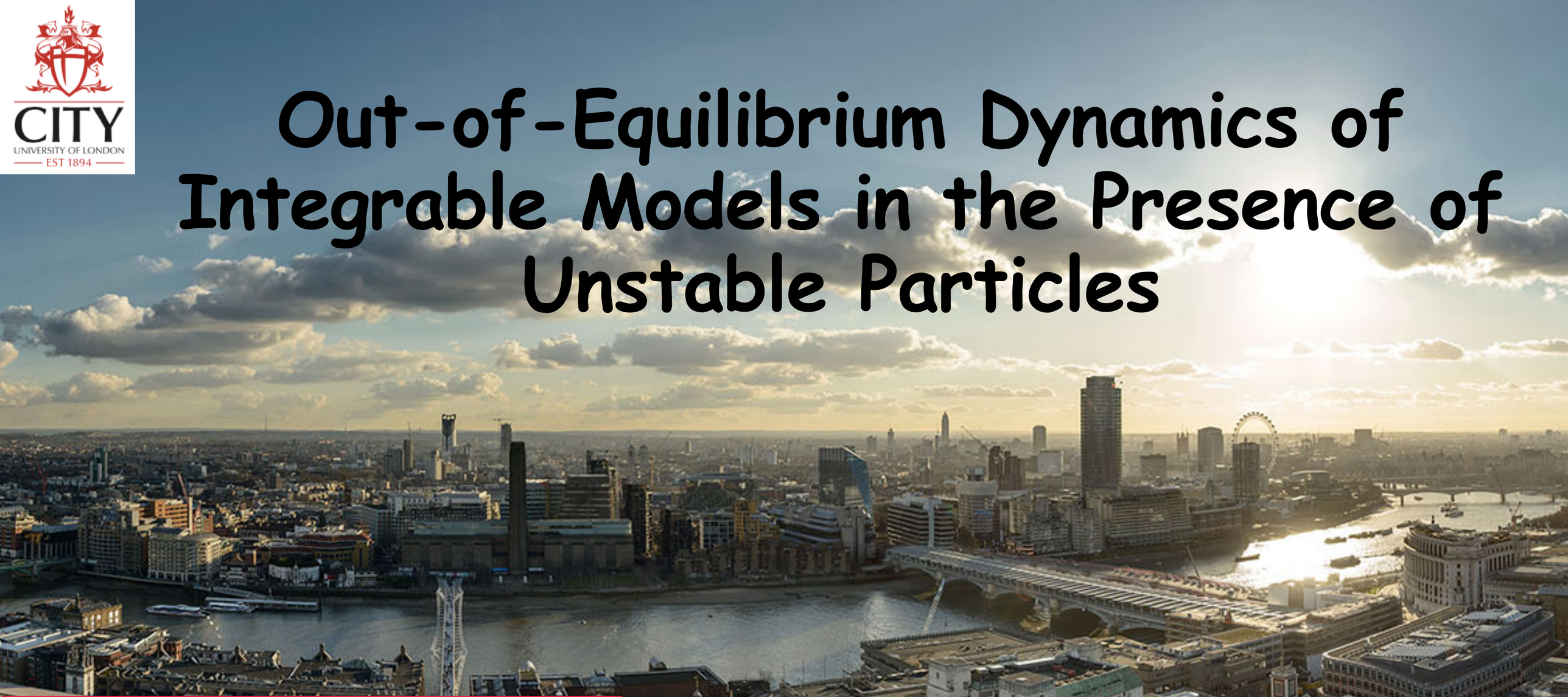
Out-of-Equilibrium Dynamics of Integrable Models in the Presence of Unstable Particles



Olalla Castro-Alvaredo
Department of Mathematics
City, University of London

KITP, Santa Barbara, CA
Talking Integrability: Spins, Fields and Strings August 29 – September 1, 2022.

1. Papers

The work I will present today contains ideas from four papers:

OC-A, Cecilia De Fazio, Benjamin Doyon and Francesco Ravanini
On the Hydrodynamics of Unstable Excitations
JHEP 09 (2020) 045; arXiv:2005.11266

OC-A, Cecilia De Fazio, Benjamin Doyon and Aleksandra A. Ziółkowska
Tails of Instability and Decay: a Hydrodynamic Perspective
SciPost Phys. **12**, 115 (2022); arXiv:2103.03735

OC-A, Cecilia De Fazio, Benjamin Doyon and Aleksandra A. Ziółkowska
Generalized Hydrodynamics of Particle Creation and Decay
JHEP **04**, 035 (2022); arXiv:2112.05462

David X. Horváth, Pasquale Calabrese and OC-A
Entanglement of Stationary States in the Presence of Unstable Quasiparticles
In Preparation...

