KINEMATIC SPACE I

Geometry, information & a holographic CFT language

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Motivation

Objective: We want to understand holography **'microscopically'**

How are **encoded** in the CFT:

- The bulk **geometry**?
- Its **dynamics**?
- The low energy **effective degrees of freedom**?

Bulk geometry: Ryu and Takayanagi gave us a useful clue.

Entanglement Entropy = Area of minimal surfaces

(Geodesic lengths in AdS_3**)**

Our proposal

- **Entanglement** structure contains **geometric data.**
- We need to build an appropriate **language** to extract this data.
- **Our proposal:** A natural language is provided by **integral geometry.**

Integral geometry is the study of geometric spaces using global properties of special classes of extended objects, i.e. geodesics.

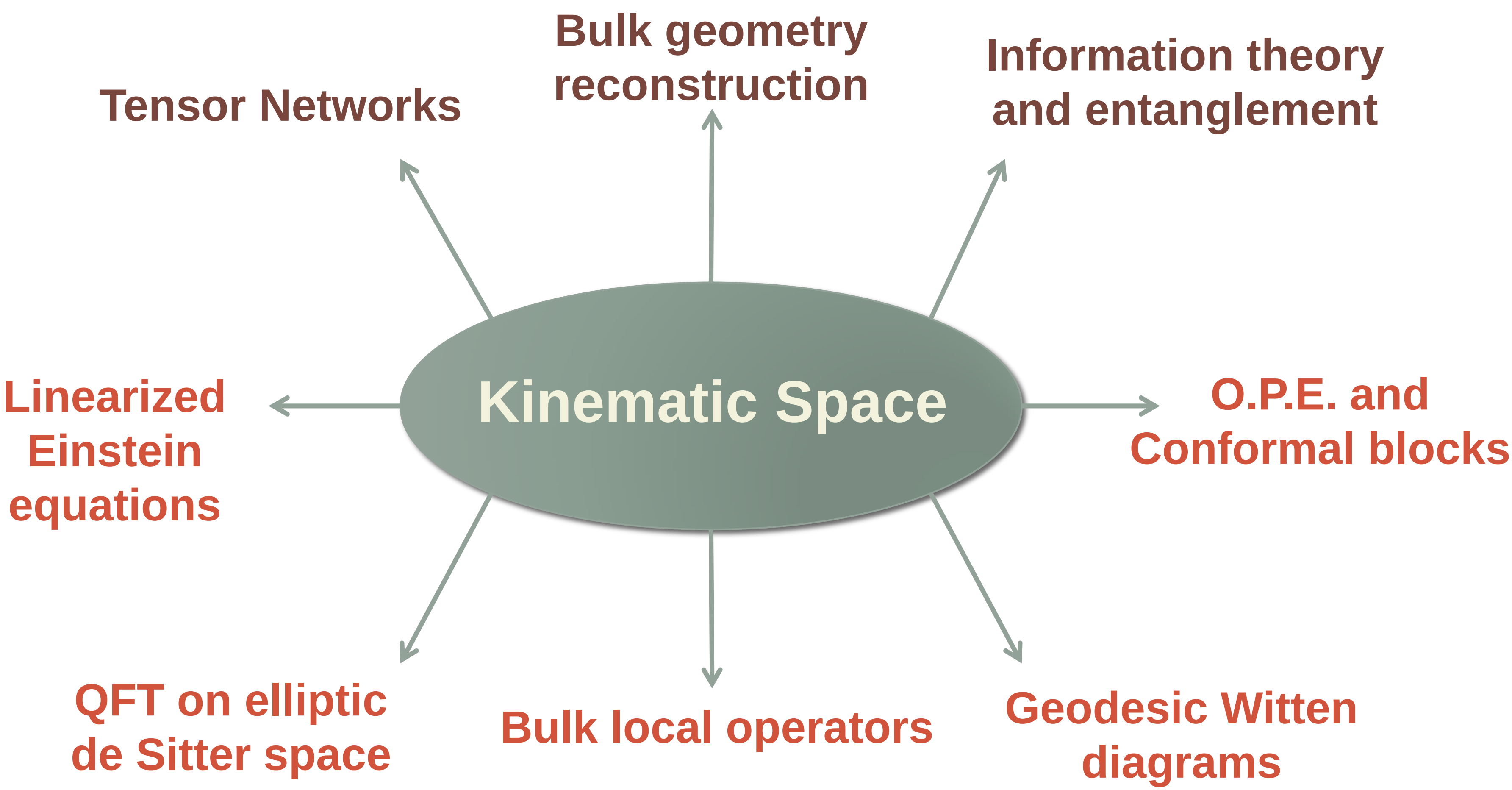
Our proposal

Space of geodesics: **Kinematic space**

Kinematic space in **holography** has a **dual nature**:

- It's a **geometric construct** useful in the **integral geometric** description of Riemannian manifolds.
- It's an **information theoretic space** that organizes the **entanglement pattern** of CFT subregions in a **geometric way.**

Uses of Kinematic Space



Outline

Review:

- Kinematic space in **integral geometry.**
- Kinematic space in **information theory.**

New developments:

- **Kinematic operators**: A kinematic basis for the CFT operators.
- **Holographic dual** of kinematic operators and **Radon transforms**.
- Relation to: OPE, conformal blocks, geodesic Witten diagrams and 'entanglement holography'.
- Interactions, 1/N and **bulk** VS **kinematic locality**.

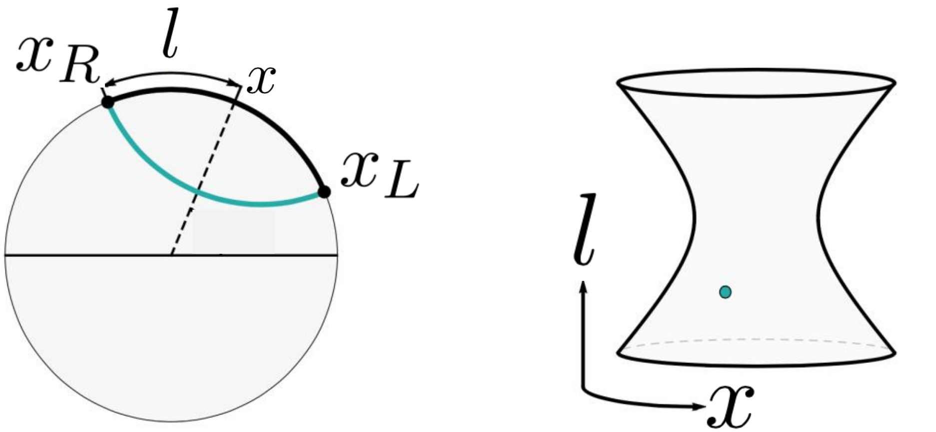
GEOMETRY FROM INFORMATION

The Kinematic Framework

What is Kinematic Space?

Integral Geometry:

- Kinematic space is the **space of geodesics** organized according to the **location** and **size** of their boundary support.



- The only information we need is the **length** of all geodesics, which defines a **function** on **kinematic space**:

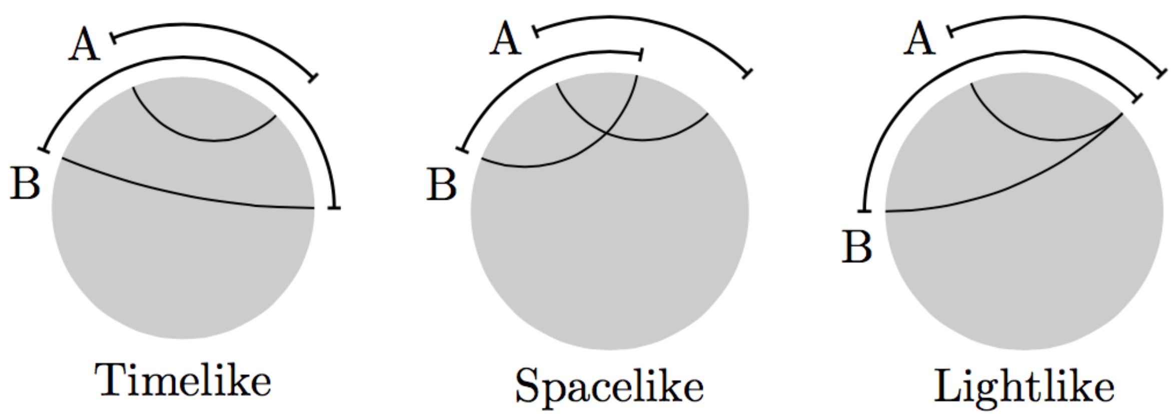
$$S(x_L, x_R) = \int_{\gamma_{LR}} ds$$

Integral Geometry

- Kinematic space in **integral geometry** is equipped with a **volume form** (Crofton form):

$$\omega = \frac{\partial^2 S(x_L, x_R)}{\partial x_L \partial x_R} dx_L \wedge dx_R$$

- Kinematic space is **partially ordered**: Containment relation of boundary intervals. This induces a natural **causal structure** on KS.



Integral Geometry

- Combining the two structures we defined a **metric**:

$$ds^2 = \frac{\partial^2 S(x_L, x_R)}{\partial x_L \partial x_R} dx_L dx_R$$

- The **endpoints** serve as **null coordinates**.

