Correlation functions & Functional Separation of Variables

Paul Ryan

Kings College, London

2202.01591 N. Gromov, N. Primi, P.R

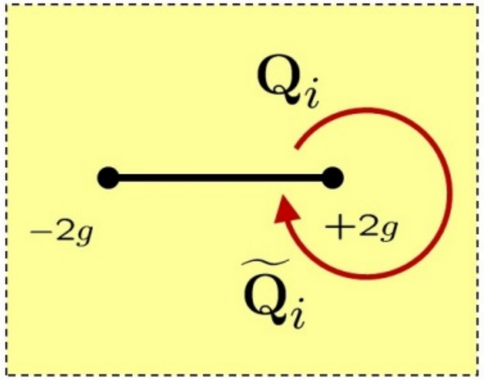
+ earlier related work with F. Levkovich-Maslyuk + D. Volin

Quantum Spectral Curve

Exact solution of spectral problem of planar N=4 SYM

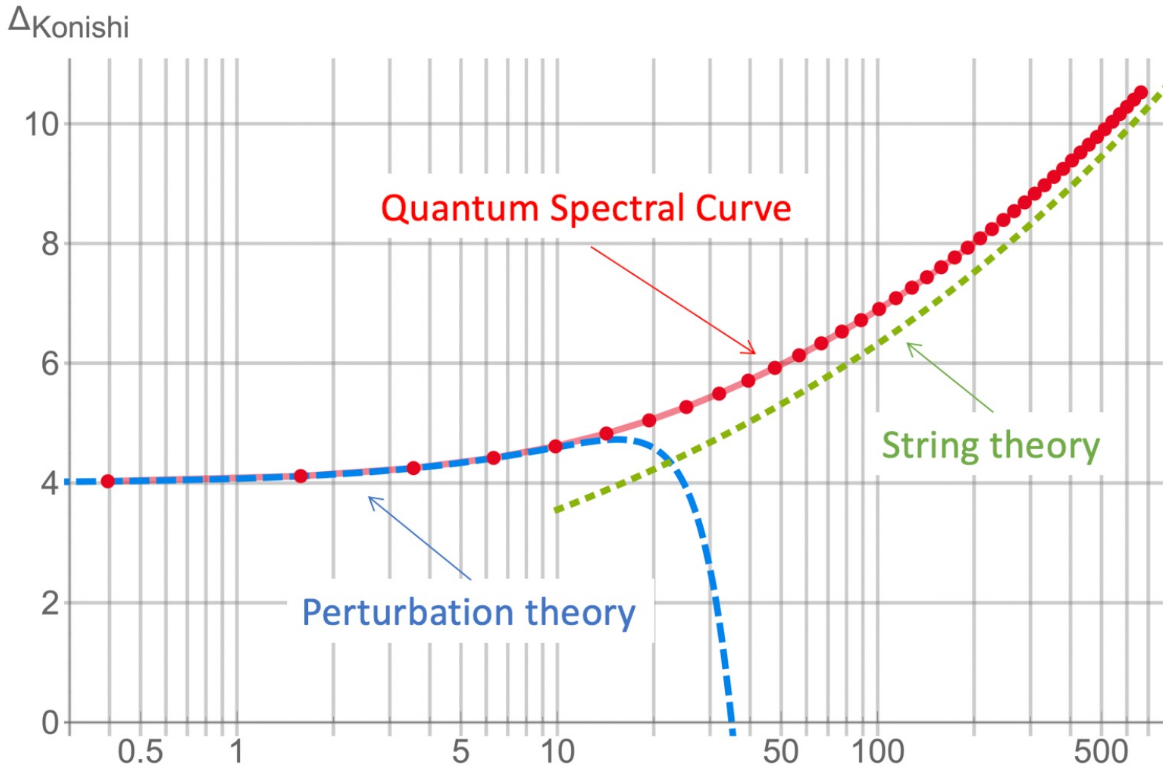
[Gromov, Kazakov, Leurent, Volin]

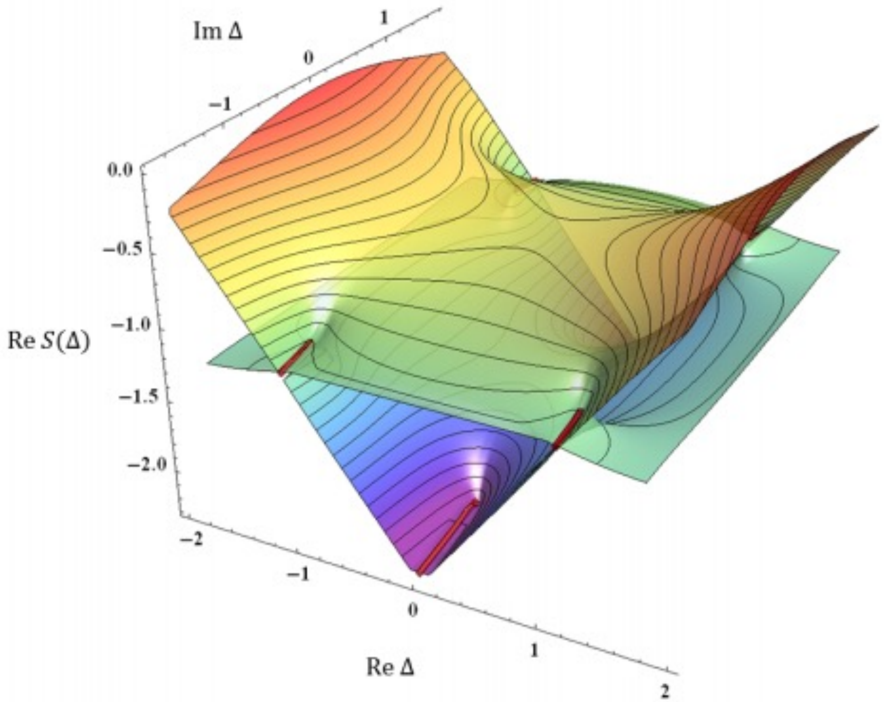
Functional relations on Q(u) + analytic requirements



