Merging Parton Showers with Fixed Order

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LoopFest III
April 1–3, 2004

Kavli Institute for Theoretical Physics

Three Different Approaches

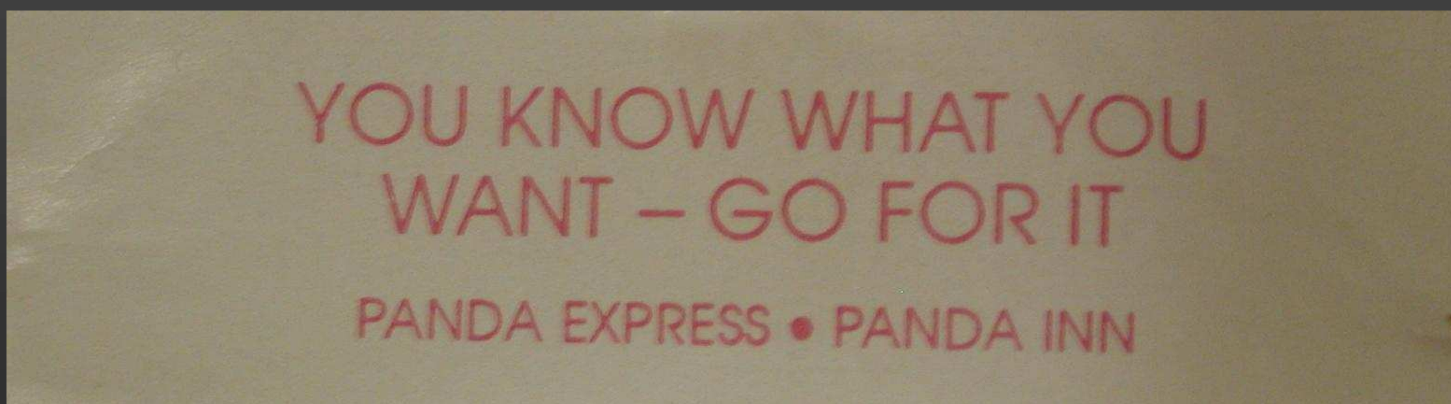
- General parton-level fixed-order calculations
  - Numerical jet programs: general observables
  - Systematic to higher order/high multiplicity in perturbation theory
  - Parton-level, approximate jet algorithm; match detector events only statistically
- Parton showers
  - General observables
  - Leading- or next-to-leading logs only, approximate for higher order/high multiplicity
  - Can hadronize & look at detector response event-by-event
- Semi-analytic calculations/resummations
  - Specific observable, for high-value targets
  - Checks on general fixed-order calculations

Problems with Parton Showers

- Don't include higher-order corrections
  - No access to precision physics
- Don't get soft logs right
  - Limited access to observables dependent on non-global logarithms
- Don't include correct wide-angle radiation
  - Don't even get LO predictions right for multijet final states
  - Wrong matrix elements for emission
  - Limitation on accessible phase space

Mergers & Acquisitions

- Want an approach which combines advantages of parton showers with fixed-order fully-differential programs



Naïve Approach

- Study n jet production
- Generate n hard partons, then shower
- Problem: double counting even at tree level
  - initial generation could have small Q^2
  - showers can generate hard radiation
- At NLO, more double counting
  - of virtual corrections
  - real integrations must go down into the IR region

Recent Work

- Multijets to LO accuracy
  - Separate matrix element from parton shower by a slicing
  - Modify matrix elements by Sudakovs
  - Subject parton showers to veto

Catani, Krauss, Kuhn, & Webber (2001)

- MC@NLO: incorporate virtual corrections
  - Modified subtraction correct to O (α_s)

Frixione & Webber (2002 –2004)

Frixione, Nason, & Webber (2003)

some related work: Kramer & Soper (2003)

Approaches to NLO Computations

- Slicing

Giele & Glover (1992)

- Subtraction

Frixione, Kunszt, & Signer (1994)

