

The travel of charge and heat in SrTiO_3

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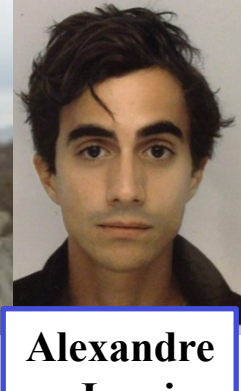
Benoît Fauqué



**Clément
Collignon**



**Lisa
Buchauer**



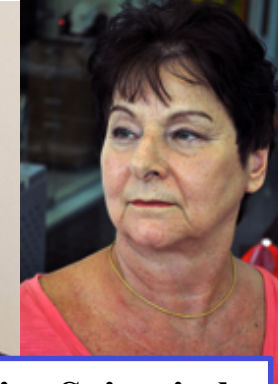
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Jaoui**



Yann Gallais
Paris-Diderot
Raman scattering

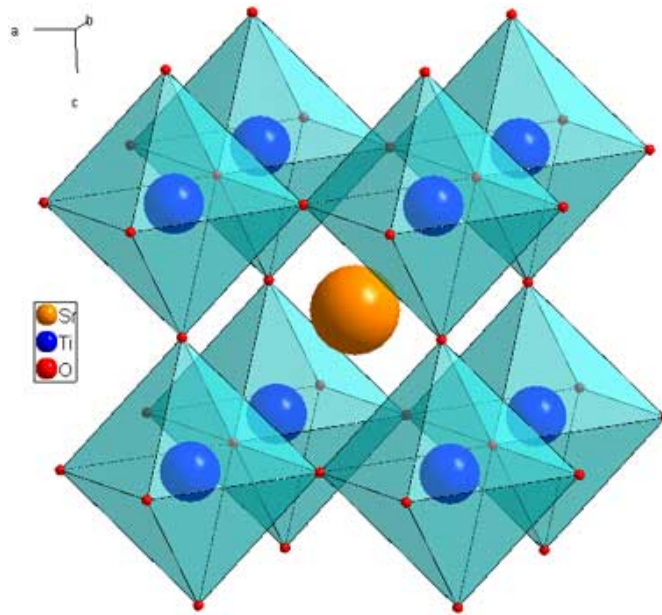


Joachim Hemberger
Cologne
Electric permittivity



Valentina Martelli, Juio Larrea & Elisa Saitovitch
CBPF, Rio de Janeiro
Thermal conductivity

Remarkable for many reasons



- TiO₆ octahedron
- SrO₁₂ cuboctahedron

- ...is superconducting upon doping *Schooley et al. (1964)*
- ...is a water splitter *Mavroides et al. (1976)*
- ...is a quantum paraelectric *Müller & Burkhard, (1979)*
- ...exhibits second sound *Koreeda et al. (2007)*
- ...displays flexoelectricity *Zubko et al. (2007)*
- ...hosts interface superconductivity *Reyren et al. (2007)*
- ...becomes magnetic with light *Rice et al. (2014)*

Outline

- Dilute Superconductivity

Intertwined with ferroelectricity

- Charge transport

Bohr radius and mobility

Low-temperature T^2 resistivity

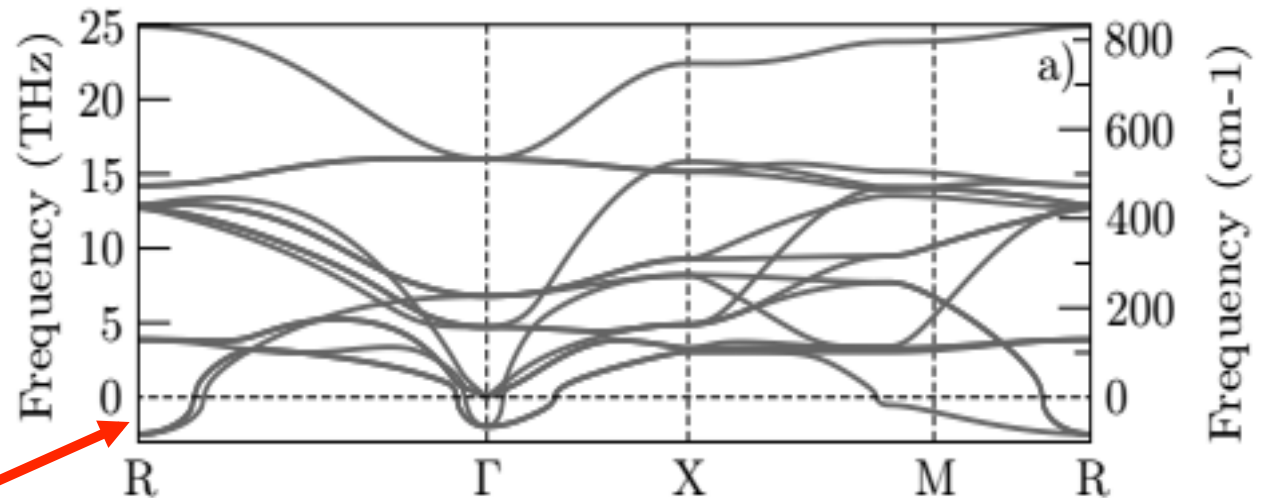
Room-temperature bad metallicity

- Heat transport

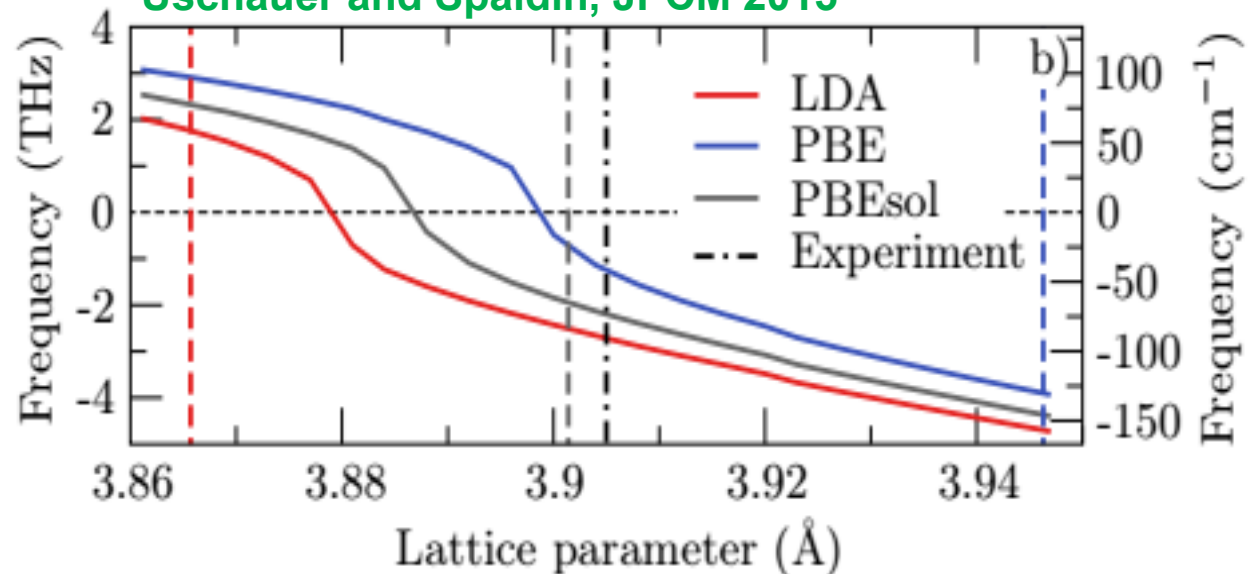
Phonon poseuille flow

Diffusivity and Planck time

Ab initio theory: this solid cannot be!



Uschauer and Spaldin, JPCM 2015



Imaginary phonon frequencies!

See also:

Lebedev, Phys. Sol. Stat. (2009)

Chodhuri *et al.*, PRB (2008)

Tadano & Tsuyenuki, PRB (2015)

Owes its very existence to zero-point quantum fluctuations!

SrTiO₃: An intrinsic quantum paraelectric below 4 K

K. A. Müller

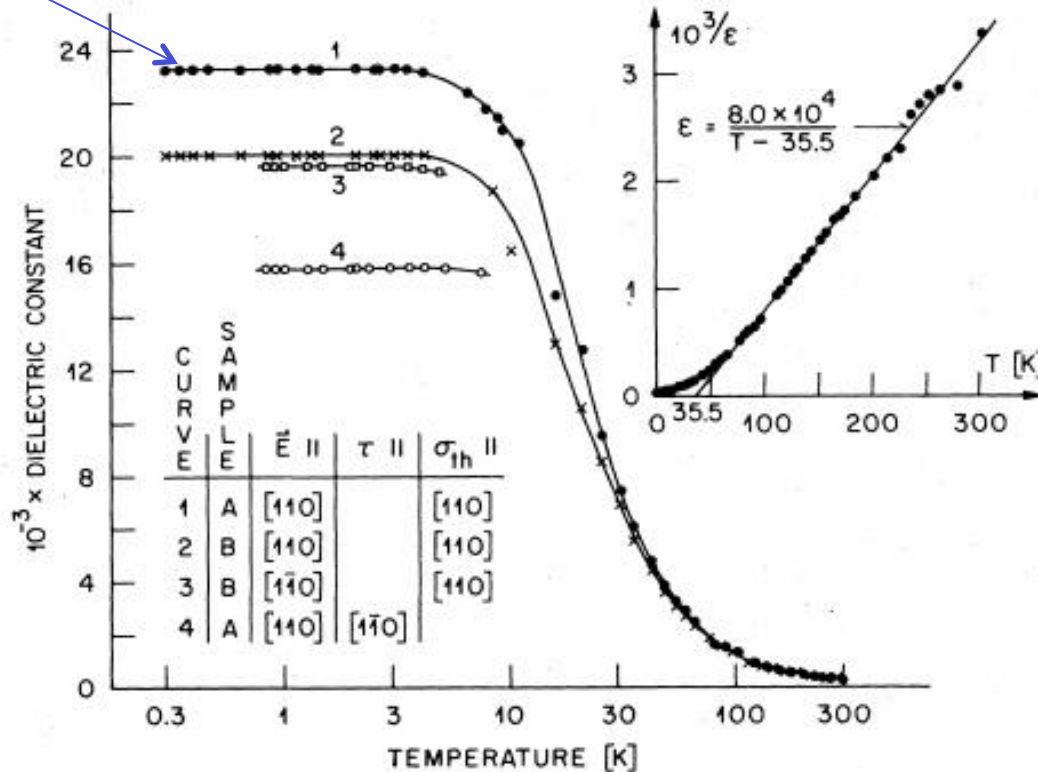
*IBM Zürich Research Laboratory,
8803 Rüschlikon, Switzerland*

H. Burkard*

*Laboratory of Solid State Physics,
ETH 8093 Zürich, Switzerland*

(Received 19 July 1976; revised manuscript received 14 August 1978)

The dielectric coefficient becomes as large as $20000\epsilon_0$!



I. Dilute superconductivity

Superconducting Transition Temperatures of Semiconducting SrTiO_3

C. S. KOONCE* AND MARVIN L. COHEN†

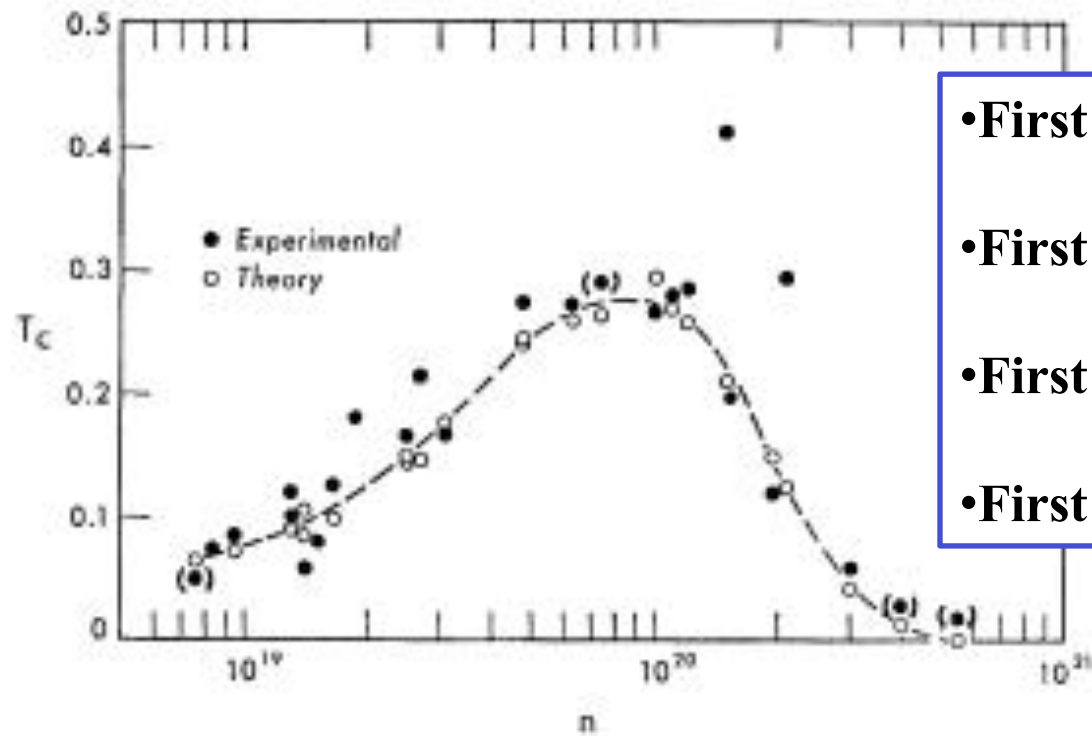
Department of Physics, University of California, Berkeley, California

AND

J. F. SHOOLEY,‡ W. R. HOSLER,§ AND E. R. PFEIFFER

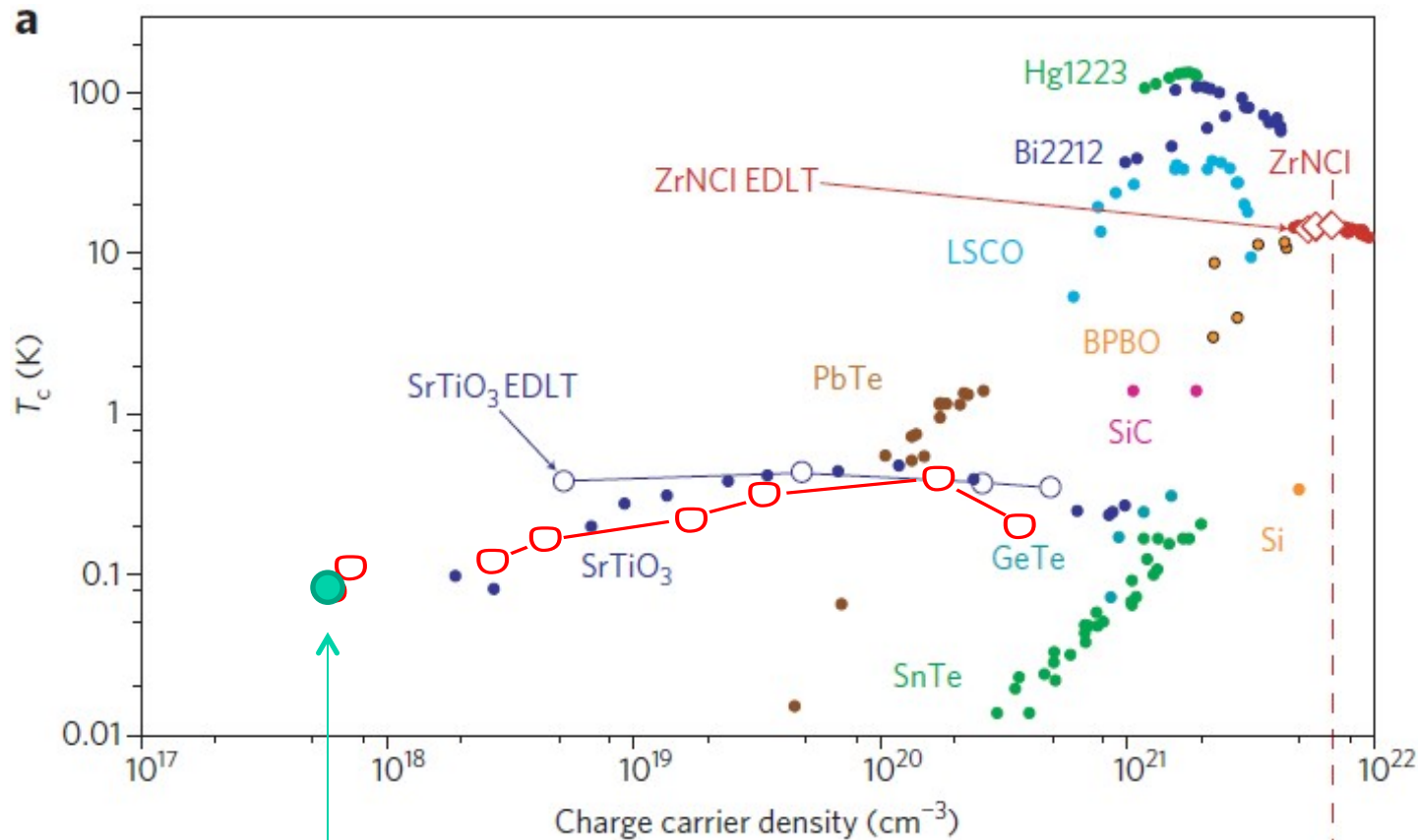
National Bureau of Standards, Washington, D. C.

