Chiral symmetry breaking and the Dirac spectrum at non-zero μ_B

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WHAT ?

- The Banks-Casher relation for $\mu_B \neq 0$

WHY ?

- Non trivial because of the sign problem.
- Improve our analytic understanding of chiral symmetry breaking.

HOW ?

- Exact solution for the spectrum of the QCD Dirac operator at $\mu_B \neq 0$ in the ϵ-regime
- A complex eigenvalue density in the complex plane due to the sign problem

Definition of the eigenvalue density

Eigenvalue equation

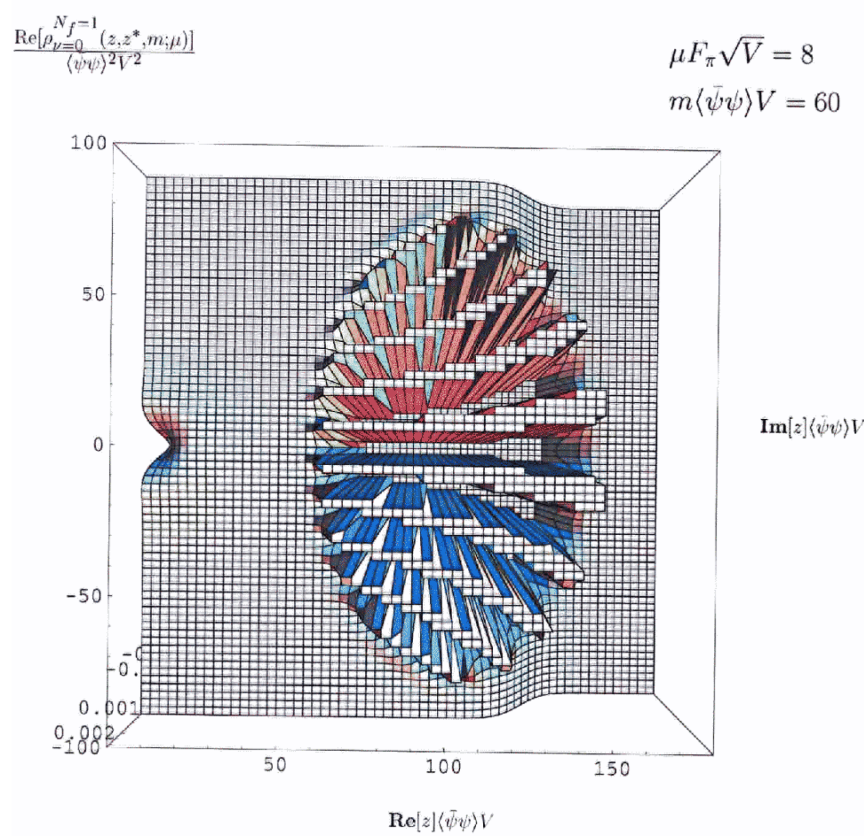
$$(D_\eta \gamma_\eta + \mu \gamma_0)\psi_j = z_j \psi_j$$

Eigenvalue density

$$\boxed{\rho^{N_f}(z, z^*, m; \mu) \equiv \left\langle \sum_j \delta^2(z - z_j) \right\rangle_{\rm QCD}}$$

$$\langle \mathcal{O} \rangle_{\rm QCD} \equiv \frac{\int dA\ \mathcal{O} \det(D_\eta\gamma_\eta + \mu\gamma_0 + m)^{N_f}\ e^{-S_{\rm YM}(A)}}{\int dA \det(D_\eta\gamma_\eta + \mu\gamma_0 + m_f)^{N_f}\ e^{-S_{\rm YM}(A)}}$$

One flavor



Osborn, PRL 93 (2004) 222001
Akemann, Osborn, Splittorff, Verbaarschot, hep-th/0411030

How to calculate the unquenched eigenvalue density

The replica way of writing the δ-functions

$$\rho^{N_f}(z, z^*, m; \mu) = \lim_{n \to 0} \frac{1}{\pi n} \partial_{z^*} \partial_z \log \mathcal{Z}^{N_f,n}(m, z, z^*; \mu)$$

generating functionals for the eigenvalue density

$$\mathcal{Z}^{N_f,n}(m, z, z^*; \mu) =$$

$$\int \mathrm{d}A \det(D_\eta \gamma_\eta + \mu \gamma_0 + m)^{N_f} |\det(D_\eta \gamma_\eta + \mu \gamma_0 + z)|^{2n} \, \mathrm{e}^{-S_{\mathrm{YM}}(A)}$$

Girko, *Theory of random determinants* (1990)
Stephanov, PRL **76**, 4472 (1996)
Fyodorov, Sommers, J.Math.Phys. **38** (1997) 1918
Akemann, Osborn, Splittorff, Verbaarschot hep-th/0411030

Central observation

The eigenvalue z and its complex conjugate z^* appears as the mass of two conjugate fermions.

very small eigenvalues $\longleftrightarrow$ *very* light quarks

$\Longrightarrow$ Chiral Perturbation Theory works *very* well.

