

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

**Status of the muon g-2
and the
theoretical uncertainty of the hadronic contributions**

F. JEGERLEHNER, DESY Zeuthen
supported by EU network EURIDICE



LoopFest 2004 , April 1 - 3, 2004, KITP, Santa Barbara CA (USA)

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

Outline of Talk:

- ① **Lepton magnetic moments: basics**
- ② **The Anomalous Magnetic Moment of the Muon**
- ③ **Standard Model Prediction for a_μ**
- ④ **Evaluation of a_μ^{had}**
- ⑤ **e^+e^- –Cross Sections via τ –Decay Spectral Functions**
- ⑥ **Outlook**

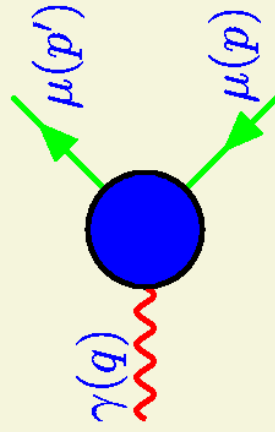
Abstract: The muon anomalous magnetic moment is one of the most precisely measured quantities in particle physics. New high precision measurements at Brookhaven indicate a 2.6 sigma deviation from the electroweak Standard Model and thus might signal a contribution from yet unknown new physics. I review the status of the theoretical predictions and in particular discuss the role of hadronic vacuum polarization effects and the light-by-light scattering contribution.

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

① Lepton magnetic moments: basics

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{s} ; \quad g_\mu = 2(1 + a_\mu)$$

Dirac: $g_\mu = 2$, a_μ muon anomaly



$$= (-ie) \bar{u}(p') \left[\gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2(q^2) \right] u(p)$$

$$F_1(0) = 1 ; \quad F_2(0) = a_\mu$$

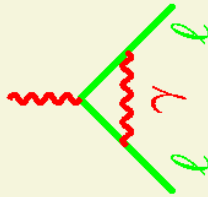
a_μ responsible for the Larmor precession
directly proportional at magic energy ~ 3.1 GeV

CERN, BNL g-2 experiments

$$\vec{\omega}_a = \frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] \stackrel{E \sim 3.1 \text{ GeV}}{\simeq} \frac{e}{m} \left[a_\mu \vec{B} \right] \text{ at "magic } \gamma \text{"}$$

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

a_ℓ : dimensionless (just a number), must vanish at tree level in any renormalizable theory \rightarrow finite prediction, testing helicity flip, most precisely measured electroweak observables. Lowest order QED/SM prediction: a_ℓ universal



$$a_e = a_\mu = a_\tau = \frac{\alpha}{2\pi} \quad \text{Schwinger 48}$$

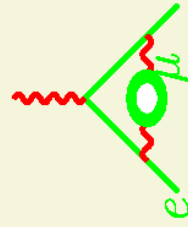
a_e : Presently most precise determination of $\alpha = e^2/4\pi$

a_μ : Enhanced sensitivity to New Physics, severe constraints to physics beyond the SM

$$a_\ell \sim \left(\frac{m_\ell}{m_{\text{NP}}} \right)^2 \Rightarrow \left(\frac{m_\mu}{m_e} \right)^2$$

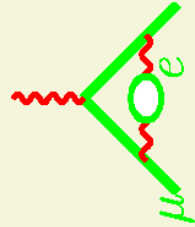
$$a_\mu \sim 40000 \text{ times more sensitive than } a_e$$

Even so a_μ is $143 \times$ less precise than a_e its still $280 \times$ more sensitive to NP
Mass effects: — HEAVY internal masses decouples



$$= \left[\frac{1}{45} \left(\frac{m_e}{m_\mu} \right)^2 + O \left(\frac{m_e^4}{m_\mu^4} \ln \frac{m_\mu}{m_e} \right) \right] \left(\frac{\alpha}{\pi} \right)^2$$

— LIGHT internal masses give rise to log's of mass ratios



$$= \left[\frac{1}{3} \ln \frac{m_\mu}{m_e} - \frac{25}{36} + O \left(\frac{m_e}{m_\mu} \right) \right] \left(\frac{\alpha}{\pi} \right)^2$$

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

Universal contributions

- 2-loop diagrams [7] with common fermion lines

$$a_\ell^{(4)} = \left[\frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2} \ln 2 + \frac{3}{4} \zeta(3) \right] \left(\frac{\alpha}{\pi} \right)^2$$

Peterman 57, Sommerfield 57

- 3-loop diagrams [72] with common fermion lines

$$a_\ell^{(6)} = \left[\frac{282259}{5184} + \frac{17101}{810} \pi^2 - \frac{298}{9} \pi^2 \ln 2 + \frac{139}{18} \zeta(3) + \frac{100}{3} \left\{ \text{Li}_4\left(\frac{1}{2}\right) + \frac{1}{24} \ln^4 2 - \frac{1}{24} \pi^2 \ln^2 2 \right\} - \frac{239}{2160} \pi^4 + \frac{83}{72} \pi^2 \zeta(3) - \frac{215}{24} \zeta(5) \right] \left(\frac{\alpha}{\pi} \right)^3$$

Laporta & Remiddi 96

- 4-loop diagrams [891] with common fermion lines numerically
Kinoshita 99, Kinoshita & Nio 02

$$a_\ell^{\text{universal}} = 0.5 \left(\frac{\alpha}{\pi} \right) - 0.3284789655 \dots \left(\frac{\alpha}{\pi} \right)^2 + 1.18124145 \dots \left(\frac{\alpha}{\pi} \right)^3 - \boxed{1.7502(384)} \left(\frac{\alpha}{\pi} \right)^4$$

Kinoshita & Nio 02 coefficient of $\left(\frac{\alpha}{\pi} \right)^4$ changed by -0.24

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

Corrections due to internal μ - and τ -loops

$$a_e = a_e^{\text{uni}} + a_e(\mu) + a_e(\tau)$$

$$a_e(\mu) = \underbrace{5.197 \times 10^{-7}}_{\mu \text{ VP}} \left(\frac{\alpha}{\pi}\right)^2 + \underbrace{(-2.1768 \times 10^{-5})}_{\mu \text{ ho VP}} + \underbrace{1.4394 \times 10^{-5}}_{\mu \text{ LBL}} \times \left(\frac{\alpha}{\pi}\right)^3$$

$$a_e(\tau) = \underbrace{1.838 \times 10^{-9}}_{\tau \text{ VP}} \left(\frac{\alpha}{\pi}\right)^2$$

Hadronic corrections: $a_e^{\text{had}} = 1.67(3) \times 10^{-12}$

Weak corrections: $a_e^{\text{weak}} = 0.03 \times 10^{-12}$

$\Rightarrow a_e$ is excellent observable for extracting α_{QED} !