(Received 5 July 1967)



- First **oxide** superconductor
- First **multi-band** superconductor
- First **semiconducting** superconductor
- First superconducting **dome**

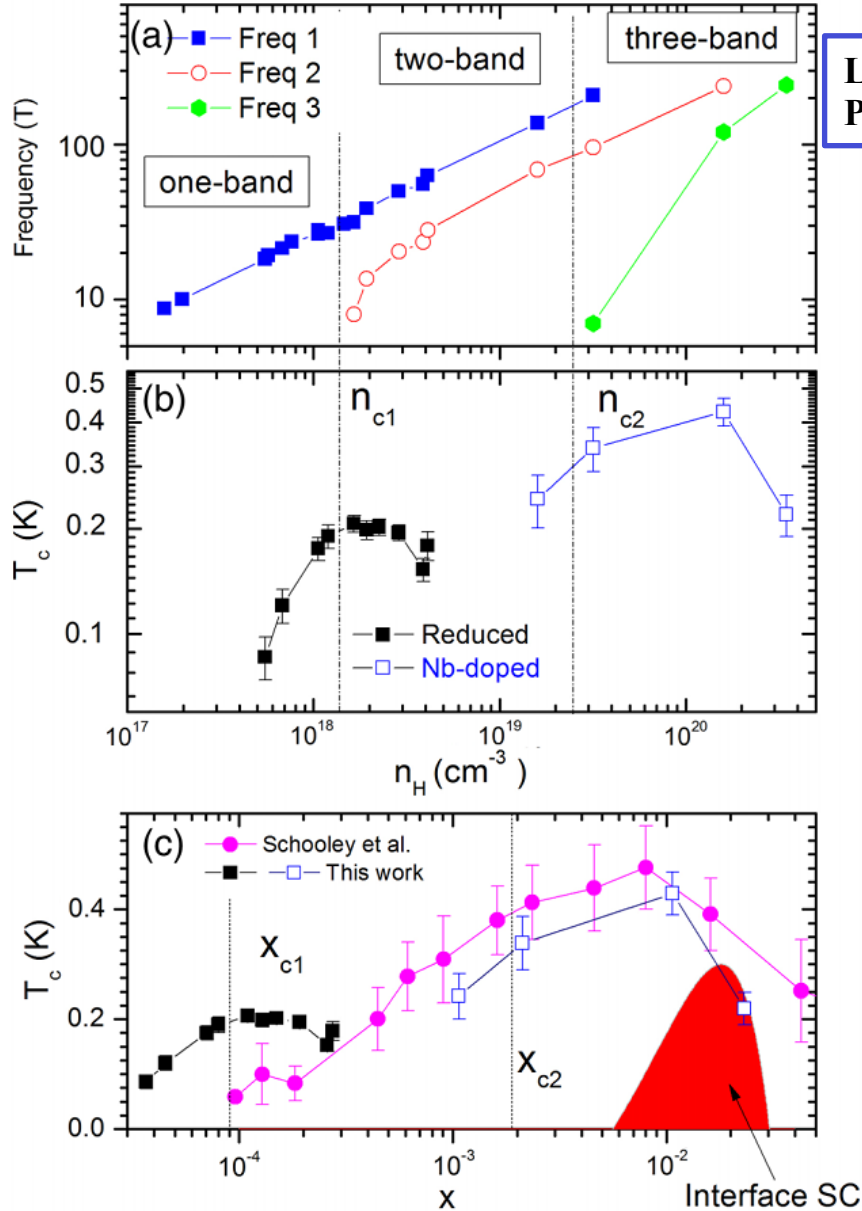
“Semiconducting” superconductors



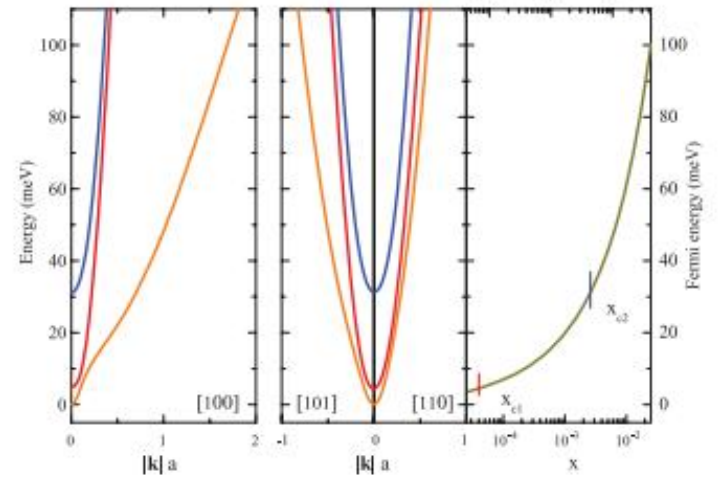
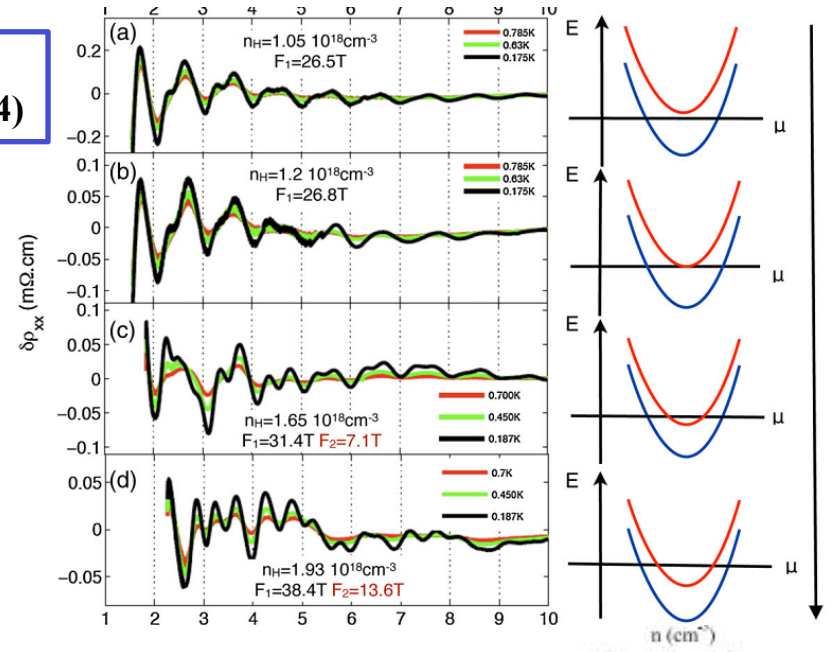
$5.5 \times 10^{17} \text{ cm}^{-3}$ – 3 K Ueno et. al., Nature Nanotechnology 6, 408 2011

By far, the most dilute superconductor currently known

Bands and domes in n-doped STO!

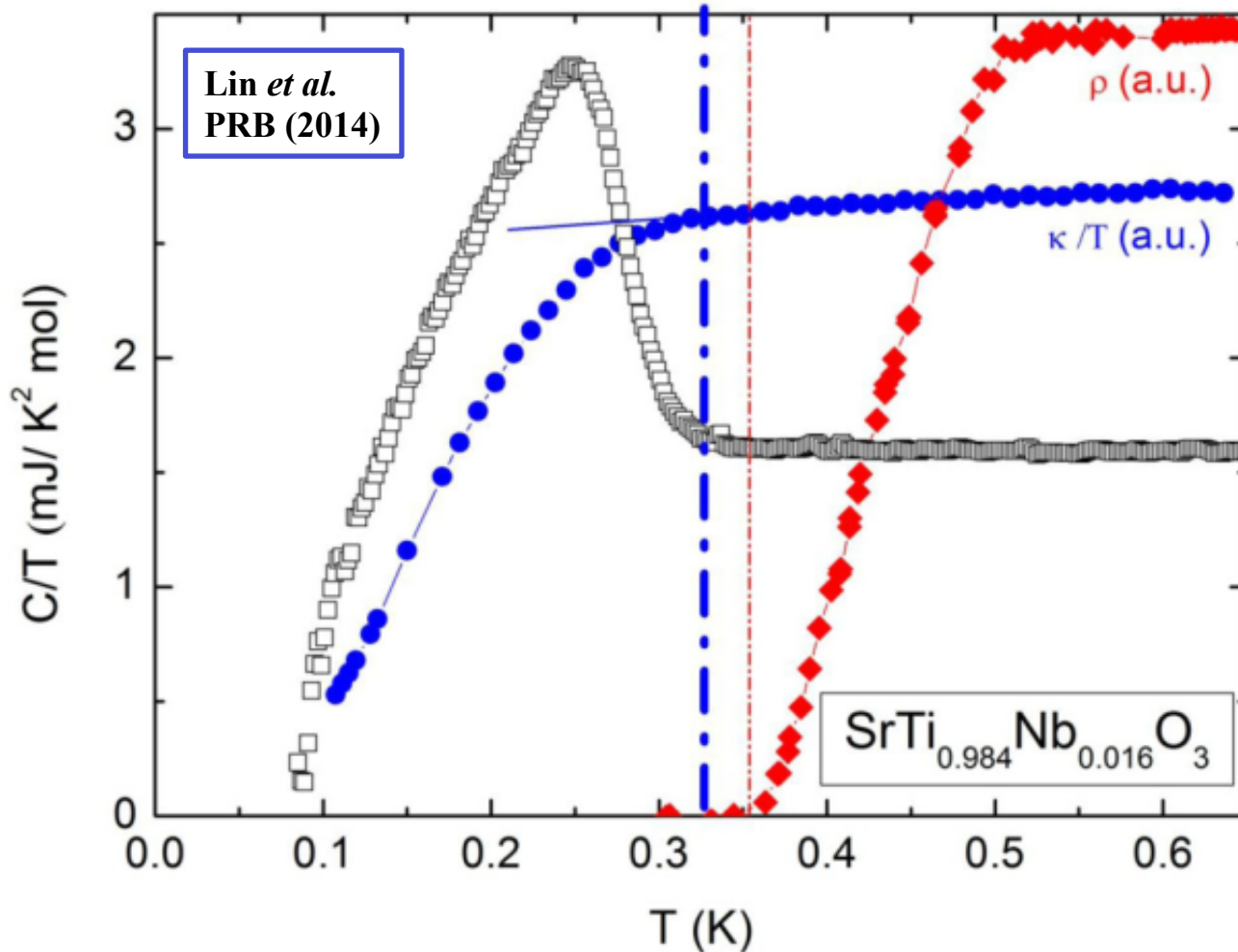


Lin *et al.*
PRL (2014)



van der Marel, van Mechelen & Mazin, PRB (2011)

Superconducting transition seen by different probes



$$\gamma = 1.55 \text{ mJ} / \text{K}^2 \text{ mol}$$

Compatible with:

$$m_1 = 3.8 m_e$$

$$m_2 = 1.8 m_e$$

$$m_3 = 1.8 m_e$$

Theory on microscopic origin of pairing

- Plasmons?

Takada 1980

P. A. Lee 2016

- A phonon Soft mode ?

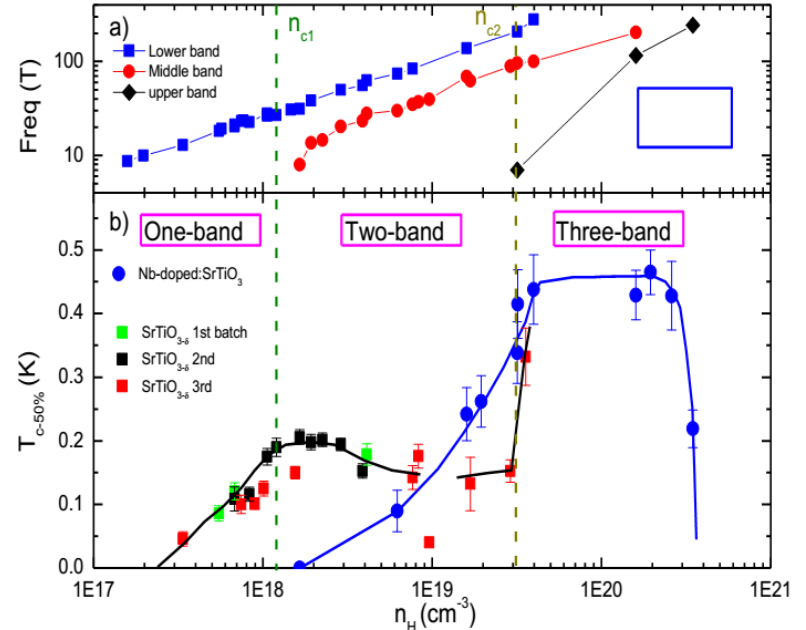
Appel 1980

- Negative U superconductivity?

Geballe and Kivelson 2014

- Longitudinal optical phonons?

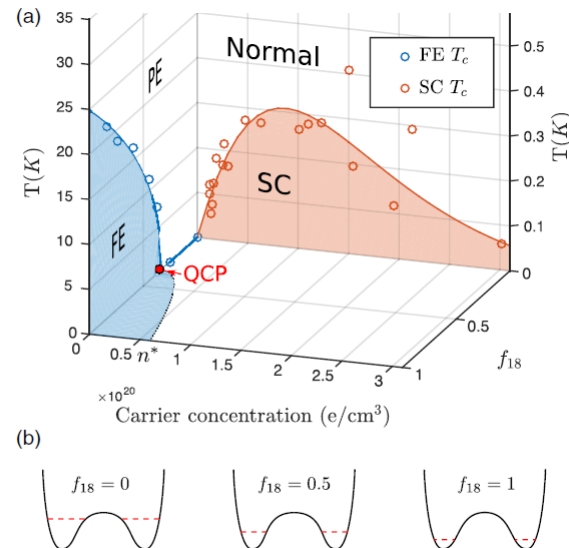
Gor'kov 2016

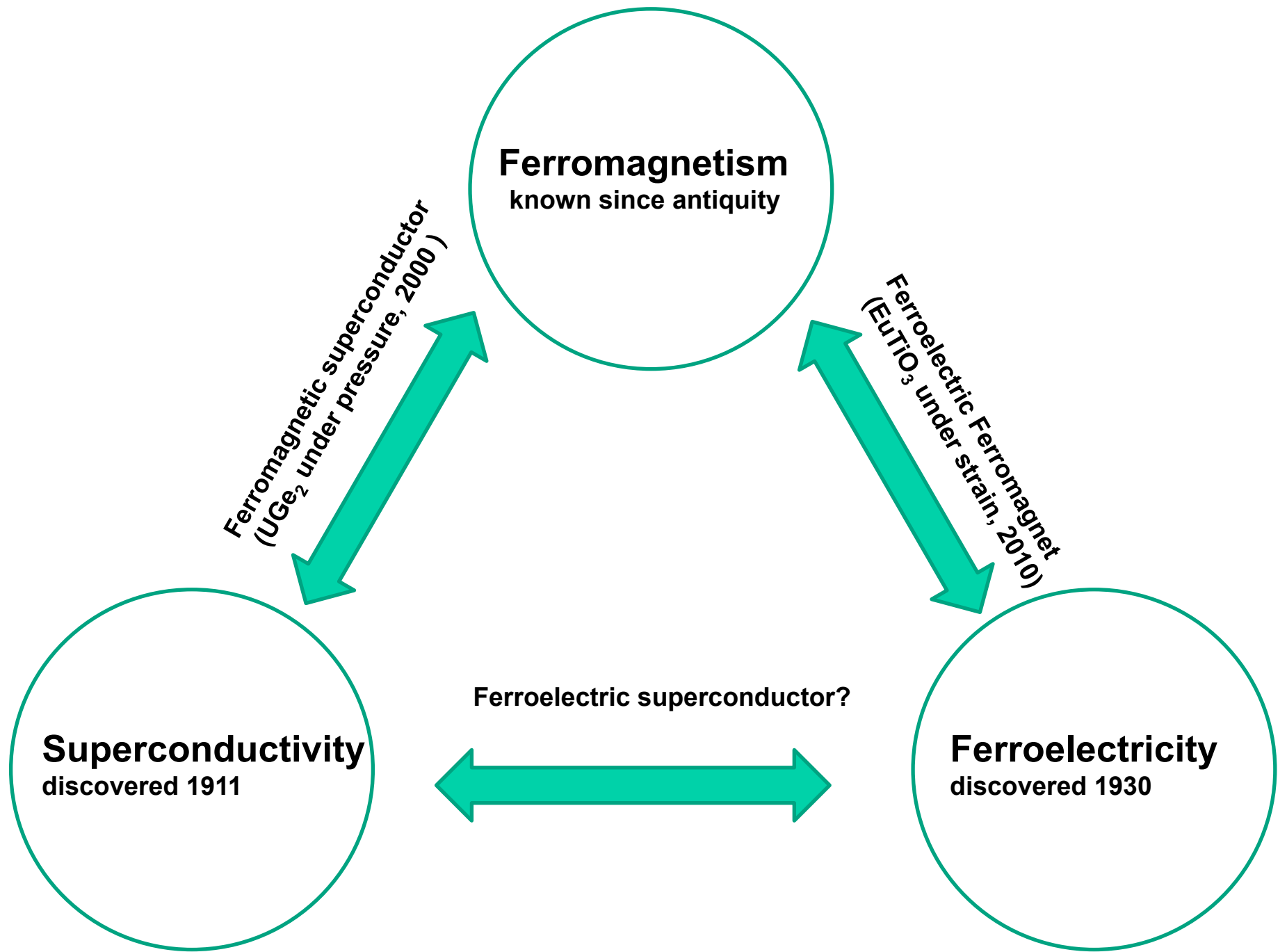


Ferroelectric quantum criticality?

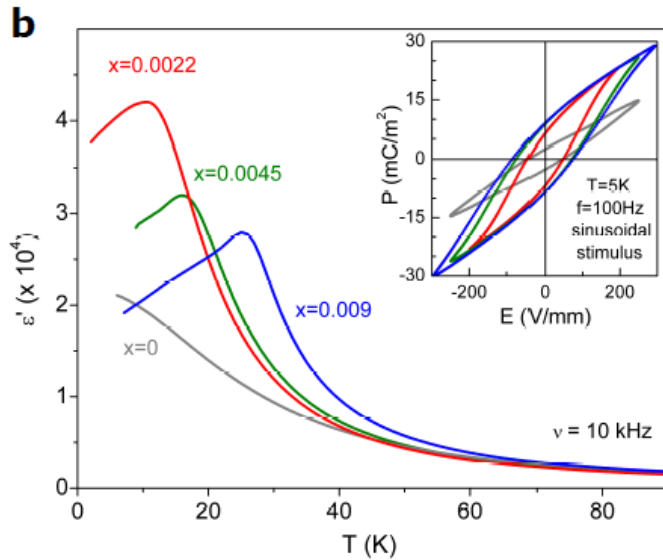
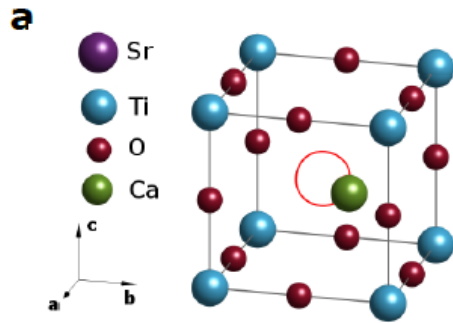
Lonzarich *et al.* 2014

Balatsky *et al.* 2015

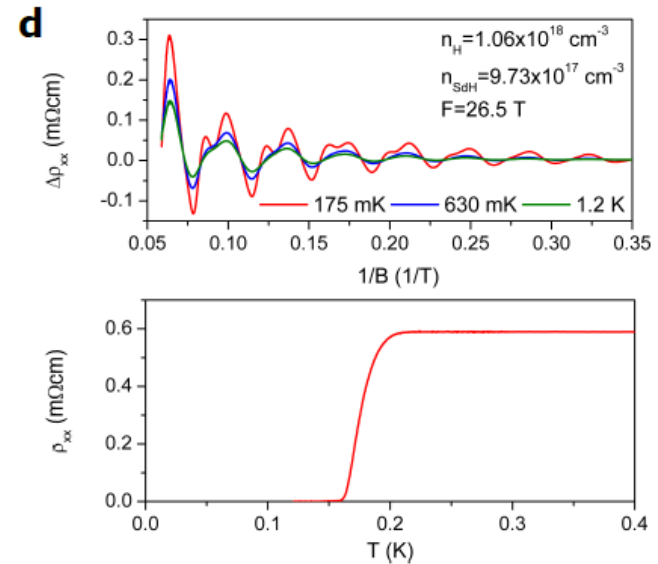
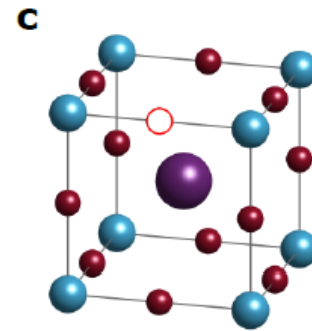




