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# Strong coupling behavior of the neutron resonance mode in unconventional superconductors [1]

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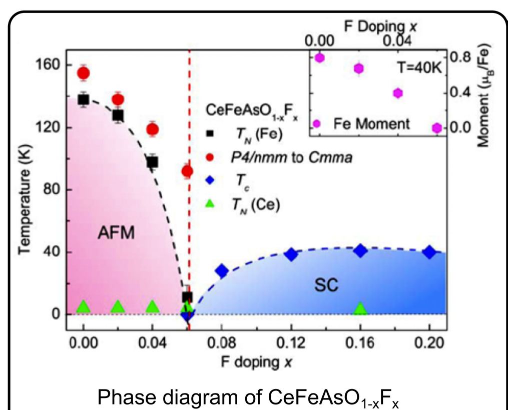
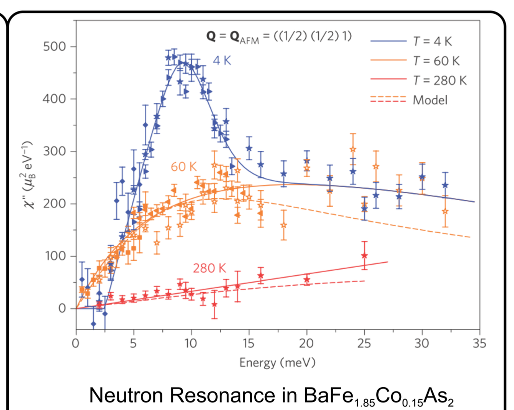
## Motivation

Unconventional superconductors (USC) [e.g. cuprates, iron pnictides] have a wide range of interesting features. The phase diagrams of these systems show an interesting regime with possible coexistence of AF and SC order depending on doping. In the SC state there is a sharp resonance in neutron scattering experiments at the antiferromagnetic (AF) vector $\boldsymbol{Q}$. The emergence of superconductivity and collective spin excitations in these materials can be described by the spin-fermion model. Within one-loop perturbation theory it is possible to give qualitative statements about the resonance energy $\Omega_{\text{res}}$ of the spin mode, their influence on the fermionic system and the SC properties such as the pairing symmetry below $T_c$. We examine the validity of the leading-order results considering both vertex and self-energy corrections and find that, in general, such correction cannot be ignored in $d = 2$ dimensions. Nevertheless, we show that a controlled perturbation theory can be performed using an $\varepsilon$-expansion around the upper critical dimension $d_{uc} = 3$.




Phase diagram of $CeFeAsO_{1-x}F_x$
[M. Norman, Physics 1, 21]




Neutron Resonance in $BaFe_{1.85}Co_{0.15}As_2$
[D. Inosov et al. , Nature 6, 178]

## Spin-fermion model

- Hamiltonian [see Review: **cond-mat/0201140**]

$$\mathcal{H} = \mathcal{H}_F^{(0)} + \mathcal{H}_{\text{Spin}}^{(0)} + g \int dx \boldsymbol{S} \cdot (\bar{\psi}_\alpha \boldsymbol{\sigma}_{\alpha,\beta} \psi_\beta)$$

- fermionic quasiparticles $\psi$ with dispersion

$$\varepsilon_{\boldsymbol{k}} = \boldsymbol{v}_F \cdot (\boldsymbol{k} - \boldsymbol{k}_F) - \mu$$

- fermionic system contains $N$ so called hot-spots $\boldsymbol{k}_F$ connected by the AFV $\boldsymbol{Q}$
- coupled bosonic spin mode $\boldsymbol{S}$ [closeness to AF QCP] with spin susceptibility

$$\boxed{\chi_{\boldsymbol{q}}(\omega) = \frac{1}{r + c_s(\boldsymbol{q} - \boldsymbol{Q})^2 - \Pi_{\boldsymbol{q}}(\omega)}},$$

