

Precision Calculations for the LHC

LHC Olympics 2006

Zvi Bern, UCLA

with Carola Berger, Lance Dixon, Darren Forde
and David Kosower

[hep-ph/0501240](#)

[hep-ph/0505055](#)

[hep-ph/0507005](#)

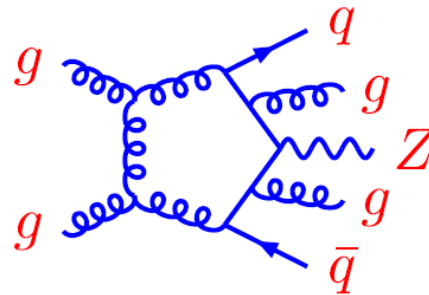
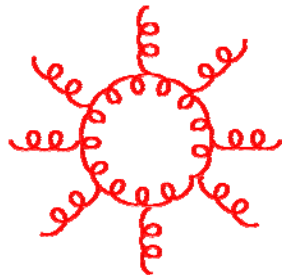
[hep-ph/0604195](#)

[hep-ph/0607014](#)

Outline

- The need for NLO calculations at the LHC
- The experimenters' wish list.
- The trouble.
- On-shell methods. Rewrite of QFT.
- Example of state-of-the art calculations

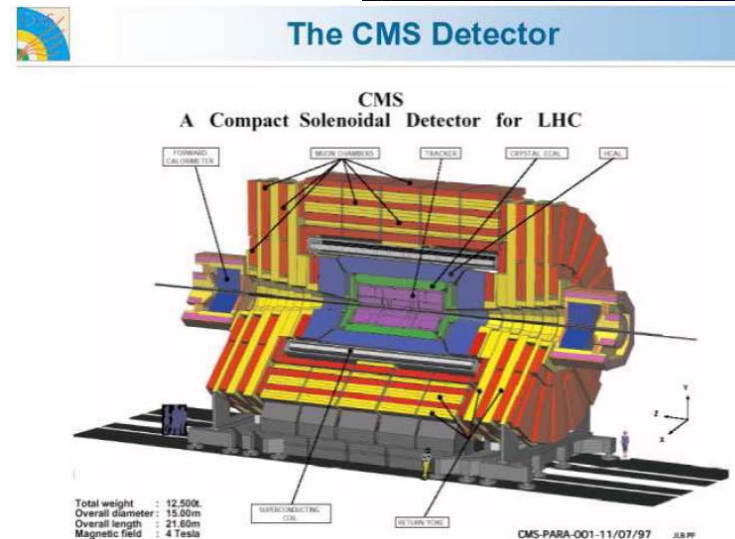
- Blackboard lecture, working through a simple example of on-shell recursion.



LHC Physics

The LHC will start operations in 2007.

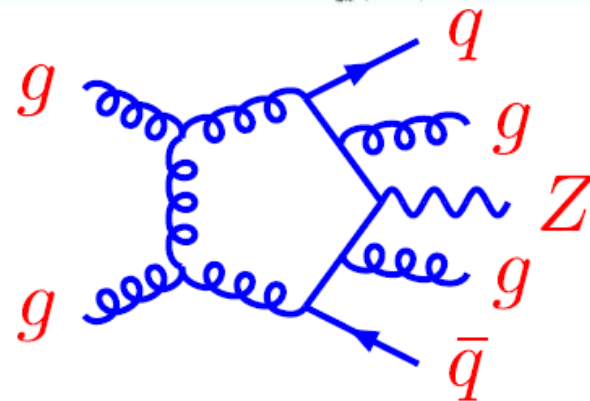
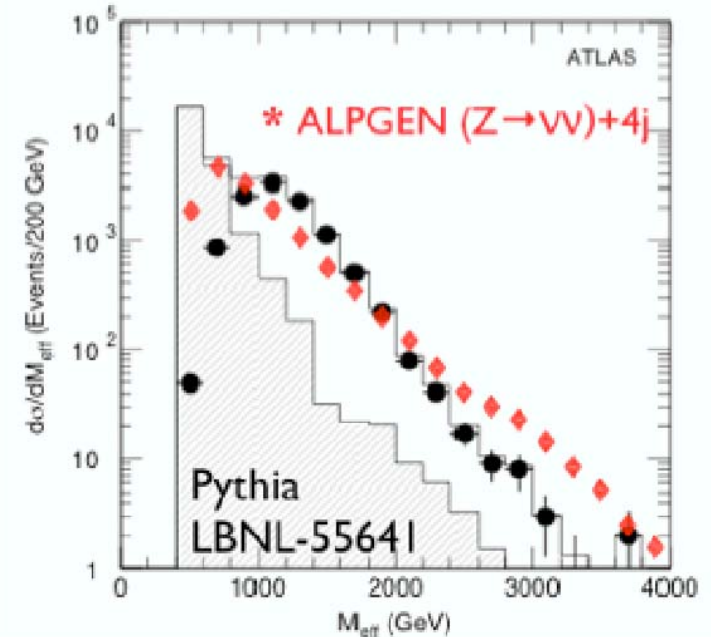
We will have lots of multi-particle processes. Want reliable predictions.



Example: Susy Search

Early ATLAS TDR studies using PYTHIA overly optimistic.