$$g = \frac{\sqrt{\lambda}}{4\pi}$$

$$Q(u) \sim u^{\Delta}$$





N=4 SYM is a conformal field theory

Scaling dimensions of local operators not enough – need three point structure constants

- QSC Q-functions = building blocks of exact wave-functions
  Should know about correlation functions

- Recently realised via hexagons [Basso, Georgoudis, Sueiro]

- Difficult to use in practice – infinite sum of an infinite number of integrals of Q-functions

- However, there are regimes where correlation functions can be expressed in terms of a finite number of Q-functions.

Correlators & QSC

(See Alessandro's and Kolya's talks)

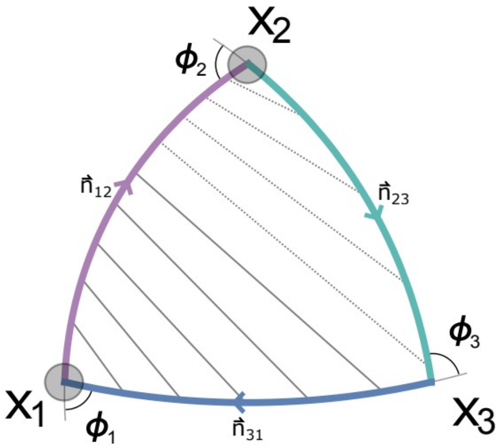
Bracket

$$\left(f \right) = \int_\gamma \mathrm{d}u\, \mu(u)\, f(u)$$

$$\left(Q_A Q_B \right) \propto \delta_{AB}$$

Cusps in ladders limit

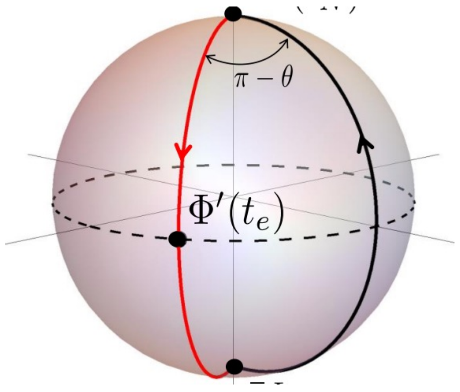
[Cavaglia, Gromov, Levkovich-Maslyuk]



$$\left(f \right) \sim \int_{c-i\infty}^{c+i\infty} \mathrm{d}u\, \frac{1}{2\pi i\, u} f(u), \quad c > 0$$

Near-BPS Wilson loops

[Komatsu, Giombi]



$$x + \frac{1}{x} = \frac{u}{g}$$

$$\left(f \right) \sim \oint \mathrm{d}u \left(x - \frac{1}{x} \right) \sinh(2\pi u)\, f(u)$$

$$\frac{\langle \Psi | \partial_\phi \left(2 \sin\phi\, \hat{D} \right) | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \pm \frac{\left(u\, Q^2 \right)}{\left(Q^2 \right)}$$

Correlators & QSC

(See Alessandro's and Kolya's talks)

Bracket

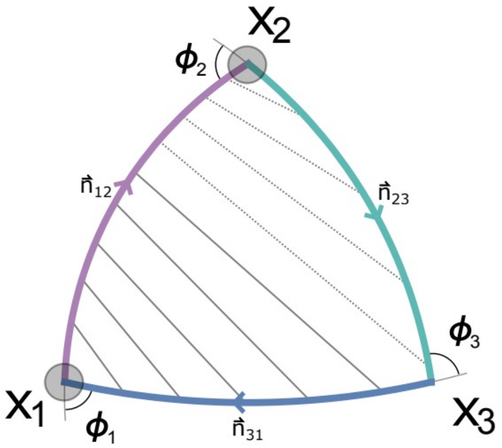
$$\left(\!\left(f \right)\!\right) = \int_\gamma du\, \mu(u)\, f(u)$$

