Resumption methods in heavy flavor production

Pavel Nadolsky

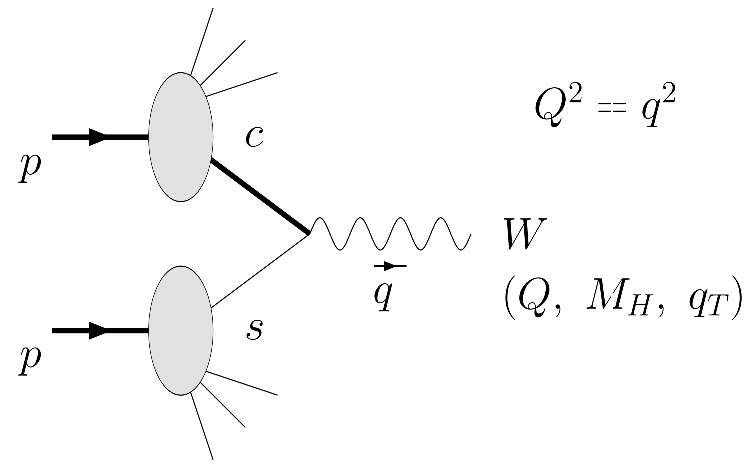
P. N., N. Kidonakis, F. I. Olness, C.-P. Yuan, Phys Rev, D67, 074015 (2003)

S. Berge, P. N., F. I. Olness, C.-P. Yuan, in preparation

- ✓ Systematic calculation of differential distributions in reactions with heavy quarks H (H=c, b, or t) in perturbative QCD in the presence of 3 distinct momentum scales (Q, M_H, and q_T)
- ✓ Relies on usage of a massive variable flavor number (VFN) factorization scheme

Heavy-flavor contributions to W & Z boson production at hadron colliders

Example: $c + s \to W^{\pm} + X$



✓ Relevant momentum scales

- heavy quark mass M_H
- Virtuality Q of the W boson
- transverse momentum q_T of of the W boson

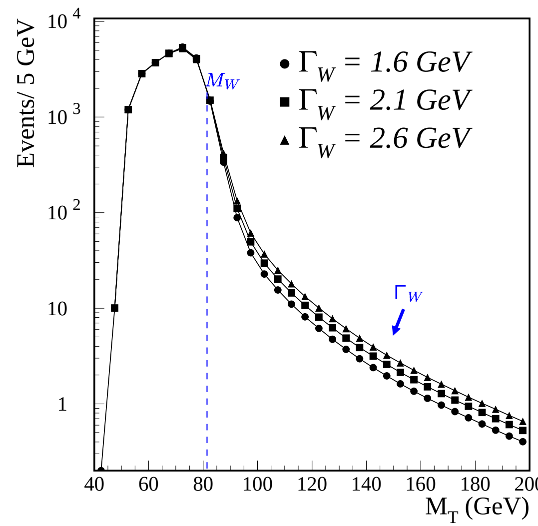
✓ Relevant kinematic region

$$\Lambda^2_{QCD} \ll M_H^2 \ll Q^2$$
$$0 \leq q_T^2$$

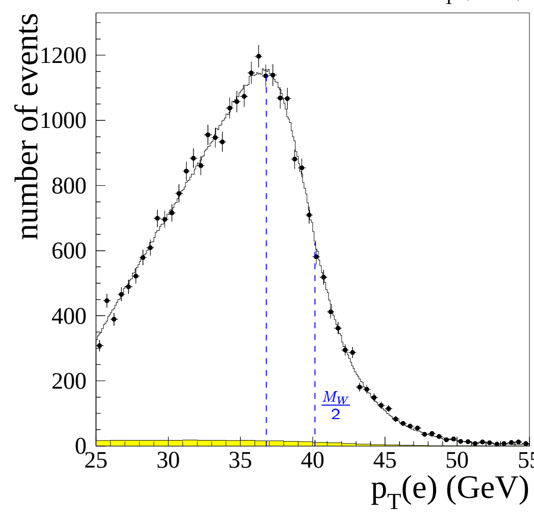
including $q_T^2 \sim M_H^2$

Measurement of W-boson mass M_W at the Tevatron

Tevatron Run-2 goal is to measure M_W with $\delta M_W \lesssim$ 30 MeV per experiment ($\delta M_W/M_W \lesssim 0.0004$)



M_W is derived from lepton distributions $d\sigma/dM_T^{\ell\nu}$ and $d\sigma/dp_T^{\ell}$, sensitive to q_T of W boson at $q_T \to 0$



The measurement of M_W relies on predicting $d\sigma/dq_T$ at $q_T \to 0$ with high precision!

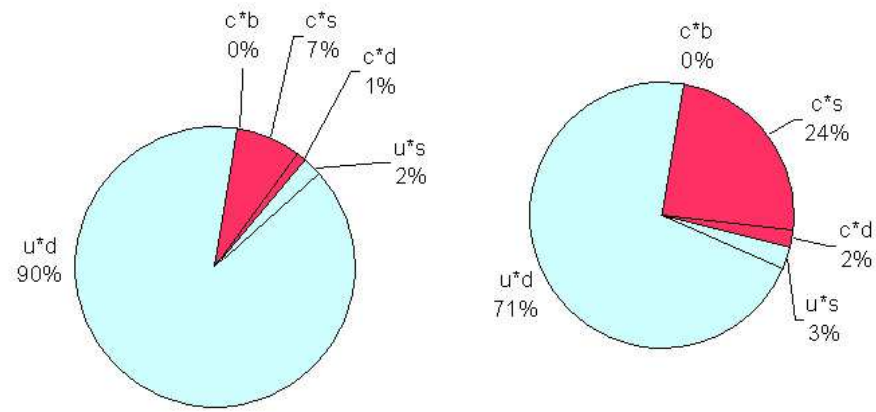
Heavy flavor contributions in vector boson production

Tevatron Run-2 (left); LHC (right)

$W^{\pm}$ boson production

Contributions of different flavors to

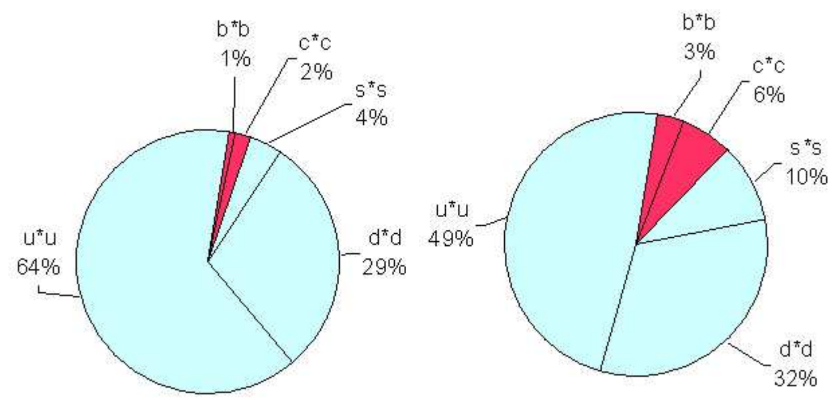
v_{CKM}^2*Parton luminosity $\sim \sigma_{\text{tot}}^{\text{Born}}$



Z boson production

Contributions of different flavors to

Parton luminosity$\sim \sigma_{\text{tot}}^{\text{Born}}$



Charm contributions in W production are sizable ($\sim$ 8% and 26%) and much larger than in Z production

It is well known how to calculate $d\sigma/dq_T$ at $q_T \to 0$ for massless quarks (resummation for $\ln^p(q_T^2/Q^2)$)