- For a global timeslice of AdS_3 it's **2D de Sitter** space

$$ds^2 = \frac{1}{l^2}(-dl^2 + dx^2)$$

where: $x_L = x - l$ and $x_R = x + l$

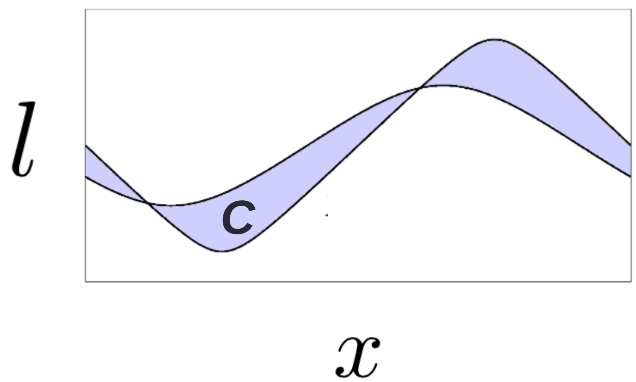
- The **spacelike boundary** of dS at $l = 0$ is identified with the **physical boundary** of the bulk.

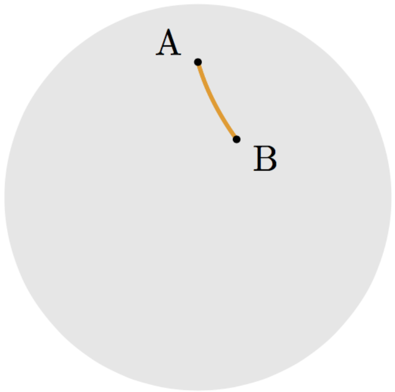
What do we learn from it?

- We can **reconstruct bulk geometry** from **boundary data** in a diffeomorphism invariant way.

- How? Crofton formula:

$$\text{Lengths} = \int_C \omega$$





- **Validity**: **Static**, **2+1D**, no conjugate points, tangent space spanned by boundary anchored geodesics.

What is Kinematic Space?

Information Theory:

- Kinematic space is the space of **intervals** (connected subregions) of the CFT.

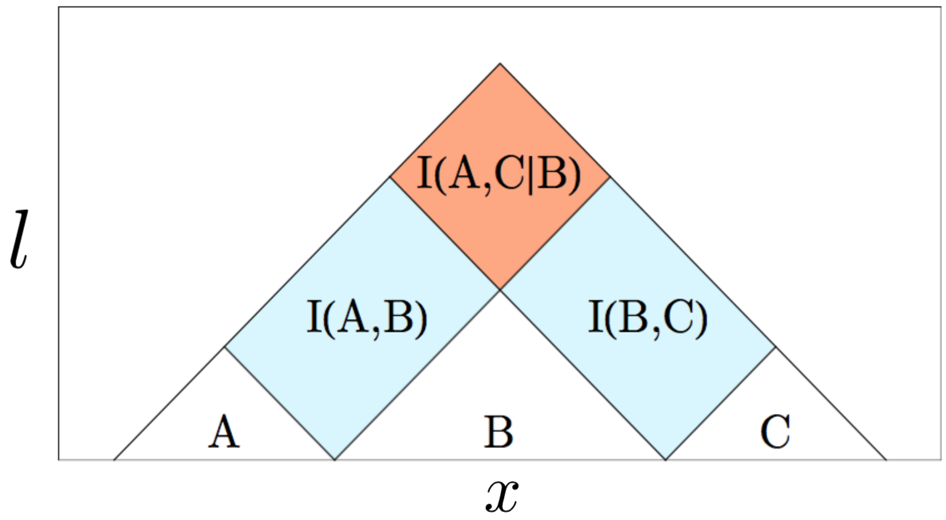
- Using RT, the **kinematic volume** is provided by **Conditional Mutual Information.**

$$d^2 V_{\text{kin}} = \frac{\partial^2 S(x_L, x_R)}{\partial x_L \partial x_R} dx_L dx_R = I\left(dx_L, dx_R | \Delta x_{LR}\right) = \text{C.M.I.}$$

- **Positivity** of the measure is guaranteed by **S.S.A.**

What do we learn from it?

