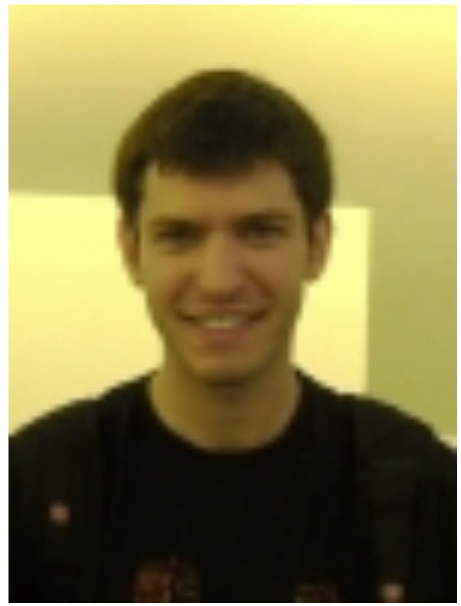
Exact critical exponents for the antiferromagnetic quantum critical metal in two dimensions

Sung-Sik Lee

McMaster University
Perimeter Institute

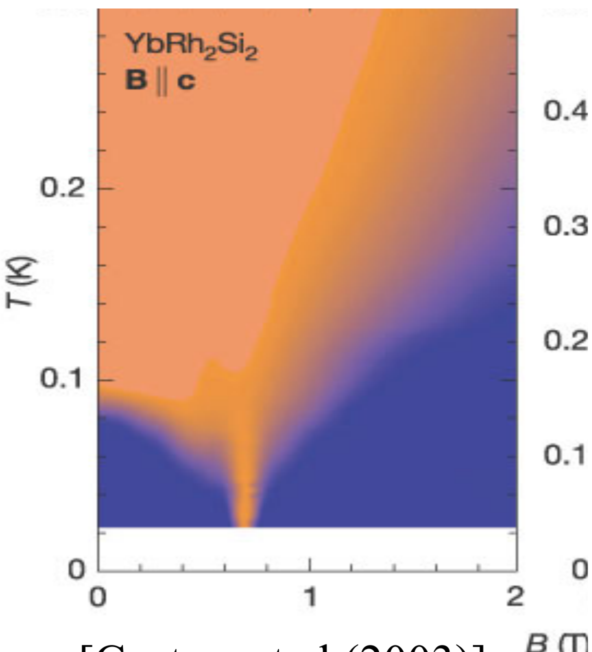
Shouvik Sur



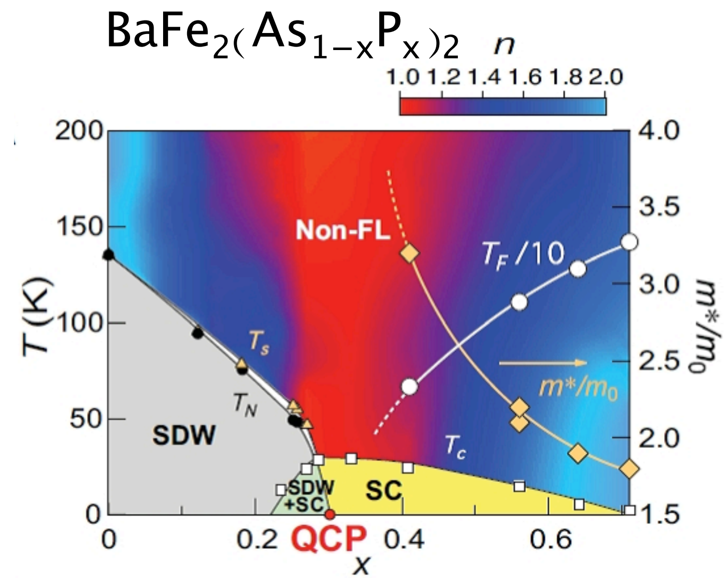
Peter Lunts

Andres Schlief

Breakdown of Fermi liquid near Quantum Critical Point



[Custers et al.(2003)]



