FUTURE USE OF RANDOM MATRIX THEORY IN LATTICE GAUGE THEORY

- THE DEEP-ROOTED SKEPTICISM
- AVOIDING THE USE OF RMT
- MANY EIGENVALUES $\longrightarrow$ RMT?
- RMT AND PHYSICAL OBSERVABLES
- STAGGERED FERMIONS AND RMT
- LARGE N_c ?

THE SKEPTICISM:

$$Z_\nu = \int dM \prod_f^{N_f} \det(M+m_f)\, e^{-N\,\mathrm{Tr}\,\underbrace{V(M^2)}_{\text{ARBITRARY (ALMOST)}}}$$

$$M \equiv \begin{pmatrix} 0 & W^\dagger \\ W & 0 \end{pmatrix}$$

$W = N \times (N+\nu)$, COMPLEX

SHURYAK, VERBAARSCHOT '94

– AN RMT OF THE $\boxed{\text{chUE}}$

IS THIS A "TOY MODEL" OF QCD?

$$Z_\nu = \int [dA_\mu]_\nu \prod_f^{N_f} \det(i\not{D}+m_f)\, e^{-S[A_\mu]}$$

WHY SHOULD WE TAKE IT SERIOUS?

ARGUMENTS PUT FORWARD:

- THE BOHIGAS CONJECTURE
- SYMMETRIES
- UNIVERSALITY
- INSTANTONS (!)

⋮ + PROBABLY MANY MORE

IF A STATEMENT NEEDS 5 DIFFERENT ARGUMENTS THEN PROBABLY NONE OF THEM ARE WORTH MUCH...

THE "REAL PROOF":

$\boxed{\text{RMT chUE}} \Longleftrightarrow \boxed{\text{CHIRAL LAGR., QCD}}$

IN THE "ϵ- REGIME" OF QCD

GASSER, LEUTWYLER
LEUTWYLER, SMILGA
SHURYAK, VERBAARSCHOT

+ 10 YEARS OF HARD WORK!

AVOIDING THE USE OF RMT:

STAY ENTIRELY WITHIN FRAMEWORK OF CHIRAL PERTURBATION THEORY!

ϵ- REGIME: THE "EXTREME" CHIRAL LIMIT

$$\frac{1}{m_\pi} > L$$

- A SYSTEMATIC CHIRAL EXPANSION
- THE LEADING TERM IS OF ZERO-MOM. MODES ~ RMT!

$\boxed{\text{WHAT RMT CAN DO: SPECTRAL CORRELATION FUNCTIONS OF DIRAC OP., INDIVIDUAL EIGENV. DISTRIBUTIONS}}$

TO DO THE SAME IN χPT WE NEED PARTIAL QUENCHING:

* "SUPERSYMMETRY"
* REPLICAS

BOTH WORK TRIVIALLY IN PERTURBATION THEORY

* SUPERSYMMETRY "  = 0 "

BERNARD, GOLTERMANN

* REPLICAS " $\lim_{N\to 0}$  = 0 "

P.D., SPLITTORFF

NON-PERTURBATIVELY (WE NEED THAT!)

A HIGHLY NON-TRIVIAL PROBLEM

* SUPERSYMMETRY: "STRAIGHTFORWARD" BUT TERRIBLY CUMBERSOME

P.D, OSBORN, TOUBLAN, VERBAARSCHOT

* REPLICAS: GREAT PROGRESS RECENTLY (PAINLEVÉ EQS, TODA EQS.)

KANZIEPER
SPLITTORFF, VERBAARSCHOT

(- RECENT APPLICATIONS TO FINITE-μ THEORIES

AKEMANN, OSBORN, SPLITTORFF, VERBAARSCHOT

(TALKS AT THE WORKSHOP))

UPSHOT:

SPECTRAL DENSITY OF THE DIRAC OPERATOR NEAR $\lambda \sim 0$

$$\rho(\zeta) = \frac{1}{2}\zeta\left(\left(J_{N_f+\nu}(\zeta)\right)^2 - J_{N_f+\nu+1}(\zeta)J_{N_f+\nu-1}(\zeta)\right)$$

$$\boxed{\zeta \equiv \lambda \Sigma V}$$

INF. VOLUME $\langle\bar{\psi}\psi\rangle$

AND

SPECTRAL 2-PT. FUNCTION $\rho(\zeta_1,\zeta_2)$

- DERIVED FROM χPT IN ϵ-REGIME

SPECTRAL n-PT FUNCTIONS $\rho(\zeta_1,\dots,\zeta_n)$ DERIVABLE FROM χPT (BUT TEDIOUS!)

COMPLETE AGREEMENT WITH RMT!

INDIVIDUAL EIGENVALUE DISTRIBUTIONS:

CAN **ALSO** BE DERIVED DIRECTLY FROM QFT, WITHOUT USE OF RMT.

DEFINE

$$E(s;y) \equiv 1 + \sum_{\ell=1}^{\infty} (-y)^{\ell} \frac{1}{\ell!} \int_0^s d\lambda_1 \cdots d\lambda_\ell \, \rho(\lambda_1, \ldots, \lambda_\ell)$$

"GAP PROBABILITY"

$$E_k(s) = (-1)^k \frac{\partial^k}{\partial y^k} E(s;y)\Big|_{y=1}$$

$$\Rightarrow \quad \boxed{\frac{1}{k!} \frac{\partial}{\partial s} E_k(s) = p_k(s) - p_{k+1}(s)}$$

EXAMPLES:

$$p_1(s) = -\frac{\partial}{\partial s} E_0(s) = \rho_1(s) - \int_0^s d\lambda \, \rho_2(\lambda, s) + \ldots$$

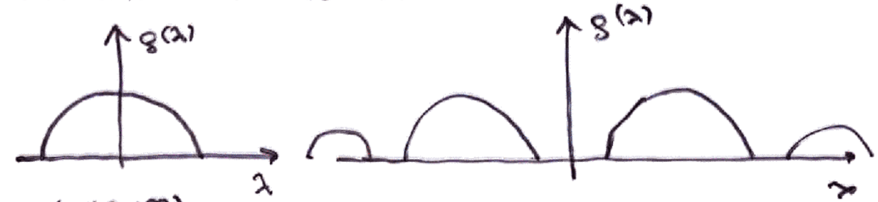
$$p_2(s) = \int_0^s d\lambda \, \rho_2(\lambda, s) + \ldots$$

ETC.

G. AKEMANN, P.D., 2004

ARE MANY EIGENVALUES ENOUGH?

TYPICAL RMT SPECTRA:

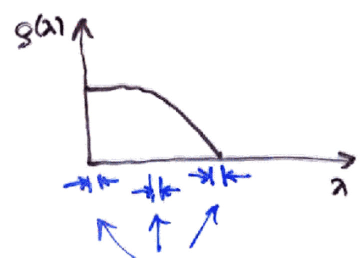


(WIGNER)

GENERIC FEATURES:
- * BOUNDED DOMAINS OF SUPPORT
- * SPECTRA DEPEND ON CHOICE OF RMT "ACTION"

- SUCH "MACROSCOPIC" FEATURES HAVE ALMOST NEVER PHYSICAL SIGNIFICANCE (THERE ARE EXCEPTIONS)

- IT IS "MICROSCOPIC" SPECTRA THAT REALLY HAVE A CHANCE:



BLOW UP EITHER OF THESE 3 REGIONS!

