Hadronic light-by-light scattering contribution to muon anomalous magnetic moment revisited

Kirill Melnikov

University of Hawaii at Manoa

In collaboration with Arkady Vainshtein

Loopfest III , April 2004

Outline

- Status of the muon anomalous magnetic moment
- Hadronic light-by-light scattering contribution:
  - The problem
  - Matching hadronic models to the OPE
  - Pseudoscalar contributions
  - Anatomy of the charged pion contribution
- Conclusions

Current status

- Results from E821 at BNL are self-consistent:

$$a_{\mu^+} = 116592030(80) \times 10^{-11}$$
$$a_{\mu^-} = 116592140(80)(30) \times 10^{-11}$$

- Current theoretical prediction

$$a_\mu^{\rm th} = \begin{cases} 116591810(80) \times 10^{-11} & e^+e^- \text{ data} \\ 116591960(70) \times 10^{-11} & \tau \text{ data} \end{cases}$$

- Experiment vs. theory: $70 - 330 \times 10^{-11}$ $(0 - 3\ \sigma)$.
- The best understood contributions: QED, EW.
- Contributions, understood "in principle": hadronic vacuum polarization.
- The least understood contribution: hadronic light-by-light.

Hadronic light-by-light: problem

- The physics of hadronic light-by-light scattering is non-perturbative. *Ab initio* calculations are not possible.
- Numerically small, comparable to the current precision on a_μ; but might be a real bottleneck in the long run.
- Problem: we are uncertain if the physics we put into the calculation is right or if something is missing. No data to compare.
- How to get a reasonable central value and a solid estimate of the theoretical uncertainty?
- Recent results:
  1. Knecht and Nyffeler: $a_\mu^{\rm lbl} = 80(40) \times 10^{-11}$;
  2. Hayakawa and Kinoshita: $a_\mu^{\rm lbl} = 90(15) \times 10^{-11}$;
  3. Bijnens, Pallante, Prades $a_\mu^{\rm lbl} = 83(32) \times 10^{-11}$;
- So is the problem solved?

Hadronic light-by-light: problem

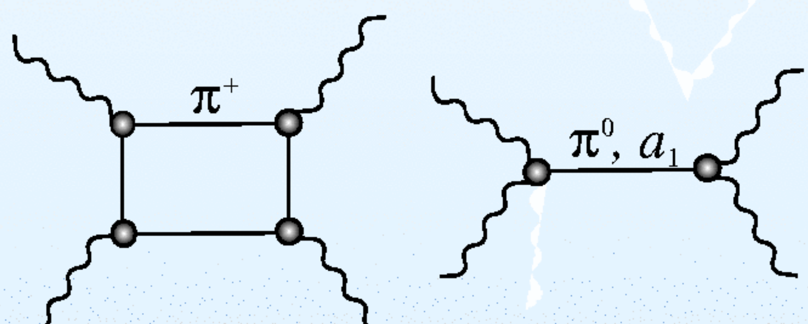
- A stricking feature of existing calculations is that the interplay between the short- and the long-distance contributions to a_μ is not discussed.
- This seems to be relevant:

$$a_\mu^{\pi_0} \approx 50 \times 10^{-11}, \quad a_\mu^{\rm quark} \approx 50 \times 10^{-11} \quad (m_q = 300~{\rm MeV}).$$

- Theoretical parameters:

$$m_\mu \sim m_\pi \ll 4\pi f_\pi \sim 1~{\rm GeV}, \qquad N_c = 3 \approx \infty.$$

- The $\pi^\pm$ contribution, formally chirally enhanced, is small.
- N_c-enhanced π_0 exchange is the largest.



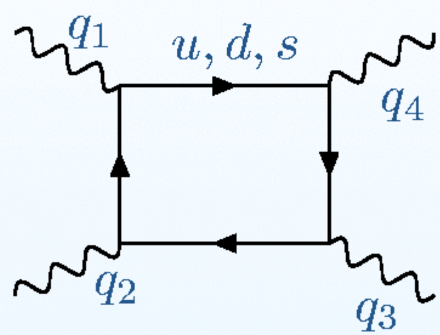
a b

Hadronic light-by-light: matching hadronic models...