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

$$\text{Most precise } \alpha_{\text{em}} \text{ from } a_e \equiv (g_e - 2)/2$$

Compare extraordinary precise measurements of electron anomalous magnetic moment

$$a_e \equiv (g_e - 2)/2$$

$$a_{e^-}^{\text{exp}} = 0.001\,159\,652\,188\,4(43), \quad a_{e^+}^{\text{exp}} = 0.001\,159\,652\,187\,9(43)$$

Dehmelt et al. 1987

with the prediction

$$a_e^{\text{SM}} = \frac{\alpha}{2\pi} - 0.328\,478\,444\,00 \left(\frac{\alpha}{\pi}\right)^2 + 1.181\,234\,017 \left(\frac{\alpha}{\pi}\right)^3 - 1.7502(384) \left(\frac{\alpha}{\pi}\right)^4 + 1.66(3) \times 10^{-12} \text{ (hadronic \& electroweak loops)}$$



currently provides the best determination of the fine structure constant

$$\alpha^{-1}(a_e) = 137.035\,998\,75(52) \text{ (3.8 ppb)}$$

* Kinoshita, Nio 2003 $[-0.24] \times (\frac{\alpha}{\pi})^4$ update announced

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

② The Anomalous Magnetic Moment of the Muon

a_μ most precise test of quantum field theory model (QED/SM)

Experimental results: from muon storage rings

history CERN 1977:

$$\text{ave } \mu^+ \text{ and } \mu^-: a_\mu^{\text{exp}} = 11\,659\,230(84) \times 10^{-10} \text{ (CERN 1977)}$$

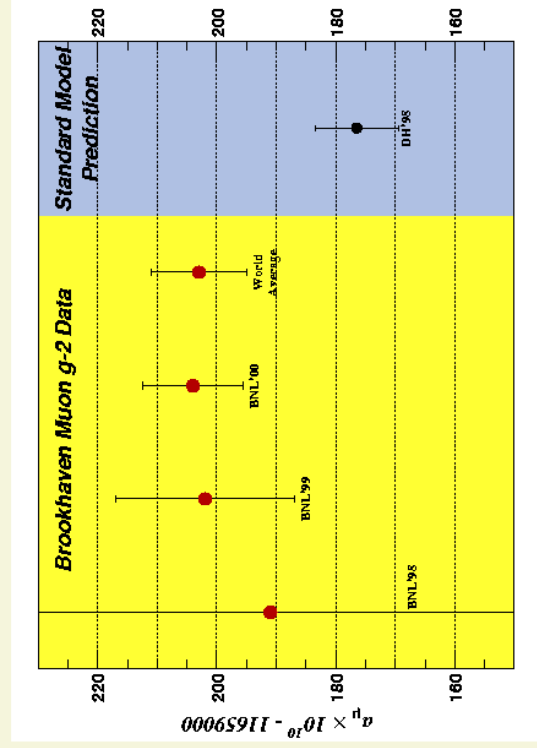
from ongoing experiment E821 at BNL 1998, ...:

$$\text{BNL 2002 with } \mu^+: a_{\mu^+} = 11\,659\,203(8) \times 10^{-10} \text{ (BNL'02+...)}$$

$$\text{BNL 2003 with } \mu^-: a_{\mu^-} = 11\,659\,214(8)(3) \times 10^{-10} \text{ (BNL'04)}$$

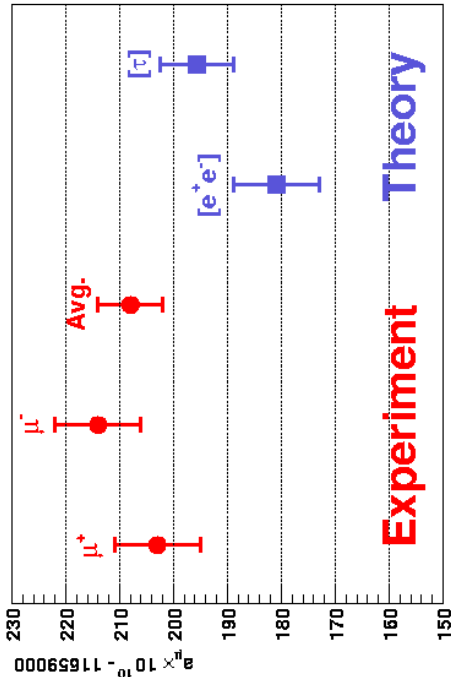
Averaged (CPT assumed):

$$a_\mu(\text{exp}) = 11\,659\,208(6) \times 10^{-10} \text{ (0.5 ppm)}$$



$(g - 2)_\mu$ and a_μ^{had}

Experiment vs. Theory Summary 2004



BNL - E821
 Muon (g-2) Collaboration
 hep-ex/0401008

$2.7 \sigma (e^+e^-) [1.4 \sigma (\tau)]$

experiment = 6×10^{-10} [5×10^{-10} stat, 4×10^{-10} syst]
 theory = 8×10^{-10} (e^+e^-) [$0.6\text{ppm} = 7 \times 10^{-10}$ (τ)]
 BNL design = 4×10^{-10}

New physics sensitivity: (example) $\Delta a_\mu^{\text{SUSY}} / a_\mu \simeq 1.25\text{ppm} \left(\frac{100\text{GeV}}{\tilde{m}} \right)^2 \tan \beta$

2-loops may amount $\sim 1.0 \sigma$
 for allowed parameter space

Heinemeyer, Stöckinger, Weiglein
 hep-ph/0312264

\tilde{m} lightest SUSY particle

$\tan \beta = \frac{v_1}{v_2}, v_i = \langle H_i \rangle ; i = 1, 2$

SUSY requires two Higgs fields

see Sven Heinemeiers talk

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

③ Standard Model Prediction for a_μ

① QED Contribution

The QED contribution to a_μ has been computed (or estimated) through **5 loops**

$$\begin{aligned} a_\mu^{\text{QED}} = & \frac{\alpha}{2\pi} + 0.765\,857\,376(27) \left(\frac{\alpha}{\pi}\right)^2 \\ & + 24.050\,508\,98(44) \left(\frac{\alpha}{\pi}\right)^3 \\ & + 126.07(41) \left(\frac{\alpha}{\pi}\right)^4 \\ & + 930(170) \left(\frac{\alpha}{\pi}\right)^5. \end{aligned}$$

Growing coefficients in the α/π expansion reflect the presence of large $\ln \frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops. Employing the value of α from a_e leads to

$$a_\mu^{\text{QED}} = 116\,584\,705.7(2.9) \times 10^{-11} \text{ (old)}$$

The current uncertainty is well below the $\pm 40 \times 10^{-11}$ ultimate experimental error anticipated from E821 and should, therefore, play no essential role in the confrontation between theory and experiment.

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

Not quite: Corrections due to internal e- and τ -loops updated

$$a_\mu = a_e^{\text{uni}} + a_\mu(m_\mu/m_e) + a_\mu(m_\mu/m_\tau) + a_\mu(m_\mu/m_e, m_\mu/m_\tau)$$

$$a_\mu(m_\mu/m_e) = 1.094\,258\,282\,8 \text{ (98)} \left(\frac{\alpha}{\pi}\right)^2 + 22.868\,379\,36 \text{ (23)} \left(\frac{\alpha}{\pi}\right)^3$$

$$+ 132.682\,3 \text{ (72)} \left(\frac{\alpha}{\pi}\right)^4$$

$$a_\mu(m_\mu/m_\tau) = 7.8059 \text{ (25)} \times 10^{-5} \left(\frac{\alpha}{\pi}\right)^2 + 36.054 \text{ (21)} \times 10^{-5} \left(\frac{\alpha}{\pi}\right)^3$$

$$+ 127.50 \text{ (41)} \left(\frac{\alpha}{\pi}\right)^4$$

$$a_\mu(m_\mu/m_e, m_\mu/m_\tau) = 52.763 \text{ (17)} \times 10^{-5} \left(\frac{\alpha}{\pi}\right)^3$$

$$+ 0.037\,594 \text{ (83)} \left(\frac{\alpha}{\pi}\right)^4$$

$$\text{with } \alpha^{-1}(\text{a.i.}) = 137.036\,000\,3 \text{ (10)} [7.4 \text{ ppb}]$$

$$a_\mu^{\text{QED}} = 116\,584\,719.35 \underbrace{(0.03)}_{\alpha^4} \underbrace{(1.15)}_{\alpha^5} \underbrace{(0.85)}_{\alpha_{\text{inp}}} \times 10^{-11}$$