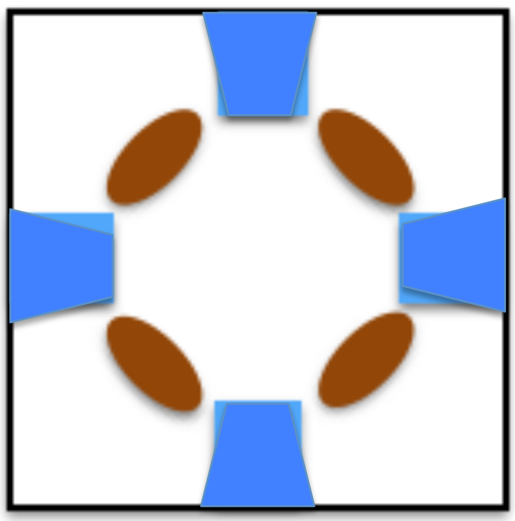
[Hashimoto et al. (2012)]

- Specific heat : C ~ T log(1/T),
- Resistivity : $\rho \sim T^n$, n < 2

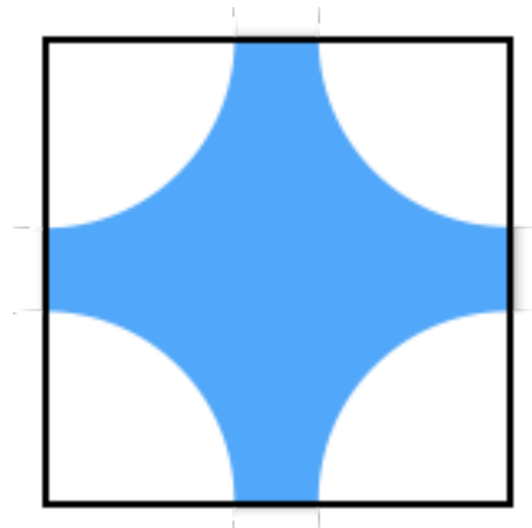
Goal : capture universal low-energy physics of metals
without well-defined quasiparticle

