Intertwined Order

S. A. Kivelson

"Theory of Intertwined Orders in High Temperature Superconductors"
E. Fradkin, SAK, and J.M. Tranquada, arXiv:1407.4480

KITP - 2014

Intertwined Order

M. Curie

L. Landau

E. Noether

P.W. Anderson

Intertwined Order

E. Fradkin and J. Tranquada

Laimei Nie and Gilles Tarjus

E. Berg, E-A. Kim, V. Oganesyan

H. Yao, W-F. Tsai, J-P. Hu, J. Robertson,

Also

Andy Mackenzie, Erica Carlson, Karin Dahmen,

Sri Raghu, Michael Lawler, Ted Geballe,
A.Kapitulnik, S-C. Zhang

V. J. Emery

Intertwined Order

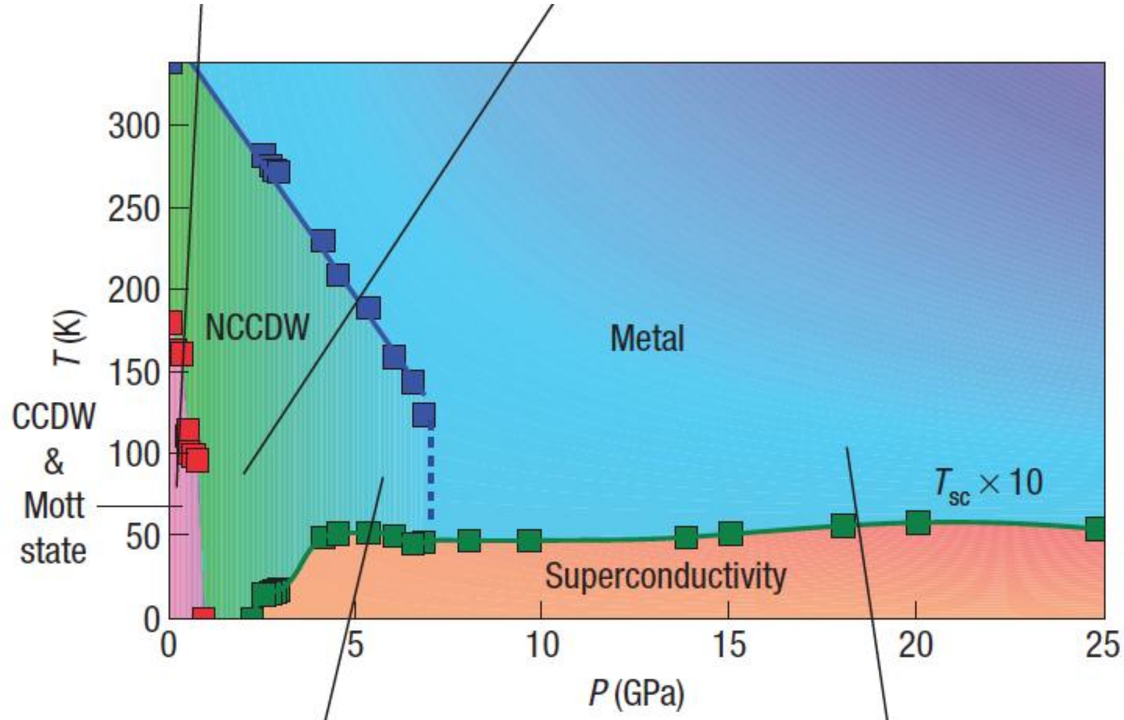
In which the same features of the microscopic physics produce multiple ordering tendencies with similar energy/temperature scales.

This leads to complex phase diagrams with multiple patterns of broken symmetry, and sometimes new types of broken symmetry phases.

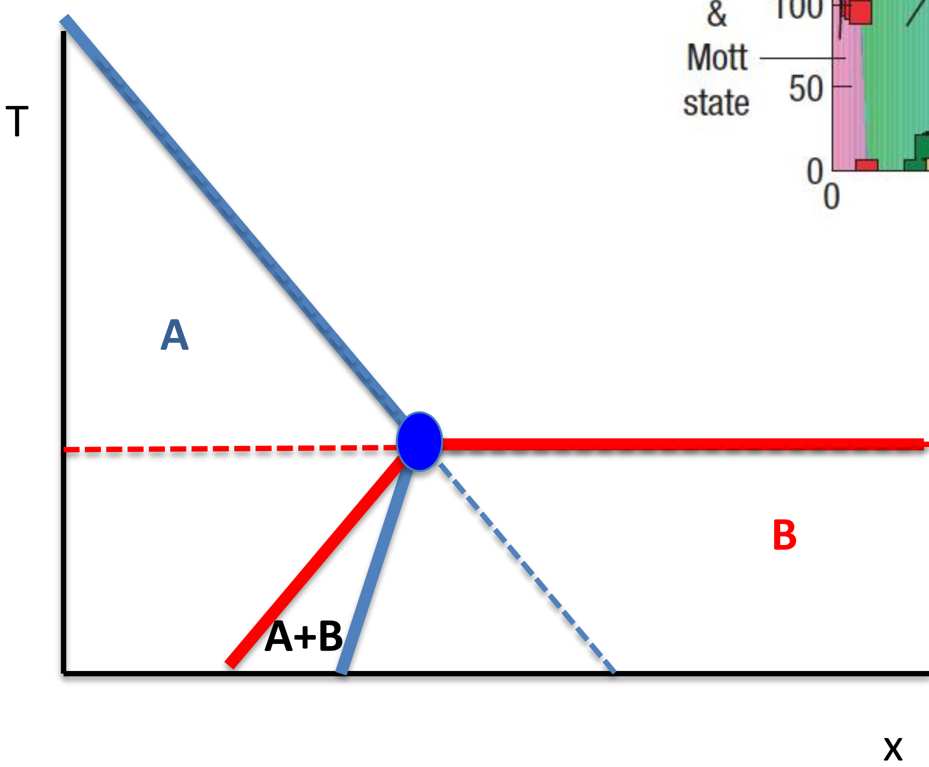
It suggests that there may exist a high energy scale at which an order-parameter "amplitude" develops which cannot really be associated with one or the other order, as it is somehow a precursor to all of them.

Contrast with "Competing Order"

Competing Order

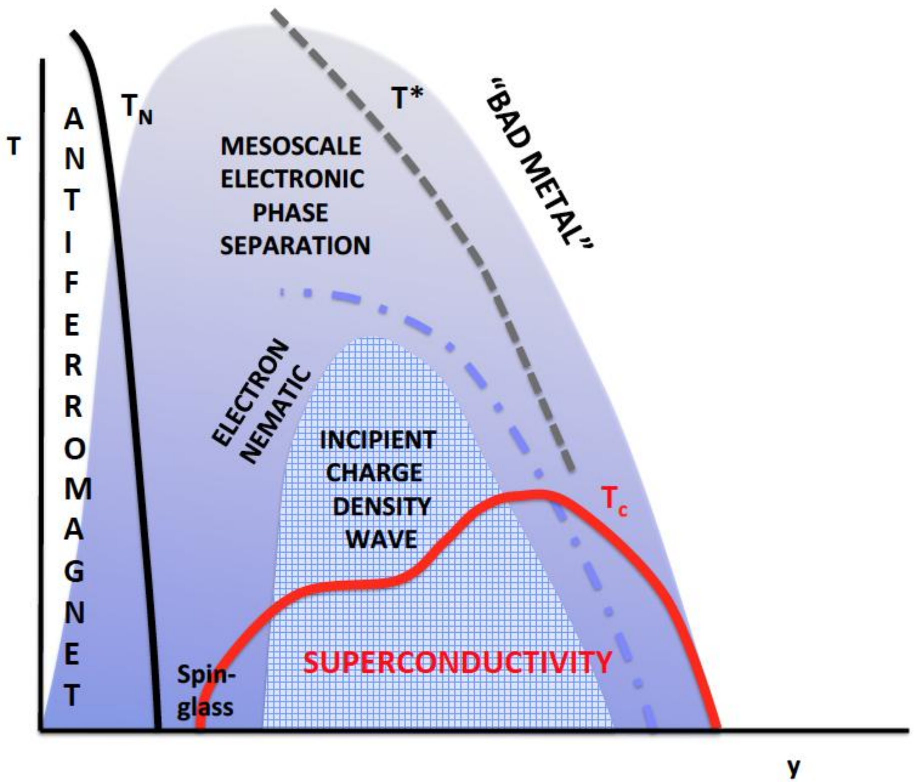


1T-TaS_2; Sipos *et al*, Nat. Mat. 2008



Intertwined Order vs. Competing Order

Cuprates (and others) have complex phase diagrams with many apparent ordering tendencies of comparable strength occuring generically



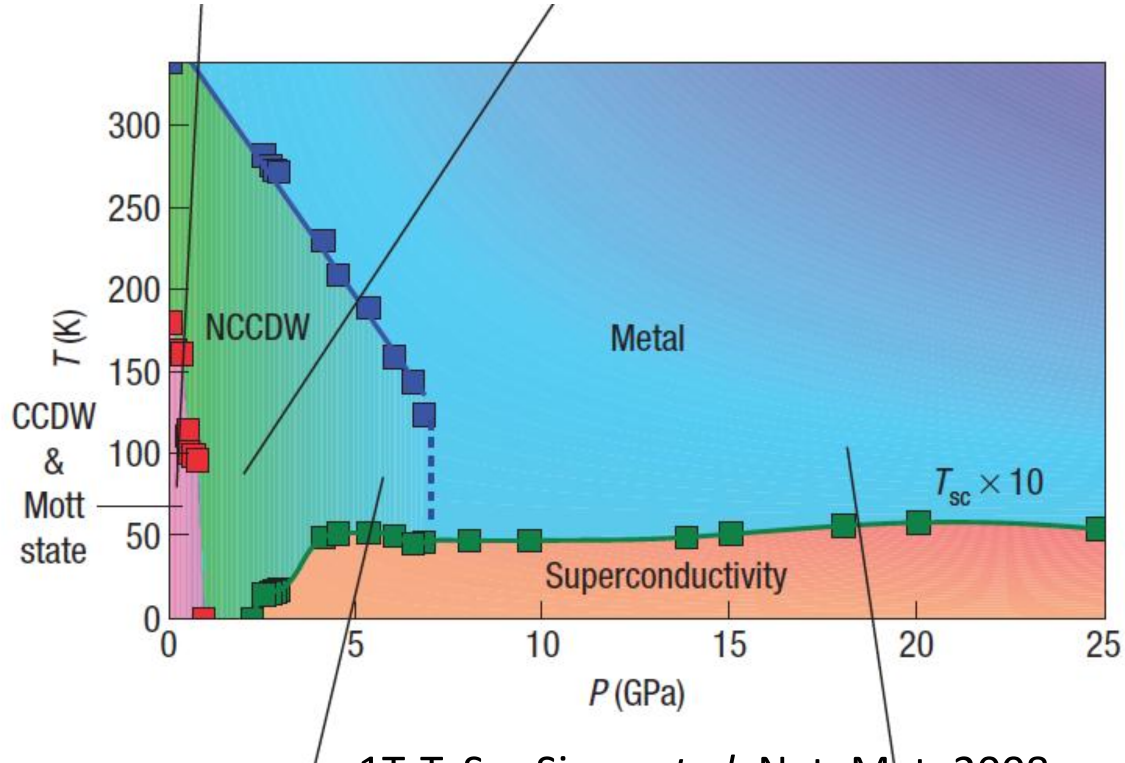
Fradkin and SAK, Nature Phys. (2012)

1) Pure competition does not produce SC "domes"

2) Real competition might be expected only near a fine-tuned multi-critical point.

