Semileptonic B-meson decay phenomenology with lattice QCD

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(On behalf of the FNAL/MILC Collaborations)

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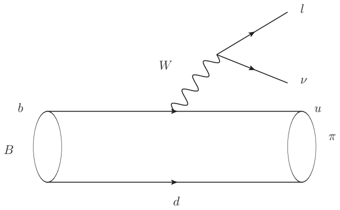
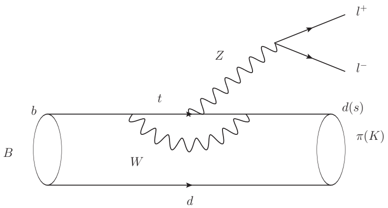
Theoretical Motivation



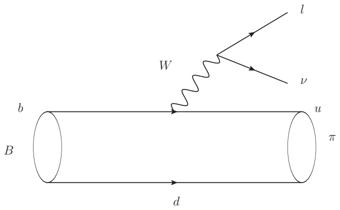
Tree-level diagram



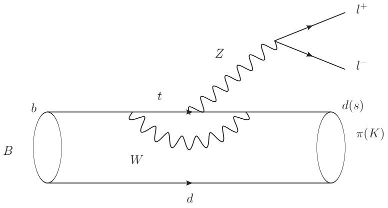
Loop-level diagram

- B-meson semileptonic decays through tree-level diagram ($b \to ul\nu$). For example, $B \to \pi l\nu$, $B_s \to Kl\nu$
- These processes are used to determine CKM matrix elements ($|V_{ub}|$ from $B \to \pi l\nu$ and $B_s \to Kl\nu$, $|V_{cb}|$ from $B \to Dl\nu$).

Theoretical Motivation



Tree-level diagram



Loop-level diagram

- B-meson semileptonic decays through loop-level diagram ($B \to K(\pi)l^+l^-$, $B \to K(\pi)\nu\bar{\nu}$)
- Standard Model contribution is suppressed in the loop-level diagram. (Suitable processes to detect physics BSM)
- Studied by many experiment groups (BABAR, Belle, CDF, LHCb, *B*-factory *etc.*)

Standard Model prediction

The effective Hamiltonian of the $b \to d(s) l^+ l^-$ transition under OPE with α_s and Λ/m_b corrections is:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V^*_{td(s)} V_{tb} \sum_{i=0}^{10} C_i(\mu) O_i(\mu) + \ldots \quad (1)$$

the Standard Model prediction can be written in a generic form:

$$\text{Theo. pred.} = (\text{prefactors}) \times (\text{CKMfactor}) \times \langle f|\hat{O}|i\rangle \quad (2)$$

- Prefactors contain the Wilson coefficients (short distance physics).
- CKM factor depends on the processes.
- Lattice QCD calculates $\langle f|\hat{O}|i\rangle$ non-perturbatively from first principle. (long distance physics)
- We use B-meson semileptonic decays to extract CKM factor or detect new physics.

Hadronic matrix elements and form factors

- Matrix elements in $B \to K(\pi)ll$ and $B \to \pi l\nu$ processes:

$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle$, $\langle B(p)|\bar{s}\sigma^{\mu\nu} b|K(k)\rangle$

$$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle = f_+(p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu) + f_0 \frac{m_B^2 - m_K^2}{q^2} q^\mu$$
$$= \sqrt{2m_B}\left[f_\parallel \frac{p^\mu}{m_B} + f_\perp k_\perp^\mu \right]$$

$$\begin{cases} f_\parallel(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^0 s|K(k)\rangle}{\sqrt{2m_B}} \\ f_\perp(E_K) = \dfrac{\langle B(p)|\bar{b}\gamma^i s|K(k)\rangle}{2\sqrt{m_B}} \dfrac{1}{p_i} \end{cases}$$

$$\begin{cases} f_0(E_K) = \dfrac{2m_B}{m_B^2 - m_K^2}\left[(m_B - E_K) f_\parallel(E_K) + (E_K^2 - m_K^2) f_\perp(E_K)\right] \\ f_+(E_K) = \dfrac{1}{\sqrt{2m_B}}\left[f_\parallel(E_K) + (m_B - E_K) f_\perp(E_K)\right] \end{cases}$$