Antiferromagnetic phase transition in metal

$$\vec{\phi} \neq 0$$



$$\vec{\phi} = 0$$

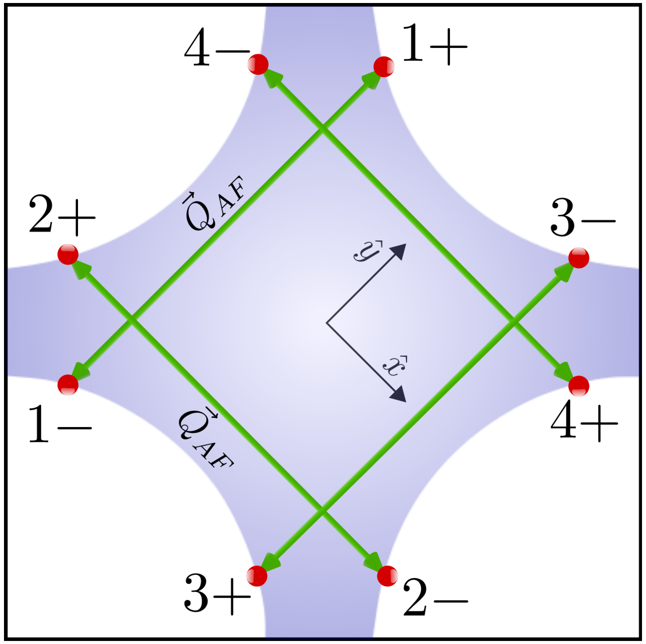


λ

Metal with antiferromagnetic order

Quantum Critical Point

Paramagnetic Metal



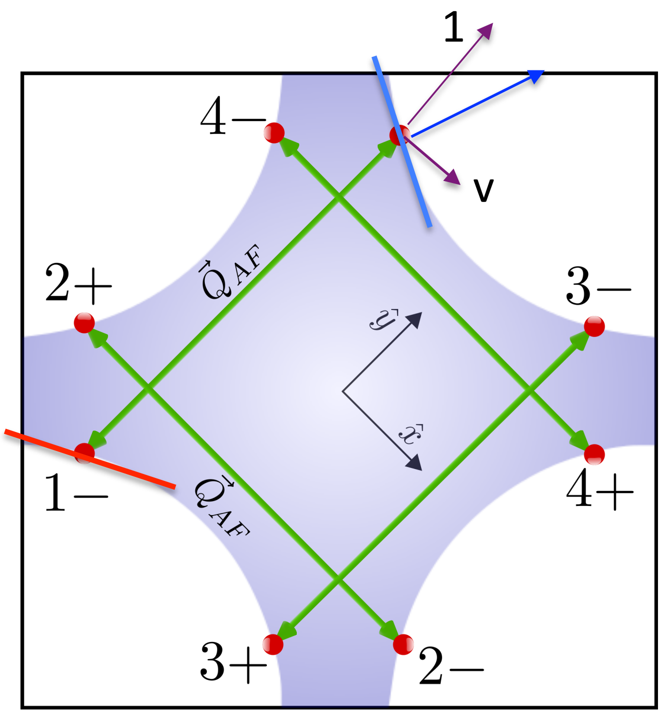
Minimal Theory

$$e_1^{\pm}(\vec{k}) = -e_3^{\pm}(\vec{k}) = v k_x \pm k_y$$

$$e_2^{\pm}(\vec{k}) = -e_4^{\pm}(\vec{k}) = \mp k_x + v k_y$$

$$\begin{aligned}\mathcal{S} = & \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k) \\ & + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q) \\ & + g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right]\end{aligned}$$

Parameters of the theory



- v : Fermi velocity perpendicular to Q_{AF}
- c : boson velocity
- g : coupling bet'n fermion and boson

- If v=0, hot spots connected by Q_{AF} are nested

Scale invariance

- At QCP, the correlation length is infinite.
- The scale invariance is characterized by a set of critical exponents
- The critical exponents determine the scaling forms of the physical observables at low energies, e.g.

$$\chi''(\omega, \vec{Q}_{AF} + \vec{q}) \sim \frac{1}{\omega^{\frac{2-\eta}{z}}} f\left(\frac{\omega}{q_x^z}, \frac{\omega}{q_y^z}\right)$$

Scaling analysis (non-interacting)

$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 \; + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k)$$

$$+\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$

$$k \to s \; k, \qquad \Psi \to s^{-2} \; \Psi, \qquad \phi \to s^{-5/2} \; \phi$$

Strong quantum fluctuations in 2+1D

$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3}\, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k)$$

$$+ \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$

$$+ \boxed{g_0} \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right]$$

Interactions are relevant at the Gaussian fixed point

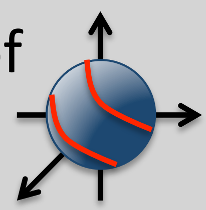
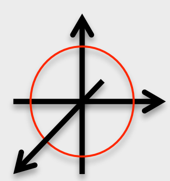
Earlier works (incomplete list)

- Fermi surfaces get nested and quasiparticle is destroyed near the hot spots [Abanov, Chubukov,Schmalian]
- The theory flows to strong coupling regime even in the large N limit [Metlitski, Sachdev]
- The field theory can be regularized by a sign-problem-free lattice model : QMC shows enhancement of d-wave SC at QCP [Berg,Metlitski, Sachdev; …]
- The precise nature of the NFL state has not been understood due to a lack of control over the theory

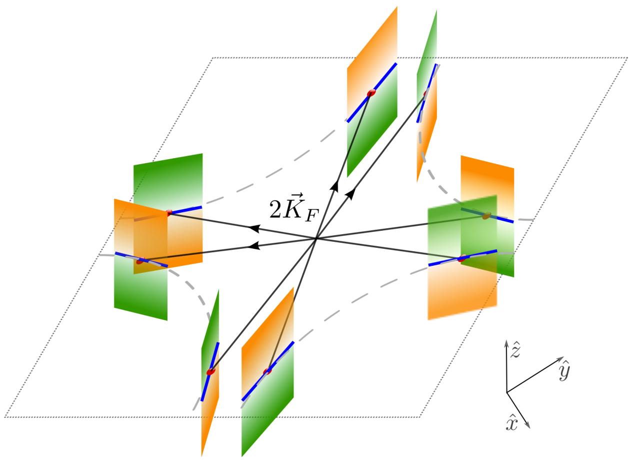
Strategy

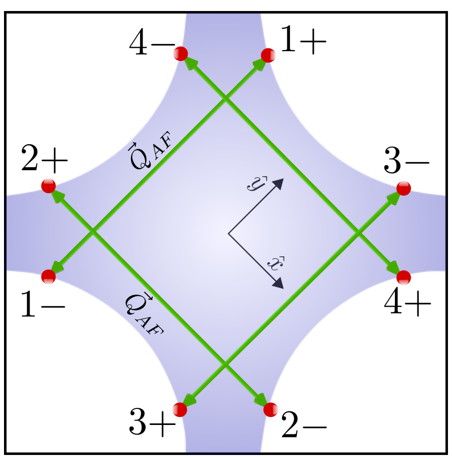
- Deform the strongly coupled theory to a limit in which quantum fluctuations can be included in a controlled way
- Use the intuition from the perturbative answer to construct a non-perturbative Ansatz for the original theory
- Check the consistency of the Ansatz