Collaborators

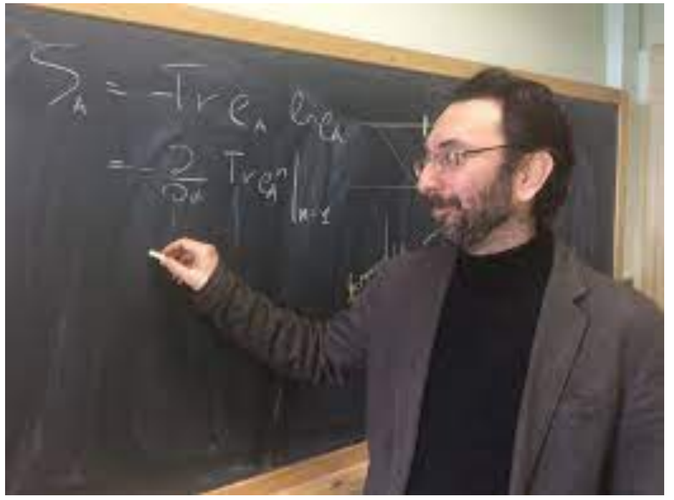
Cecilia

Aleksandra

David

Benjamin

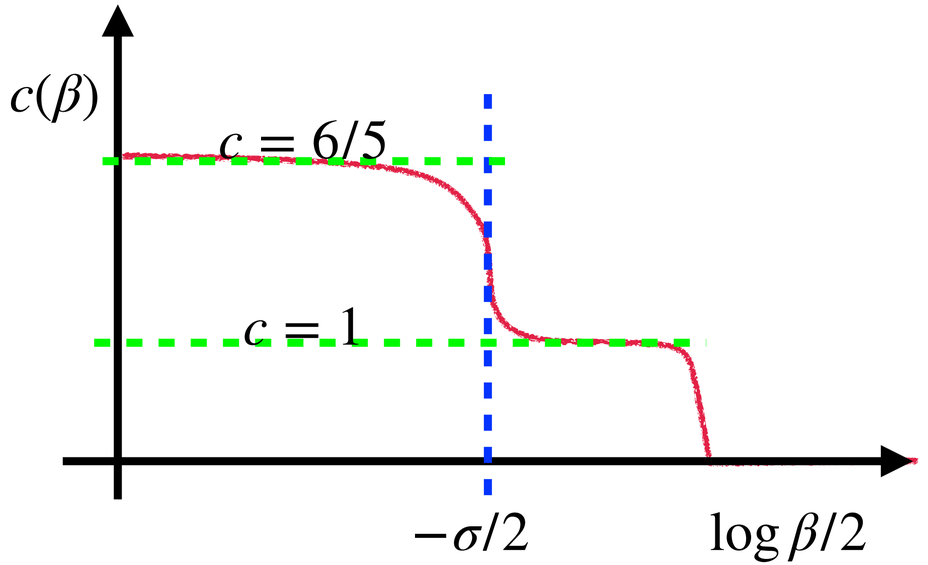
Francesco



Pasquale

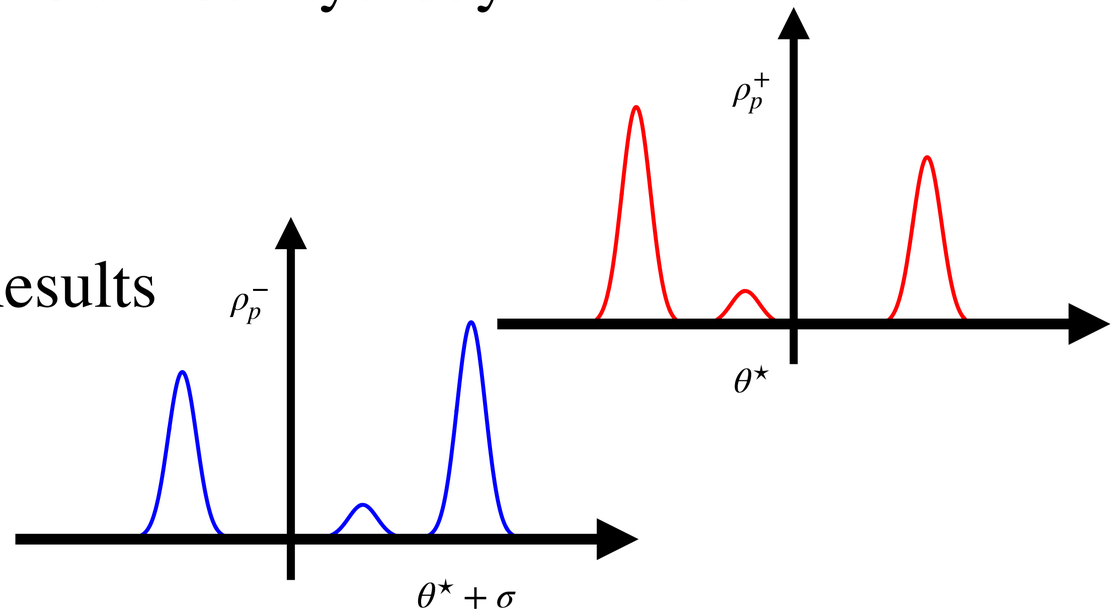
2. Structure

This talk is organized as follows:

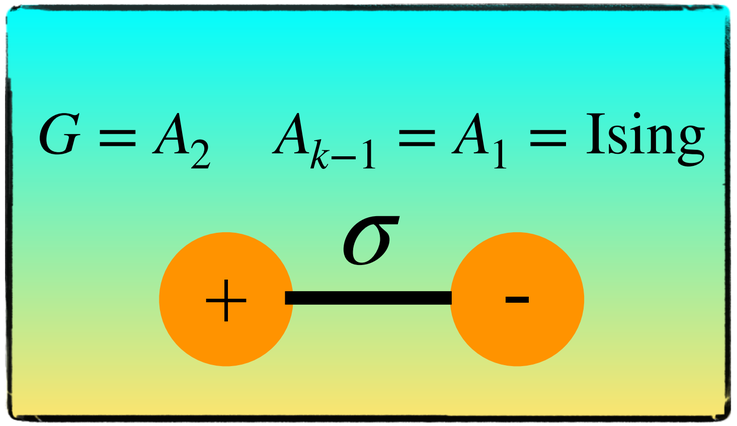


[OC-A, Fring, Korff & Miramontes'01]

- The $SU(3)_2$-Homogeneous sine-Gordon Model
- Generalized Hydrodynamics
- Results



3. Our Model: $SU(3)_2 - HSG$



- Our model is the simplest non-trivial theory in the series. We have two (stable, asymptotic) particles which form one unstable particle.

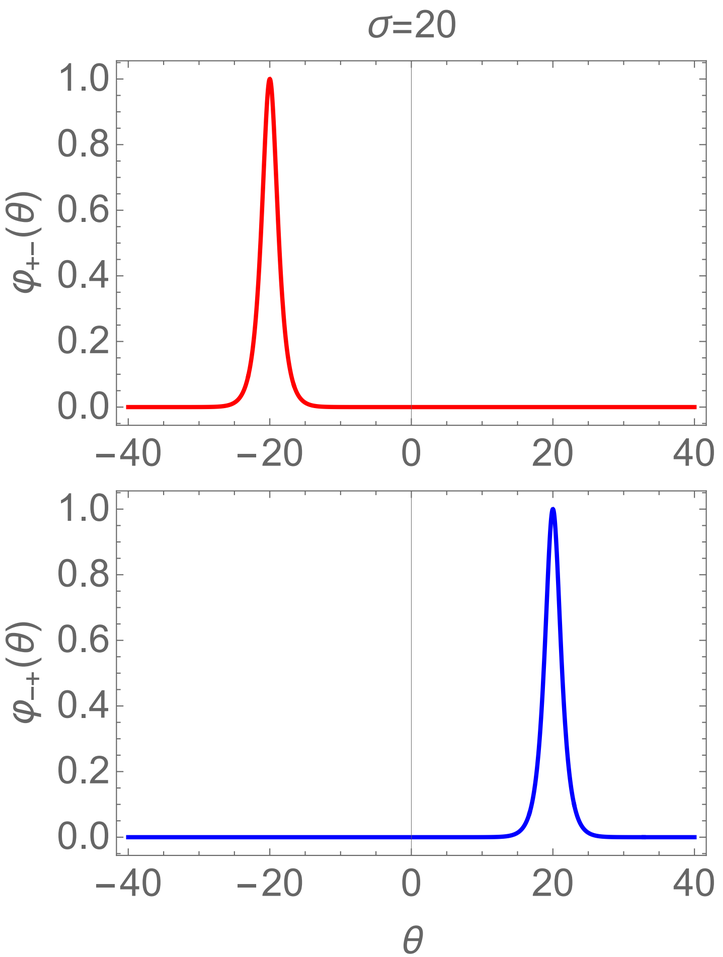
$$S_{\pm\pm}(\theta) = -1, \quad S_{\pm\mp}(\theta) = \pm \tanh \frac{1}{2}\left(\theta \pm \sigma - \frac{i\pi}{2}\right)$$