$\text{Sr}_{1-x}\text{Ca}_x\text{TiO}_3$ ($0.002 < x < 0.02$)
Ferroelectric

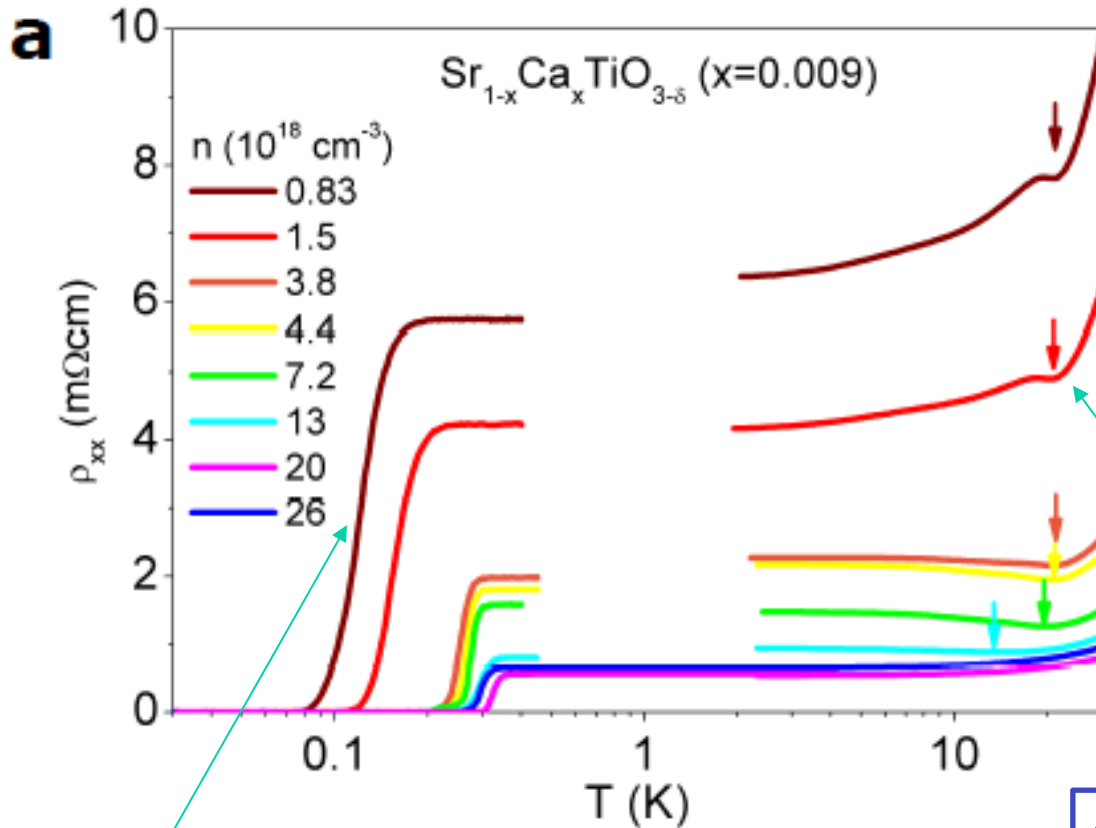


$\text{SrTiO}_{3-\delta}$ ($10^{-5} < \delta < 10^{-2}$)
Metal & superconductor

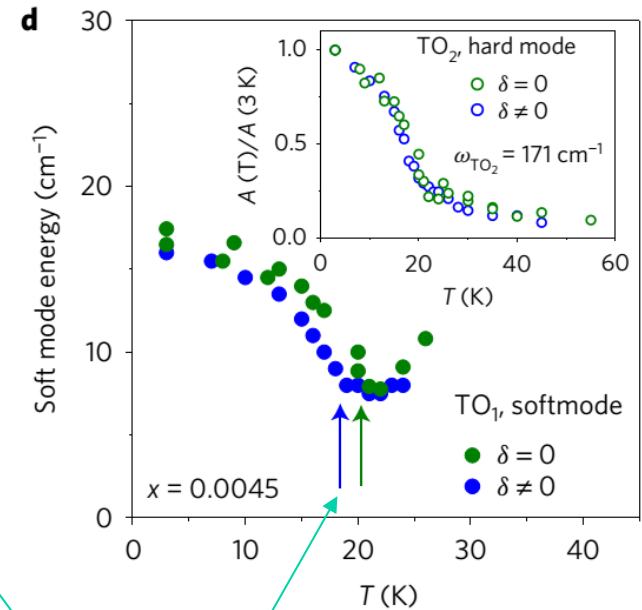


What happens to these two orders if both substitutions happen at the same time?

Two phase transitions in metallic samples

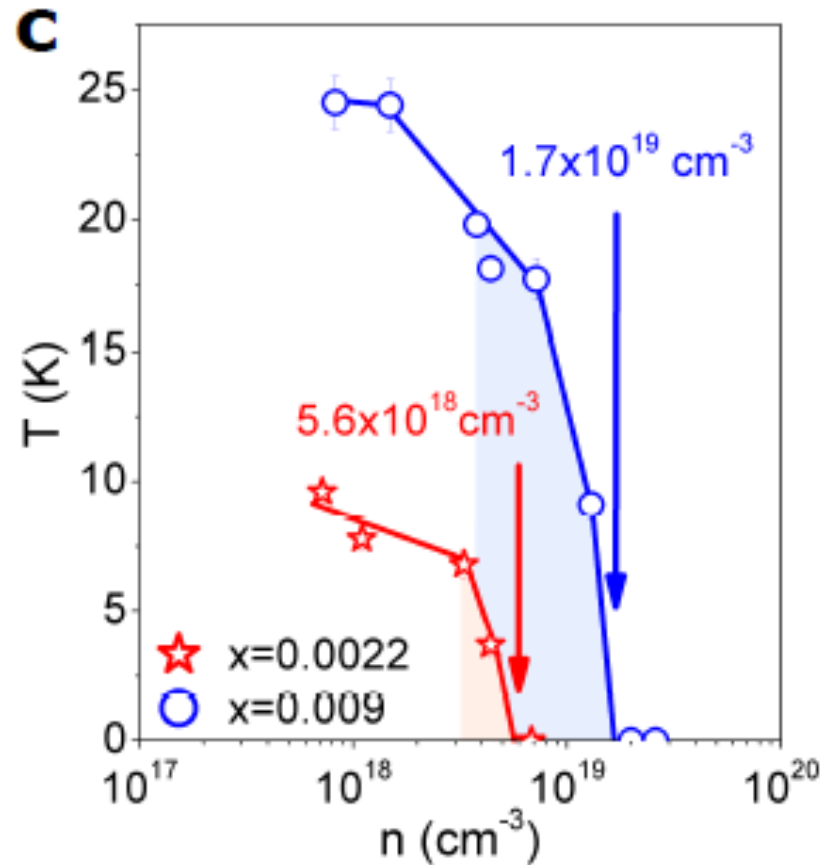
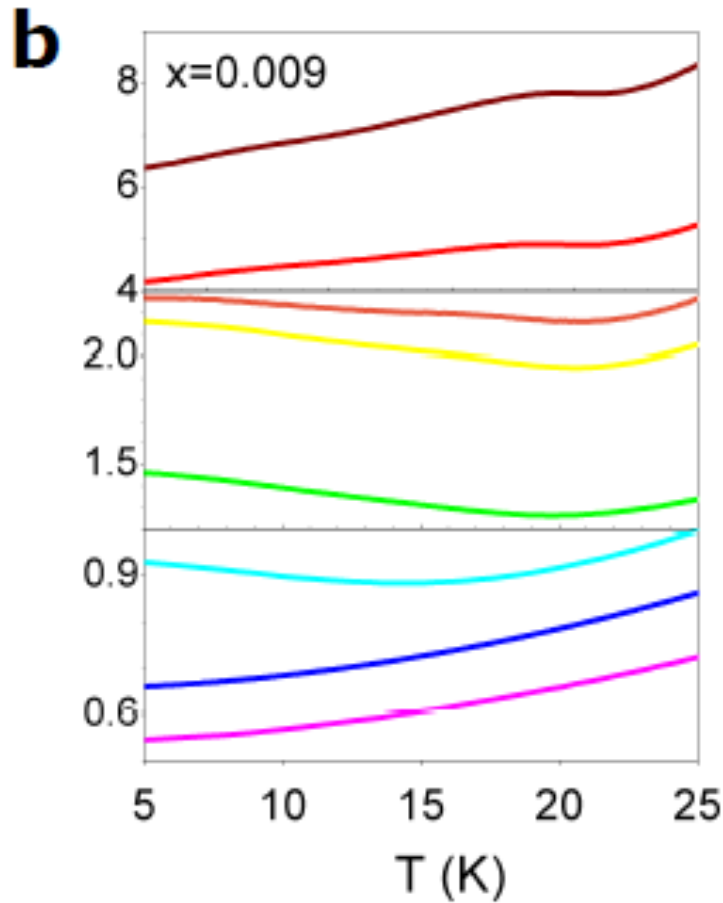


Superconducting transition



- Ferroelectric-like transition
- Raman scattering: no difference between the insulator and the dilute metal!

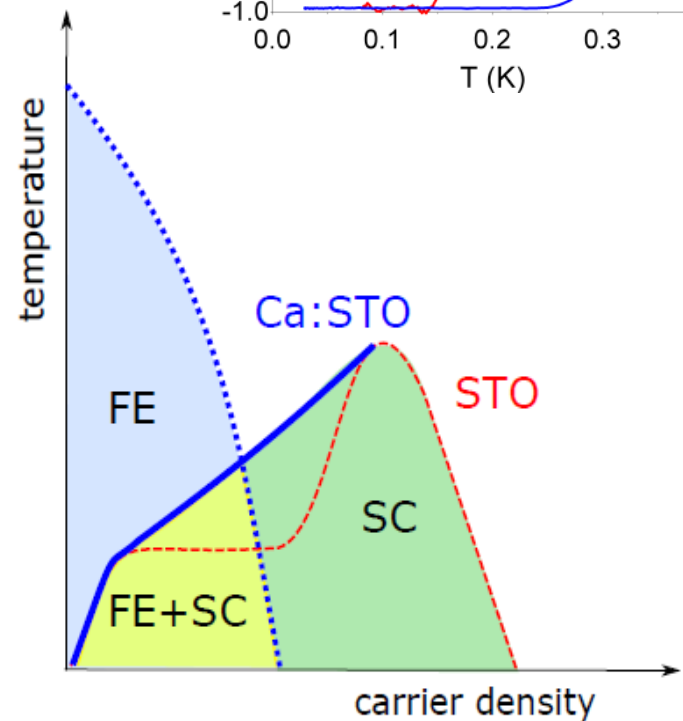
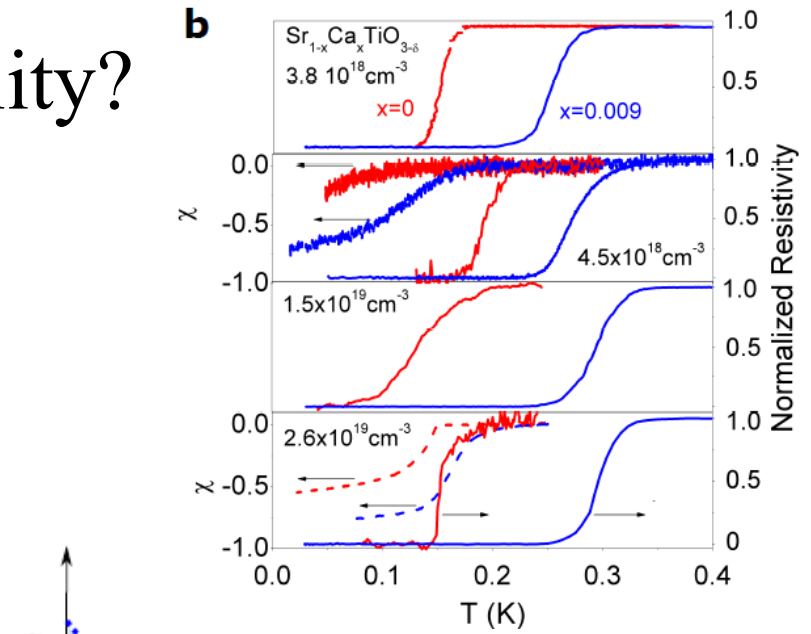
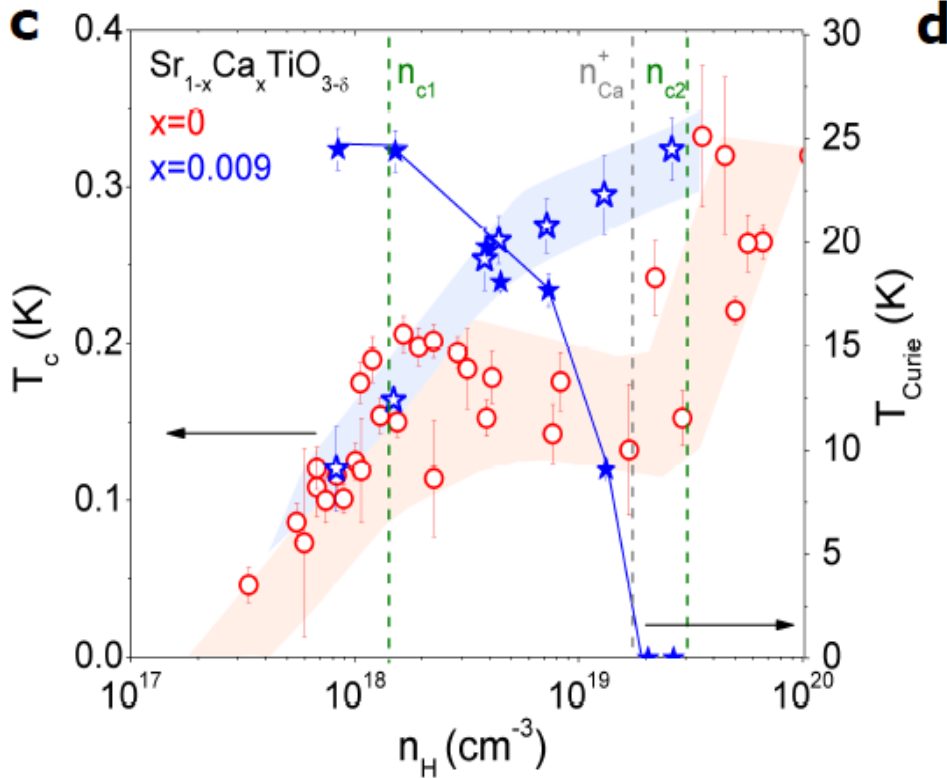
Destruction of the ferroelectric order



A Quantum Phase Transition at a critical doping, which scales with Ca content

Ferroelectric quantum criticality?

Rischau *et al.*, Nature Physics (2017)

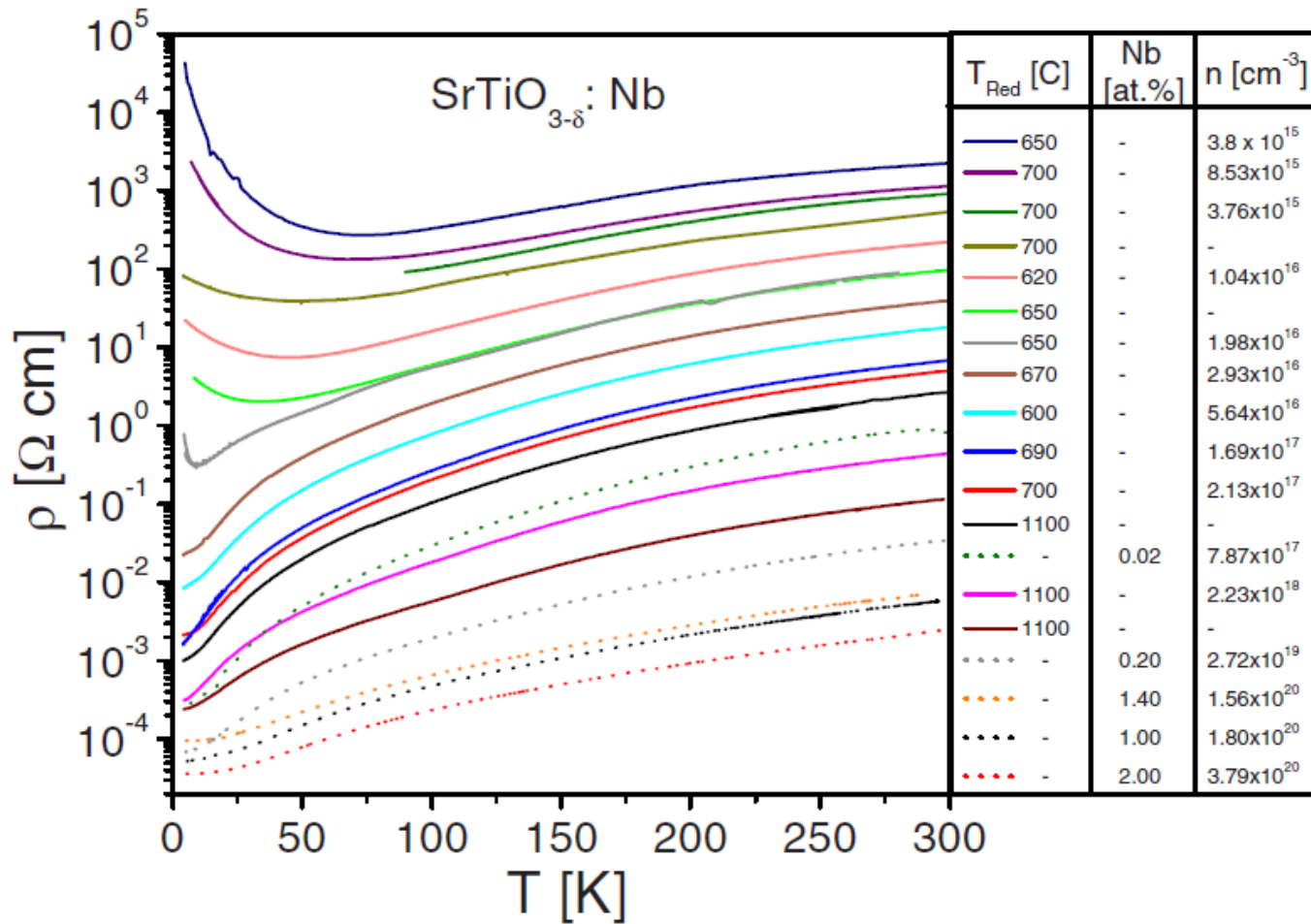


See also van der Marel's recent work on O^{18} substitution!

II. Mysteries of charge transport

Carrier density can be tuned over a wide range

Spinelli et al. PRB (2010)



$3.8 \cdot 10^{15} \text{ cm}^{-3}$

Reduced

Nb-doped

$3.8 \cdot 10^{20} \text{ cm}^{-3}$

Do we understand:

- Zero-temperature conductivity?

Yes, Mobility is high because Bohr radius is long!

- Conductivity below the degeneracy temperature?

Maybe, T-square resistivity is expected in a Fermi liquid, but...

- Conductivity at room-temperature and above?

No, the Drude picture leads to an unbearable paradox!

