Holography for N=2* on S^4

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N=4 SYM

N=2* theory in flat space

massless vector mult.

& massive hypermult.

gravitational dual

Pilch-Warner flow w/ flat-sliced domain walls.

Pilch-Warner (2000)

Buchel-Peet-Polchinski (2000)

Evans-Johnson-Petrini (2000)

Localization

N=2* theory on S^4

Free energy F

Pestun (2007)

Buchel-Russo-Zarembo (2013)

gravitational dual?

On-shell action $S_{\text{on-shell}}$

Localization

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large N and large 't Hooft coupling $\lambda = g^2_{\mathrm{YM}} N$

mass of hyper

radius of S^4

Euler const

$$F_{S^4} = -\frac{N^2}{2}(1 + m^2 a^2) \log \frac{\lambda (1 + m^2 a^2) e^{2\gamma + \frac{1}{2}}}{16\pi^2}$$

Ambiguities due to UV subtractions are eliminated in 3rd derivative:

$$\frac{d^3 F_{S^4}}{d(ma)^3} = -2N^2 \frac{ma(m^2a^2 + 3)}{(m^2a^2 + 1)^2}$$

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$$\frac{d^3 F_{S^4}}{d(ma)^3} = -2N^2 \frac{ma(m^2a^2+3)}{(m^2a^2+1)^2}$$

$$= N^2(-6ma + 10m^3a^3 + \dots)$$

$$\text{for } ma \ll 1$$

Buchel initiated holographic study.

Showed that m^3a^3-term in the free energy was **NOT** matched by S^4-sliced flow in the 5d Pilch-Warner supergravity model. Buchel (2013)

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In this talk:

- Explain why Pilch-Warner does not capture flow of N=2* theory on S^4.
- Construct flow in suitable truncation of 5d N=8 supergravity.
- Use holographic renormalization *to exactly match full functional form of the universal part of the free energy.*

This offers a precision-test of the gauge-gravity duality in the context of a Euclidean non-conformal setting.

Plan

- $\mathcal{N}=2^*$ on S^4
- Gravity dual
- Holographic renormalization
- Comments

Based on 1311.1508 with
Nikolay Bobev, Dan Freedman, and Silviu Pufu

Review of N=2*: setup in flat space

N=4 SYM

$$A_\mu, \qquad \underbrace{\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,}_{\text{4 gluinos}} \qquad \underbrace{X_1, X_2, X_3, X_4, X_5, X_6}_{\text{6 scalars}}$$

Global R-symmetry: $SU(4)_R \sim SO(6)_R$

In N=2 formulation:

N=2 vector multiplet: $A_\mu, \quad \begin{array}{c} \psi_1 = \lambda_3 \\ \psi_2 = \lambda_4 \end{array}, \quad Z_3 = \frac{1}{\sqrt{2}}(X_3 + iX_6)$

N=2 hypermultiplet: $\begin{array}{c} \chi_1 = \lambda_1 \\ \chi_2 = \lambda_2 \end{array}, \quad \begin{array}{c} Z_1 = \frac{1}{\sqrt{2}}(X_1 + iX_4) \\ Z_2 = \frac{1}{\sqrt{2}}(X_2 + iX_5) \end{array}$

Add mass for hyper $\Longrightarrow$ **N=2* theory**

Review of N=2*: global symmetries

$SU(4)_R \;\sim\; SO(6)_R \xrightarrow{\text{N=2}}$

	$SU(2)_V$	$SU(2)_H$	$U(1)_R$
A_μ	0	0	0
Φ	0	0	$+2$
$\psi_{1,2}$	$1/2$	0	$+1$
$\tilde{\psi}_{1,2}$	$1/2$	0	-1
$Z_{1,2}$	$1/2^\dagger$	$1/2$	0
$\chi_{1,2}$	0	$1/2$	-1
$\tilde{\chi}_{1,2}$	0	$1/2$	$+1$

Mass term for N=2* in flat space

$$\mathcal{L}_m^{\mathbb{R}^4} = m^2\,\mathrm{tr}\left(|Z_1|^2 + |Z_2|^2\right) + m\,\mathrm{tr}\left(\chi_1\chi_1 + \chi_2\chi_2 + \mathrm{h.c.}\right)$$

breaks $SU(2)_V \times SU(2)_H \times U(1)_R \Longrightarrow SU(2)_V \times U(1)_{H}$