SBχS

Low energy effective $\mathcal{L}$

- uniquely determined by the symmetries
(and the scheme)

Finite volume low energy QCD

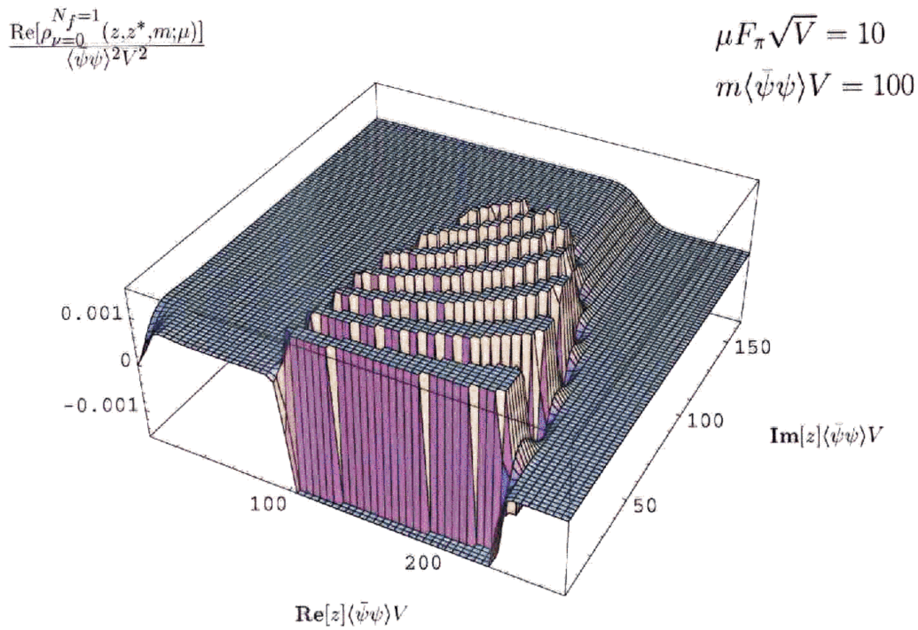
ϵ-limit at non-zero chemical potential:

$$\frac{1}{m_\pi} \sim \frac{1}{\mu} \sim \sqrt{V}$$

$$Z^{N_f,n} = \int_{U(N_f+2n)} \mathcal{D}U \times e^{-\frac{VF_\pi^2\mu^2}{4}\mathrm{Tr}([U,B][U^\dagger,B]) + \frac{\langle\bar\psi\psi\rangle V}{2}\mathrm{Tr}(UM+M^\dagger U^\dagger)}$$

Gasser, Leutwyler, Ann.Phys. **158** (1984) 142
Leutwyler, Smilga, PRD **46** (1992) 5607
Kogut, Stephanov, Toublan, PLB **464** (1999) 183
Son, Stephanov, PRL 86 (2001) 592
Toublan, Verbaarschot, Int.J.Mod.Phys. B15 (2001) 1404
Splittorff, Verbaarschot, NPB 683 (2004) 467
Akemann, Fyodorov, Vernizzi, NPB 694 (2004) 59

One flavor



AMPLITUDE: $e^{F(x,y;\mu)V}$; MAX VALUE ~ 14000

PERIOD: $\sim 1/V$

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The chiral condensate from the eigenvalue density

The chiral condensate from the eigenvalue density

$$\langle\bar{\psi}\psi\rangle(m) = \frac{1}{V}\partial_m \log Z_{N_f}(m;\mu)$$
$$= \frac{1}{V}\int dxdy \frac{\rho_{N_f}(x,y,m;\mu)}{x+iy+m}$$

The oscillations of the density are responsible for chiral symmetry breaking

Osborn, Splittorff, Verbaarschot, hep-th/0501210

Do the y-integral first

Structure:

$$\rho_{corr} = e^{-V[y^2+f(x,m;\mu)]}e^{iVy\,g(x,m;\mu)}$$

In the $y = a + ib$-plane

$$\rho_{corr} = e^{-V[a^2-b^2+f(x,m;\mu)]-Vb\,g(x,m;\mu)} \times e^{iVa\,g(x,m;\mu)-iV2ab}$$

For $m < x < 2\mu^2 F_\pi^2/\langle\bar{\psi}\psi\rangle$:

ρ_{corr} suppressed in strip to the left of the pole.

Deform the contour into this strip

$\Rightarrow \int dy \frac{\rho_{corr}}{x+iy+m}$ is given by the pole.

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The contribution from the pole

The residue at the pole is

$$\rho_{corr}(z = m) = -\frac{V \langle \bar{\psi}\psi \rangle^2}{4\pi\mu^2}$$

independent of x simply because

$$\rho_{N_f}(z = m) = 0$$

and

$$\rho_{N_f}(z) \equiv \rho_{N_f=0}(z) + \rho_{corr}(z)$$

μ-independent chiral condensate from a complex & oscillating μ-dependent density

Osborn, Splittorff, Verbaarschot, hep-th/0501210

Conclusion

- An oscillatory spectral density can give a discontinuity of the chiral condensate.
- The entire oscillating region of the complex eigenvalue plane contributes to the chiral condensate
- Uncovered using: The exact eigenvalue density of the QCD Dirac operator for fixed

$$m\langle \bar{\psi}\psi \rangle V, \quad z\langle \bar{\psi}\psi \rangle V, \quad \mu_B^2 F_\pi^2 V$$

which has oscillations with a period of order $1/V$ and an amplitude $\sim e^{\#V}$