Competing Order

Usually, where orders compete, one strongly out-competes the other.



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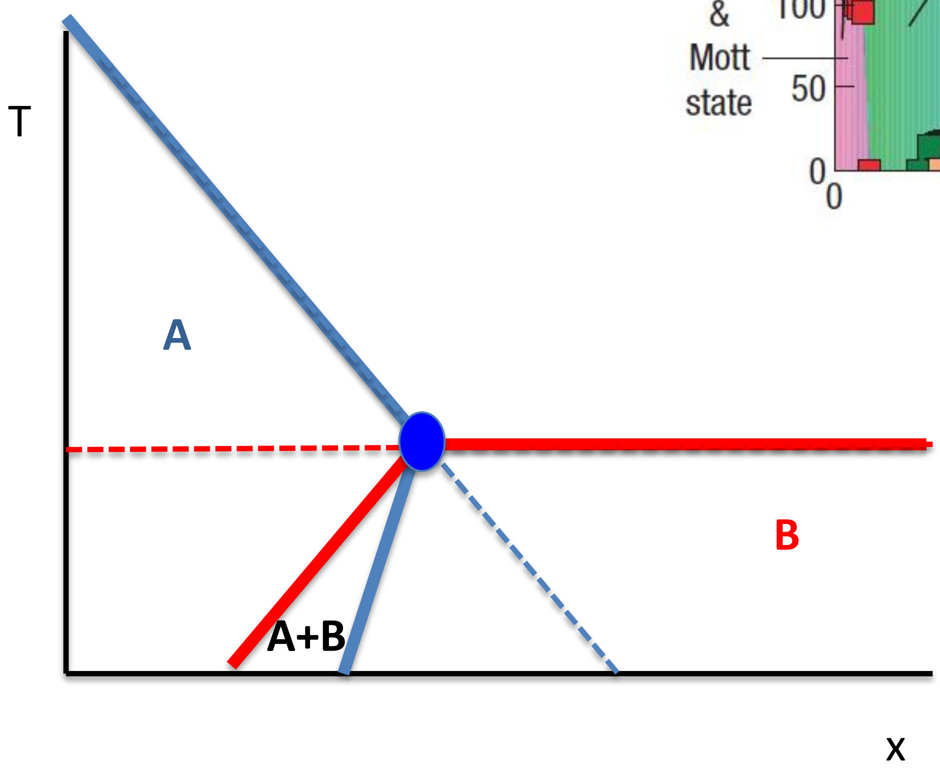

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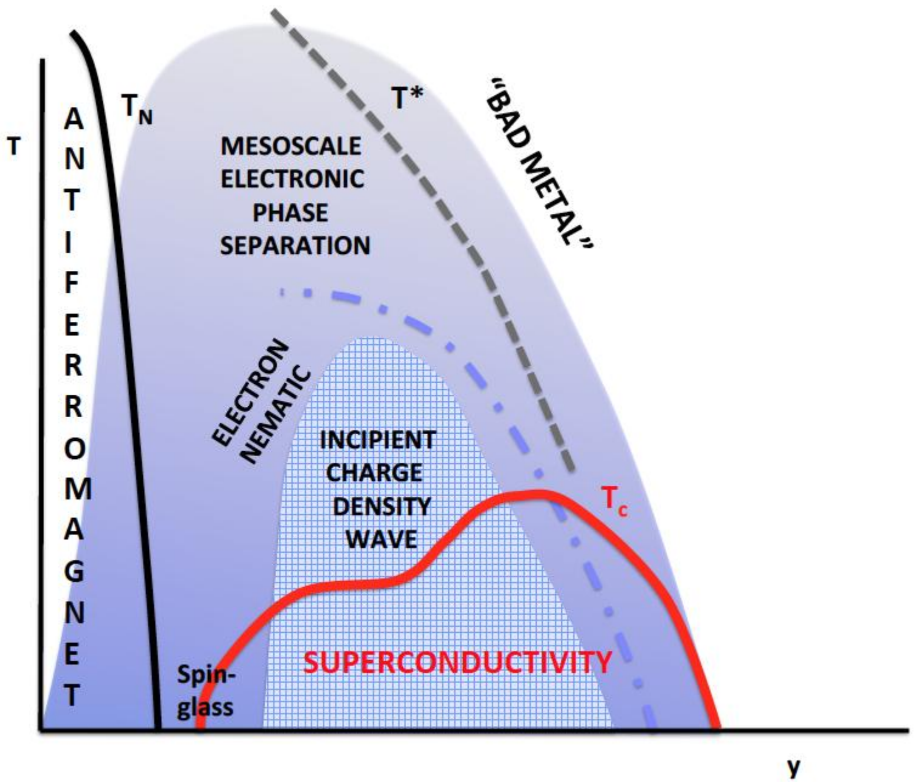


Fradkin and SAK, Nature Phys. (2012)

1) Pure competition does not produce SC "domes"

2) Real competition might be expected only near a fine-tuned multi-critical point.

2b) Multi-critical points do not generically have higher symmetries
e.g. AF + SC has O(3) x U(1) not SO(5)
CDW + SC has U(1) x U(1) not SU(2)

3) Complexity of the phase diagram is a general signature of intertwined orders.

4) Will discuss a specific proposal in which it arises from an underlying "parent" state with many broken symmetries.

Intertwined Order

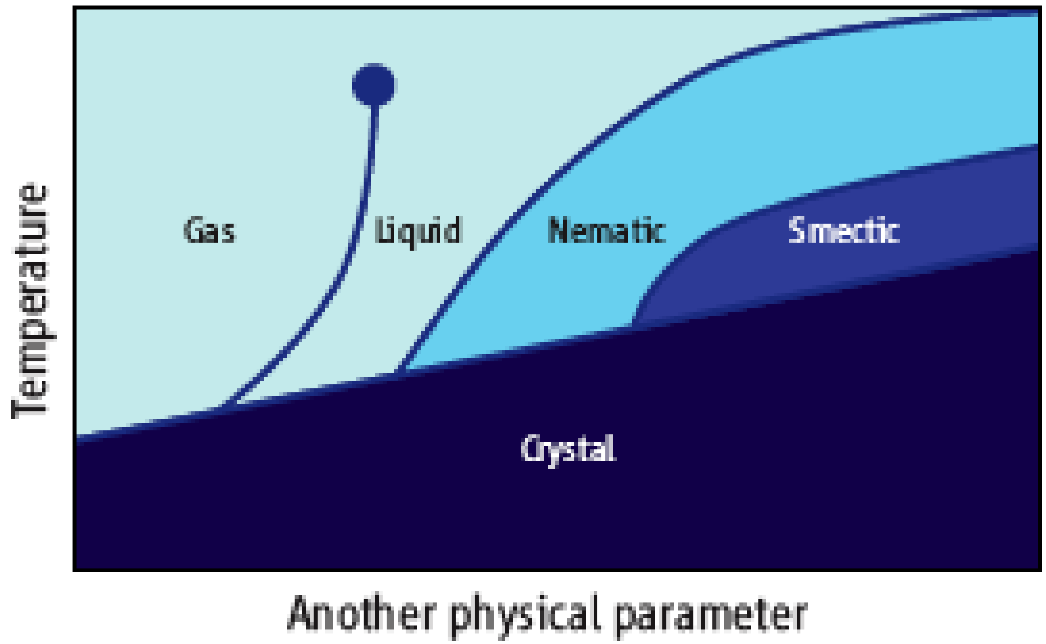
When the same features of the microscopic physics produce multiple ordering tendencies with similar energy/temperature scales.

Vestigial Order

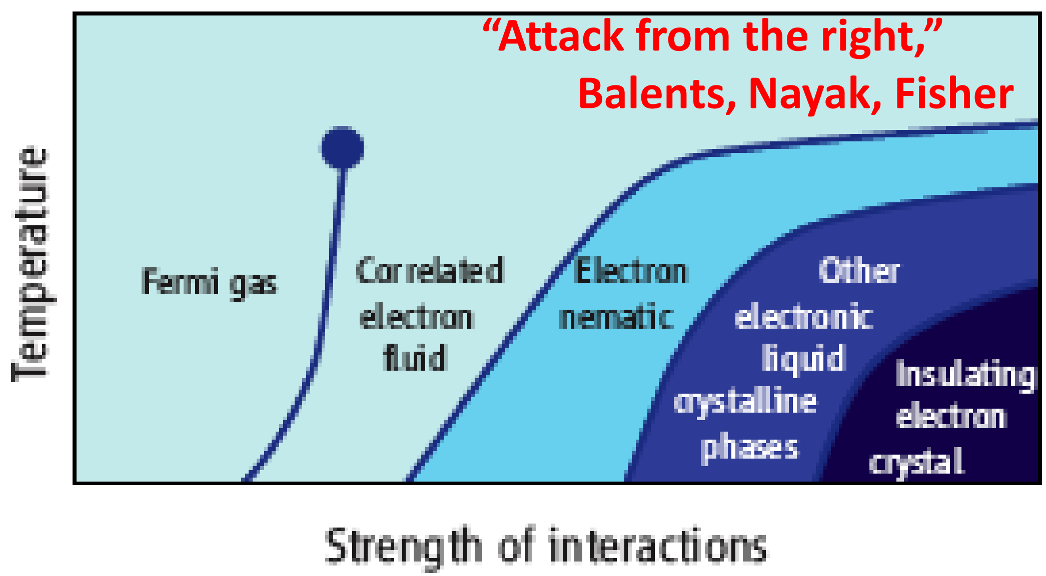
A system in which there is a sequence of transitions separating an ordered (broken symmetry) state from a disordered (symmetric) state, intermediate phases that restore some but not all of the symmetries can be said to have "vestigial order."