Catani & Seymour (1996)

$$\int_{\substack{\text{real} \\ \text{singular}} \times \text{hard}} \text{Exact}_{n+1} - \text{Approx}$$

$$+ \int_{\substack{\text{real} \\ \text{singular}}} \text{Singular} \int_{\text{hard}} \text{Exact}_{n} + V \int_{\text{hard}} \text{Exact}_{n}$$

Color Decomposition

$$\mathcal{A}_n^{\rm tree}(\{k_i, \lambda_i, a_i\}) = \sum_{\sigma \in S_n/Z_n} {\rm Tr}(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}})\, A_n^{\rm tree}(\sigma(1^{\lambda_1}, \ldots, n^{\lambda_n}))$$

where S_n/Z_n is the sum over non-cyclic permutations.

Universal Factorization

- In the soft limit,

$$A_n^{\rm tree}(\ldots, a, s^{\lambda_s}, b, \ldots) \xrightarrow{k_s \to 0} {\rm Soft}^{\rm tree}(a, s^{\lambda_s}, b)\, A_{n-1}^{\rm tree}(\ldots, a, b, \ldots)$$

- In the collinear limit,

$$A_n^{\rm tree}(\ldots, a^{\lambda_a}, b^{\lambda_b}, \ldots) \xrightarrow{a \| b} \sum_{\lambda=\pm} {\rm Split}_{-\lambda}^{\rm tree}(a^{\lambda_a}, b^{\lambda_b}; z)\, A_{n-1}^{\rm tree}(\ldots, (a+b)^{\lambda}, \ldots)$$

Dipole Factorization

Catani & Seymour (1996)

- Unify soft & collinear limits of squared matrix element

$$|A_{n+1}|^2 \xrightarrow{i \text{ or } j \text{ soft or } i\|j} \sum_{k\neq i,j} \frac{1}{s_{ij}} V_{ij,k} |A_n|^2$$

- Add soft limits to collinear factorization

$$i+j \longrightarrow \text{ emitter } \widetilde{ij}, \qquad \tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}} p_k^\mu$$

$$k \longrightarrow \text{ spectator } \tilde{k}, \qquad \tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}} p_k^\mu$$

$$y_{ij,k} = \frac{p_i \cdot p_j}{p_i \cdot p_j + p_j \cdot p_k + p_i \cdot p_k}, \qquad \tilde{z}_l = \frac{p_l \cdot \tilde{p}_k}{\tilde{p}_{ij} \cdot \tilde{p}_k}$$

Dipole Factorization

$$V_{ij,k}^{\mu\nu} = g^{\mu\nu}\left(\frac{1}{1-\tilde{z}_i(1-y_{ij,k})} + \frac{1}{1-\tilde{z}_j(1-y_{ij,k})} - 2\right)$$
$$+ \frac{(1-\epsilon)}{p_i \cdot p_j}\left(\tilde{z}_i p_i^\mu - \tilde{z}_j p_j^\mu\right) \cdot \left(\tilde{z}_i p_i^\nu - \tilde{z}_j p_j^\nu\right)$$

- Leads to subtraction method at NLO
- Double-real subtraction terms now known at NNLO
- Virtual-real mixed subtraction term and integral also known

Weinzierl (2003)

Gehrmann-De Ridder, Gehrmann, & Heinrich (2003)

Gehrmann-De Ridder, Gehrmann, & Glover (2004)

Antenna Factorization

DAK (1998)

- Similarities to Catani–Seymour dipole factorization
- Combine soft & collinear limits at amplitude level
- Add collinear "wings" to soft "core"
- Use tree-level current J, computed recursively