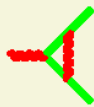
$$\text{shift by } +13.7 \times 10^{-11}$$

Kinoshita, Nio 04

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

# of loops	$a_\mu^{\text{QED}} \times 10^{11}$
1	116140972.87 (0.44)
2	413217.60 (0.02)
3	30141.90 (0.00)
4	367.01 (1.19)
5	6.29 (1.15)
tot	116584705.66 (2.80)

① 1 diagram



Schwinger 1948

② 7 diagrams



Peterman 1957, Sommerfield 1957

③ 72 diagrams



Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996

④ about 1000 diagrams



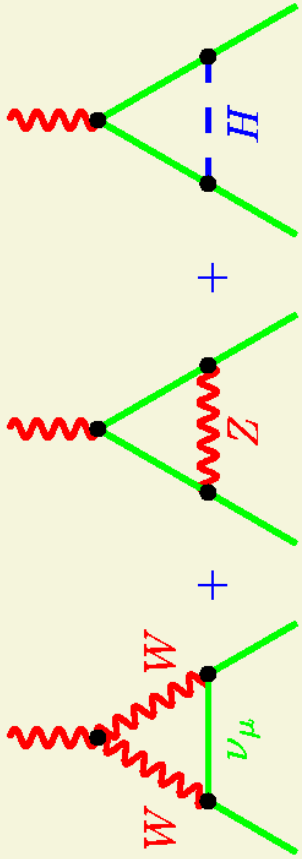
Kinoshita 1999, Kinoshita, Nio 2004

⑤ RG estimate

Czarnecki, Marciano 00

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

② Weak Contributions



$$a_\mu^{\text{weak}(1)} = (195 \pm 0) \times 10^{-11}$$

Brodsky, Sullivan 67, ...,
 Bardeen, Gastmans, Lautrup 72
 Higgs contribution tiny!

$$a_\mu^{\text{weak}(2)} = -(44 \pm 4) \times 10^{-11}$$

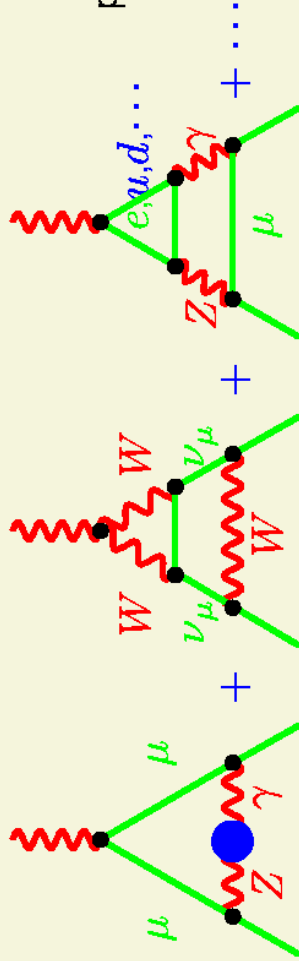
Kukhto et al 92

potentially large terms $\sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_{q,\mu}}$

Peris, Perrottet, de Rafael 95

quark-lepton (triangle anomaly) partial

Czarnecki, Krause, Marciano 96 full result



Most recent evaluations: improved hadronic part (beyond QPM)

$$a_\mu^{\text{weak}} = (152 \pm 1[\text{had}] \pm ?) \times 10^{-11}$$

(Knecht, Peris, Perrottet, de Rafael 02)

$$a_\mu^{\text{weak}} = (154 \pm 1[\text{had}] \pm 2[m_H, m_t, 3 - \text{loop}]) \times 10^{-11}$$

(Czarnecki, Marciano, Vainshtein 02)

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

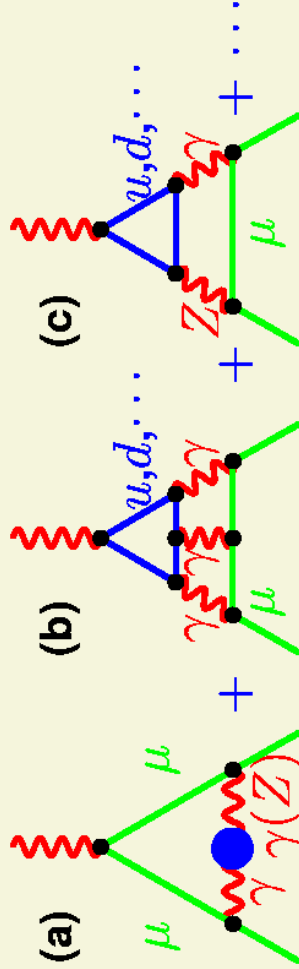
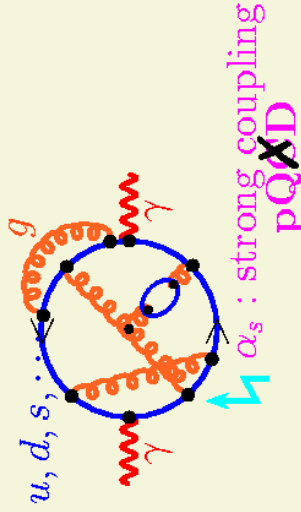
Hadronic Contributions

General problem in electroweak precision physics:
contributions from hadrons (quark loops) at low energy scales

Leptons



Quarks



(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$

(b) Hadronic light-by-light scattering $O(\alpha^3)$

(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$

Light quark loops \rightarrow Hadronic “blob”

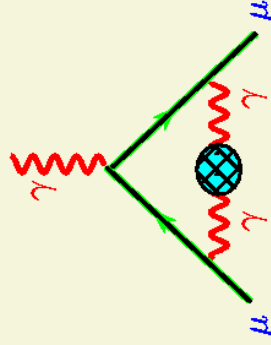
$(g - 2)_\mu$ and a_μ^{had}

④ Evaluation of a_μ^{had}

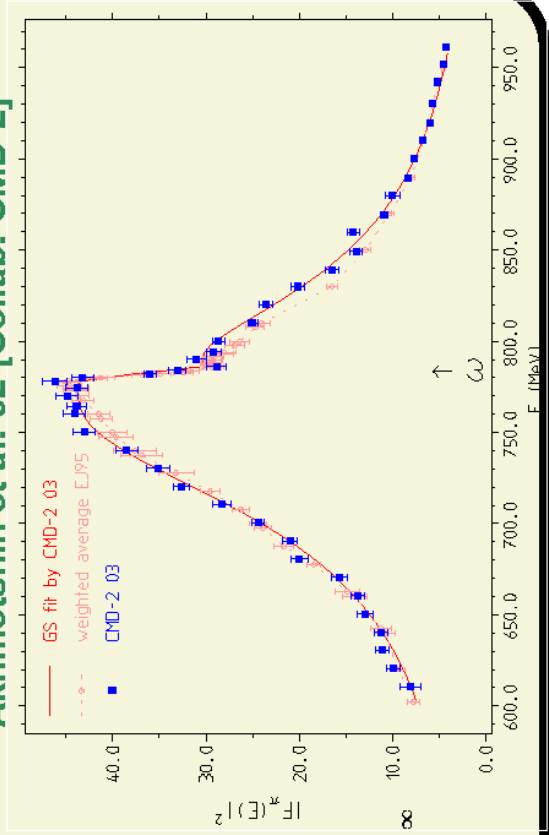
Leading non-perturbative hadronic contributions a_μ^{had} can be obtained in terms of

$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}) / \frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