- **Maps out** the single-interval **entanglement pattern** of the state and endows it with an **information geometry.**

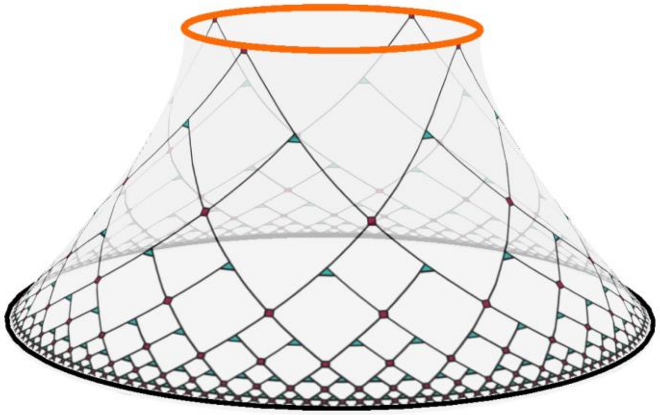


- Using Crofton formula: **Information theoretic** interpretation for **every bulk curve**, generalizing RT.

Tensor Networks from Kinematic Space

The **kinematic organization of information** is realized in **tensor network** representations of the state

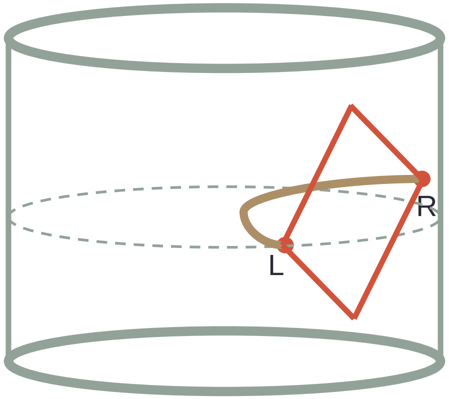
For the ground state, **MERA** is a successful realization of the **kinematic geometry.**



For other states, the kinematic organization of entanglement is achieved by generalizing MERA to an **iterative compression network**.

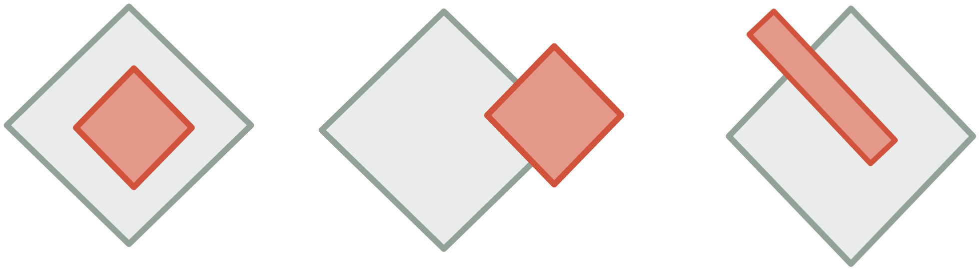
Generalizing: 4D Kinematic Space

- A natural/necessary **generalization** is to consider the space of **all spacelike geodesics**.
- Generalized kinematic space is **4-D**.
- Each geodesic is now canonically associated to boundary **causal diamonds.**
- Convenient **parametrization**: **Null coordinates** of the two endpoints: $z_L, \bar{z}_L$ and $z_R, \bar{z}_R$



Generalizing: 4D Kinematic Space

Kinematic causality now is given by the **containment relation** of the **causal diamonds** on the boundary, yielding **2 auxiliary time directions**.



The **metric** is determined by:

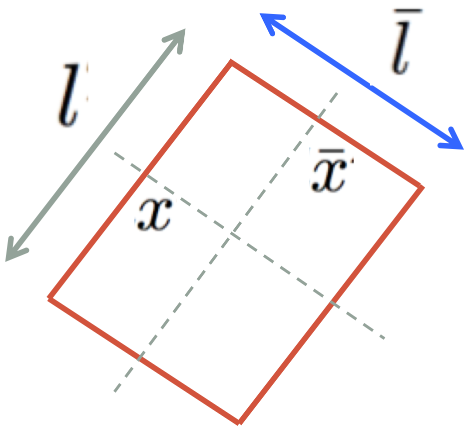
- **SO(2,2)** invariance
- Need to **reproduce** the previous metric on the **appropriate 2D slice**.

$$ds^2 = \frac{\partial^2 S(x_L, x_R)}{\partial x_L^\mu \partial x_R^\nu} dx_L^\mu dx_R^\nu$$

Generalizing: 4D Kinematic Space

Kinematic metric for AdS_3:

$$ds^2 = \frac{1}{l^2}(-dl^2 + dx^2) + \frac{1}{\bar{l}^2}(-d\bar{l}^2 + d\bar{x}^2)$$



$$l = \frac{z_R - z_L}{2} \qquad x = \frac{z_L + z_R}{2}$$

$$\bar{l} = \frac{\bar{z}_R - \bar{z}_L}{2} \qquad \bar{x} = \frac{\bar{z}_L + \bar{z}_R}{2}$$