$$\left(\!\left(Q_A Q_B \right)\!\right) \propto \delta_{AB}$$

Natural object in Functional Separation of Variables

Cusps in ladders limit

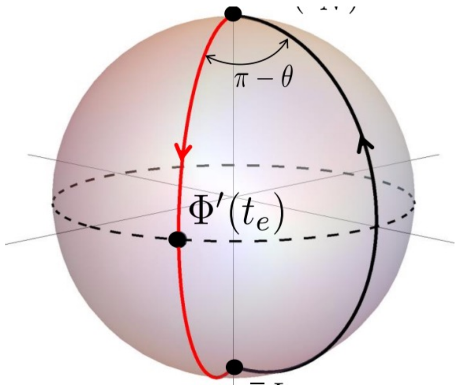
[Cavaglia, Gromov, Levkovich-Maslyuk]



$$\left(\!\left(f \right)\!\right) \sim \int_{c-i\infty}^{c+i\infty} du\, \frac{1}{2\pi i\, u} f(u), \quad c > 0$$

Near-BPS Wilson loops

[Komatsu, Giombi]



$$x + \frac{1}{x} = \frac{u}{g}$$

$$\left(\!\left(f \right)\!\right) \sim \oint du \left(x - \frac{1}{x} \right) \sinh(2\pi u)\, f(u)$$

$$\frac{\langle \Psi | \partial_\phi \left(2 \sin\phi\, \hat{D} \right) | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \pm \frac{\left(\!\left(u\, Q^2 \right)\!\right)}{\left(\!\left(Q^2 \right)\!\right)}$$

Separation of Variables (SoV)

[Sklyanin, ‘85]

Identify SoV basis $\langle \mathsf{x} |$

$$\Psi(\mathsf{x}) = \langle \mathsf{x} | \Psi \rangle \sim \prod_{\alpha=1}^{L} Q(\mathsf{x}_\alpha)$$

Extremely powerful in rank 1 models (SL(2))

- One-point functions [Lukyanov]
- Hexagons [Jiang, Komatsu, Kostov, Serban]
- Boundary overlaps [Caetano, Komatsu + Gombor, Pozsgay]

- Matrix elements [Smirnov]
- Basso-Dixon correlators [Derkachov, Kazakov, Olivucci]

[+ many, many others]

Only fully extended to higher-rank recently

[Cavaglia, Gromov, Levkovich-Maslyuk, Sizov + Liashyk, Slavnov + Maillet, Niccoli, Vignoli + PR, Volin + Derkachov, Olivucci]

See earlier work of Sklyanin and Smirnov

Lesson: operator-based SoV approach of Sklyanin should be supplemented with Functional SoV

Goal: Systematic study of correlators using Functional SoV

Punchline

This talk: mostly su(2)

- Studied SL(N) spin chains – finite / infinite – dim representations

- Identified special set of operators – ``Principal operators” $\hat{I}^{(r)}_{b,\beta}$ L x (N-1) x (N+1) operators

- Generate all observables + play especially nicely with SoV

- Compute all possible form-factors – determinants of Q-functions

Punchline

This talk: mostly su(2)

- Studied SL(N) spin chains – finite / infinite – dim representations

- Identified special set of operators – ``Principal operators” $\hat{I}^{(r)}_{b,\beta}$ L x (N-1) x (N+1) operators

- Generate all observables + play especially nicely with SoV

- Compute all possible form-factors – determinants of Q-functions

$$\left(f \right) = \int_\gamma du\, \mu(u)\, f(u)$$

$$\langle \Psi_A | \Psi_B \rangle = \frac{1}{\mathcal{N}} \begin{vmatrix} \left(Q_{12}^{A-} Q_1^B \right) & \left(Q_{12}^{A+} Q_1^B \right) \\ \left(Q_{13}^{A-} Q_1^B \right) & \left(Q_{13}^{A+} Q_1^B \right) \end{vmatrix} \qquad \langle \Psi_A | \hat{I}^{(1)}_{1,1} | \Psi_B \rangle = \frac{1}{\mathcal{N}} \begin{vmatrix} \left(Q_{12}^{A-}\, u\, Q_1^B \right) & \left(Q_{12}^{A+} Q_1^B \right) \\ \left(Q_{13}^{A-}\, u\, Q_1^B \right) & \left(Q_{13}^{A+} Q_1^B \right) \end{vmatrix}$$

Functional SoV

Functional SoV: SoV directly at level of functional equations (Baxter TQ equation)

Quantum spectral curve for su(2) XXX spin chain

$$\tau(u)Q(u) = \left(u + \frac{i}{2}\right)^L Q(u-i) + \left(u - \frac{i}{2}\right)^L Q(u+i)$$

Transfer matrix conserved charges

Baxter polynomial

Main idea:

Orthogonality

$$\langle \Psi_A | \Psi_B \rangle \propto \delta_{AB}$$

States completely fixed by Q-functions

How to reformulate orthogonality directly at the level of Q-functions?