[Miramontes & Fernández-Pousa'00]

$$-i\frac{d}{d\theta}\log(S(\theta))$$

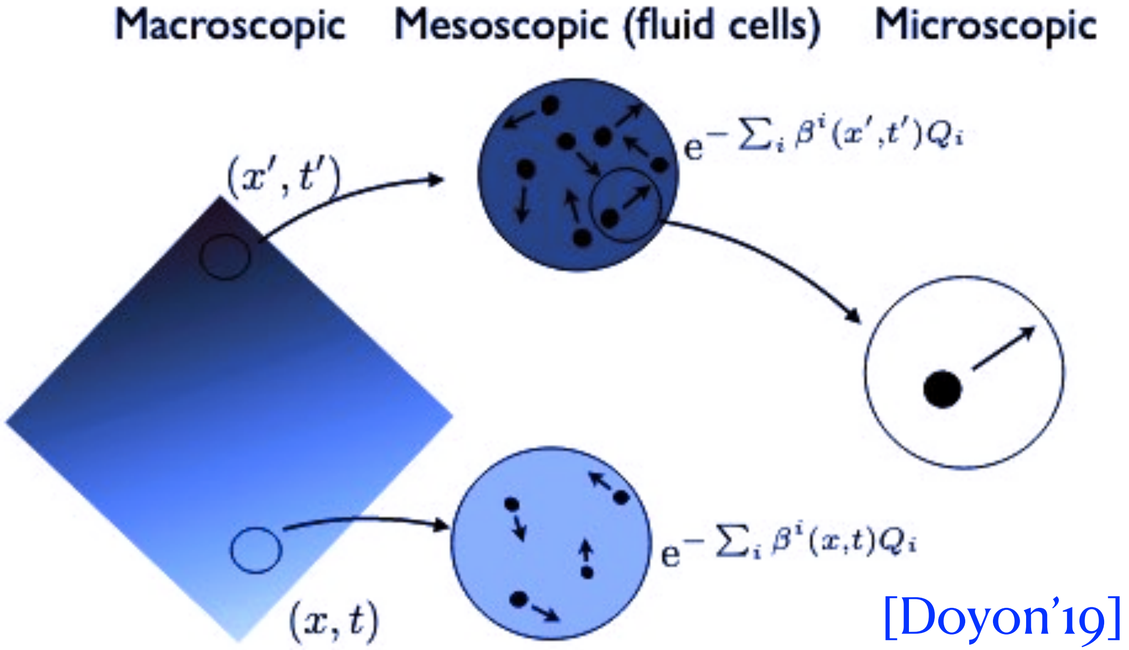
- For $\sigma \to \infty$ we have two free fermions
- For $\sigma \gg 1$ the mass of the unstable particle is $\propto me^{|\sigma|/2}$

$$\varphi_{\pm\pm}(\theta) = 0, \quad \varphi_{\pm\mp}(\theta) = \operatorname{sech}(\theta \pm \sigma)$$



4. Generalized Hydrodynamics (GHD)

- GHD is a technique that allows us to describe the large time out-of-equilibrium dynamics of many-body quantum systems. The concept of fluid cells plays a prominent role.



[Doyon'19]

- In conventional hydrodynamics this means that local averages on a fluid cell are described by a Gibbs ensemble $Z = e^{-\beta H}$

$$\Rightarrow e^{\sum_i \beta_i Q_i} \quad \text{GGE}$$

- The input here are conservation laws and the principle of emergent hydrodynamics (local entropy maximization).

- In integrable models, this is generalized to GGEs, leading to the GHD principle first proposed in [O.C-A, Doyon & Yoshimura'16; Bertini, Collura, De Nardis & Fagotti'16]. For a pedagogical review see [Doyon, Lecture notes on Generalised Hydrodynamics'19]

5. Quasi-Particle Description

- GHD requires an effective description of quasi-particle excitations. In integrable quantum field theory this is provided by the Thermodynamic Bethe Ansatz [Zamolodchikov'90] and its generalization to GGEs [Fioretto & Mussardo'10; Mossel & Caux'12].

For a single particle theory

$$\varepsilon(\theta) = \sum_i \beta_i h_i(\theta) - \varphi \star L(\theta) \quad \underset{\partial\beta_i}{\Leftrightarrow} \quad h_i^{\mathrm{dr}}(\theta) = h_i(\theta) + \varphi \star (n\, h_i^{\mathrm{dr}})(\theta)$$

kernel

convolution

$L(\theta) = \log(1 + e^{-\varepsilon(\theta)})$

$h_0(\theta) = 1$
$h_1(\theta) = e(\theta) = \cosh\theta$
$h_2(\theta) = p(\theta) = \sinh\theta$

- In a GGE with $\beta_i := \beta_i(x,t)$ all quantities depend on space-time and the key GHD equation is: $\partial_t n(x,t,\theta) + v^{\mathrm{eff}}(x,t,\theta)\partial_x n(x,t,\theta) = 0$ where

$$n(x,t,\theta) = \frac{1}{1+e^{\varepsilon(x,t,\theta)}} \quad \text{occupation number}$$

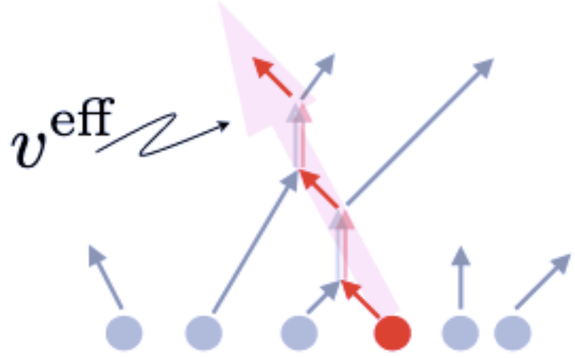
$$v^{\mathrm{eff}}(x,t,\theta) := \frac{(e')^{\mathrm{dr}}(x,t,\theta)}{(p')^{\mathrm{dr}}(x,t,\theta)} \quad \text{effective velocity}$$

6. Averages

- The occupation numbers characterised the state. Then, GHD provides a method to compute correlation functions of local operators. The average current and average density associated to a conserved quantity q_i with eigenvalue $h_i(\theta)$ are given by:

$$\mathtt{j}_i = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\theta\, p(\theta) h_i^{\mathrm{dr}}(x,t,\theta) n(x,t,\theta), \qquad \mathtt{q}_i = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\theta\, e(\theta) h_i^{\mathrm{dr}}(x,t,\theta) n(x,t,\theta)$$

- The effective velocity is the velocity of propagation due to interaction. $v^{\mathrm{eff}}(\theta) = \tanh\theta$ for free theories
- In this talk I will focus on just a few functions:

