $\mathbb{D}$

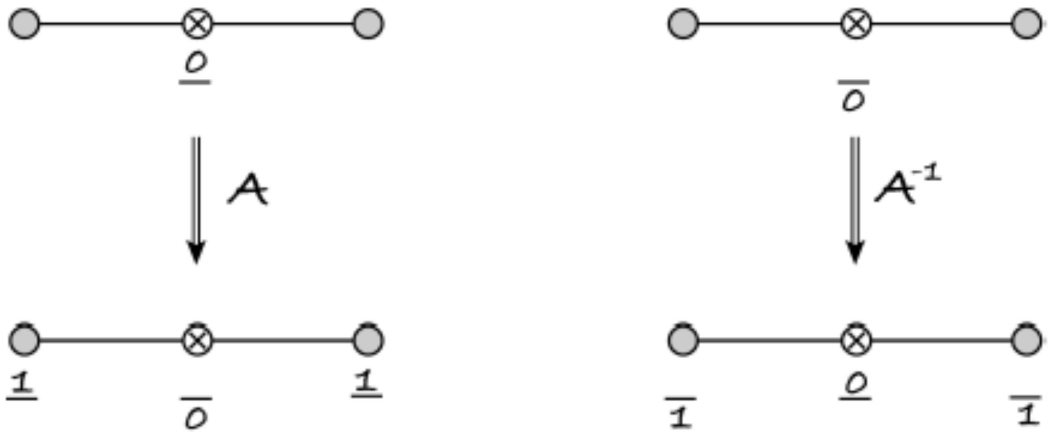
Fermionic duality after node a: $Q_a \to \widetilde{Q}_a$

$$Q_a(0)\mathbb{D} = Q_{a-1}(i/2)\, Q_{a+1}(i/2)\, \frac{\widetilde{\mathbb{D}}}{\widetilde{Q}_a(0)}$$

Found numerically
Analytical proof wanted

C.K., Müller, Zarembo '20

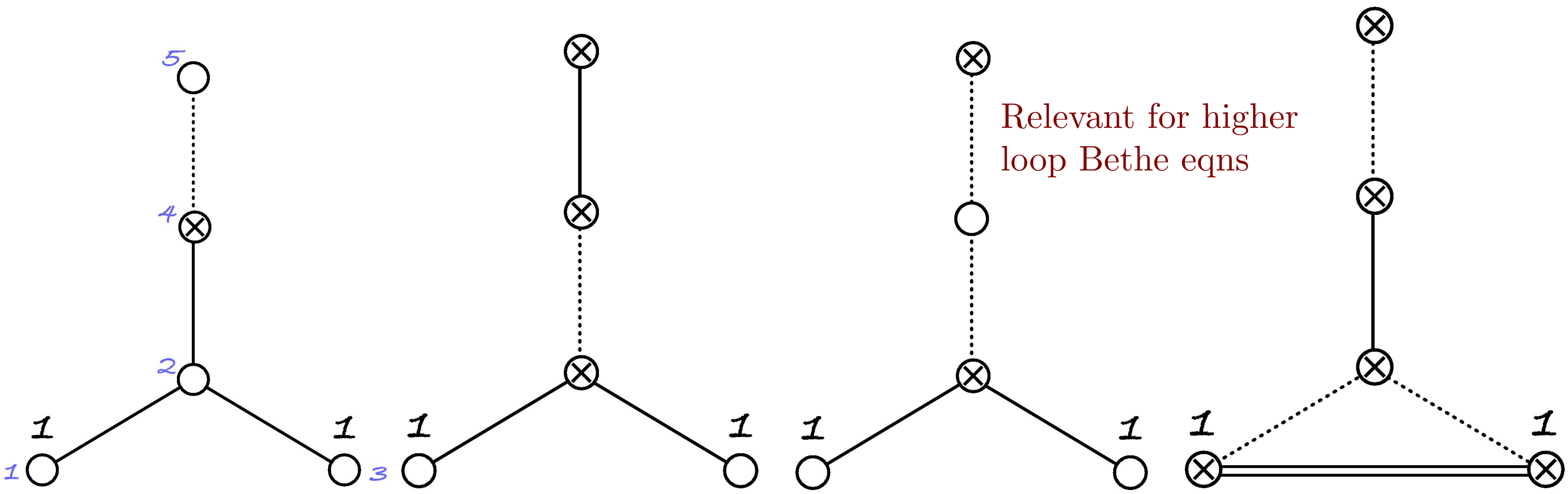
OBS: Covariance of overlap formula which involves $Q_a(0)\mathbb{D}$ or $\widetilde{\mathbb{D}}/\widetilde{Q}_a(0)$



Covariance of overlap formulas very constraining

The full Osp(6|4) spin chain of ABJM theory

Possible Dynkin diagrams



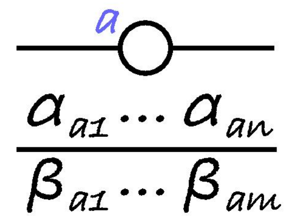
All connected via fermionic dualities

Idea: Determine the complete overlap formula by requiring covariance under fermionic duality

Fixing overlap by covariance requirement

Assume factorized formula (possibly a sum of such terms)

$$C = \sqrt{\prod_n \frac{\prod_j Q_n(i\alpha_{aj}/2)}{\prod_k Q_n(i\beta_{ak}/2)} \mathbb{D}}$$