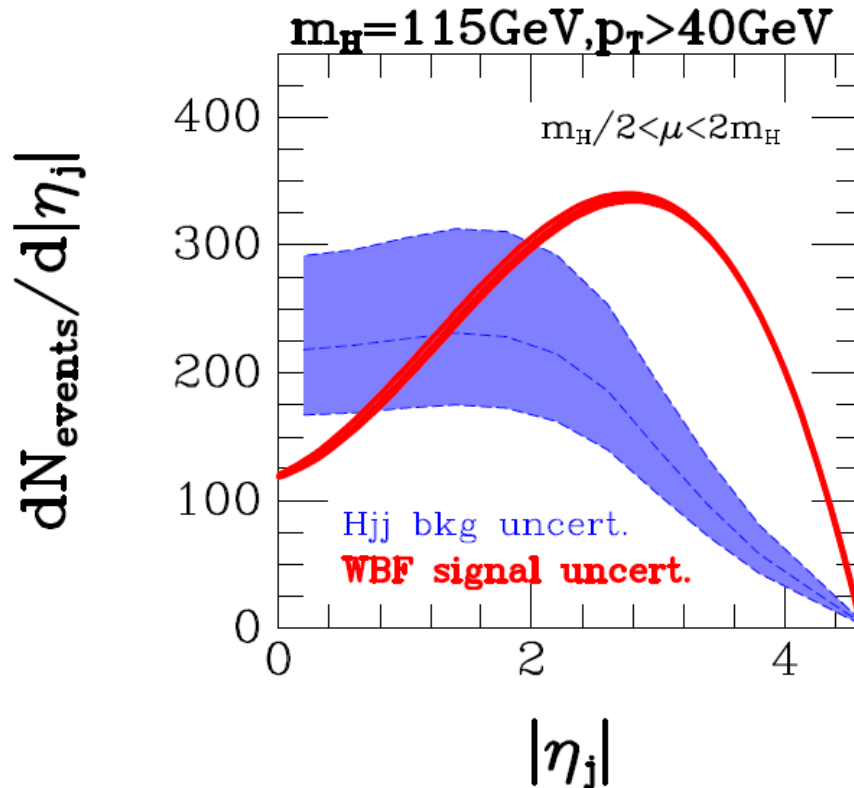
- ALPGEN is based on LO matrix elements and much better at modeling hard jets.
- What will disagreement between ALPGEN and data mean? Hard to tell. **Need NLO.**



We need $pp \rightarrow Z + 4 \text{ jets}$ at NLO

Example: Higgs + 2 jets from Weak Boson Fusion

Figy, Oleari and Zeppenfeld
Berger and Campbell
Campbell, Ellis and Zanderighi



From
Berger and Campbell
hep-ph/0403194
MCFM used

Purpose: After discovery of Higgs Boson measure HWW coupling

Background uncertainty is reduced with an NLO calculation.

LHC Experimenters' NLO Wishlist

process ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \rightarrow V V \text{ jet}$	$t\bar{t}H$, new physics
2. $pp \rightarrow H + 2 \text{ jets}$	H production by vector boson fusion (VBF)
3. $pp \rightarrow t\bar{t} b\bar{b}$	$t\bar{t}H$
4. $pp \rightarrow t\bar{t} + 2 \text{ jets}$	$t\bar{t}H$
5. $pp \rightarrow V V b\bar{b}$	VBF $\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics
6. $pp \rightarrow V V + 2 \text{ jets}$	VBF $\rightarrow H \rightarrow VV$
7. $pp \rightarrow V + 3 \text{ jets}$	various new physics signatures
8. $pp \rightarrow V V V$	SUSY tripleton

Les Houches 2005

Large number of high multiplicity processes that we need to compute.

- Numerical approaches. Recent progress.

Binoth and Heinrich Kaur; Giele, Glover, Zanderighi; Binoth, Guillet, Heinrich, Pilon, Schubert; Soper and Nagy; Ellis, Giele and Zanderighi; Anastasiou and Daleo; Czakon; Binoth, Heinrich and Ciccolini

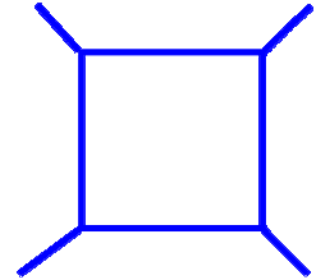
- In this talk analytic on-shell methods: spinors, twistors, unitarity method, on-shell bootstrap approach.

Bern, Dixon, Dunbar, Kosower; Bern and Morgan; Cachazo, Svrcek and Witten; Bern, Dixon, Kosower; Bedford, Brandhuber, Spence, Travaglini; Britto, Cachazo, Feng and Witten; Berger, Bern, Dixon, Kosower, Forde; Xiao, Yang and Zhu

