Dynamic universality class of the QCD critical point

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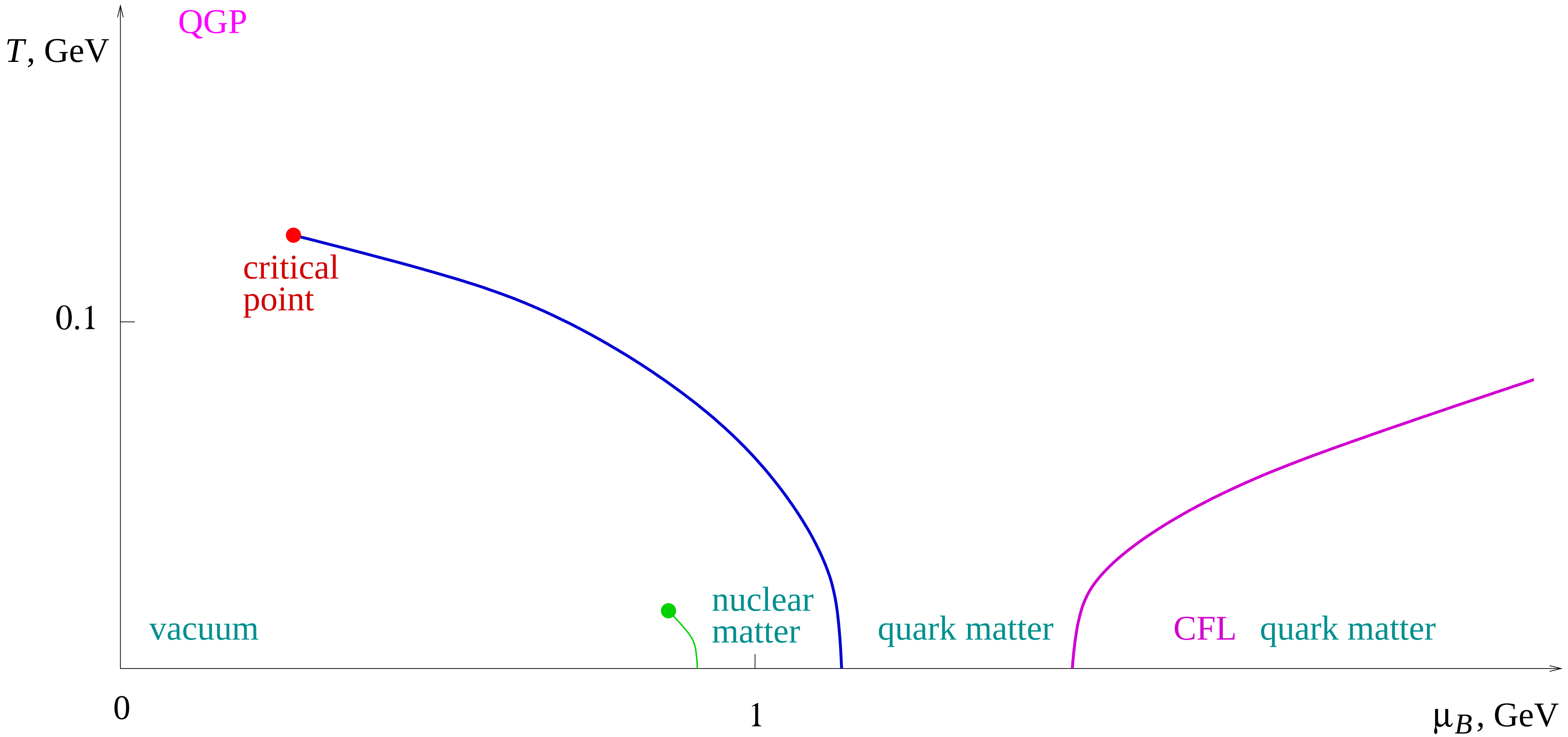
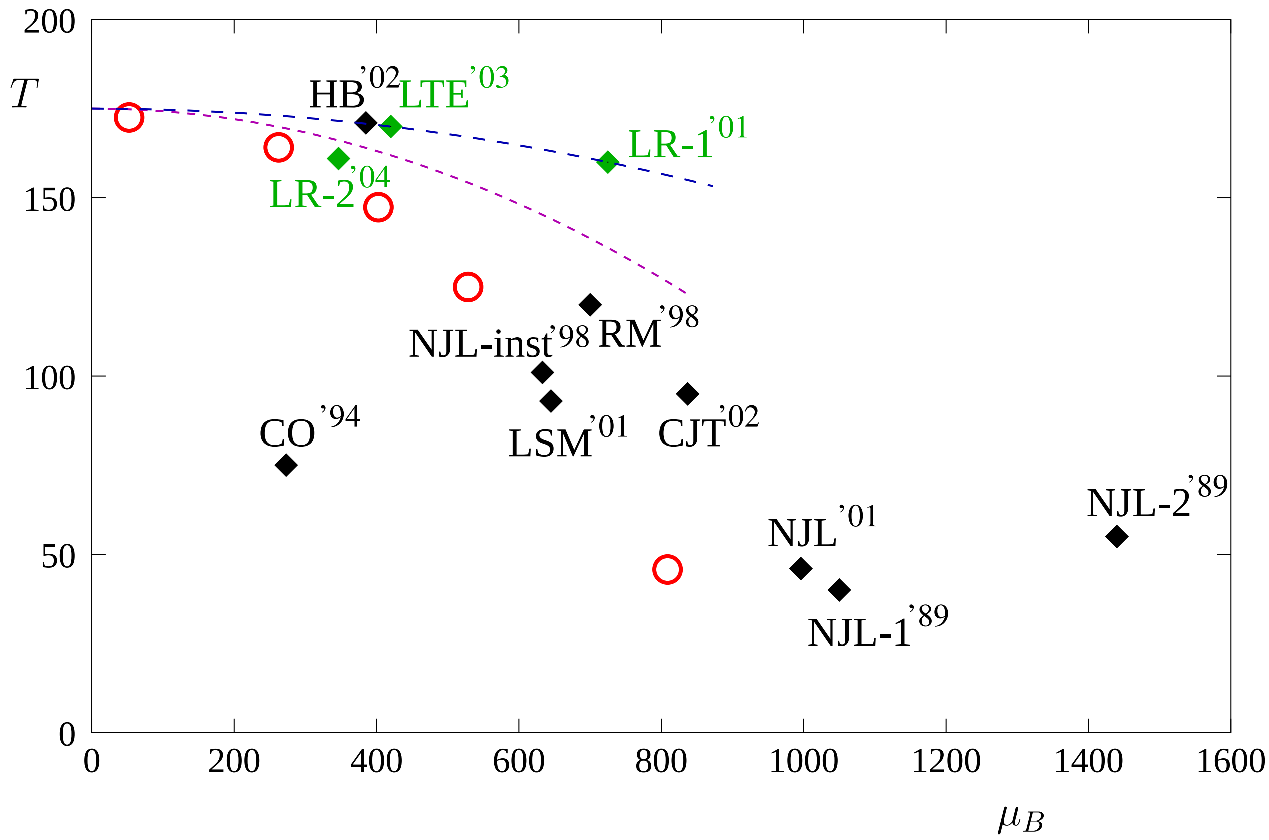
QCD critical point