Hadronic matrix elements and form factors

Semileptonic $B \to K$ transition from tensor current:

$$q_\nu \langle K(k)|\bar{s}\sigma^{\mu\nu} b|B(p)\rangle = \frac{i f_T}{m_B + m_K}\left[q^2(p^\mu + k^\mu) - (m_B^2 - m_K^2)q^\mu\right]$$

Solve for f_T:

$$f_T = \frac{m_B + m_K}{\sqrt{2m_B}} \frac{\langle K(k)|ib\sigma^{0i}s|B(p)\rangle}{\sqrt{2m_B}k^i}$$

Lattice ensembles used in $B \to K(\pi)$ works

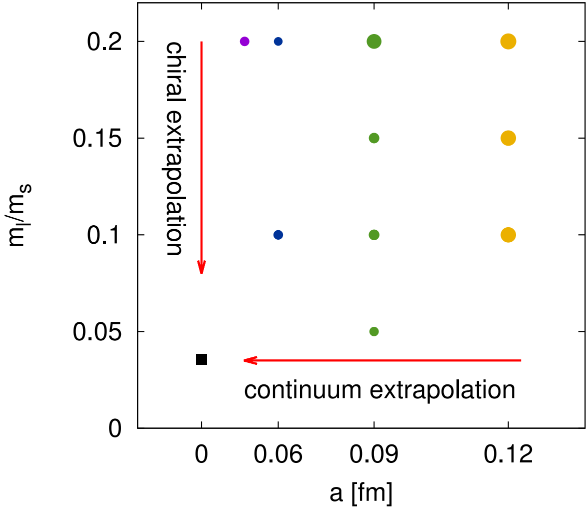


Figure: Ensembles of QCD gauge field configurations used in the simulations.

$f_{\parallel}$, $f_{\perp}$ chiral-continuum extrapolations

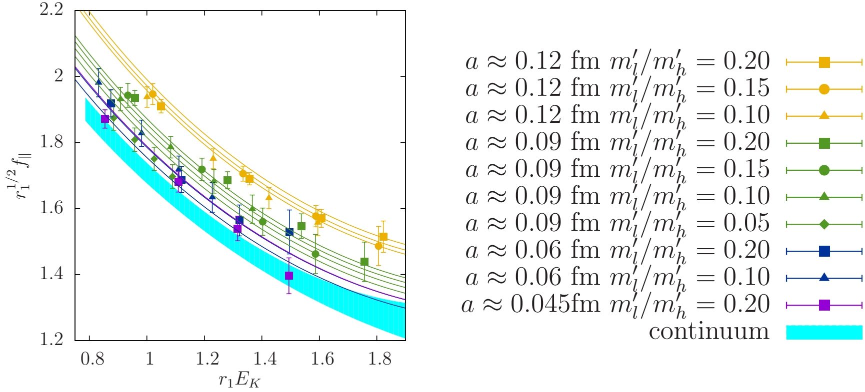


Figure: Chiral-continuum extrapolation on $f_{\parallel}$ (left).

- The heavy-meson chiral perturbation theory (HMChPT) is used as an effective theory of QCD in the extrapolation.
- Lattice-QCD simulations are in the low $E_{K(\pi)}$ region.

z-expansion on $B \to K(\pi)$ semileptonic decay form factors

The from factors from chiral-continuum extrapolations are valid only in low E_k regime, because

- Form factors computed on the lattice are mostly in low E_k regime. (Data range is limited.)
- ChPT is valid only in low E_k regime. (Extrapolation range is limited.)

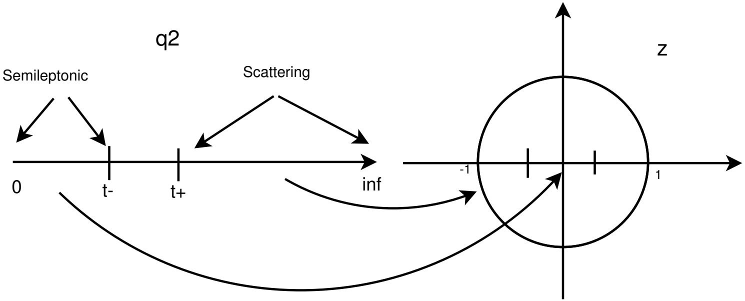
q^2 and $E_{K(\pi)}$ relation:

- Form factors are functions of E_k or $q^2 = (p_B - p_K)^2 = m_B^2 + m_K^2 - 2m_B E_K$
- We have extrapolated the continuum form factors in the high q^2 region.
- Our continuum form factors are in the range of 17$\sim q^2_{\rm max}$.

