Bethe ansatz
and quantum computing

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Prologue: Simulation of quantum systems with quantum computers

“Can you do it [simulate a quantum system] with a new kind of computer — a quantum computer?”
- Richard Feynman, 1981

“Feynman was on the right track when he suggested using quantum computers to solve problems in quantum physics and chemistry. That is still the most important application we can clearly foresee…”
- John Preskill, 2021

“This [the simulation of quantum physics and chemistry] was not only Feynman’s original vision, but it’s probably still, even after all these years, our best shot (at least that we know) at an economically useful application of quantum computers.”
- Scott Aaronson, 2022
Prologue: Simulation of quantum systems with quantum computers is a problem worth thinking about.

Today’s talk is about an effort to partially address this problem.
Outline

1. Heisenberg quantum spin chain

2. Brief review of Bethe ansatz

3. Bethe states on a quantum computer
   arXiv: 2103.13388  J.Van Dyke, G. Barron, N. Mayhall, E. Barnes, S. Economou
   arXiv: 2201.03021  W. Li, M. Okay, RN
   arXiv: 2109.05607  J.Van Dyke, E. Barnes, S. Economou, RN

4. Outlook
I. Heisenberg quantum spin chain
Deceptively simple model of magnetism (1928)

- L spin-1/2 spins arranged in a circle
- each spin interacts with its neighbor

Hamiltonian

\[ H = -\frac{1}{2} \sum_{n=0}^{L-1} (\vec{\sigma}_n \cdot \vec{\sigma}_{n+1} - I) \]

\[ \vec{\sigma}_L \equiv \vec{\sigma}_0 \]

periodic boundary conditions

\[ \sum_{j=1}^{3} \sigma_n^j \sigma_{n+1} \]

Pauli spin matrices

\[ \vec{\sigma} = (\sigma^x, \sigma^y, \sigma^z) \]

\[ \mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

can be realized experimentally; connections with CFT, AdS/CFT, …

Problem: find eigenvalues (\( E \)) and eigenvectors (\( \psi \)) of \( H \)

\[ H \psi = E \psi \]

Brute force doesn’t go very far…
2. Brief review of Bethe ansatz
Coordinate Bethe ansatz (1931)

Eigenvectors are multi-particle (“magnon”) states

ground (0-particle) state:

\[ |\psi_0\rangle = \left( \begin{array}{c} 1 \\ 0 \end{array} \right)^\otimes L = |\uparrow \cdots \uparrow\rangle \]

\[ H|\psi_0\rangle = 0 \]

1-particle state:

\[ |\psi(k)\rangle = \sum_{x=0}^{L-1} e^{ikx} |\uparrow \cdots \uparrow\rangle \]

\[ H|\psi(k)\rangle = e(k)|\psi(k)\rangle \]

\[ e(k) = 4 \sin^2\left(\frac{k}{2}\right) \]

I-particle energy

provided

\[ e^{ikL} = 1 \]
2-particle state:

\[ |\psi(k_0, k_1)\rangle = \sum_{0 \leq x_0 < x_1 \leq L-1} \left[ A^{(01)} e^{i(k_0 x_0 + k_1 x_1)} - A^{(10)} e^{i(k_1 x_0 + k_0 x_1)} \right] |\uparrow \cdots \downarrow \cdots \uparrow\rangle \]

\[ A^{(10)} = -S(k_1, k_0) A^{(01)} \]

\[ S(k_1, k_0) = \frac{u(k_1) - u(k_0) + i}{u(k_1) - u(k_0) - i} \]

\[ u(k) = \frac{1}{2} \cot \left( \frac{k}{2} \right) \]

\[ H |\psi(k_0, k_1)\rangle = E |\psi(k_0, k_1)\rangle \]

\[ E = e(k_0) + e(k_1) \]

provided \[ \begin{cases} e^{ik_0 L} = S(k_0, k_1) \\ e^{ik_1 L} = S(k_1, k_0) \end{cases} \]
M-particle state:

\[ |\psi(k_0, \ldots, k_{M-1})\rangle = \sum_{P \in \text{Perm}} \varepsilon_P A^{(P)} |k_{P(0)}, \ldots, k_{P(M-1)}\rangle \]