- **Formal relation to composite operators in field-theories**

Correlated fluids viewed as partially melted crystals



Sometimes, "melting" can occur in a cascade of transitions – in each a subset of the symmetries are restored.



Fradkin and Kivelson, Science **327**, 5962 (2010).

Theory of Vestigial Nematic Order

We will consider a concrete model problem,
similar to models considered in the context of
the Fe-based high temperature superconductors
and striped phases of curpates

L. Nie, G. Tarjus, and SAK, PNAS (2013)

Fang, Yao, Tsai, Hu, and Kivelson, PRB **77**, 224509 (2008)
Xu, Muller, and Sachdev , PRB **78**, 020501 (2008).

Incommensurate CDW Order

$$\rho(\vec{r}) = \bar{\rho} + \left[\psi_x(\vec{r}) e^{i\vec{Q}_x \cdot r} + \psi_y(\vec{r}) e^{i\vec{Q}_y \cdot r} + \mathrm{c.c.}\right] + \dots$$

$$\mathcal{F} = \frac{r}{2}\left[|\psi_x|^2 + |\psi_y|^2\right] + \frac{u}{4}\left[|\psi_x|^2 + |\psi_y|^2\right]^2 + \frac{\gamma}{2}|\psi_x|^2|\psi_y|^2 + \dots$$

Classical field theory description

$$\psi_a \to \psi_a e^{i\theta_a} \quad \Rightarrow \quad \mathrm{SO(2)} \times \mathrm{SO(2)} \;\; \text{symmetry}$$

$$\psi_x \leftrightarrow \psi_y \;\; \text{and} \;\; \mathrm{x} \leftrightarrow \mathrm{y} \quad \Rightarrow \quad \mathrm{Z}_2 \;\; \text{symmetry}$$

L. Nie, G. Tarjus, and SAK PNAS (2013)

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$$\text{Stripe ordered phase} : |\langle\psi_x\rangle| \neq 0 \text{ or } |\langle\psi_y\rangle| \neq 0$$

$$\text{so } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle \neq 0$$

$$\text{Checkerboard ordered phase} : |\langle\psi_x\rangle| = |\langle\psi_y\rangle| \neq 0$$

$$\text{with } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle = 0$$

$$\text{Nematic phase} : |\langle\psi_x\rangle| = 0 \text{ and } |\langle\psi_y\rangle| = 0$$

$$\text{but } \langle\left[|\psi_x|^2 - |\psi_y|^2\right]\rangle \equiv \mathcal{N} \neq 0$$

L. Nie, G. Tarjus, and SAK PNAS (2013)

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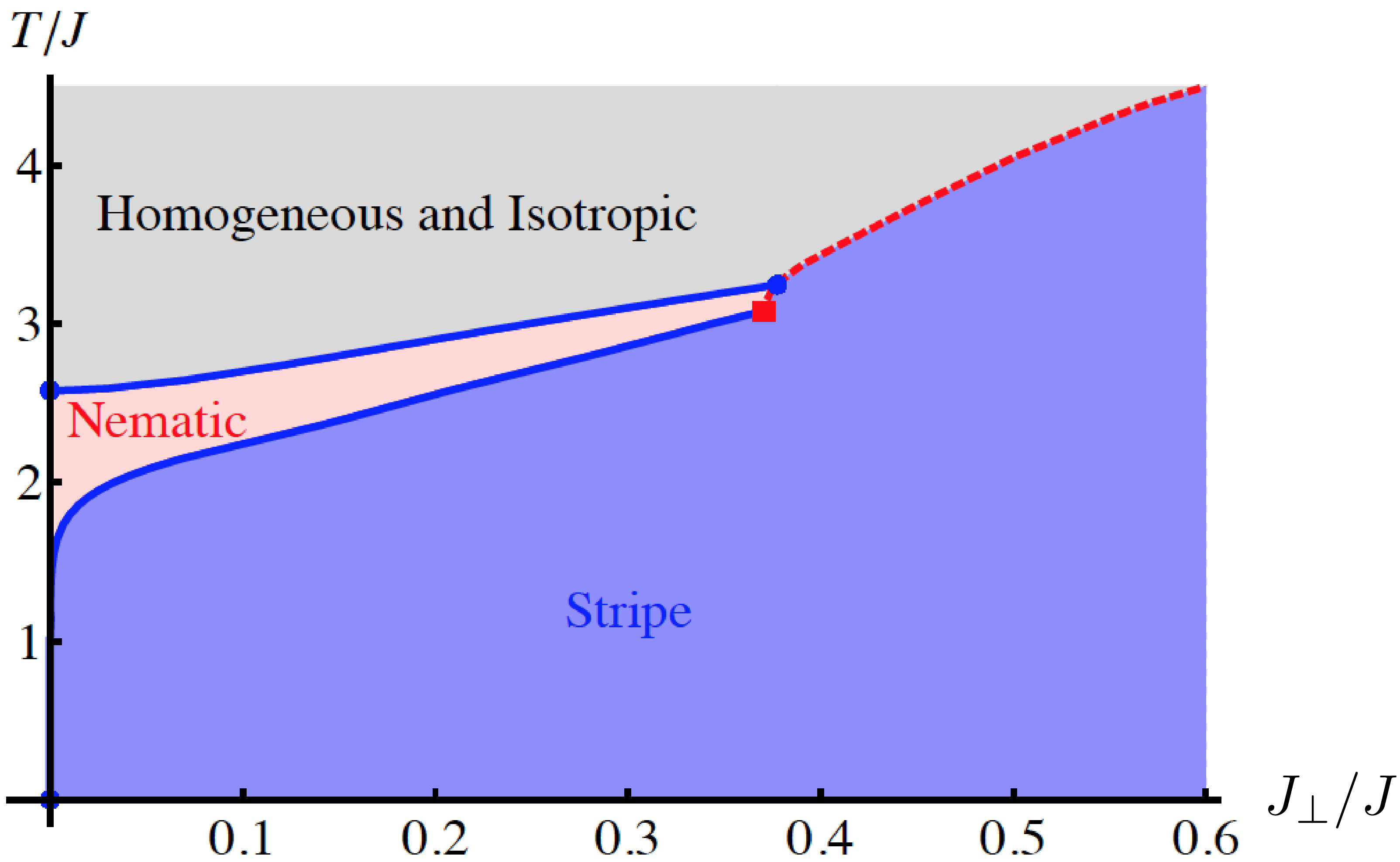
Solve using Feynman variational approach which becomes exact in generalization to large N limit -

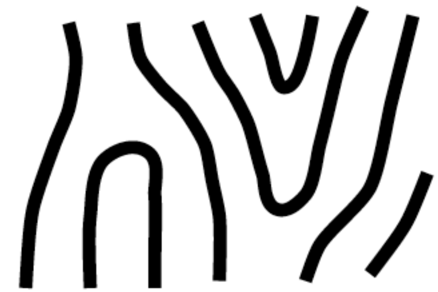
$$SO(N) \times SO(N) \times Z_2$$

L. Nie, G. Tarjus, and SAK PNAS (2013)



L. Nie, G. Tarjus, and SAK PNAS (2013)

Nemattic Phase as “Fluctuating Stripes”



An example of a composite order parameter or as a vestige of a “nearby” CDW

Fully ordered state:	$\langle \mathbf{\Phi} \rangle \neq 0$	$\langle \mathbf{\Phi}^\dagger \tau_\alpha \mathbf{\Phi} \rangle \neq 0$
Partially ordered state:	$\langle \mathbf{\Phi} \rangle = 0$	$\langle \mathbf{\Phi}^\dagger \tau_\alpha \mathbf{\Phi} \rangle \neq 0$
Disordered state:	$\langle \mathbf{\Phi} \rangle = 0$	$\langle \mathbf{\Phi}^\dagger \tau_\alpha \mathbf{\Phi} \rangle = 0$

This view of partially ordered states is beyond mean-field theory.

Effect of Quenched Randomness

The random field problem is relevant to problems involving the breaking of pure spatial symmetries.

The absence of random-fields is a special feature of

superconductivity, ferromagnetism, and certain forms of commensurate antiferromagnetism.

Effect of Quenched Randomness

Random Field problem in Statistical Mechanics

$$H[\mathbf{S}] = H_0[\mathbf{S}] + \sum_j \mathbf{h}_j \cdot \mathbf{S}_j$$

$$\overline{\mathbf{h}_j} = 0 \qquad \overline{\mathbf{h}_j \mathbf{h}_i} = \sigma^2 \delta_{ij}$$

D=2 is lower critical dimension for Ising (Z_2) model (and presumably other models with discrete broken symmetries)