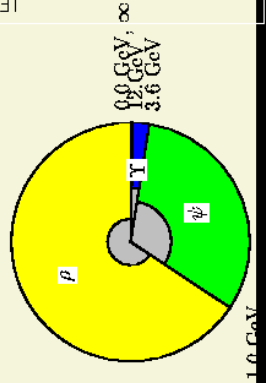
$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\text{cut}}^2} ds \frac{R_\gamma^{\text{data}}(s) \hat{K}(s)}{s^2} + \int_{E_{\text{cut}}^2}^{\infty} ds \frac{R_\gamma^{\text{PQCD}}(s) \hat{K}(s)}{s^2} \right)$$



Data: Akhmetshin et al. 02 [Collab. CMD-2]



- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 67\%$ of error on a_μ^{had} comes from region $4m_\pi^2 < m_\pi^2 < M_\Phi^2$



$a_\mu^{\text{had}(1)} = (694.8 \pm 8.6) 10^{-10}$
 e^+e^- -data based

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

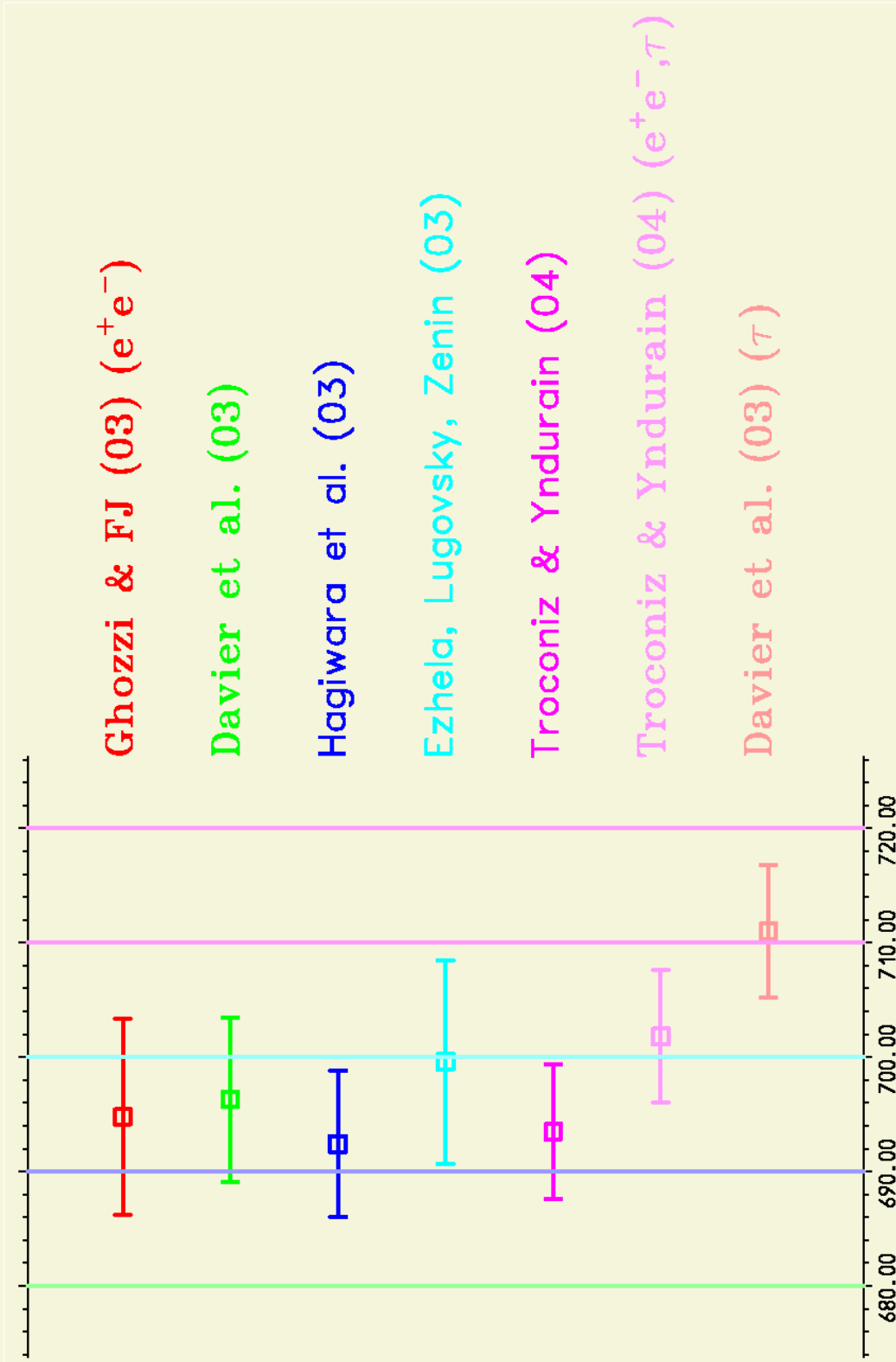
Recent evaluations of a_μ^{had}

Only results based on most recent data **CMD-2 03, BES II 01** shown

data	$a_\mu^{\text{had(1)}} \times 10^{10}$	Ref.	hep-ph
e^+e^-	694.8[8.6]	FJ,GJ 03	0310181
e^+e^-	696.3[7.2](6.2) _{exp} (3.6) _{rad}	DEHZ 03	0308213
e^+e^-	692.4[6.4](5.9) _{exp} (2.4) _{rad}	HMNT 03	0312250
e^+e^-	699.6[8.9](8.5) _{exp} (1.9) _{rad} (2.0) _{proc}	ELZ 03	0312114
τ	711.0[5.8](5.0) _{exp} (0.8) _{rad} (2.8) _{SU(2)}	DEHZ 03	0308213
e^+e^-	693.5[5.9](5.0) _{exp} (1.0) _{rad} (3.0) _{$\ell \times \ell$}	TY 04	0402285
$e^+e^- + \tau$	701.8[5.8](4.9) _{exp} (1.0) _{rad} (3.0) _{$\ell \times \ell$}	TY 04	0402285

Differences in errors mainly by utilizing more or less theory: pQCD, SR, low energy QCD methods

$(g - 2)_\mu$ and a_μ^{had}

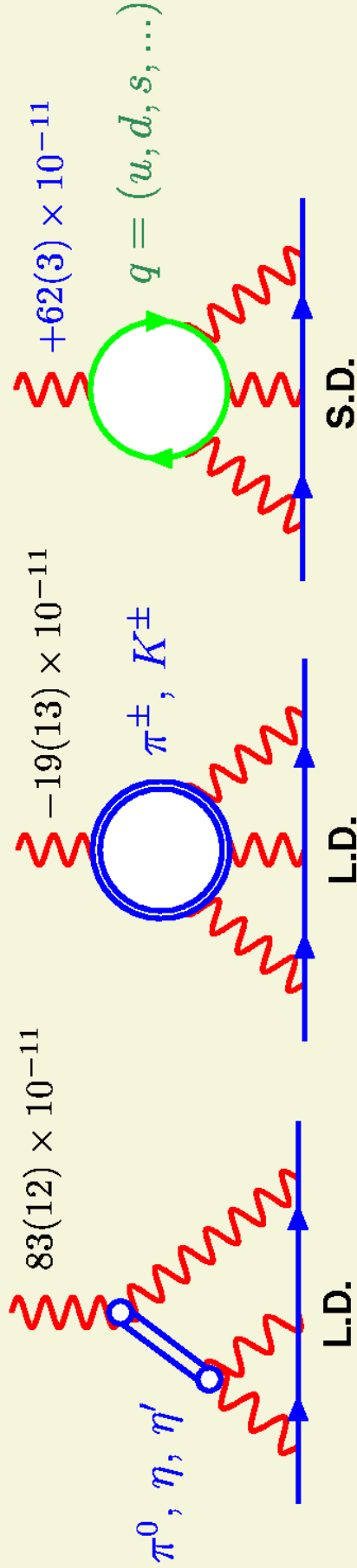


$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

- Higher order hadronic contributions $a_\mu^{\text{had}(2)} = -(100 \pm 6) \times 10^{-11}$ (Krause 96)
 $a_\mu^{\text{had}(2)} = -(98 \pm 1) \times 10^{-11}$ (Hagiwara et al. 03)