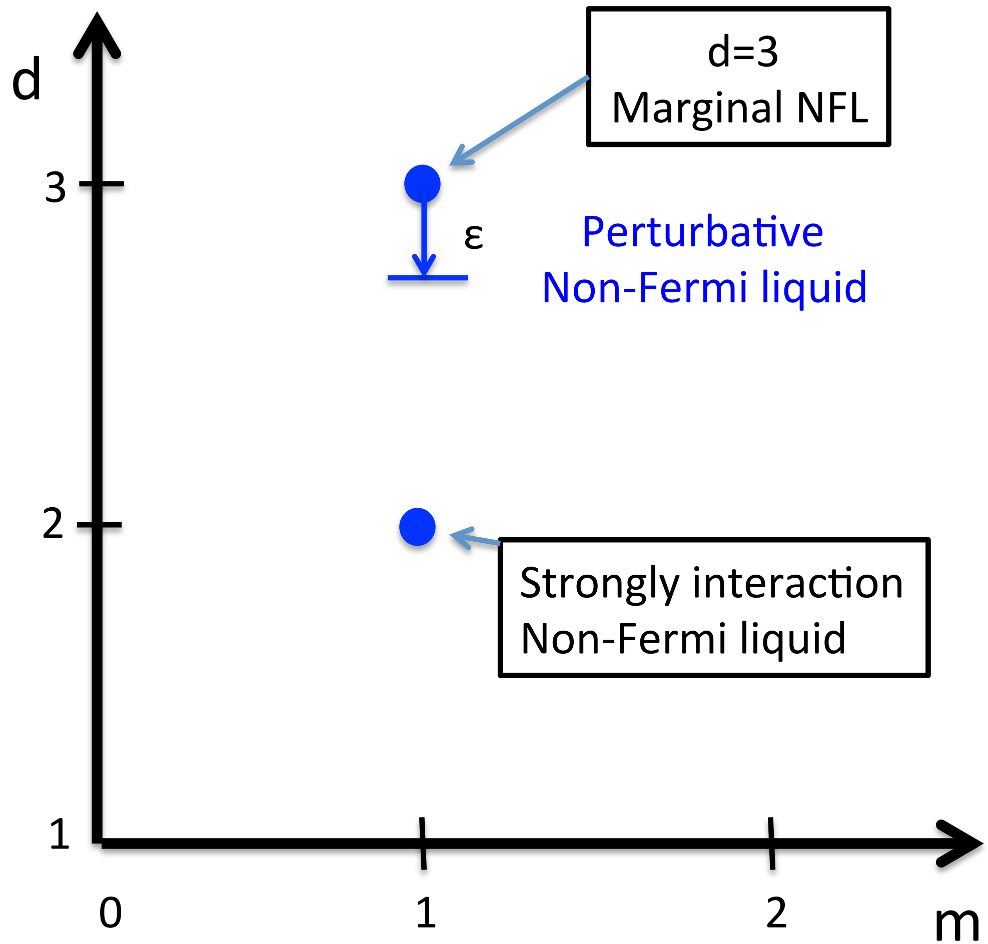
Strategies for taming quantum fluctuations

	Deformation schemes	Pro	Con
1/N	Increase the # of flavors	Symmetry, locality	Not controlled
Dynamical tuning	Modify the dispersion $$\|q\|^2\phi^2 \rightarrow \|q\|^{1+\epsilon}\phi^2$$	symmetry	Locality lost
Dim. reg.	Tune the number of dimensions 	Symmetry, locality	spurious scale (size of 'hot line')
Co-dim. reg.	Tune the number of co-dimensions 	locality	Some symmetry broken

A continuous interpolation between 2d Fermi surface and 3d metal with line nodes







[Sur, Lee (14)]

Lesson from the ε-expansion

[Sur, Lee (14); Lunts, Andres, Lee(17)]