- In the large-N_c limit, QCD is described by an infinite sum of narrow resonances, in any channel, for any observable.
- Leading pQCD contribution to hadronic light-by-light scattering is the quark box diagram, leading in N_c.
- Give up on infinite number of resonances; try to match pQCD amplitudes and the large-N_c hadronic amplitudes with small number of resonances.
- Selection criterion: Acceptable low-energy hadronic models for $\gamma^*\gamma^*$ scattering must extrapolate to pQCD amplitudes for $q^2 \gg \Lambda^2_{\rm QCD}$.
- So we are going to:
  1. compute $\gamma^*\gamma^*$ scattering through the quark loop in pQCD;
  2. construct the low-energy model, with resonance exchanges, e.g. $\pi_0, \ , \eta, \ \eta', \sigma, a_1, f_1, f_1^*, \ etc.$;
  3. adjust the model in such a way, that for photons with large virtualities, the pQCD result for each isospin-parity channel is reproduced at a reasonable level.

Hadronic light-by-light: $\gamma^*\gamma^* \to \gamma^*\gamma$

- LBL scattering through the quark loop is N_c-enhanced.



$$\sim \alpha^2 N_c \sum Q_i^4$$

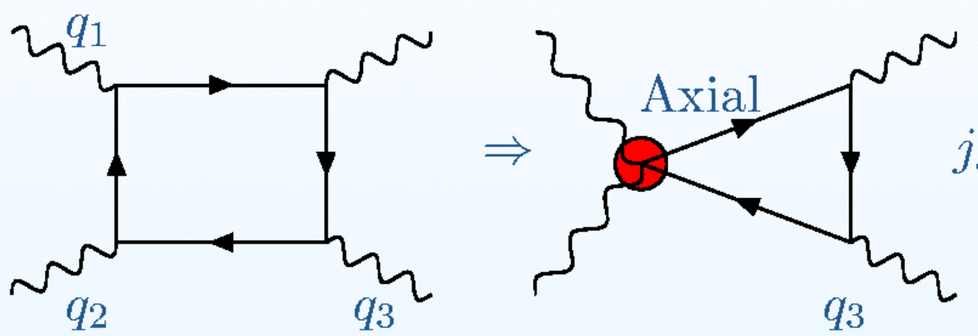
$$\mathcal{M} = \alpha^2 N_c \,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A} = \alpha^2 N_c \,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta}\epsilon_1^{\mu_1}\,\epsilon_2^{\mu_2}\,\epsilon_3^{\mu_3}\,f^{\gamma\delta}$$

$$= -e^3 \int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-iq_1x-iq_2y}\,\epsilon_1^{\mu_1}\,\epsilon_2^{\mu_2}\,\epsilon_3^{\mu_3}\,\langle 0|T\,\{j_{\mu_1}(x)\,j_{\mu_2}(y)\,j_{\mu_3}(0)\}\,|\gamma\rangle,$$

- $q_1 + q_2 + q_3 = 0$. Choose q_i^2 as three independent variables.
- Complicated function of q_i^2; 19 gauge-invariant structures, 7 form factors.
- Two distinct kinematic regimes: $q_i^2 \ll q_j^2 \sim q_k^2$ and $q_i^2 \sim q_j^2 \sim q_k^2$.
- The first one gives an easy connection to “hadronic world”.

Hadronic light-by-light: the OPE

- Consider $q_1 \sim q_2 \gg q_3$.



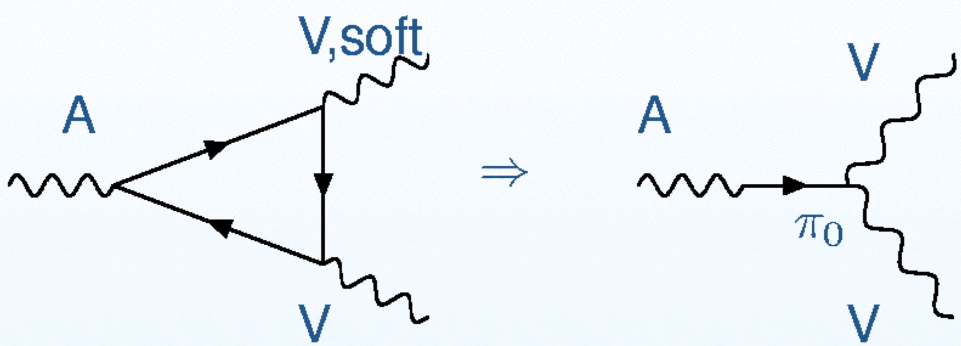
$$j_{5\rho} = \sum_{a=3,8,0} \frac{\mathrm{Tr}[\lambda_a \hat{Q}^2]}{\mathrm{Tr}[\lambda_a^2]} \, j_{5\rho}^{(a)}$$