M! terms Bethe state

\[ \varepsilon_P = \pm 1 \]

signature of \( P \)

\[ |k_0, \ldots, k_{M-1}\rangle = \sum_{0 \leq x_0 < \ldots < x_{M-1} \leq L-1} e^{i(k_0x_0 + \ldots + k_{M-1}x_{M-1})} | \uparrow \ldots \downarrow \ldots \uparrow \rangle \]

\[ x_0 \ x_{M-1} \]

\[ A^{(...ij...)} = -S(k_i, k_j)A^{(...ji...)} \Rightarrow A^{(P)} = A^{(I)} \times \text{product of 2-particle S-matrices} \]

\[ H|\psi(k_0, \ldots, k_{M-1})\rangle = E|\psi(k_0, \ldots, k_{M-1})\rangle \]

\[ E = \sum_{j=0}^{M-1} e(k_j) \]

“exact solution”!

provided

\[ e^{ik_j L} = \prod_{l=0; l \neq j}^{M-1} S(k_j, k_l), \quad j = 0, \ldots, M - 1 \]

Bethe equations
SU(2) symmetry $\Rightarrow$ degeneracy $L - 2M + 1$

\[
M \leq \frac{L}{2}
\]

Example: $L = 4$ \quad $M = 0, 1, 2$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$k_j$</th>
<th>$E$</th>
<th>degeneracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{1}{2}\pi$</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{3}{2}\pi$</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>$\pi$</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>$\pm i\infty$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>$\pm \frac{2}{3}\pi$</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

Total: $16 = 2^4 = 2^L$ \quad “complete” \quad √
Remarks

- Bethe’s solution is possible because the model is *quantum integrable*
  
  There are infinitely-many such models, corresponding to solutions of Yang-Baxter equation

- Heisenberg model is prototype

- Bethe equations (BE) are generally hard to solve
  
  complete set of solutions of BE up to \( L=14 \)

- Significant effort is generally still required to explicitly compute quantities of physical interest

  “exact solution” \(\xrightarrow{\text{}}\) “exact half-solution” 😊

Can quantum computers help?
3. Bethe states on a quantum computer
“Quantum computer”:

- device with \( n \) “qubits”: 2-state systems

- can initialize each qubit
  \[ |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

- can perform unitary transformations on qubits (decomposed into 1-qubit & 2-qubit unitary “gates”)

- can perform projective measurements
  \[ |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]
  “computational basis”

Currently: \( n \sim 10^2 \) IBM, Google, … “noisy” - make many errors!

Dream: \( n \sim 10^4 \) fault-tolerant

In the meantime, can test algorithms (“quantum circuits”) using noiseless simulators

Here: IBM Qiskit simulators \( n \sim 30 \)
idea: • solve Bethe equations classically for real \( \{ k_0, \ldots, k_{M-1} \} \) (easy!) (classically hard!)

• use quantum computer to construct exact eigenstates using coordinate Bethe ansatz

Schematic diagram of quantum circuit:

Prepare “system” qubits in Dicke state

\[
\frac{1}{\sqrt{\binom{L}{M}}} \sum_{0 \leq x_0 < \ldots < x_{M-1} \leq L-1} |x_0 \downarrow \cdots \downarrow x_{M-1} \uparrow\rangle
\]

[Van Dyke et al 2021]

[Bärtschi, Eidenbenz 2019; Mukherjee et al 2020, …]
idea: • solve Bethe equations classically for real \( \{k_0, \ldots, k_{M-1}\} \) (easy!)  
• use quantum computer to construct exact eigenstates using coordinate Bethe ansatz (classically hard!)

