Magnetic Field Transport in Thin Accretion Disks

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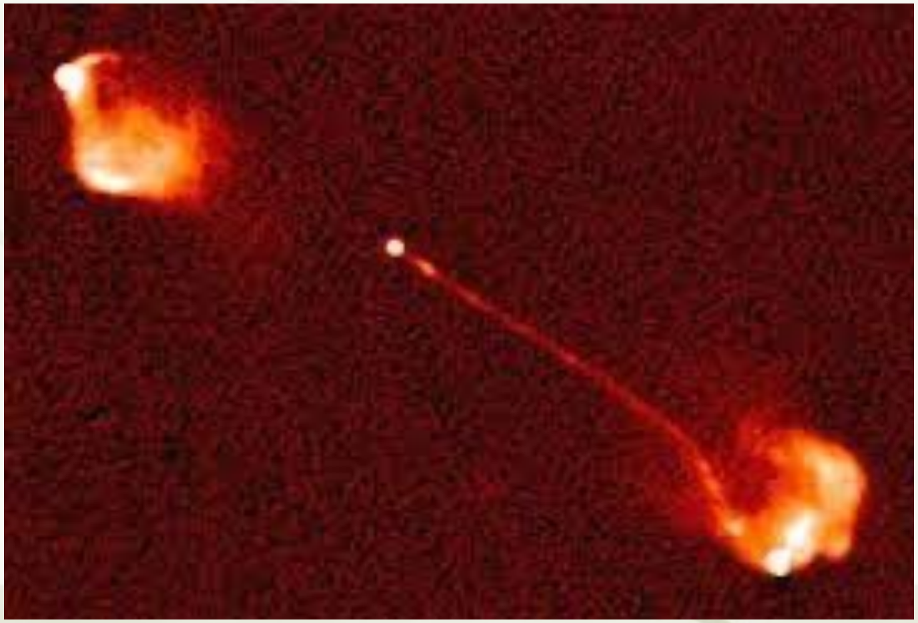