Mott criterion for metal-insulator transition

● ← STO

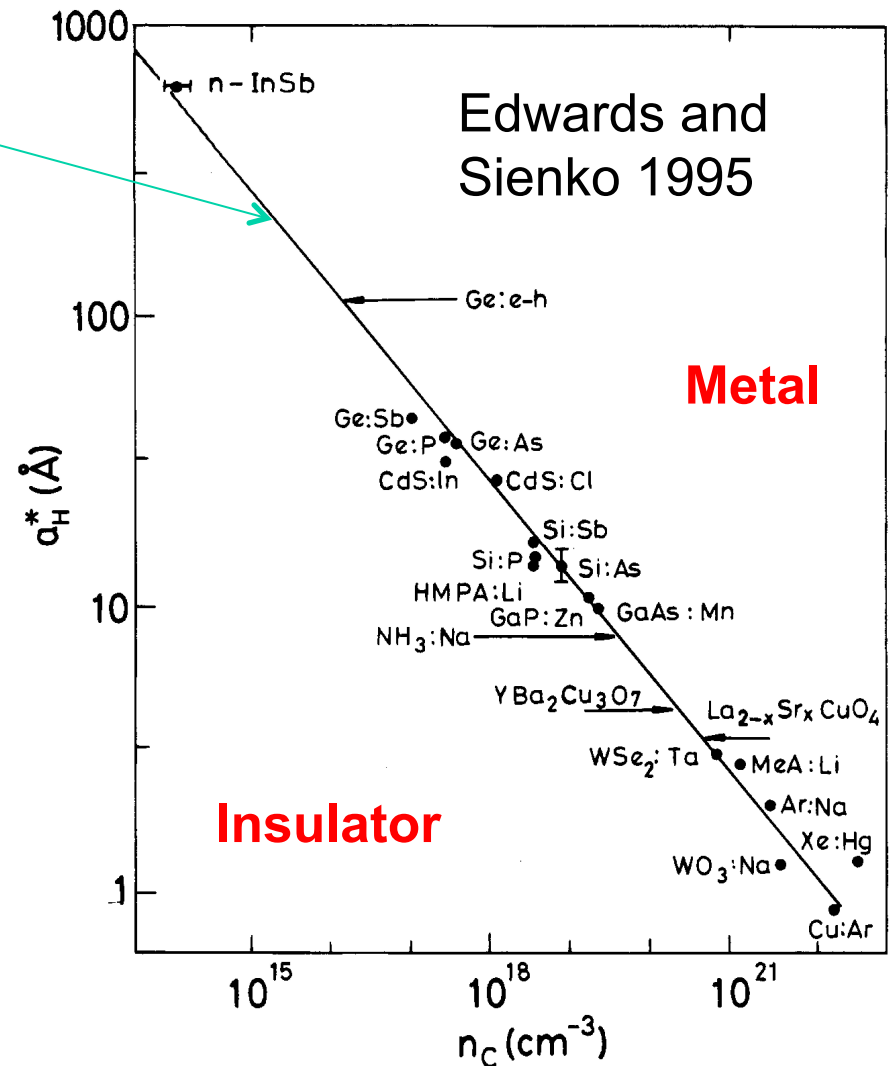
$$a_B n^{1/3} \cong 0.25$$

In STO

$$m^* = 1.8 m_e$$

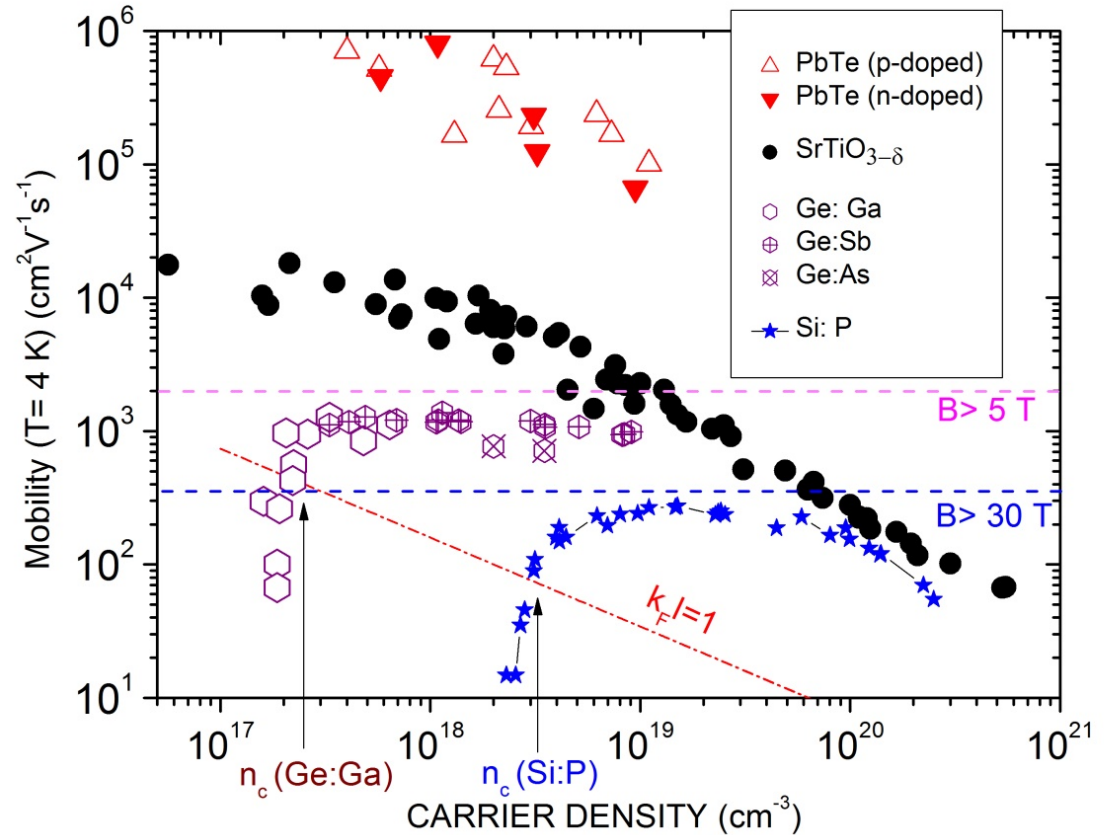
$$\epsilon = 1.8 \cdot 10^4$$

$$a_B = 600 \text{ \AA}$$



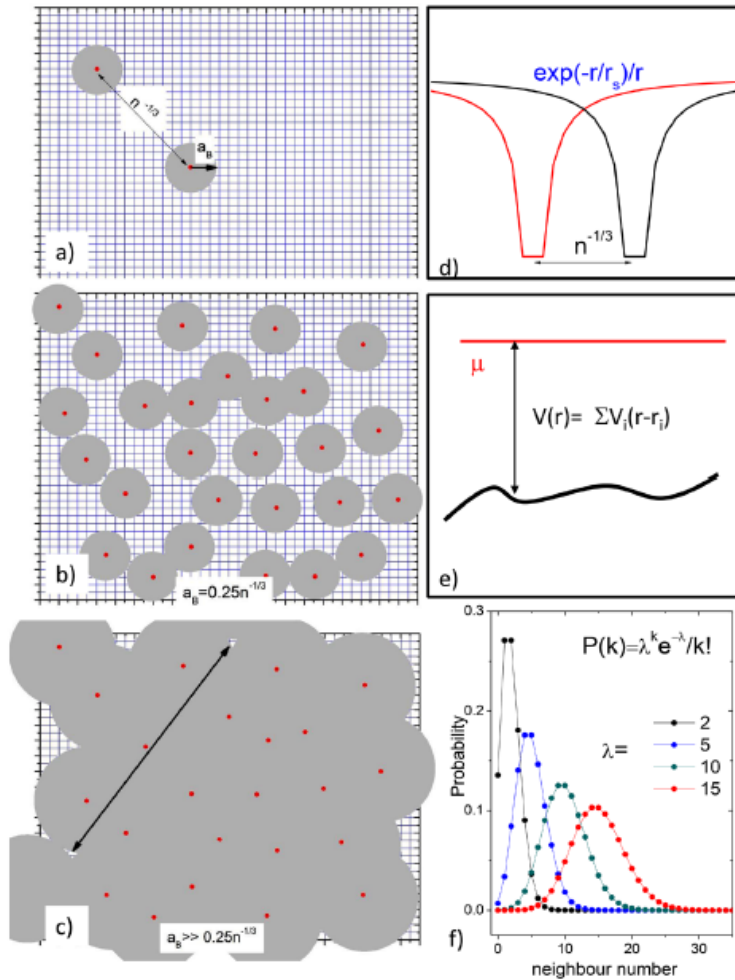
Electron mobility and the Bohr radius in shallow Fermi seas

	Dielec. (ϵ_r/ϵ_0)	m^*/m_0	Bohr radius (nm)
STO	20000	1.8	600
PbTe	1000	0.07	800
Si	12.5	0.45	1.5
Ge	16	0.24	3.5

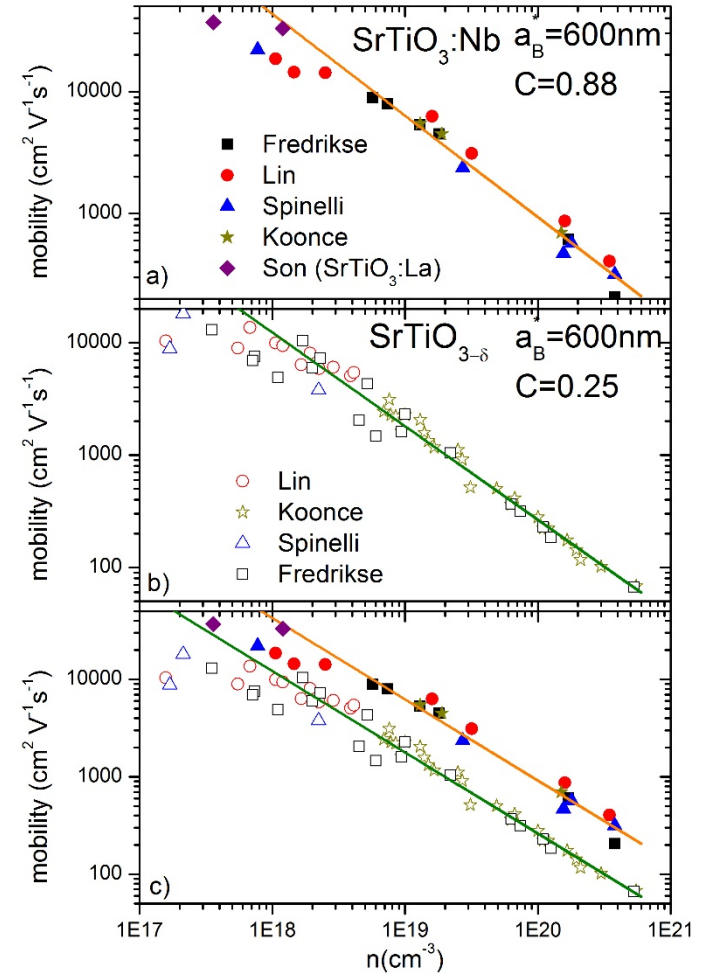


The longer the Bohr radius, the higher the mobility!

Electron mobility depends on Bohr radius



Probability of islands when the Fermi seafloor is carved by a random distribution of dopants

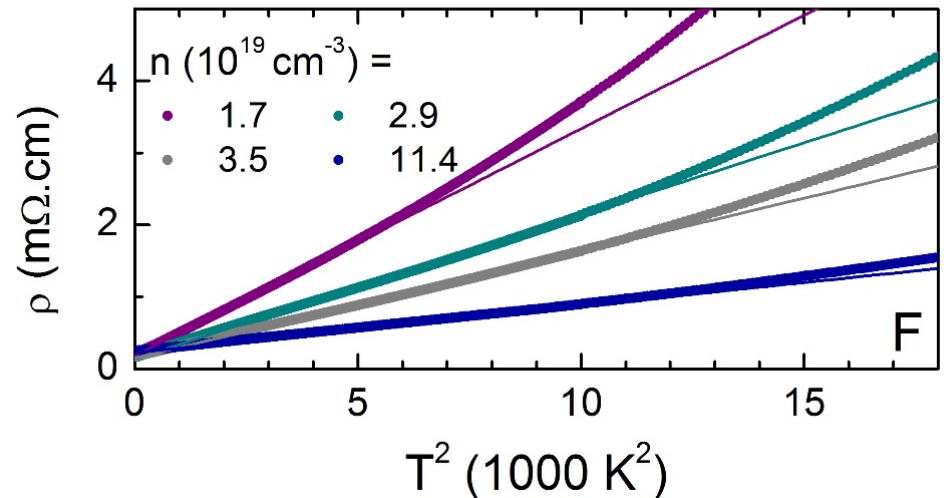
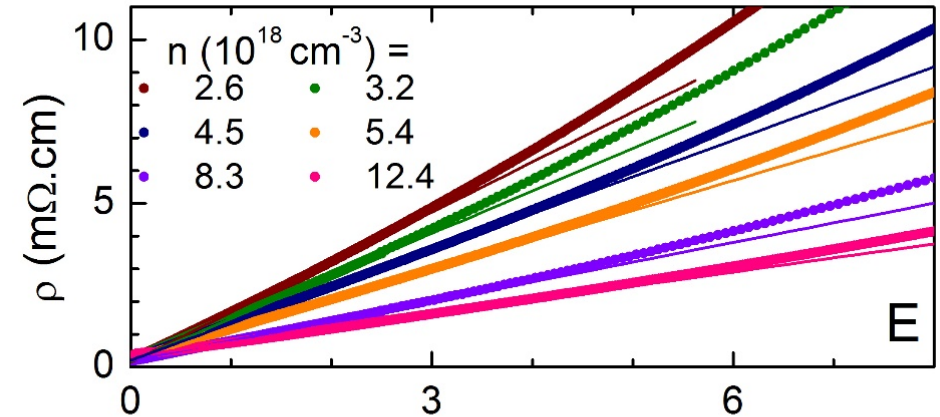
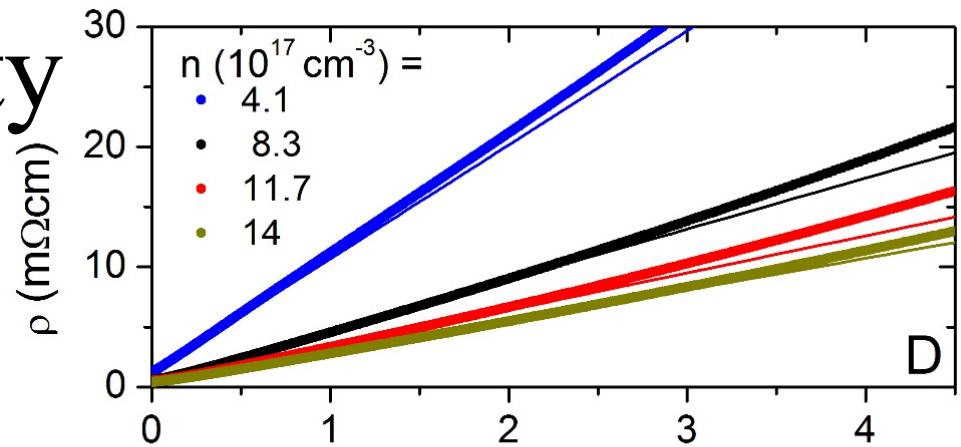


$$\mu \propto a_B^{-1/2} n^{-5/6}$$

T-square resistivity

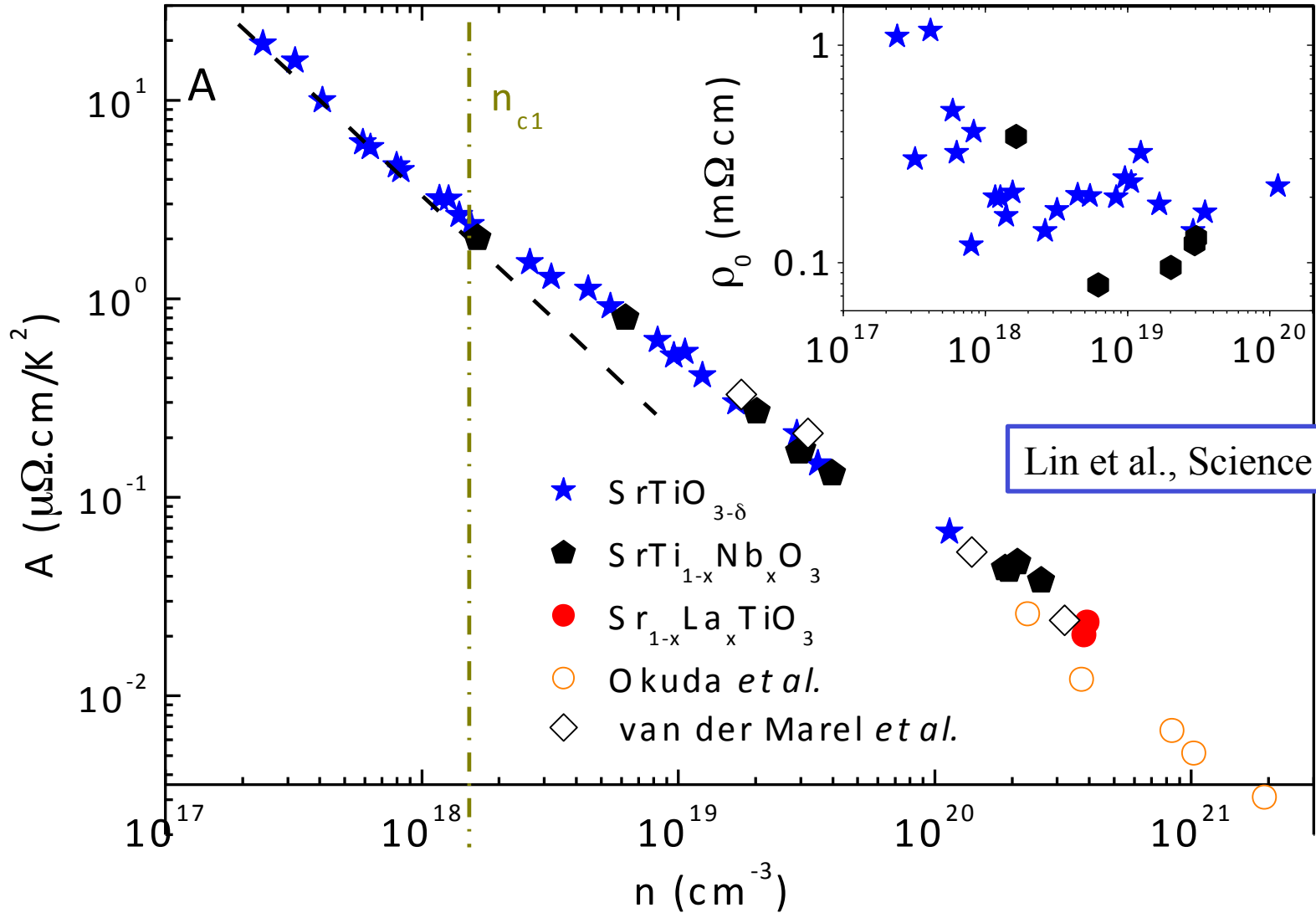
$$\rho = \rho_0 + AT^2$$

- Resistivity remains T^2 even for $n \sim 10^{-5}$
- The prefactor of T^2 resistivity decreases with increasing doping.