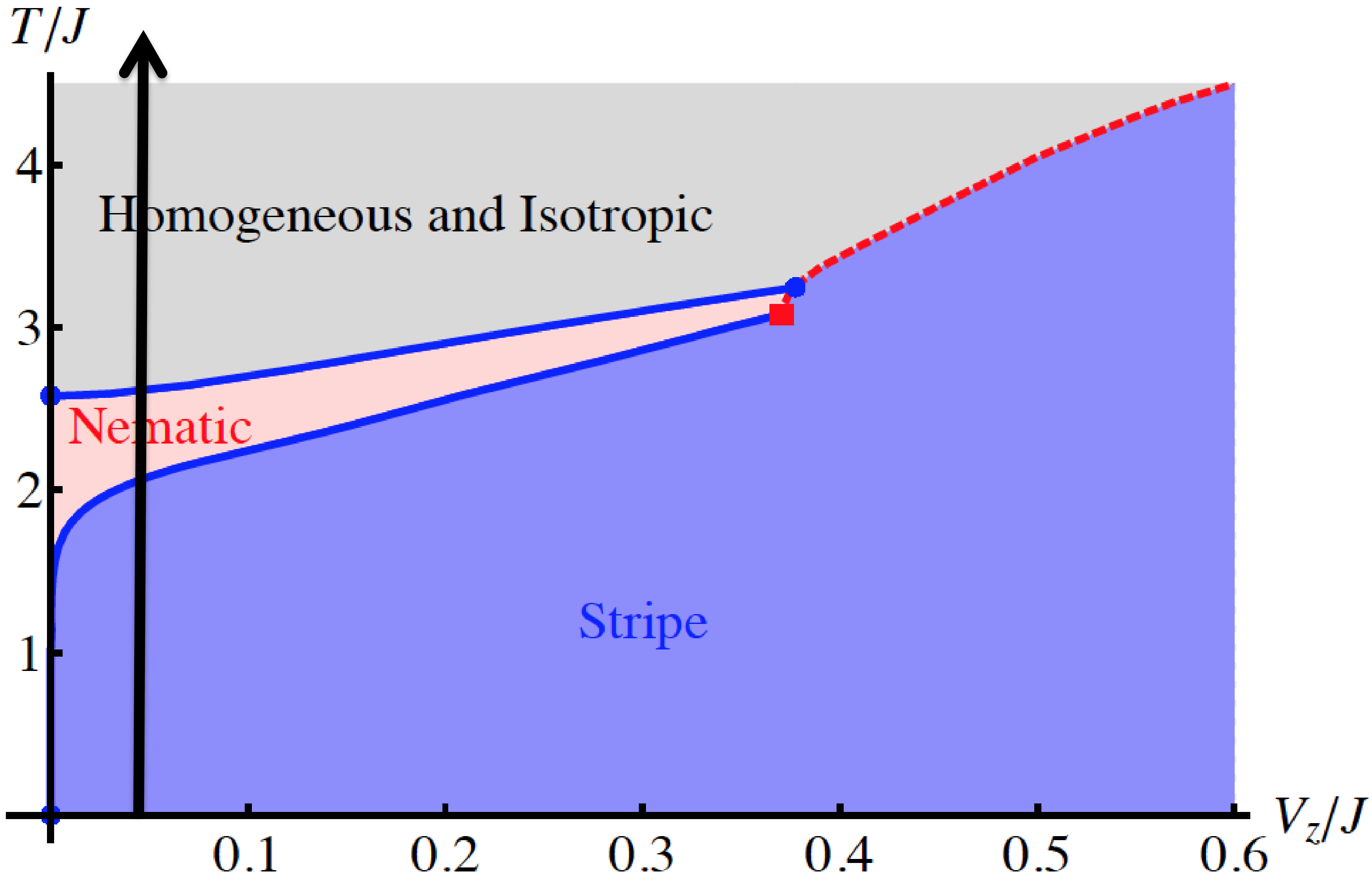
D=4 is the lower critical dimension for Heisenberg (SO(3)) model and most models with continuous broken symmetries.

(There is the possible subtlety of a “Bragg glass” phase for D=3 and XY (SO(2)) symmetry.)

Incommensurate Stripe Order

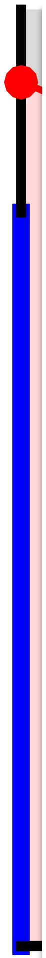
Assume a stripe-ordered ground-state and V_z << J

L. Nie, G. Tarjus, and SAK (2013)



T/J
4
3
2
1
Homogeneous and Isotropic
Nematic
Stripe
0.1
0.2
0.3
0.4
0.5
0.6
V_z/J

“Vestigial” Nematic Order

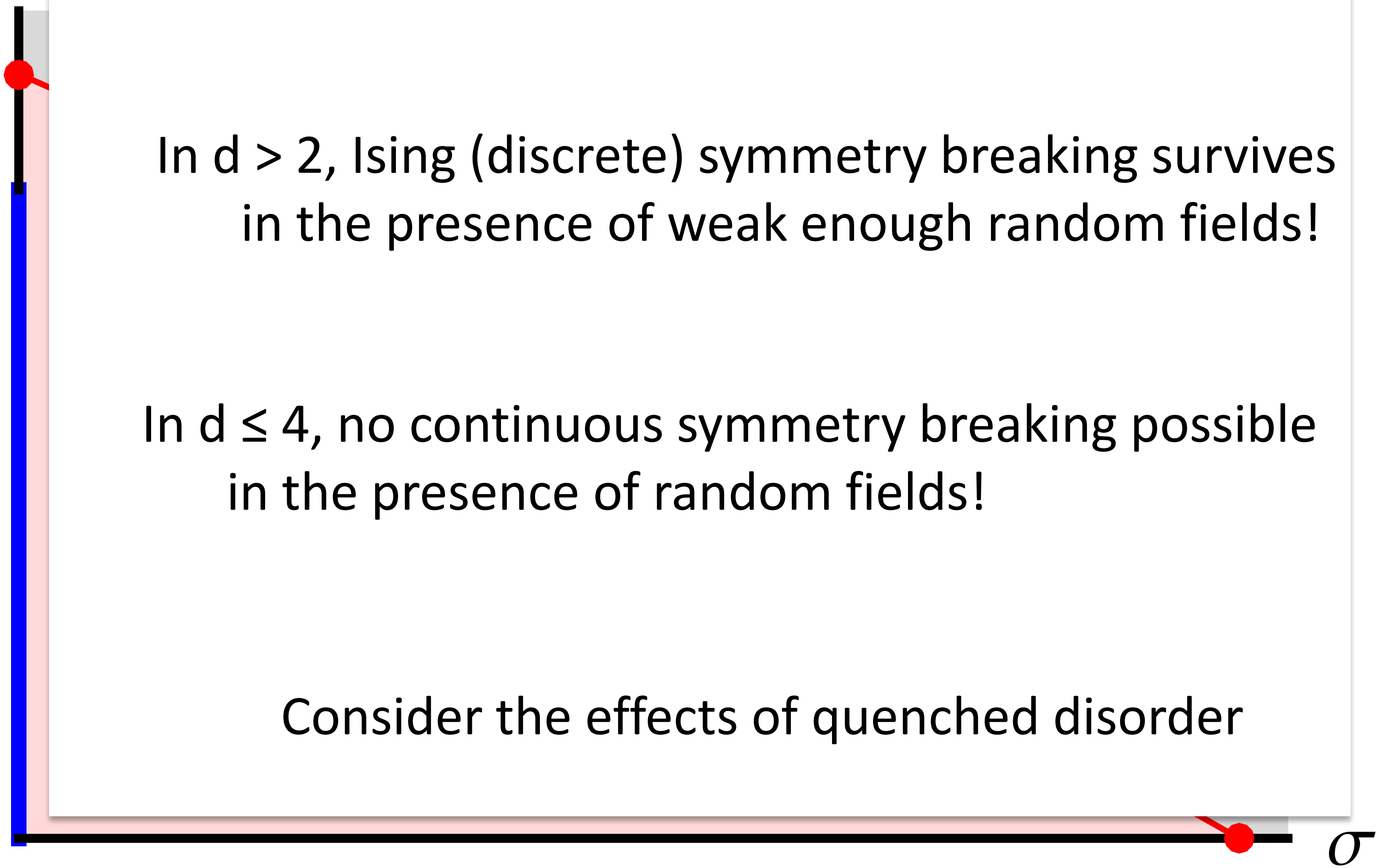


The “soft” fluctuational modes of the CDW cause it to melt at a lower temperature than the nematic order.

Assume a stripe-ordered ground-state and $V_z << J$

L. Nie, G. Tarjus, and SAK (2013)

“Vestigial” Nematic Order



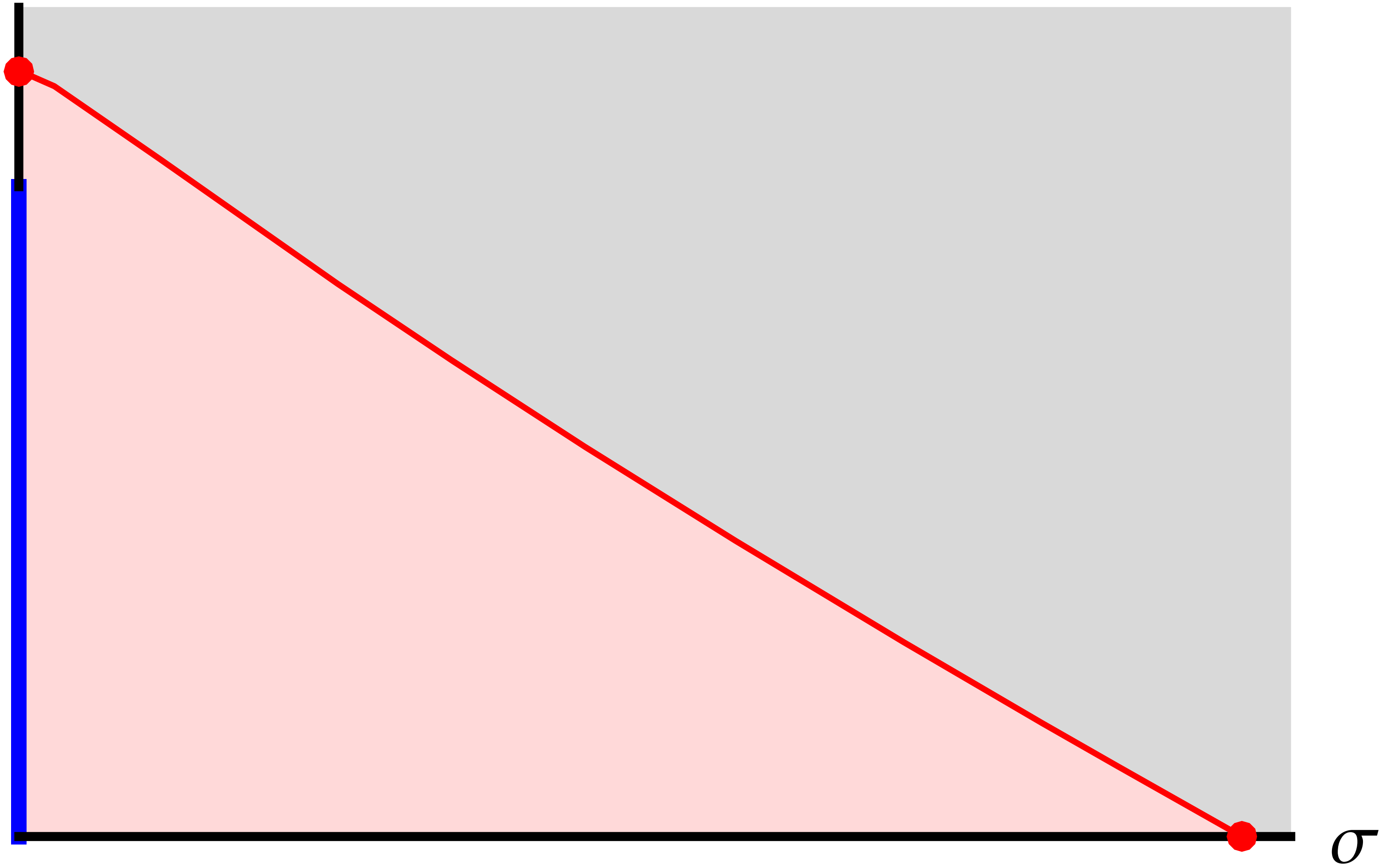
In d > 2, Ising (discrete) symmetry breaking survives in the presence of weak enough random fields!