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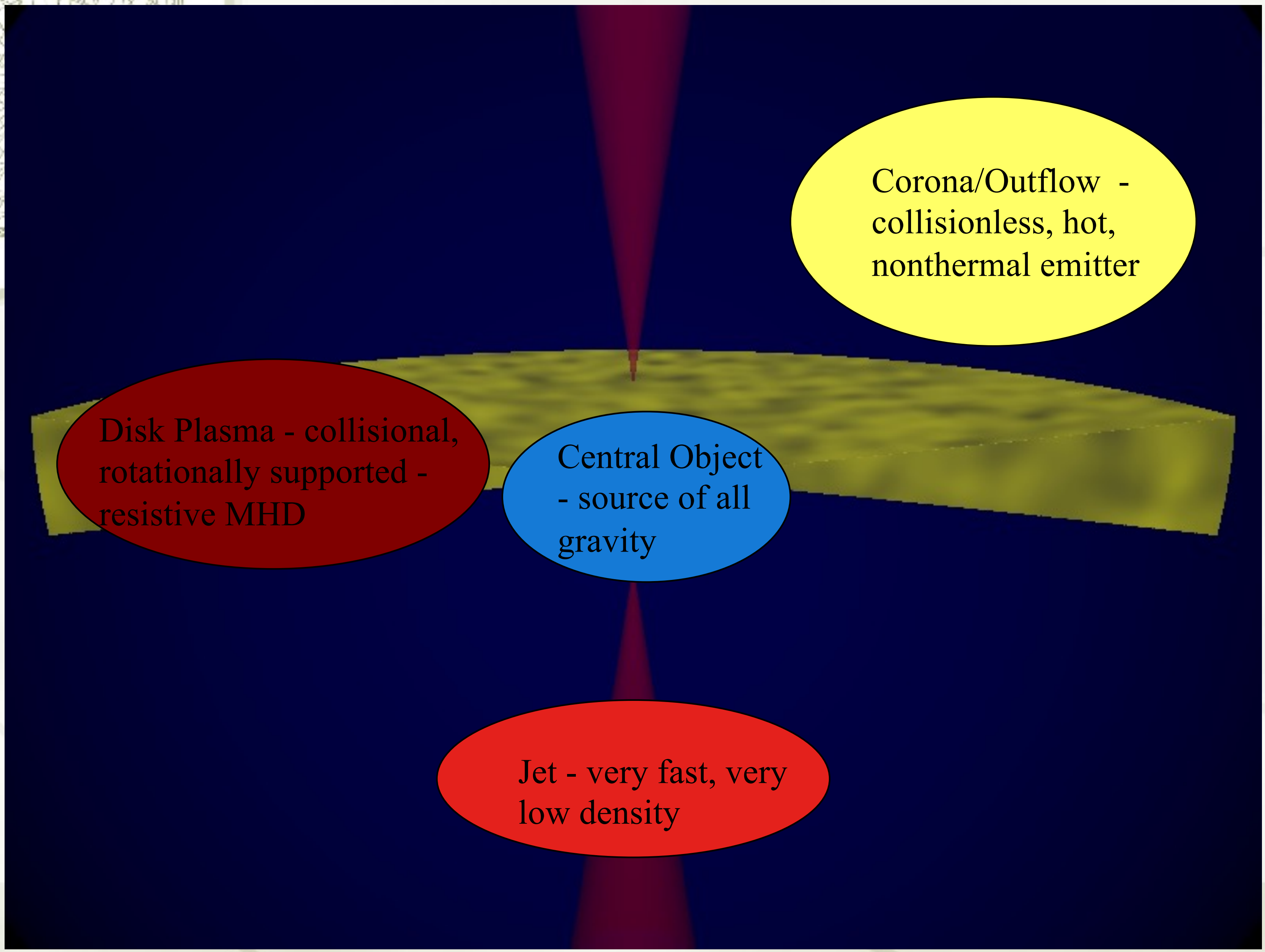
What is the problem and why is it important?