Fermionic duality transformation after node a

Compatible with all data

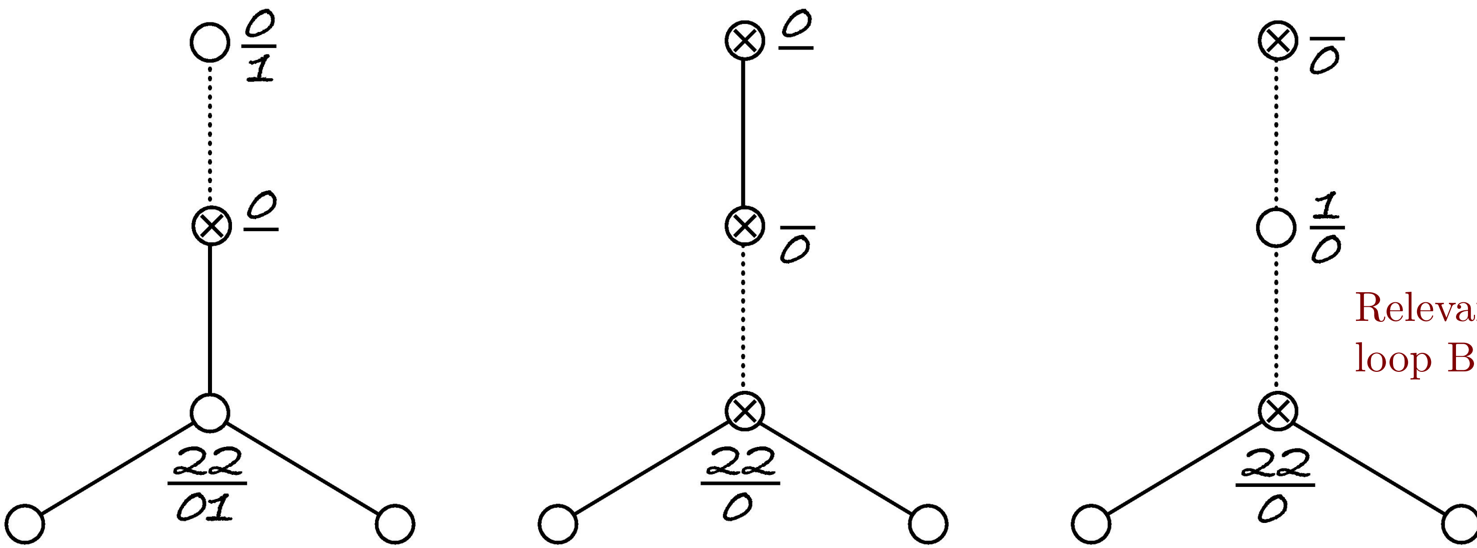
$$Q_a(0)\ \mathbb{D} = \prod_{\text{b:neigbour}} Q_b(i/2)\ \frac{\widetilde{\mathbb{D}}}{\widetilde{Q}_a(0)}$$

Shown numerically, holds semi-on-shell

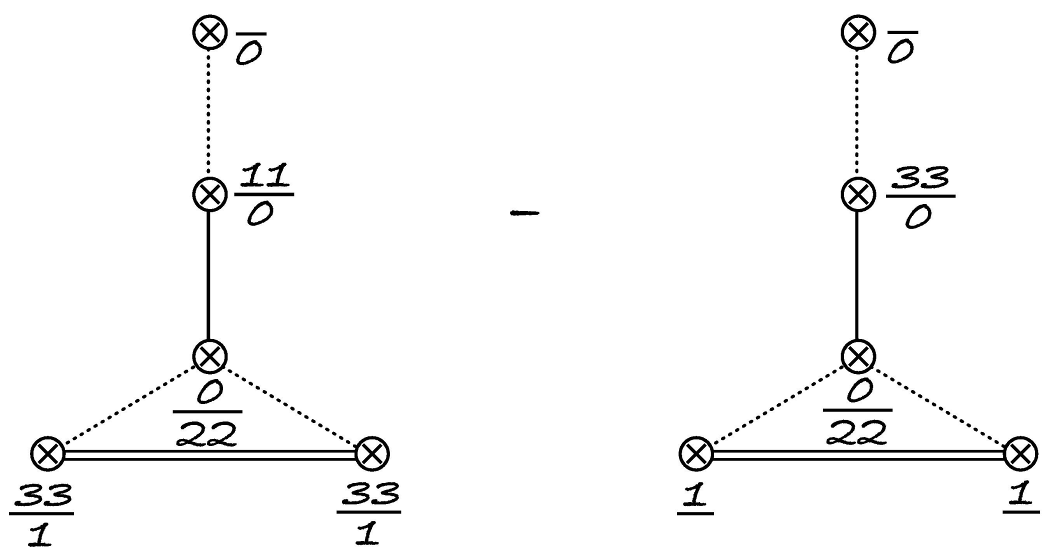
C.K., Müller & Zarembo '21, C.K., Vu & Zarembo '21,

Overlap formula in different gradings

C.K., Vu
& Zarembo '21,



Relevant for higher loop Bethe eqns



Future Directions

- Proof of the duality transformation formula for $\mathbb{D}$
- Bootstrap the formula to higher loop orders as has been done for $\mathcal{N} = 4$ SYM.

  Buhl-Mortensen, de Leeuw, Ipsen, C.K. & Wilhelm`17, Bajnok & Gombor `20, Komatsu & Wang `20
- Derive the TBA for overlaps (Finite size effects)
- Other integrable defect set-ups (Coulomb branch of $\mathcal{N} = 4$ SYM, co-dimension 2 defects, defects in $\mathrm{AdS}_3/\mathrm{CFT}_2$.)

Thank you !