- Hadronic light-by-light scattering $a_\mu^{\text{lbl}} = (80 \pm 40) \times 10^{-11}$ (Nyffeler 02)
 $a_\mu^{\text{lbl}} = (136 \pm 25) \times 10^{-11}$ (Melnikov & Vainshtein 03) shift by $+56 \times 10^{-11}$

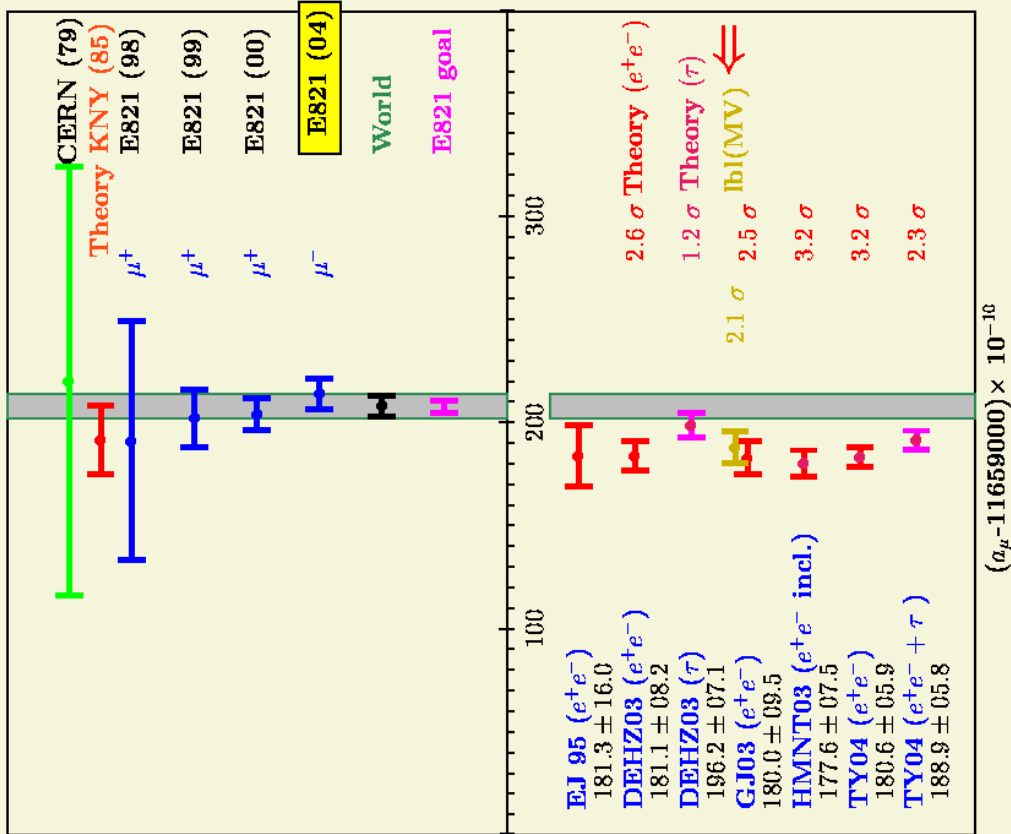


Low energy effective theory: e.g. ENJL

see Kirill Melnikov's talk

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

a_μ Summary: experiment vs. theory

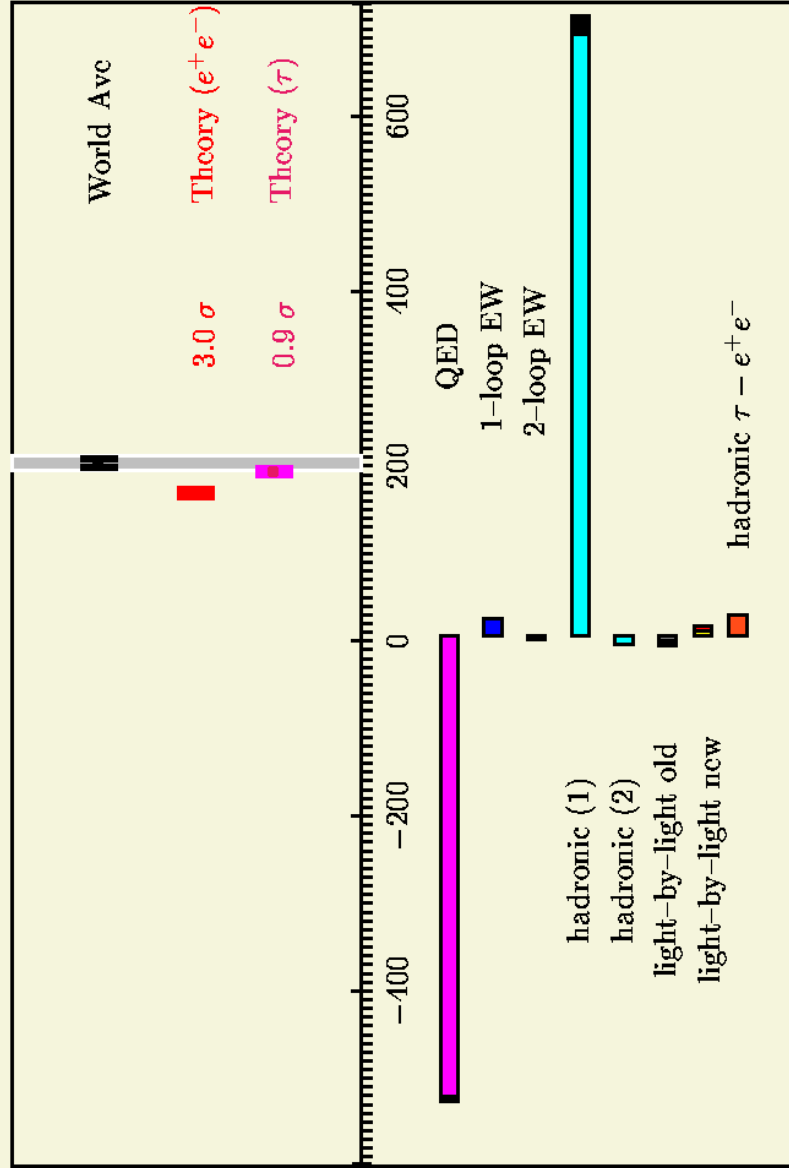


Given theory results only differ by $a_{\mu}^{\text{had}(1)}$!