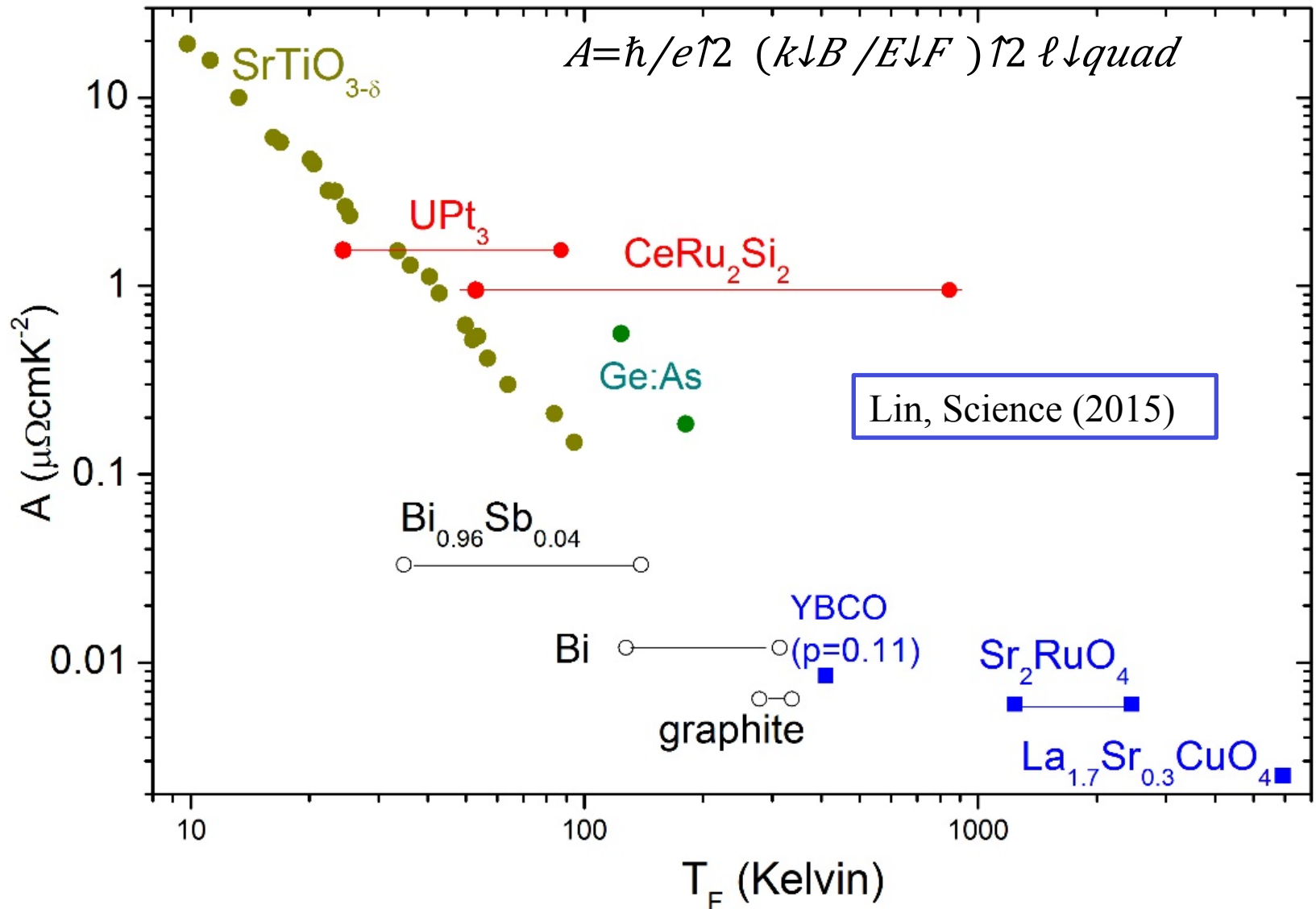
Lin et al., Science (2015)

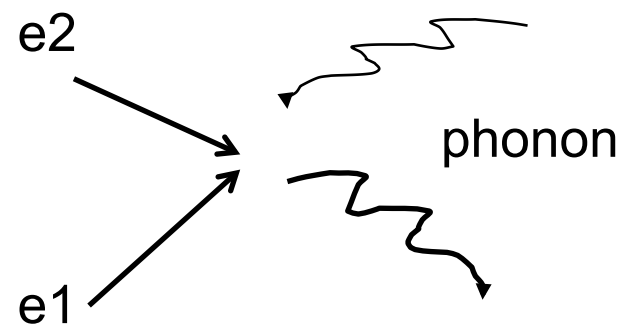
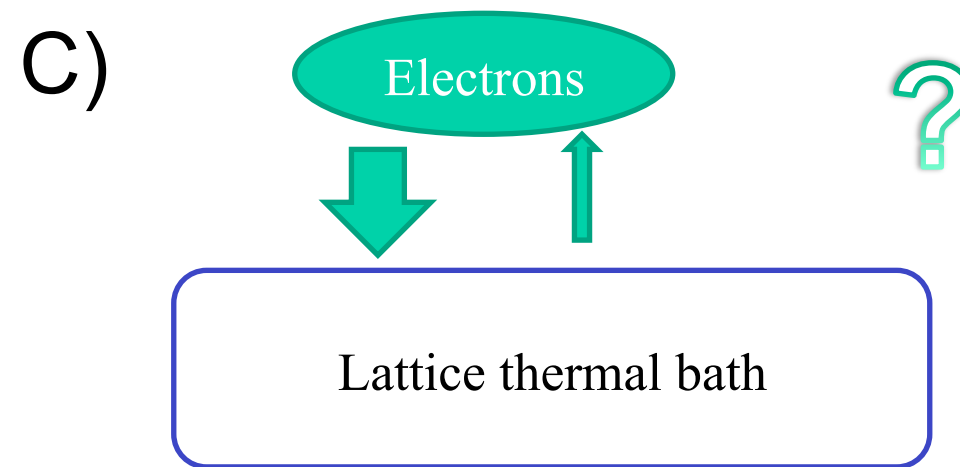
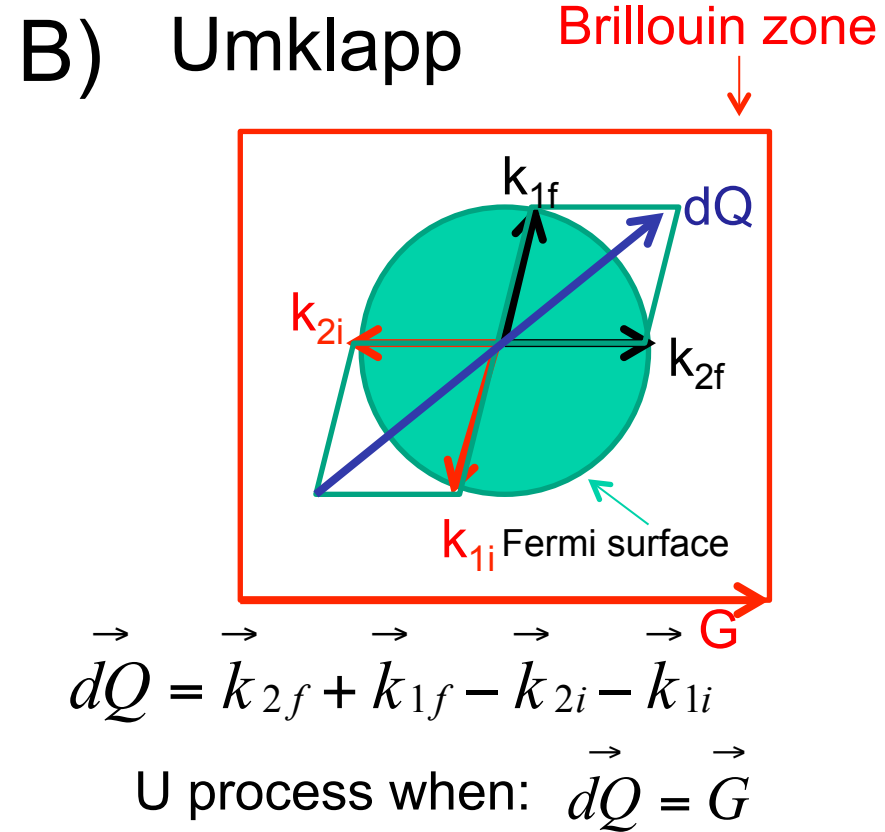
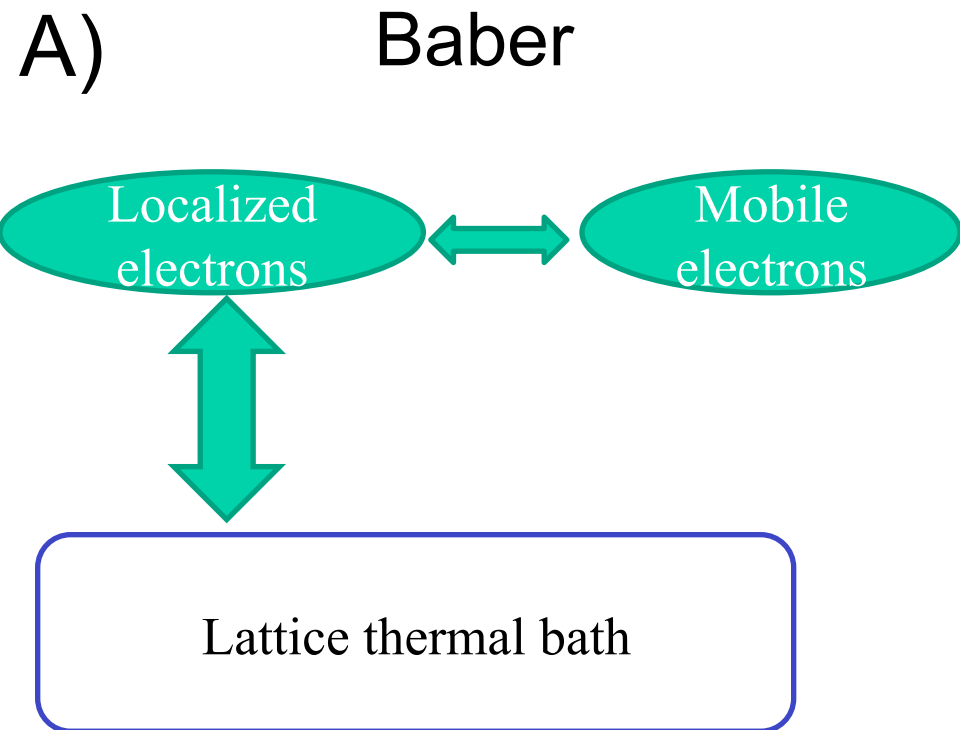
T-square resistivity



A can be tuned over four orders of magnitude!

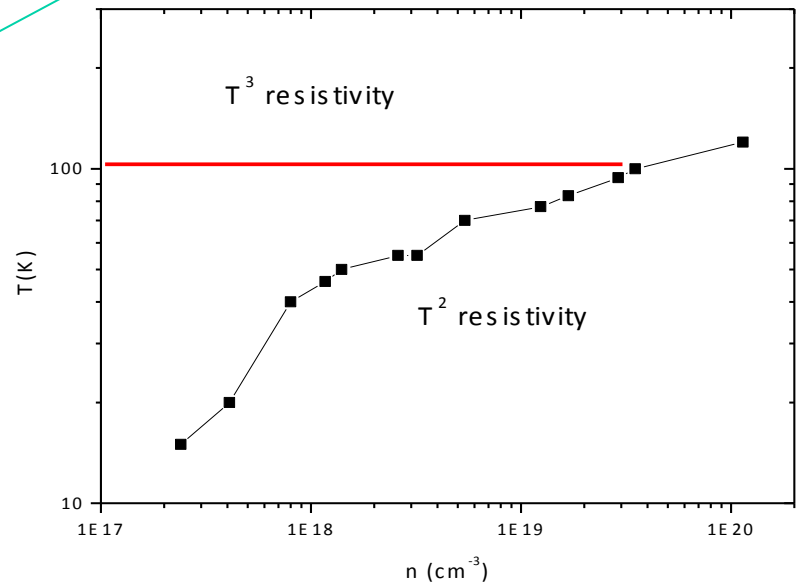
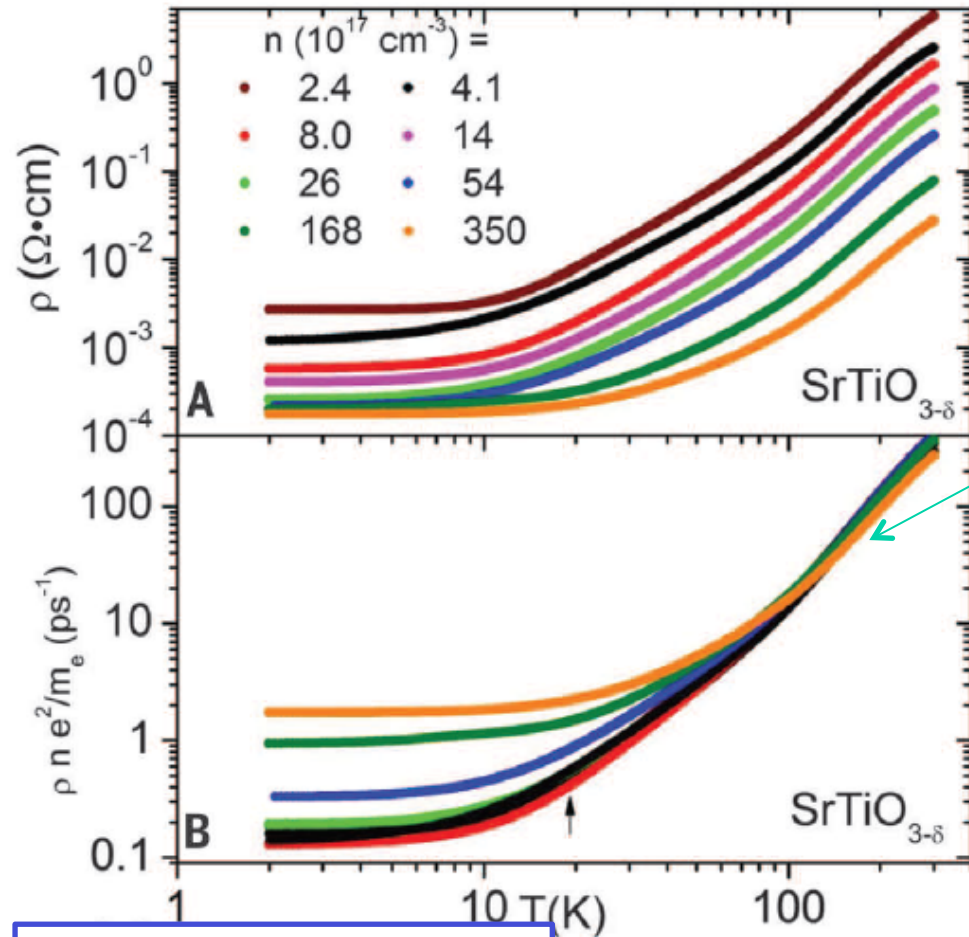
T-square resistivity





How can e-e scattering relax momentum in absence of A and B ?

High temperature cubic resistivity



Lin et al., Science (2015)

Electron Mobility in Semiconducting Strontium Titanate

O. N. TUFT AND P. W. CHAPMAN
 Honeywell Corporate Research Center, Hopkins, Minnesota
 (Received 18 October 1966)

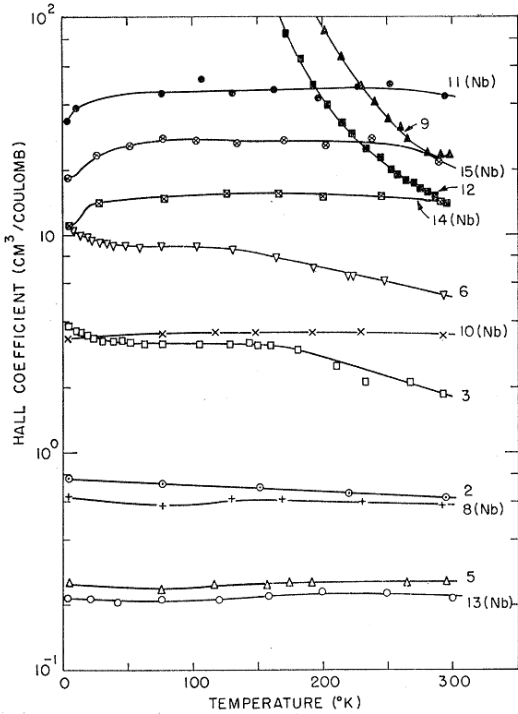


FIG. 2. The temperature dependence of the Hall coefficient in semiconducting SrTiO₃.

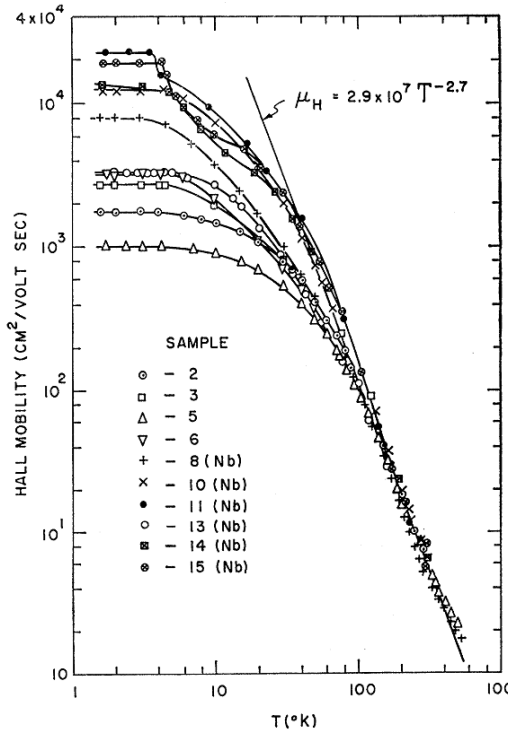
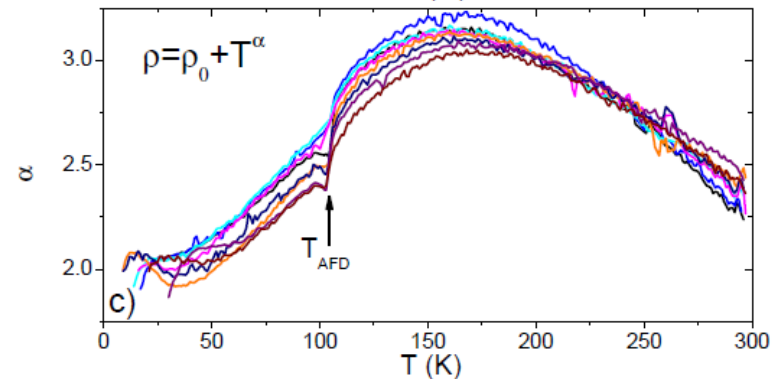
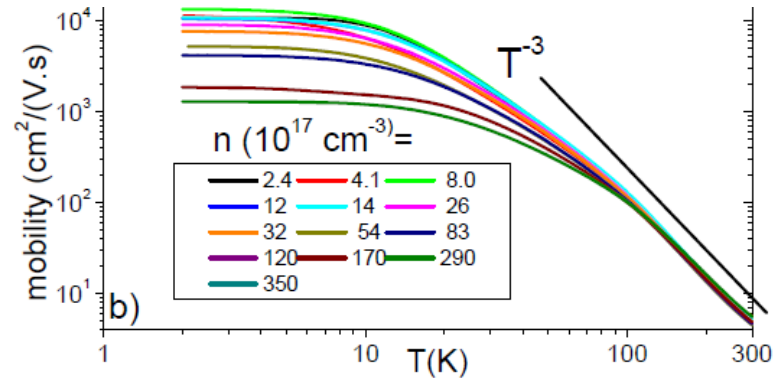


FIG. 4. The temperature dependence of the Hall mobility in semiconducting SrTiO₃.