We need z-expansion as a model independent extrapolation method to get form factors in low q^2 range.

z-expansion on $B \to K(\pi)$ semileptonic decay form factors

$q^2 = (p_B - p_K)^2 = m_B^2 + m_K^2 - 2m_B E_K$



- z-expansion maps q^2 to z by:

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad t_\pm = (m_B \pm m_K)^2$$

- Choose $t_0 = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right)$ such that $z << 1$
- Expand form factors as a function of z.

z-expansion on $B \to K(\pi)$ semileptonic decay form factors

We expand form factors as a function of z in BCL formalism. (PRD **79**, 013008)

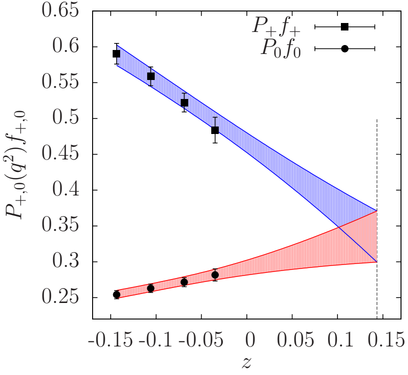
$$f_{+,T}(q^2) = \frac{1}{P(q^2)} \sum_{k=0}^{K-1} b_k \left[z^k - (-1)^{k-K} \frac{k}{K} z^K \right],$$

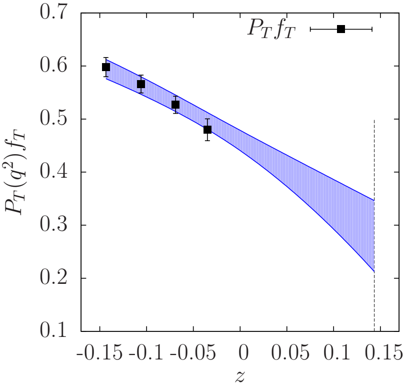
$$f_0(q^2) = \frac{1}{P(q^2)} \sum_{k=0}^{K-1} b_k z^k,$$

- $P(q^2) = 1 - \frac{q^2}{m_R^2}$ and m_R is used to count the pole in form factors.
- Unitarity constrains ensure $\sum_{j,k=0}^{\infty} B_{jk} b_j b_k \leq 1$
- We found three or four terms are enough for a good description of the lattice data and well controlled truncation error.

z-expansion on $B \to Kll$ form factors

z-expansion fit result as a function of $z = z(q^2)$.





z-expansion on $B \to Kll$ form factors

z-expansion fit result as a function of q^2.

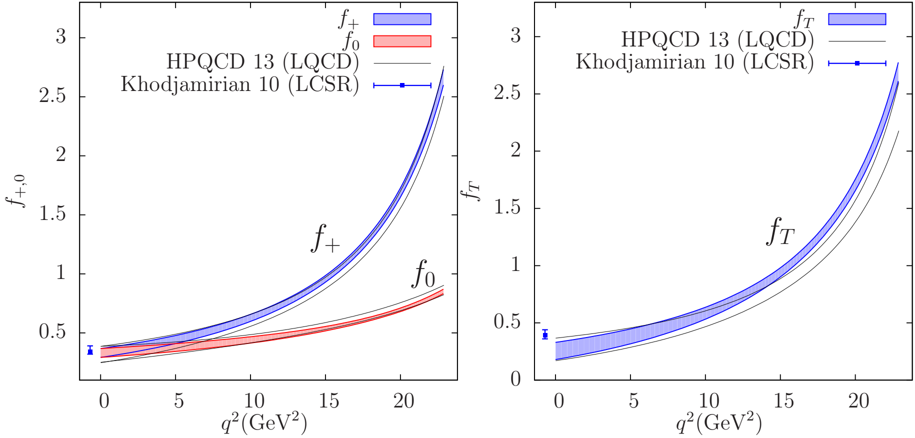


Figure: Comparison of the form factors. HPQCD and LCSR results are from arXiv:1006.4945 and arXiv:1306.2384.