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

a_μ : type and size of contributions

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had}(1)} + a_\mu^{\text{had}(2)} + a_\mu^{\text{weak}(1)} + a_\mu^{\text{weak}(2)} + a_\mu^{\text{lbl}} (+ a_\mu^{\text{new physics}})$$

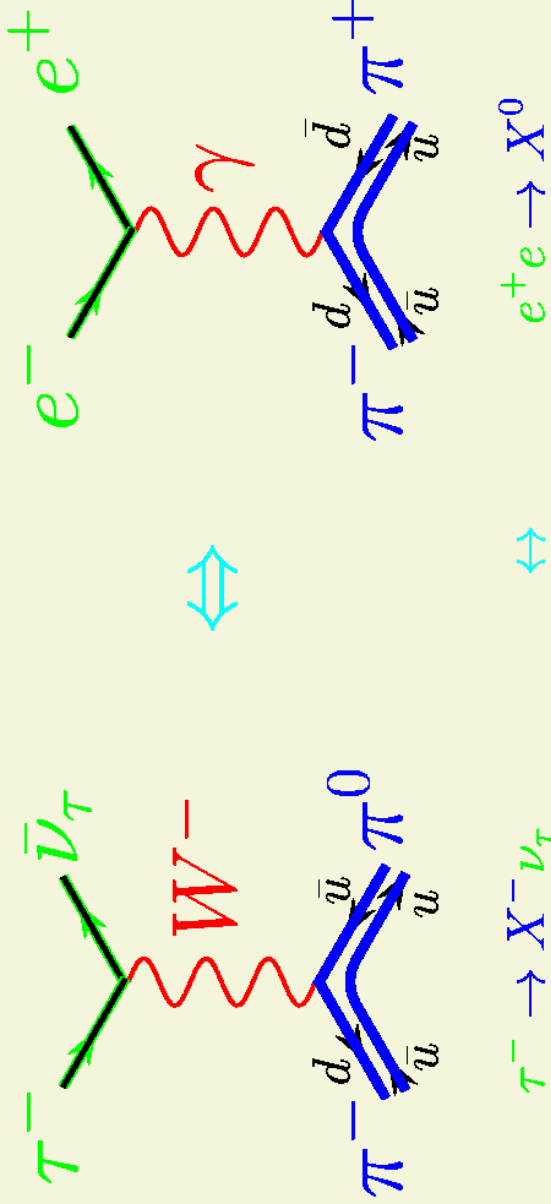


All kind of physics meets !

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

⑤ e^+e^- –cross sections via τ –decay spectral functions

The iso-vector part of $\sigma(e^+e^- \rightarrow \text{hadrons})$ may be calculated by a iso-spin rotation from τ –decay spectra (to the extent that CVC is valid)



X^- and X^0 are hadronic states related by iso-spin rotation.

The e^+e^- cross-section is then given by

$$\sigma_{e^+e^- \rightarrow X^0}^{I=1} = \frac{4\pi\alpha^2}{s} v_1 X^- , \quad \sqrt{s} \leq M_\tau$$

in terms of the τ spectral function v_1 .

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

- Additional “ e^+e^- ” data: τ -data + CVC **ALEPH, OPAL, CLEO (ADH96, DEHZ02)**
- mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ -resonance contribution)
- which is dominating in a_μ^{had} (72%)

$I = 1 \sim 75\%$; $I = 0 \sim 25\% \implies \tau$ -data cannot replace e^+e^- -data

Alemany, Davier, Höcker 95:

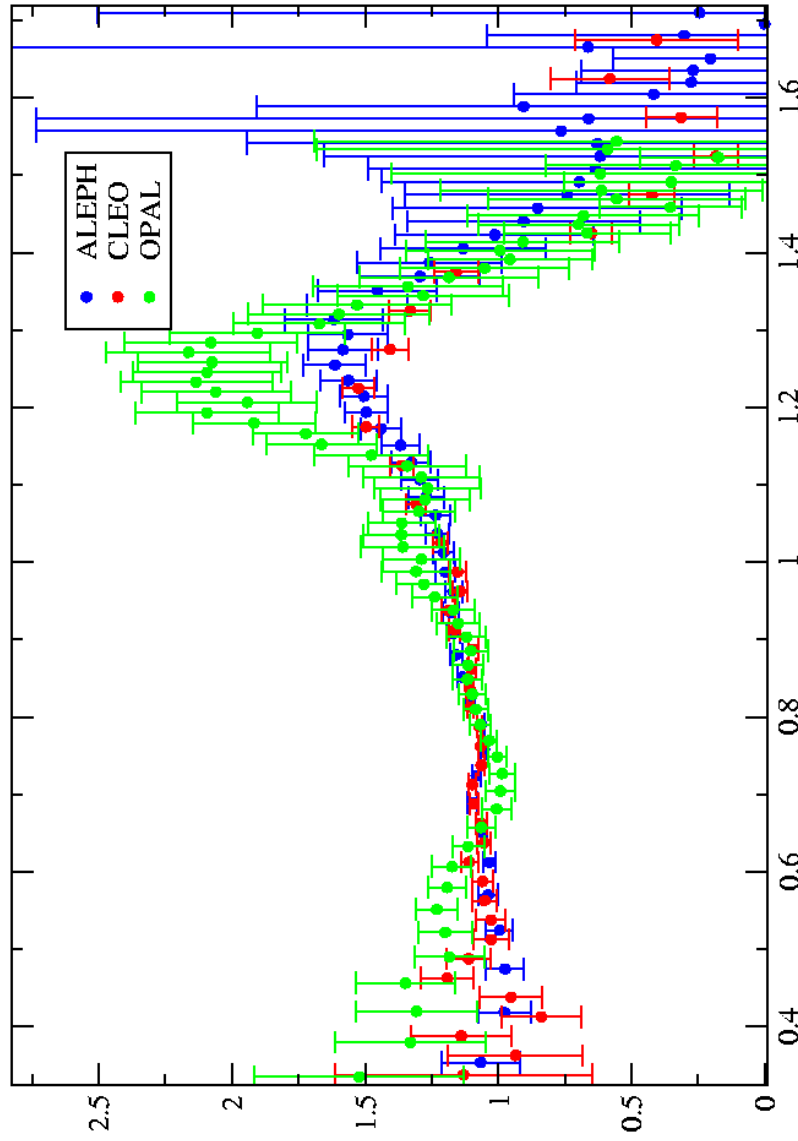
$$\delta a_\mu : 15.6 \times 10^{-10} \rightarrow 10.2 \times 10^{-10}$$

$$\delta \Delta\alpha : 0.00067 \rightarrow 0.00065$$

All kind of isospin breaking effects have to be taken into account !!!

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

- Comparison of τ -data:



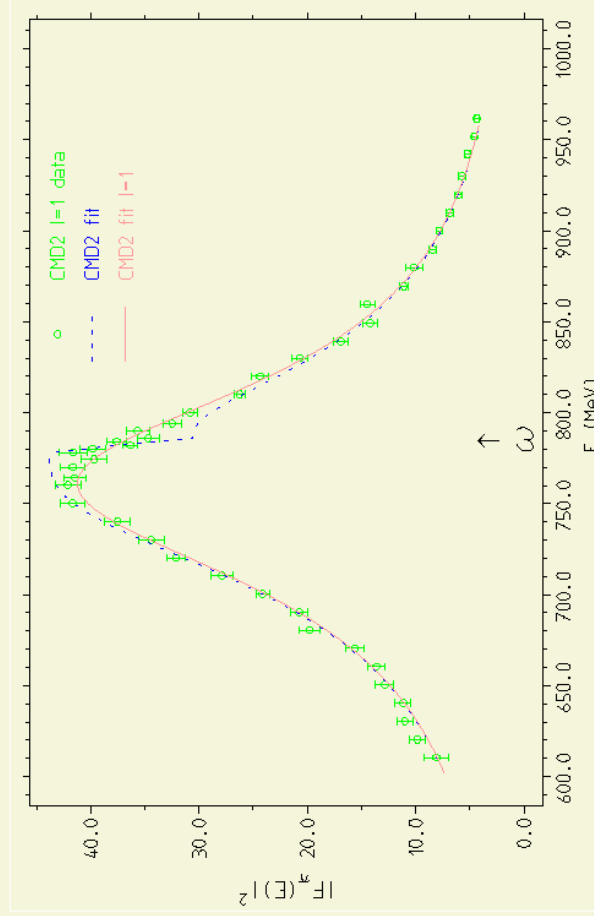
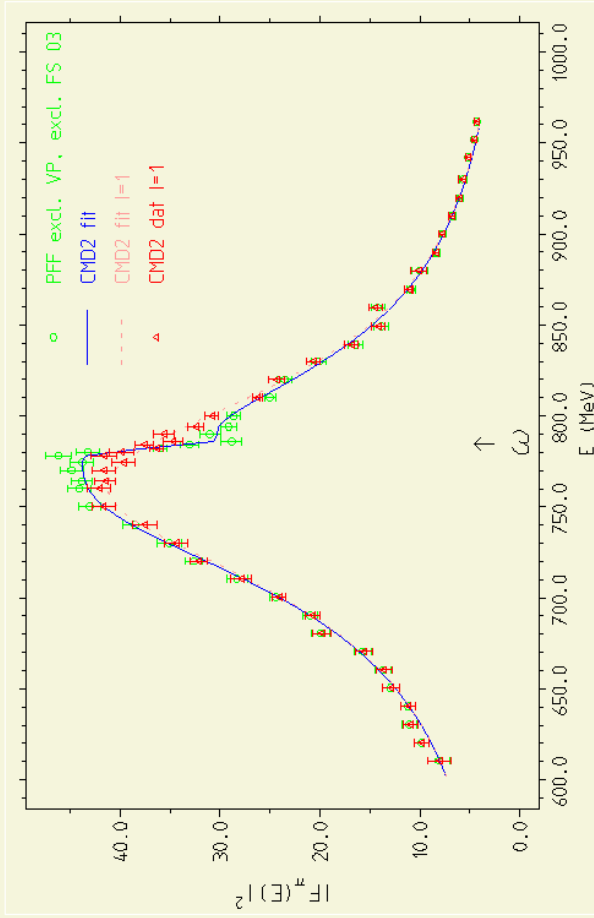
τ -data may be not so easy; DELPHI, L3 could not measure τ spectral-functions;

ALEPH vs. OPAL no good agreement.

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

Comparing e^+e^- and τ data: use Gounaris-Sakurai for parametrization

1) no VP, no FSR, undo $\rho - \omega$ -mixing



Fit for m_{ρ^0} and Γ_{ρ^0} at fixed background:

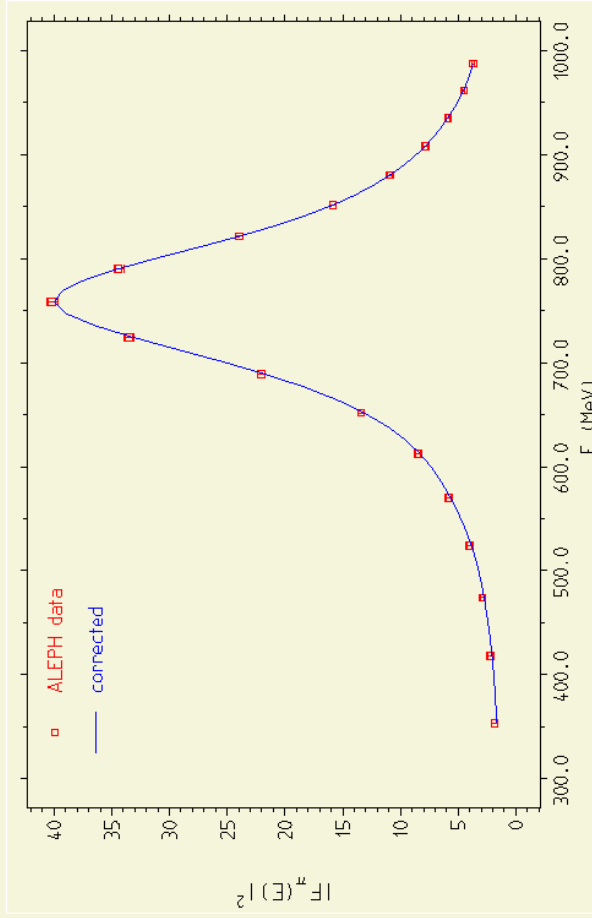
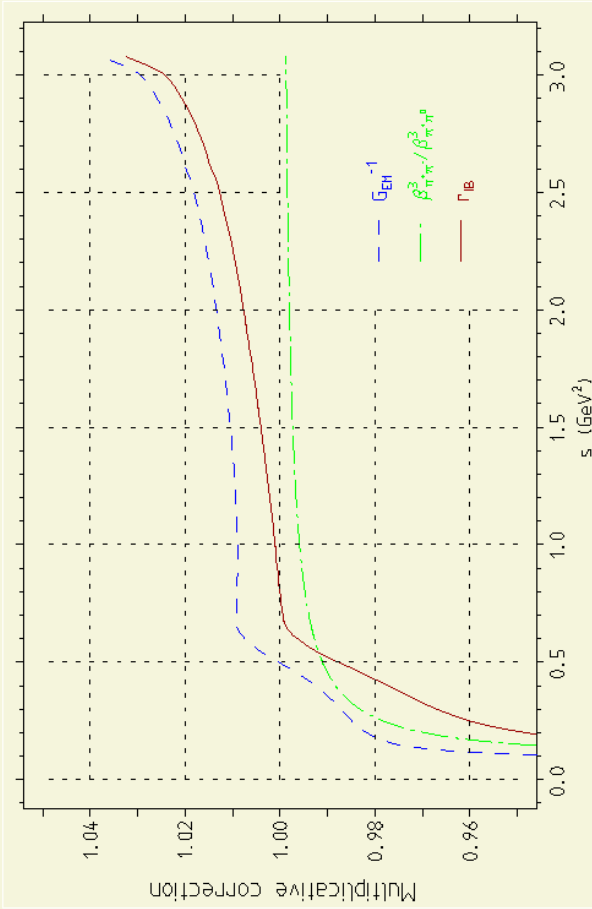
$$m_{\rho^0} = 772.5 \pm 0.6, \quad \Gamma_{\rho^0} = 148.1 \pm 1.0$$

(in agreement with M. Davier's fit)

CMD-2 data for $|F_\pi|^2$ in $\rho - \omega$ region together with Gounaris-Sakurai fit. Left before subtraction right after subtraction of the ω .

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

2) Iso-spin corrections applied to the τ data: left the corrections (CEN 02), with



$$r_{\text{IB}}(s) = \frac{1}{G_{\text{EM}}(s)} \frac{\beta_{\pi^-\pi^+}^3}{\beta_{\pi^-\pi^0}^3} \frac{S_{\text{EW}}(\text{old})}{S_{\text{EW}}(\text{new})}$$

and right the effect on the ALEPH data.