Locating the QCD critical point



Signatures: event-wise fluctuations.

Susceptibilities diverge $\Rightarrow$ fluctuations grow towards the critical point.

Magnitude of fluctuations

Scaling and universality of critical phenomena: $\chi \sim \xi^{\text{power}}$.

How big can ξ grow?

Limits:

- Proximity of the critical point
- Finite size of the system $\xi < 6$ fm.
- Finite *time*: $\tau \sim 10$ fm.

$$\xi \sim \tau^{1/z}$$

z – dynamical critical exponent.

Dynamic scaling and universality

Hohenberg, Halperin Rev. Mod. Phys. 49, 435 (1977)

Near the c.p. Typical relaxation time scale in the system diverges (critical slowing down):

$$\tau \sim \xi^z$$

(dynamic scaling).

The exponent z is determined by the *dynamic* universality class.

Systems with equivalent *static* critical behavior are not always in the same *dynamic* universality class.

Example: Ising model and liquid-gas phase transition.

Simple reason: the order parameter (density) in the latter is a conserved quantity $\Rightarrow$ relaxes slower at large distance scales (by diffusion), then in the Ising model (local relaxation).

Dynamic universality class of QCD critical point

Son, MS, hep-ph/0401052

Statics: symmetry and dimensionality $\rightarrow$ Ising model (same as liquid-gas).

Dynamic universality class depends on relevant hydrodynamic modes.

Modes which relax arbitrarily slowly: densities of conserved quantities, order parameter.

- The fluctuations of the energy and momentum densities: $\varepsilon \equiv T^{00} - \langle T^{00} \rangle$, and $\pi^i \equiv T^{0i}$;
- The fluctuations of the baryon number density, $n \equiv \bar{q}\gamma^0 q - \langle \bar{q}\gamma^0 q \rangle$;
- The chiral condensate $\sigma \equiv \bar{q}q - \langle \bar{q}q \rangle$.