Example of difficulty

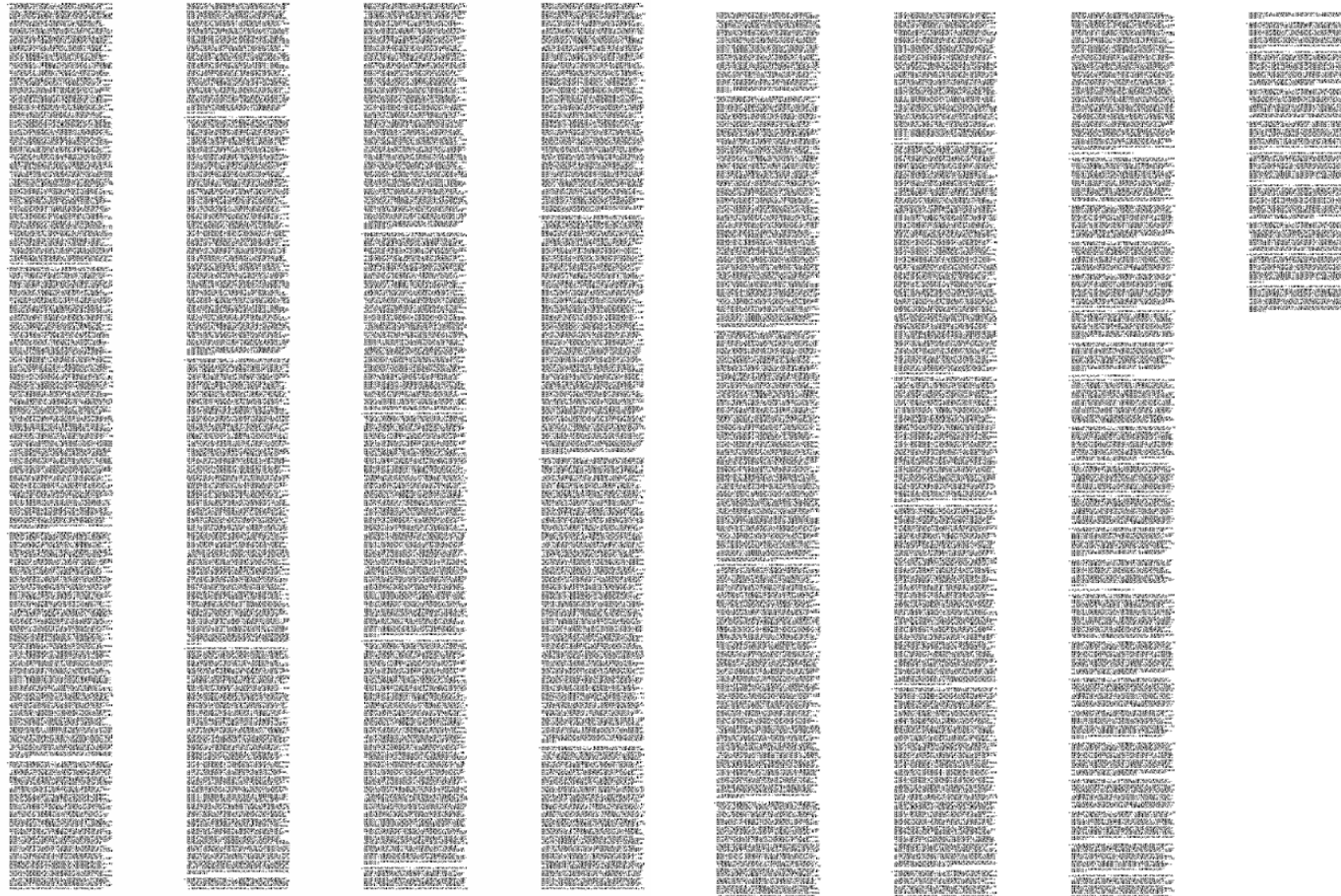
Consider a tensor integral:

$$\int \frac{d^{4-2\epsilon} \ell}{(2\pi)^{4-\epsilon}} \frac{\ell^\mu \ell^\nu \ell^\rho \ell^\lambda}{\ell^2 (\ell - k_1)^2 (\ell - k_1 - k_2)^2 (\ell + k_4)^2}$$



Evaluate this integral via Passarino-Veltman reduction. Result is ...

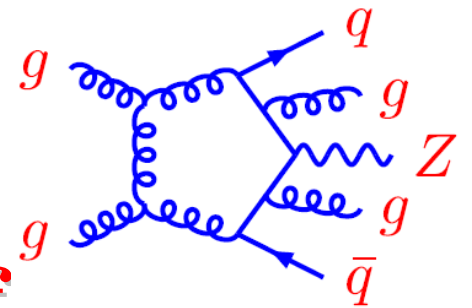
Result of performing the integration



**Numerical stability is a key issue.
Clearly, there should be a better way**

What we need

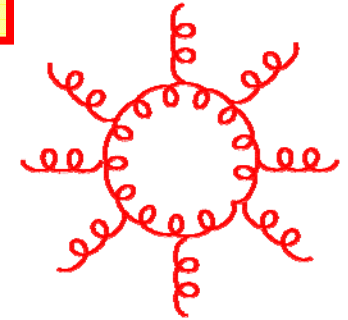
- Numerical stability.
- Scalable to large numbers of external partons.
- A general solution that applies to any process.
- Can be automated.



What we're dreaming of

- A technique where computations undergo modest growth in complexity with increasing number of legs.
- Compact analytic expressions “that fit on a page”.

Progress Towards the Dream



Results with on-shell methods:

- Many QCD amplitudes with $n > 5$ legs.
Berger, Bern, Dixon, Forde and Kosower
- Certain log contributions via on-shell recursion.
Bern; Bjerrum-Bohr; Dunbar, Ita
- Improved ways to obtain logarithmic contributions via unitarity method.
Britto, Cachazo, Feng; Britto, Feng and Mastrolia

Key Feature: Modest growth in complexity as n increases. No unwanted Gram dets.

Spinors and Twistors

Witten

Spinor helicity for gluon polarizations in QCD:

$$\epsilon_{\mu}^{+}(k; q) = \frac{\langle q^{-} | \gamma_{\mu} | k^{-} \rangle}{\sqrt{2} \langle q k \rangle}, \quad \epsilon_{\mu}^{-}(k, q) = \frac{\langle q^{+} | \gamma_{\mu} | k^{+} \rangle}{\sqrt{2} [k q]}$$

$$\epsilon^{ab} \lambda_{ja} \lambda_{lb} \longleftrightarrow \langle j l \rangle = \langle k_{j-} | k_{l+} \rangle = \sqrt{2k_j \cdot k_l} e^{i\phi} = \frac{1}{2} \bar{u}(k_j) (1 + \gamma_5) u(k_l)$$

$$\epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_j^{\dot{a}} \tilde{\lambda}_l^{\dot{b}} \longleftrightarrow [j l] = \langle k_{j+} | k_{l-} \rangle = -\sqrt{2k_j \cdot k_l} e^{-i\phi} = \frac{1}{2} \bar{u}(k_j) (1 - \gamma_5) u(k_l)$$

Penrose Twistor Transform:

$$\tilde{A}(\lambda_i, \mu_i) = \int \prod_i \frac{d^2 \tilde{\lambda}_i}{(2\pi)^2} \exp\left(\sum_j \mu_j^{\dot{a}} \tilde{\lambda}_{j\dot{a}}\right) A(\lambda_i, \tilde{\lambda}_i)$$

Early work from Nair

Witten's remarkable twistor-space link:

Witten; Roiban, Spradlin and Volovich

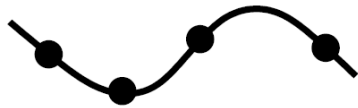
QCD scattering amplitudes \longleftrightarrow Topological String Theory

Key implication: Scattering amplitudes have a much much simpler structure than anyone would have believed.