$\Rightarrow$ $r \sim \xi^{-2}$ determines the distance to AF QCP

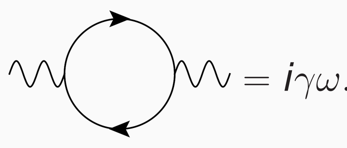
## Perturbation theory

In what follows we assume $g$ to be small compared to the corresponding fermionic scales, which implies smallness of the dimensionless parameter

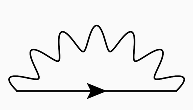
$$\boxed{\gamma = \frac{g^2 N}{2\pi v_F^2}}.$$

## Normal-state analysis

- Coupling to gapless fermionic quasiparticles gives rise to a damped bosonic one-loop self-energy at the AFV $\boldsymbol{Q}$ [2]

$$\Pi_{\boldsymbol{Q}}(\omega) =$$  $$= i\gamma\omega.$$

- The corresponding fermionic self-energy at the hot-spots

$$\Sigma_{\boldsymbol{k}_F}(i\Omega_n) =$$  $$= \begin{cases} -i\Omega_n \lambda & , \text{ if } |\Omega_n| \ll \omega_{\text{sf}} \\ -i\Omega_n \left|\frac{\bar{\Omega}}{\Omega_n}\right|^{\varepsilon/2} & , \text{ if } |\Omega_n| \gg \omega_{\text{sf}} \end{cases},$$

is separated by the characteristic frequency

$$\boxed{\omega_{\text{sf}} = r/\gamma}$$

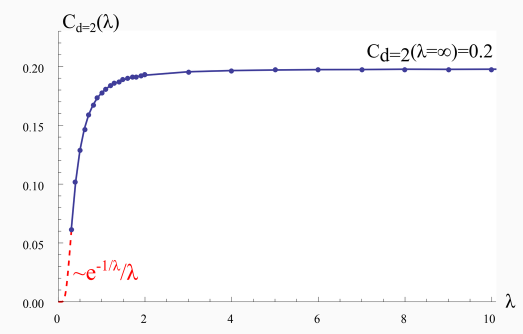
into a Fermi-liquid regime $\Sigma \sim \Omega$ and a quantum critical regime $\Sigma \sim \sqrt{\Omega}$. Here, we defined the dimensionless pairing parameter $\lambda$ and the energy scale $\bar{\Omega}$

$$\lambda = \frac{3g^2}{4\pi v_F \sqrt{r c_s}} \qquad \bar{\Omega} = \frac{9g^2}{2\pi N c_s} \qquad (d = 2)$$

## Superconducting transition

Superconductivity due to boson exchange with unconventional pairing symmetry [3]

$$\boxed{\Delta_{\boldsymbol{k}_F + \boldsymbol{Q}} = -\Delta_{\boldsymbol{k}_F} \equiv \Delta}.$$




Solving the Eliashberg-equations the transition temperature is given by

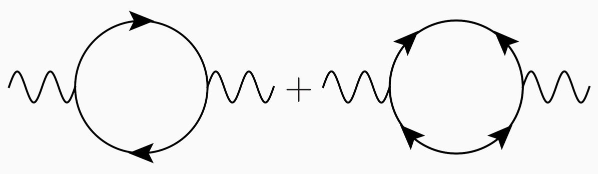
$$T_c = \bar{\Omega} \cdot C(\lambda)$$

with dimensionless function

$$C(\lambda) = \begin{cases} \sim \frac{e^{-1/\lambda}}{\lambda} & \lambda \ll 1, (T_c \ll \omega_{\text{sf}}) \\ \approx 0.2 & \lambda \gg 1, (T_c \gg \omega_{\text{sf}}) \end{cases}$$

## Resonance mode in SC at one-loop

- One-loop boson self-energy in superconducting state at $\boldsymbol{Q}$

$$\Pi_{\boldsymbol{Q}}^{(1)}(\omega) =$$  $$\qquad (1)$$

obtains for $T = 0$ a discontinuity in the imaginary part at $\omega = 2\Delta$ for unconventional gap symmetries [2]