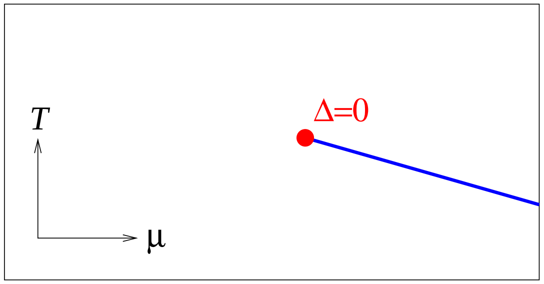
Is it liquid gas (H), or is it Ising model (A), or is it another universality class altogether? (Berdnikov, Rajagopal: model C)

Statics

σ and n mix.

$$F[\sigma, n] = \int d\boldsymbol{x} \left[V(\sigma, n) + \frac{a}{2}(\boldsymbol{\nabla}\sigma)^2 + b(\boldsymbol{\nabla}\sigma)(\boldsymbol{\nabla}n) + \frac{c}{2}(\boldsymbol{\nabla}n)^2 \right],$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2 + \text{higher orders.}$$



At the critical point $\Delta \equiv AC - B^2 \to 0$.

One flat direction: $(\sigma, n) \sim (-B, A)$. Only one critical mode.

Susceptibilities, e.g.:

$$\langle \sigma^2_{\boldsymbol{q}\to 0} \rangle = \frac{TC}{\Delta}, \qquad \langle n^2_{\boldsymbol{q}\to 0} \rangle = T\chi_B = \frac{TA}{\Delta}.$$

Either σ or n can be used as an order parameter (both jump across 1st order phase transition).

Dynamic equations

$$
\begin{aligned}
\dot{\sigma} &= -\Gamma \frac{\delta F}{\delta \sigma} + \tilde{\lambda} \boldsymbol{\nabla}^2 \frac{\delta F}{\delta n} + \xi_\sigma \,, \\
\dot{n} &= \tilde{\lambda} \boldsymbol{\nabla}^2 \frac{\delta F}{\delta \sigma} + \lambda \boldsymbol{\nabla}^2 \frac{\delta F}{\delta n} + \xi_n \,.
\end{aligned}
$$

with (so that $P[\sigma, n] \sim \exp(-F[\sigma, n]/T)$)

$$
\begin{aligned}
\langle \xi_\sigma(x) \xi_\sigma(y) \rangle &= 2T\Gamma \delta^4(x-y) \,, \\
\langle \xi_\sigma(x) \xi_n(y) \rangle &= -2T\tilde{\lambda}\delta(t-t') \boldsymbol{\nabla}^2 \delta^3(\boldsymbol{x}-\boldsymbol{y}) \,, \\
\langle \xi_n(x) \xi_n(y) \rangle &= -2T\lambda\delta(t-t') \boldsymbol{\nabla}^2 \delta^3(\boldsymbol{x}-\boldsymbol{y}) \,.
\end{aligned}
$$

Using F (and to leading order in $\boldsymbol{\nabla}$):

$$
\begin{aligned}
\dot{\sigma} &= -\Gamma A \sigma - \Gamma B n \,, \\
\dot{n} &= (\tilde{\lambda} A + \lambda B) \boldsymbol{\nabla}^2 \sigma + (\tilde{\lambda} B + \lambda C) \boldsymbol{\nabla}^2 n \,.
\end{aligned}
$$

($+$ noise)

Modes

$$\det\begin{vmatrix} \Gamma A - i\omega & \Gamma B \\ (\tilde{\lambda}A + \lambda B)\boldsymbol{q}^2 & (\tilde{\lambda}B + \lambda C)\boldsymbol{q}^2 - i\omega \end{vmatrix} = 0\,.$$

Near critical point:

$$\begin{aligned} \omega_1 &= -i\lambda\frac{\Delta}{A}\boldsymbol{q}^2\,. \\ \omega_2 &= -i\Gamma A\,. \end{aligned}$$

Diffusive: $(\sigma, n) \sim (-B, A)$, relaxational: $(\sigma, n) \sim (1, 0)$.

Critical mode (flat direction) is the diffusive mode, $\sigma = (-B/A)n$.

Sigma alone ($n = 0$) relaxes on a finite time scale *even at c.p* (Γ is finite).

Conclusion: only one hydrodynamic mode after σ and n mixing, and it is diffusive.

Diffusion constant:

$$D = \lambda\frac{\Delta}{A} = \lambda C - \lambda\frac{B^2}{A}\,.$$

$D \to 0$ at c.p.