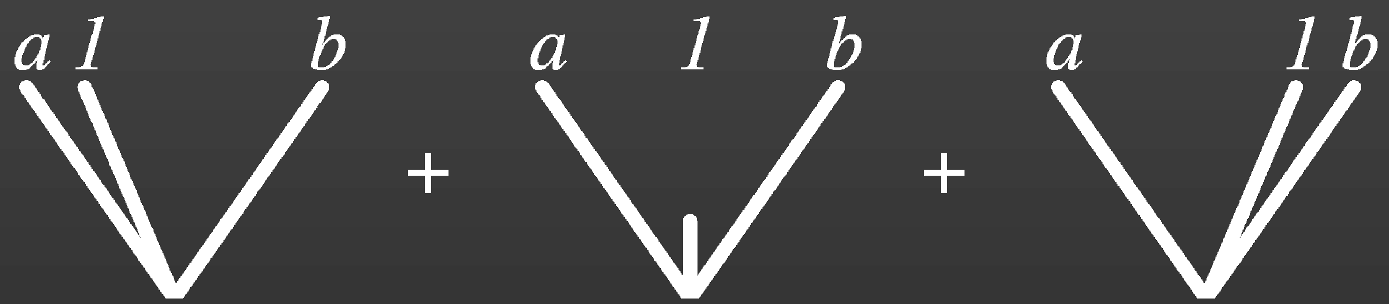
Berends & Giele (1988), Dixon (1995)

$$\mathrm{Ant}(\hat{a},\hat{b} \leftarrow a,1,b) = J(a,1;\hat{a})J(b;\hat{b}) + J(a;\hat{a})J(1,b;\hat{b})$$

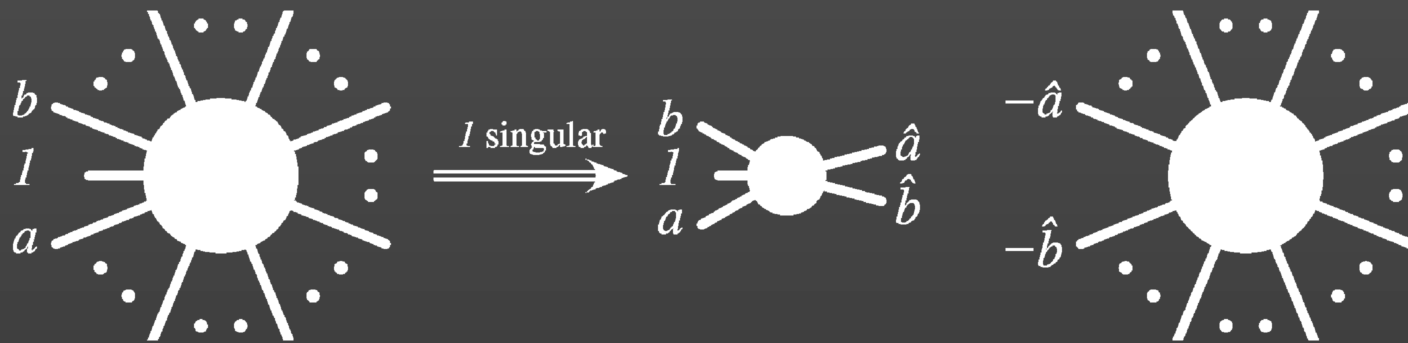
- Remap n momenta to two massless momenta