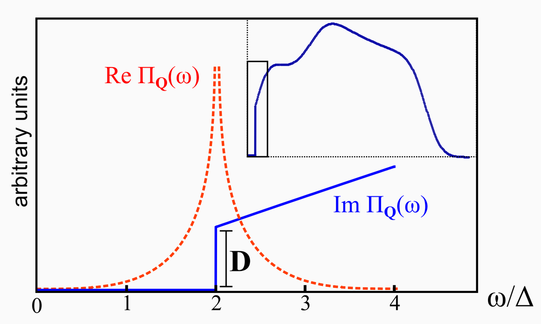
$$\boxed{D_0 = \lim_{\eta \to 0} \textbf{Im}\, \Pi_{\boldsymbol{Q}}^{(1)}(2\Delta + \eta) = \begin{cases} \pi\gamma\Delta & \text{for } \Delta_{\boldsymbol{k}_F+\boldsymbol{Q}} = -\Delta_{\boldsymbol{k}_F} \\ 0 & \text{for } \Delta_{\boldsymbol{k}_F+\boldsymbol{Q}} = \Delta_{\boldsymbol{k}_F} \end{cases}}$$

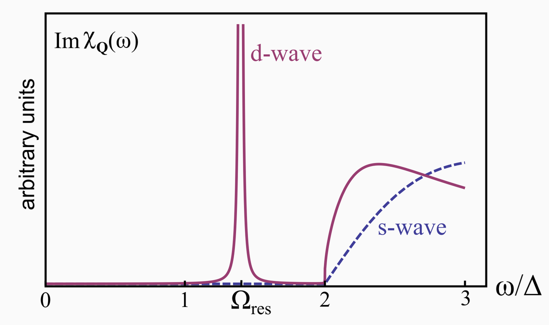
- $\textbf{Im}\, \Pi_{\boldsymbol{Q}}^{(1)}(\omega < 2\Delta) = 0$ due to the gapped fermionic quasiparticles.
- Generalization using $\Delta_{\boldsymbol{k}_F} = \Delta_1 e^{i\phi_1}, \Delta_{\boldsymbol{k}_F+\boldsymbol{Q}} = \Delta_2 e^{i\phi_2}$:

$$D_0 = \lim_{\eta \to 0} \textbf{Im}\, \Pi_{\boldsymbol{Q}}^{(1)}(\Delta_1 + \Delta_2 + \eta) = \pi\gamma\sqrt{\Delta_1\Delta_2} \sin^2\left(\frac{\phi_1 - \phi_2}{2}\right).$$

- Discontinuity in the imaginary part leads to logarithmic divergence $\textbf{Re}\, \Pi_{\boldsymbol{Q}}^{(1)}(\omega \approx 2\Delta) = -\gamma\Delta \log(\frac{\omega - 2\Delta}{2\Delta})$ in the real part. Thus we get for $\phi_1 \neq \phi_2$ a guaranteed resonance at