Massless approximation ignores terms $\sim M_H^2/q_T^2$, which may be important at small q_T

$$\frac{1}{q_T^2 + M_H^2} = \frac{1}{q_T^2}\left(\frac{1}{1+\frac{M_H^2}{q_T^2}}\right)$$

Estimate of mass effects would require simultaneous calculation of the sums

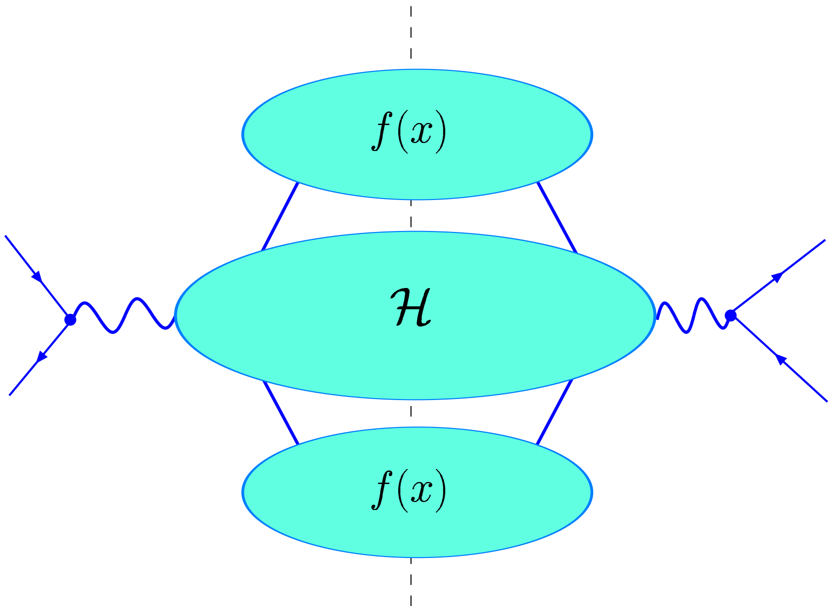
$$\sum_{k=1}^{\infty} \alpha_s^k \sum_p \ln^p(Q^2/M_H^2) \text{ and } \sum_{k'=1}^{\infty} \alpha_s^{k'} \sum_{p'} \ln^{p'}(q_T^2/Q^2)$$

QCD factorization at small q_T and $Q^2 \gg \{m_q^2\}$

(Collins, Soper, Sterman, 1985)

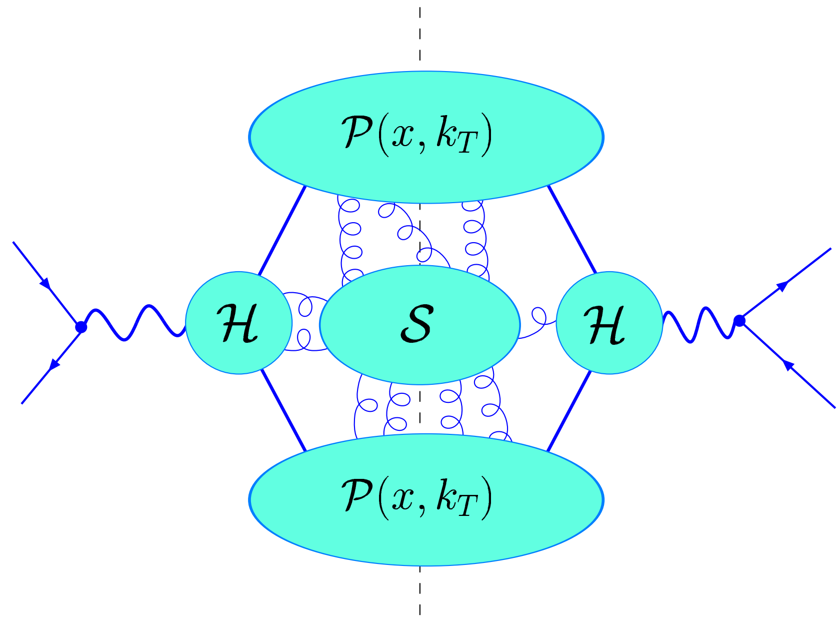
Finite-order (FO) factorization

$\Lambda^2_{QCD} \ll q_T^2 \sim Q^2$

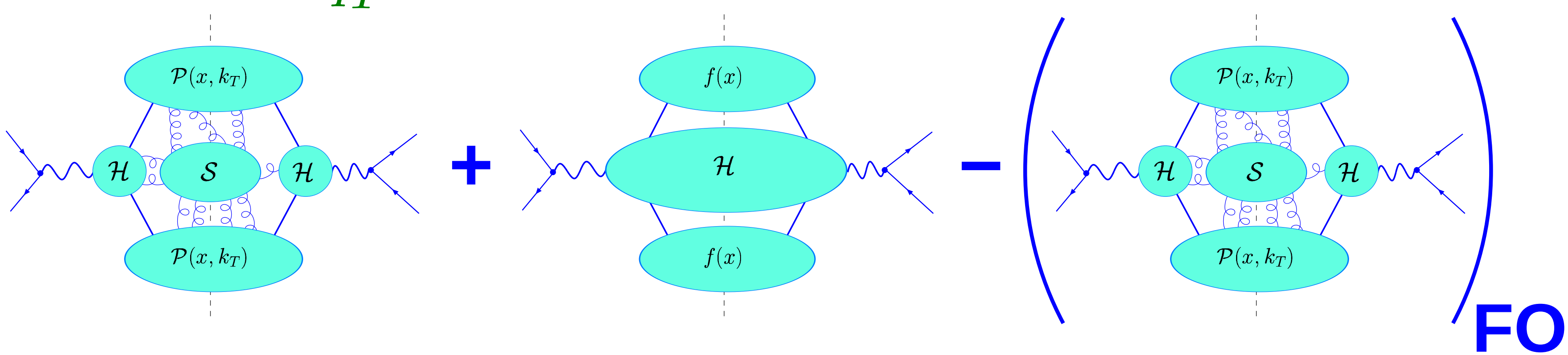


Small-q_T factorization

$\Lambda^2_{QCD} \ll q_T^2 \ll Q^2$



Solution for all q_T:



Factorization at $q_T \ll Q$

Realized in the space of the impact parameter b (conjugate to q_T)

$$\left.\frac{d\sigma_{AB\to VX}}{dQ^2 dy dq_T^2}\right|_{q_T^2 \ll Q^2} \sim \sum_{a,b=g,\overset{(-)}{u},\overset{(-)}{d},\ldots} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{q}_T\cdot\vec{b}} \widetilde{W}_{ab}(b,Q,x_A,x_B)$$

In the perturbative region ($b^2 \ll 1/\Lambda_{QCD}^2$):

$$\widetilde{W}_{ab}(b,Q,x_A,x_B) \approx \sum_j v_j\, e^{-S(b,Q)} \overline{\mathcal{P}}_a(x_A,b)\overline{\mathcal{P}}_b(x_B,b),$$

where

$$\overline{\mathcal{P}}_a(x,b) = \int d^{n-2}\vec{k}_T e^{-i\vec{k}_T\cdot\vec{b}} \mathcal{P}_a(x,\vec{k}_T)$$