Collinear Wings: Single Emission

- Merge $a \,||\, 1$, 1 soft, and $1 \,||\, b$



Antenna Factorization

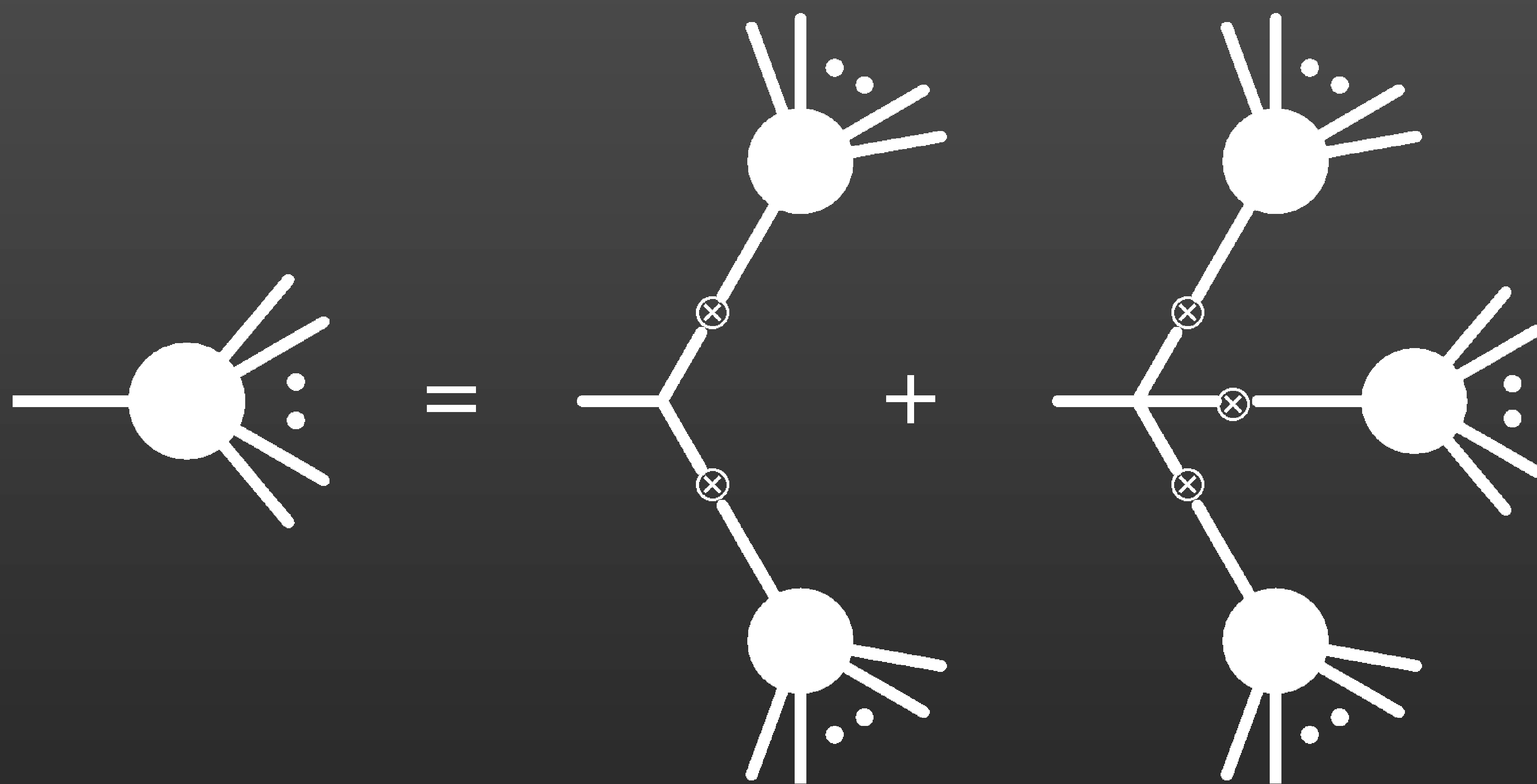


- In any singular limit ($\Delta(a,1,b)/s_{ab}{}^3 \to 0$),

$$A_n(\ldots,a,1,b,\ldots) \xrightarrow{k_1 \text{ singular}} \sum_{\text{ph. pol.}\lambda_{a,b}} \text{Ant}(\hat{a}^{\lambda_a},\hat{b}^{\lambda_b} \leftarrow a,1,b) A_{n-1}(\ldots,-k_{\hat{a}}^{-\lambda_a},-k_{\hat{b}}^{-\lambda_b},\ldots)$$

Currents from Recurrence Relations

Berends & Giele (1988), Dixon (1995)