Lin *et al.*
 NPJ Quant. Mat. 2017



Similar data by

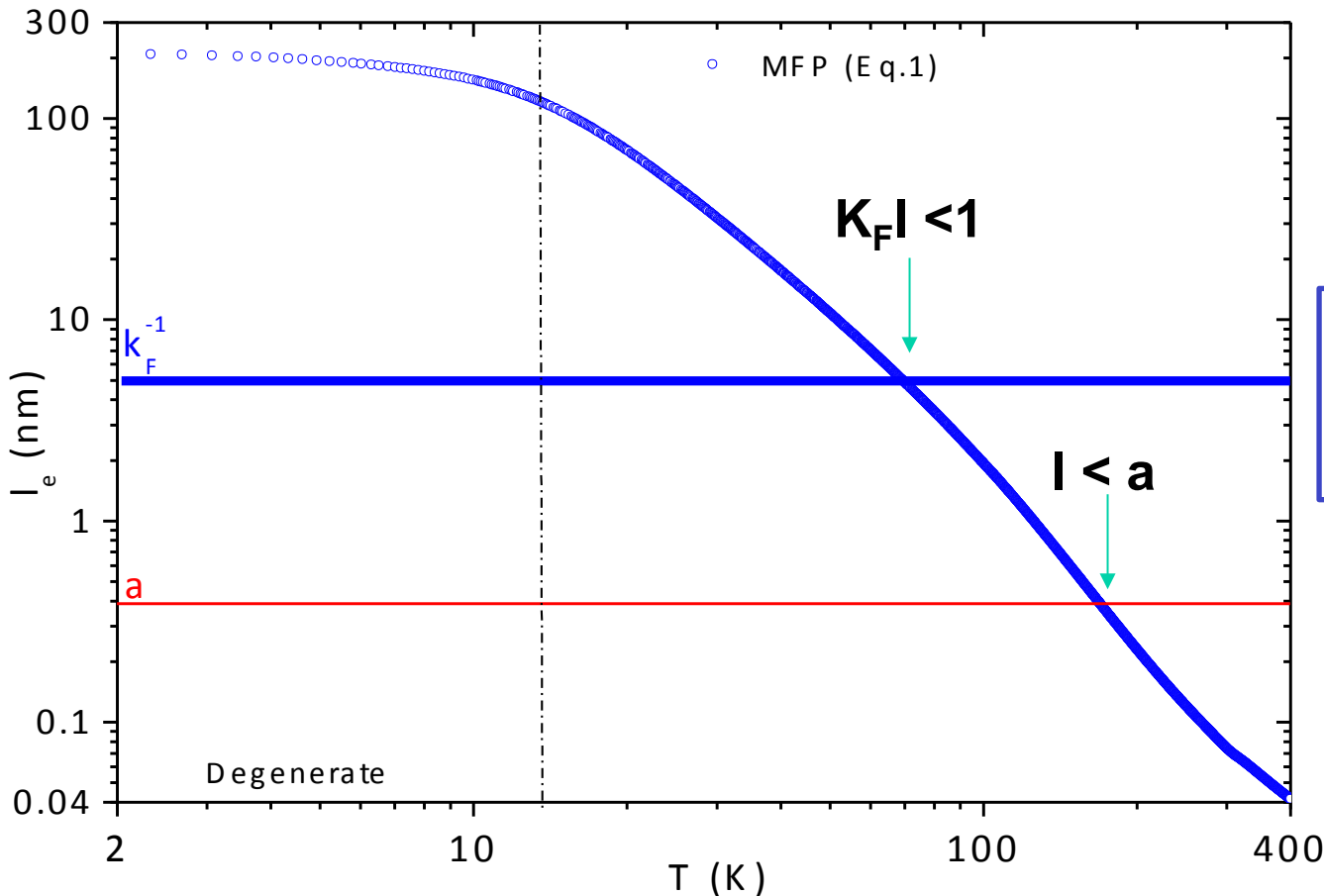
- Susanne Stemmer group, Santa Barbara
- Chris Leighton group, Minesotta
- Bhattacharya et al., Argonne

Drude conductivity

$$\sigma = \frac{ne^2\tau}{m^*} = \frac{1}{3\pi^2} \frac{e^2}{\hbar} k_F^2 \ell_e$$

Knowing the carrier concentration, one can extract mean-free-path!

SrTiO_{3-δ} $n = 3.7 \cdot 10^{17} \text{ cm}^{-3}$

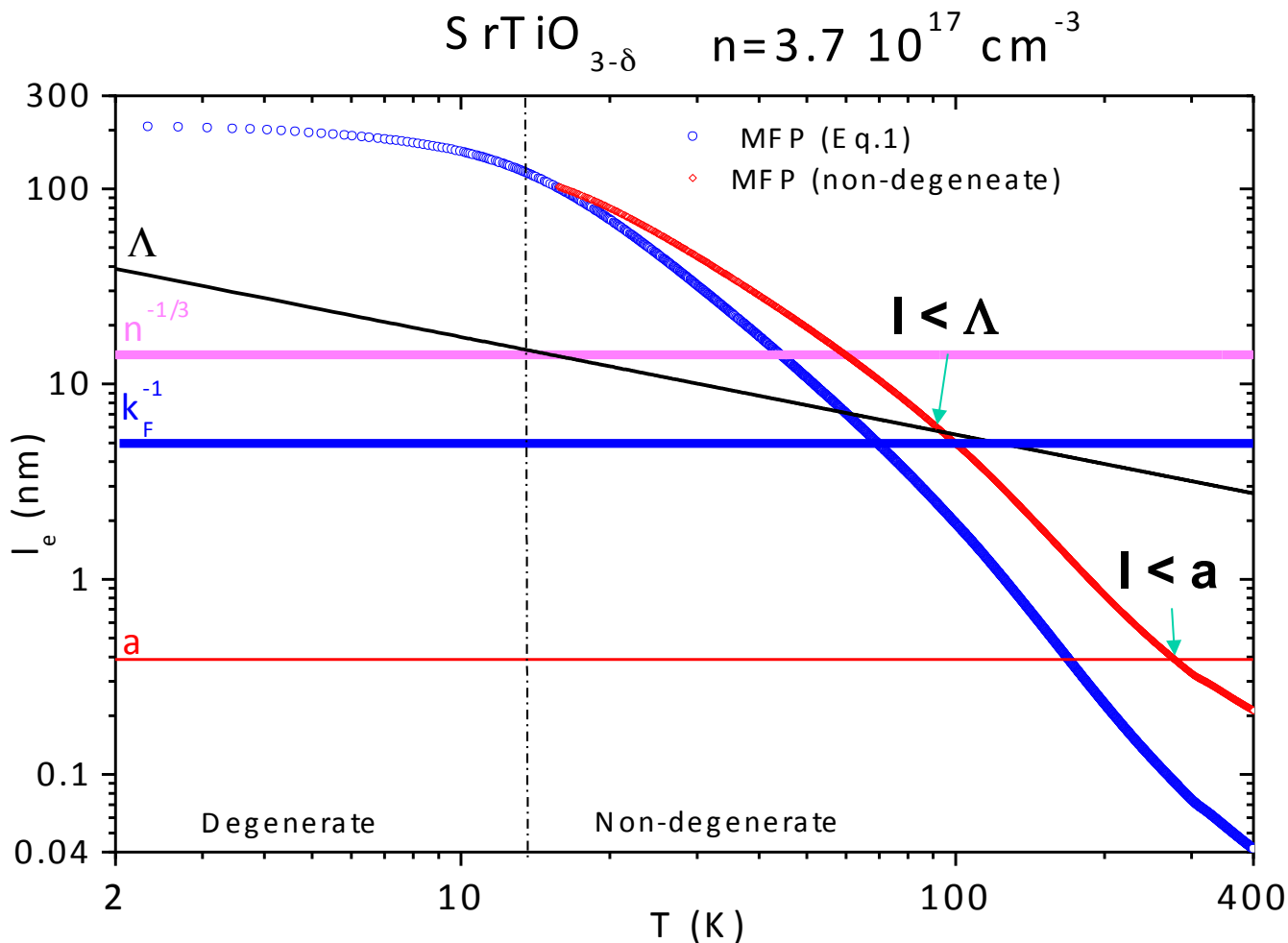


Mean-free-path below two conceivable floors!

Drude conductivity

$$\sigma = \frac{ne^2\tau}{m^*} = \frac{1}{3\pi^2} \frac{e^2}{\hbar} k_F^2 \ell_e$$

For non-degenerate carriers, velocity is thermal (and not Fermi).

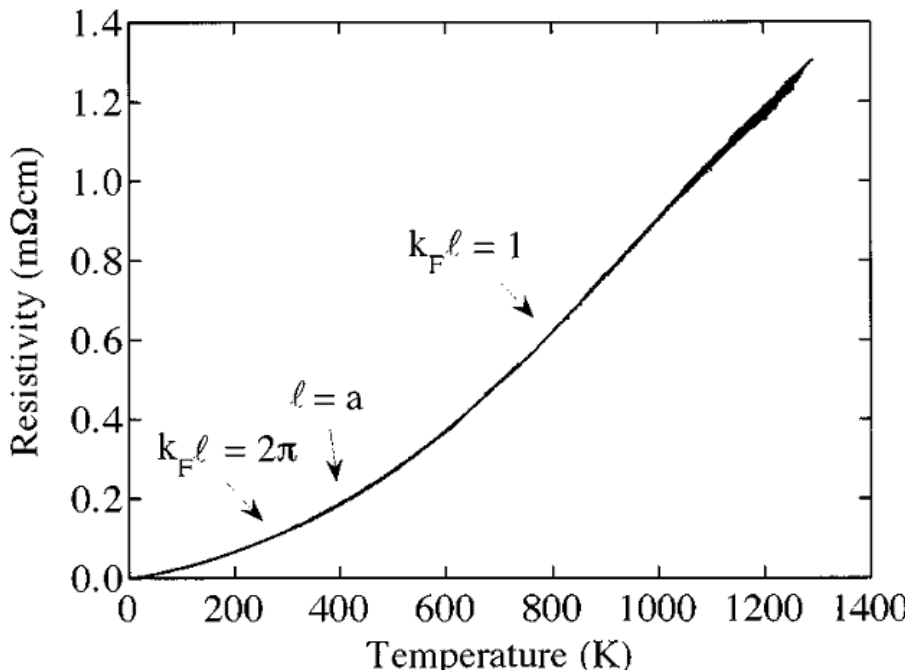


$$v \downarrow F = \hbar k \downarrow F / m \uparrow *$$

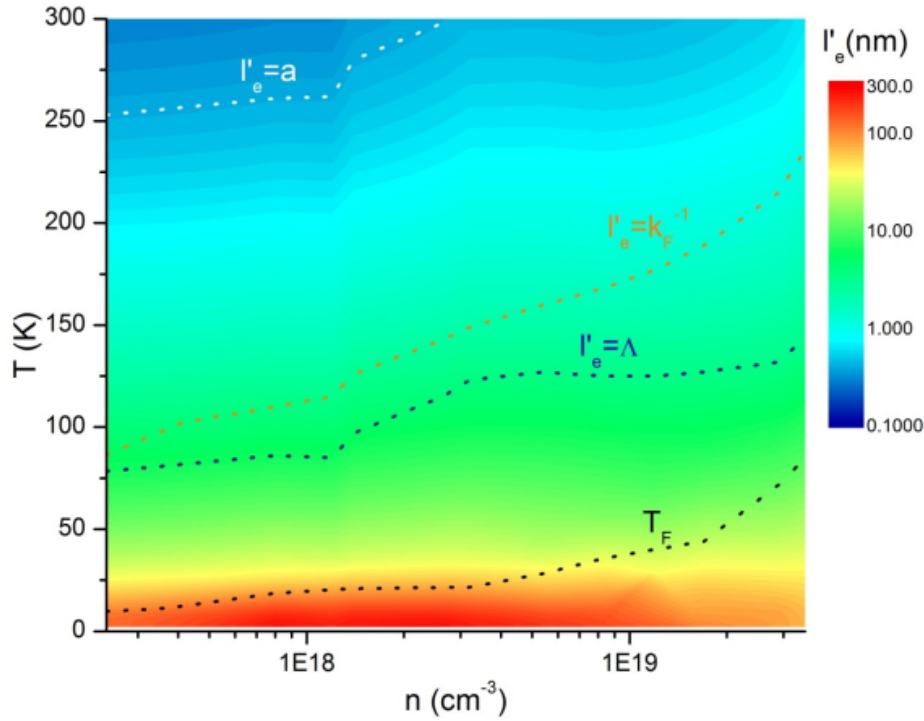
$$v \downarrow T = \sqrt{2 k_B T / m}$$

Mean-free-path falls below any conceivable floor!

The MIR limit exceeded at cryogenic temperatures!



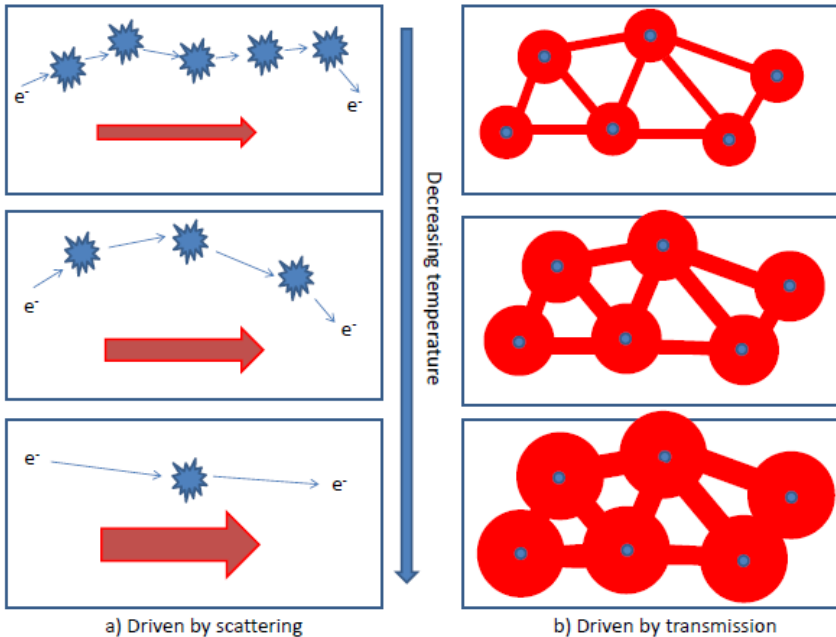
Sr_2RuO_4
 Tyler et al. 1998



$\text{SrTiO}_{3-\delta}$

Compound	k_F (nm^{-1})	$\rho(400\text{K})$ ($\text{m}\Omega\text{cm}$)	$\ell(400\text{K})(\text{nm})$	$k_F \ell$ (400K)	$T_{k_F \ell = 1}$ (K)	Reference
$\text{La}_{1.72}\text{Sr}_{0.18}\text{CuO}_4$	6.6	0.32	0.35	2.2	850	[4, 34]
$\text{YBa}_2\text{Cu}_4\text{O}_8$	1.4	0.36	0.8	1.1	450	[35, 36]
Sr_2RuO_4	5.6	0.2	0.38	2.1	800	[37]
$\text{SrTiO}_{3-\delta}$ ($n=4 \cdot 10^{17} \text{ cm}^{-3}$)	0.24	5900	0.03	0.07	70	This work

Two pictures of metallicity



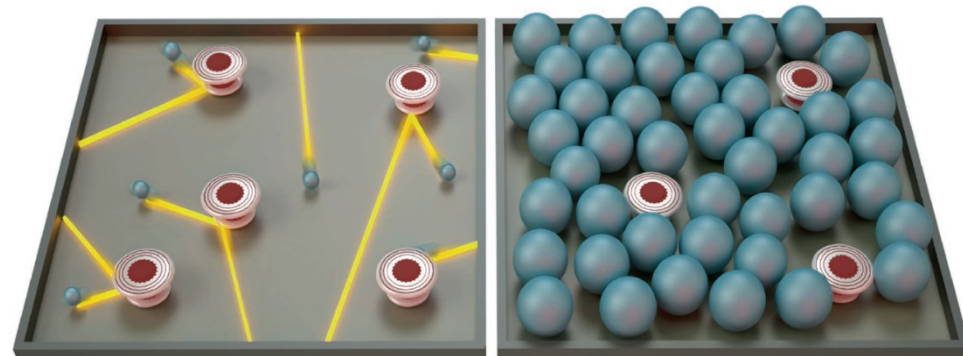
Lattice polarizability

PHYSICS

Electrons go with the flow in exotic material systems

Electronic hydrodynamic flow—making electrons flow like a fluid—has been observed

By Jan Zaanen



Distinguishing different flow regimes. (Left) In conventional metals, the flow of electrical current is due to electrons (balls) moving independently as a consequence of quantum physics while scattering against crystal imperfections (bumpers). (Right) In normal fluids such as water, the molecules collide with each other, equilibrating in a macroscopic fluid that is described by the theory of hydrodynamics. Electrons in particular solids that form strongly interacting quantum systems are also found to exhibit hydrodynamic transport properties (1–3).