and

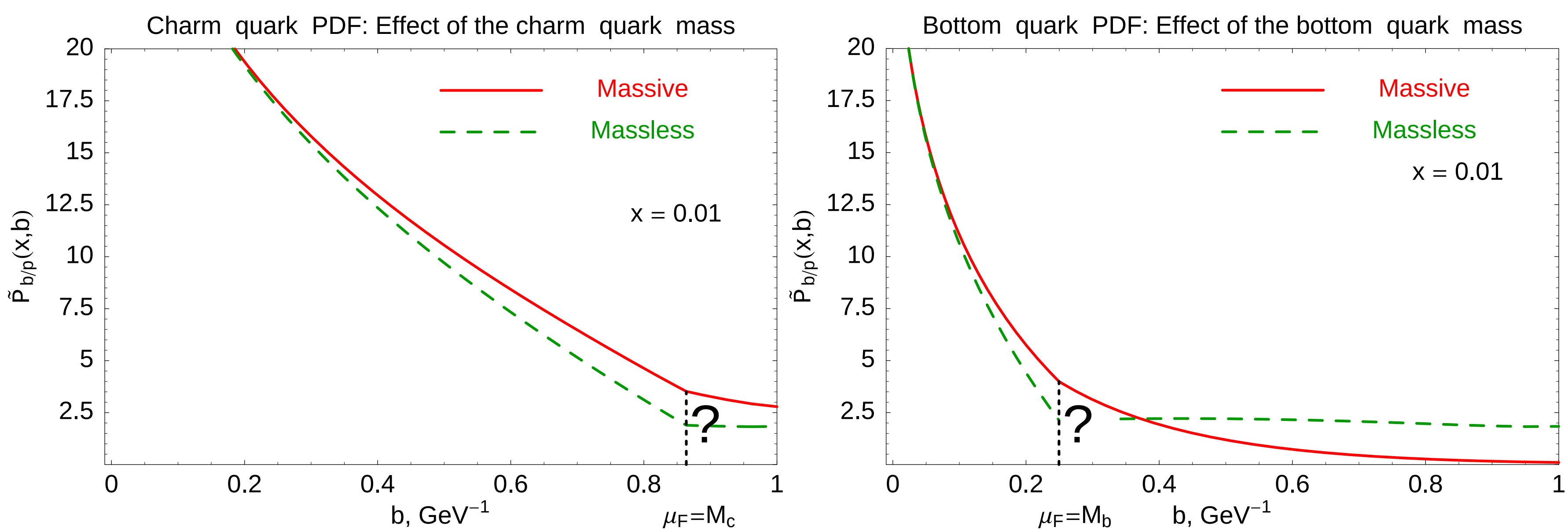
$$\overline{\mathcal{P}}_a(x,b) \equiv \sum_{i=g,u,d\ldots} \left[\mathcal{C}_{ai}\otimes f_i\right](x_A,\mu_F,b)$$

$S(b,Q)$ and $\mathcal{C}_{ai}(x_A,\mu_F b)$ are calculable in PQCD

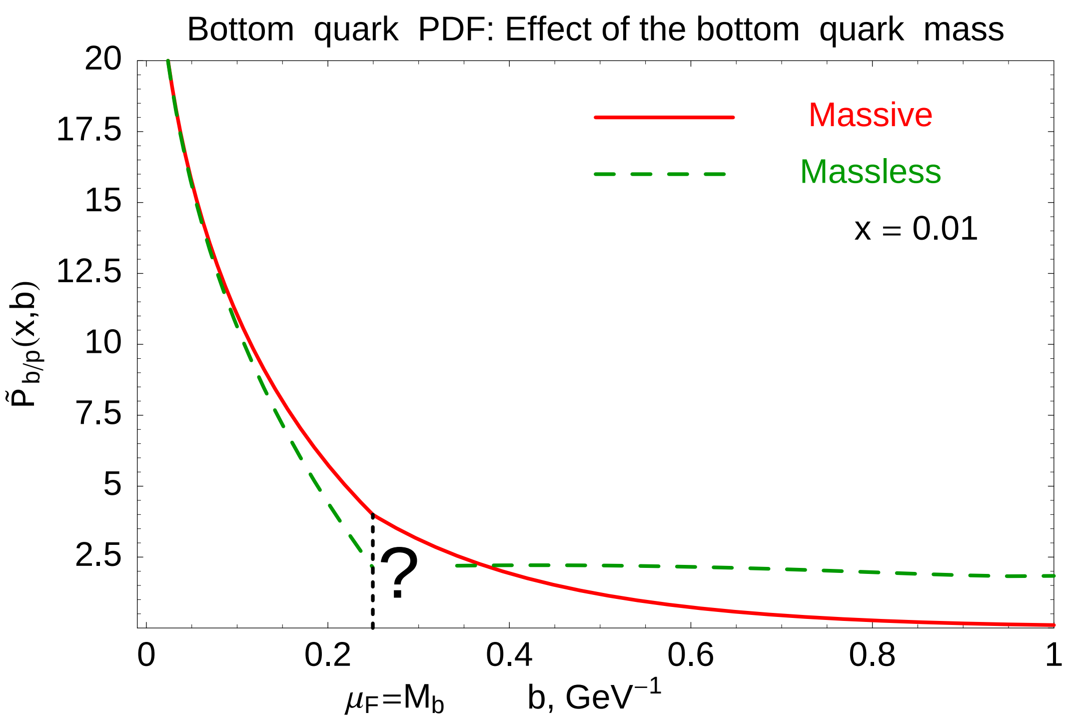
Failure of the massless approximation at $b \geq 1/M_H$

To calculate the Fourier integral, $\overline{\mathcal{P}}_a(x,b)$ must be defined in the whole range $0 \leq b \leq \infty$

$\overline{\mathcal{P}}_a(x,b)$ at $\mathcal{O}(\alpha_s)$ in the massless and massive VFN schemes



Failure of the massless approximation at $b \geq 1/M_H$ (continued)



$\mu_F \sim 1/b$

Massless $\overline{\mathcal{P}}_a(x,b)$

- ✓ underestimates the mass-dependent result at $b \lesssim 1/M_H$
- ✓ is ill-defined at $b \gtrsim 1/M_H$

Massive $\overline{\mathcal{P}}_a(x,b)$

- ✓ reduces to the massless result at $b^2 \ll 1/M_H^2$ ($\mu_F^2 \gg M_H^2$)
- ✓ vanishes at $b^2 \gg 1/M_H^2$ (decoupling)
- ✓ is automatically continuous at the switching point ($\mu_F = M_b$)

q_T resummation for DIS heavy flavor production

$$ep \to eQX$$

Relevant momentum scales:

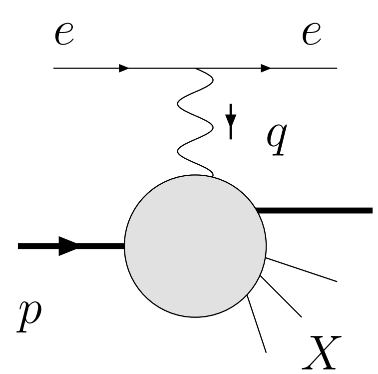
$$1\,\mathrm{GeV}^2 \ll M_H^2 \le Q^2;\ q_T^2$$

(P. N., Kidonakis, Olness, Yuan, Phys. Rev. D67, 074015 (2003))

Heavy flavor production in semi-inclusive deep inelastic scattering (SIDIS)

$e + p \rightarrow e + c + X$

$e + p \rightarrow e + b + X$



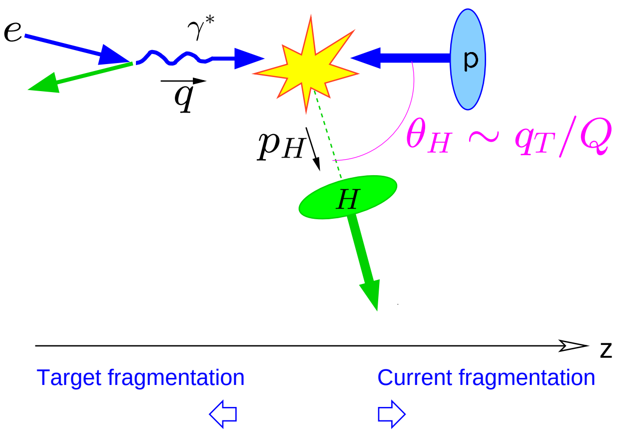
$Q^2 = -q^2$

$(M_H, \; q_T)$

$$\Lambda^2_{QCD} \ll M_H^2 \;\; \leq \;\; Q^2 < \infty$$
$$0 \;\; \leq \;\; q_T^2$$

q_T is a rescaled transverse momentum, related to the polar angle in the Breit frame