Reconstruction Functions

- Map three momenta to two massless momenta
- Conserve momentum
- Ensure no subleading terms contribute to singular limits

$$k_{\hat{a}} = -\frac{1}{2(K^2 - s_{1b})}\left[(1+\rho)K^2 - 2s_{1b}r_1\right] k_a - r_1 k_1$$
$$- \frac{1}{2(K^2 - s_{a1})}\left[(1-\rho)K^2 - 2s_{1a}r_1\right] k_b\,,$$

$$k_{\hat{b}} = -\frac{1}{2(K^2 - s_{1b})}\left[(1-\rho)K^2 - 2s_{1b}(1-r_1)\right] k_a - (1-r_1)k_1$$
$$- \frac{1}{2(K^2 - s_{a1})}\left[(1+\rho)K^2 - 2s_{1a}(1-r_1)\right] k_b\,,$$

$$r_1 = \frac{s_{1b}}{s_{1a}+s_{1b}}, \qquad \rho = \sqrt{1 + \frac{4r_1(1-r_1)s_{1a}s_{1b}}{K^2 s_{ab}}}$$

- Generalizes to m singular emissions: $m \to 2$

Limiting Values

$$k_{\hat{a}} = -(k_a + k_1), k_{\hat{b}} = -k_b, \qquad \text{when } s_{a1} = 0, s_{1b} \neq 0\,;$$
$$k_{\hat{a}} = -k_a, k_{\hat{b}} = -(k_1 + k_b), \qquad \text{when } s_{a1} \neq 0, s_{1b} = 0\,;$$
$$k_{\hat{a}} = -k_a, k_{\hat{b}} = -k_b, \qquad \text{when } s_{a1} = 0 = s_{1b}\,.$$

Gluon Examples

- A particular helicity

$$\mathrm{Ant}(\hat{a}^+, \hat{b}^+ \leftarrow a^-, 1^+, b^-) = \frac{\langle a\, b\rangle^3}{\langle a\, 1\rangle\; \langle \hat{a}\, \hat{b}\rangle^2\; \langle 1\, b\rangle} \xrightarrow{k_1 \to 0} \frac{\langle a\, b\rangle}{\langle a\, 1\rangle\; \langle 1\, b\rangle}$$

$$\xrightarrow{a \| 1} \frac{(1-z)^2}{\sqrt{z(1-z)}\; \langle a\, 1\rangle}$$

- Helicity-summed square (factorization of squared matrix element)

$$2\frac{\left(K^2(s_{a1}+s_{1b})+s_{ab}^2\right)^2}{s_{a1}s_{1b}s_{ab}(K^2)^2} \xrightarrow{k_1 \to 0} 2\frac{K^2}{s_{a1}s_{1b}}$$

$$\xrightarrow{a \| 1} 2\frac{\left(1-z+z^2\right)^2}{s_{a1}z(1-z)}$$

Multiple Emission Antenna

- Generalizes single-emission,

$$\mathrm{Ant}(\hat{a},\hat{b} \leftarrow a,1,\ldots,m,b) = \sum_{j=0}^{m} J(a,1,\ldots,j;\hat{a}) J(j+1,\ldots,m,b;\hat{b})$$

- Factorization,

$$A_n(\ldots,a,1,\ldots,m,b,\ldots) \longrightarrow \sum_{\text{ph. pol.}\lambda_{a,b}}$$

$$\mathrm{Ant}(\hat{a}^{\lambda_a},\hat{b}^{\lambda_b} \leftarrow a,1,\ldots,m,b) A_{n-m}(\ldots,-k_{\hat{a}}^{-\lambda_a},-k_{\hat{b}}^{-\lambda_b},\ldots)$$