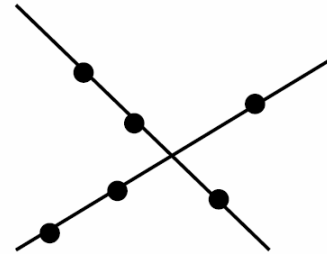
Amazing Simplicity

Witten conjectured that in twistor –space gauge theory amplitudes have delta-function support on curves of degree:

$$d = q - 1 + L, \quad q = \# \text{ negative helicities}, \quad L = \# \text{ loops},$$



Connected picture

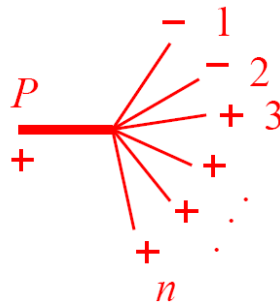


Disconnected picture

Structures imply an amazing simplicity in the scattering amplitudes.

MHV vertices for building amplitudes

Cachazo, Svrcek and Witten



Witten

Roiban, Spradlin and Volovich

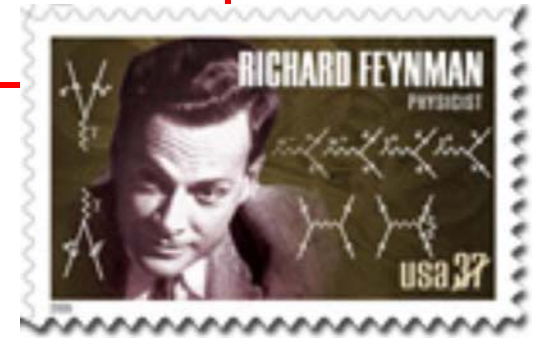
Cachazo, Svrcek and Witten

Gukov, Motl and Neitzke

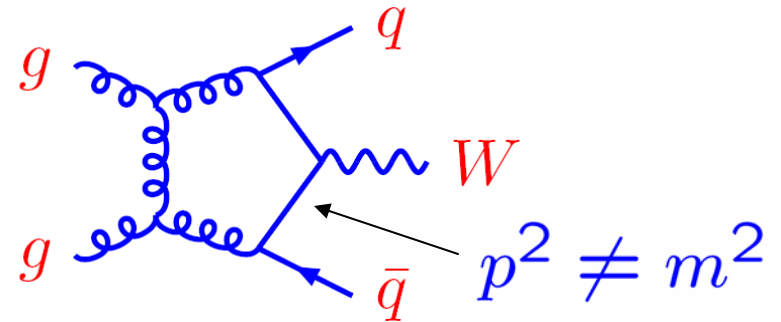
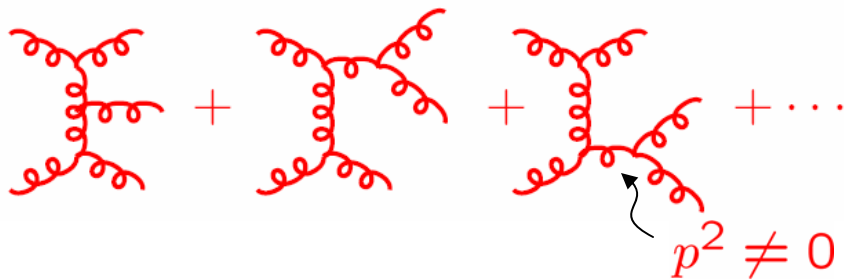
Bena, Bern and Kosower

Much of the simplicity survives addition of masses or loops

Why are Feynman diagrams clumsy for high-multiplicity processes?

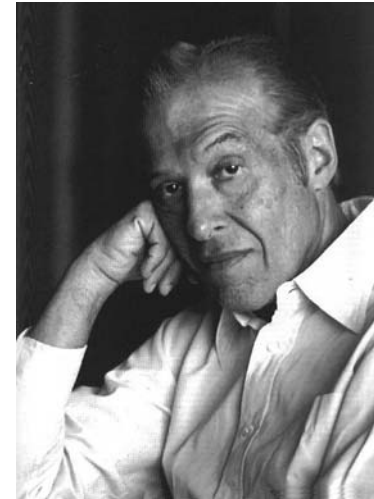
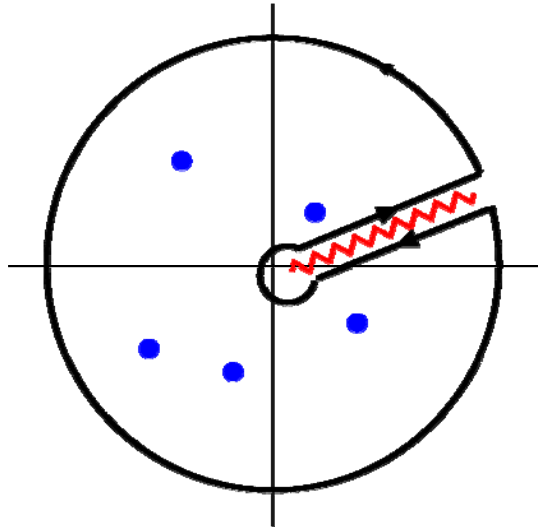


- Vertices and propagators involve gauge-dependent off-shell states. Origin of the complexity.



- To get at root cause of the trouble we must rewrite perturbative quantum field theory.

- **All steps should be in terms of gauge invariant on-shell states. $p^2 = m^2$**
- **Radical rewriting of perturbative QCD needed.**



“One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane.”