Semi-inclusive DIS in $\gamma^* p$ c.m. frame

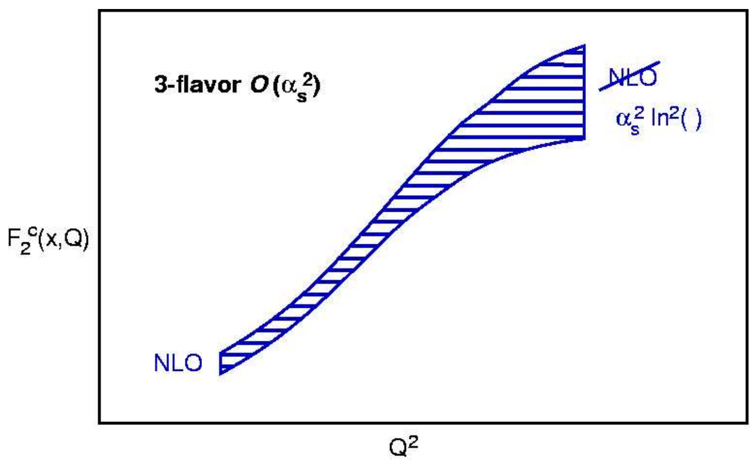


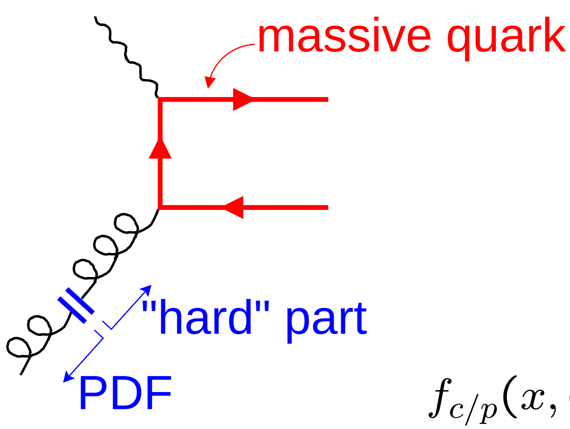
Factorization for k_T-integrated PDF's $f_a(x, Q^2)$

Are heavy quarks active partons?

Fixed Flavor Number (FFN) scheme

$M_H \neq 0$ for all $Q^2 \geq M_H^2$

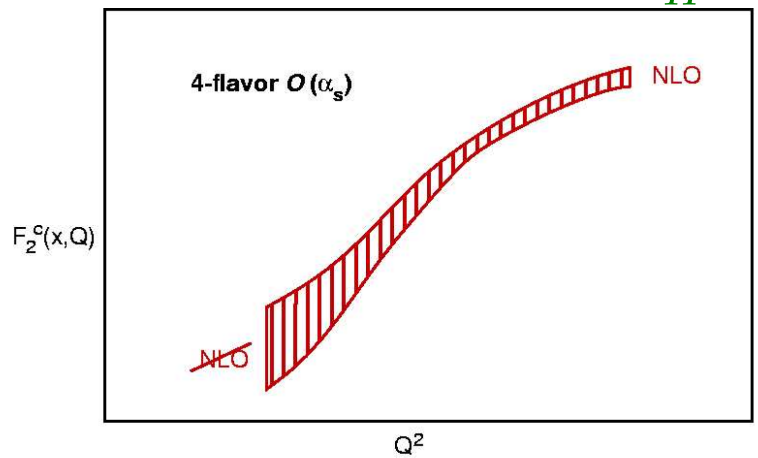


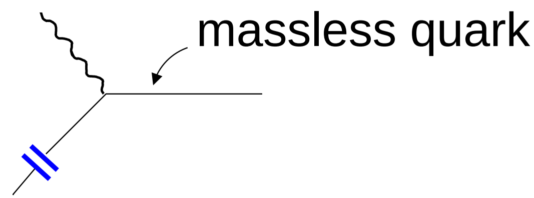


$$f_{c/p}(x,Q^2) \equiv 0$$

Massless Variable Flavor Number scheme

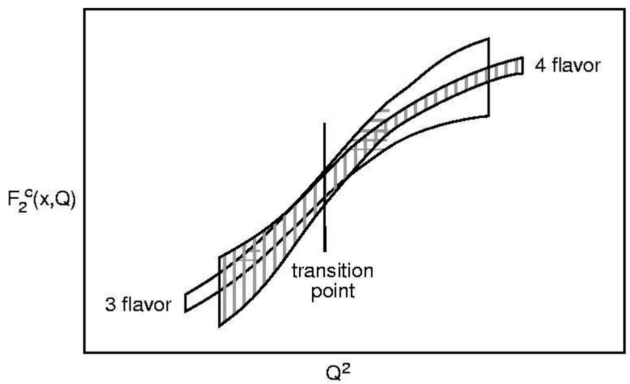
$M_H = 0$ for all $Q^2 \geq M_H^2$





$$f_{c/p}(x,Q^2) \sim \sum_{m,n=1}^{\infty} \alpha_S^n v_{nm} \frac{\ln^m(Q^2/M_H^2)}{m!}$$

Massive variable flavor number schemes

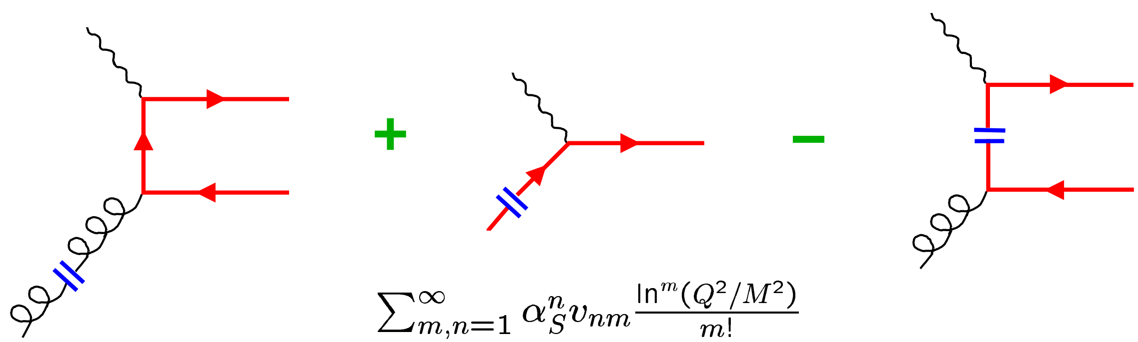


(Collins, 1998; Aivasis et al.; Chuvakin et al.; Thorne, Roberts; Kniehl et al.; Buza et al.; Cacciari et al.; ...)

$$F_2^c(x,Q,M_H) = \sigma_0 \sum_a \int_\zeta^1 \frac{d\xi}{\xi} C_{c\leftarrow a}\left(\frac{x}{\xi},\frac{Q}{\mu_F},\frac{M_H}{Q}\right) f_a\left(\xi,\frac{\mu_F}{M_H}\right) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)$$