Schematic diagram of quantum circuit:

```
<table>
<thead>
<tr>
<th>System qubits</th>
<th>Permutation-label qubits</th>
<th>Faucet qubits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prepare Dicke state</td>
<td></td>
</tr>
<tr>
<td>0^L \rightarrow^L</td>
<td>\rightarrow^i</td>
<td>\rightarrow^0</td>
</tr>
<tr>
<td>0^M^2 \rightarrow^M^2</td>
<td>\rightarrow</td>
<td>\rightarrow</td>
</tr>
<tr>
<td>0^M \rightarrow^M</td>
<td>\rightarrow</td>
<td>\rightarrow</td>
</tr>
</tbody>
</table>
```

Prepare "permutation-label" qubits in state

\[
\frac{1}{\sqrt{M!}} \sum_{P} \varepsilon_P A_P |P\rangle
\]

|P\rangle encode permutation P
idea: • solve Bethe equations classically for real $\{k_0, \ldots, k_{M-1}\}$ (easy!)

• use quantum computer to construct exact eigenstates using coordinate Bethe ansatz (classically hard!)

Schematic diagram of quantum circuit:

```
“system” qubits |0⟩⊗L  →  ^L Prepare Dicke state

“permutation-label” qubits |0⟩⊗M²  →  ^M² Create permutation labels, and apply phases $\varepsilon_P A_P$

“faucet” qubits |0⟩⊗M  →  ^M Apply phases $e^{i \sum_{j=0}^{M-1} k_{P(j)} x_j}$ using “faucet” method

Reverse permutation labels (without phases)

Apply phases $e^{i \sum_j k_{P(j)} x_j}$ to “permutation-label” qubits
```
idea: • solve Bethe equations classically for real \( \{ k_0, \ldots, k_{M-1} \} \) (easy!)

• use quantum computer to construct exact eigenstates using coordinate Bethe ansatz (classically hard!)

Schematic diagram of quantum circuit:

```
|“system” qubits | \( |0\rangle^\otimes L \) Prepare Dicke state |
|“permutation-label” qubits | \( |0\rangle^\otimes M^2 \) Create permutation labels, and apply phases \( e^{i \sum_{j=0}^{M-1} k_j x_j} \) using “faucet” method |
|“faucet” qubits | \( |0\rangle^\otimes M \) Reverse permutation labels (without phases) |
```

Reverse of step 2, but without phases
idea: • solve Bethe equations classically for real \( \{ k_0, \ldots, k_{M-1} \} \) (easy!)

• use quantum computer to construct exact eigenstates using coordinate Bethe ansatz (classically hard!)

Schematic diagram of quantum circuit:

- **“system” qubits**
  \[ |0\rangle^\otimes L \]
  - Prepare Dicke state

- **“permutation-label” qubits**
  \[ |0\rangle^\otimes M^2 \]
  - Create permutation labels, and apply phases \( \varepsilon_{P} A_{P} \)

- **“faucet” qubits**
  \[ |0\rangle^\otimes M \]
  - Apply phases \( e^{i \sum_{j=0}^{M-1} k_{P(j)} x_j} \) using “faucet” method

- **Output**
  \[ |\tilde{0}\rangle \]
  - Reverse permutation labels (without phases)

\[ |\Psi\rangle = \alpha |00\cdots\rangle |\tilde{\phi}\rangle + \ldots \]

\(|\tilde{\phi}\rangle \) normalized Bethe state
idea: • solve Bethe equations classically for real \( \{k_0, \ldots, k_{M-1}\} \) (easy!)

• use quantum computer to construct exact eigenstates using coordinate Bethe ansatz (classically hard!)

Schematic diagram of quantum circuit:

- **“system” qubits**: Prepare Dicke state
- **“permutation-label” qubits**: Create permutation labels, and apply phases \( e^{i \sum_{j=0}^{M-1} k_p x_j} \) using “faucet” method
- **“faucet” qubits**: Measure “permutation-label” qubits

\[
|\Psi\rangle = \alpha|00\ldots0\rangle|\tilde{\phi}\rangle + \ldots
\]
idea: 
- solve Bethe equations classically for real \( \{k_0, \ldots, k_{M-1}\} \) (easy!) 
- use quantum computer to construct exact eigenstates using coordinate Bethe ansatz (classically hard!)