J. Schwinger in “Particles, Sources and Fields” Vol 1

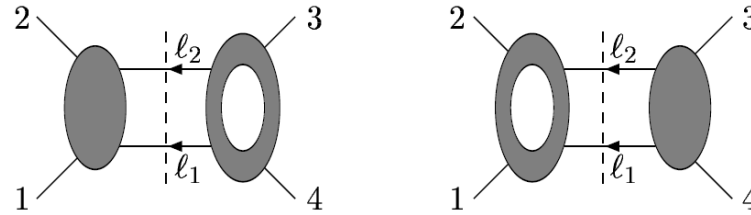
On-shell Formalisms

With on-shell formalisms we can exploit analytic properties.

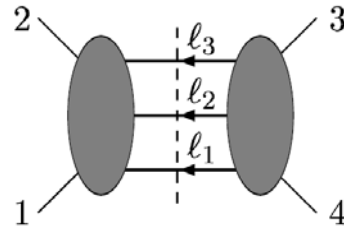
- **Curiously, a practical on-shell formalism was constructed at loop level prior to tree level: unitarity method.** (1994)
Bern, Dixon, Dunbar, Kosower
- **Solution at tree-level had to await Witten's twistor inspiration.** (2004)
 - **MHV vertices** Cachazo, Svrcek Witten; Brandhuber, Spence, Travaglini
 - **On-shell recursion** Britto, Cachazo, Feng, Witten
- **Combining unitarity method with on-shell recursion gives loop-level on-shell bootstrap.** (2006)
Berger, Bern, Dixon, Forde, Kosower

Unitarity Method

Two-particle cut:



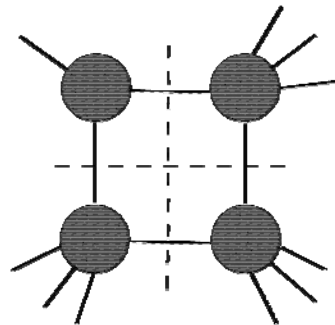
Three- particle cut:



$$2 \operatorname{Im} \left[\text{Diagram of a box integral with a vertical dashed green line} \right] = \int_{d\text{LIPS}} \left[\text{Diagram of two tree-level diagrams with red arrows pointing to internal lines labeled 'on-shell'} \right]$$

Generalized unitarity:

Bern, Dixon and Kosower



As observed by Britto, Cachazo and Feng quadruple cut freezes integral:

Coefficients of box integrals always easy.

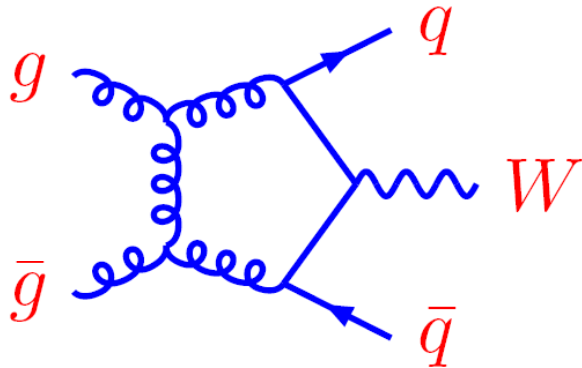
Generalized cut interpreted as cut propagators not canceling.

A number of recent improvements to method

Britto, Buchbinder, Cachazo and Feng; Berger, Bern, Dixon, Forde and Kosower; Britto, Feng and Mastrolia; Xiao, Yang and Zhu

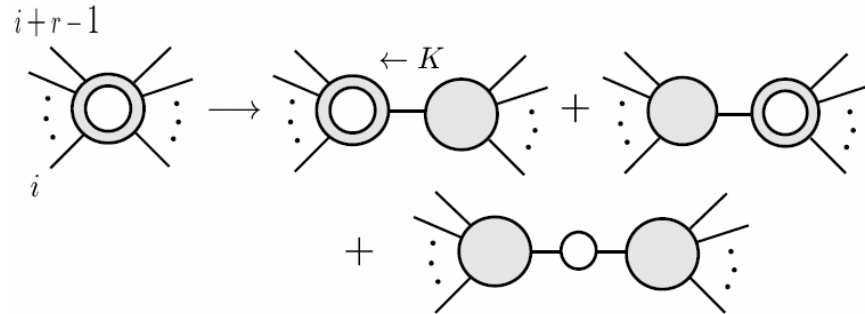
Early On-Shell Bootstrap

Bern, Dixon, Kosower
hep-ph/9708239



$$e^+e^- \rightarrow Z \rightarrow 4 \text{ partons}$$

$$pp \rightarrow W, Z + 2 \text{ partons}$$



Early Approach:

- Use Unitarity Method with $D = 4$ helicity states. Efficient means for obtaining logs and polylogs.
- Use **factorization properties** to find rational function contributions.

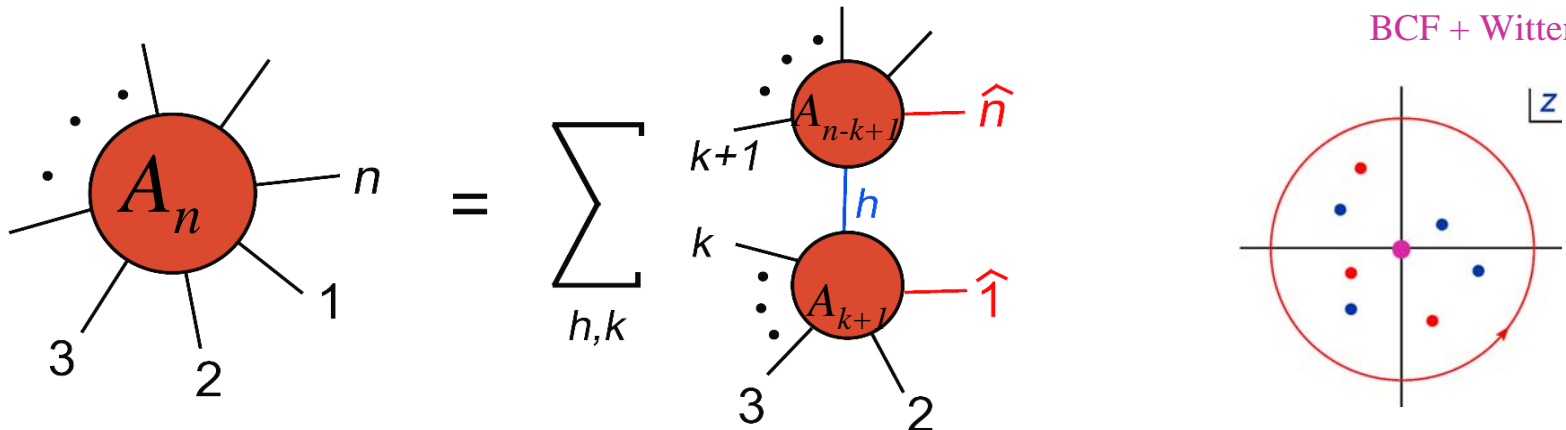
Key problems preventing widespread applications:

- Difficult to find rational functions with desired factorization properties.
- Systematization unclear – key problem.

Tree-Level On-Shell Recursion

New representations of tree amplitudes from IR consistency of one-loop amplitudes in $N = 4$ super-Yang-Mills theory. Bern, Del Duca, Dixon, Kosower; Roiban, Spradlin, Volovich

Using intuition from twistors and generalized unitarity: Britto, Cachazo, Feng BCF + Witten



$$p_1^\mu(z) = p_1^\mu - \frac{z}{2} \langle 1^- | \gamma^\mu | n^- \rangle$$

$$p_n^\mu(z) = p_n^\mu + \frac{z}{2} \langle 1^- | \gamma^\mu | n^- \rangle$$

On-shell conditions maintained by shift.

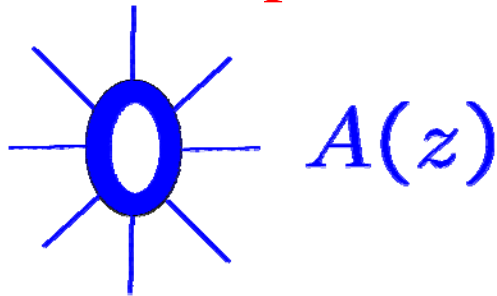
Proof relies on so little. Power comes from generality

- Cauchy's theorem
- Basic field theory factorization properties

Construction of Loop Amplitudes

Berger, Bern, Dixon, Forde, Kosower

Shifted amplitude function of a complex parameter



$$p_1^\mu(z) = p_1^\mu - \frac{z}{2} \langle 1^- | \gamma^\mu | 2^- \rangle$$

$$p_2^\mu(z) = p_2^\mu + \frac{z}{2} \langle 1^- | \gamma^\mu | 2^- \rangle$$

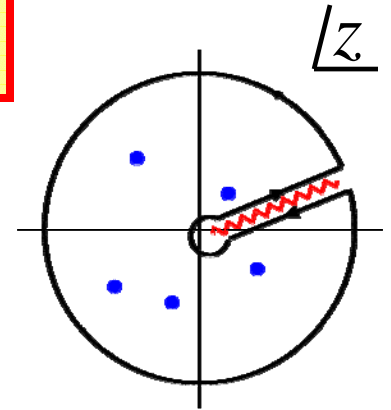
Shift maintains on-shellness and momentum conservation

$$A(z) = \sum \text{polylog terms} \leftarrow \text{Use unitarity method (in special cases on-shell recursion)}$$

$$+ \sum_i \frac{\text{Res}_i}{(z - z_i)} \leftarrow \text{Use on-shell recursion}$$

$$+ \sum_i a_i z^i \leftarrow \text{Use auxiliary on-shell recursion in another variable}$$

Loop-Level Recursion



New Features:

- Presence of branch cuts.
- Unreal poles – new poles appear with complex momenta.

$$\frac{[ab]}{\langle ab \rangle}$$

Pure phase for real momenta

- Double poles. $\frac{[ab]}{\langle ab \rangle^2}$
- Spurious singularities that cancel only against polylogs.
- Double count between cut and recursive contributions.

On shell bootstrap deals with these features.

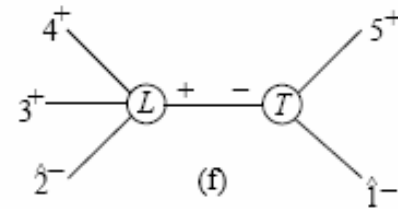
One-Loop Five-Point Example

The most challenging part was rational function terms – at the end of chain of integral reductions.

Assume we already have log terms computed from $D = 4$ cuts.

Only one non-vanishing recursive diagram: $- - + + +$

$$D_5 = -\frac{1}{3} \frac{[24][35]^3}{\langle 34 \rangle [12][15][23]^2}.$$

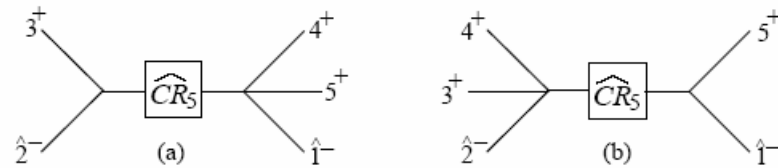


Tree-like calculations

Only two double-count diagrams:

$$O_5^{(a)} = \frac{1}{6} \frac{\langle 12 \rangle^2 \langle 14 \rangle [34]}{\langle 15 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle [23]}$$

$$O_5^{(b)} = \frac{1}{6} \frac{\langle 14 \rangle [34][35] (\langle 14 \rangle [34] - \langle 15 \rangle [35])}{\langle 15 \rangle \langle 34 \rangle \langle 45 \rangle [15][23]^2}$$



These are computed by taking residues

Rational function terms obtained from tree-like calculation!

