Mirror Symmetry and Langlands Duality II

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Moral: SYZ duality is expected to imply a homological mirror symmetry relationship between the two manifolds

Today: hyperKähler case ($\mathcal{N}=(4,4)$ worldsheet)

Last time: (p,q) $\cancel{H^{(p,q)}}$ $\quad H^q_{\bar\partial}(X, \Lambda^p TX^*) \qquad p \geq 0,\ q \geq 0$

$$H^q_{\bar\partial}(X, \Lambda^p TX) \qquad (-p, q)$$

Gepner construction

anticipated by G. Anderson, early 1980's

$X \subseteq \mathbb{CP}^N$ Fermat of degree d

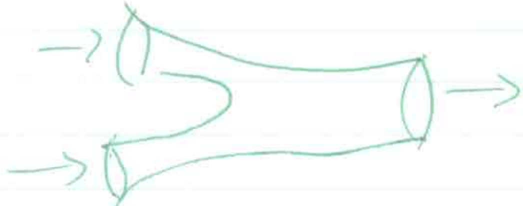
$$H^{N-1}(X) = \Big(\otimes \big(\underbrace{H^{a,b} \oplus H^{b,a}}_{a,b \in \mathbb{Q}}\big)\Big)^G$$

D-branes and open string boundary conditions

Type II string theories have Ramond-Ramond charges

$\Sigma \longrightarrow X^{10}$

closed string worldsheet



D-branes Open strings can have boundary on D-branes

On boundary, need a gauge field

Geometry of D-branes is constrained by insisting on SUSY

A-branes B-branes

½ of worldsheet SUSY preserved

Fuk(X) · CY ⊃ ½ dim'l subspace, Lagrangian + gauge field (flat)
$D^b(X)$ · CY ⊃ holo submanifold + holo bundle

~~HyperKähler~~

CY manifolds depend on continuous parameters

$$\int_\Sigma \|d\varphi\|^2 \mathrm{dA} + \int_\Sigma \varphi^*(B)$$

metric on X

$\varphi : \Sigma \to X$

Kähler metric, chosen cx structure

Ricci-flat (conformal)

Yau's Thm holonomy $SU(n)$
Kähler class $\rightsquigarrow$! Ricci flat metric
+ Complex structure
+ $B \in H^2(X, \mathbb{R}/\mathbb{Z})$

holomorphic automorphisms

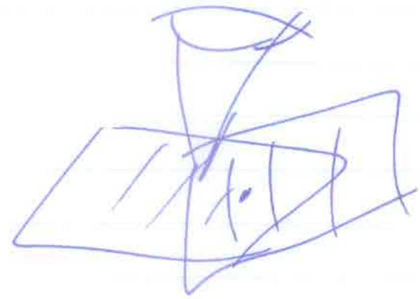
Kähler Cone

$$\left(\mathcal{K} + iH^2(X, \mathbb{R}/\mathbb{Z})\right)^{Aut(X)}$$

$$\subseteq H^2(X, \mathbb{C}/\mathbb{Z})$$

$\mathcal{M}_{Cx}$

Mirror Symmetry gives local isomorphism



$U \longrightarrow \hat{U}$

(Cone + vector space) mod translations

$$(\mathcal{K} + iV)/V_{\mathbb{Z}}$$

holonomy $Sp(n)$ acts on $\mathbb{R}^{4n}$

choose a cx structure compatible w/ Riemannian metric
fix Kähler class $\longrightarrow$! Ricci flat metric

ω = Kähler form

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Re α, Im α = real + imaginary parts of holo 2-form α
α non-degen $\dim H^{2,0}(X) = 1$

X compact

$$\int_X \omega^{2n} = 1 \,, \quad \int_X \mathrm{Re}\, \alpha^{2n} = 1 \,, \quad \int_X \mathrm{Im}\, \alpha^{2n} = 1$$

S^2 of cx structures compatible w/ metric

ω, Re α, Im α $\in \mathbb{R}^3 \supseteq S^2$

$\mathbb{R}^3 \subseteq H^2(X, \mathbb{R})$ ~~preserved~~ by
determined by metric

$$\underbrace{(3, b_2 - 3)}$$

Hodge index thm

pos def subspace

metric + 2-form

$\downarrow$

pos 3-plane in $\mathbb{R}^{3, b^2-3}$

$$Gr^+(3, b_2) = O(3, b_2-3) / O(3) \times O(b_2-3)$$

<u>period map</u>

$$\mathcal{M}_{\text{metrics}} \longrightarrow \Gamma \backslash Gr^+(3, b^2)$$

$$\text{metric} + 2\text{form} \longrightarrow \tilde{\Gamma} \backslash Gr^+(4, b^2+1)$$

<u>String Theory moduli space</u>

$$\Gamma \backslash O(4, k-4) / O(4) \times O(k)$$

X $\quad$ X'

K3 moduli space

Γ arithmetic parabolic

mirror identification

provides Γ_{par} to $\Gamma = \Gamma_{SO(4,20;\mathbb{Z})}$

SYZ Duality

$$\begin{tikzcd} X^{4n} \arrow[d, "\pi"'] \\ B \end{tikzcd} \qquad \pi^{-1}(b) = T^{2n} = \text{Lagrangian } \omega$$

$$\qquad\qquad\qquad\qquad\qquad\qquad I$$

Mirror fibration

$$\begin{tikzcd} \tilde{X}' \arrow[d, "\pi'"'] \\ B' \end{tikzcd}$$

J holo submanifold

$4n = 4 \quad (X = K3)$

T^2 holo

elliptically fibered K3

almost every K3 is elliptically fibered

$$\begin{tikzcd} X \arrow[d] & X' \arrow[d] \\ B \arrow[r, equal] & B' \end{tikzcd}$$