Semileptonic B-meson decay phenomenology

Outline:

Tree-level process:

- $B \to \pi l \nu$ and $|V_{ub}|$ determination. (arXiv:1503.07839)

Loop-level process:

- $B \to K l^+ l^-$ ($l = e, \mu, \tau$)
- $B \to \pi l^+ l^-$ ($l = e, \mu, \tau$) (arXiv:1503.07839, arXiv:1507.01618)
- $B \to \pi \nu \bar{\nu}$, $B \to K \nu \bar{\nu}$

$B \to \pi l \nu$ semileptonic decay and $|V_{ub}|$

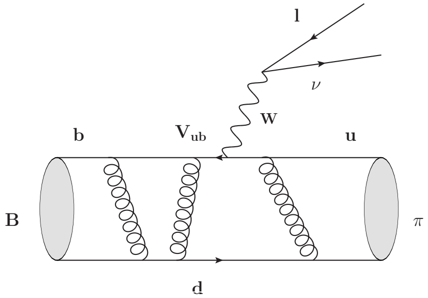


Figure: $B \to \pi l \nu$ exclusive decay process.

$$\frac{d\Gamma}{dq^2} \propto |V_{ub}|^2 |f_+(q^2)|^2 \quad \text{Exp.}$$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right] + f_0(q^2)\frac{M^2 - m^2}{q^2} q^\mu$$

$$q^2 = (p_B - p_\pi)^2 = M_B^2 + M_\pi^2 - 2 M_B M_\pi E_\pi$$

$B \to \pi l \nu$ semileptonic decay and $|V_{ub}|$

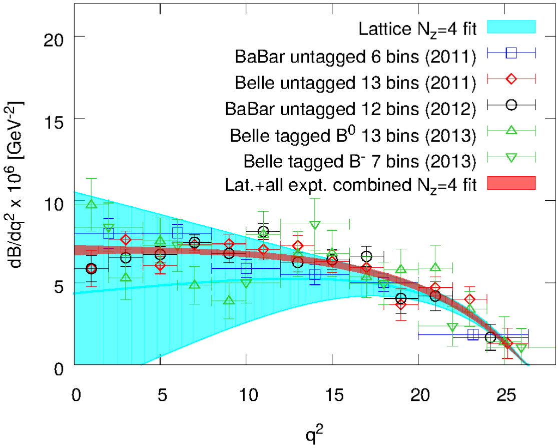


Figure: Combined fit of lattice-QCD form factors and experimental data. (arXiv:1503.07839) The combined fit of experimental data and lattice-QCD data significantly reduced the form factor's error at low q^2.

$B \to \pi l \nu$ semileptonic decay and $|V_{ub}|$

Three ways to determine $|V_{ub}|$ via B-meson decay ($b \to u$):

- Leptonic decay ($B \to \tau\nu$)
  Lattice QCD can provide decay constant (f_B).
- Semileptonic inclusive process (no specified final state): $B \to X_u l \nu$
  Perturbation theory is applicable, but it has model dependent error.
- Semileptonic exclusive process (specify a final state): $B \to \pi l \nu$ etc.
  Lattice QCD can help in the determination of form factors.

$B \to \pi l \nu$ semileptonic decay and $|V_{ub}|$

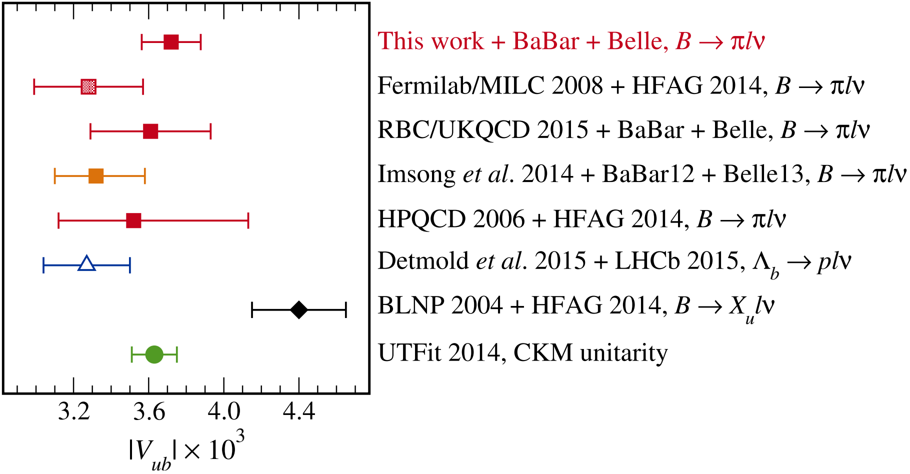


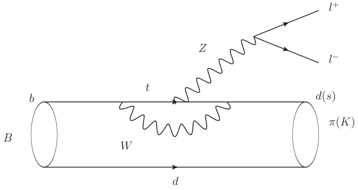
Figure: Comparison of the $|V_{ub}|$ results from different determinations. (arXiv:1503.07839)