In d ≤ 4, no continuous symmetry breaking possible in the presence of random fields!

Consider the effects of quenched disorder

L. Nie, G. Tarjus, and SAK (2013)

“Vestigial” Nematic Order



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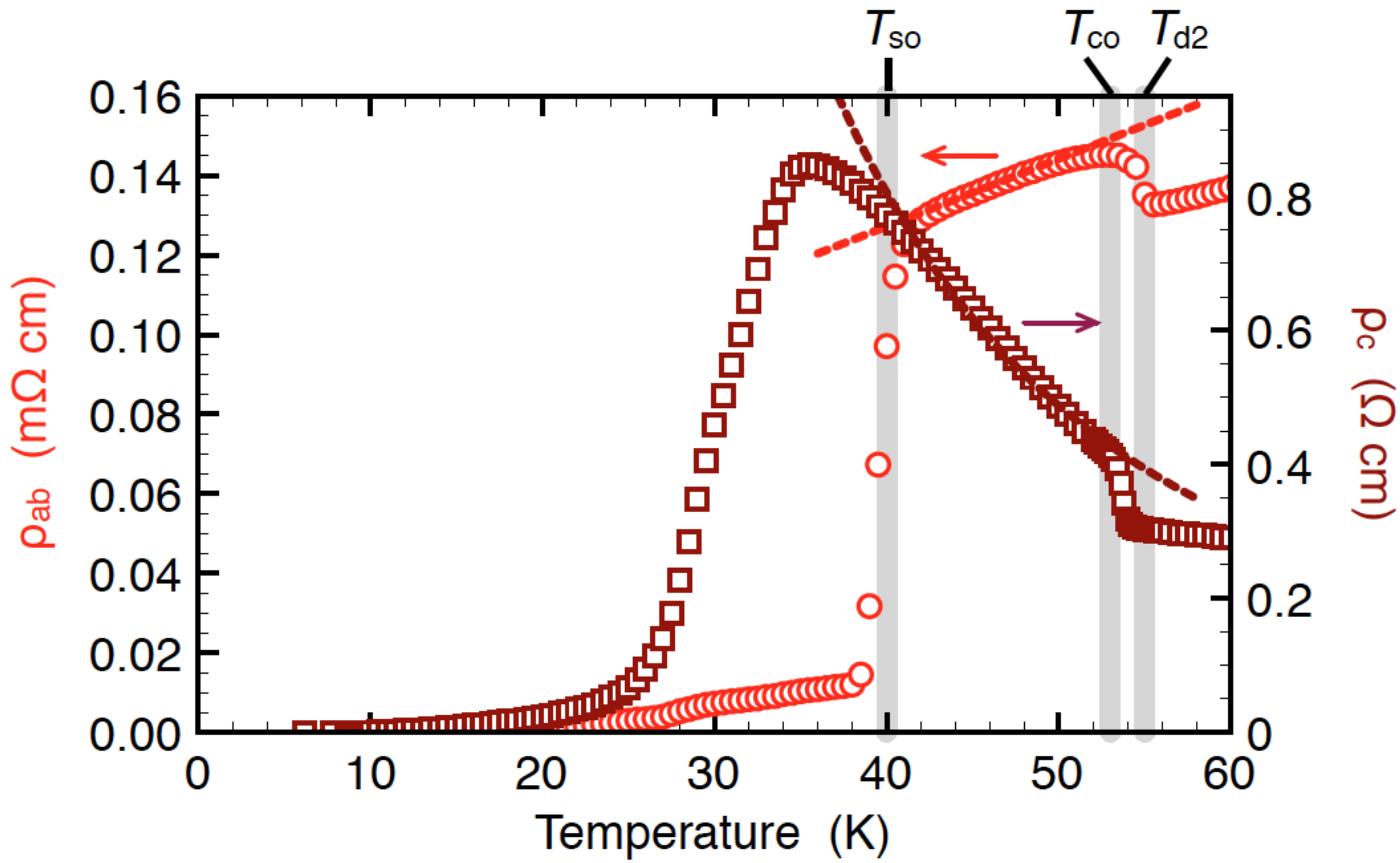
Pair-Density Wave as an Intertwined “Parent” Order

The “fully ordered” version of this state intertwines CDW, SDW and superconducting orders –

Evidence of the existence of a new state of matter in LBCO with x=1/8

Q. Li, M. Huecker, G.D. Gu, A. M. Tsvelik, and J. M. Tranquada, "Two-dimensional superconducting fluctuations in stripe-ordered LBCO," Phys. Rev. Lett**.** **99**, 067001 (2007).

In plane and interplane resisitivity of LBCO with x=1/8



J. Tranquada *et al*, PRB **78**, 174529 (2009).

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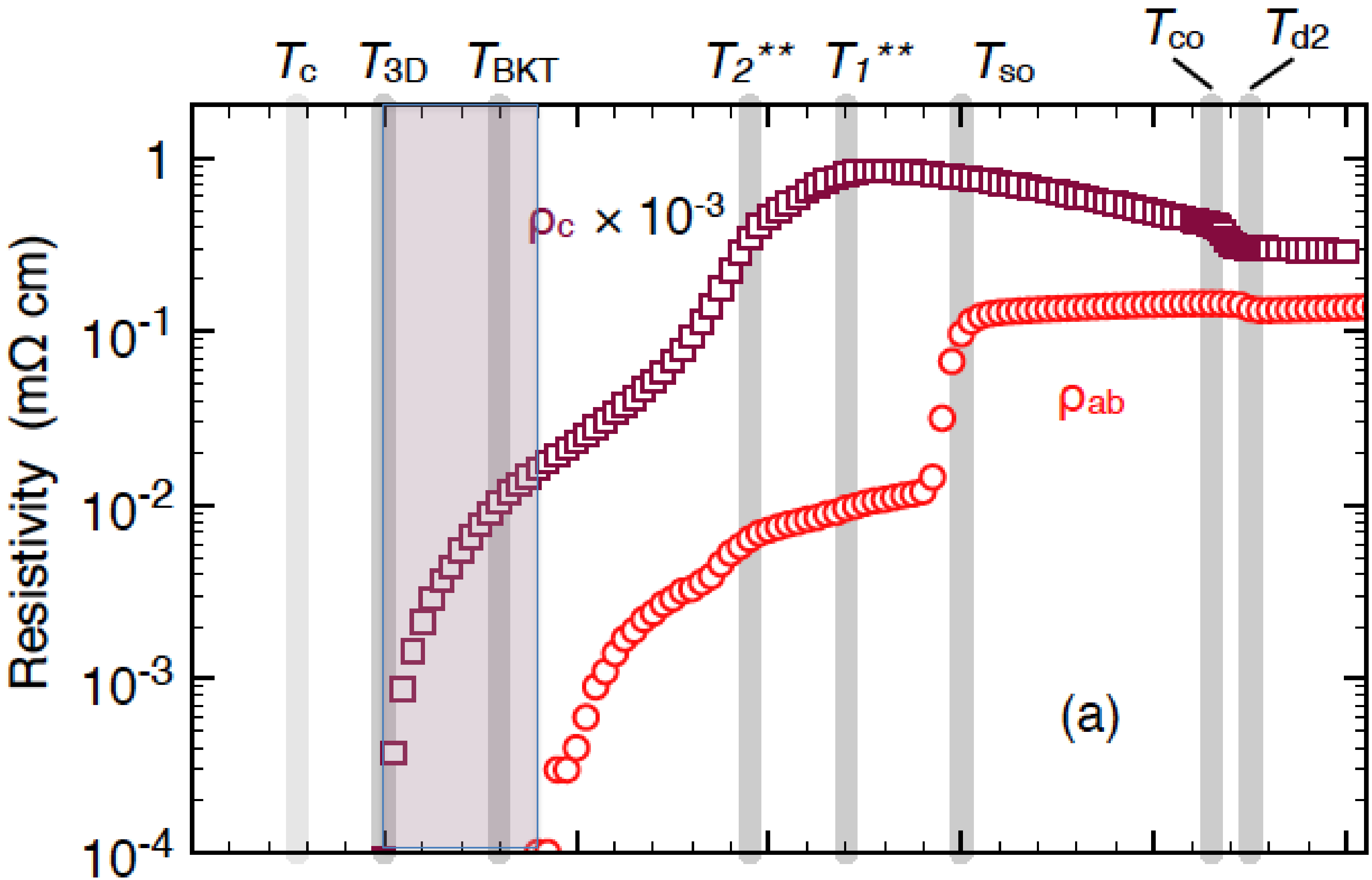
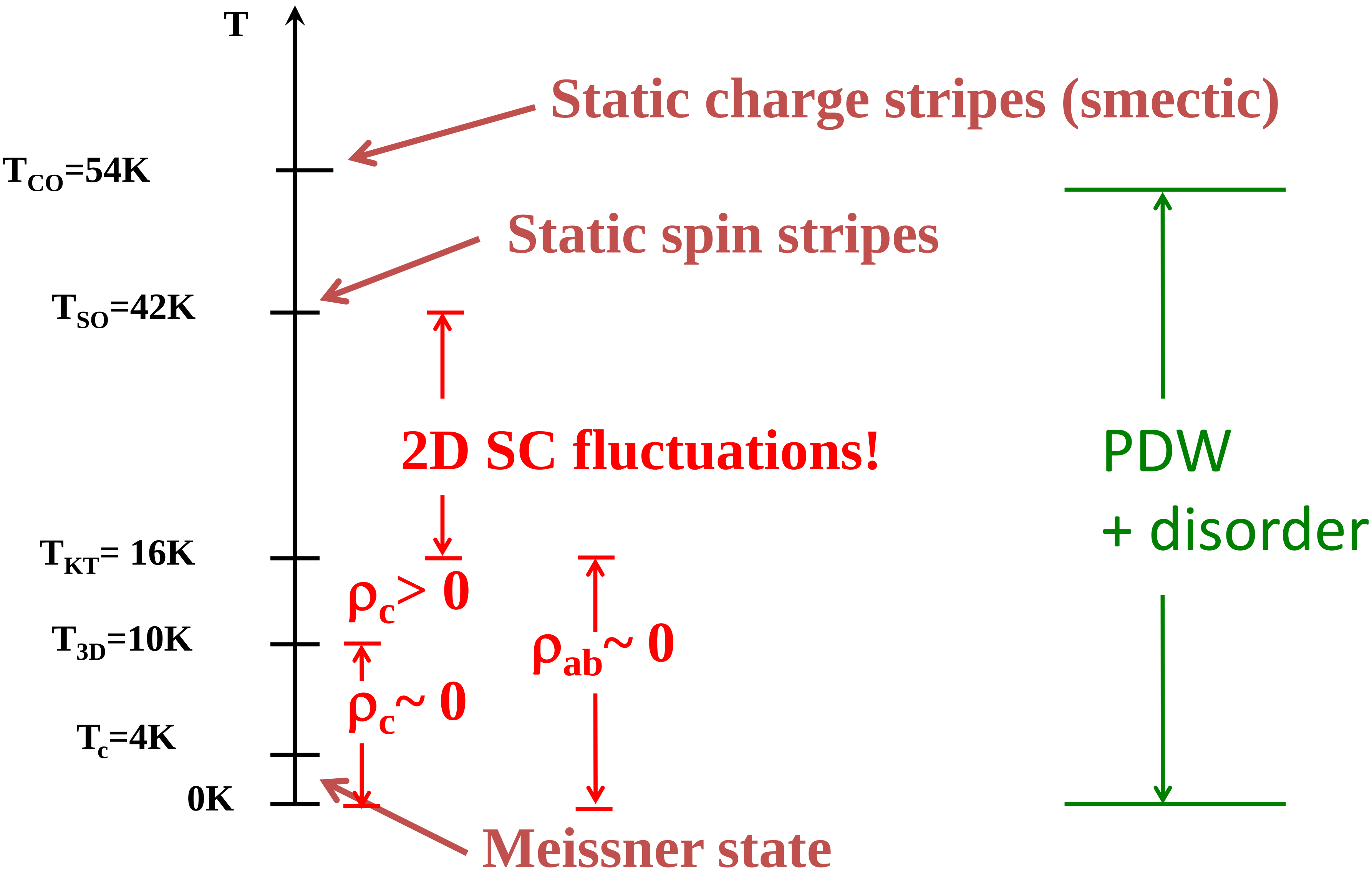