Electron viscosity

Theory of universal incoherent metallic transport

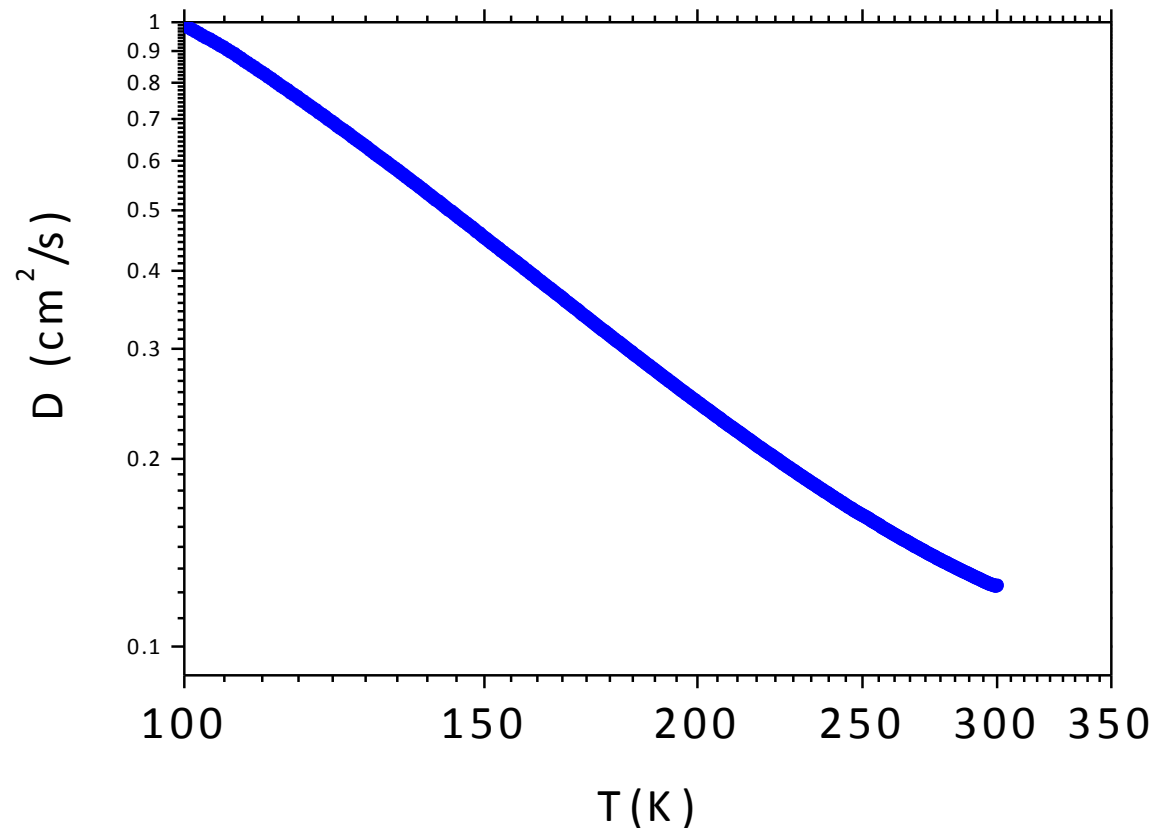
Sean A. Hartnoll

An upper boundary to the magnitude of diffusion constant

$$D \lesssim \hbar v_F^2 / (k_B T)$$

Charge diffusion constant can be extracted from mobility

$$D = \frac{\mu k_B T}{e}$$



Charge diffusion constant in dilute metallic STO

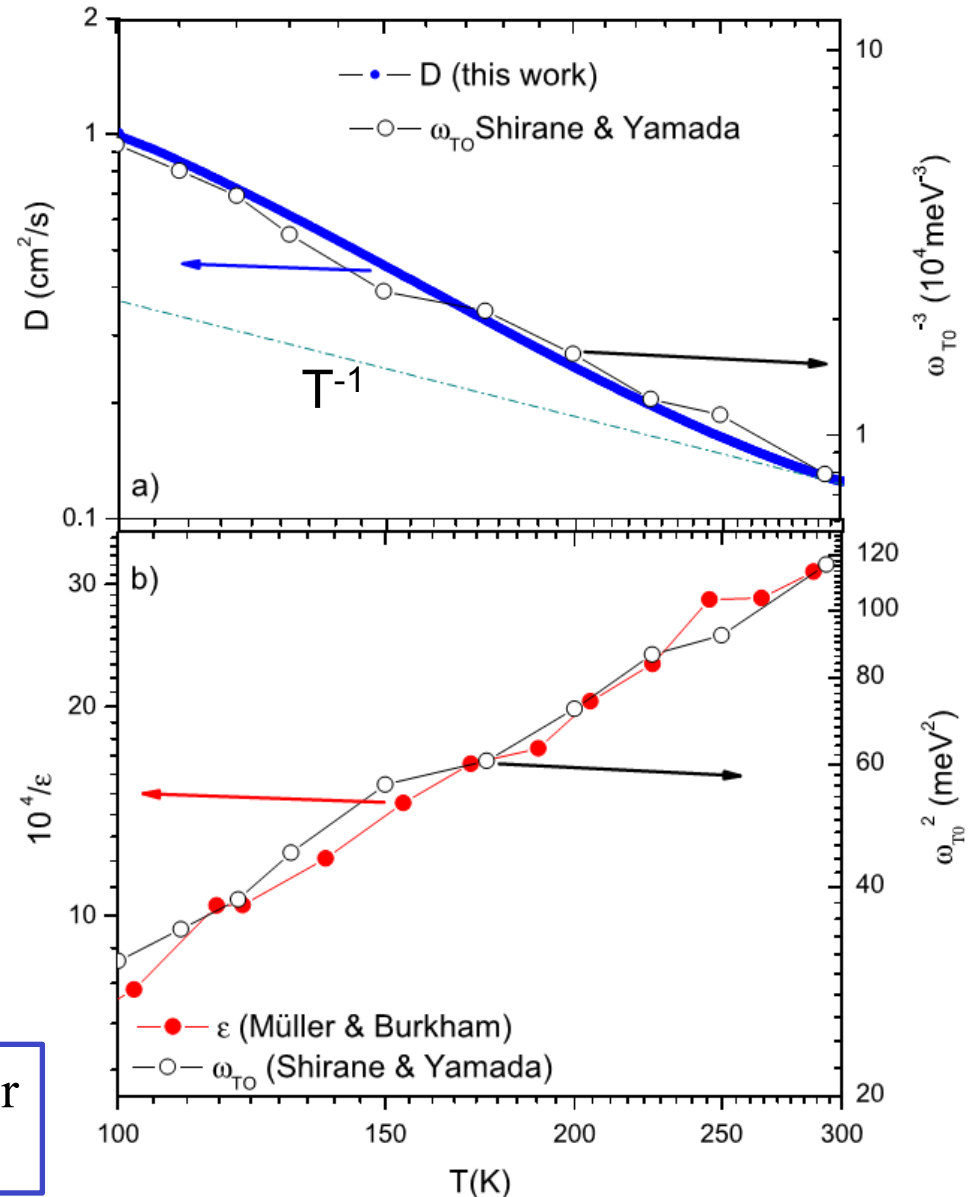
- decreases faster than T^{-1}
- tracks permittivity (or TO frequency)

$$D = \frac{\mu k_B T}{e}$$

$$\mu k_B T \propto \omega_{TO}^{-3} \propto \epsilon / \omega_{TO}$$

$$\frac{\epsilon_0}{\epsilon_\infty} = \left(\frac{\omega_L}{\omega_T} \right)^2$$

Lyddane-Sachs-Teller
scaling

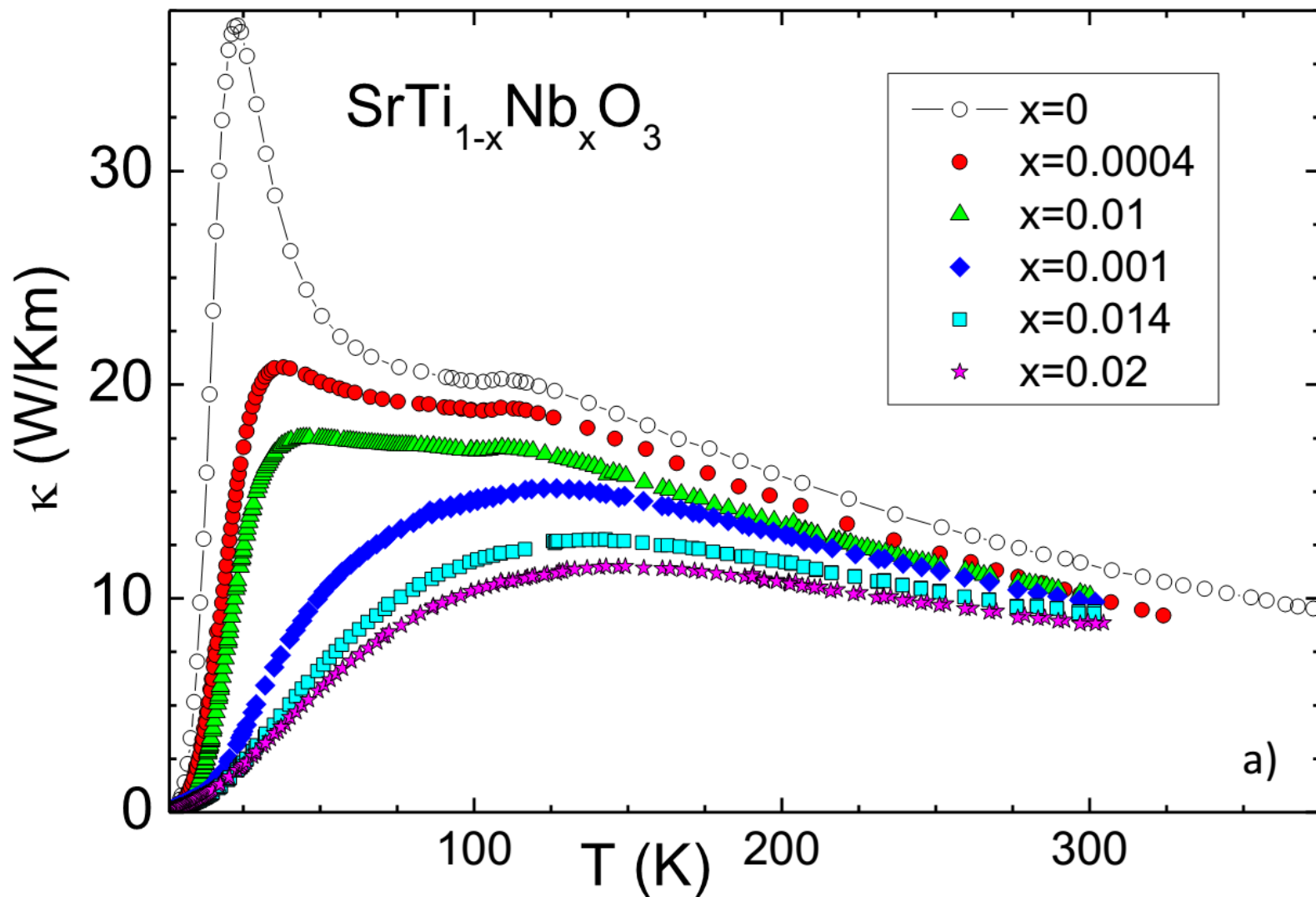


III. Heat transport

Poiseuille flow

Thermal diffusivity

Martelli et al. unpublished



Thermal conductivity of insulators

$$\kappa = 1/3 C v l$$

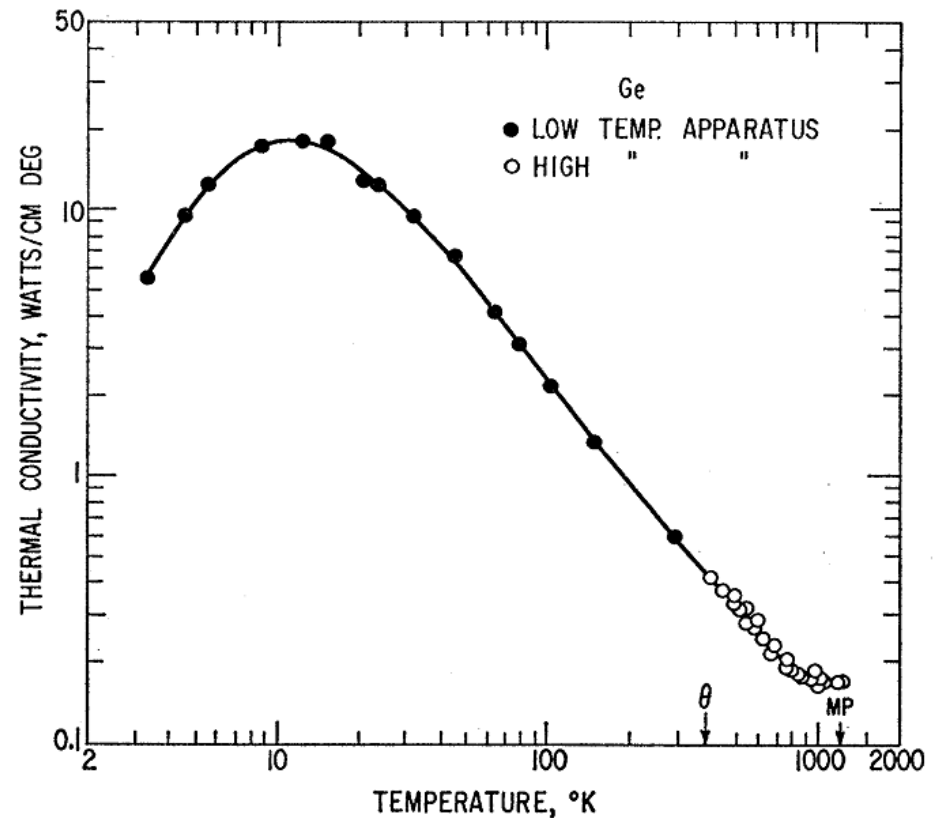
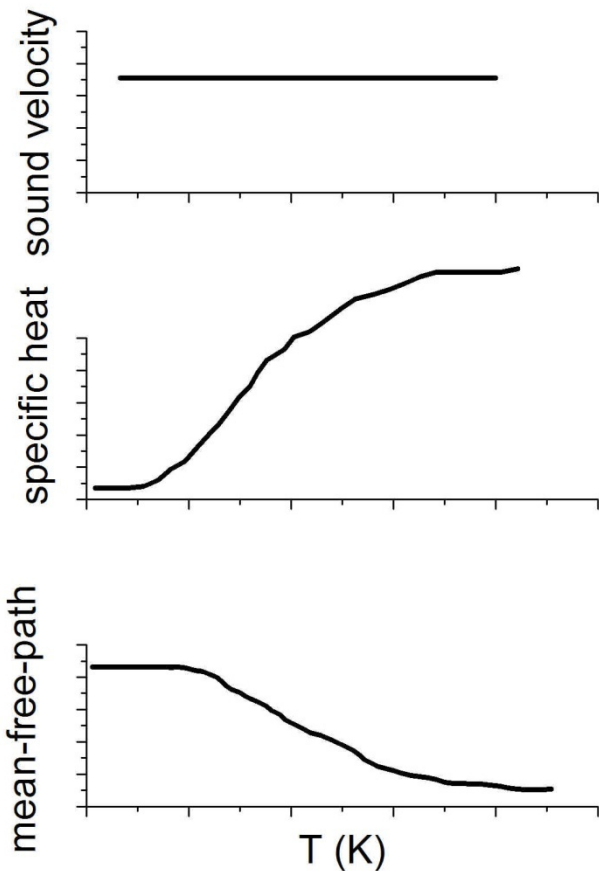


FIG. 5. The low- and high-temperature K versus T results for Ge.

Phonon hydrodynamics

Table 1 | Classification of thermal transport regimes.

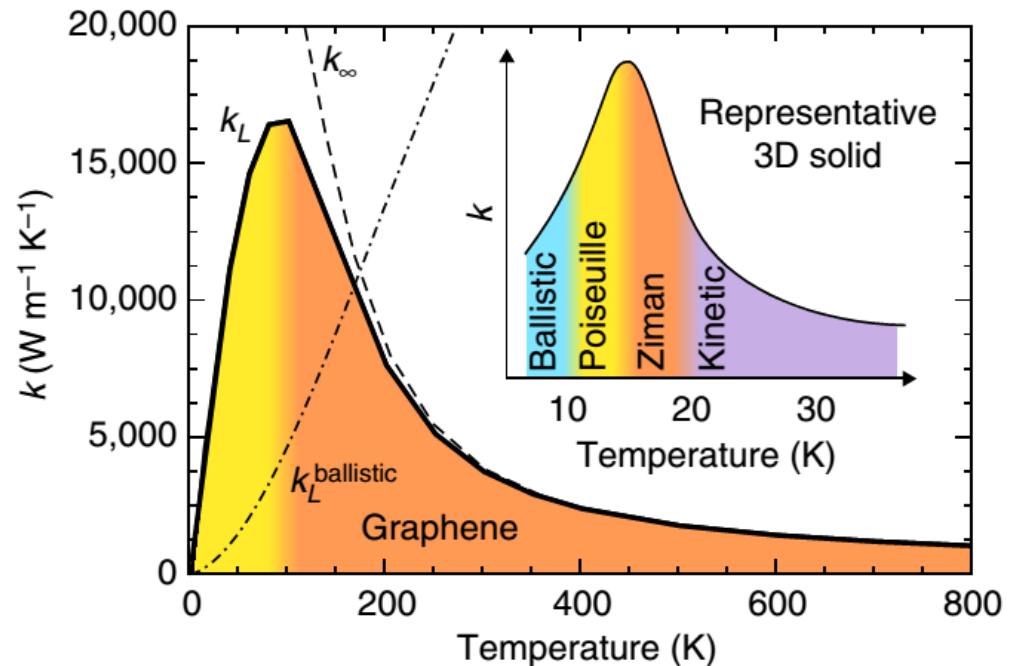
Ballistic	$\mathcal{E} \gg \mathcal{N}$ and $\mathcal{E} \gg \mathcal{R}$
Poiseuille	$\mathcal{N} \gg \mathcal{E} \gg \mathcal{R}$
Ziman	$\mathcal{N} \gg \mathcal{R} \gg \mathcal{E}$
Kinetic	$\mathcal{R} \gg \mathcal{N}$ and $\mathcal{R} \gg \mathcal{E}$

Classification of different regimes of thermal conductivity as a function of the linewidths of different scattering events: normal (\mathcal{N}), resistive (\mathcal{R} —combining both Umklapp and isotopic) and extrinsic (\mathcal{E}). Poiseuille and Ziman hydrodynamics are characterized by dominant \mathcal{N} scattering against all other mechanisms.

A. Cepellotti et al.,
Nature Commun. 2014

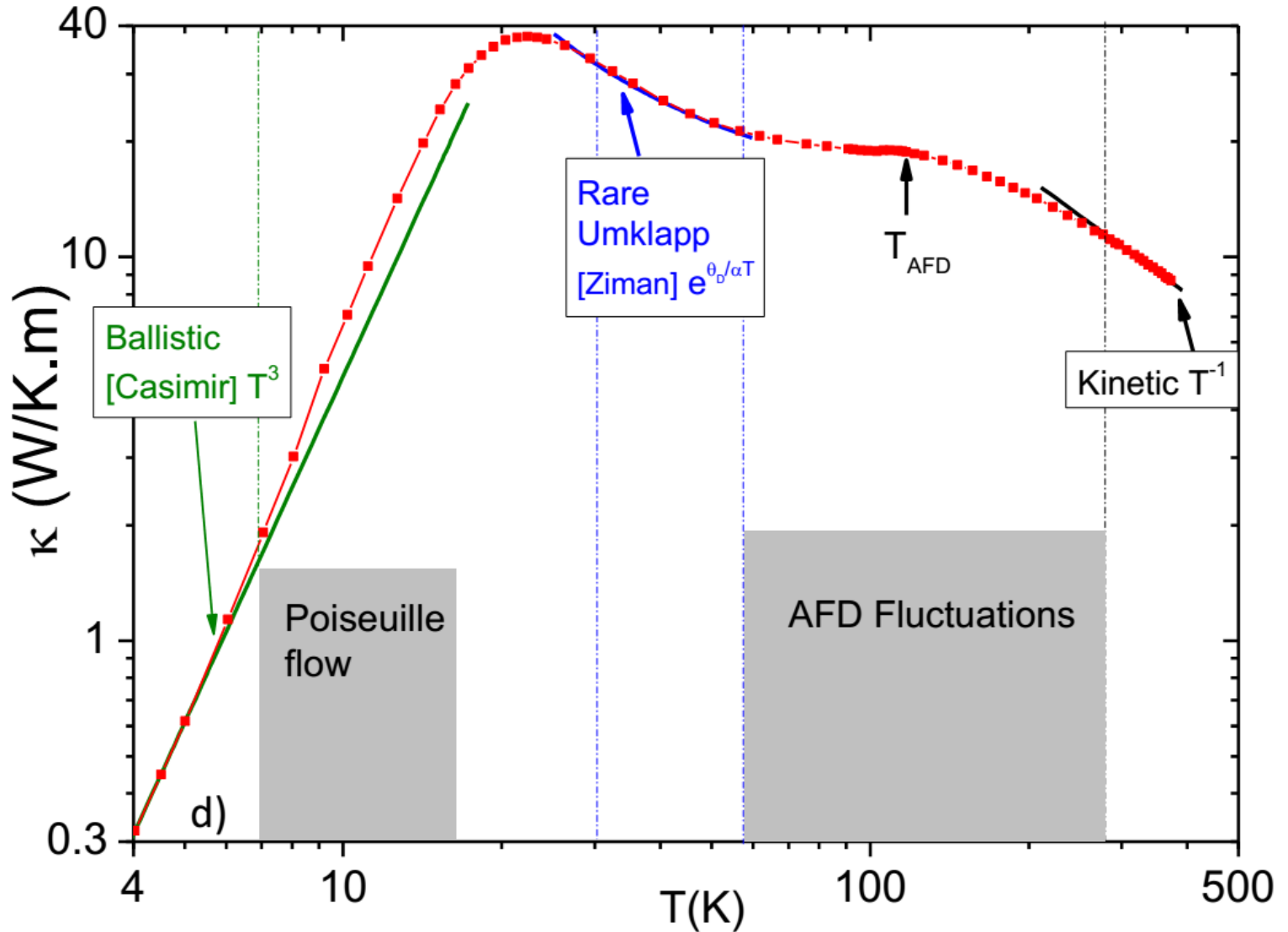
See also:

- Guyer & Krumhansl (1966)
- Gurzhi (1968)
- Beck, Meyer & Thellung (1974)



Regimes of heat transport in strontium titanate

Martelli et al. unpublished



Poiseuille flow and second sound

Normal scattering >> Extrinsic scattering >> Resistive scattering

Observed only in a handful of solids:

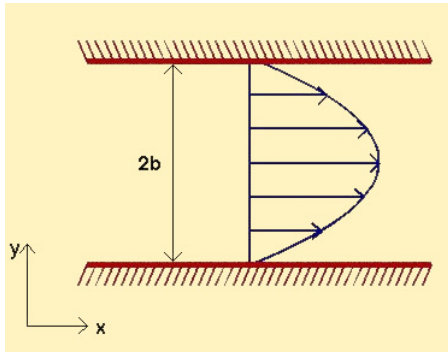
^4He for SS [15, 47, 48] and PF [13, 14, 49, 50] ,
 ^3He for SS [51] and PF [52] ,
NaF for SS [53 to 56] ,
Bi for SS [57] and PF [58] .