$$\boxed{\Omega_{\text{res}} = 2\Delta(1 - e^{-\frac{\pi r}{D_0}})} \qquad (2)$$

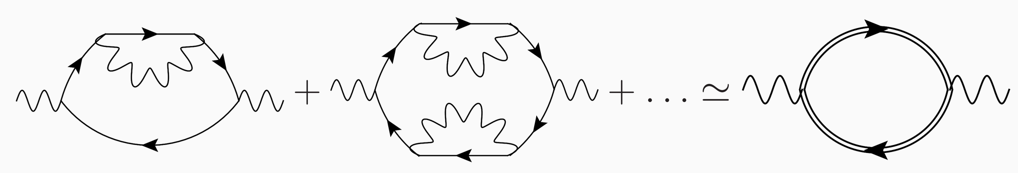



**Note: Discontinuity $D$ is the only important feature to describe resonance mode!**

## Self-energy corrections

- Self-energy corrections beyond the one-loop calculation

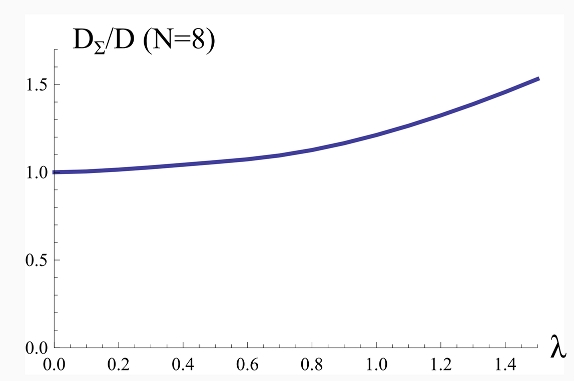


- $\lambda \simeq 1, \lambda \ll 1$: include corrections by a Taylor expansion of renormalized dispersion and gap

$$\tilde{\epsilon}_{\boldsymbol{k}} \approx \epsilon_{\boldsymbol{k}} + \nu_m \epsilon_{\boldsymbol{k}+\boldsymbol{Q}},$$
$$\Delta_{\boldsymbol{k}}(\omega) \approx \Delta + \Delta_f(\omega - \Delta).$$

Discontinuity with self-energy:

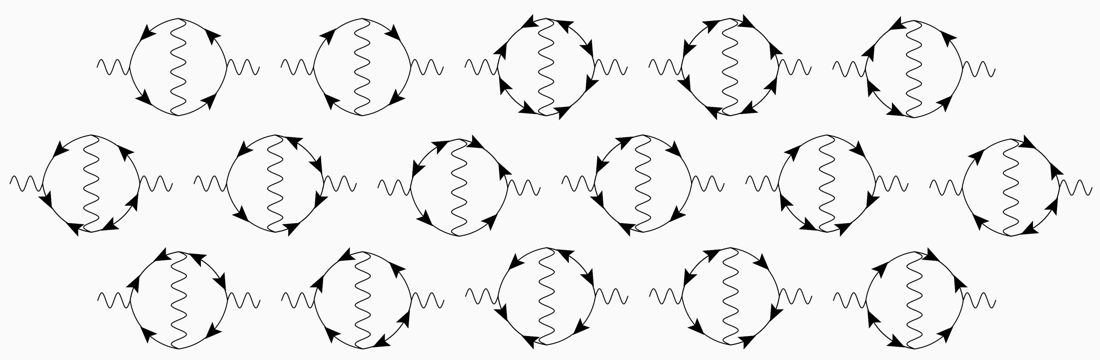
$$\boxed{D_\Sigma = \frac{D_0}{(1 - \nu_m)^2(1 - \Delta_f)}}$$




- Coefficients $\nu_m, \Delta_f \sim g^2$ only for the weak-coupling regime $\lambda \ll 1 \Rightarrow$
- self-energy corrections are of order one in the physical regime $\lambda \sim 1! \Rightarrow$ controlled in the large $N$ limit since $\nu_m, \Delta_f \sim 1/N$ .

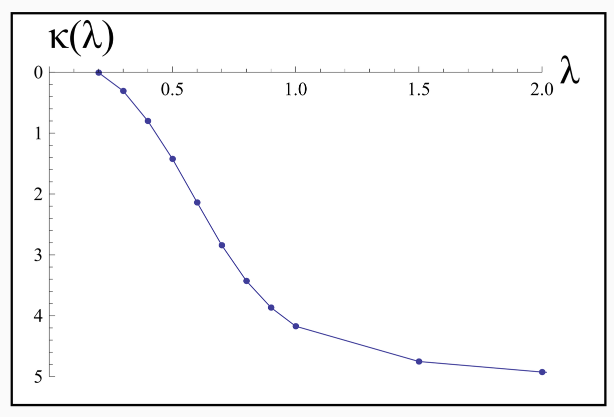
## Vertex corrections

- Consider vertex corrections (VC) beyond one-loop



- VC to discontinuity

$$\delta D_{\text{VC}} = \begin{cases} \frac{D_0}{N}\kappa(\lambda) & \text{for } \Delta_{\boldsymbol{k}_F+\boldsymbol{Q}} = -\Delta_{\boldsymbol{k}_F} \\ 0 & \text{for } \Delta_{\boldsymbol{k}_F+\boldsymbol{Q}} = \Delta_{\boldsymbol{k}_F} \end{cases} \qquad (3)$$




$\rightarrow$ Calculation only controllable for weak coupling $\lambda \ll 1$

$\rightarrow$ $\lambda \simeq 1$: VC are not small . Physical origin of strong-coupling behavior: QC spin-fluctuations with $\omega > \omega_{\text{sf}}$ contributing to the above diagrams.