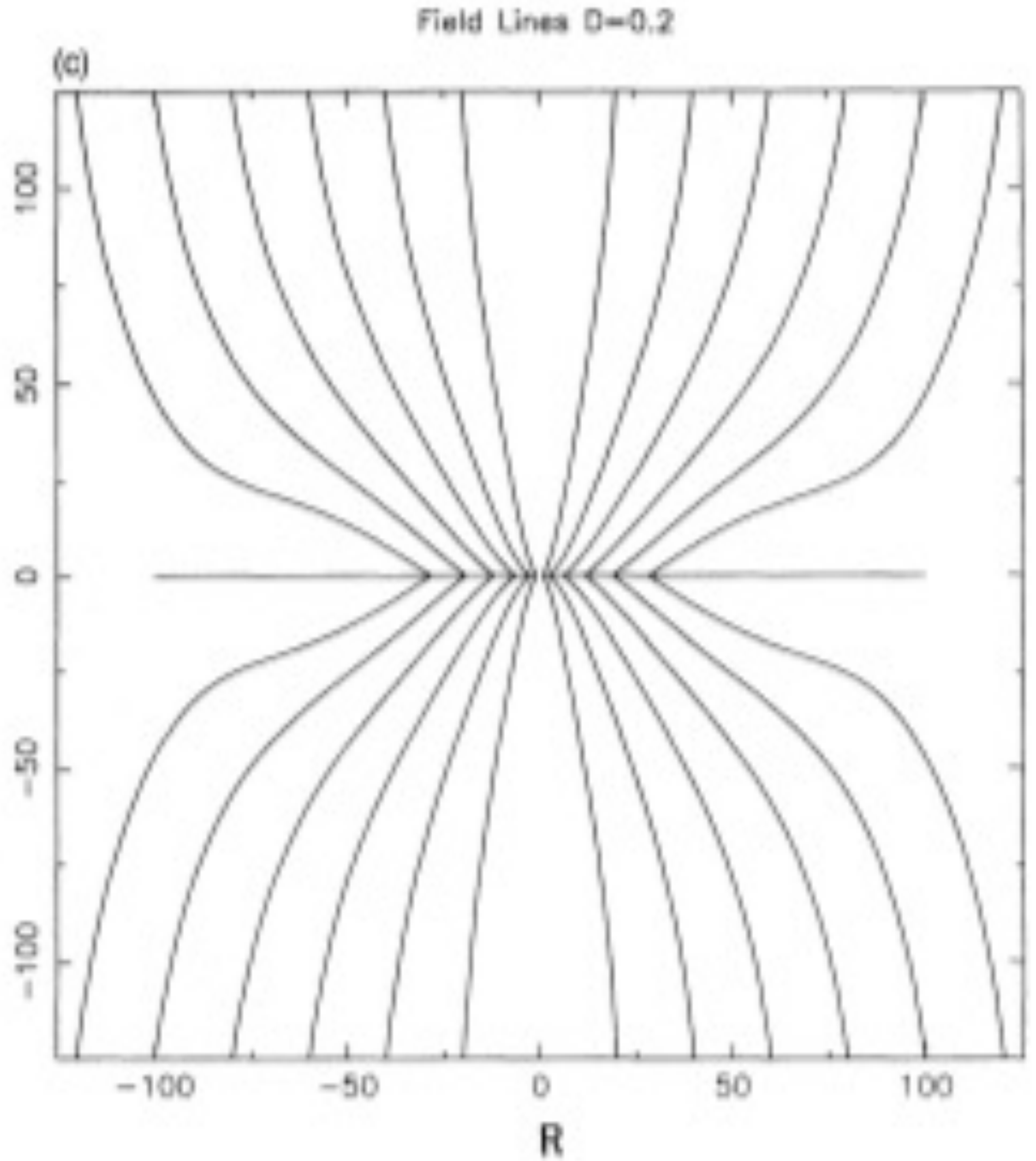
- Geometrically thin accretion disks are common around (black holes – large and small, white dwarfs, neutron stars, protostars)
- This geometry makes it very unlikely that a disk dynamo can generate significant poloidal flux
- A poloidal field is necessary to generate jets and launch strong winds – both of which are common features of accretion disks.
- Simulations suggest that a strong poloidal field may be a critical component of the hot state of variable disks