$\lim_{Q\to\infty} C$ exists; no terms $\mathcal{O}(M_H/Q)$ in the remainder

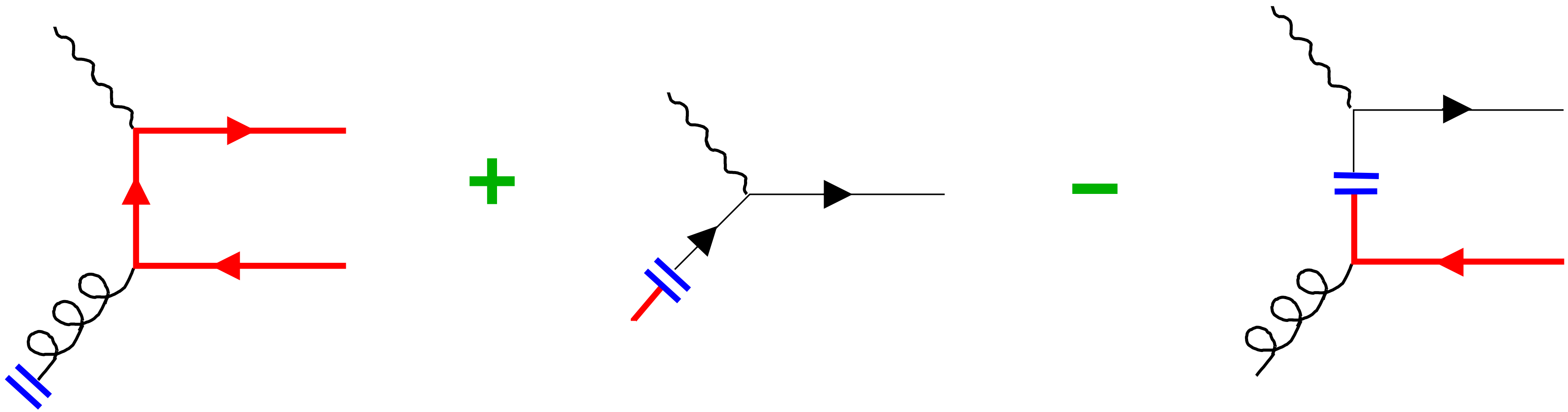
Aivasis, Collins, Olness, Tung (ACOT, 1994)



$\sum_{m,n=1}^{\infty} \alpha_S^n v_{nm} \frac{\ln^m(Q^2/M^2)}{m!}$

Simplified ACOT scheme

(Collins, 1998; Kramer, Olness, Soper, 2000)



- ✓ Set $M_H = 0$ in coefficient functions for incoming heavy quarks
- ✓ Not an approximation (exact factorization scheme)
- ✓ Significantly simplifies calculations
- ✓ Quark-initiated process from the massless calculation
- ✓ Close to the conventional ACOT scheme numerically

The Analytic Result for $F_2^H(x,Q^2)$

Which would you prefer to calculate?

ACOT

$$
\begin{aligned}
\hat{f}_1^q(\hat{s}_1) &= \frac{8}{\Delta'^2}\Bigg\{ -\Delta^2 (S_+\Sigma_{++} - 2m_1 m_2 S_-) I_q + 2m_1 m_2 S_- \left(\frac{1}{\hat{s}_1}(\Delta'^2 + 4m_2^2\Sigma_{+-}) \right. \\
&+ 2\Sigma_{+-} - \Sigma_{-+} + \frac{\Sigma_{++}+\hat{s}_1}{2} + \frac{\hat{s}_1 + m_2^2}{\Delta'\hat{s}_1}\left[\Delta'^2 + 2\Sigma_{+-}\Sigma_{++} + (m_2^2+Q^2)\,\hat{s}_1\right] I_q' \Big) \\
&+ S_+\Bigg(\frac{-m_2^2\Sigma_{++}}{(\hat{s}_1+m_2^2)\hat{s}_1}(\Delta^2 + 4m_2^2\Sigma_{+-}) - \frac{1}{4(\hat{s}_1+m_2^2)}\Big[3\Sigma_{++}^2\Sigma_{-+} + 4m_2^2(10\Sigma_{+-}\Sigma_{+-} \\
&- \Sigma_{-+}\Sigma_{++} - m_1^2\Sigma_{++}) + \hat{s}_1(-7\Sigma_{++}\Sigma_{-+} + 18\Delta^2 - 4m_1^2(7Q^2 - 4m_2^2 + 7m_1^2)) \\
&+ 3\hat{s}_1^2(\Sigma_{+-} - 2m_1^2) - \hat{s}_1^3\Big] + \frac{\hat{s}_1+m_2^2}{2\Delta'}\left[\frac{-2}{\hat{s}_1}\Sigma_{++}(\Delta^2 + 2\Sigma_{+-}\Sigma_{++}) \right. \\
&+ \left.(4m_1^2 m_2^2 - 7\Sigma_{+-}\Sigma_{++}) - 4\Sigma_{+-}\hat{s}_1 - \hat{s}_1^2\right] I_q' \Bigg)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\hat{f}_2^q(\hat{s}_1) &= \frac{16}{\Delta'^2}\Bigg\{ -2\Delta^4 S_+ I_q + 2m_1 m_2 S_- \left(\frac{\hat{s}_1+m_2^2}{\Delta'}\left(\Delta'^2 - 6m_2^2Q^2\right) I_q' \right. \\
&- \frac{\Delta'^2(\hat{s}_1+\Sigma_{+-})}{2(\hat{s}_1+m_2^2)} + \left(2\Delta'^2 - 3Q^2(\hat{s}_1+\Sigma_{+-})\right)\Big) + S_+\Big(-2(\Delta^2 - 6m_1^2Q^2)(\hat{s}_1+m_2^2) \\
&- 2(m_1^2+m_2^2)\hat{s}_1^2 - 9m_2^2\Sigma_{+-}^2 + \Delta^2(2\Sigma_{++} - m_2^2) + 2\hat{s}_1(2\Delta^2 + (m_1^2 - 5m_2^2)\Sigma_{+-}) \\
&+ \frac{(\Delta'^2 - 6Q^2(m_2^2+\hat{s}_1))\Sigma_{++}(\hat{s}_1+\Sigma_{+-})}{2(\hat{s}_1+m_2^2)} - \frac{2\Delta^2}{\hat{s}_1}(\Delta^2 + 2(2m_2^2+\hat{s}_1)\Sigma_{+-}) \\
&+ \frac{\hat{s}_1+m_2^2}{\Delta'}\Big[\frac{-2}{\hat{s}_1}\Delta^2(\Delta^2 + 2\Sigma_{+-}\Sigma_{++}) - 2\hat{s}_1(\Delta^2 - 6m_1^2Q^2) \\
&- (\Delta'^2 - 18m_1^2Q^2)\Sigma_{++} - 2\Delta^2(\Sigma_{++} + 2\Sigma_{+-})\Big] I_q' \Big)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\hat{f}_3^q(\hat{s}_1) &= \frac{16}{\Delta'^2}\Bigg\{ -2\Delta^2 R_- I_q + 2m_1 m_2 R_-\left(1 - \frac{\Sigma_{-+}}{\hat{s}_1} + \frac{(\hat{s}_1+m_2^2)(\hat{s}_1+\Sigma_{+-})}{\Delta'\hat{s}_1} I_q'\right) \\
&+ R_+\Bigg(\Sigma_{-+} - 3\Sigma_{+-} - \frac{2}{\hat{s}_1}(\Delta^2 + 2m_2^2\Sigma_{+-}) - \frac{(\hat{s}_1 - \Sigma_{-+})(\hat{s}_1+\Sigma_{++})}{2(\hat{s}_1+m_2^2)} \\
&+ \frac{\hat{s}_1+m_2^2}{\Delta'\hat{s}_1}\left[-\hat{s}_1^2 + 4(m_1^2\Sigma_{-+} - \Delta^2) - 3\hat{s}_1\Sigma_{+-}\right] I_q'\Bigg)\Bigg\}
\end{aligned}
$$

with

$$
I_q = \ln\left(\frac{\Sigma_{++} + \hat{s}_1 - \Delta'}{\Sigma_{++} + \hat{s}_1 + \Delta'}\right)
$$

and

$$
I_q' = \left(\frac{\hat{s}_1 + 2m_2^2}{\hat{s}_1^2} + \frac{\hat{s}_1 + m_2^2}{\Delta'\hat{s}_1^2}\Sigma_{++} I_q\right).
$$