- ε-expansion ≠ loop expansion
- Emergent quasi-locality with a hierarchy in velocities

$$v, c \to 0 \ \left(\frac{v}{c} \to 0\right), \quad g \to 0 \left(\frac{g^2}{v} \to O(\epsilon)\right)$$

- Collective mode is strongly damped by particle-hole excitation and acquire an O(ε) anomalous dimension
- Fermions remain largely coherent

Ansatz in 2+1D : Interaction driven scaling

Scaling which leaves the interaction marginal at the expense of dropping kinetic energy as irrelevant term

$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 \ + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k)$$

$$+\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$

$$+g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right]$$

Ansatz in 2+1D :
Interaction driven scaling

Drop the boson kinetic term because collective mode is damped by the particle-hole excitations

$$\mathcal{S} = \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k)$$

$$+\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2|\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q)$$

$$+g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right]$$

Ansatz in 2+1D : Interaction driven scaling

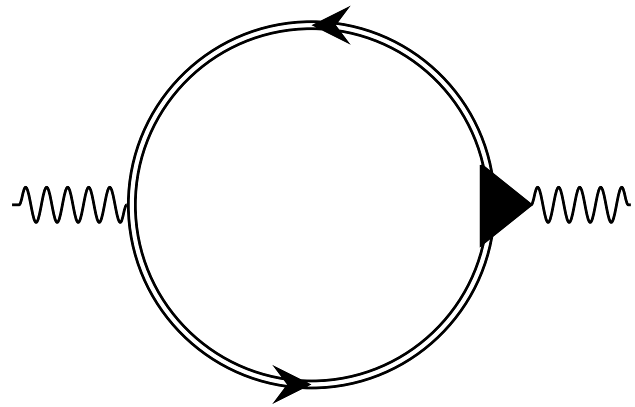
$$[k_0] = [k_x] = [k_y] = 1,$$

$$[\psi(k)] = [\phi(k)] = -2.$$

- Electron keeps the classical scaling dimension
- Collective mode has a large anomalous dimension
- It turns out that these are exact

[Andres, Lunts, Lee (17)]