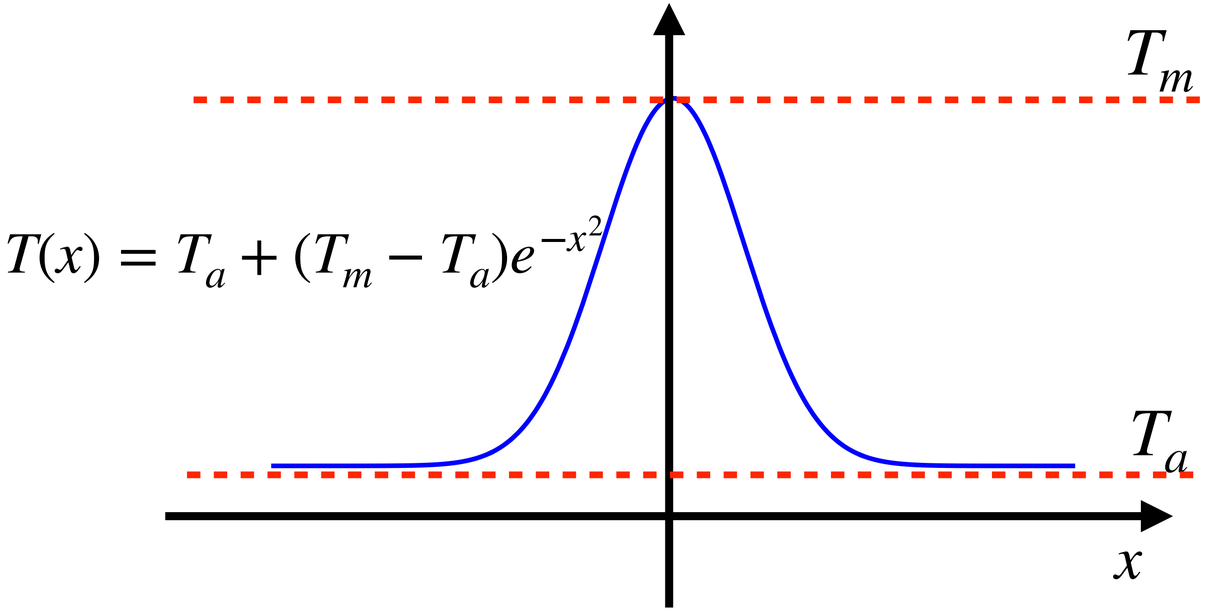


$2\pi\rho_p(x,t,\theta) = e^{\mathrm{dr}}(\theta) n(x,t,\theta)$
spectral particle density

$\mathtt{q}_0(x,t) = \int_{-\infty}^{\infty} d\theta \rho_p(x,t,\theta)$ particle density

7. Out-of-Equilibrium

- In our recent paper we considered an out-of-equilibrium situation where the initial state is characterised by a gaussian temperature profile in space.

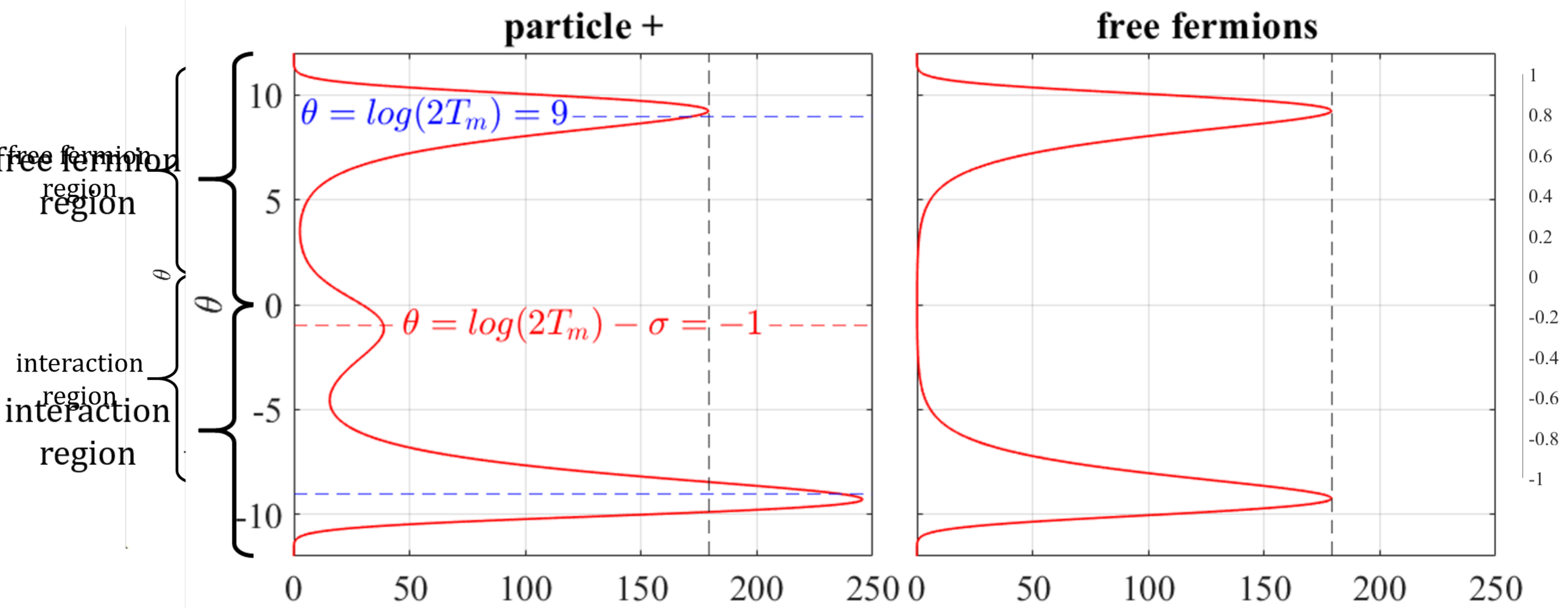


- We found that this is an ideal set-up in which to find signatures not only of the formation of unstable particles but also of their decay under time evolution.

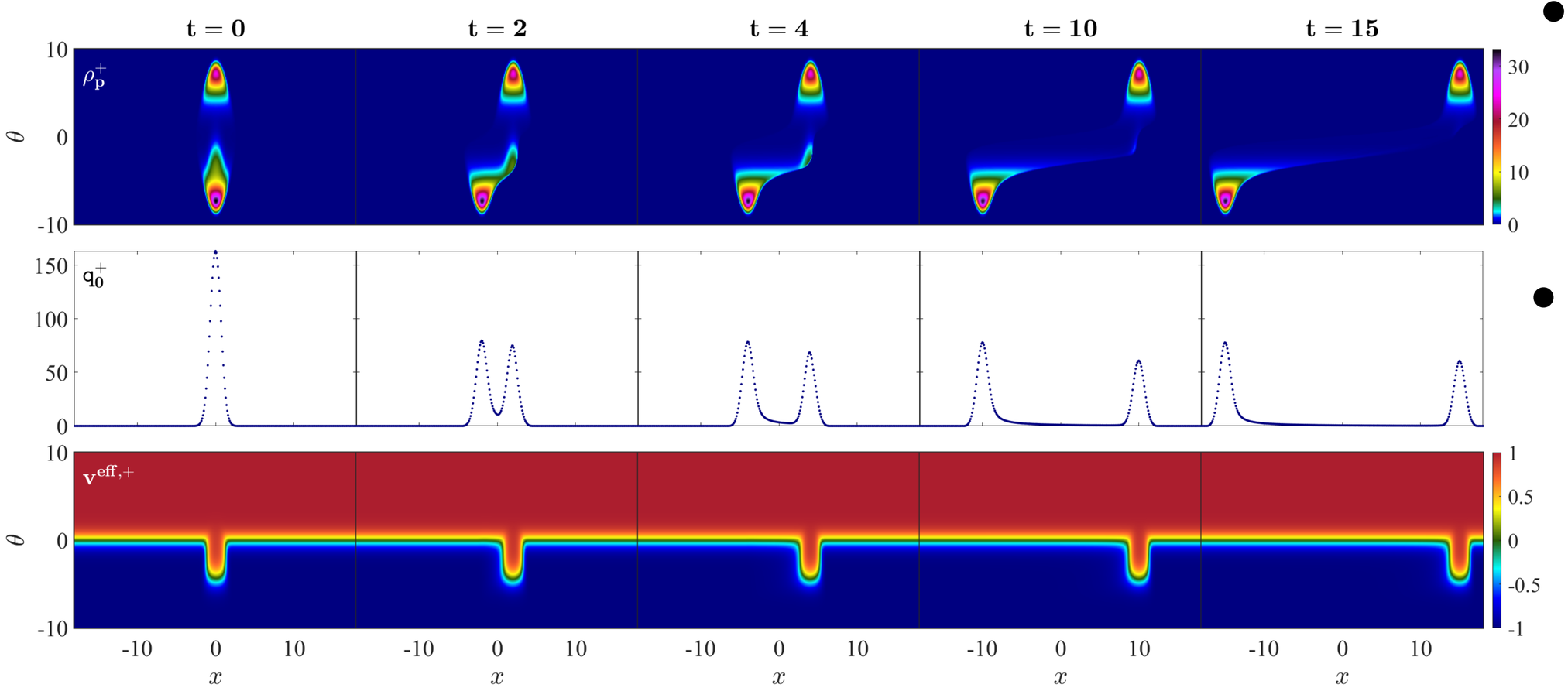
- It turns out that the presence ($T_a \neq 0$) or absence ($T_a = 0$) of a bath reveals different phenomena.