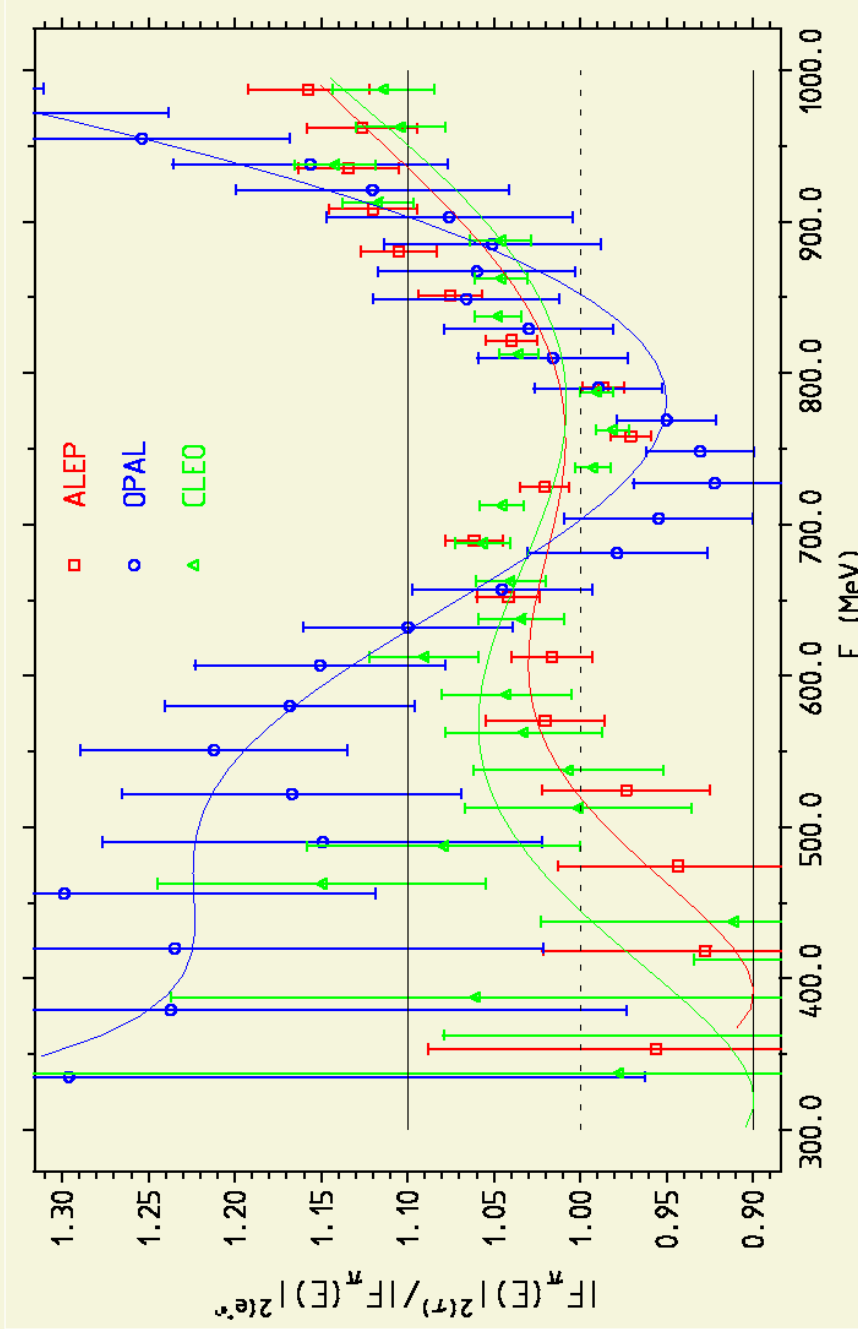
Fit ALEPH data for m_{ρ^-} and Γ_{ρ^-} at fixed background:

$$m_{\rho^-} = 776.0 \pm 0.7, \quad \Gamma_{\rho^-} = 150.6 \pm 1.3$$

(M. Davier's fit, covariance matrix!)

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

The ratio: S. Ghozzi, FJ, hep-ph/0308117



Can fit ratio to 1 ± 0.001 by letting float m_ρ and Γ_ρ in th GS-formula in numerator \Rightarrow yields back

$$m_{\rho^0} \text{ and } \Gamma_{\rho^0}$$

not completely trivial as it works over the range shown.

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

- Two sets of reasonably consistent data: τ -data ALEPH, CLEO vs. CMD-2, KLOE
- What effect is able to give 10% in a resonance tail? Answer: a shift of the energy by about 1%. Could be a problem of energy calibration, but this is very unlikely. Rather the physical resonance parameters have no reason to be identical!
- In spite of possible experimental problems, there is no reason to expect the neutral channel parameters to be the same as the charged current ones. Example, pions: $m_{\pi^\pm} - m_{\pi^0} = 4.5935 \pm 0.0005 \text{ MeV}$. Why should this be very different for the ρ a similar (same quark content) bound state?
- Usual argument $\Delta m_\rho^2 = \Delta m_\pi^2$ from a sum rule yields $m_{\rho^-} - m_{\rho^0} = \frac{1}{2} \frac{\Delta m_\pi^2}{m_{\rho^0}} \sim 0.02 \text{ MeV} !$.
- This is based on an **assumption** which easily could be wrong!
- Our conclusion: the observed discrepancy is a so far unaccounted iso-spin breaking effect, which the τ -data have to be corrected for!
- Relevant for calculating a_μ^{had} are the e^+e^- -data in first place.
- How to include the τ -data? Should know Δm_ρ and $\Delta \Gamma_\rho$ from elsewhere. Other

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

parameters like $m_{\rho'}$ and $\Gamma_{\rho'}$ are affected as well, but this is a higher order effect.

- In spite that the peak is shifted downwards and the s^{-2} weight in the integral which yields a_μ^{had} the latter does not go up, it rather goes down, because the width also substantially goes down and over-compensates the effect of the mass shift.
- If you don't like the idea: notice that the $\frac{\partial a_\mu^{\text{had}}}{\partial m_\rho[\Gamma_\rho]}$ is very large so that this quantity is very sensitive to the parameters.

My conclusion:

**we are back to one prediction for a_μ
at 2σ from the experimental value!**

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

⑥ Outlook

Summary:

- the new a_{μ^-} measurements is in very good agreement with the a_{μ^+} value in accordance with CPT and the combined result with an error of 60×10^{-11} not so far from the original goal of 40×10^{-11} .
- the increasing precision required and triggered a lot of new efforts to understand and/or settle the observed “discrepancy”
- still a big challenge on both the experimental side (e^+e^- cross sections) and on the theory side (also Lattice QCD could become an important tool ([Blum 03](#), [Schierholz et al. 03](#))).
- with unprecedented precision of the g-2 of the Muon experiment new obstacles showed up in many places which require more efforts on the theory side.
- remember situation similar to the one with CERN experiment 25 years ago (error in theory, leptonic lbl)

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

- e^+e^- -databased data seem to be related more directly to what we need in the evaluation of a_μ^{had}
- τ -data require more reliable estimates of isospin breaking effects
- a conservative estimate is

$$|a_\mu^{\text{exp}} - a_\mu^{\text{SM}}| = (28 \pm 11) \times 10^{-10}$$

Still an exciting situation, some room for new physics, serious challenges for theory and experiment

- Need better e^+e^- -data: KLOE is now finalizing their measurement by the radiative return method
- similar results expected from the B -factories
- ongoing upgrades: Novosibirsk, Beijing
- τ -charm factory
- able to perform an energy scan between 2 and 3.6 GeV
- would satisfy requirements of future precision experiments g-2, GigaZ,...

$$(g - 2)_\mu \text{ and } a_\mu^{\text{had}}$$

- Last but not least: need further progress in theory
- ✗ more careful study of isospin breaking in τ -data vs. e^+e^- -data
- ✗ constraints to $F_\pi(s)$ from χ PT, unitarity and analyticity below the ρ
- ✗ still a theoretical challenge: the hadronic light-by-light scattering contribution !
- ✗ Concerning τ -data vs. e^+e^- -data discrepancy: need more careful check of radiative corrections which have been applied !
- ✗ Further progress in radiative corrections calculations needed for the processes involved in R -measurements
- ✗ Further progress in determination of QCD parameters crucial for improving PQCD part

Big experimental challenge: attempt cross-section measurements at 1% level up to $J/\psi[\Upsilon]$!!! crucial for $\alpha_{\text{QED}}(M_Z)$ at GigaZ.