Twisted su(2) spin chain: Spectrum and Baxter equation

Technically simpler to regulate (deform) the model a bit

Inhomogeneity $\theta_1, \ldots, \theta_L$ $\longrightarrow$ $Q_\theta(u) = \prod_{\alpha=1}^{L}(u - \theta_\alpha)$

Twist $\lambda_1,\ \lambda_2$ $\longrightarrow$ $\chi_1 = \lambda_1 + \lambda_2,\ \chi_2 = \lambda_1\lambda_2$ characters

Transfer matrix $\tau(u) = \chi_1 u^L + \sum_{\beta=1}^{L} u^{\beta-1} I_\beta$

$$Q_\theta\left(u + \frac{i}{2}\right) Q(u - i) - \tau(u) Q(u) + \chi_2\, Q_\theta\left(u - \frac{i}{2}\right) Q(u + i) = 0$$

Recast Baxter equation as a finite-difference operator [Cavaglia, Gromov, Levkovich-Maslyuk]

$$\mathcal{O} = Q_\theta\left(u+\frac{i}{2}\right)D^{-2} - \tau(u) + \chi_2\, Q_\theta\left(u-\frac{i}{2}\right)D^{2} \longrightarrow \mathcal{O}Q(u)=0$$

$$D^{\pm 1} f(u) = f\left(u \pm \frac{i}{2}\right)$$

Introduce the following brackets $\langle f(u)\rangle_\alpha = \oint \mathrm{d}u\, \mu_\alpha(u)\, f(u)$ $\qquad \mu_\alpha(u) = \prod_{\beta\neq\alpha} \frac{1-e^{2\pi(u-\theta_\beta+\frac{i}{2})}}{Q_\theta(u+\frac{i}{2})Q_\theta(u-\frac{i}{2})}$

Self-adjointness

$$\langle \chi_2^{-iu}\, f\, \mathcal{O}\, g\rangle_\alpha = \langle \chi_2^{-iu}\, g\, \mathcal{O}\, f\rangle_\alpha$$

$$\langle f\, \mathcal{O}\, Q\rangle_\alpha = 0 \qquad \langle \bar{Q}(u)\, \mathcal{O}\, f\rangle_\alpha = 0 \qquad \bar{Q}(u) = \chi_2^{-iu} Q(u)$$

Functional orthogonality

Consider two distinct states $|\Psi_A\rangle$ $|\Psi_B\rangle$

$\downarrow$ $\downarrow$

Corresponding Q-functions Q_A Q_B

$$\langle \bar{Q}_B (\mathcal{O}_A - \mathcal{O}_B) Q_A \rangle_\alpha = 0$$

Expand $\mathcal{O} = Q_\theta\left(u + \frac{i}{2}\right) D^{-2} - \tau(u) + \chi_2 Q_\theta\left(u - \frac{i}{2}\right) D^2$

$$\tau_A(u) = \chi_1 u^L + \sum_{\beta=1}^{L} u^{\beta-1} I_\beta^A$$

$$\sum_{\beta=1}^{L} \langle \bar{Q}_B u^{\beta-1} Q_A \rangle_\alpha (I_\beta^A - I_\beta^B) = 0$$

$$\langle f(u) \rangle_\alpha = \oint du\, \mu_\alpha(u)\, f(u)$$

Linear system must have vanishing det

$$\langle \Psi_B | \Psi_A \rangle = \frac{1}{\mathcal{N}} \det_{1 \le \alpha, \beta \le L} \langle \bar{Q}_B u^{\beta-1} Q_A \rangle_\alpha$$

[Cavaglia, Gromov, Levkovich-Maslyuk, PR, Volin]

Observables

Functional SoV leads to scalar product

What other observables can we extract?

Recall Hellmann-Feynman theorem

[Cavaglia, Gromov, Levkovich-Maslyuk, PR]

$$H|\Psi\rangle = E|\Psi\rangle$$

$$\langle\Psi|H = E\langle\Psi|$$

$$\langle\Psi|\partial_p H|\Psi\rangle = \partial_p E\langle\Psi|\Psi\rangle$$

Functional orthogonality allows to compute at fixed value of p!

Diagonal Form-factors

Functional orthogonality leads to not only scalar product but a whole host of diagonal form-factors!

p some parameter of the model, twist, inhomogeneity …

$$p \to p + \delta_p \qquad \langle \bar{Q}(\mathcal{O} + \delta\mathcal{O})(Q + \delta Q)\rangle_\alpha = 0 \longrightarrow \boxed{\langle \bar{Q}\, \partial_p \mathcal{O}\, Q\rangle_\alpha = 0}$$

$$\mathcal{O} = Q_\theta\left(u + \frac{i}{2}\right) D^{-2} - \tau(u) + \chi_2\, Q_\theta\left(u - \frac{i}{2}\right) D^2$$

$$\mathcal{O} = Y - \sum_{\beta=1}^{L} I_\beta u^{\beta-1} \qquad Y = Q_\theta\left(u + \frac{i}{2}\right) D^{-2} - \chi_1\, u^L + \chi_2\, Q_\theta\left(u - \frac{i}{2}\right) D^2$$

Inhomogeneous linear system for $\partial_p I_\beta$

Solvable by Cramer's rule!