A Theoretical Cartoon:



Magnetic Fields?

- The disk supports an internal dynamo that generates a toroidal field. The magnetorotational instability drives angular momentum outward and matter inward, and also drives the dynamo that maintains the toroidal field.
- Accretion of externally generated field? (Supplied by the local interstellar medium or by companion stars in binary systems.) Requires field lines to be pulled inward as the gas accretes - consistent with numerical simulations.

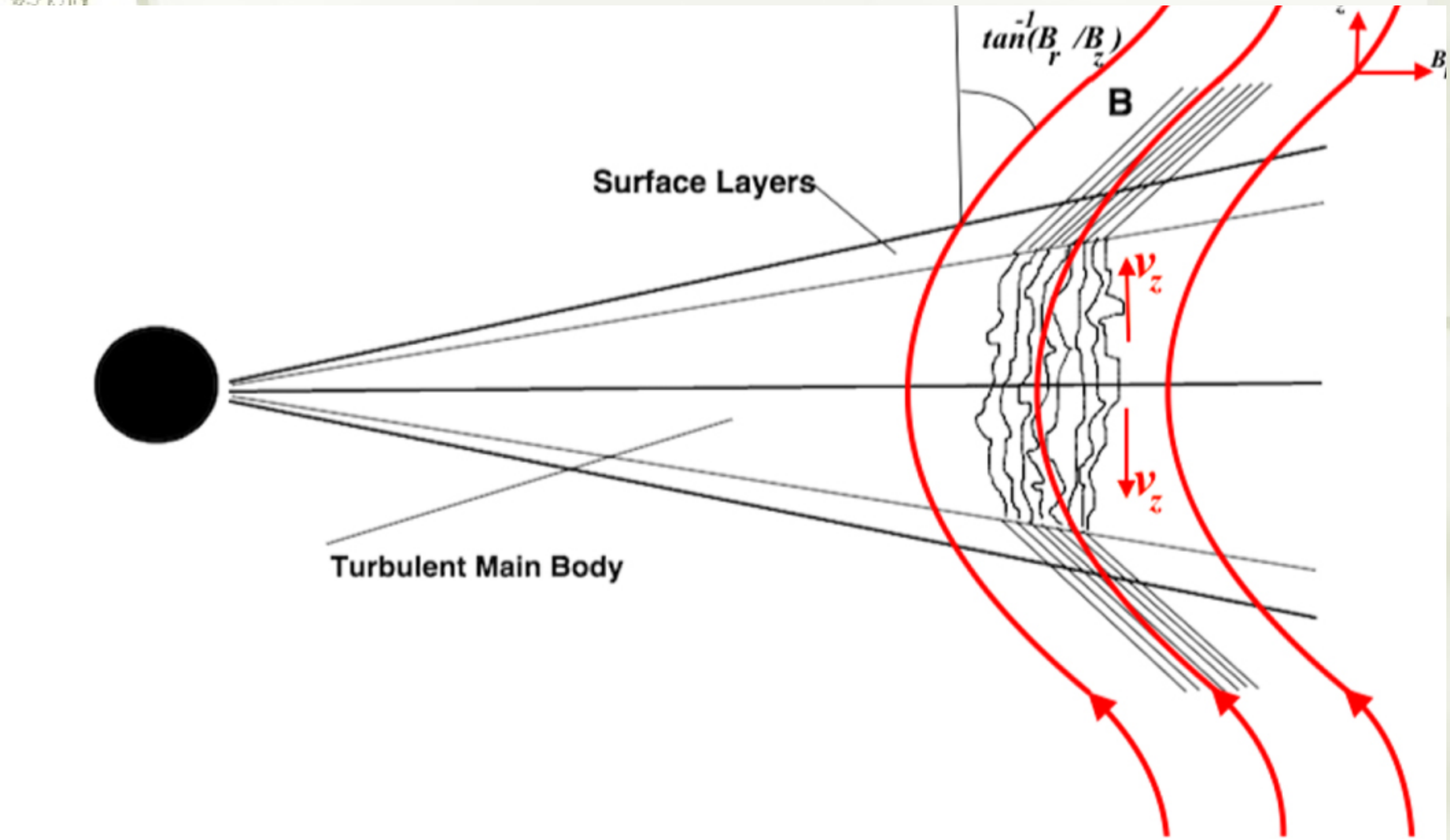


From Lubow and Pringle 1994

The simplest wrong answer – flux freezing

$$\partial_t \vec{B} = \nabla \times (\vec{v} \times \vec{B} + \vec{R})$$

- The resistive term is negligible but the curl is not always small. A dense network of reconnection regions plus stochastic wandering of field lines in turbulence means that flux freezing fails in turbulent media.



B
Surface Layers
Turbulent Main Body

A Comprehensive Approach

- The drift of the magnetic field in the presence of turbulence and background gradients is not simply described by buoyancy and turbulent diffusion.
- The condition for global equilibrium is that the curl of the electric field vanish everywhere, and that the azimuthal component vanish.

$$(\vec{v} \times \vec{B})_{\phi} = 0$$

- Here we are concerned with the toroidal component which describes vertical motion of a radial field and radial motion of a vertical field.

What is the vertical drift?

- After allowing for background motions e.g. the accretion flow, we take the time derivative of $\langle \vec{V} \times \vec{B} \rangle_{\varphi}$ and multiply by the eddy correlation time. Any piece that is proportion to the horizontal background field helps determine the vertical drift.
- The final result includes buoyancy, velocity "pumping" towards regions of lower velocity, magnetic field pumping towards regions of higher field concentration, and a drift towards lower entropy volumes.

Technical comments

- In addition we need to calculate the volume averaged fluid velocity, which is obtained by the condition that the mass flux average to zero.
- We need to include the pressure term, which is derived from an inverse Laplacian and depends on eddy shape.