$B \to \pi l \nu$ semileptonic decay and $|V_{ub}|$

Summary:

- The current lattice QCD+experimental data combined analysis gives very accurate form factors(f_+, f_0) in the whole q^2 range for $B \to \pi l \nu$ semileptonic decay.
- The small tension ($\sim 2\sigma$) between inclusive and exclusive determination of $|V_{ub}|$ still exists in the more accurate result.
- Two ongoing projects about $B \to \pi l \nu$ by FNAL/MILC (HISQ val+HISQ sea) and HPQCD (HISQ val+Asqtad sea) will provide more results for comparison in the future.
- The studies on the $B_s \to K l \nu$ process can also provide an independent determination of $|V_{ub}|$ in the future. (arXiv:1406.2279, arXiv:1312.3197)

$B \to \pi(K) l^+ l^-$ semileptonic decay



Loop-level diagram

- The $B \to \pi l^+ l^-$ decay was first observed in 2012 by LHCb (arXiv:1210.2645).
- The branching function of the $B \to \pi \mu^+ \mu^-$ process is

$$\mathcal{B}(B^+ \to \pi^+ \mu^+ \mu^-) = [2.3 \pm 0.6(stat.) \pm 0.1(syst.)] \times 10^{-8} \quad (3)$$

- The ratio of $B^+ \to \pi^+ \mu^+ \mu^-$ to $B^+ \to K^+ \mu^+ \mu^-$ is

$$\frac{\mathcal{B}(B^+ \to \pi^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)} = 0.053 \pm 0.014(stat.) \pm 0.001(syst.) \quad (4)$$

- More detailed results are in arXiv:1509.00414.

Theoretical results of the $B \to \pi l^+ l^-$ semileptonic decay

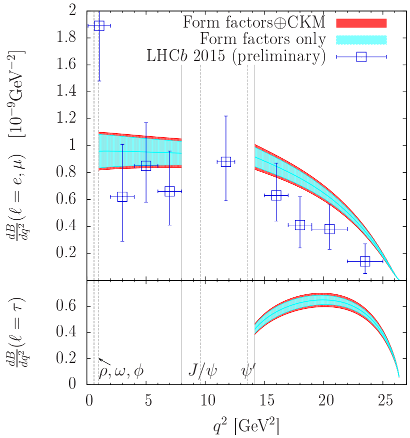
Some papers on the standard Model predictions (The form factors were calculated in different methods.):

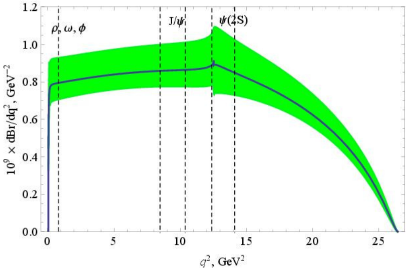
- Wen-Fei Wang *et al.* 1207.0265 (QCD factorization)
- Ahmed Ali *et al.* 1312.2523 (lattice, LCSR, $B \to \pi l \nu$ exp.)
- R. N. Faustov *et al.* 1403.4466 (relativistic quark model)
- Wei-Shu Hou *et al.* 1403.7410 (LCSR)
- Zuo-Hong Li *et al.* 1411.0466 (LCSR)
- Christian Hambrock *et al.* 1506.07760 (LCSR)

Compared with these works, we use

- The most recent f_+ and f_0 from MILC(asqtad) lattice ensembles and experiments (arXiv:1503.07839).
- First result of f_T in $B \to \pi l^+ l^-$ process from lattice-QCD calculation (arXiv:1507.01618)
- NNLO Wilson coefficients (arXiv:0512066).