Diagonal Form-factors

$$\partial_p I_\beta = \frac{[\beta \to \langle \bar{Q}\, \partial_p Y\, Q\rangle]}{\det\limits_{1\le\alpha,\beta\le L} \langle \bar{Q} u^{\beta-1} Q\rangle_\alpha}$$

$$\begin{vmatrix} \langle \bar{Q}\, Q\rangle_1 & \langle \bar{Q}\, u\, Q\rangle_1 \\ \langle \bar{Q}\, Q\rangle_2 & \langle \bar{Q}\, u\, Q\rangle_2 \end{vmatrix} \longrightarrow$$

$$[1 \to \langle \bar{Q}\, \partial_p Y\, Q\rangle] = \begin{vmatrix} \langle \bar{Q}\, \partial_p Y\, Q\rangle_1 & \langle \bar{Q}\, u\, Q\rangle_1 \\ \langle \bar{Q}\, \partial_p Y\, Q\rangle_2 & \langle \bar{Q}\, u\, Q\rangle_2 \end{vmatrix}$$

$$[2 \to \langle \bar{Q}\, \partial_p Y\, Q\rangle] = \begin{vmatrix} \langle \bar{Q}\, Q\rangle_1 & \langle \bar{Q}\, \partial_p Y\, Q\rangle_1 \\ \langle \bar{Q}\, Q\rangle_2 & \langle \bar{Q}\, \partial_p Y\, Q\rangle_2 \end{vmatrix}$$

$$\boxed{\langle \Psi | \partial_p \hat{I}_\beta | \Psi \rangle = \frac{1}{\mathcal{N}} [\beta \to \langle \bar{Q}\, \partial_p Y\, Q\rangle]}$$

Includes local operators + 3pt function with Lagrangian insertion in fishnets

Can we extend to off-diagonal correlators?

Yes! For appropriate choice of p – twist eigenvalues $\lambda_1,\ \lambda_2$

$$\langle\Psi|\partial_p \hat{I}_\beta|\Psi\rangle = \frac{1}{\mathcal{N}}[\beta \to \langle\bar{Q}\,\partial_p Y\,Q\rangle]$$

Survives a number of upgrades!

$$\langle\Psi_A|\partial^{n_1}_{\lambda_1}\partial^{n_2}_{\lambda_2}\hat{I}_\beta|\Psi_B\rangle = \frac{1}{\mathcal{N}}[\beta \to \langle\bar{Q}_A\,\partial^{n_1}_{\lambda_1}\partial^{n_2}_{\lambda_2}Y\,Q_B\rangle]$$

Can do even better! $\langle\Psi_A|$ and $|\Psi_B\rangle$ need not even be Hamiltonian eigenstates!

More general class of states - ``factorizable" states   Incl. off-shell Bethe states

Generate full algebra of observables!

Functional can only take us so far…

… we need some operatorial input!

Spin chain – algebraic construction

$$E_{12} = S^+, \quad E_{21} = S^-$$

$$E_{11} = \frac{1}{2} + S_z, \quad E_{22} = \frac{1}{2} - S_z$$

Lax operator $\mathcal{L}_{ij}(u) = u\,\delta_{ij} + iE_{ji}$

Monodromy matrix $T_{ij}(u) = \sum_{i_1 \dots i_{L-1}} \mathcal{L}_{ii_1}(u-\theta_1) \otimes \cdots \otimes \mathcal{L}_{i_{L-1}j}(u-\theta_L)$

Yangian algebra $-i(u-v)[T_{jk}(u), T_{lm}(v)] = T_{lk}(v)T_{jm}(u) - T_{lk}(u)T_{jm}(v)$

Transfer matrix $t(u) = \sum_{ij} T_{ij}(u) G_{ji}$

Generates integrals of motion $[t(u), t(v)] = 0$

What's a good twist?

Natural guess - diagonal

$$g = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

GL(2) covariance lets us choose any twist we like with the same eigenvalues

Much more convenient

$$G = \begin{pmatrix} \chi_1 & -\chi_2 \\ 1 & 0 \end{pmatrix}$$

$$\operatorname{tr} G = \chi_1 = \lambda_1 + \lambda_2$$

$$\det G = \chi_2 = \lambda_1 \lambda_2$$

$$t(u) = T_{12}(u) + \chi_1 T_{11}(u) - \chi_2 T_{21}(u)$$

SoV bases independent of twist!

[Ryan, Volin]

Serve to factorise wave functions of different Hamiltonians!

[Gromov, Levkovich-Maslyuk, Ryan]

Principal operators

$$t(u) = T_{12}(u) + \chi_1 T_{11}(u) - \chi_2 T_{21}(u)$$

$$t(u) = \chi_1 u^L + \sum_{\beta=1}^{L} \hat{I}_\beta u^{\beta-1}$$

Generate remaining operator $T_{22}(u)$

Now integrals of motion admit character expansion

$$\hat{I}_\beta \longrightarrow \hat{I}_\beta^{(0)} + \chi_1 \hat{I}_\beta^{(1)} + \chi_2 \hat{I}_\beta^{(2)}$$

$$t(u) \longrightarrow \mathbb{P}_0(u) + \chi_1 \mathbb{P}_1(u) + \chi_2 \mathbb{P}_2(u)$$