S-ACOT

$$
\begin{aligned}
C_2^{(Va)(1)} &= C_2(F)\frac{x}{2}\left[\frac{1+x^2}{1-x}\left(\ln\frac{(1-x)}{x} - \frac{3}{4}\right) + \frac{1}{4}(9+5x)\right]_+ , \\
C_L^{(Va)(1)} &= \frac{1}{2x}C_2^{(Va)(1)} - C_2(F)\frac{1}{2}x , \\
C_3^{(Va)(1)} &= \frac{1}{x}C_2^{(Va)(1)} - C_2(F)(1+x) ,
\end{aligned}
$$

Resummation for heavy flavors: our procedure

1. Start from the resummed form factor for $m_q \neq 0$, any b *(CSS, 1985)*

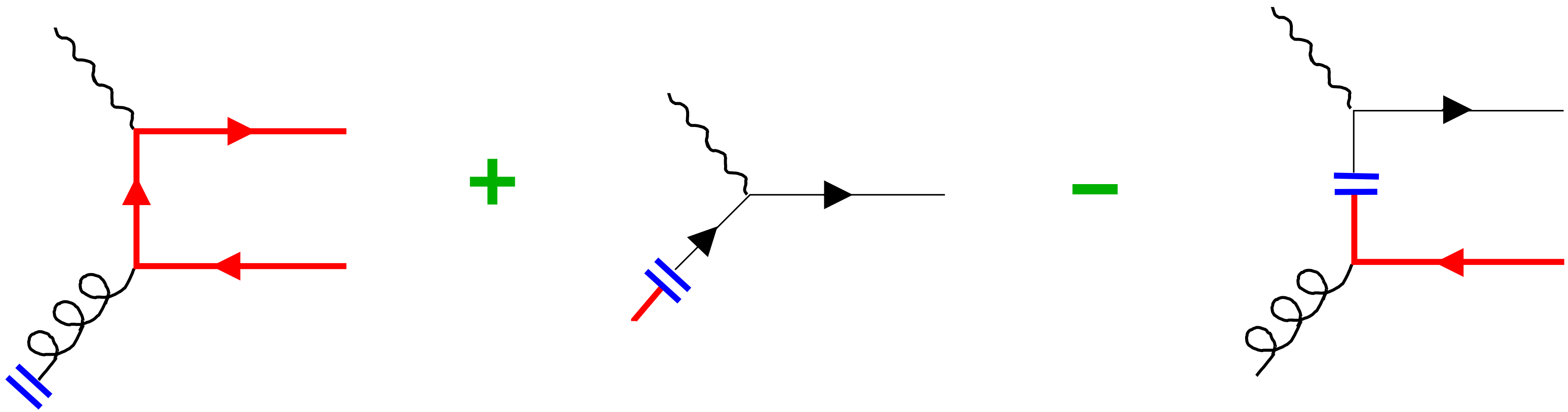
$$\widetilde{W}(b,Q,x,z) = \sum_j e_j^2\, \overline{\mathscr{P}}^{out}_{H/j}(z,b,\{m_q\})\, \overline{\mathscr{P}}^{in}_{j/A}(x,b,\{m_q\})$$
$$\times \exp\left\{ -\int_{C_1^2/b^2}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2 Q^2}{\bar{\mu}^2}\right) \mathcal{A}(m_q) + \mathcal{B}(m_q)\right]\right\}.$$

2. Calculate $\widetilde{W}$ for $M_H \neq 0$ from its definition using the simplified ACOT scheme

3. $M_H^2 \gg \Lambda^2_{QCD}$: perturbative QCD is applicable

$$\overline{\mathscr{P}}^{in}_{j/A}(x,b,\{m_q\})\Big|_{\Lambda^2 \ll 1/b^2 \sim M_H^2} = \sum_a \int_x^1 \frac{d\xi}{\xi} \mathcal{C}_{j/a}\left(\frac{x}{\xi}, \mu_F b, bM_H\right) f_{a/A}\left(\xi, \frac{\mu_F}{M}\right)$$

Simplified ACOT factorization scheme

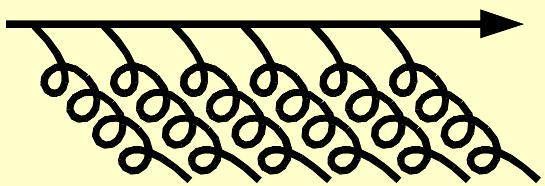


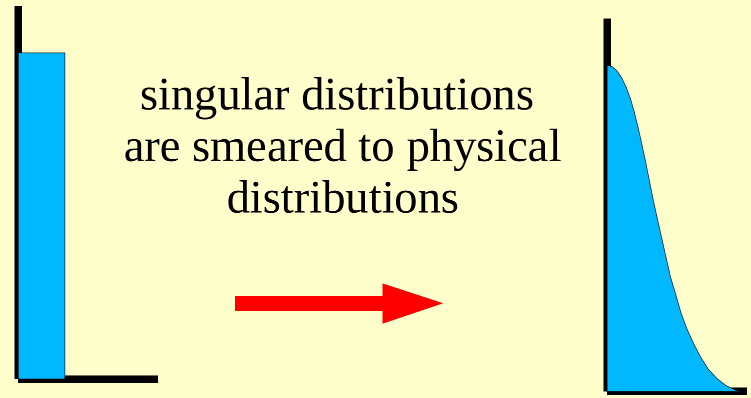
Set $M_H = 0$ in hard subgraphs with incoming heavy quarks

Only graphs with explicit flavor creation retain $M_H \neq 0$

M_H is dropped in the Sudakov factor $S(b, Q)$ and functions $\mathcal{C}^{in}_{jq}(x, b\mu_F)$, $\mathcal{C}^{out}_{bj}(z, b\mu_F)$ with incoming heavy quarks

Sudakov Resummation of Soft Gluon Radiation

 $= \; e^{-S}$

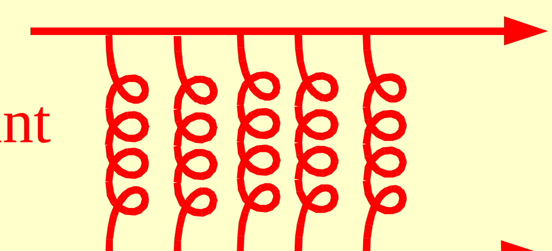


$$S_{ba}(b, Q, M_H) = \int \frac{d\mu^2}{\mu^2} \left\{ A(\alpha_s, M_H) \ln\left(\frac{Q^2}{\mu^2}\right) + B(\alpha_s, M_H) \right\} + S_{Non-Pert}$$