J. Tranquada *et al*, PRB **78**, 174529 (2009).

Cascade of transitions/crossovers in $La_{1.877}Ba_{.125}CuO_4$

Discussed by Berg, Fradkin, Kivelson, & Tranquada, New J. Phys. **11**, 115004 (2009)



PDW or non-zero Q pairing

$$\Delta(\vec{r}) = \Delta_{\vec{Q}}\ e^{i\vec{Q}\cdot\vec{r}} + \Delta_{-\vec{Q}}\ e^{-i\vec{Q}\cdot\vec{r}} + \ldots$$

$$\Delta(\vec{r},\vec{r}') = F(\vec{\rho})\left[\Delta_{\vec{Q}}\ e^{i\vec{Q}\cdot\vec{R}} + \Delta_{-\vec{Q}}\ e^{-i\vec{Q}\cdot\vec{R}}\right] + \ldots$$

$$\vec{R} = \frac{1}{2}(\vec{r}+\vec{r}') \quad \vec{\rho} = \vec{r}-\vec{r}'$$

$$\int d\vec{R}\Delta(\vec{R}+\vec{\rho}/2, \vec{R}-\vec{\rho}/2) = 0$$

From a broken symmetry point of view, this is identical to “Amperian pairing”

There may be important differences in that we will discuss pairing of electrons, not “spinons” so there are no “emergent gauge fields” etc.

Can strong interactions give rise to a PDW?

Macroscopic array of π-junctions - Berg *et al*, PRL (2007)

Kondo-Heisenberg chain - Berg *et al*, PRL (2010)

Various 2-leg ladders Jaefari and Fradkin (2010)

HF-BCS mean field theory with strong attractive V
- Loder *et al* PRB (2010) and PRL (2011)

Related to FFLO states but without net magnetization
– see extensive discussion in L. Radzihovsky PRA **84** (2011),

.

Also various works of Agterberg *et al*
starting with Tsunetsugu and Agterberg (2008)

Can strong interactions give rise to a PDW?

Variational treatments of the 2D t-J model:

Himeda, Kato, and Ogata, PRL (2002).

Corboz, White, Vidal, and Troyer, PRB **84**, 041108 (2011).

Corboz, Rice, and Troyer, PRL **113**, 046402 (2014).

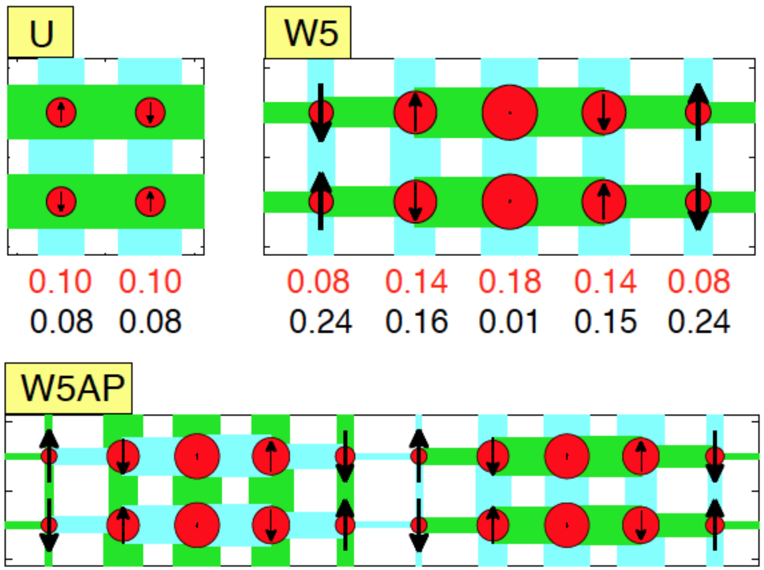
"Best" variational calculations to date

Uses new methods imported from quantum information theory

Can strong interactions give rise to a PDW?

Corboz, White, Vidal, and Troyer, PRB **84**, 041108 (2011).

Corboz, Rice, and Troyer, PRL **113**, 046402 (2014).



$E_{PDW} - E_{stripe} \sim 0.001t\ x$

$E_{unif} - E_{stripe} \sim 0.01t\ x$

Stripe period is strong function of J/t

- unrelated to FS nesting!

Find three different phases all VERY close in energy

Uniform d-wave SC:
- coexists with AF for $x < x_c$
- with no AF for $x > x_c$
- $x_c \sim 0.1$ is a function of J/t

Striped d-wave-like SC
- coexists with CDW
- SDW with $2Q_{sdw}=Q_{cdw}$

d-wave-like PDW which
- coexists with CDW
- SDW with $2Q_{sdw}=Q_{cdw}$

CDW order as a vestige of PDW order:

$$\rho_{2\vec{Q}} \equiv \Delta^{\star}_{\vec{Q}}\Delta_{\vec{Q}}$$ Composite order parameter

$$\tilde{\rho}_{2\vec{Q}} \equiv \mathbf{S}_{\vec{Q}} \cdot \mathbf{S}_{\vec{Q}}$$ Composite order parameter

From partial disordering of a PDW ground-state, can obtain:

PDW+CDW+SDW with or without C4 symmetry breaking

CDW+SDW with or without C4 symmetry breaking

CDW with or without C4 symmetry breaking

Uniform charge 4e SC with or without C4 symmetry breaking

Nematic

Various loop current states with or without PDW order

Agerberg (2011);Agterberg, Melchert, Kashyap (2014); Wang, Chubukov and Nandkishore(2014)

Some remarks concerning CDW order

For $d > 1$, generically CDW requires $V\rho(E_F) \gtrsim 1$ so $V \gtrsim W$

Correspondingly, CDW order generically is not dominated by electronic states in a small shell about the Fermi surface!

Typically $\chi(\vec{Q}, T) \sim \rho(E_F)$ so usual mean field condition

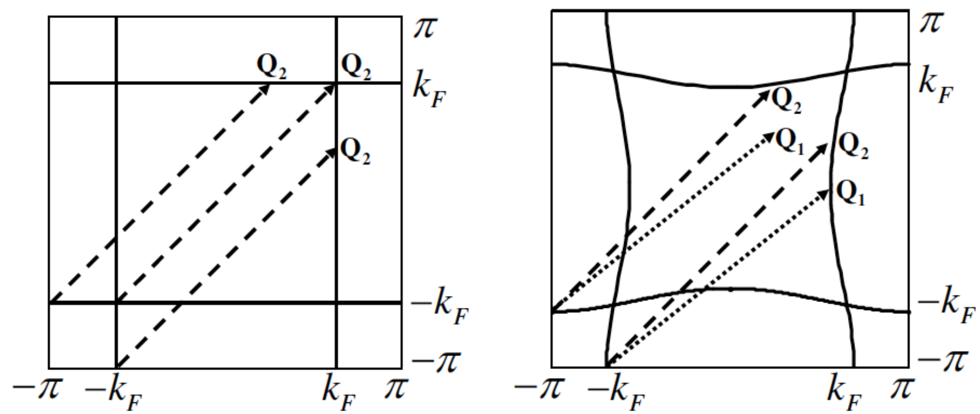
$V\chi(\vec{Q}, T_c) = 1$ requires large V

In fact maximum $\chi(\vec{Q}, T)$ often not the ordering vector

As Sachdev showed yesterday, this is the case in the cuprates.

Some remarks concerning CDW order

Rare earth tritellurides – p-band metals (so very low “Mottness”) have substantial nesting due to hidden 1D character.



Here CDW ordering occurs at subsidiary maximum of x (for reasons that can be understood from weak coupling) leading to stripe CDW order.