- The solution by the method of characteristics of the GHD equations in this situation was presented in [Doyon, Spohn & Yoshimura'18]. In our work we used the numerical solution of GHD as implemented in [Møller & Schmiedmayer'20] known as iFluid.

8. (Local) Equilibrium



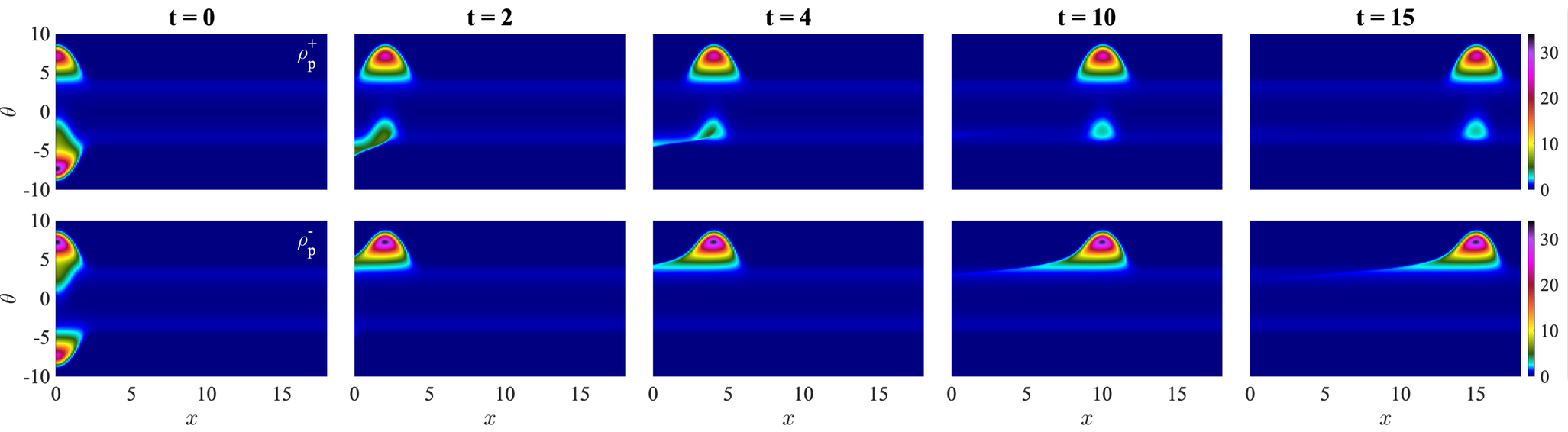
9. Dynamics in the Absence of a Bath



- Here T_m is such that unstable particles are present in the initial state.
- We see the decay of the S peak of the spectral density and the formation of a tail. We can associate this tail with the decay of unstable particles.

Here, $\sigma = 10, \log(2T_m) = 7$ and we are looking at the spectral density, particle density and effective velocity of particle +. Those of particle - are related, for instance $v^{\mathrm{eff},+}(x,t,\theta) = -v^{\mathrm{eff},-}(-x,t,-\theta)$

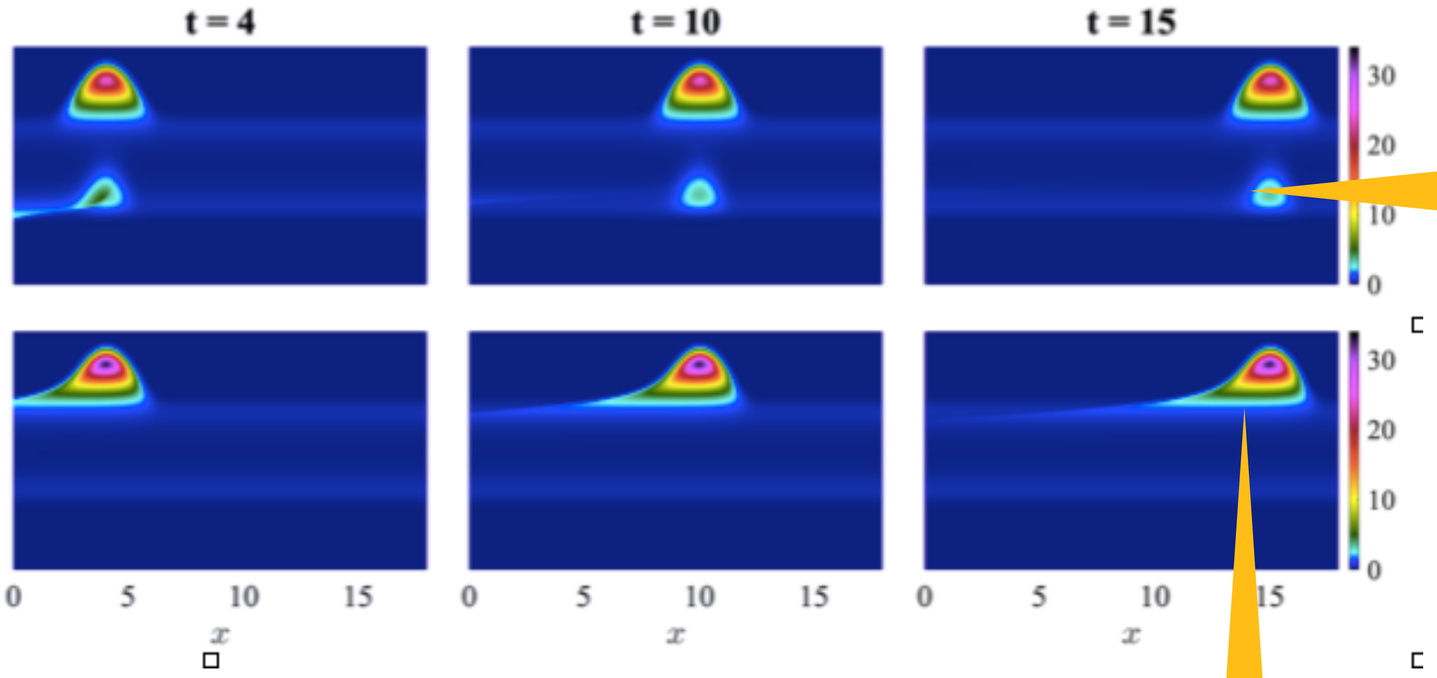
10. Dynamics in the Presence of a Cold Bath



Here, $\sigma = 10$, $\log(2T_m) = 7$, $\log(2T_a) = 3$ and and we are looking at the spectral densities of particles + and - for $\theta > 0$

- T_m is such that unstable particles are present in the initial state and T_a is a temperature which, at equilibrium, would correspond to the free fermion regime.
- We still see a bit of a tail in $\rho_p^-(x, t, \theta > 0)$ but the main feature is that there is now a persisting peak of the spectral density $\rho_p^+(x, t, \theta > 0)$ that is "riding" on top of the bath ridges and apparently moving at speed +1. Why?