$\Rightarrow$ DO NOT GET DISCOURAGED IF THE MACROSCOPIC SPECTRUM LOOKS TOTALLY WRONG!

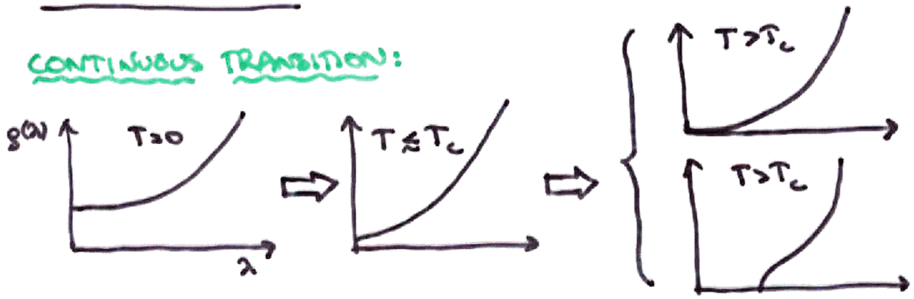
TEMPTING AREA FOR RMT :

$\boxed{\text{THE CHIRAL TRANSITION}}$

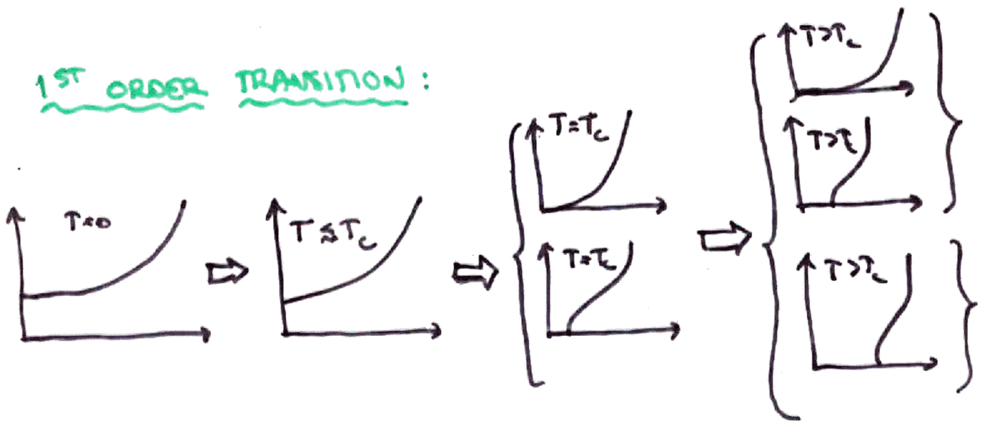
FARCHIONI, DE FORCRAND, HIP, LANG, SPLITTORFF 2000
P.D., HELLER, NICLASEN, RUMMUKAINEN 2000

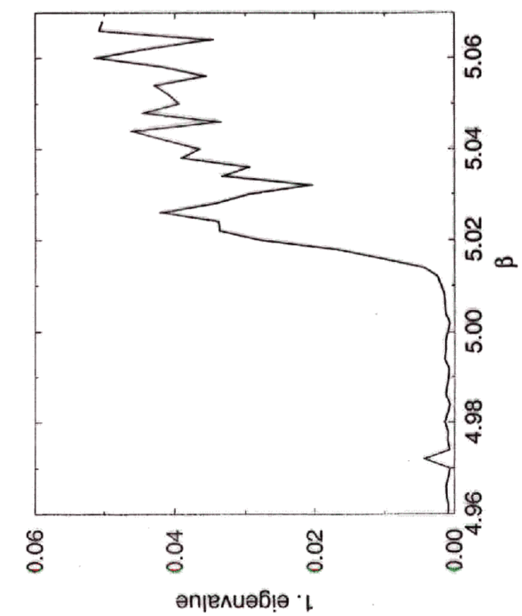
SCHEMATICALLY :

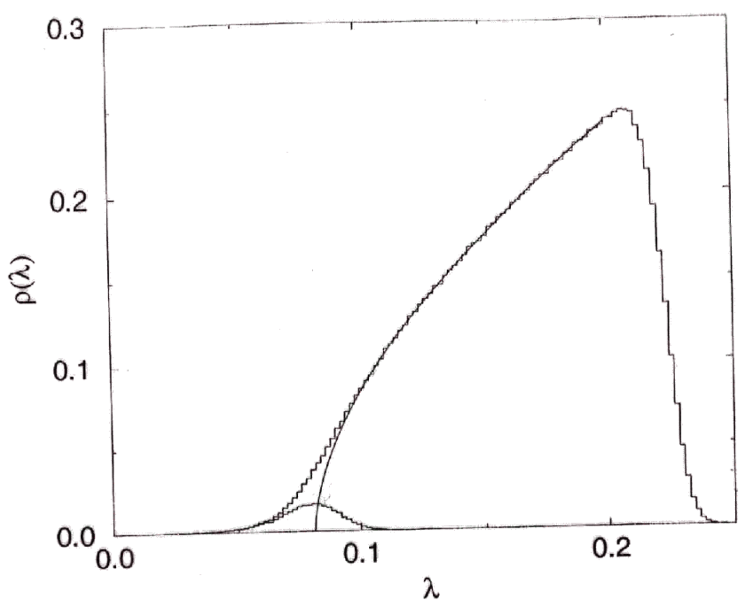
CONTINUOUS TRANSITION:

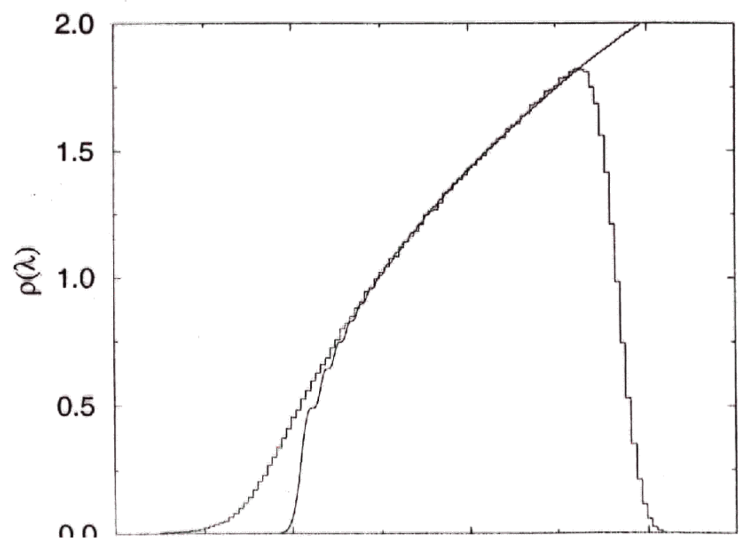


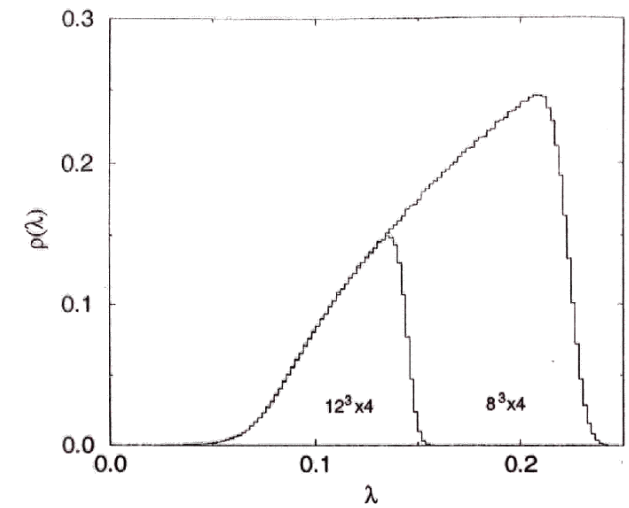
1^{ST} ORDER TRANSITION :