$$i \int \mathrm{d}^4x \, \mathrm{d}^4y \, \mathrm{e}^{-iq_1x - iq_2y} \, T\{j_{\mu_1}(x), j_{\mu_2}(y)\} = \int \mathrm{d}^4z \, \mathrm{e}^{-i(q_1+q_2)z} \, \frac{2i}{\hat{q}^2} \, \epsilon_{\mu_1\mu_2\delta\rho} \, \hat{q}^\delta j_5^\rho(z) + \cdots .$$

- The remaining calculation of the light-by-light scattering amplitude involves the triangle VVA diagram with soft vector current

$$T_{\mu_3\rho}^{(a)} = i\,\langle 0| \int \mathrm{d}^4z \, \mathrm{e}^{iq_3z} \, T\{j_{5\rho}^{(a)}(z) \, j_{\mu_3}(0)\} |\gamma\rangle.$$

Hadronic light-by-light: the OPE



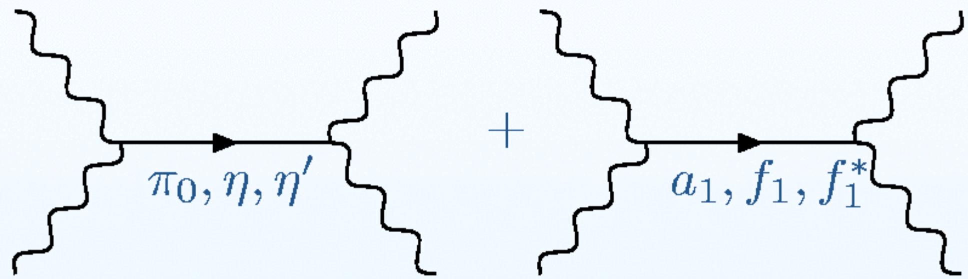
- $T^{(a)}_{\mu_3\rho}$ is expressed through longitudinal and transverse structure functions:

$$T^{(a)}_{\mu_3\rho} = -\frac{ie\, N_c \mathrm{Tr}\,[\lambda_a \hat{Q}^2]}{4\pi^2}\left\{ w_L^{(a)}(q_3^2)\, q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3} + w_T^{(a)}(q_3^2)\left(-q_3^2 \tilde{f}_{\mu_3\rho} + q_{3\mu_3} q_3^\sigma \tilde{f}_{\sigma\rho} - q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\rho} \right.\right.$$

- For massless quarks: $w_L^{(a)}(q^2) = 2w_T^{(a)}(q^2) = -\dfrac{2}{q^2}$.
- $w_L(q^2)$ for non-singlet axial current can be continued to $q^2 \le \Lambda^2_{\mathrm{QCD}}$ without modifications if no explicit chiral symmetry breaking is present.
- Direct connection to π_0 and η from pQCD domain.

Hadronic light-by-light: The model

- For large q^2, acceptable hadronic model should match the OPE prediction for the light-by-light scattering amplitude.



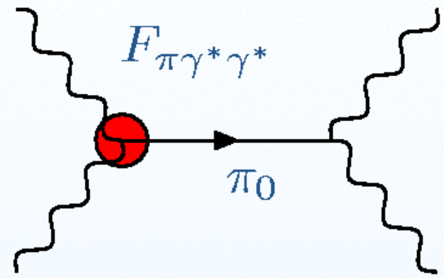
$$\mathcal{A} = \mathcal{A}_{\rm PS} + \mathcal{A}_{\rm PV} + \text{permutations}$$

$$\mathcal{A}_{\rm PS} = \sum_{a=3,8,0} W^{(a)} \phi_L^{(a)}(q_1^2, q_2^2) w_L^{(a)}(q_3^2) \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\}$$

$$\mathcal{A}_{\rm PV} = \sum_{a=3,8,0} W^{(a)} \phi_T^{(a)}(q_1^2, q_2^2) w_T^{(a)}(q_3^2) \Big(\{q_2 f_2 \tilde{f}_1 \tilde{f} f_3 q_3\} + \{q_1 f_1 \tilde{f}_2 \tilde{f} f_3 q_3\} + \frac{q_1^2 + q_2^2}{4} \{f_2 \tilde{f}_1\}\{$$

- Pseudoscalar and pseudovector exchanges are included, to conform with the limit from the OPE. No scalars and vectors.