Review of N=2*: holography

Mass term for N=2* in flat space $SU(2)_V \times U(1)_H$

$$\mathcal{L}_m^{\mathbb{R}^4} = m^2 \,\mathrm{tr}\left(|Z_1|^2 + |Z_2|^2\right) + m\,\mathrm{tr}\left(\chi_1\chi_1 + \chi_2\chi_2 + \mathrm{h.c.}\right)$$

dimension 2 operator

dual scalar ϕ

dimension 3 operator

dual scalar ψ

5d Pilch-Warner flow has flat domain walls

$$ds^2 = dr^2 + e^{2A(r)}\delta_{ij}dx^i dx^j$$

and non-trivial radial profiles for the two scalars

Type IIB lift: $SU(2)_V \times U(1)_H \times U(1)_Y$

Now put the theory on S^4

Euclidean formalism for supergravity

Festuccia and Seiberg (2011)
“Rigid SUSY in curved spacetime”

Freedman and Pufu (2013)
“Holography of F-maximization”

On S^4: N=4 SYM

N=4 SYM is conformal,

so just need conformal coupling for the scalars:

$$\mathcal{L}^{S^4}_{\mathcal{N}=4} = \mathcal{L}^{\mathbb{R}^4}_{\mathcal{N}=4}\Big|_{\eta_{\mu\nu}\to g_{\mu\nu}} + \frac{2}{a^2}\,\mathrm{tr}\Big(\,|Z_1|^2 + |Z_2|^2 + |Z_3|^2\,\Big)$$

where a is the radius of the sphere.

On S^4: N=2* SYM

N=2* theory is ***NOT*** conformal,

so in addition to conformal coupling for the scalars,

$$\frac{2}{a^2}\,\mathrm{tr}\big(\,|Z_1|^2+|Z_2|^2+|Z_3|^2\,\big)$$

the presence of the mass terms

$$m^2\,\mathrm{tr}\left(|Z_1|^2+|Z_2|^2\right)+m\,\mathrm{tr}\left(\chi_1\chi_1+\chi_2\chi_2+\mathrm{h.c.}\right)$$

requires another term

$$\frac{im}{2a}\,\mathrm{tr}\left(Z_1^2+Z_2^2+\mathrm{h.c.}\right)$$

in order for supersymmetry to be preserved.

SUSY transf w/ S^4 Killing spinors $\nabla_\mu\epsilon_\pm=\pm\frac{i}{2a}\sigma_\mu\tilde{\epsilon}_\pm$

On S^4: N=2* SYM

Consequences of $$\frac{im}{2a}\,\mathrm{tr}\left(Z_1^2+Z_2^2+\mathrm{h.c.}\right)$$

1) $SU(2)_V \times U(1)_H$ is broken to $U(1)_V \times U(1)_H$

2) The gravity dual can be expected to involve one more scalar dual to this dimension 2 operator.

$$\Downarrow$$

dual scalar χ

The third scalar turns out to be necessary for the gravitational flow dual to N=2* theory on S^4.

This is why the two-scalar Pilch-Warner model does not capture this flow on S^4.

Holographic dual of N=2* SYM on S^4

Fields $g_{\mu\nu}$, ϕ, ψ, and χ

with the three scalars dual to the three operators

$$\mathcal{O}_\phi = \mathrm{tr}\big(|Z_1|^2 + |Z_2|^2\big)\,, \quad \mathcal{O}_\psi = \mathrm{tr}\big(\chi_1\chi_1 + \chi_2\chi_2 + \mathrm{h.c.}\big)\,, \quad \mathcal{O}_\chi = \mathrm{tr}\big(Z_1^2 + Z_2^2 + \mathrm{h.c.}\big)\,.$$

symmetry $U(1)_V \times U(1)_H \times U(1)_Y$

Intriligator (1998)

Buchel-Peet-Polchinski (2000)

bonus symmetry at large-N

Truncation of N=8 gauged supergravity in 5d:

$$\mathcal{L} = \frac{1}{2\kappa^2}\left[-R + \frac{12\partial_\mu\eta\partial^\mu\eta}{\eta^2} + \frac{4\,\partial_\mu z\partial^\mu\tilde{z}}{(1-z\tilde{z})^2} + V\right],$$

$$V \equiv -\frac{4}{L^2}\left(\frac{1}{\eta^4} + 2\eta^2\frac{1+z\tilde{z}}{1-z\tilde{z}} + \frac{\eta^8}{4}\frac{(z-\tilde{z})^2}{(1-z\tilde{z})^2}\right).$$

$$\eta = e^{\phi/\sqrt{6}}$$

$$z = \frac{1}{\sqrt{2}}(\chi + i\psi)$$

$$\tilde{z} = \frac{1}{\sqrt{2}}(\chi - i\psi)$$

Holographic dual of N=2* SYM on S^4

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1) Bulk theory:

$$V = -3 - 2\phi^2 - 2\chi^2 - \frac{3}{2}\psi^2 + \ \ldots$$

Scale dimension $\Delta = 2 + \sqrt{4 + m^2}.$

Two scaling dimension 2, one 3.