$\mathbb{P}_r(u)$ - Principal operators

We will compute

$$\langle \Psi_A | \mathbb{P}_r(u) | \Psi_B \rangle$$

Starting point is the scalar product

$$\langle f(u)\rangle_\alpha = \oint \mathrm{d}u\, \mu_\alpha(u)\, f(u)$$

$$\langle \Psi_B | \Psi_A \rangle = \frac{1}{\mathcal{N}} \det_{1\le\alpha,\beta\le L} \langle \bar{Q}_B u^{\beta-1} Q_A \rangle_\alpha$$

Now consider the trivial equality

$$\frac{1}{\mathcal{N}}[\beta' \to \langle \bar{Q}_B \mathcal{O}_A Q_A \rangle] = 0$$

$$Y = Q_\theta\left(u+\frac{i}{2}\right) D^{-2} - \chi_1 u^L + \chi_2 Q_\theta\left(u-\frac{i}{2}\right) D^2$$

and expand

$$\langle \Psi_B | \Psi_A \rangle I^A_{\beta'} = -\frac{1}{\mathcal{N}}[\beta' \to \langle \bar{Q}_B Y Q_A \rangle]$$

Matrix element of integral of motion

$$\hat{I}_{\beta'} |\Psi_A\rangle = I^A_{\beta'} |\Psi_A\rangle$$

$$\langle \Psi_B | \hat{I}_{\beta'} | \Psi_A \rangle = -\frac{1}{\mathcal{N}}[\beta' \to \langle \bar{Q}_B Y Q_A \rangle]$$

$$\langle\Psi_B|\hat{I}_{\beta'}|\Psi_A\rangle = -\frac{1}{\mathcal{N}}[\beta' \to \langle\bar{Q}_B Y Q_A\rangle]$$

Principal operators

$$\hat{I}_\beta \longrightarrow \hat{I}^{(0)}_\beta + \chi_1 \hat{I}^{(1)}_\beta + \chi_2 \hat{I}^{(2)}_\beta$$

$$Y = Q_\theta\left(u+\frac{i}{2}\right)D^{-2} - \chi_1\, u^L + \chi_2\, Q_\theta\left(u-\frac{i}{2}\right)D^2$$

$$Y = \mathcal{O}_{(1)} + \chi_1\,\mathcal{O}_{(1)} + \chi_2\,\mathcal{O}_{(2)}$$

Now lhs and rhs linear in characters

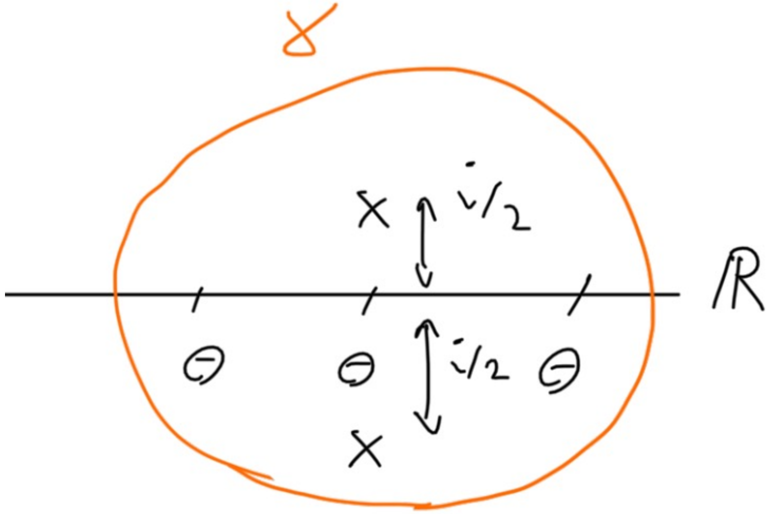
Character projection!

$$\langle\Psi_B|\hat{I}^{(r)}_{\beta'}|\Psi_A\rangle = -\frac{1}{\mathcal{N}}[\beta' \to \langle\bar{Q}_B\mathcal{O}_{(r)}Q_A\rangle]$$

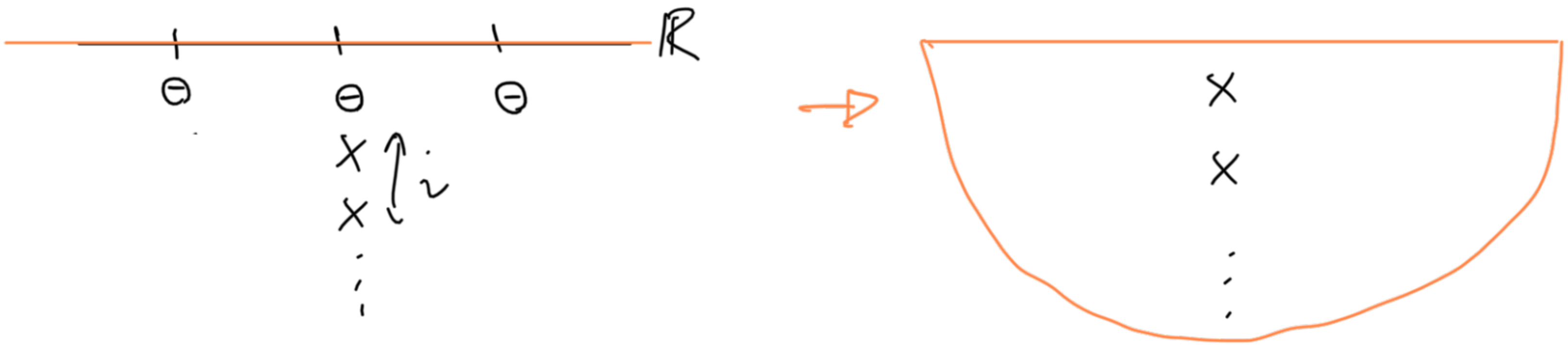
Caveat: Twist (character) dependence in Q-functions and eigenstates.
Can be made rigorous using twist-independence of SoV basis.