Standard Model predictions





- Left: FNAL/MILC $B \to \pi$ lattice data + exp (arXiv:1503.07839)
- Right: (arXiv:1312.2523): old FNAL/MILC $B \to \pi$ lattice data (arXiv:0811.3640) + HPQCD's $B \to K$ lattice data(arXiv:1306.2384) + exp + LCSR + model

Standard Model predictions

Table: Standard-Model predictions for $B^+ \to \pi^+ \ell^+ \ell^-$ partial branching fractions. Errors shown are from the CKM elements, form factors, variation of the high and low matching scales, and the quadrature sum of all other contributions, respectively. (arXiv:1507.01618)

$[q^2_{\text{min}}, q^2_{\text{max}}]$ (GeV^2)	BR($B^+ \to \pi^+ \ell^+ \ell^-$) $\times 10^9$ $\ell = e, \mu$	$\ell = \tau$
[0.1, 2.0]	1.81(11,24,6,2)	
[2.0, 4.0]	1.92(11,22,6,3)	
[4.0, 6.0]	1.91(11,20,6,3)	
[6.0, 8.0]	1.89(11,18,5,3)	
[15, 17]	1.69(10,13,3,5)	1.11(7,8,2,4)
[17, 19]	1.52(9,10,2,4)	1.25(8,8,2,3)
[19, 22]	1.84(11,11,3,5)	1.93(12,10,4,5)
[22, 25]	1.07(6,6,3,3)	1.59(10,7,4,4)
[1, 6]	4.78(29,54,15,6)	
[15, 22]	5.05(30,34,7,15)	4.29(26,25,7,12)
$[4m_\ell^2, 26.4]$	20.4(1.2,1.6,0.3,0.5)	

Standard Model predictions of $B \to K l^+ l^-$ process

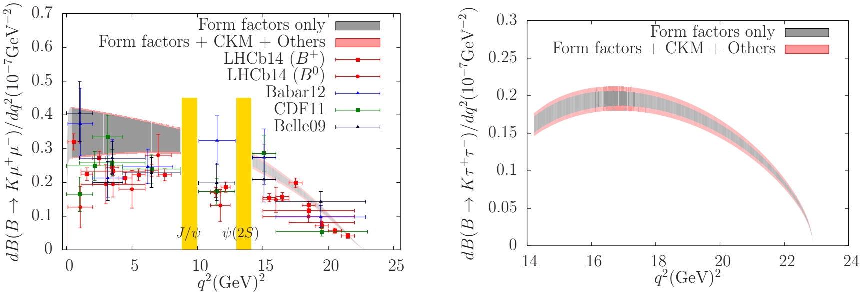


Figure: Standard-Model differential branching fraction (gray band) for $B \to K\mu^+\mu^-$ decay (left) and $B \to K\tau^+\tau^-$ (right) using the form factors obtained from lattice QCD. Experimental results for $B \to K\mu^+\mu^-$ are from Belle (arXiv:0904.0770), CDF (arXiv:1107.3753), BaBar (arXiv:1204.3933), and LHCb (arXiv:1403.8044). The BaBar, Belle, and CDF experiments report isospin-averaged measurements, while LHCb separately reports results for B^+ and B^0 decays.

Ratios of $B \to \pi ll / B \to Kll$ observables

The Standard-Model predicts the ratios between $B \to \pi ll$ and $B \to Kll$ partially-integrated branching ratios:

$$\frac{\mathrm{BR}(B^+ \to \pi^+\mu^+\mu^-)}{\mathrm{BR}(B^+ \to K^+\mu^+\mu^-)} = \left|\frac{V_{td}}{V_{ts}}\right|^2 F^{\pi/K} \tag{5}$$

- The ratio can be used to be compared with experiments. (check new physics)
- This ratio privides an independent determination of the $|\frac{V_{td}}{V_{ts}}|$ (Assuming Standard-Model is correct.). The results can be compared with the $|\frac{V_{td}}{V_{ts}}|$ determined from B mixing or CKM fitter.
- Lattice-QCD calculation can privide $F^{\pi/K}$.

Ratios of $B \to \pi ll / B \to K ll$ observables

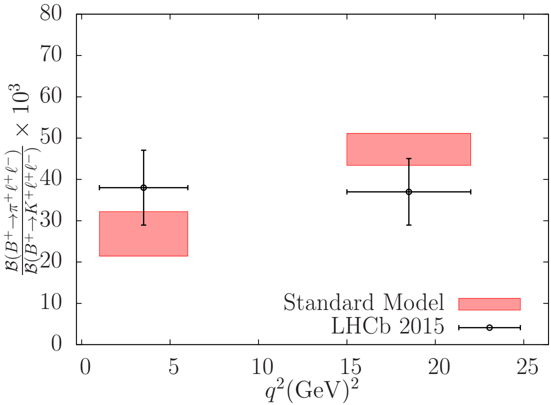


Figure: Ratio of partially-integrated branching ratios $\text{BR}(B \to \pi \ell^+ \ell^-)/\text{BR}(B \to K \ell^+ \ell^-)$ in the Standard Model using the lattice form factors, compared with experimental measurements from LHCb(arXiv:1509.00414). The errors in the Standard-Model results are dominated by the form-factor uncertainties.