$\rightarrow$ Large $N$ theory not applicable as shown by Lee [4] ($1/N$ expansion breaks down)

**$\rightarrow$ One-loop calculation is not controlled as vertex corrections are of order unity!**

## Controlling the calculation via $\varepsilon$-expansion

- Similar to Mross et al. [5] we perform $\varepsilon$-expansion around upper critical dimension $d_{uc} = 3$

$$\varepsilon = 3 - d$$

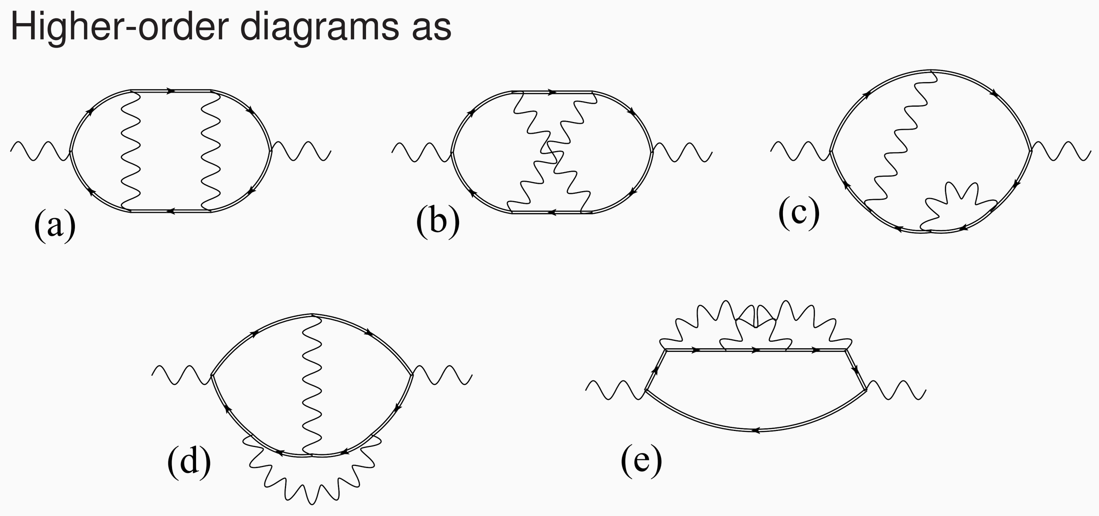
- only bosonic field depends on the additional dimension in $z$-direction (fermionic QP still confined to xy-plane)
- Vertex corrections to discontinuity

$$\frac{\delta D_{\text{VC}}}{D_0} \propto \gamma^\alpha \propto g^{2\alpha} \qquad \text{with } \alpha = \frac{2}{\epsilon} - 2 > 0$$

- $\varepsilon \ll 1$: Exponent $\alpha$ is large and vertex corrections are small (for $d = 3$ exponentially small) **$\rightarrow$ One-loop calculation can be controlled within $\varepsilon$-expansion!**

## Phase sensitivity beyond two-loop

Higher-order diagrams as




for gap symmetry for $\Delta_{\boldsymbol{k}_F+\boldsymbol{Q}} = \Delta_{\boldsymbol{k}_F}$ also show no discontinuity

$\Rightarrow$ Expect resonance mode to be a general feature of USC.

## Acknowledgments

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## References

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[4] S.-S. Lee: PRB **80**, 165102 (2009)
[4] D. Mross *et al.* , PRB **82**, 045121 (2010)