Self-consistent boson propagator

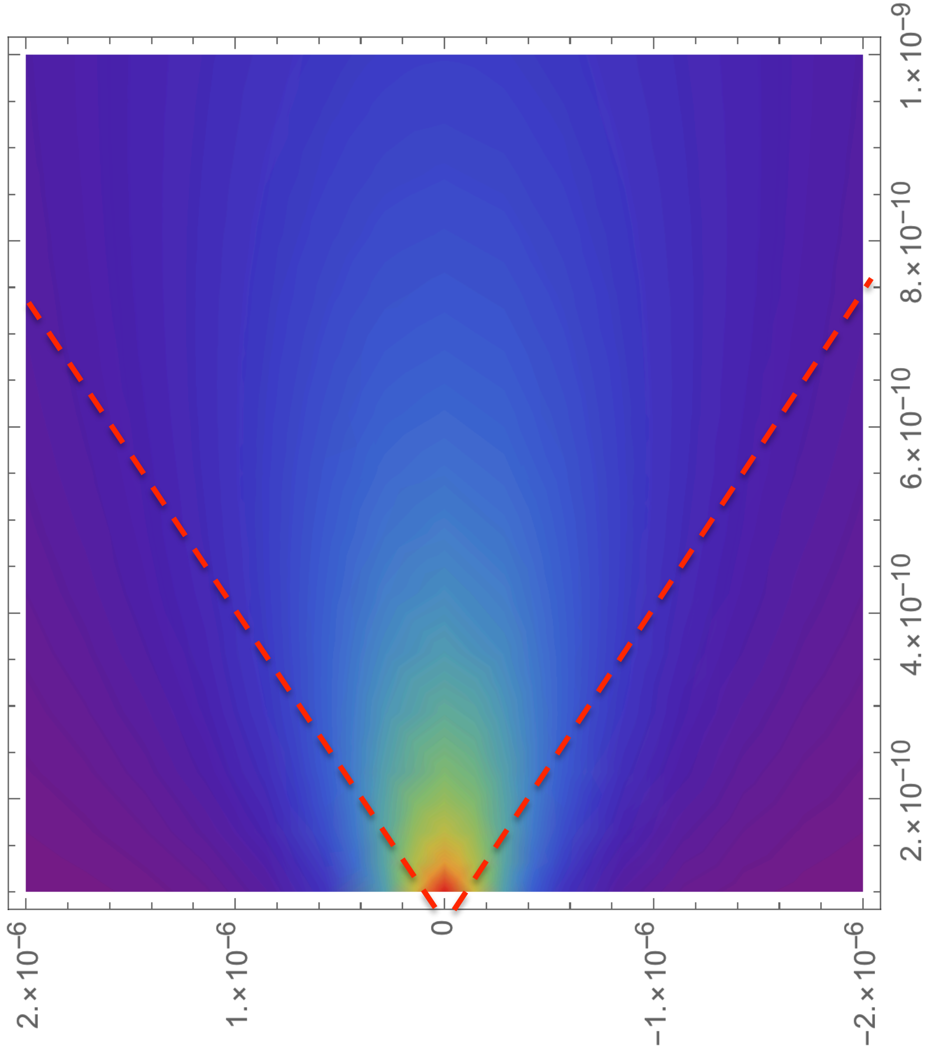


$$D(q)^{-1} = m_{CT} - \pi v \sum_n \int dk \; \mathrm{Tr}\left[\gamma_1 G_{\bar{n}}(k+q)\Gamma(k,q)G_n(k)\right]$$

- In general, it is hard to solve the self-consistent equation because G(k), Γ(k,q) depend on D(q)
- However, in the small v limit, this can be solved
- Furthermore, v dynamically flows to zero in the IR

Dynamical Susceptibility $\chi''(\omega, \vec{Q}_{AF} + \vec{q})$

ω



$$\Delta q \sim \frac{\omega}{\sqrt{v}}$$

q

- Incoherent peak centered at Q_{AF} at all ω
- The width in momentum space scales linearly in energy

Spectral function at the hot spots

$$A(\omega) \sim \frac{1}{\omega e^{2\sqrt{3}\frac{\left(\log\frac{1}{\omega}\right)^{1/2}}{\log\log\frac{1}{\omega}}}}$$

- Weak deviation from Fermi liquid

Divergent correlation length

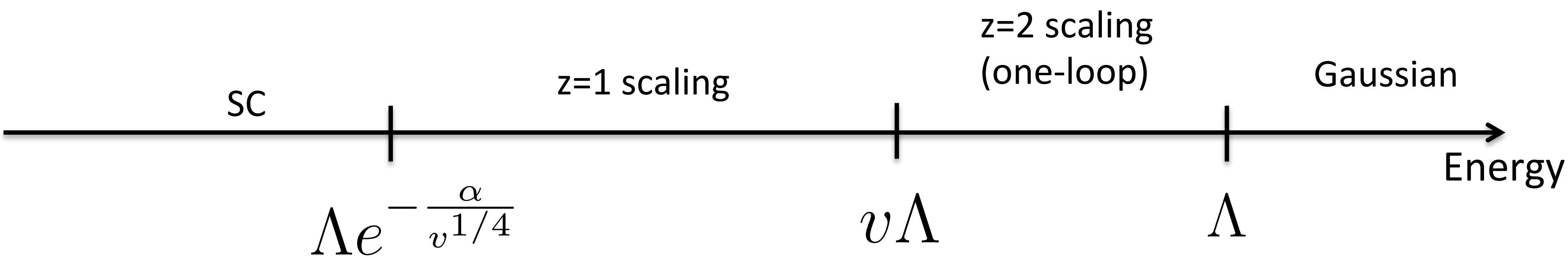
$$\xi \sim (\lambda - \lambda_c)^{-1} e^{\frac{2}{\sqrt{3}}\left(\log \frac{1}{|\lambda - \lambda_c|}\right)^{1/2}}$$

Specific heat

$$c \sim T e^{2\sqrt{3}\frac{\left(\log \frac{1}{T}\right)^{1/2}}{\log \log \frac{1}{T}}}$$

Superconductivity

- In the low T limit, d-wave superconductivity is enhanced
- Hierarchy in energy scales



- The crossover energy scales are sensitive to the bare value of v, which can be experimentally tested
- In the small v limit, there is a large window for the z=1 critical scaling

Summary

- Low Antiferromagnetic critical metal in 2+1D
  - Exact critical exponents are predicted based on a non-perturbative solution
  - Precise measurements are needed to test the predictions
- Open problems
  - How general is the interaction-driven scaling ?
  - AF critical metal with different symmetries [e.g. U(1) symmetric theory] [Gerlach, Schattner, Berg, Trebst]