Lepton flavor violation in $B \to K(\pi)ll$ process

Lepton-flavor-violating effect in the $B \to K(\pi)ll$ process is defined as:

$$R^{\mu,e}(q_1^2, q_2^2) = \frac{\int_{q_1^2}^{q_2^2} dq^2 \, d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2}{\int_{q_1^2}^{q_2^2} dq^2 \, d\mathrm{BR}(B \to Ke^+e^-)/dq^2}, \tag{6}$$

- $R^{\mu,e}$ is close 1 in Standard Model for $B \to Kll$ and $B \to \pi ll$ processes.
- BaBar found $R_K^{\mu,e} = 1.00(^{+31}_{-25})(7)$ in the union of $[0.1, 8.12]\mathrm{GeV}^2$ and $[10.11, q^2_{\mathrm{max}}]\mathrm{GeV}^2$. (arXiv:1204.3933)
- Bell found $R_K^{\mu,e} = 1.03(^{+19}_{-6})$ in the full q^2 range. (arXiv:0904.0770)
- LHCb found $R_K^{\mu,e} = 0.745(^{+90}_{-74})(36)$ which is 2.6σ away from 1. (arXiv:1406.6482)
- New physics models and lepton flavor violation. (arXiv:1411.0565, arXiv:1508.07009, *etc.*)

Lepton favor violation in $B \to K(\pi)ll$ process

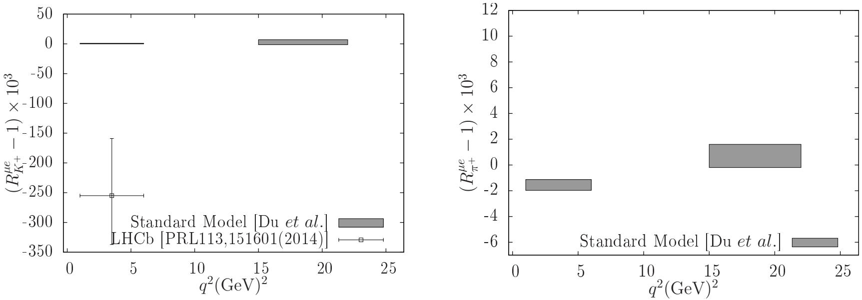
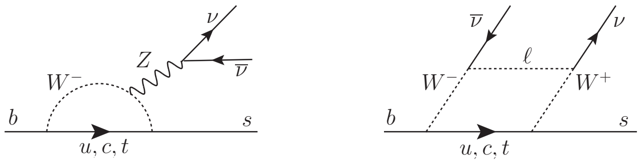


Figure: Standard-Model lepton-flavor-violating ratios $\left(R^{\mu e}_{K^+} - 1\right)$ (left) and $\left(R^{\mu e}_{\pi^+} - 1\right)$ (right) for $(q^2_{\mathrm{min}}, q^2_{\mathrm{max}}) = (1\mathrm{GeV}^2, 6\mathrm{GeV}^2)$ and $(15\mathrm{GeV}^2, 22\mathrm{GeV}^2)$ using the lattice form factors. Our result is consistent with HPQCD's (arXiv:1306.0434), but different from LHCb's experimental data.

$B \to K\nu\bar{\nu}$ and $B \to \pi\nu\bar{\nu}$

Feynman diagrams of the $b \to s\nu\bar{\nu}$ transition (from arXiv:1303.3719):



Experimental results:

- BaBar: $\mathrm{BR}(B^+ \to K^+\nu\bar{\nu}) < 3.2 \times 10^{-5}$ (90% CL) (arxiv:1303.7465)
- Belle: $\mathrm{BR}(B^+ \to K^+\nu\bar{\nu}) < 5.5 \times 10^{-5}$ (90% CL) (arxiv:1303.3719)
- Belle: $\mathrm{BR}(B^+ \to \pi^+\nu\bar{\nu}) < 9.8 \times 10^{-5}$ (90% CL) (arxiv:1303.3719)

Theoretical results:

- Wolfgang Altmannshofer *et al.* 0902.0160 (LCSR)
- Andrzej J. Buras *et al.* 1409.4557 (lattice, LCSR)
- Christian Hambrock *et al.* 1506.07760 (LCSR)

Theoretical studies of the $B \to K(\pi)\nu\bar{\nu}$

In the Standard Model, the decay rate for $B \to K(\pi)\nu\bar{\nu}$ is: (arXiv:1409.4557, arXiv:0902.0160)

$$\frac{dB(B \to K(\pi)\nu\bar{\nu})}{dq^2} = 3\tau_B |N_{K(\pi)}|^2 \frac{X_t^2}{(\sin^2\theta_W)^4} \rho_{K(\pi)}(q^2) , \qquad (7)$$

where the numerical coefficient $N_{K(\pi)}$ depends upon the relevant CKM factors and $\rho_{K(\pi)}$ is the rescaled hadronic form factor:

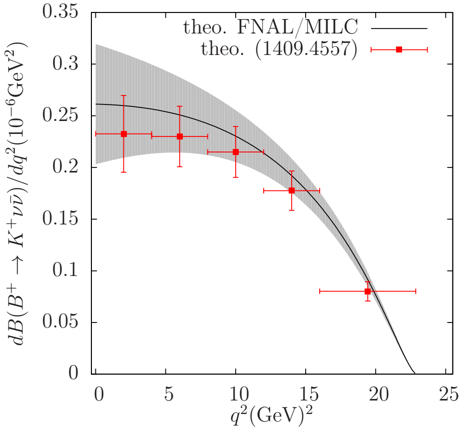
$$N_{K(\pi)} = V_{tb} V^*_{ts(d)} \frac{G_F \alpha_{EW}}{16\pi^2} \sqrt{\frac{M_B}{3\pi}} , \qquad (8)$$

$$\rho_{K(\pi)}(q^2) = \frac{\lambda^{3/2}(q^2)}{M_B^4} f_+^2(q^2) . \qquad (9)$$

- The form factor f_+ is the same as in the $B \to K(\pi) l^+ l^-$ lattice-QCD calculations.

Theoretical studies of the $B \to K\nu\bar{\nu}$

(Preliminary)



- Grey band: theoretical result from FNAL/MILC $B \to K$ form factor
- Red points: theoretical result from and lattice-QCD plus LCSR form factor results (arXiv:1409.4557).

Summary

- The form factors in the semileptonic B-meson decay processes can be computed by lattice-QCD accurately.
- The $B \to \pi l^+ l^-$ form factors and theoretical predictions are available. (1503.07839, 1507.01618)
- The $B \to K l^+ l^-$ form factors and other theoretical predictions will be finished soon.
- Theoretical predictions in the $B \to \pi l \nu$, $B \to K(\pi) l^+ l^-$, $B \to K(\pi) \nu \bar{\nu}$ process are shown and compared with the experiments. There are a few minor discrepancies. ($|V_{ub}|$, dB/dq^2, lepton flavor violation, etc.)
- New experimental results could be available soon. (arXiv:1509.00414)
- More studies on the B-meson semileptonic decay form factors on the HISQ ensembles are under investigation.

Backup Slides

Standard Model predictions

Theoretical prediction of dB/dq^2 in high q^2 region:

$$\frac{dB}{dq^2} = \frac{G_F^2\alpha^2|V_{tb}V_{td}^*|^2}{2^7\pi^5}|\boldsymbol{k}|\beta_+ \left\{ \frac{2}{3}|\boldsymbol{k}|^2\beta_+^2 \left| C_{10}^{\text{eff}} f_+(q^2)\right|^2 + \frac{m_l^2(M_B^2-M_K^2)^2}{q^2M_B^2}\left|C_{10}^{\text{eff}} f_0(q^2)\right|^2 + |\boldsymbol{k}|^2\left[1-\frac{1}{3}\beta_+^2\right]\left|C_9^{\text{eff}} f_+(q^2) + 2C_7^{\text{eff}}\frac{m_b}{M_B+M_K}f_T(q^2)\right|^2 \right\}, \quad (10)$$

where G_F, α, and V_{tq} are the Fermi constant, the (QED) fine structure constant, and CKM matrix elements, respectively, $|\boldsymbol{k}| = \sqrt{E_K^2 - M_K^2}$ is the kaon momentum in the B-meson rest frame, and $\beta_+^2 = 1 - 4m_l^2/q^2$, with m_l the lepton mass.