Schematic diagram of quantum circuit:

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<td>apply phases using “faucet” method</td>
</tr>
</tbody>
</table>
```

Measure “permutation-label” qubits

If \(|00\ldots0\rangle\) then “system” qubits are in Bethe state \(|\tilde{\phi}\rangle\) Success! 😊

Algorithm is probabilistic

“success probability” \( \equiv |\alpha|^2 \) Value?

\[ |\Psi\rangle = \alpha |00\ldots0\rangle |\tilde{\phi}\rangle + \ldots \]
$|\alpha|^2 = \frac{(L - M)!}{L! M!} \det G$

G: “Gaudin matrix” \hspace{1cm} $M \times M$

$G_{m,n} = \delta_{m,n} \left[ L - \sum_l \frac{4 (1 - \cos k_l)}{(n,l)(l,n)} \right] + \frac{4 (1 - \cos k_m)}{(n,m)(m,n)}$

$(j,l) := 2 - e^{-ik_j} - e^{ik_l}$

Large $L$ (fixed $M$):

$$\lim_{L \to \infty} |\alpha|^2 = \frac{1}{M!}$$
Spin-spin correlation functions

\[ \langle \psi_0 | \sigma_0^z \sigma_l^z | \psi_0 \rangle , \quad l = 1, 2, \ldots, \frac{L}{2} \]

\[ |\psi_0\rangle \text{ normalized } \text{anti-ferromagnetic} \text{ ground state (Hamiltonian } -H \text{) } \]

\[ M = \frac{L}{2} \text{ real Bethe roots} \]

To compute an expectation value on a quantum computer:

• perform multiple “shots” How many? For error \( \epsilon \), need \( N \leq N_{\text{max}} \), need

\[ N_{\text{max}} := \frac{1}{|\alpha|^2 \epsilon^2} \]

• usually, can use built-in functionality

Not here, since \( |\psi_0\rangle \) is produced probabilistically!

Can proceed by measuring all qubits, and then appropriately combining the corresponding probabilities

[Kitanine, Maillet, Terras 1999; Kitanine, Maillet, Slavnov, Terras 2002; Lukyanov, Terras 2003; Caux, Maillet 2005, …]
\[ \langle \psi_0 | \sigma_0^z \sigma_l^z | \psi_0 \rangle, \quad l = 1, 2, \ldots, \frac{L}{2} \]

\[ M = \frac{L}{2} \]

<table>
<thead>
<tr>
<th>(l)</th>
<th>(L = 4)</th>
<th>(L = 6)</th>
<th>(L = 8)</th>
<th>(L = \infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\text{th})</td>
<td>(\text{exp})</td>
<td>(\text{th})</td>
<td>(\text{exp})</td>
</tr>
<tr>
<td></td>
<td>(</td>
<td>\alpha</td>
<td>^2 = 0.5) &amp; (N = 2 \times 10^4)</td>
<td>(</td>
</tr>
<tr>
<td>1</td>
<td>-0.666667 &amp; -0.6657 \pm 0.0076</td>
<td>-0.622839 &amp; -0.6230 \pm 0.0072</td>
<td>-0.608516 &amp; -0.6091 \pm 0.0077</td>
<td>-0.590863</td>
</tr>
<tr>
<td>2</td>
<td>0.333333 &amp; (0.3309 \pm 0.0090)</td>
<td>0.27735 &amp; 0.2777 \pm 0.0086</td>
<td>0.261037 &amp; 0.2602 \pm 0.0086</td>
<td>0.242719</td>
</tr>
<tr>
<td>3</td>
<td>- &amp; -</td>
<td>-0.309022 &amp; -0.3102 \pm 0.0090</td>
<td>-0.251937 &amp; -0.2519 \pm 0.0088</td>
<td>-0.200995</td>
</tr>
<tr>
<td>4</td>
<td>- &amp; -</td>
<td>- &amp; -</td>