No integral reductions. No unwanted “Grim” dets. 21

Six-Point Example

$$\hat{R}_6^{\text{scalar}}(1^-, 2^-, 3^+, 4^+, 5^+, 6^+)$$

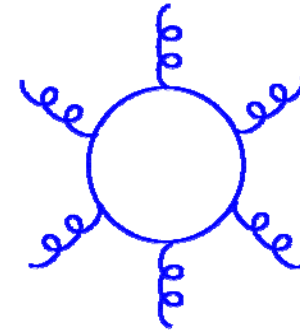
$$\begin{aligned} \hat{R}_6 = \frac{1}{6} & \left\{ -2 \frac{\langle 35 \rangle [35] \langle 4^- | (1+2) | 3^- \rangle \langle 4^- | (1+2) | 6^- \rangle \langle 5^- | (1+2) | 6^- \rangle}{[12] \langle 34 \rangle^2 \langle 45 \rangle^2 [61] \langle 5^- | (3+4) | 2^- \rangle \langle 6^- | (1+2) | 3^- \rangle} \right. \\ & - 2 \frac{\langle 35 \rangle [36] \langle 4^- | (1+2) | 6^- \rangle^2}{[12] \langle 34 \rangle^2 \langle 45 \rangle^2 [61] \langle 5^- | (3+4) | 2^- \rangle} \\ & + 2 \frac{\langle 12 \rangle \langle 24 \rangle \langle 35 \rangle [35]^2 [56] \langle 5^- | (1+2) | 6^- \rangle}{\langle 34 \rangle^2 \langle 45 \rangle [61] \langle 2^- | (1+6) | 5^- \rangle \langle 5^- | (3+4) | 2^- \rangle \langle 6^- | (1+2) | 3^- \rangle} \\ & + 2 \frac{\langle 12 \rangle^2 [35]^2 (\langle 5^- | (3+4) | 2 | 1^+ \rangle + \langle 5^- | 35 | 1^+ \rangle)}{\langle 34 \rangle \langle 45 \rangle \langle 61 \rangle \langle 2^- | (1+6) | 5^- \rangle \langle 5^- | (3+4) | 2^- \rangle \langle 6^- | (1+2) | 3^- \rangle} \\ & - \frac{\langle 12 \rangle^3 \langle 35 \rangle [46] [56]}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 1^- | (2+3) | 4^- \rangle \langle 3^- | (1+2) | 6^- \rangle} + 2 \frac{[36]^3}{[12] [23] \langle 45 \rangle^2 [61]} \\ & - \frac{[56] \langle 5^- | (1+2) | 6^- \rangle^2 (2 \langle 4^- | (3+5) (1+2) | 5^+ \rangle + \langle 12 \rangle [12] \langle 45 \rangle)}{[12] \langle 34 \rangle \langle 45 \rangle^2 \langle 56 \rangle [61] \langle 3^- | (1+2) | 6^- \rangle \langle 5^- | (3+4) | 2^- \rangle} \\ & + 2 \frac{\langle 15 \rangle^2 [34]^2 [56] (\langle 16 \rangle [34] \langle 45 \rangle - \langle 1^- | (2+4) | 3^- \rangle \langle 56 \rangle)}{[23] \langle 45 \rangle \langle 56 \rangle^2 s_{234} \langle 1^- | (2+3) | 4^- \rangle \langle 5^- | (3+4) | 2^- \rangle} \\ & - \frac{\langle 12 \rangle \langle 15 \rangle [34] [56] \langle 1^- | (5+6) (3+4) | 5^+ \rangle}{\langle 34 \rangle \langle 45 \rangle \langle 56 \rangle s_{234} \langle 1^- | (2+3) | 4^- \rangle \langle 5^- | (3+4) | 2^- \rangle} \\ & + 2 \frac{\langle 35 \rangle \langle 1^- | (2+4) | 3^- \rangle^3}{[23] \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle s_{234} \langle 5^- | (3+4) | 2^- \rangle} \\ & - \frac{\langle 12 \rangle \langle 1^- | (2+4) | 3^- \rangle (2 \langle 1^- | (2+4) | 3^- \rangle + \langle 1^- | 4 | 3^- \rangle)}{[23] \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle s_{234}} \\ & + 2 \frac{\langle 12 \rangle^3 [46]^2 \langle 5^- | (4+6) | 5^- \rangle}{\langle 23 \rangle \langle 45 \rangle \langle 56 \rangle s_{123} \langle 1^- | (2+3) | 4^- \rangle \langle 3^- | (1+2) | 6^- \rangle} \\ & + 2 \frac{\langle 12 \rangle^3 [35]^2 \langle 4^- | (3+5) | 4^- \rangle}{\langle 34 \rangle \langle 45 \rangle \langle 61 \rangle s_{612} \langle 2^- | (1+6) | 5^- \rangle \langle 6^- | (1+2) | 3^- \rangle} \\ & \left. - \frac{\langle 12 \rangle^2}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle} \left[\frac{\langle 1^- | 4 | 3^- \rangle}{[23]} + \frac{\langle 2^- | 5 | 6^- \rangle}{[61]} \right] \right\} \end{aligned}$$

$$\langle a^- | (b+c) | d^- \rangle \equiv \langle ab \rangle [bd] + \langle ac \rangle [cd]$$

$$A_{n;1}^{\text{fermion}} = A_{n;1}^{\mathcal{N}=1} - A_{n;1}^{\text{scalar}}$$

$$A_{n;1}^{\text{gluon}} = A_{n;1}^{\mathcal{N}=4} - 4A_{n;1}^{\mathcal{N}=1} + A_{n;1}^{\text{scalar}}$$

Rational function parts
of scalar loops were by far
most difficult to calculate.



Using on-shell bootstrap
rational parts are given
by tree-like calculations.
No integral reductions.