2) Truncation $z = -\tilde{z}$

gives Pilch-Warner model with flat-sliced domain wall solutions.

Holographic dual of N=2* SYM on S^4

3) We want S^4-sliced domain wall solutions. Ansatz:

$$ds^2 = L^2 e^{2A(r)} ds^2_{S^4} + dr^2$$

$$\eta = \eta(r) \qquad z = z(r) \qquad \tilde{z} = \tilde{z}(r)$$

Note: Euclidean solution, z and $\tilde{z}$ are indep!

BPS equations:

$$z' = \frac{3\eta'(z\tilde{z}-1)\left[2(z+\tilde{z})+\eta^6(z-\tilde{z})\right]}{2\eta\left[\eta^6\left(\tilde{z}^2-1\right)+\tilde{z}^2+1\right]}\,,$$

$$\tilde{z}' = \frac{3\eta'(z\tilde{z}-1)\left[2(z+\tilde{z})-\eta^6(z-\tilde{z})\right]}{2\eta\left[\eta^6\left(z^2-1\right)+z^2+1\right]}\,,$$

$$(\eta')^2 = \frac{\left[\eta^6\left(z^2-1\right)+z^2+1\right]\left[\eta^6\left(\tilde{z}^2-1\right)+\tilde{z}^2+1\right]}{9L^2\eta^2(z\tilde{z}-1)^2}$$

$$e^{2A} = \frac{(z\tilde{z}-1)^2\left[\eta^6\left(z^2-1\right)+z^2+1\right]\left[\eta^6\left(\tilde{z}^2-1\right)+\tilde{z}^2+1\right]}{\eta^8\left(z^2-\tilde{z}^2\right)^2}$$

(imply EOM)

Holographic dual of N=2* SYM on S^4

Have not found analytic solution to BPS eqs, but can analyze UV and IR behavior:

UV behavior: $r \to \infty.$

Solution approaches Euclidean AdS_5 (scalars $\to$ 0)

$$ds_5^2 = dr^2 + L^2 \sinh^2\left(\frac{r}{L}\right) ds_{S^4}^2$$

Solving the BPS eqs iteratively order by order gives

$$e^{2A} = \frac{e^{2r}}{4} + \frac{1}{6}(\mu^2 - 3) + \ \ldots \qquad \eta = 1 + e^{-2r}\left[\frac{2\mu^2}{3} r + \frac{\mu(\mu+v)}{3}\right] + \ldots$$

$$\frac{1}{2}(z+\tilde{z}) = e^{-2r}\left[2\mu\, r + v\right] + \ \ldots \qquad \frac{1}{2}(z-\tilde{z}) = \mp\mu\, e^{-r} \mp \ \ldots$$

Two parameters: μ mass, v vev

Holographic dual of N=2* SYM on S^4

The holographic dual of N=2* SYM should depend on
Just one dimensionless parameter: ma

IR behavior:

Solution approaches Euclidean flat space smoothly as S^4 shrinks to zero size and scalars $\rightarrow$ constants.

Solve BPS equations iteratively as $r \rightarrow 0$

Smoothness condition gives 1-parameter family

$$\eta = \eta_0 + O(r^2)$$

$$\frac{1}{2}(z+\tilde{z}) = \sqrt{\frac{\eta_0^6-1}{\eta_0^6+1}}\,\frac{\eta_0^6}{\eta_0^6+2} + O(r^2)$$