Result: In Simplified-ACOT scheme, we obtain:

$$A(\alpha_s, M_H) = A(\alpha_s, 0)$$
$$B(\alpha_s, M_H) = B(\alpha_s, 0)$$

Why: Dominant



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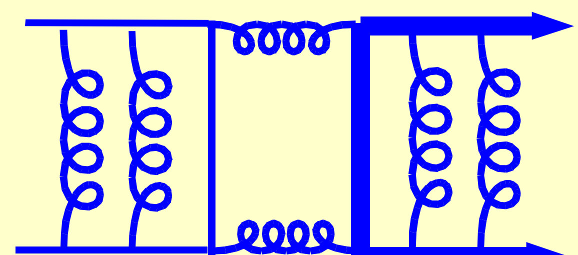


$$\sigma = \text{RES} + \text{FO} - \text{SUBTRACTION}$$

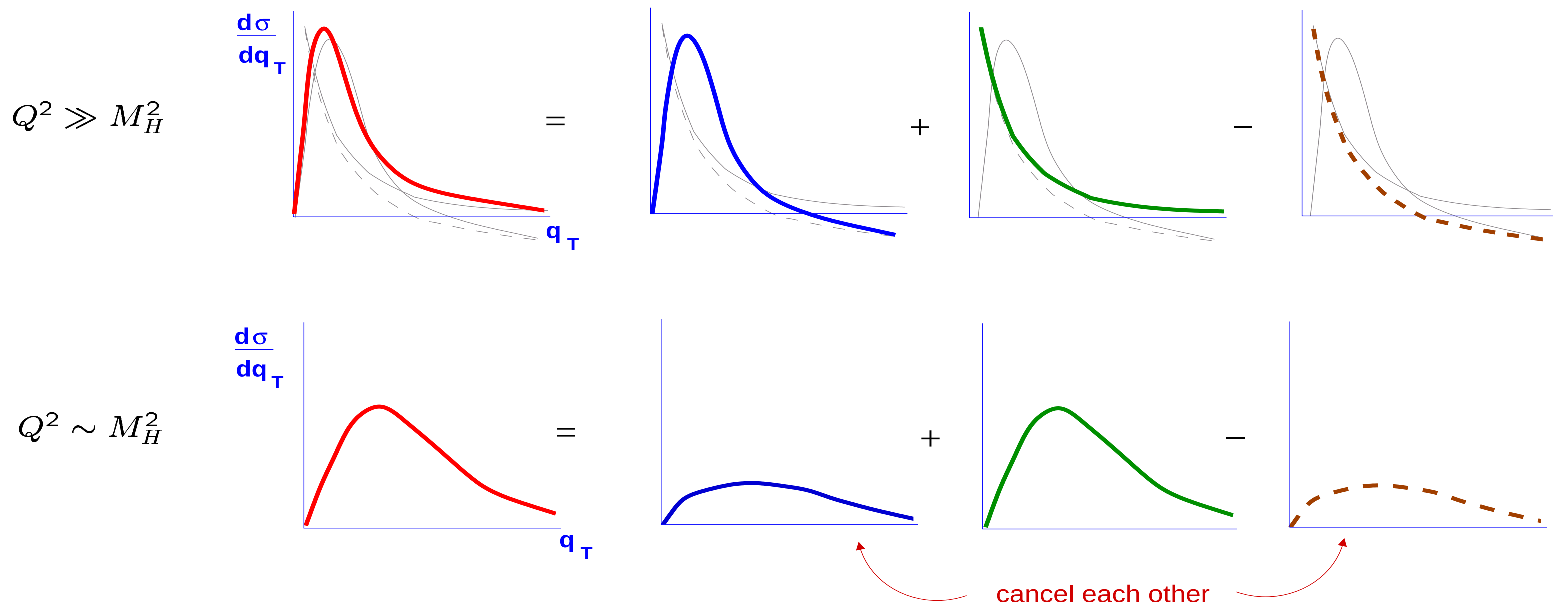
RES: $\sum_{m,n=1}^{\infty} \alpha_S^n v_{nm} \frac{\ln^m (Q^2/M^2)}{m!}$

$\frac{1}{q_T^2} \sum_{n=1}^{\infty} \alpha_S^n \times$
$\times \sum_{m=0}^{2n-1} w_{nm} \ln^m (q_T^2/Q^2)$

Calculate according to its definition for $M_H \neq 0$

Calculate in perturbative QCD at order α_S^n

Calculate as an expansion of RES up to order α_S^n



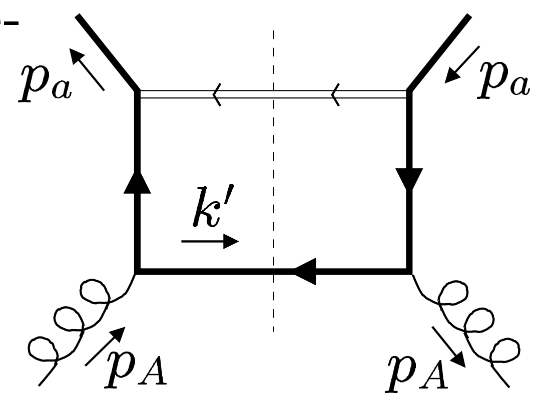
σ contains all terms $\propto (M_H/Q)^n$ at $Q \sim M_H$ and correctly sums large logs at $Q^2 \gg M_H^2$

$\mathcal{O}(\alpha_S)$ resummed cross section

Simplified ACOT scheme was used

$\therefore M_H \neq 0$ only in the gluon-initiated channels

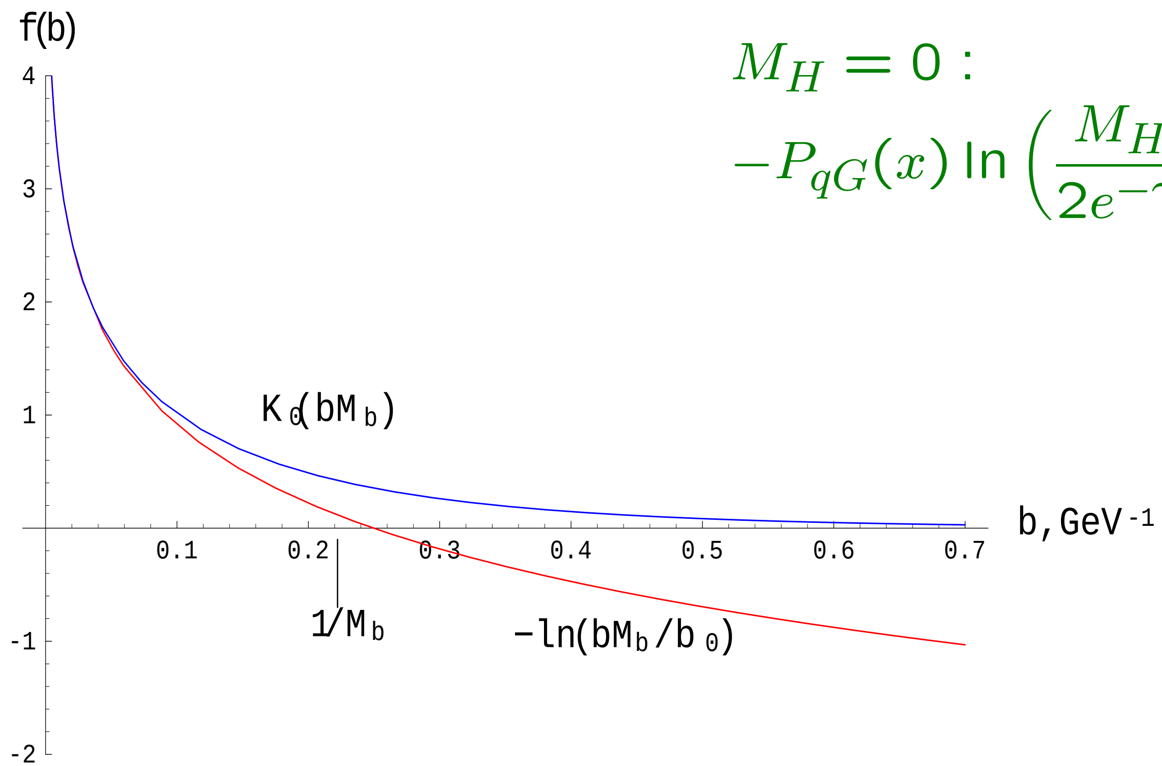
Only $\mathcal{C}^{in}_{jg}(x, b, M, \mu_F)$ retains mass
☞ derived from the k_T-dependent PDF as described by Collins & Soper, 1981



$$\mathscr{P}^{in}_{j/A}(x, \vec{k}_T, M, \zeta) = \int \frac{dy^- d\vec{y}_T}{(2\pi)^2} e^{-ixp^+y^- + i\vec{k}_T\cdot\vec{y}_T}$$
$$\times \quad \langle p|\bar{\psi}_j(0, y^-, \vec{y}_T)\frac{\gamma^+}{2}\psi(0)|p\rangle$$

in the gauge $\eta \cdot A = 0$, $\eta^2 < 0$; $\zeta = (p\cdot\eta)/|\eta|^2 \to \infty$