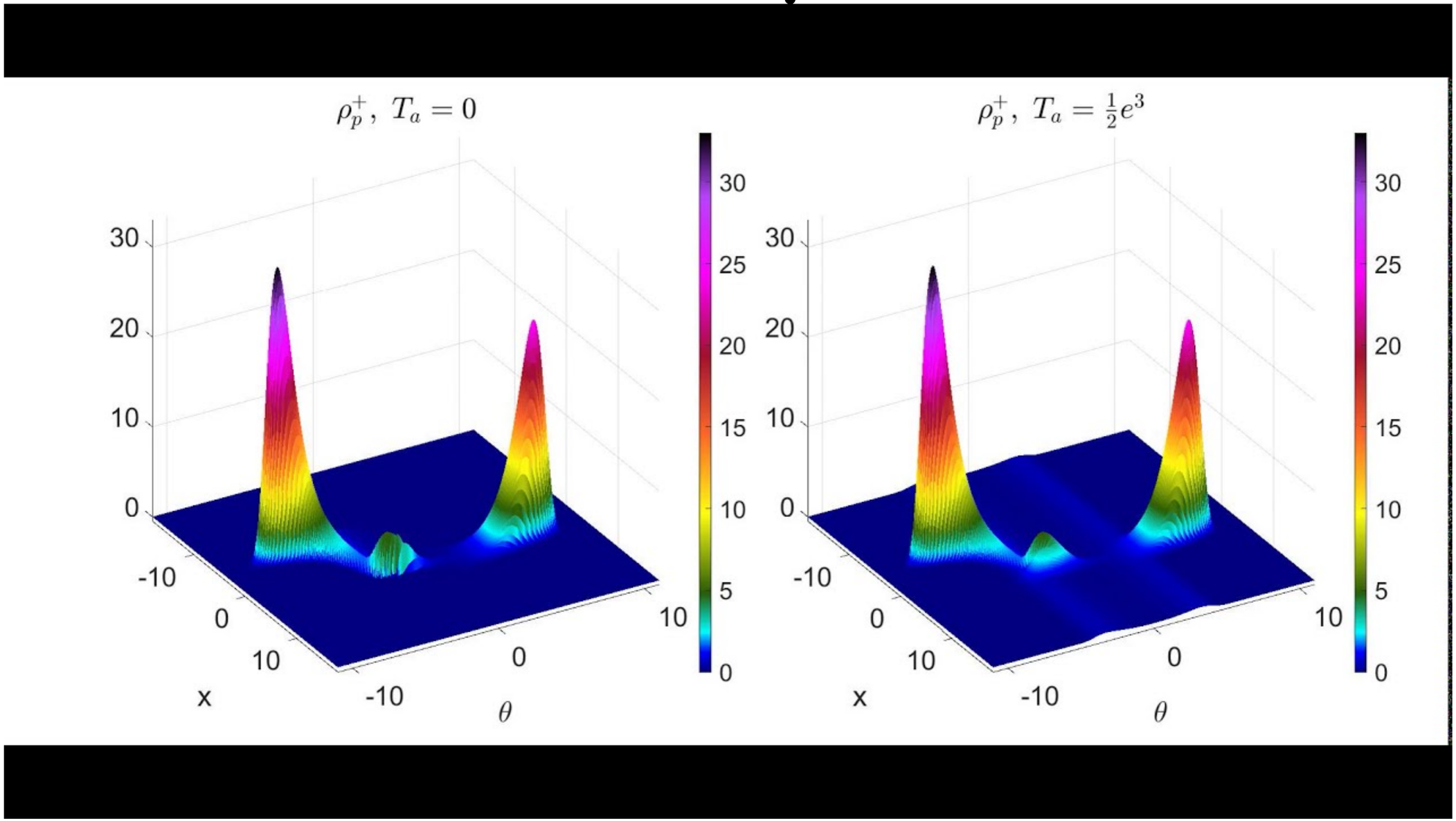
11. Magnetic Fluid Effect



The bath interacts with the "magnet", is being pulled along by it, allowing for a persistent density of particles to travel with it

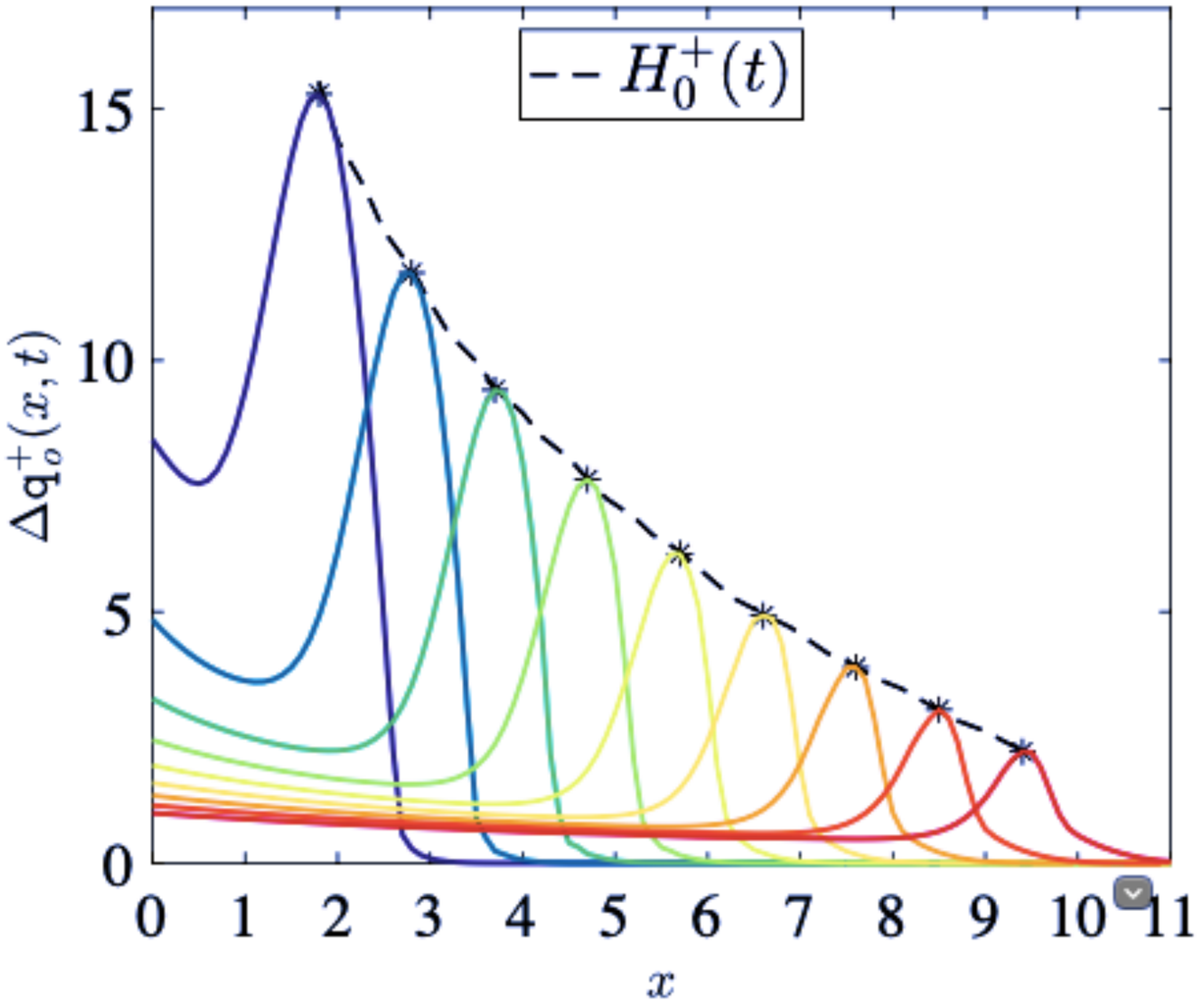
Magnet-like interacting peak moving at speed +1