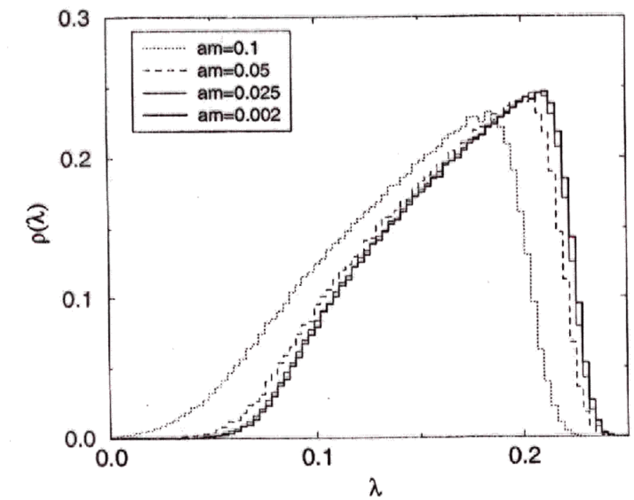





ρ(λ)
λ
0.0
0.1
0.2
0.3
0.0
0.1
0.2


ρ(λ)
2.0
1.5
1.0
0.5
0.0


ρ(λ)
λ
0.0
0.1
0.2
0.3
0.0
0.1
0.2
12³x4
8³x4


am=0.1
am=0.05
am=0.025
am=0.002
ρ(λ)
λ
0.0
0.1
0.2
0.3
0.0
0.1
0.2

NUMERICAL OBSERVATIONS

- ONE SINGLE DIRAC EIGENVALUE IS AN EXTREMELY GOOD INDICATOR OF THE PHASE TRANSITION
- THE TAIL APPROACHES A LIMITING DISTRIBUTION AS $V_3 \to \infty$
- THE TAIL APPROACHES A LIMITING DISTRIBUTION AS $m_q \to 0$
- THE TAIL IS VERY WELL FIT BY

$$\rho(\lambda) \propto (\lambda - \lambda_0)^{1/2}$$

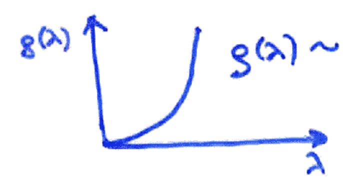
- THE TAIL DOES NOT APPEAR TO BE DESCRIBED BY THE RMT FORMULA

$$\rho(\lambda) \sim (\lambda-\lambda_0)V^{2/3}\left(Ai((\lambda-\lambda_0)V^{2/3})\right)^2 + \left(Ai'((\lambda-\lambda_0)V^{2/3})\right)^2$$

POSSIBLE EXPLANATIONS

- WHY SHOULD RMT BE RELEVANT HERE ANYWAY?
- THERE MAY BE NO GAP AT ALL, i.e. NO "SOFT EDGE"
- PERHAPS RMT WILL ONLY DESCRIBE DATA WHEN THE CORRELATION LENGHT DIVERGES AT $T=T_c$ (i.e. FOR CONTINUOUS PHASE TRANSITIONS)

THERE ARE "MULTICRITICAL" RMT'S FOR



$\rho(\lambda) \sim \lambda^{2m}$

ONE UNIVERSALITY CLASS FOR EACH m!

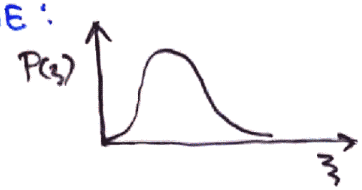
AKEMANN, P.D., MAGNEA, NISHIGAKI '99

RMT AND PHYSICAL OBSERVABLES:

- RMT IS LEADING-ORDER TERM OF ϵ-EXPANSION FOR THE CHIRAL LAGR.:

$$\mathcal{L} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial_\mu U^\dagger) - \frac{\Sigma}{2}\mathrm{Re}\,\mathrm{Tr}(\mathcal{M}e^{i\theta/N_f}U) + \ldots$$

- ALL PARAMETERS CAN BE EXTRACTED FROM DISTRIBUTION OF, SAY, 1ST EIGENVALUE:



(BUT BEYOND Σ AND F THIS IS NOT PRACTICAL)

- THE ϵ-EXPANSION OF χPT IS AN EXPANSION ABOUT RMT

$\Rightarrow$ USE THE ϵ-EXPANSION TO EXTRACT ALL LOW-ENERGY CONSTANTS OF QCD!

REMINDER:

ϵ-REGIME OF QCD:

* $m_\pi L < 1$
* TOPOLOGY: DIFFERENT PREDICTIONS FOR EACH ν
* ANALYTICAL PREDICTIONS FOR CORRELATION FUNCTIONS
* ALTHOUGH AT FINITE V ALL PARAMETERS ENTERING ARE INFINITE-VOLUME PARAMETERS

USE RMT TO CHECK WHEN STAGGERED FERMIONS ARE "GOOD"

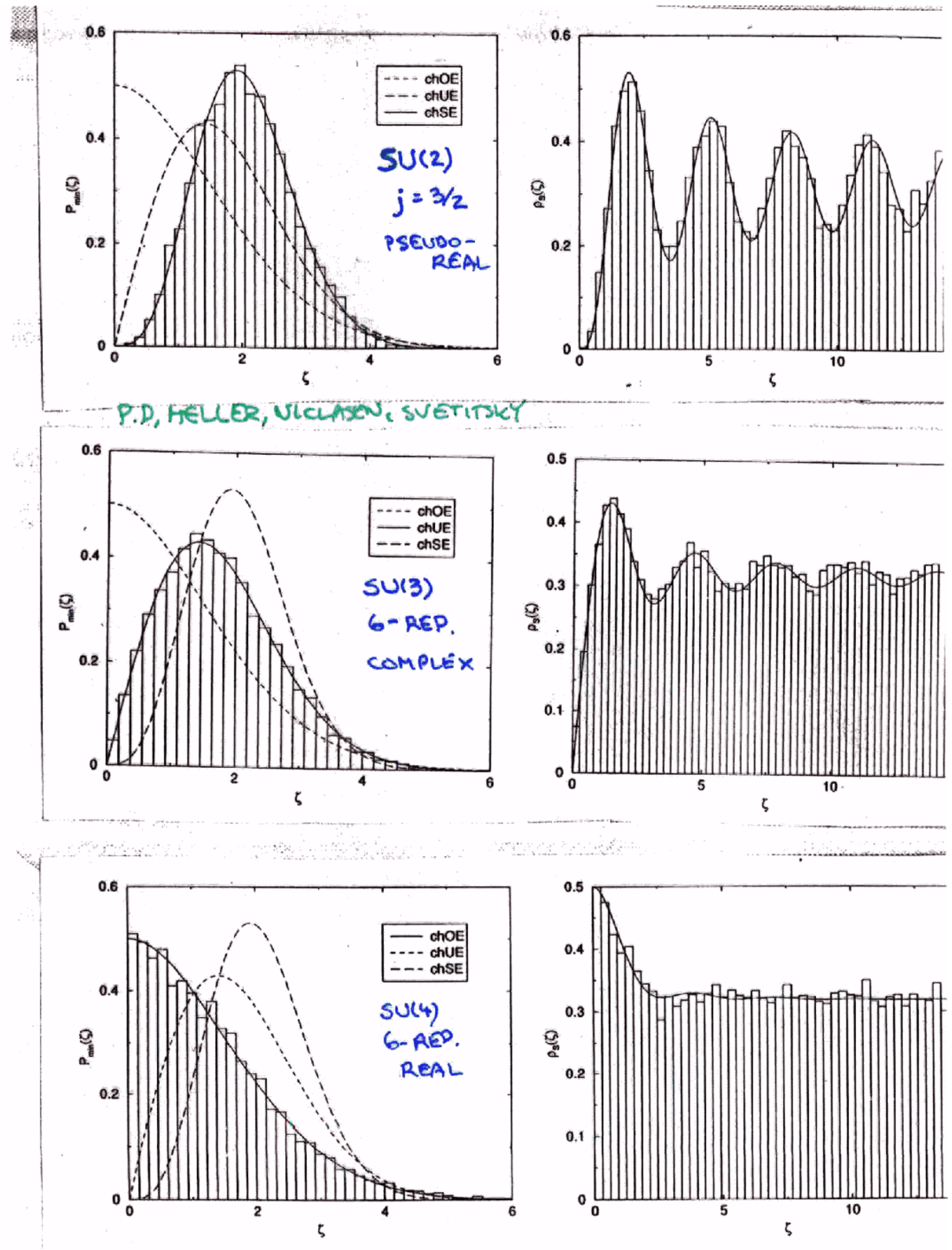
- WRONG CHIRAL SYMMETRY BREAKING
- WHEN DOES 1 BECOME 4 ?