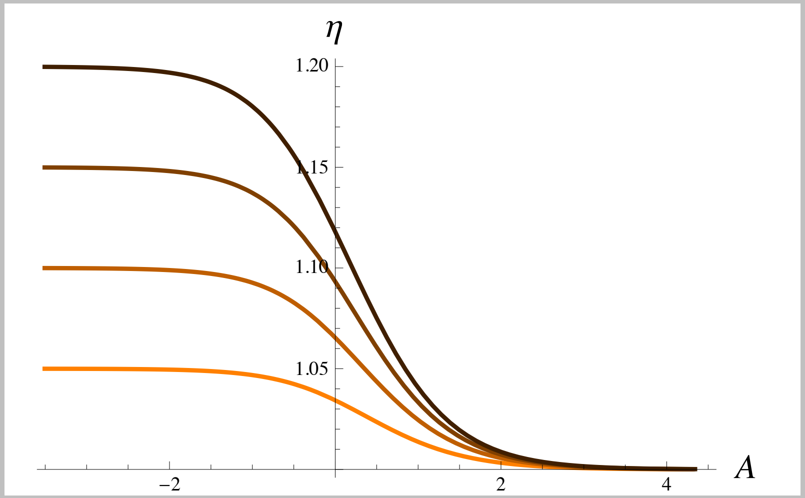
$$e^{2A} = O(r^2)$$

etc

Holographic dual of N=2* SYM on S^4

Now match

UV and IR behavior



Numerical solution interpolates between UV and IR region: fixes the two UV parameters in terms of the single IR parameter:

$$\mu(\eta_0) \quad \text{and} \quad v(\eta_0)$$

Extract from the numerics

$$\boxed{v(\mu) = -2\mu - \mu \log(1 - \mu^2)}$$

Official Old San Marcos Pass. 3 miles, 1165 ft.

JoeP Personal Record 23:02min. Feb 26, 2014.

Recap:

We have constructed a candidate dual for the RG flow of N=2* on S^4

Next is calculation of the *on-shell action.*

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We have constructed a candidate dual for the RG flow of N=2* on S^4

Next is calculation of the *on-shell action.*

Divergent, but standard systematic technique for handling it with ***infinite* counterterms**:

Holographic renormalization

Leaves ambiguity of ***finite* counterterms**

Holographic renormalization

Finite counterterms tricky.

For *flat-sliced* domain walls, one can use the **Bogomolnyi trick** to determine the counterterms.

Idea:

Supergravity theory with several scalars, Kahler potential, and scalar potential given in terms of superpotential as

$$V = \frac{1}{2}K^{ij}\partial_i W\,\partial_j W - \frac{4}{3}W^2$$

gives BPS eqs

the action can be rearranged to sum of squares:

$$S = \int d^4x \int^{r_0} \left(e^{4A}\left[-3\left(A' - \frac{2}{3}W\right)^2 + \frac{1}{2}K_{ij}\left(\phi^{i\prime} - K^{il}\partial_l W\right)\left(\phi^{j\prime} - K^{jm}\partial_m W\right)\right] - \frac{d}{dr}\left(e^{4A}W\right)\right)$$

Bdr counterterms thus fixed by SUSY:

$$S_W = \int d^4x\, e^{4A(r_0)}\, W\big(\phi_i(r_0)\big)$$

Holographic renormalization

- Our flows have S^4-slicing.
  The flat-sliced limit is only consistent for $\tilde{z} = \pm z$

- There is no superpotential W for our scalar potential.

No Bogo. for us!!?

Holographic renormalization

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- There is no superpotential W for our scalar potential.

No Bogo. for us!!?

Idea:
Our scalar potential has an approximate superpotential, valid near the boundary:

$$W = \frac{3}{2} + \phi^2 + \frac{1}{2}\psi^2 + \frac{1}{2}\chi^2 + \sqrt{\frac{2}{3}}\phi\,\psi^2 + \frac{1}{4}\psi^4$$

Use it to determine finite counterterms for flat-sliced case.

Call upon *counterterm universality!*

$$S_{\rm susy} = \int d^4x\,\sqrt{\gamma}\,W,$$

Satisfies multiple checks & truncations

On-shell action & the free energy

$$S_{\rm ren} \;=\; S_{\rm 5D} + S_{\rm GH} + S_{\rm susy}\,, \qquad \text{with} \qquad S_{\rm susy} = \int d^4x\,\sqrt{\gamma}\,W.$$

Show that the derivative of the free energy with respect to the source parameter is given in terms of 1-point functions:

$$\frac{dF}{d\mu} = \frac{N^2}{2\pi^2}\int d^4x\,\sqrt{g_0}\left(\langle\mathcal{O}_\psi\rangle\frac{\partial\psi_0}{\partial\mu} + \langle\mathcal{O}_\phi\rangle\frac{\partial\phi_0}{\partial\mu} + \langle\mathcal{O}_\chi\rangle\frac{\partial\chi_0}{\partial\mu}\right)$$

where

$$\langle\mathcal{O}_\psi\rangle = \lim_{\epsilon\to 0}\frac{1}{\epsilon^{3/2}}\frac{1}{\sqrt{\gamma}}\frac{\delta S_{\rm ren}}{\delta\psi} \qquad \text{and} \qquad \langle\mathcal{O}_\phi\rangle = \lim_{\epsilon\to 0}\frac{\log\epsilon}{\epsilon}\frac{1}{\sqrt{\gamma}}\frac{\delta S_{\rm ren}}{\delta\phi}$$

and we have used $1/4\pi G_5 = N^2/2\pi^2$.