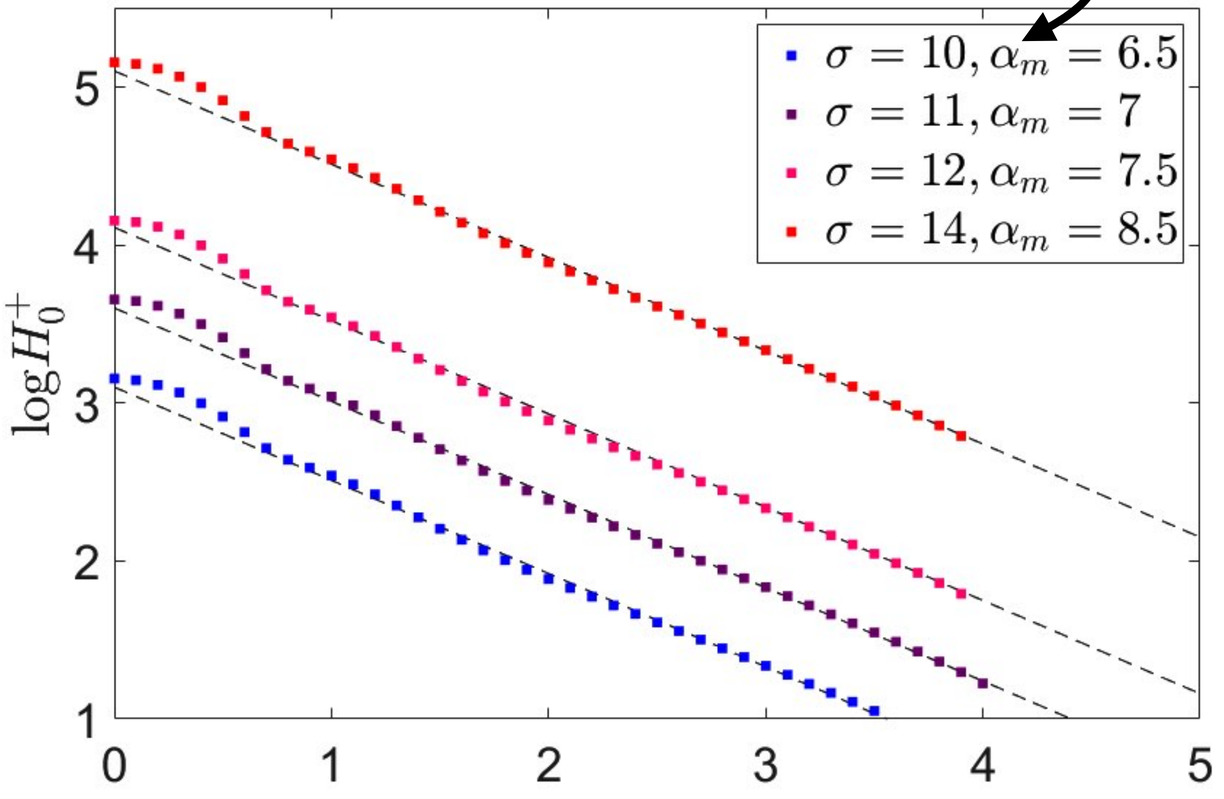
12. No Bath vs Bath Dynamics



13. Quantifying Decay

$$\alpha_m = \log(2T_m)$$



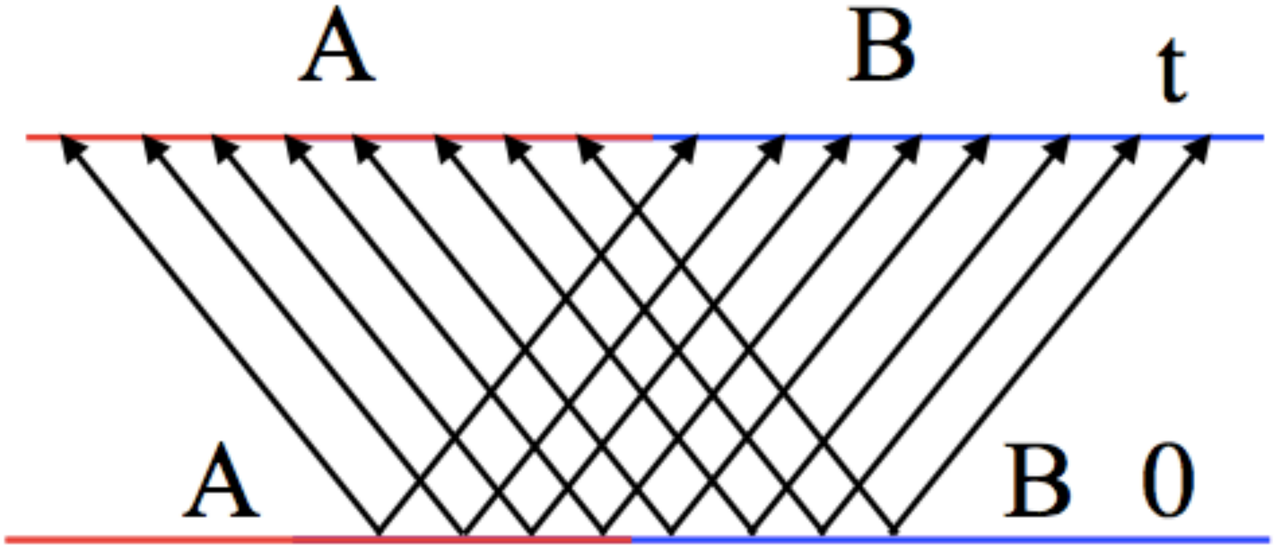


$$H_0^+(t) \approx Ae^{-\frac{t}{f(\kappa)}} \quad \text{with} \quad \kappa = \log(2T_m) - \frac{\sigma}{2}$$