Vertical Drift

- The result is:

$$\frac{\tau}{2}\left(\frac{1}{4\pi\rho}\frac{\partial\langle b_z^2\rangle}{\partial z}-\frac{\partial\langle v_z^2\rangle}{\partial z}\right)$$
$$-\frac{\tau}{\gamma L_p}\frac{1}{4\pi\rho}\left(\langle b_x^2\rangle-\left\langle b_x(\mathcal{R}_z^2+\frac{1}{2}\mathcal{R}_x^2)b_x\right\rangle\right)$$
$$+\frac{\tau}{L_s}\left(\frac{1}{4\pi\rho}\left\langle b_z(\mathcal{R}_x^2+\frac{1}{2}\mathcal{R}_z^2)b_z\right\rangle-\langle v_z^2\rangle\right).$$

Vertical Drift

- In the limit of isotropic turbulence and equipartition between magnetic and kinetic energies we get

$$V_z \approx -\frac{\langle v_z^2 \rangle}{L_S}\tau$$

- All things being equal this would tend to confine the magnetic field, but MRI induced turbulence is highly anisotropic with more magnetic than kinetic energy.

Diffusion Terms

- Naively one expects diffusion terms to be diffusive, i.e. $-\varepsilon_{ijk}\left(\langle v_j v_l\rangle + \langle b_j b_l\rangle\right)\tau\partial_l B_k$
- Magnetic fluctuations couple to the magnetic field gradient through the induced pressure terms so that the real magnetc contribution is more complicated

$$-\varepsilon_{ijk}\langle b_j b_k\rangle\tau\left[\delta_{ks} - 2Q_{ks}\right]\partial_l B_s$$

- One also needs to account for the magnetic helicity back reaction, which mixes diffusive terms and changes their magnitudes by factors of order unity.

Field Equilibrium

- For turbulence driven by the MRI, buoyancy is strongly dominant over other vertical drift mechanisms.
- This gives a field solution

$$\frac{B_r}{B_z} \approx N \frac{z}{r} \left(\frac{P_0}{P} \right)^N$$

The Field Equilibrium

- At some point, far from the midplane, the forces due to the poloidal field will dominate the dynamics and the bending angle stop growing.
- When the field is weak the resulting bending angle will be very large.
- This will happen sooner for a stronger field, so accretion and bending are easier for a weak field than a strong one.

The Field Equilibrium

- The most relevant limit is when the pressure gradient due to the field is stronger than the background gradient

$$P_{\lim} \approx \frac{NB_{\varphi}^{2}}{8\pi}$$

- Far from the midplane the azimuthal field is only slightly greater than the radial field.

The Field Equilibrium

- The dimensionless parameter that governs the solution is $\frac{P_0}{B_z^2}$
- This implies that accretion of magnetic fields will be more efficient at high mass accretion rates, since the backreaction of the accreted field will be negligible to smaller radii

The Field Equilibrium

- The strength of the accreted field will rise to small radii until one reaches a radius where the backreaction is very large.
- At some point this can drive a rise in the internal angular momentum transport, but this will not qualitatively change the disk structure or accretion proces

The Field Equilibrium

- The critical radius is set by the condition that angular momentum transport into the corona is more important than internal angular momentum transport

$$-\frac{B_z B_\varphi}{4\pi} \geq \alpha \frac{H}{r} P$$

- As we get near this radius the angular momentum flux expulsion, and the orbital energy dissipation, will be maximum far from the disk midplane.

The Field Equilibrium

- In order to solve the global equilibrium one needs some ansatz for the field outside the disk. If we take the popular choice that the external field scales as r^{-2} we have a very steep, and unrealistic, scaling of the vertical field with radius.
- This leads to strong outward diffusion of the embedded vertical field and a smaller bending angle (replace accretion velocity with accretion minus diffusion).

The Field Equilibrium

- Now we need to invoke some condition at the critical radius to get the overall amplitude of the bending.
- For self-consistency we need a bending angle of order unity, with most of the accreted flux moved inside this radius.
- The critical radius is then

$$R_{crit} \sim R\left(\frac{B_{ext}^2 R}{\dot{M}\Omega(R)}\right)^{2/3}$$

Conclusions

- Thin disks can efficiently suck up poloidal fields in their environment and transport them to small radii.
- For strong fields, weak mass flow, or a large ratio of outer to inner radius, this will lead to a magnetically dominated inner disk. This is much more likely for black hole accretion disks than white dwarf systems.