Numerical results for n gluons

Choose specific points in phase-space – see [hep-ph/0604195](https://arxiv.org/abs/hep-ph/0604195)

Scalar loop contributions

6 points

Helicity	1/ε	ε ⁰
++++++	0	0.1024706290 + i 0.5198025397
--++++	0	2.749806130 + i 1.750985849
---+++	- 9.370119558 + i 1.547789294	- 45.80779561 + i 13.03695870
----++	- 0.2614534328 - i 0.6288641470	0.3883482043 - i 5.830791857

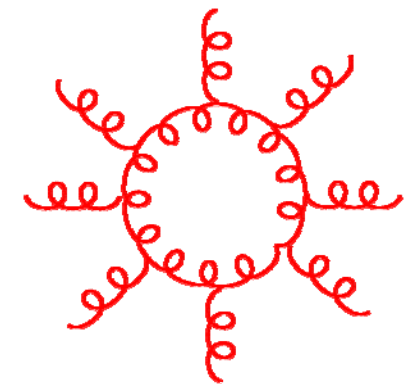
7 points

+++++++	0	0.1815778027 + i 1.941357266
-+++++	0	22.52927821 + i 5.464377788
--++++	- 34.85372799 + i 15.11569825	- 176.2169235 + i 87.93931019
---+++	0.3564513374 - i 0.4914226070	0.7087164424 - i 11.32916632

8 points

+++++++	0	- 0.0009856214410 + i 0.002143695508
-+++++	0	0.001078316199 + i 0.03129931739
---++++	- 0.05330088846 - i 0.04051789981	0.05513350697 + i 0.1659518861
----+++	- 0.003622640270 - i 0.0007910999246	0.02719752089 - i 0.02586206549
-----	- 0.002273559586 - i 0.001209645382	0.01154855076 - i 0.0008935357840

amusing count



+ 3,017,489
other diagrams

Modest growth in complexity as number of legs increases

At 6 points these agree with numerical results of Ellis, Giele and Zanderighi

What should be done?

- **Attack items on experimenters' wishlist.**
- **Automation for general processes.**
- **Assembly of full cross-sections, e.g., Catani-Seymour formalism.**
- **Massive loops -- tree recursion understood.**
Badger, Glover, Khoze, Svrcek
- **First principles derivation of formalism:**
Large z behavior of loop amplitudes.
General understanding of unreal poles.
- **Connection to Lagrangian**
Vaman and Yao; Draggiotis, Kleiss, Lazopolous, Papadopoulos

Other Applications

On-shell methods applied in a variety of problems:

- **Computations of two-loop 2 to 2 QCD amplitudes.**

Bern, Dixon, Kosower
Bern, De Freitas, Dixon

- **Ansatz for planar MHV amplitudes to *all* loop orders in $N = 4$ super-Yang-Mills.**

Anastasiou, Bern, Dixon, Kosower
Bern, Dixon, Smirnov
Cachazo, Spradlin and Volovich

- **Applications to gravity.**

Bern, Dixon, Dunbar, Perelstein and Rozowsky
Bern, Bjerrum-Bohr and Dunbar
Brandhuber, Spence and Travaglini
Cachazo and Svrcek

Summary

- We need bold action to provide the full range of NLO calculations for the LHC.
- On-shell bootstrap – unitarity and factorization.
- New results for one-loop $n \geq 6$ gluons.
- Explicit numerical results for up to eight gluons.
- Technology should apply as is to external mass cases.
- Important issues remain: automation, massive loops, construction of physical cross sections with experimental cuts.

Experimenters' wish list awaits us!

Experimenter's Wish List

Les Houches 2005

process ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \rightarrow V V \text{ jet}$	$t\bar{t}H$, new physics
2. $pp \rightarrow H + 2 \text{ jets}$	H production by vector boson fusion (VBF)
3. $pp \rightarrow t\bar{t} b\bar{b}$	$t\bar{t}H$
4. $pp \rightarrow t\bar{t} + 2 \text{ jets}$	$t\bar{t}H$
5. $pp \rightarrow V V b\bar{b}$	VBF $\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics
6. $pp \rightarrow V V + 2 \text{ jets}$	VBF $\rightarrow H \rightarrow VV$
7. $pp \rightarrow V + 3 \text{ jets}$	various new physics signatures
8. $pp \rightarrow V V V$	SUSY tripleton

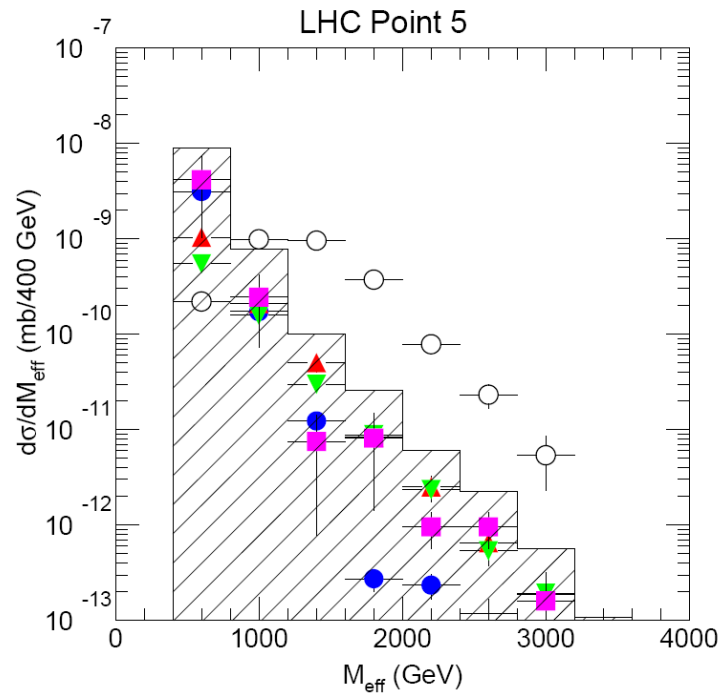
Complete List

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

Extra Transparencies

Incorrect Plot from ATLAS TDR



Background is
actually 10 times
larger.

figure 20-4 M_{eff} distribution for the Point 5 signal (open circles) and for the sum of all Standard Model backgrounds (histogram); the latter includes $t\bar{t}$ (solid circles), W + jets (triangles), Z + jets (downward triangles), and QCD jets (squares).