STAGGERED:

REP. r	COSET	RMT
PSEUDO-REAL	U(2N)/SO(2N)	chSE
COMPLEX	U(N)	chUE
REAL	U(2N)/Sp(2N)	chOE

CF. CONTINUUM FERMIONS:

REP. r	COSET	RMT
PSEUDO-REAL	$SU(2N_f)/Sp(2N_f)$	chOE
COMPLEX	$SU(N_f)$	chUE
REAL	$SU(2N_f)/SO(2N_f)$	chSE



LATTICE GAUGE THEORY WOULD HAVE EVOLVED **VERY DIFFERENTLY** IF QUARKS HAD CARRIED REAL OR PSEUDO-REAL REPRESENTATIONS !

$\rightsquigarrow$ COMPLETELY WRONG SET OF GOLDSTONE BOSONS...

EXAMPLE :

PSEUDO-REAL STAGGERED $U(2N)/O(2N)$

$\Rightarrow$ $\boxed{N(2N+1) \text{ GOLDSTONES}}$

SHOULD SWAP WITH $U(2N)/Sp(2N)$

OF $\boxed{N(2N-1) \text{ GOLDSTONES}}$

EXACTLY MASSLESS ~~SHOULD~~ GOLDSTONES AT ANY FINITE LATTICE SPACING SHOULD BECOME MASSIVE RIGHT AT THE CONTINUUM LIMIT ?

(NO!)

THE THEORY SAVES ITSELF BY THE

$\boxed{\text{“}1 \to 4 \text{ TRICK”}}$

* GOLDSTONES REMAIN GOLDSTONES
* NEW MASSLESS STATES COME DOWN

BUT THERE IS ALSO THE U(1)-FACTOR:

$U(2N)/SO(2N) \to SU(8N_f)/Sp(8N_f)$

$U(2N)/Sp(2N) \to SU(8N_f)/SO(8N_f)$

THIS EXTRA U(1)-FACTOR IS WHY STAGGERED FERMIONS FAR FROM THE CONTINUUM "DO NOT SEE TOPOLOGY"

PARTITION FUNCTION AT FIXED TOPOLOGY: (chUE)

$$Z_\theta = \sum_{n=-\infty}^{\infty} e^{in\theta} Z_n$$

$$\Rightarrow \quad Z_\nu = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, e^{-i\nu\theta} Z$$

i.e.

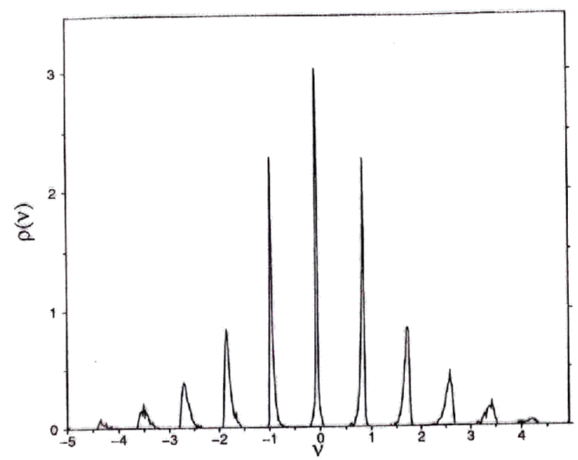
$$Z = \int_{SU(N_f)} dU\, e^{mV\Sigma\, \mathrm{Re}(\mathrm{Tr}(e^{i\theta/N_f} MU))}$$

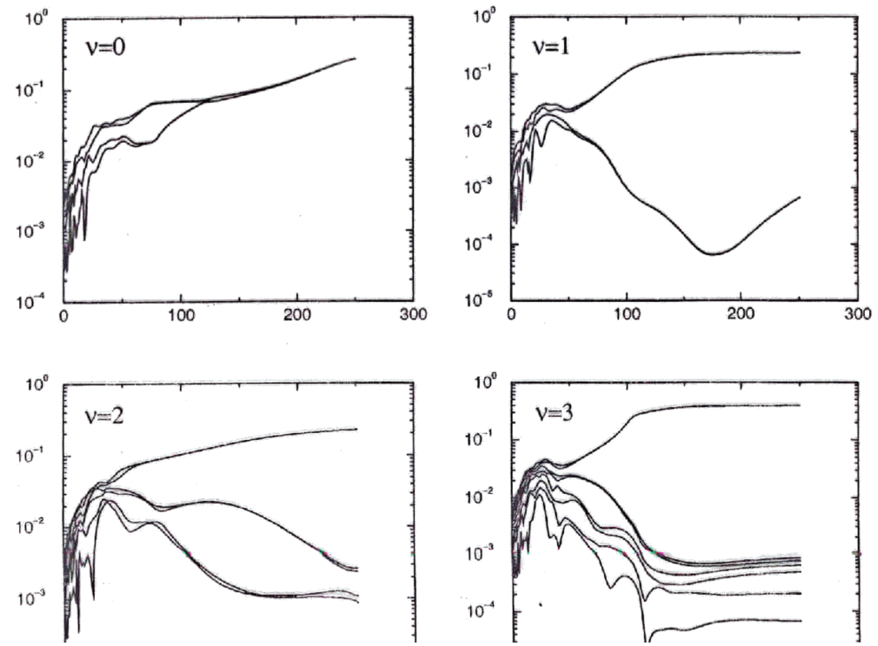
$$\Rightarrow \quad Z_\nu = \int_{U(N_f)} dU\, (\det U)^\nu e^{mV\Sigma\, \mathrm{Re}\mathrm{Tr}(MU)}$$