Upgrading: non-compact spin -s

$$\langle f(u)\rangle_\alpha = \oint \mathrm{d}u\, \mu_\alpha(u)\, f(u) \qquad \mu_\alpha(u) = \prod_{\beta\neq\alpha} \frac{1-e^{2\pi(u-\theta_\beta+\frac{i}{2})}}{Q_\theta(u+\frac{i}{2})Q_\theta(u-\frac{i}{2})}$$



$$\langle f(u)\rangle_\alpha = \int_{-\infty}^{+\infty} \mathrm{d}u\, \mu_\alpha(u)\, f(u) \qquad \mu_\alpha = \frac{\varepsilon}{1-e^{2\pi(u-\theta_\alpha-i\mathbf{s})}} \qquad \varepsilon = \prod_{\beta=1}^{L} \frac{\Gamma(\mathbf{s}-i(u-\theta_\beta))}{\Gamma(1-\mathbf{s}-i(u-\theta_\beta))}$$



Higher-rank SU(N) / SL(N) spin chains

Our approach extends to higher rank SU(N) and SL(N) with minimal effort

Just need to extend range of indices!!

Integrals of motion $\hat{I}_\beta \longrightarrow \hat{I}_{b,\beta}$

$$a, b \in \{1, 2, \ldots, N-1\}$$

Dual Q-functions $\bar{Q}(u) \longrightarrow \bar{Q}^a(u)$

$$D^{\pm 1} f(u) = f\left(u \pm \frac{i}{2}\right)$$

Scalar product

$$\langle \Psi_A | \Psi_B \rangle = \frac{1}{\mathcal{N}} \det_{(a,\alpha),(b,\beta)} \langle Q_B u^{\beta-1} D^{N-2b} \bar{Q}^a_A \rangle_\alpha$$

$$\langle f(u) \rangle_\alpha = \oint \mathrm{d}u \, \mu_\alpha(u) \, f(u)$$

Higher-rank SU(N) / SL(N) spin chains

Characters $\chi_1, \chi_2 \longrightarrow \chi_1, \ldots, \chi_N$

Transfer matrices $t(u) \longrightarrow t_a(u) \qquad a = 1, 2, \ldots, N-1$

Principal operators $\mathbb{P}_r(u) \longrightarrow \mathbb{P}_{a,r}(u)$

$$\mathbb{P}_{a,r}(u) = \delta_{ar} u^L + \sum_{\beta=1}^{L} \hat{I}_{a,\beta}^{(r)} u^{\beta-1}$$

$$\langle \Psi_A | \hat{I}_{b,\beta}^{(r)} | \Psi_B \rangle = \frac{1}{\mathcal{N}} [(b, \beta) \to \langle Q_B \mathcal{O}_{(r)} \bar{Q}_A^a \rangle]$$

$$\langle \Psi_A | \hat{I}^{(r)}_{b,\beta} | \Psi_B \rangle = \frac{1}{\mathcal{N}} [(b,\beta) \to \langle Q_B \mathcal{O}_{(r)} \bar{Q}^a_A \rangle]$$

Unlike SU(2) spin chain - need not one but two SoV bases! [See also Martin, Smirnov '15]

$$\Psi(\mathsf{x}) = \langle \mathsf{x} | \Psi \rangle = \prod_{\alpha=1}^{L} \prod_{a=1}^{N-1} Q_1(\mathsf{x}_{\alpha,a})$$

$$\Psi(\mathsf{y}) = \langle \Psi | \mathsf{y} \rangle = \prod_{\alpha=1}^{L} \det_{1 \le a,b \le N-1} \bar{Q}^a(\mathsf{y}_{\alpha,b})$$

[Gromov, Levkovich-Maslyuk, Ryan, Volin]

Determinant for form-factor can be expressed as a sum over wave functions

$$\langle \Psi_A | \hat{I}^{(r)}_{b,\beta} | \Psi_B \rangle = \sum_{\mathsf{x},\mathsf{y}} \Psi_A(\mathsf{y}) \langle \mathsf{y} | \hat{I}^{(r)}_{b,\beta} | \mathsf{x} \rangle \Psi_B(\mathsf{x})$$

Can extract matrix elements in SoV basis, hence can compute correlators with any number of insertions, only needing Q-functions of the two external states.