Approximate Radiation

- Logs come from near-singular radiation
- Approximate matrix element for large number of emissions — but start the approximation at n partons, not at 2

$$|A_{n+j}|^2 \to |A_n|^2 |\,\mathrm{Ant}(2 \to j+2)|^2$$

- Correspond to intra-/inter-jet radiation

Strong Ordering

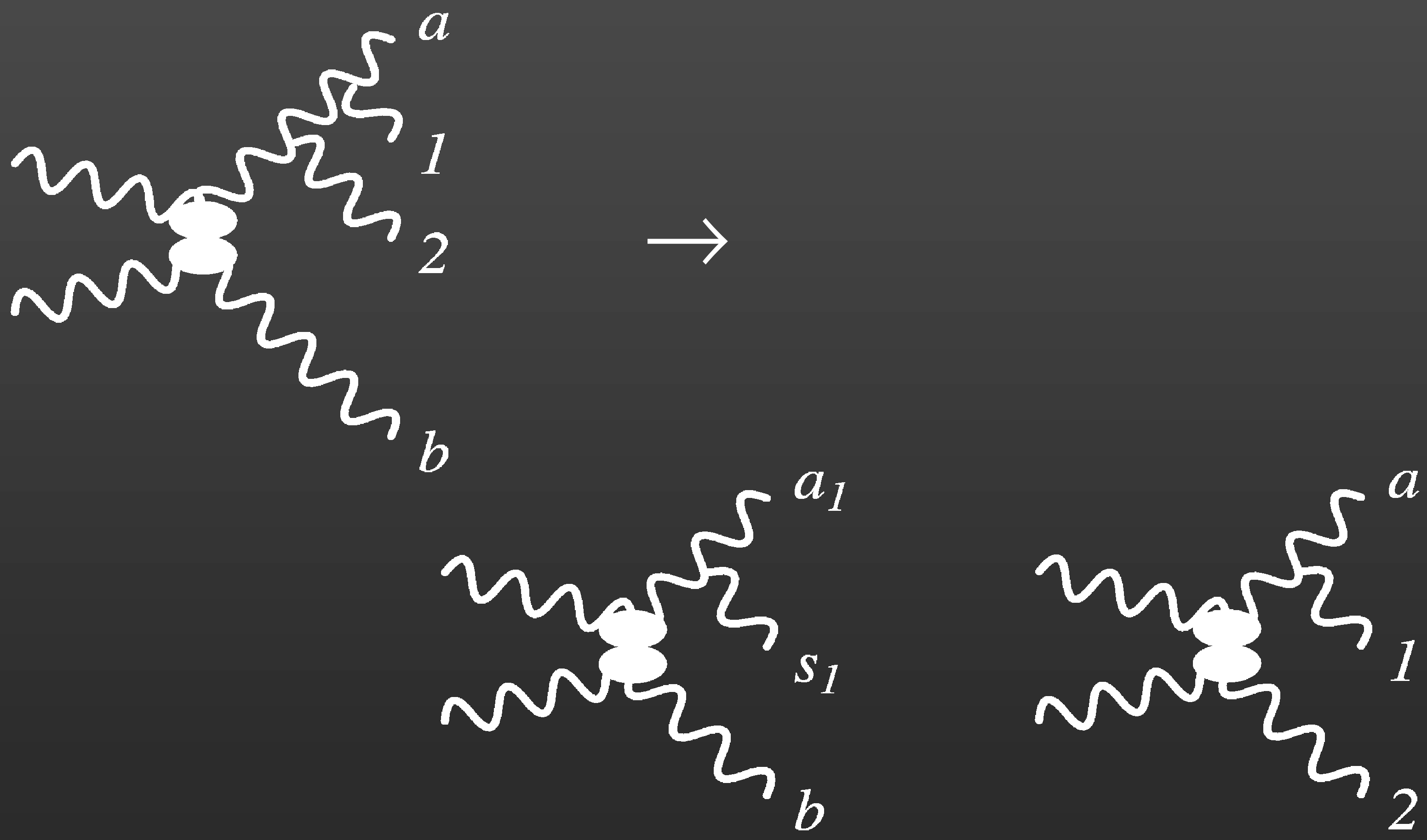
- Leading logs come from strongly-ordered emission

$$E_{j+1} \ll E_j$$

$$\theta_{j+1} \ll \theta_j$$

$$L_s(a,1,2,b) \equiv \left[\frac{G\begin{pmatrix} a,1,b \\ a,1,b \end{pmatrix} G\begin{pmatrix} a,2,b \\ a,2,b \end{pmatrix} - G^2\begin{pmatrix} a,1,b \\ a,2,b \end{pmatrix}}{G^2\begin{pmatrix} a,k_1+k_2,b \\ a,k_1+k_2,b \end{pmatrix}} \right]^{1/4}$$