Phonon Hydrodynamics in Solids

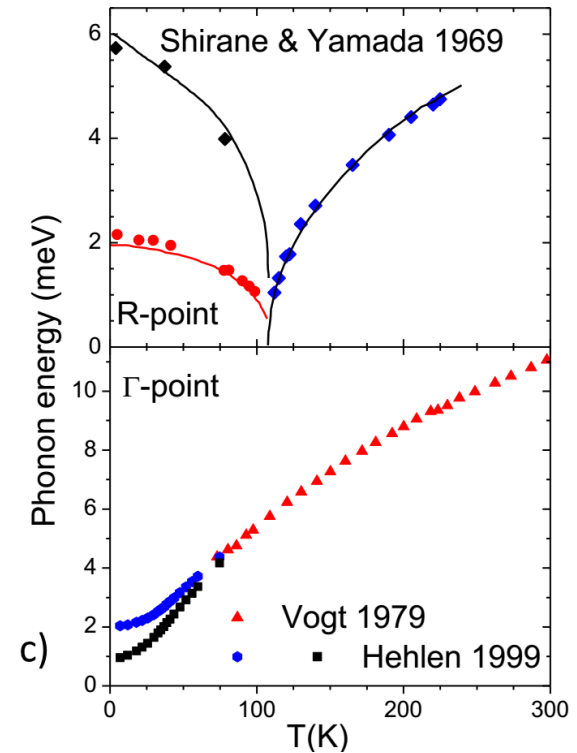
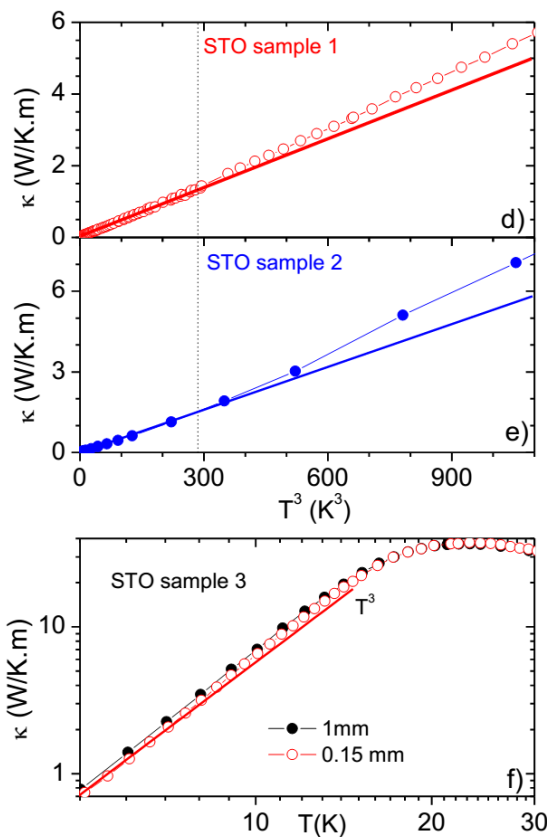
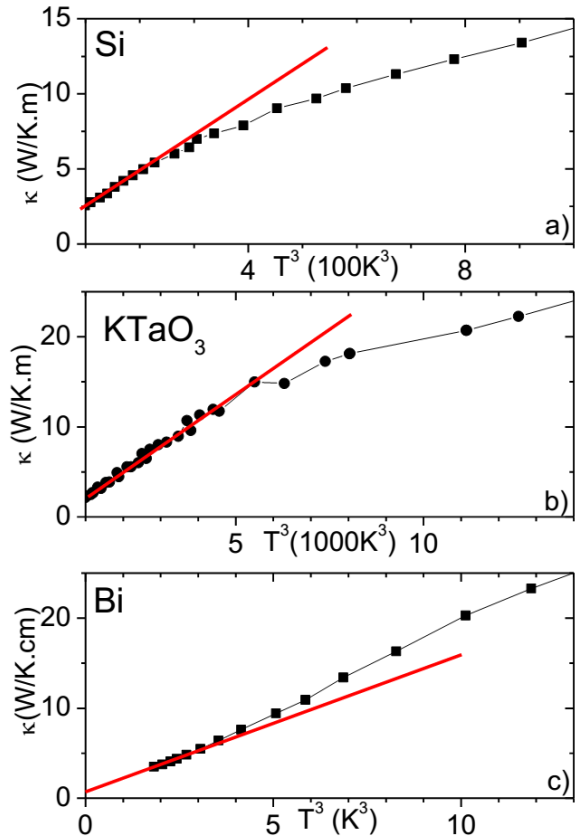
phys. stat. sol. (a) **24**, 11 (1974)

H. BECK (a)¹), P. F. MEIER (b)²), and A. THELLUNG (c)

Hallmark of Poiseuille flow: faster than T^3 thermal conductivity



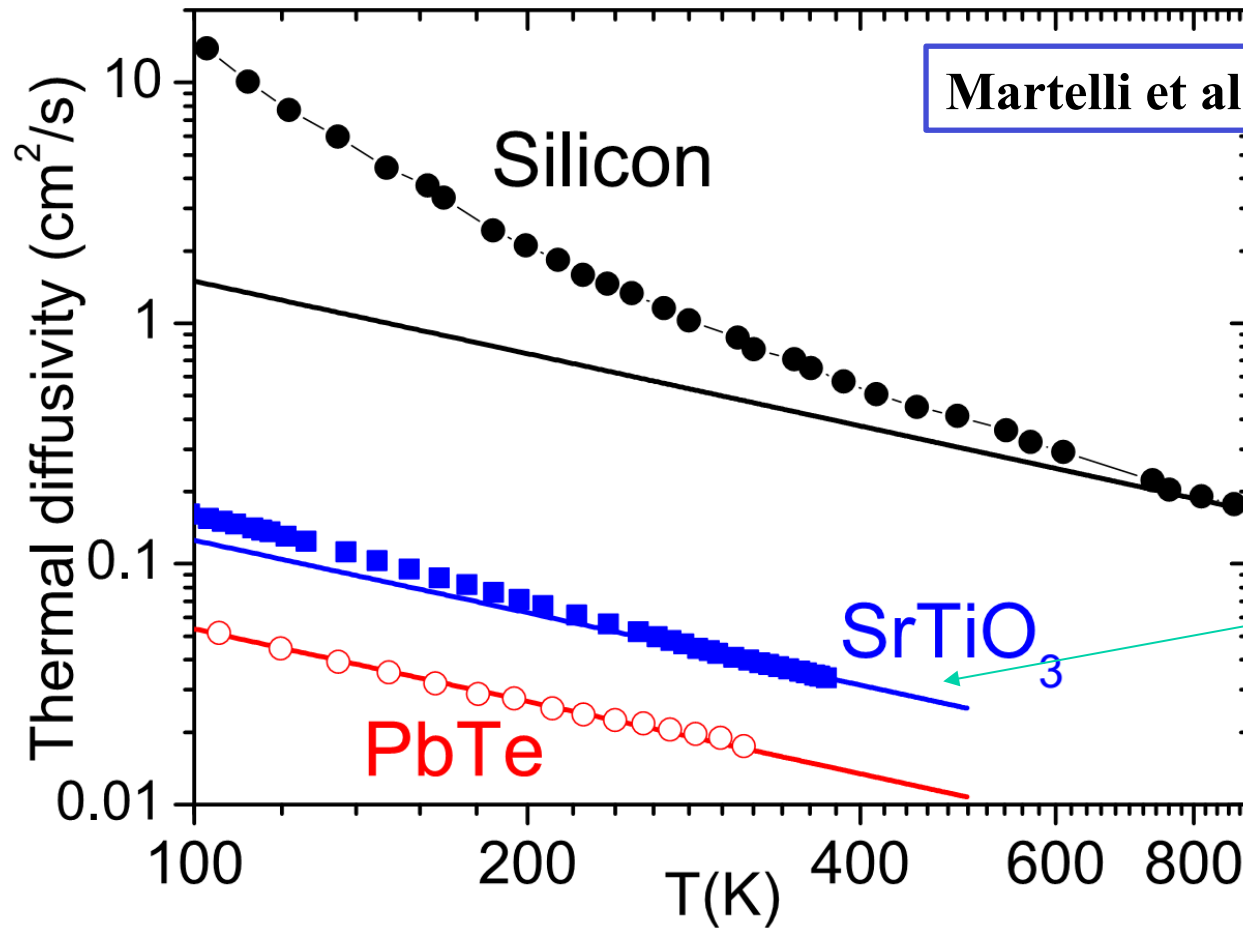
Normal scattering events are multiplied due to soft modes



Previously seen only in Bi and He crystals

High-temperature thermal diffusivity

$$D = \kappa / C$$



Martelli et al. unpublished

Silicon

SrTiO₃

PbTe

$$D = 2\pi/3 s v \downarrow s \uparrow 2$$

Planck time:

$$\tau \downarrow P = \hbar / k \downarrow B T$$

Umklapp ph-ph scattering

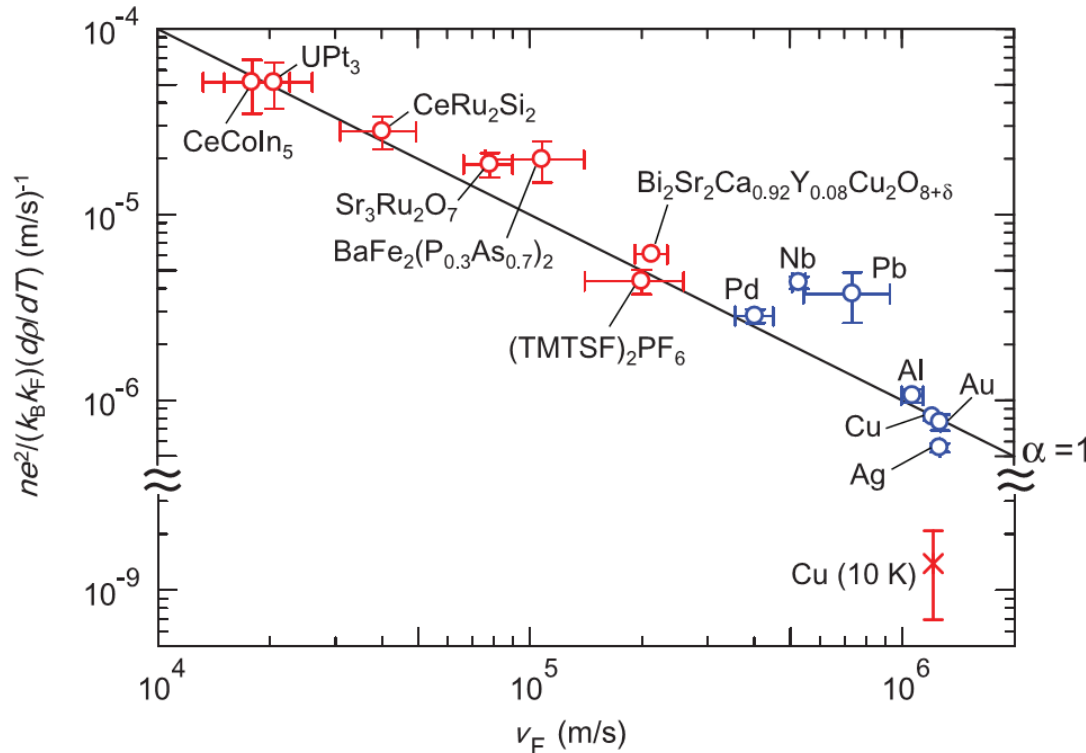
Planck time detected in metallic charge conductivity

Similarity of Scattering Rates in Metals Showing T -Linear Resistivity

J. A. N. Bruin,¹ H. Sakai,¹ R. S. Perry,² A. P. Mackenzie¹

Science 2013

$$\tau \propto P = \hbar / k_B T$$

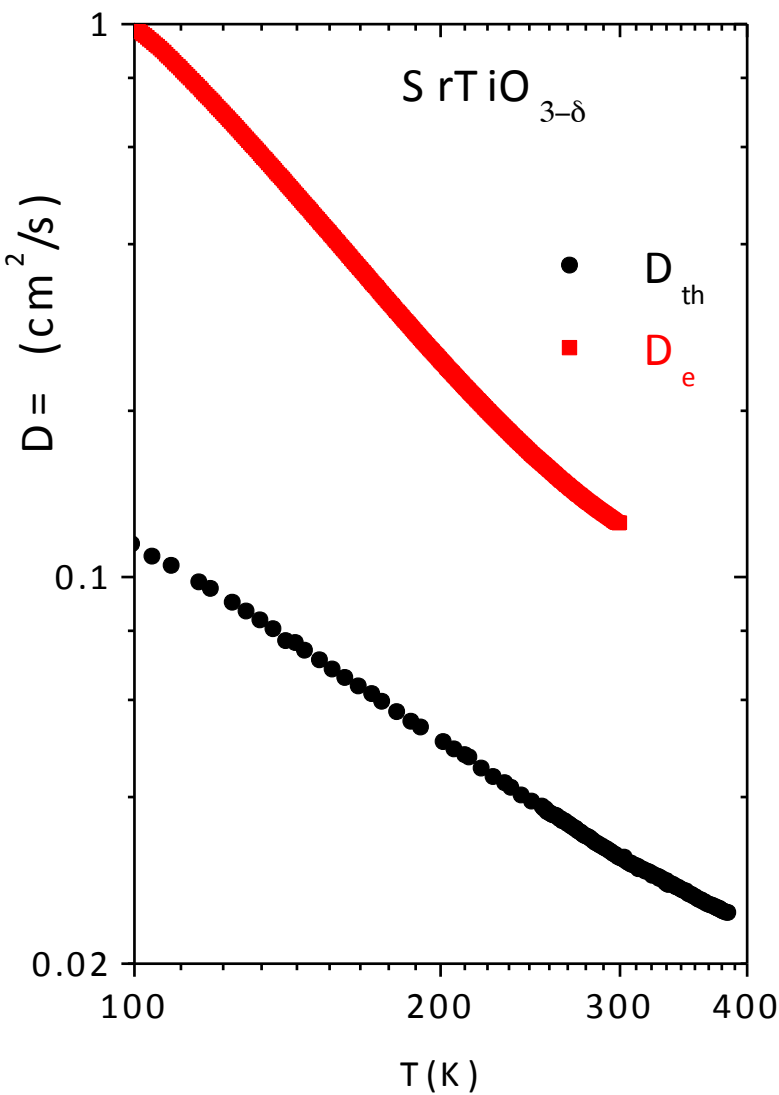


$$\sigma = \frac{ne^2\tau}{m^*} = \frac{ne^2\tau v_F}{\hbar k_F}$$

$$\alpha = \frac{\rho}{T} \frac{e^2}{2\pi k_B d} \sum_i M_i k_{Fi} v_{Fi}$$

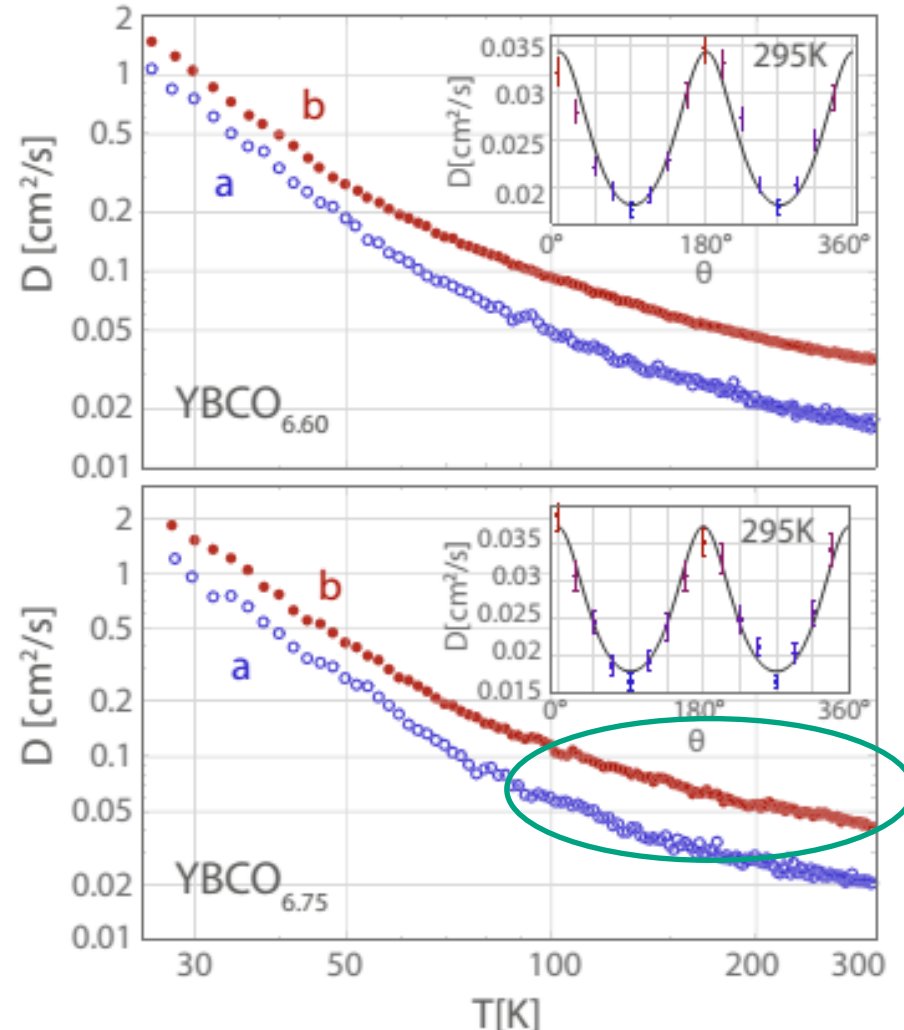
First postulated by Sachdev and Zaanen in the aughties

Thermal and electrical diffusivity



Anomalous thermal diffusivity in underdoped YBa₂Cu₃O_{6+x}

Jiecheng Zhang^{a,b}, Eli M. Levenson-Falk^{a,b}, B. J. Ramshaw^c, D. A. Bonn^{d,e}, Ruixing Liang^{d,e}, W. N. Hardy^{d,e}, Sean A. Hartnoll^b, and Abaron Kanitnik^{a,b,f,1}



Charge diffuses faster, but not much!

Summary

- With one oxygen out of 10^5 missing, SrTiO_3 becomes an intriguing superconductor in the vicinity of an intertwined ferroelectric order.
- Its metallicity cannot be accounted by Boltzmann-Drude picture. Polarons may be in charge. But we have no theory.
- The host insulator displays Poiseuille flow of phonons, only seen in a handful of solids, thanks to the presence of soft modes.
- Charge and heat diffusivity are to be put under scrutiny.