$$\frac{dF}{d\mu} \;=\; \frac{N^2}{2\pi^2}\,\mathrm{vol}_0(S^4)\Big(4\mu - 12v(\mu)\Big) \;=\; N^2\Big(\frac{1}{3}\mu - v(\mu)\Big)$$

On-shell action & the free energy

Take two more derivatives:

$$\frac{d^3F}{d\mu^3} = -N^2 v''(\mu) = -2N^2 \frac{\mu(3-\mu^2)}{(1-\mu^2)^2}$$

using

$$v(\mu) = -2\mu - \mu\log(1-\mu^2)$$

from the UV/IR match in the BPS flow solution.

Compare with the field theory result:

$$\frac{d^3F_{S^4}}{d(ma)^3} = -2N^2\frac{ma(m^2a^2+3)}{(m^2a^2+1)^2}$$

Perfect match after identification $\mu = \pm ima.$

Comments

1) Finite counterterms were key for cancelation of μ^3 terms

Could we have found a match without using supersymmetry and universality argument to fix finite counterterms?

Yes.

One can list the possible ambiguous finite counterterms, such as

$$R[\gamma]\,\psi^2\,, \qquad (\log\epsilon)^{-1}\psi^2\phi\,, \qquad \psi^4 \qquad \text{etc}$$

and calculate their potential contribution to $\dfrac{dF}{d\mu}$

Turns out that they contribute only μ or μ^3, but never $v(\mu)$

So all ambiguity of finite counterterms eliminated in $\dfrac{d^5F}{d\mu^5}$

$\Longrightarrow$ Full functional match with field theory result.

2) Why $\dfrac{d^3F}{d\mu^3}$?

Superconformal theory on S^4 has free energy of the form

small distance cutoff ϵ.

$$F = \alpha_2 \frac{a^2}{\epsilon^2} + \alpha_0 - a_{\rm anom} \log \frac{a}{\epsilon} + \mathcal{O}(\epsilon/a)$$

For N=2* theory on S^4, *F* can also depend on dim'less $m^2\epsilon^2$

For small $m^2\epsilon^2$, the coefficients of the non-universal terms can be expressed as

$$\alpha_2 = \tilde{\alpha}_2 + m^2\epsilon^2\beta_2 + O(m^4\epsilon^4) \quad \text{and} \quad \alpha_0 = \tilde{\alpha}_0 + O(m^2\epsilon^2)$$

So the non-universal contributions $\tilde{\alpha}_2 \dfrac{a^2}{\epsilon^2} + \tilde{\alpha}_0 + \beta_2 m^2 a^2$

are eliminated in $\dfrac{d^3F}{d(ma)^3}$

3) $$\frac{d^3 F_{S^4}}{d(ma)^3} = -2N^2 \frac{ma(m^2a^2+3)}{(m^2a^2+1)^2}$$

What is special about $m^2a^2 = -1$ **?**

Recall the mass terms in N=2* on S^4:

$$\frac{2}{a^2}\,\mathrm{tr}\left(|Z_1|^2 + |Z_2|^2 + |Z_3|^2 \right) \; + \; m^2\,\mathrm{tr}\left(|Z_1|^2 + |Z_2|^2\right) \; + \; \frac{im}{2a}\,\mathrm{tr}\left(Z_1^2 + Z_2^2 + \mathrm{h.c.}\right)$$

Write $Z = \frac{1}{\sqrt{2}}(A+iB)$ to find

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\left[\left(\frac{2}{a^2} + i\frac{m}{a} + m^2\right)A^2 + \left(\frac{2}{a^2} - i\frac{m}{a} + m^2\right)B^2\right]$$
$$= \frac{1}{2a^2}\left[(1+ima)(2-ima)A^2 + (1-ima)(2+ima)B^2\right]$$

$$-1 < ima < 2 \qquad\qquad -2 < ima < 1$$

So $ma = \pm i$ is the tachyon threshold!

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Happy Birthday, Joe!

thank you

for all the good you have done for us