$$f(\kappa) \approx e^{2\kappa} \sim \left(\frac{T_m}{M}\right)^2$$

14. Entropy in the Stationary State

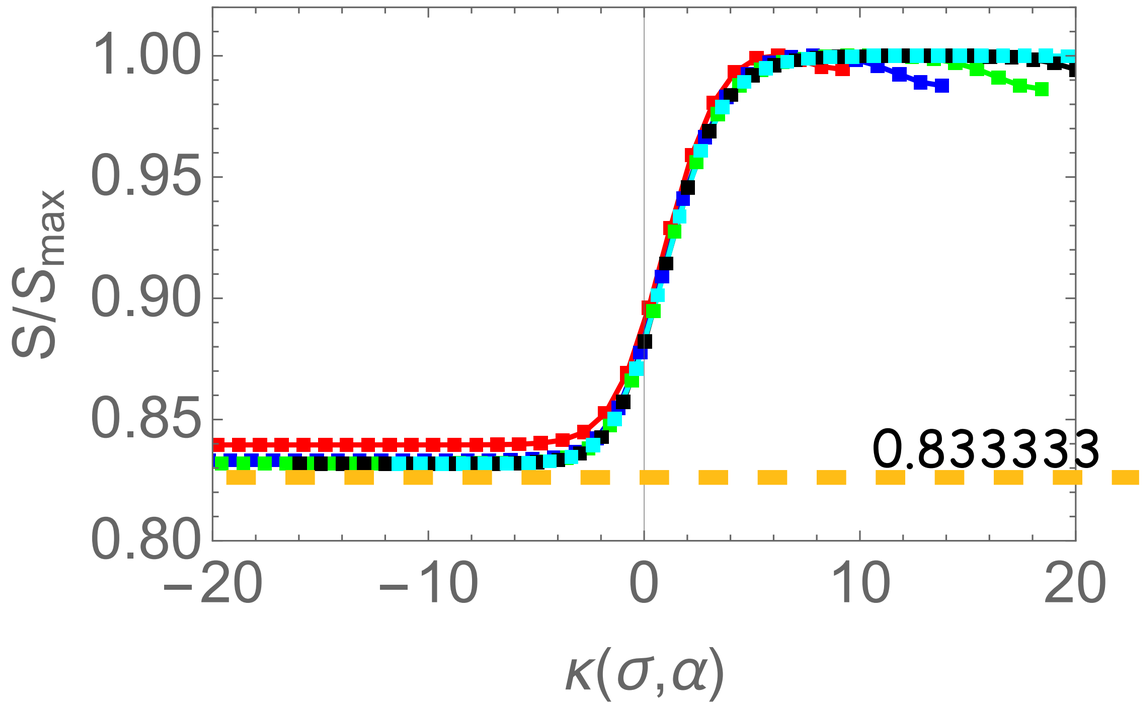
- In recent work we have looked at other types of quenches and seen that the unstable particles also leave their trace when we look at the value of the entanglement entropy in the long-time stationary state.
- There is a well-established picture [Calabrese & Cardy'05, Alba & Calabrese'17] that tells us that if we look at the entanglement entropy following a "global quantum quench" we will see the following structure:

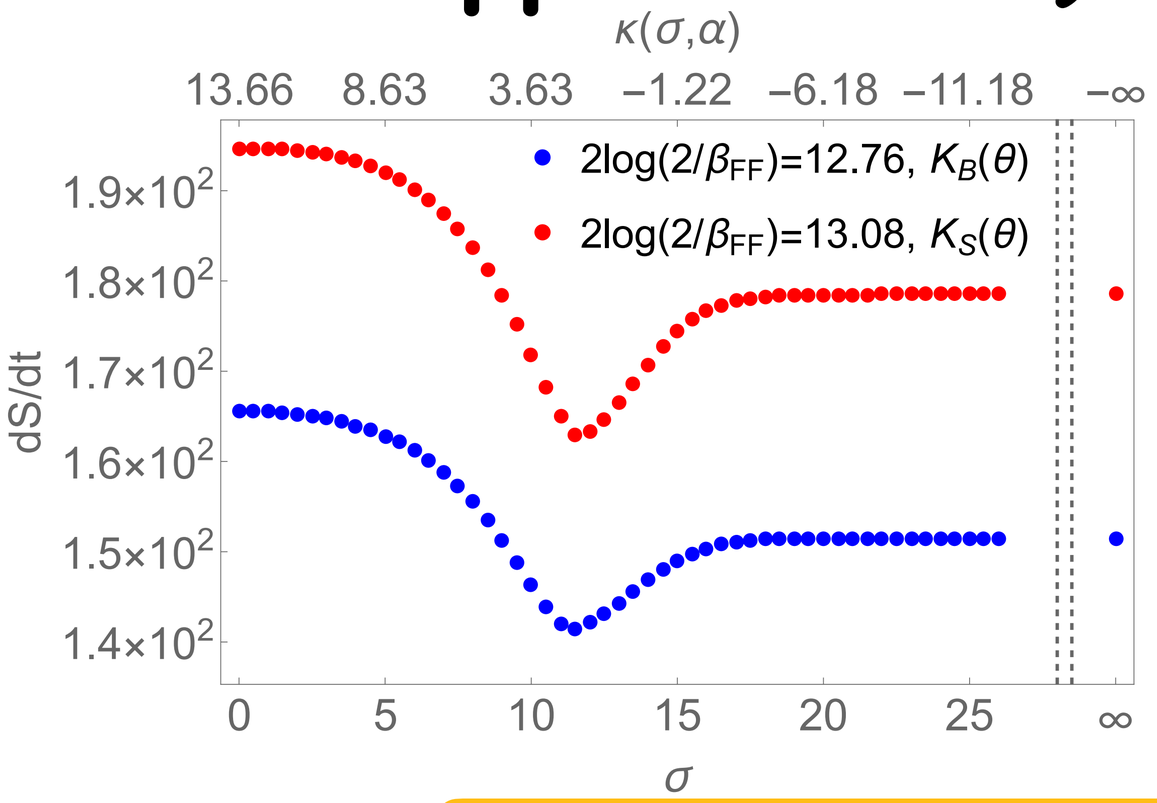


$$S(L,t) \sim Rt\big|_{t<2v} + P\big|_{t>2v} L$$

$$\frac{S}{L} = P, \qquad \frac{dS}{dt} = R$$

15. Some Results (More to Appear Soon)





1. There exists an energy scale κ which relates to the formation of the unstable particle. As a function of this scale S/L has two plateaux, separating the free fermion and the interacting regimes

2. At approximately $\kappa = 0$ the entropy growth rate has a minimum (dynamical Gibbs paradox?)

[Collura, Kormos, Tákacs'19; Lencsés, Pomponio and Tákacs'20]

16. Conclusions

- GHD can offer new insights into scattering, decay and formation of particles.
- These new insights are particularly interesting for a model where parity is broken and where unstable particles affect the dynamics.
- In our work we have identified signatures of the formation and decay of such particles. We think that they might be universal markers of the formation and decay of particles.
- Some of these signatures are: tails and additional maxima of the spectral density, an additional plateau of the effective velocities, a local minimum of the entropy growth rate.
- We observe that for generic quenches, there is a finite density of unstable particles that survive in the long term so that even if the particles themselves have short lifetimes, a finite density persist long-term and affects quantities even after equilibration.
- The decay rate of the density of unstable particles is a function of the universal ratio T_m/M, that is the ratio of the overall energy scale against the mass of the unstable particle. As expected, decay is slower, the higher the energy scale (the closer to the UV).

Thank
you!