H. Yao *et al*, PRB (2006)

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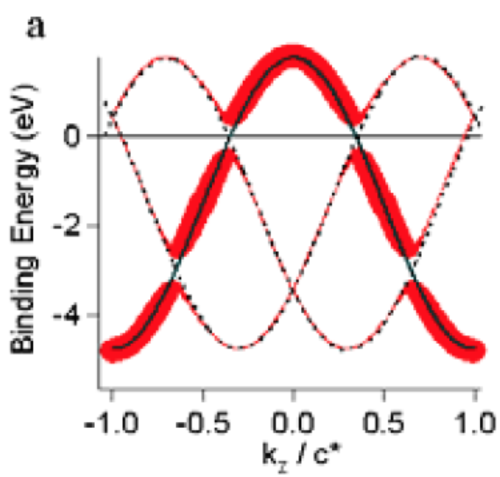
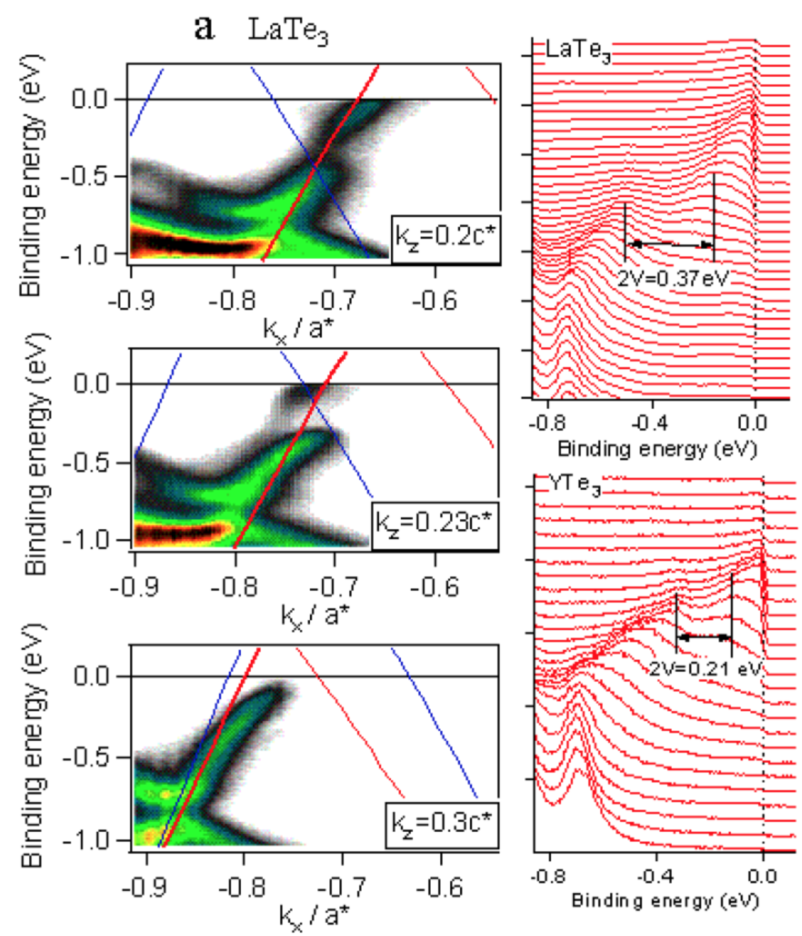




Determining the CDW gap from ARPES looking below the FS where

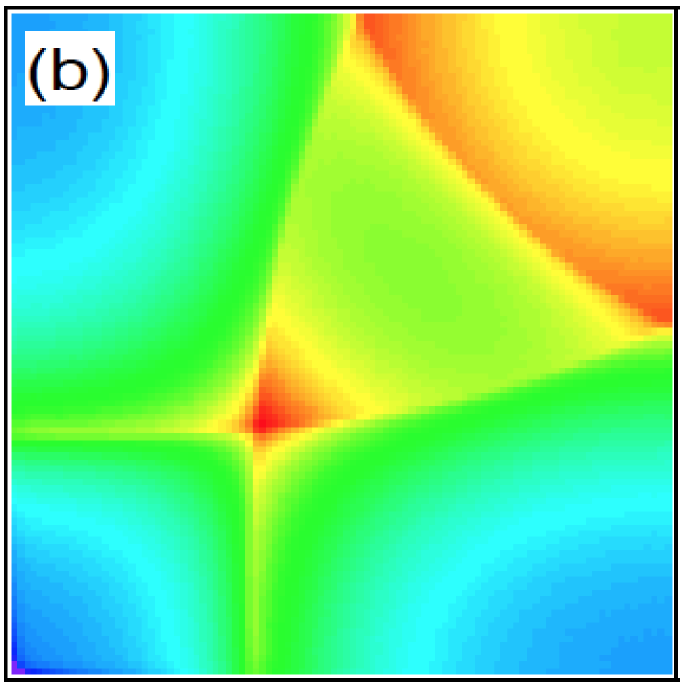
$\varepsilon_{\mathbf{k}}=\varepsilon_{\mathbf{k-Q}}$

V. Brouet et al, PRB (2008)

Some remarks concerning CDW order

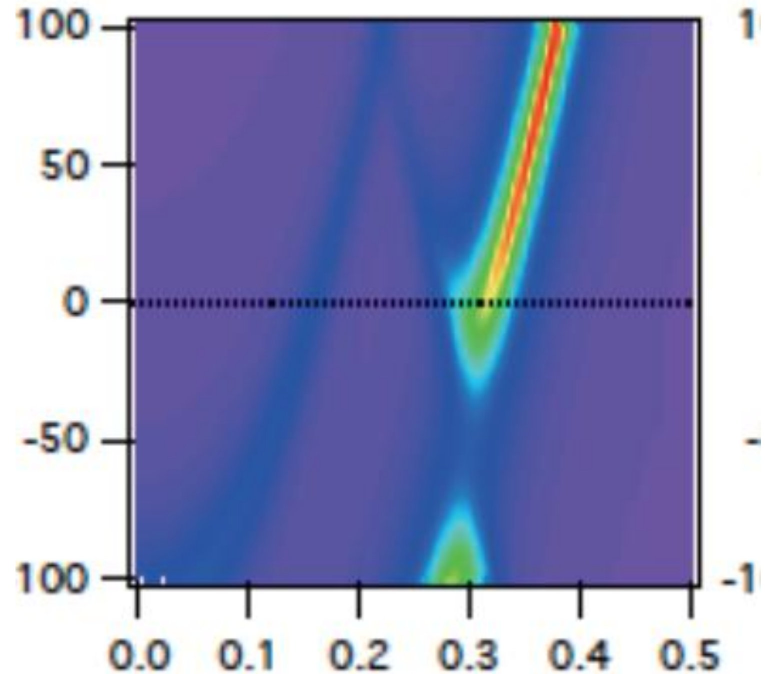
$\chi(\mathbf{Q})$

Q_y



Q_x

E



k_y/π

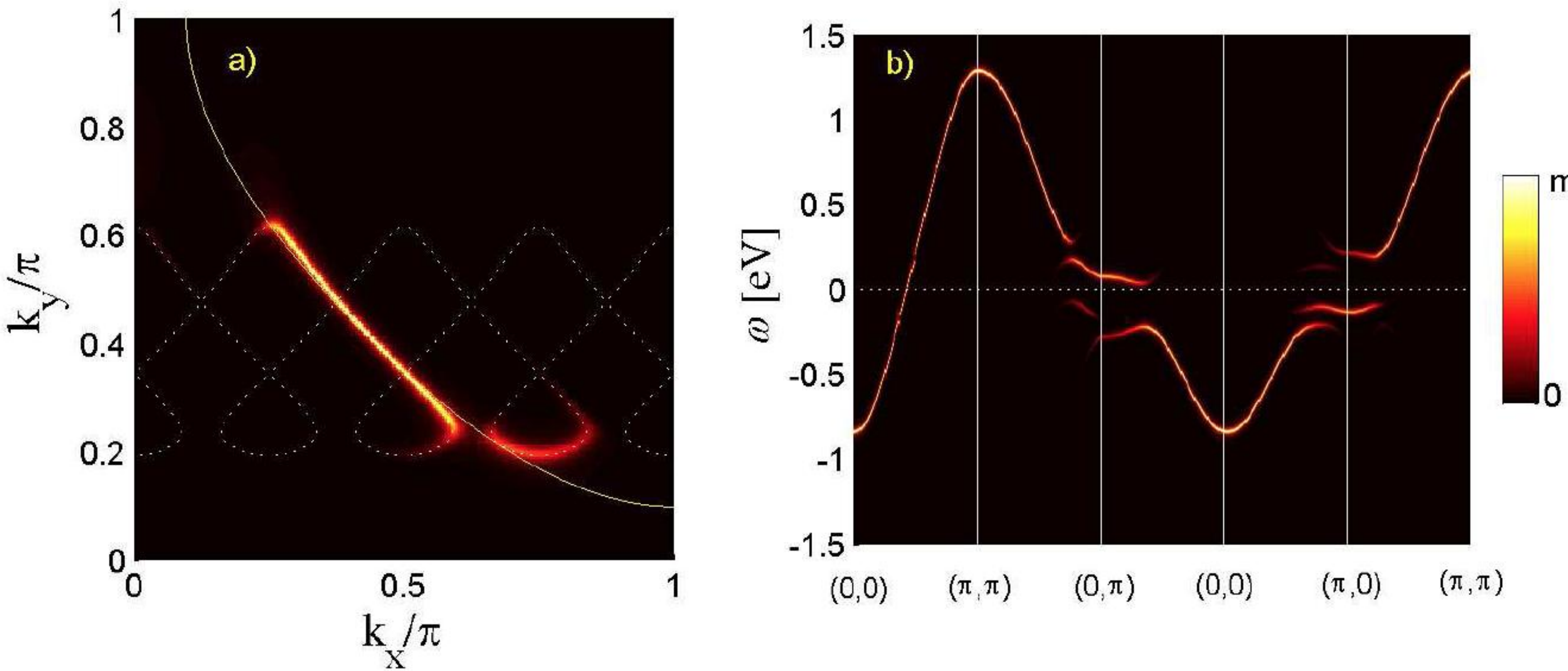
Susceptibility for Bi2212 band structure