Further Factorization



Subtraction Terms

- Antenna functions serve as subtraction terms in NLO or NNLO calculations, here as well
- Each antenna is the subtraction in a slice of phase space defined by the middle parton being softest overall

$$\Theta_j = 1 \Longleftrightarrow P_j = \frac{s_{j-1,j}s_{j,j+1}}{s_{j-1,j,j+1}} \text{ smallest of } P_k$$

Real Emission Contributions

- Subtract softest emission from n-point

$$|A_n|^2 - \sum_{\text{slice } j} \Theta_j |A_{n-1}|^2 |\,\text{Ant}_j\,|^2$$

- Add it back in to the lower-order term
- Add in approximations from $>n$ partons

$$|A_n|^2 |\,\text{Ant}_{\text{inner}}\,|^2 \cdots |\,\text{Ant}_{\text{really inner}}\,|^2$$

- Subtract & add approximations with more & more nested antennae

$$\sum_{\text{slice } j} \Theta_j |A_{m-1}|^2 |\,\text{Ant}_j\,|^2 |\,\text{Ant}_{\text{inner}}\,|^2 \cdots |\,\text{Ant}_{\text{really inner}}\,|^2$$

Matrix Element

- Build up matrix element with more & more emissions in strong-ordering approximation

$$\left[|A_m|^2 - \delta_{m\neq 2} \sum_{\text{slice } j} \Theta_j |A_{m-1}|^2 |\,\text{Ant}_j\,|^2 \right] |\,\text{Ant}_{\text{inner}}\,|^2 |\,\text{Ant}_{\text{inner}'}\,|^2$$

$$\times \sum_{\substack{\text{all nested} \\ \text{antennae}}} |\,\text{Ant}\,|^2 \cdots |\,\text{Ant}\,|^2$$

- Put in virtual corrections using unitarity
- Sum up all orders $\Rightarrow$ Sudakov factor

Evolution variable

- Convenient choice

$$s_A = \frac{4 s_{a1} s_{1b}}{K^2} \qquad \longrightarrow p_\perp^2$$

ARIADNE

- Sudakov factor $\Delta(s_0, s)$

$$\exp\left[-\alpha_s N \int ds_{a1} ds_{1b}\, \Theta\big(s - s_A\big)\, |\,\mathrm{Ant}\,|^2 (s_{a1}, s_{1b})\right]$$

Parton Showering

- Generate hard event, with up to n partons, according to

$$|A_m|^2 - \delta_{m\neq 2} \sum_{\text{slice } j} \Theta_j |A_{m-1}|^2 |\,\text{Ant}_j\,|^2, \qquad m = 2, \ldots, n$$

- Softest antenna fixes the starting scale
- Pick new scale according to Sudakov $\Delta(s_0,s)$
- Generate emission by remapping 2 $\rightarrow$ 3
  - partons kept massless
  - momentum conserved exactly
- Repeat until cut-off scale ~ 1-2 GeV is reached
- Hadronize

Summary

- Combining higher-order matrix element calculations with parton-showers would
  - fulfill long-standing requests from experimenters
  - have a direct impact on collider physics analyses
- Recent years have seen a number of proposals
- Antenna-based formalism can provide a uniform framework for addressing these issues
  - exact phase space
  - automatic momentum conservation
  - massless partons
  - all LL & NLL including non-global logs
- Numerical tests & program under way