Diffusion constant and critical indices

$D = \lambda \frac{\Delta}{A}$ and $\chi_\mathsf{B} = \frac{A}{\Delta}$:

$$D = \lambda \chi_\mathsf{B}^{-1}$$

$$\mu(\boldsymbol{x}) = \chi_\mathsf{B}^{-1} n(\boldsymbol{x}) \qquad \rightarrow \qquad \boldsymbol{E} = -\boldsymbol{\nabla}\mu \qquad \rightarrow \qquad \boldsymbol{j}_n = \lambda \boldsymbol{E} = -\lambda \chi_\mathsf{B}^{-1} \boldsymbol{\nabla} n$$

Typical relaxation (diffusion) time

$$\tau \sim D^{-1} \xi^2,$$

but $D^{-1} \sim \chi_\mathsf{B} \sim \xi^{2-\eta} \qquad \rightarrow \qquad \tau \sim \xi^{4-\eta} \qquad \rightarrow \qquad z = 4 - \eta$.

This is model B.

Coupling/mixing with energy-momentum

Modes to consider:

- ε, $\boldsymbol{\pi}$
- n
- σ

Only one combination of n and σ is truly hydrodynamic $\longrightarrow$ ε, $\boldsymbol{\pi}$ and n.

Same as in the liquid-gas dynamic universality class.

This is model H.

Model H

Review results (Hohenberg, Halperin):

$$
\begin{aligned}
D = \lambda \chi_{\mathsf{B}}^{-1} \quad &\sim \quad \xi^{x_\lambda} \chi_{\mathsf{B}}^{-1}, \\
\bar{\eta} \quad &\sim \quad \xi^{x_\eta}.
\end{aligned}
$$

$$
x_\lambda + x_\eta = 4 - d - \eta \,.
$$

$$
x_\eta \quad = \quad \frac{1}{19}\epsilon(1 + 0.238\epsilon + \cdots) \approx 0.065 \,, \qquad (1)
$$

$$
x_\lambda \quad = \quad \frac{18}{19}\epsilon(1 - 0.033\epsilon + \cdots) \approx 0.916 \,, \qquad (2)
$$

$$
D \sim \xi^{-2+\eta+x_\lambda},
$$

$$
z = 4 - \eta - x_\lambda \approx 3 \,.
$$

Conclusion

- Mixing between σ and n (and ε, $\pi_{||}$) leaves only one hydrodynamic mode – diffusive.
- Dynamic universality class of QCD c.p. is that of model H.
- $z \approx 3$ (> 2 of model A, but < 4 of model B).
- Finite time constraint is rather strong: $\xi_{\text{max}} \sim \tau^{1/z}$ is not too large. Observability: fluctuations are $\chi \sim \xi^2$.

Appendix

Divergence of $\lambda\bar\eta$

$$\boldsymbol{E} = -\boldsymbol{\nabla}\mu \qquad \rightarrow \qquad \boldsymbol{j}_n = \lambda\boldsymbol{E} \qquad \text{(in model B)}$$

$$\boldsymbol{f}_{\text{appl}} \sim n\boldsymbol{E}L^d$$

$$\boldsymbol{f}_{\text{visc}} + \boldsymbol{f}_{\text{appl}} = 0$$

$$\boldsymbol{f}_{\text{visc}} \sim -\bar\eta\boldsymbol{v}L^{d-2}$$

$$\boldsymbol{j}_n = n\boldsymbol{v} \sim \frac{n^2}{\bar\eta}L^2\,\boldsymbol{E}\,. \qquad \text{(in model H)}$$

$L \rightarrow \infty$ divergence is cut off at $L \sim \xi$, because n is correlated only up to this scale.

$\langle n^2\rangle = T\chi_{\text{B}}/L^d$ for $L \gg \xi$

$$\lambda\bar\eta \sim \langle n^2\rangle\xi^2 \sim \chi_{\text{B}}\xi^{2-d} \sim \xi^{4-d-\eta},$$

Real time correlation functions

$$\langle \sigma^2_{\omega \boldsymbol{q}} \rangle = \frac{2T\Gamma}{\omega^2 + \Gamma^2 A^2} + \frac{2TB^2\lambda \boldsymbol{q}^2}{A^2(\omega^2 + D^2\boldsymbol{q}^4)} ,$$

$$\langle n^2_{\omega \boldsymbol{q}} \rangle = \frac{2T\lambda \boldsymbol{q}^2}{\omega^2 + D^2\boldsymbol{q}^4} .$$