Hadronic light-by-light: pseudoscalar exchanges



- The π_0 exchange is the cleanest example.

$$w_L^{(3)}(q^2) = \frac{2}{q^2+m_\pi^2}, \quad \phi_L^{(3)}(0,0) = \frac{N_c}{4\pi^2 F_\pi^2},$$

$$\mathcal{A}_{\pi^0} \quad = \quad -\frac{N_c W^{(3)}}{2\pi^2 F_\pi^2} \frac{F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)}{q_3^2+m_\pi^2} \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\} + \text{permutations}$$

- Comparing $\mathcal{A}_{\pi^0}$ to $\mathcal{A}_{\rm OPE}$ we derive correct asymptotics for the pion form factor

$$\lim_{q^2 \gg \Lambda_{\rm QCD}^2} F_{\pi\gamma^*\gamma^*}(q^2, q^2) = \frac{8\pi^2 F_\pi^2}{N_c\, q^2}.$$

Hadronic light-by-light: pseudoscalar exchanges

- The pion exchange asymptotically saturates the OPE light-by-light scattering amplitude in the isotriplet pseudoscalar channel.
- Form factor for $\pi_0\gamma^*\gamma_{\rm soft}$ vertex is absent. This ensures the correct asymptotic behavior of the light-by-light scattering amplitude at large q^2.
- Parameterization of the pion form factor, consistent with OPE constraints:

$$F_{\pi\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{4\pi^2 F_\pi^2}{N_c}\,\frac{q_1^2q_2^2(q_1^2+q_2^2) - h_2 q_1^2 q_2^2 + h_5(q_1^2+q_2^2) + (N_c M_1^4 M_2^4/4\pi^2 F_\pi^2)}{(q_1^2+M_1^2)(q_1^2+M_2^2)(q_2^2+M_1^2)(q_2^2+M_2^2)}$$

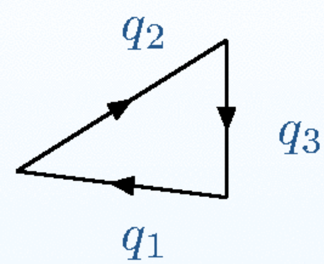
- $h_2 \approx -10\ {\rm GeV}^2$ instead of $0\ {\rm GeV}^2$; from $\mathcal{O}(q^{-4})$ asymptotics of the pion form factor.
- The result: the pion pole contribution increases

$$58 \times 10^{-11} \to 76.5 \times 10^{-11}, \quad (30\%\ \text{increase})$$

- Similar situation occurs for η and η' (mixing etc.):

$$\delta a_\mu^\eta = \delta a_\mu^{\eta'} = 18 \times 10^{-11}.$$

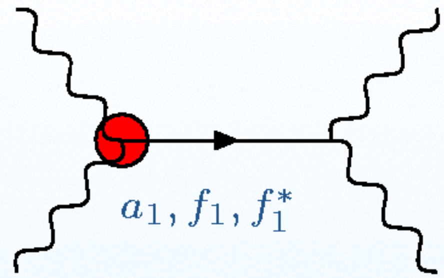
Hadronic light-by-light: check of the model



- Matching has been performed for “degenerate” triangle: $q_1 \sim q_2 \gg q_3$.
- Non-degenerate triangle: take $q_1 = q_2^2 = q_3^2 = q^2 \gg \Lambda_{\rm QCD}^2$ and compare with the honest pQCD result – agreement to 30 percent.
- Other problems with the model, like incorrect scaling w.r.t. $q_{1,2}$ in certain channels can be fixed; changes small.
- Bottom line: the major effect comes from correct asymptotic behavior:

$$\frac{1}{q^2} \quad \text{compared to} \quad \frac{1}{q^4}.$$

Hadronic light-by-light: pseudovector contribution



$\sim \phi(q_1,q_2) w_{\rm T}(q_3)$

$m_{a_1} = 1260$ MeV

$m_{f_1} = 1285$ MeV

$m_{f_1^*} = 1420$ MeV

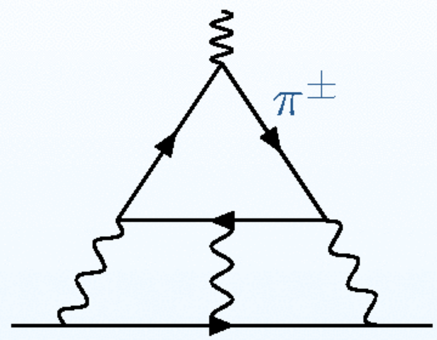
- For form factors $\phi(q_1,q_2)$ we use:

$$\phi(q_1,q_2) = -\frac{4}{(q_1^2+m^2)(q_2^2+m^2)}, \quad m = m_{\rho,\omega,\phi}$$

- For spectral densities: $w_T^{(3)} = \frac{1}{m_{a_1}^2 - m_\rho^2}\left(\frac{m_{a_1}^2 - m_\pi^2}{q^2+m_\rho^2} - \frac{m_\rho^2 - m_\pi^2}{q^2+m_{a_1}^2}\right).$
- Unclear how to assign isospin numbers to f_1, f_1^*. But:

$$a_\mu^{\rm PV} = (5.7 + 15.6 + 0.8)\times 10^{-11} = 22\times 10^{-11}, \quad (f_1^* \sim s\bar{s})$$
$$a_\mu^{\rm PV} = (5.7 + 1.9 + 9.7)\times 10^{-11} = 17\times 10^{-11}. \quad (f_1^* \sim u\bar{u} + d\bar{d} + s\bar{s})$$

Hadronic light-by-light: anatomy of $\pi^\pm$ box



Chirally enhanced

$$\delta a_\mu^{\pi^\pm} \sim \left(\frac{\alpha}{\pi}\right)^3 \frac{m_\mu^2}{m_\pi^2}$$

$$\sim 200 - 700 \times 10^{-11}$$

- But, rather small numerically ($\sim -50 \times 10^{-11}$) in reality and changes strongly when large masses (e.g. M_ρ) are introduced:

$$\delta a_\mu^{\pi^\pm} = -(4.5 - 19) \times 10^{-11}$$

- Can the diagram converge at larger values of momenta than the pion mass?
- Check by explicitely constructing an expansion using $m_\pi \sim m_\mu \ll M_\rho$.

Hadronic light-by-light: anatomy of $\pi^{\pm}$ box

- The result for $(m_\mu = m_\pi)$

$$a_\mu^\pi \approx (-69 + 54 + 18 - 8 - 1 + ..) \times 10^{-11}.$$

- Anzats:

$$a_\mu^\pi(m_\pi = m_\mu) \approx \left(\frac{\alpha}{\pi}\right)^3 \frac{m_\pi^2}{\mu^2} \left(c_1 + c_2 \frac{\mu^2}{M_\rho^2} + c_3 \frac{\mu^4}{M_\rho^4} + \ldots \right).$$

- Set $c_1 = 1$, determine μ and determine $c_{i>1}$. If of order unity, then self-consistent.

$$\mu = 4.25 m_\pi \sim 500 \text{ MeV} \sim M_\rho,$$

$$c_1 = 1, \quad c_2 \approx 1.3, \quad c_3 \approx 0.75, \quad c_4 \approx -0.6.$$

- Conclusion: $\pi^{\pm}$ contribution is not chirally enhanced; just “one of many” $\mathcal{O}(N_c^{(0)})$ contributions.

Hadronic light-by-light: conclusions

- Our result for the light-by-light:

$$a_\mu^{\rm lbl} = 136(25) \times 10^{-11} \qquad \sim 50\% \text{ increase.}$$

- Given the error bars, no numerical inconsistency with previous results.
- Better matching between short- and long- distance degrees of freedom.
- Consistency check: fair agreement with π_0 + massive quark model.
- The increase in the central value is “real”, since it comes from the well-identified effect.
- No decrease in the uncertainty. The uncertainty quoted above is subjective.
- Moves the SM result (e^+e^- data) closer to experimental value:

$$a_\mu^{\mu^-} - a_\mu^{\rm th} = 270(113) \times 10^{-11},$$
$$a_\mu^{\mu^+} - a_\mu^{\rm th} = 160(113) \times 10^{-11}.$$

With the τ data, there is no discrepancy at all.