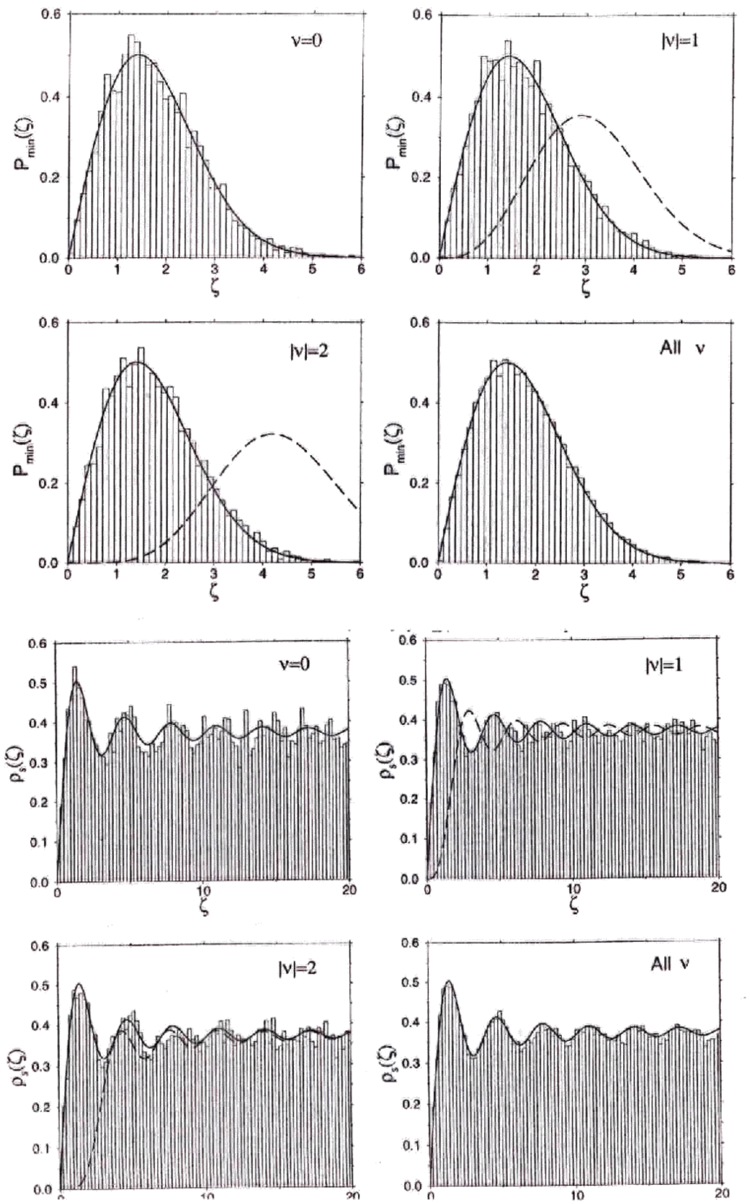
$$\Rightarrow \quad Z_0 = \int_{U(N_f)} dU\, e^{mV\Sigma\, \mathrm{Re}\mathrm{Tr}(MU)}$$

= THE EFFECTIVE PARTITION FUNCTION FOR STAGGERED FERMIONS ON <u>ANY</u> TOP. CHARGE!

HELLER, P.D., NICLASEN, RUMMUKAINEN '99







THE PROBLEM HAS RECENTLY BEEN REVISITED

DÜRR, HOELBLING, WENGER 2004
FOLLANA, HART, DAVIES 2004
WONG, WOLOSHYN 2004

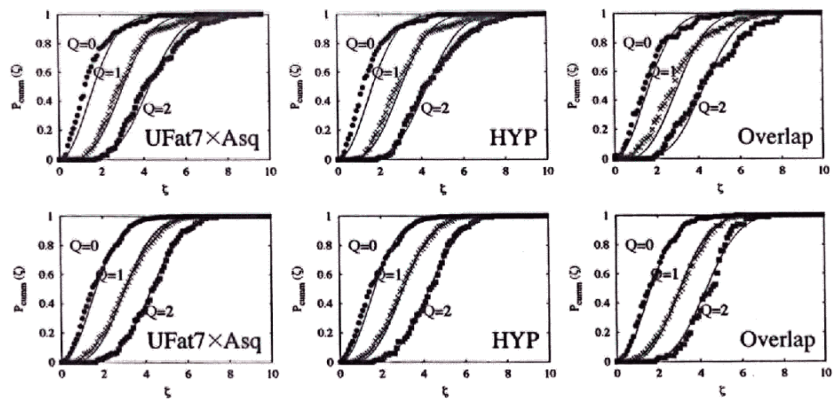
THE ISSUE IS:

HOW MUCH CAN

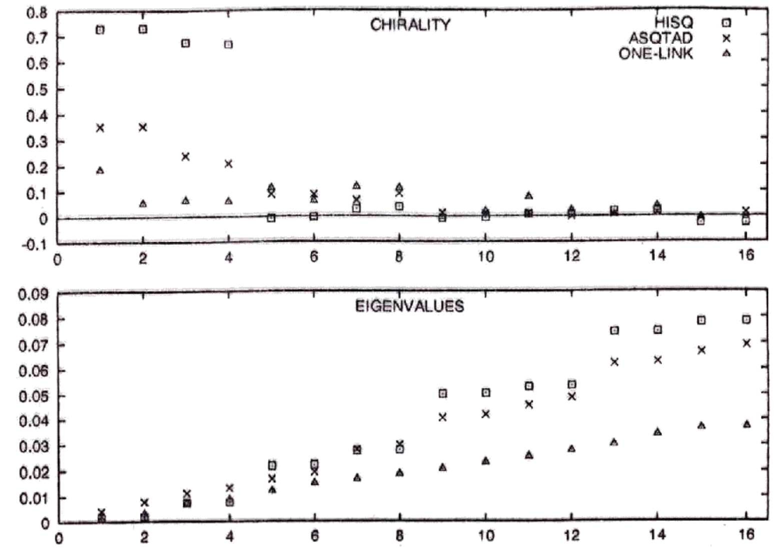
HISQ-HYP-ASQTAD-TADPOLE IMPROVED-n-STEP-SMEARING-SYMANZIK IMPROVED ACTIONS + DIRAC OPERATORS

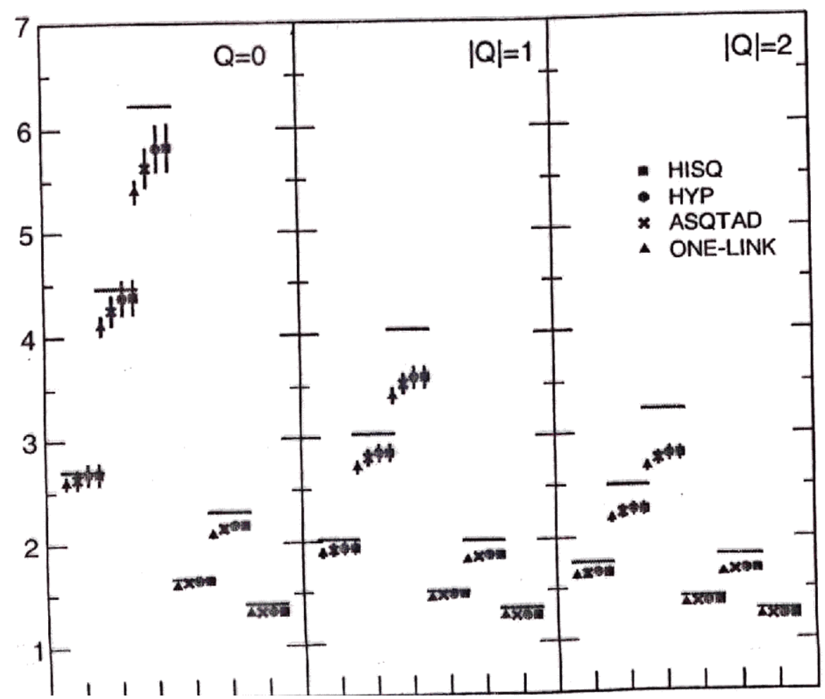
IMPROVE THE SITUATION?