<td>0.198831 &amp; 0.1988 \pm 0.0093</td>
<td>0.138611</td>
</tr>
</tbody>
</table>

**experimental (exp):** 100 trials of \(N\) shots on Qiskit qasm simulator (no noise), with \(\epsilon = 0.01\)

As \(L\) increases, \(|\alpha|^2\) decreases, so \(N\) increases \(N \approx \frac{1}{|\alpha|^2 \epsilon^2}\) standard deviations \(< \epsilon\) \(\checkmark\)

**theoretical (th):** Mathematica

Correlation functions for small $L$ values are feasible!

How about for larger $L$ values?

Success probability $|\alpha|^2$ decreases exponentially with $L$

For $L=40$:
$|\alpha|^2 \sim 5 \times 10^{-20} \implies N \sim 2 \times 10^{23}$  
not feasible ☹️

Nevertheless, it should be feasible to use this quantum circuit to study other states with small $M$ values, even for moderate $L$ values
Open spin chain (i.e. with boundaries) non-trivial generalization of formalism for closed spin chains

\[ N = 2 : \]

\[ |\psi(k_0, k_1)\rangle = \sum_{0 \leq x_0 < x_1 \leq L-1} \left[ A(k_0, k_1) e^{i(k_0 x_0 + k_1 x_1)} - A(-k_0, k_1) e^{i(-k_0 x_0 + k_1 x_1)} \right. \]

\[ - A(k_0, -k_1) e^{i(k_0 x_0 - k_1 x_1)} + A(-k_0, -k_1) e^{i(-k_0 x_0 - k_1 x_1)} \]

\[ - A(k_1, k_0) e^{i(k_1 x_0 + k_0 x_1)} + A(-k_1, k_0) e^{i(-k_1 x_0 + k_0 x_1)} \]

\[ + A(k_1, -k_0) e^{i(k_1 x_0 - k_0 x_1)} - A(-k_1, -k_0) e^{i(-k_1 x_0 - k_0 x_1)} \]

\[ | \uparrow \ldots \downarrow \ldots \downarrow \ldots \uparrow \rangle \]

\[ x_0 \quad x_1 \]

\[ |\psi(k_0, \ldots, k_{M-1})\rangle = \sum_{0 \leq x_1 < \ldots < x_{M-1} \leq L-1} \sum_{P} \varepsilon_{P} A(k_0, \ldots, k_{M-1}) e^{i(k_0 x_0 + \ldots + k_{M-1} x_{M-1})} | \uparrow \ldots \downarrow \ldots \downarrow \ldots \uparrow \rangle \]

\[ x_0 \quad x_{M-1} \]

all permutations and negations of \( \{k_0, \ldots, k_{M-1}\} \)
Open spin chain (i.e. with boundaries)

non-trivial generalization of formalism for closed spin chains

\[ M = 2 : \]
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|\psi(k_0, k_1)\rangle = \sum_{0 \leq x_0 < x_1 \leq L-1} \left[ A(k_0, k_1)e^{i(k_0x_0+k_1x_1)} - A(-k_0, k_1)e^{i(-k_0x_0+k_1x_1)} \right.
\]
\[
- A(k_0, -k_1)e^{i(k_0x_0-k_1x_1)} + A(-k_0, -k_1)e^{i(-k_0x_0-k_1x_1)}
\]
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- A(k_1, k_0)e^{i(k_1x_0+k_0x_1)} + A(-k_1, k_0)e^{i(-k_1x_0+k_0x_1)}
\]
\[
+ A(k_1, -k_0)e^{i(k_1x_0-k_0x_1)} - A(-k_1, -k_0)e^{i(-k_1x_0-k_0x_1)} \left| \uparrow \ldots \downarrow \ldots \downarrow \ldots \uparrow \right\rangle_{x_0 \ x_1}
\]

\[ H = -\frac{1}{2} \sum_{n=0}^{L-2} (\bar{\sigma}_n \cdot \bar{\sigma}_{n+1} - I) \]

[Van Dyke, Barnes, Economou, RN 2021]