Comparison

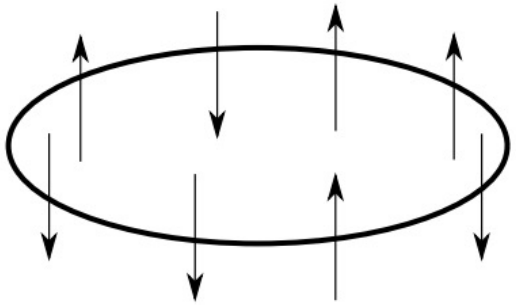
Traditional methods

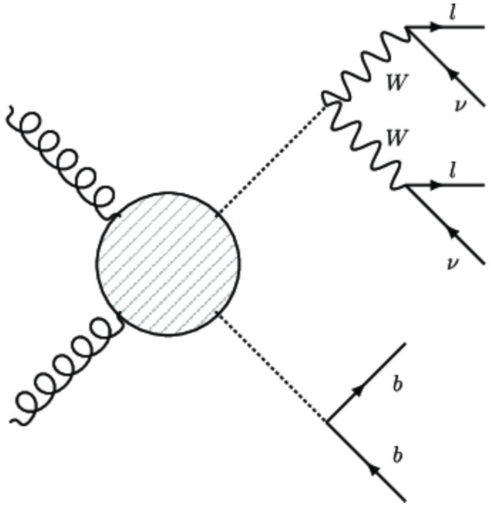
- Bethe roots
- Requires highest-weight state
- Okay for SL(2), much harder for higher rank
- Multiple insertions very difficult, need to know Bethe roots for every state

SoV methods

- Q-functions
- Doesn't care
- All SL(N) on same footing
- Simple in SoV basis

Spin chains are nice…





… but what about 4D QFT?

4D conformal fishnet theory

Still holds: $$\langle \Psi_A | \Psi_B \rangle = \frac{1}{\mathcal{N}} \det_{(a,\alpha),(b,\beta)} \langle Q_B u^{\beta-1} D^{N-2b} \bar{Q}_A^a \rangle_\alpha$$

Main difference – Q-functions have poles

Not necessarily clear which combinations to choose

Already interesting results for diagonal form factors [Cavaglia, Gromov, Levkovich-Maslyuk]

$$\partial_{\xi^2} \Delta = \frac{2i \int_| du \, \frac{p_a^\uparrow q_a^\downarrow}{(u-\theta)}}{\int_| du \, p_a^\uparrow \left(\mathcal{L}_1 q_a^{\downarrow++} + \mathcal{L}_2 q_a^{\downarrow--} + \mathcal{L}_4 \frac{u \, q_a^\downarrow}{(u-\theta)} \right)} \, ,$$

N=4 supersymmetric Yang-Mills

Still have the key relation: [Cavaglia, Gromov, Levkovich-Maslyuk]

$$\langle \bar{Q}_B(\mathcal{O}_A - \mathcal{O}_B)Q_A \rangle_\alpha = 0$$

Main difference vs fishnets: N=4 SYM dual to sigma model

infinite number independent of integrals of motion!

Determinants of infinite size – should reduce to finite size for each fixed order in perturbation theory

Transfer matrix no longer polynomial – how to find a good basis of integrals of motion?

Summary

New approach to correlation functions in high-rank integrable systems

Based on Functional separation of variables (FSoV)

Identifies distinguished set of operators (principal) which generate all observables

Can compute Hamiltonian + SoV matrix elements – allows to consider correlators with any number of insertions!

Approach trivially extends to any rank!

Outlook 1 – What can FSoV do for you?

- (Limits of) Gaudin models / Conformal blocks (Volker's talk)
  Read off wave functions from functional scalar product?

- Boundary overlaps (Charlotte's + Shota's talk)
  FSoV gives scalar product as determinant in Q-functions
  Can be factorised into parts related to Gaudin matrix

  [Caetano, Komatsu
  + Cavaglia, Gromov, Levkovich-Maslyuk]

- ODE / IM correspondence (Davide's talk)
  FSoV on ODE side = correlators in IQFT?

- Dynamical spin chains (Elli's talk)
  FSoV for dynamical spin chains = correlators in 4D N=2 SCFTs

Outlook 2

- Supersymmetry SL(M|N) – Hubbard model as testing ground for N=4 SYM?
  QSC known [Cavaglia, Cornagliotto, Mattelliano, Tateo + Ekhammar, Volin]

- Comparison with recent hexagon proposal for short operators [Basso, Georgoudis, Sueiro]

- Other algebras so(2r), etc
  Spin chain QSC extensively studied recently [Ferrando, Frassek, Kazakov + Ekhammar, Shu, Volin]

- Interpretation of principal operators in fishnets + compute correlations functions
  [ongoing with Cavaglia, Gromov, Levkovich-Maslyuk, PR]

- Open boundary conditions – dual to cusped Wilson line in N=4 SYM in ladders limit
  [ongoing with Gromov, Primi, PR]