9



WONG, WOLOSHYN 2004

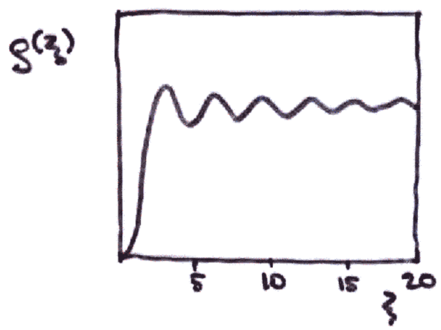




BUT WE HAVE JUST SEEN THAT "NAIVE" STAGGERED FERMIONS FAIL COMPLETELY TO SEE TOPOLOGY!

HOW CAN THIS BE RECONCILED?

CONSIDER, E.G. $|Q| = 1$:

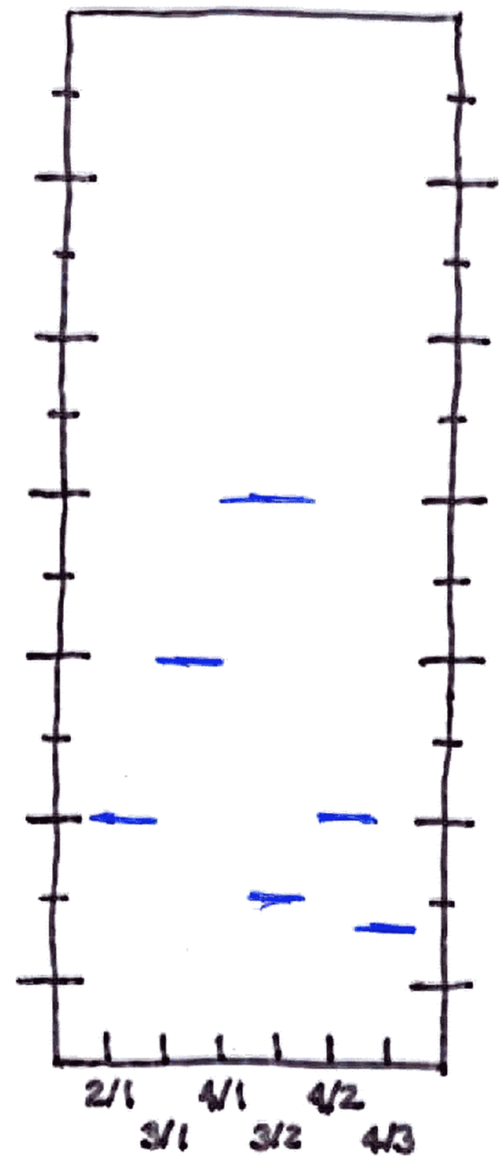


THE EIGENVALUES ARE SPACED ALMOST

EQUIDISTANTLY

(SIMPLE LEVEL REPULSION, MATHEMATICALLY A SIMPLE PROPERTY OF BESSEL FUNCTIONS)

LET'S FILL OUT A $|Q|=1$ PLOT:

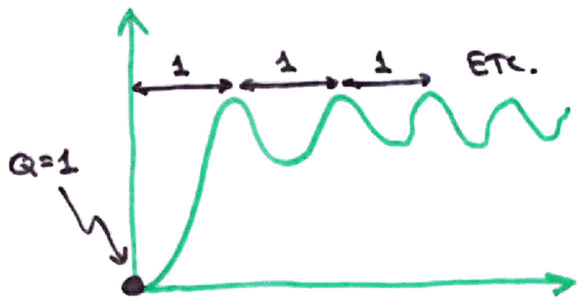


WHAT ABOUT $Q=0$ AND $|Q|=2$ RATIOS ?

A SMALL CHEAT : THIS SIMPLE ARGUMENT WORKED SO WELL BECAUSE $|Q|=1$

IT'S JUST LEVEL REPULSION:

$|Q|=1 \Rightarrow$ ONE "CHARGE" AT THE ORIGIN



SO JUST FOR $|Q|=1$:

$$\text{"}m/n\text{"} \simeq \frac{m}{n}$$

- FOR ALL OTHER Q'S

$$\text{"}m/n\text{"} \longrightarrow \frac{m}{n} \quad ; \; m,n \text{ LARGE}$$

WHAT ABOUT $Q=0$?

REPULSION AT THE ORIGIN COMES FROM THE CHIRAL PARTNER "ON THE OTHER SIDE"

$\Rightarrow$

LARGE DEVIATIONS FOR SMALL m,n :

"2/1" = 3

"3/1" = 5

"4/1" = 7

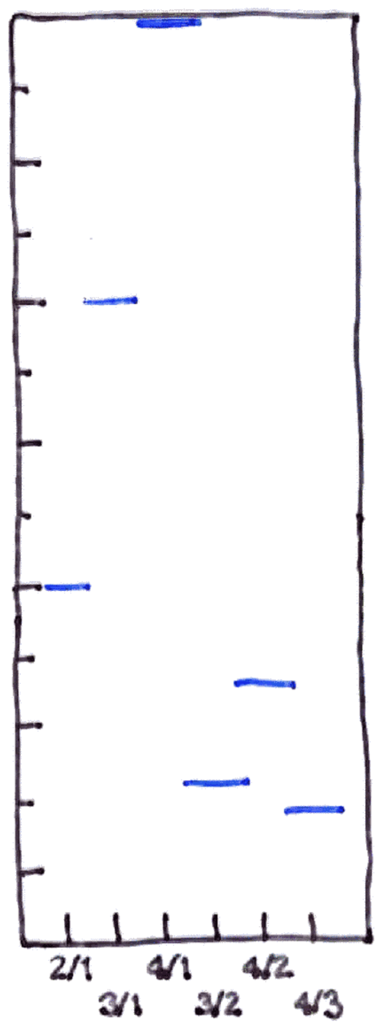
"3/2" = 1.66

"4/2" = 2.33

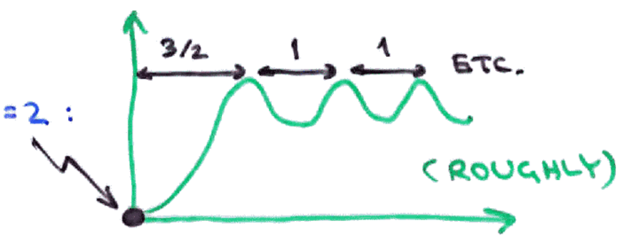
"4/3" = 1.4

ETC.