\[ |\psi(k_0, \ldots, k_{M-1})\rangle = \sum_{0 \leq x_1 < \ldots < x_{M-1} \leq L-1} \sum_{P \in \mathcal{P}} \epsilon(P) A(k_0, \ldots, k_{M-1})e^{i(k_0x_0+\ldots+k_{M-1}x_{M-1})} \left| \uparrow \ldots \downarrow \ldots \downarrow \ldots \uparrow \right\rangle_{x_0 \ x_{M-1}} \]

all permutations and negations of \( \{k_0, \ldots, k_{M-1}\} \) changes sign at each such “mutation”

[Gaudin '71, Alcaraz et al.'87]
Open spin chain (i.e. with boundaries) 

non-trivial generalization of formalism for closed spin chains

\[ H = -\frac{1}{2} \sum_{n=0}^{L-2} (\vec{\sigma}_n \cdot \vec{\sigma}_{n+1} - I) \]

\[ \text{M} = 2 : \]

\[ |\psi(k_0, k_1)\rangle = \sum_{0 \leq x_0 < x_1 \leq L-1} \left[ A(k_0, k_1)e^{i(k_0 x_0 + k_1 x_1)} - A(-k_0, k_1)e^{i(-k_0 x_0 + k_1 x_1)} \right. \]

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\[ x_0 \quad x_{M-1} \]

depends on both bulk and boundary S-matrices
Open spin chain (i.e. with boundaries)

V. D. Van Dyke, B. Barnes, and E. Economou, Rev. New 2021

A non-trivial generalization of formalism for closed spin chains in

\[ H = -\frac{1}{2} \sum_{n=0}^{L-2} (\hat{\sigma}_n \cdot \hat{\sigma}_{n+1} - I) \]

[Van Dyke '71, Alcaraz et al. '87]

\[ |\psi(k_0, k_1)\rangle = \sum_{0 \leq x_0 < x_1 \leq L-1} \left[ A(k_0, k_1) e^{i(k_0 x_0 + k_1 x_1)} - A(-k_0, k_1) e^{i(-k_0 x_0 + k_1 x_1)} \right. \]

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\[ + A(k_1, -k_0) e^{i(k_1 x_0 - k_0 x_1)} - A(-k_1, -k_0) e^{i(-k_1 x_0 - k_0 x_1)} \] \[ \uparrow \Downarrow \cdots \Downarrow \cdots \uparrow \]

\[ x_0 \quad x_1 \]

\[ |\psi(k_0, \ldots, k_{M-1})\rangle = \sum_{0 \leq x_1 < \ldots < x_{M-1} \leq L-1} \sum_{P} \epsilon P A(k_0, \ldots, k_{M-1}) e^{i(k_0 x_0 + \ldots + k_{M-1} x_{M-1})} \uparrow \Downarrow \cdots \Downarrow \uparrow \]

\[ x_0 \quad x_{M-1} \]

Hamiltonian can include boundary terms \( h \sigma_0^z + h' \sigma_{L-1}^z \) and anisotropy

Formulated quantum circuit for preparing Bethe states with real Bethe roots

- \( M \) additional qubits for reflections 
  \[ \text{total # qubits} = L + M^2 + 2M \]
- probabilistic
4. Outlook
• Integrable models are attractive candidates for simulation on quantum computers (“half-solved”, benchmarking)

• Quantum circuits for preparing exact eigenstates of XXX models \[ \text{total # qubits} \sim L + M^2 \]

• *Probabilistic* success probability \( \sim \frac{1}{M!} \) \( \Rightarrow \) small \( M \) is feasible

• Large \( M \) ?

• Complex Bethe roots?

• Higher-rank models?

• Other approaches?

  • algebraic Bethe ansatz \( \text{arXiv: 2202.04673} \) Sopena, Gordon, García-Martín, Sierra, López

  • variational approaches (no Bethe ansatz) \( \text{arXiv: 2207.09994} \) Yu, Zhao, Wei

Thank you for your attention!