Dispersion for kx=0.6p and $\mathbf{Q}=2\pi(0,.3)$

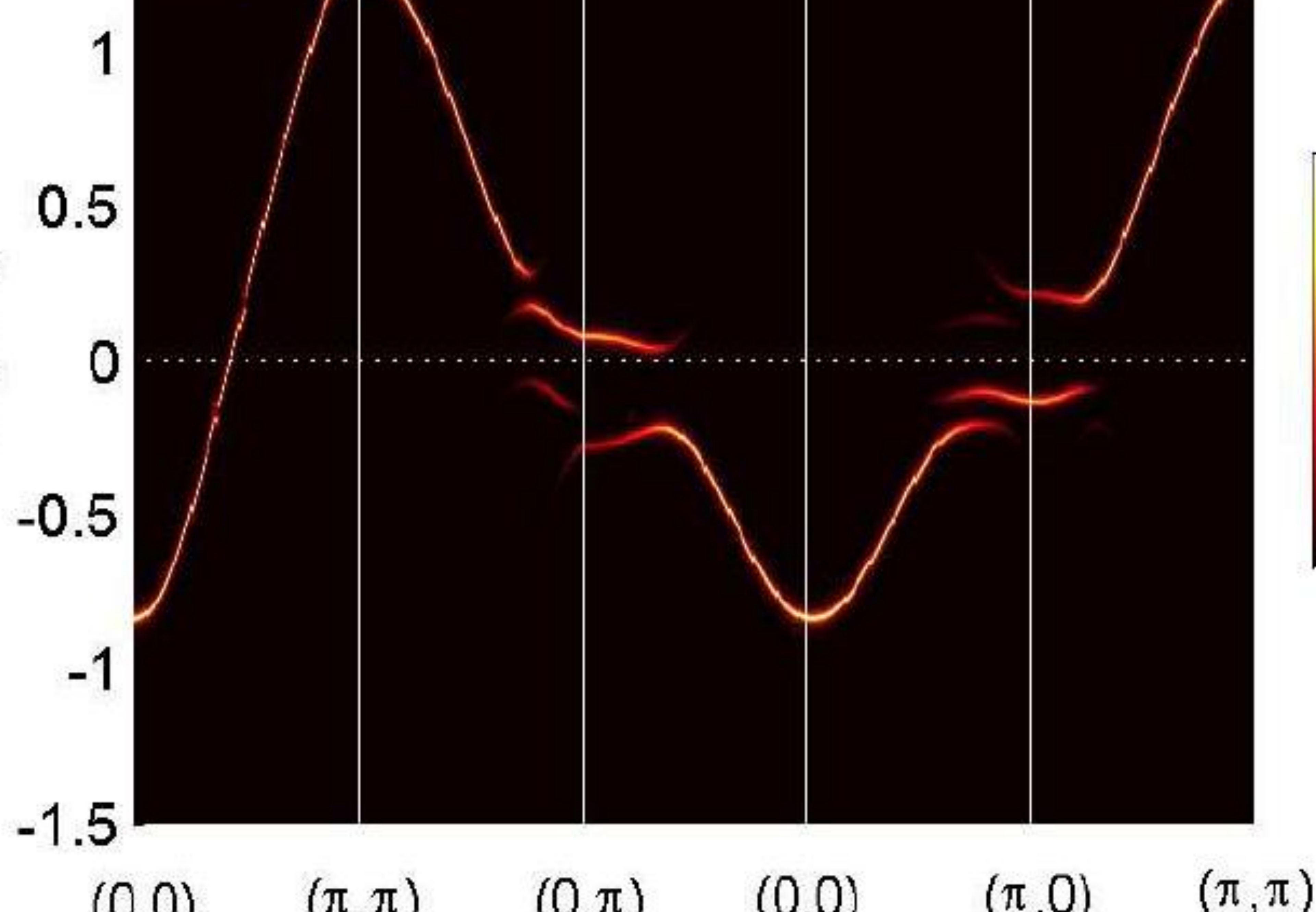
M. Norman, PRB (2007) and slides from Brazil

Some remarks concerning CDW order

Contrast with PDW where CDW component is “weak”



From Berg *et al*, New J. Phys. **11**, 115004 (2009) with **Q**=2π(1/8,0) so 2**Q**=2π(1/4,0)



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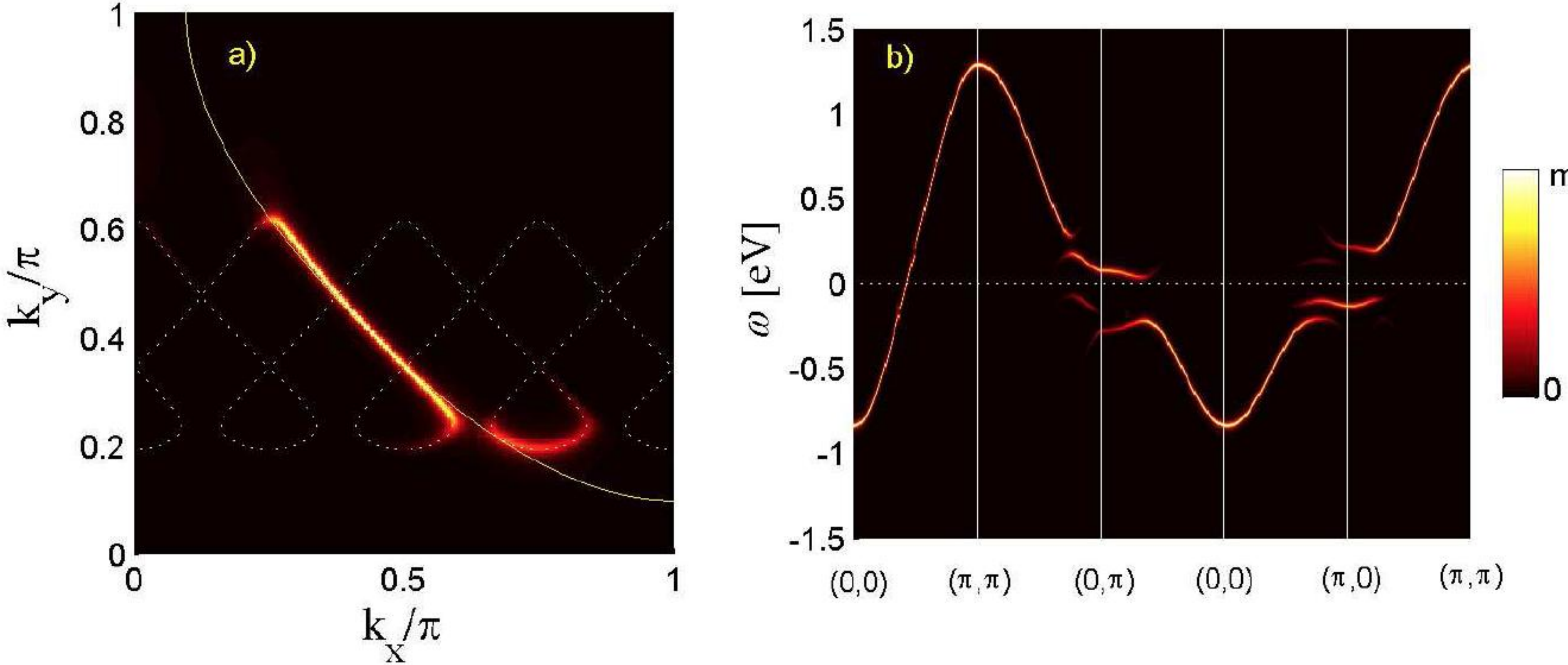

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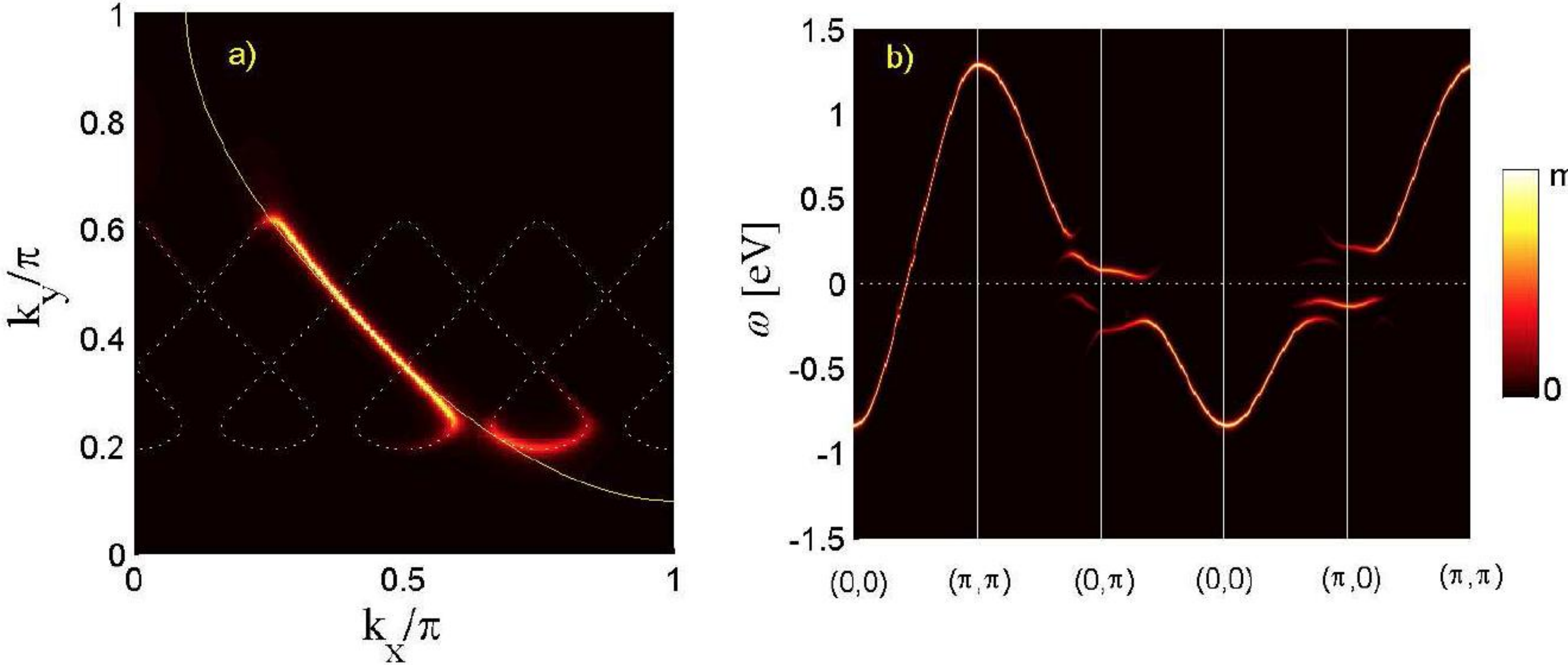

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The "fully ordered" version of this state intertwines CDW, SDW and superconducting orders –

Compelling evidence that it exists in LBCO, t-J model, and …

It is possible that CDW, nematic, and other orders can be viewed as vestiges of underlying PDW tendencies.

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