ROUGH Q=0 RMT PLOT :



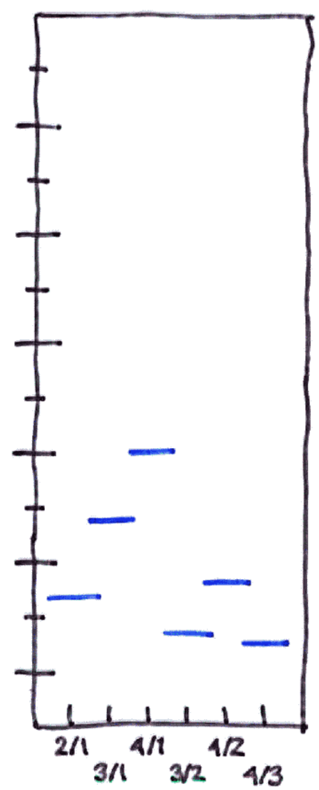
SIMILARLY FOR |Q|=2 :



⇒ "2/1" = 5/3 = 1.66

"3/1" = 7/3 = 2.33

⋮ ETC. ETC.

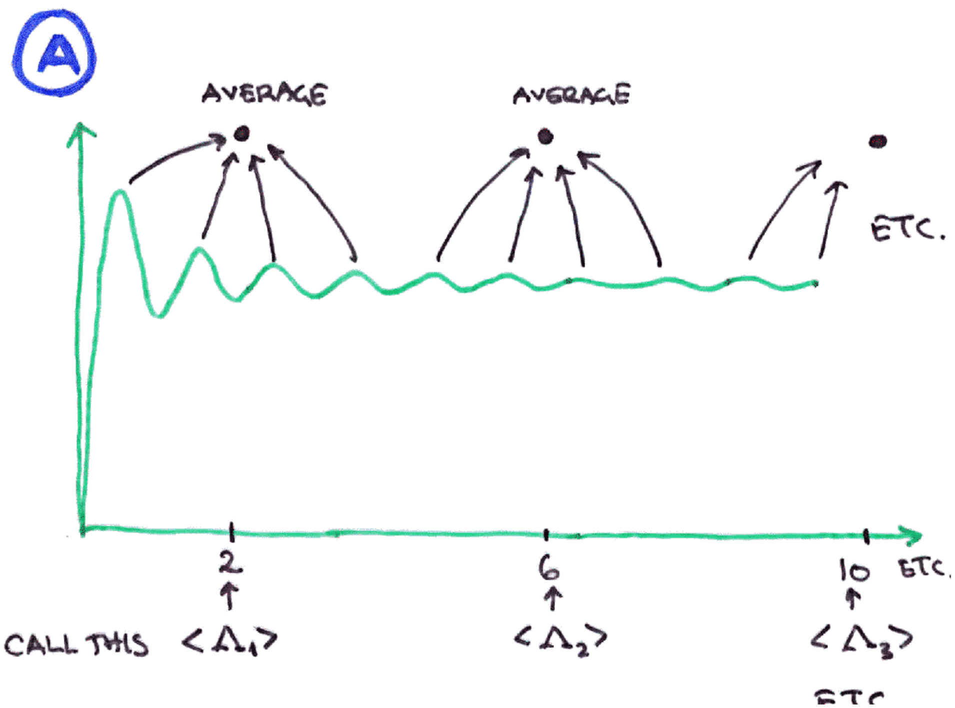


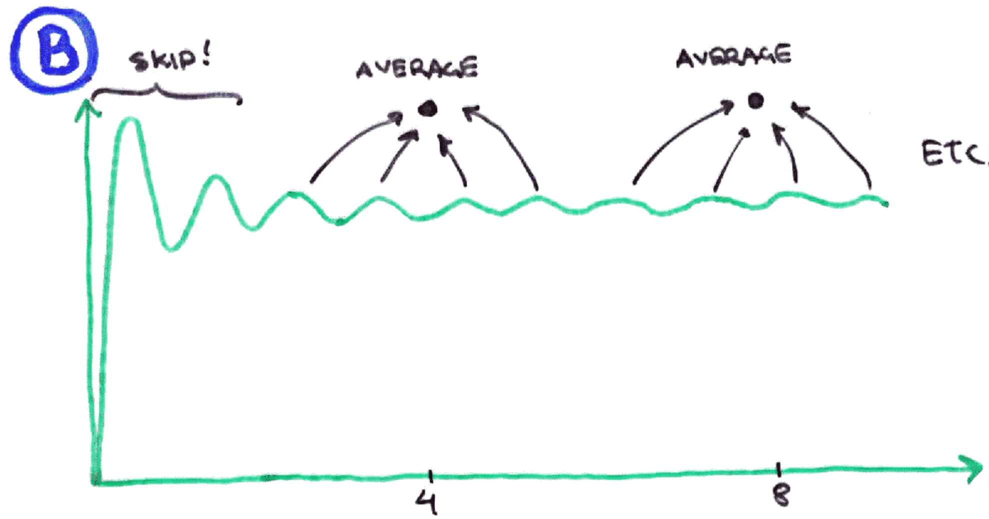
THIS WAS JUST A SIMPLE INTUITIVE PICTURE OF THE RMT RESULTS

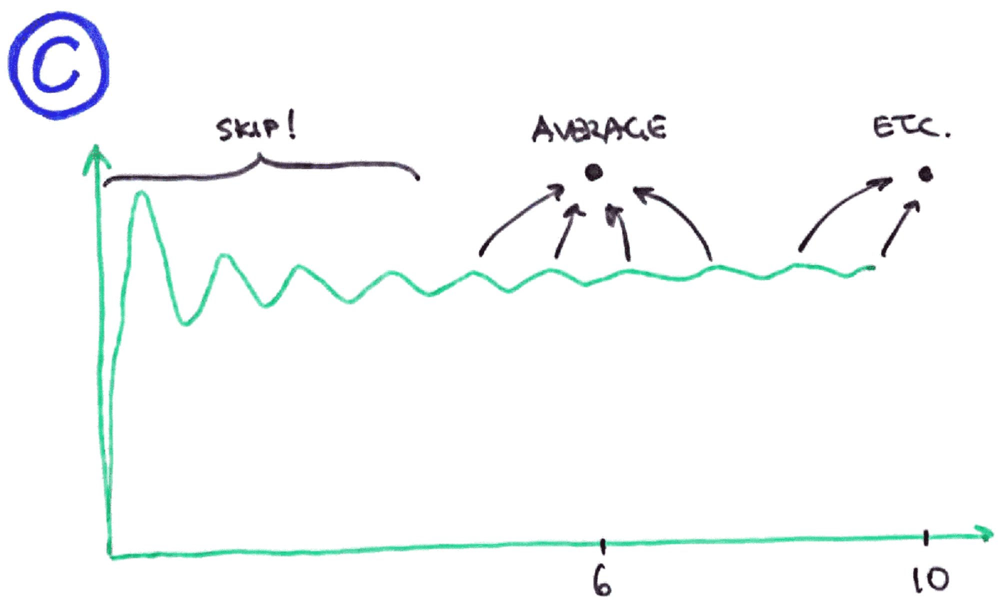
LET'S NOW DO AN AMUSING EXERCISE:

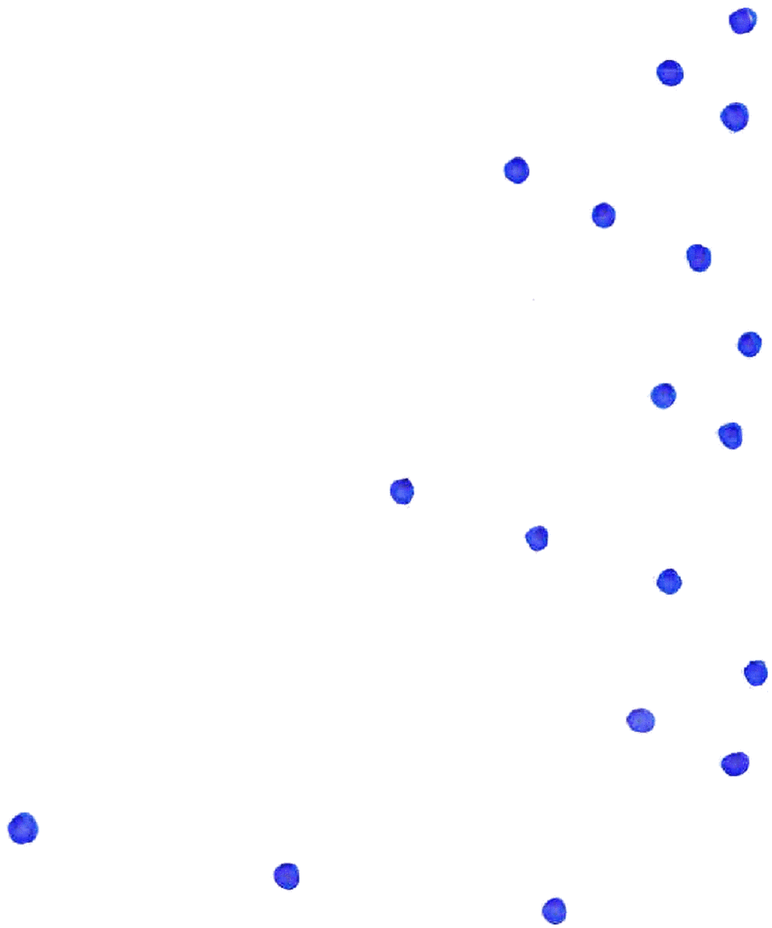
WE TAKE PERFECT **Q = 0** DATA (RMT, CONTINUUM FERMIONS, OR WHATEVER)

AND RE-PLOT THEM IN 3 DIFFERENT WAYS:









SEEMS WORTHWHILE:

- TAKE PERFECT Q=0 DATA AND DO THIS PROCEDURE CAREFULLY
- RE-DO THE FOLLANA-HART-DAVIES PLOT BUT SHUFFLE THE GAUGE CONFIGURATIONS AROUND – IS THE PLOT UNCHANGED?
- PLOT THE DISTRIBUTIONS OF Δ_i, NOT JUST $\langle \Delta_i \rangle$ – DO THEY FIT RMT?
- TO CHECK THAT IT'S NOT JUST A FLUKE, RE-DO EVERYTHING WITH, SAY, ADJOINT FERMIONS – WHICH ENSEMBLE FITS?

LARGE N_c

LARGE-N_c IS ALREADY AN RMT!

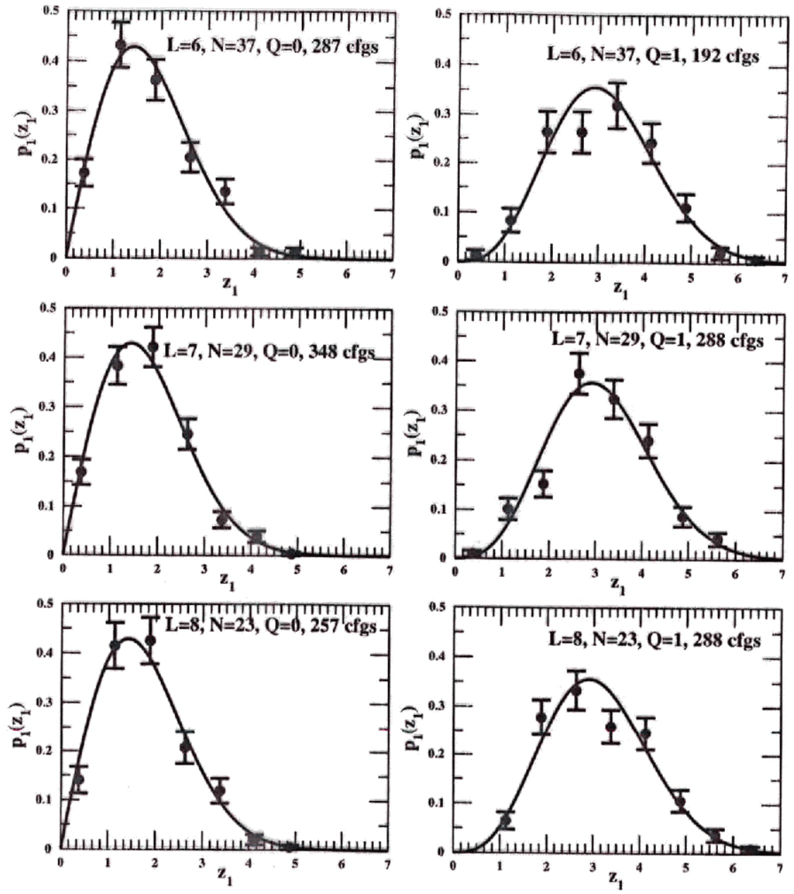
(E-K REDUCTION, "PARTIAL REDUCTION" OF KISKIS, NARAYANAN, NEUBERGER)

CHIRAL RMT: IT WORKS!

VOLUME $V \longrightarrow N_c$

(UNDERSTANDABLE FROM RMT POINT OF VIEW, BUT ALSO FROM CHIRAL LAGRANGIAN)

THERE MAY BE NEW RMT APPLICATIONS WAITING HERE...



SOME DOs AND DON'Ts :

- MAKE USE OF ALL THE INFORMATION FROM RMT

- COMPARE WITH THE DISTRIBUTION RATHER THAN THE AVERAGE :

* MANY FUNCTIONS HAVE THE SAME AVERAGE !

* WHAT IF THERE IS UNEXPECTED FEATURES IN THE DISTRIBUTIONS? (BUMPS, WIGGLES, ...)

- DO NOT INTEGRATE OR TAKE (INVERSE) MOMENTS OF $\rho(\xi)$

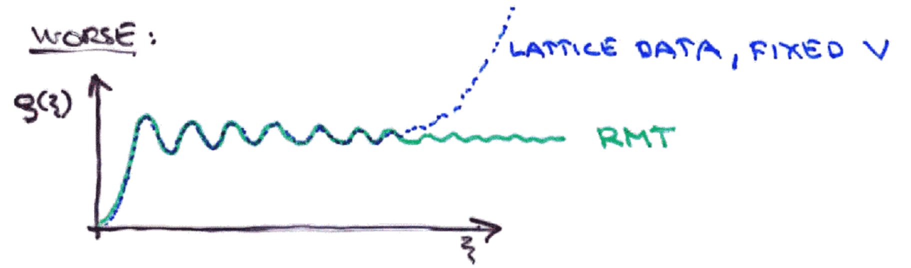
* FOR FIXED VOLUME V $\rho(\xi)$ WILL NOT BE CORRECT FOR $\xi \to \infty$

EXAMPLE :

LOTS OF STRUCTURE HERE !

$$\Sigma(m) = \underbrace{2m\int_0^\infty d\lambda \, \frac{\rho(\lambda;m)}{\lambda^2+m^2}}_{\text{VERY LITTLE STRUCTURE HERE!}}$$

WORSE :



SINCE WE NEVER TAKE $V \to \infty$ THE AGREEMENT GOES AWAY COMPLETELY AFTER A SMALL # OF OSCILLATIONS

$\Rightarrow$ $\boxed{\text{THE INTEGRATED FORMULA IS NOT AS ACCURATE}}$