$M_H \neq 0$ suppresses contributions from $1/b \lesssim M_H$



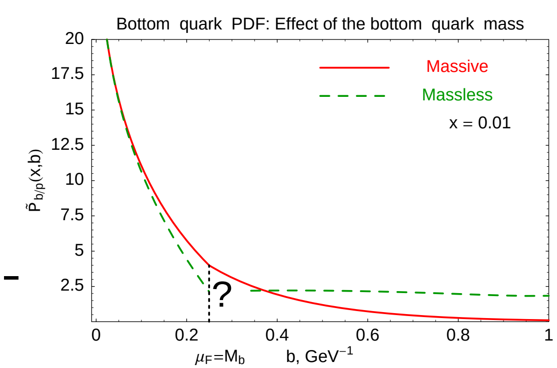
$M_H = 0$:

$-P_{qG}(x) \ln \left(\frac{M_H b}{2e^{-\gamma_E}} \right)$

$M_H \neq 0$:

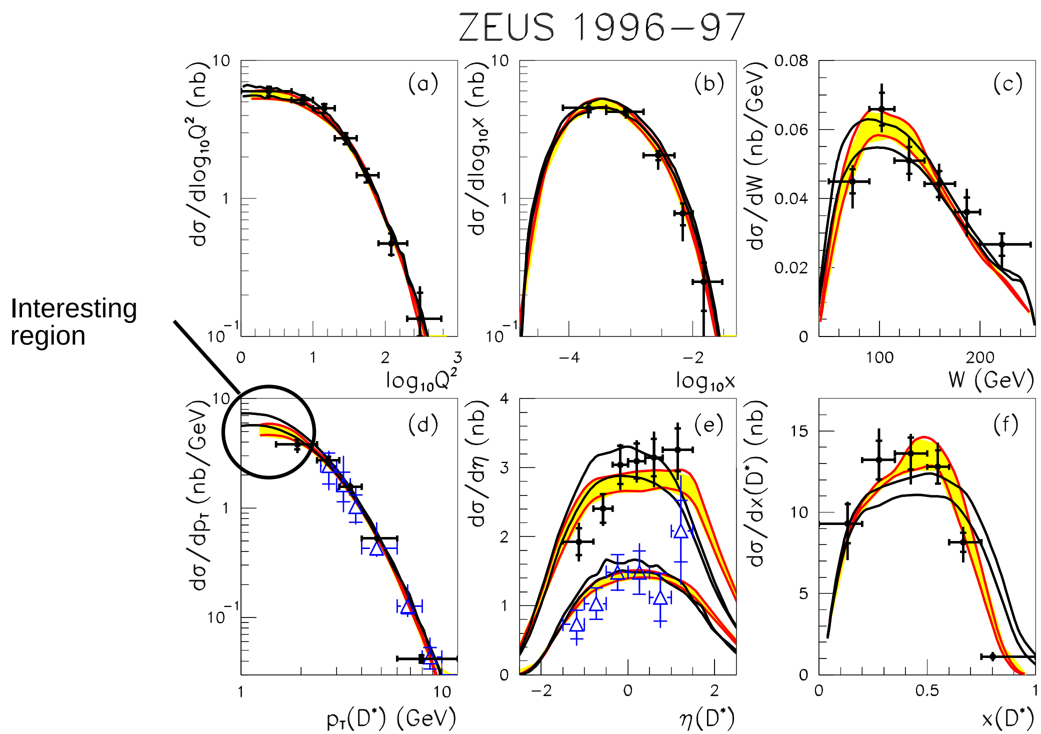
$P_{qG}(x) K_0 (M_H b)$

(modified Bessel function)



For $M_H^2 \gg \Lambda_{QCD}^2$, the resummed cross section can be calculated without the non-perturbative input from $b \sim \Lambda_{QCD}$!

Heavy flavor production at ep collider HERA



30 – 70% of published cross section for charm production is reconstructed by extrapolation using a theoretical model (currently 3-flavor number factorization scheme)

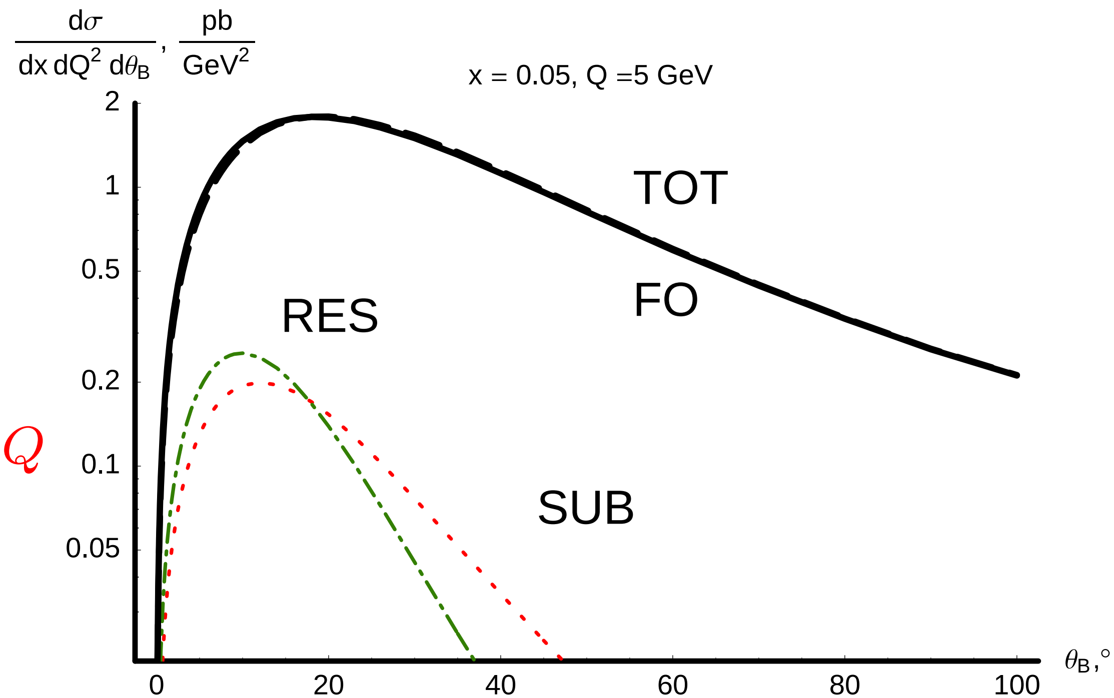
Resummed cross section can be used for consistent extrapolation of differential distributions in the whole range $M_H \leq Q \leq \infty$

⇒Monte-Carlo integrator in the S-ACOT scheme

Distributions of bottom quarks in the $\gamma^* p$ c.m. frame
Threshold region $Q = 5$ GeV

$Q^2 \sim M_b^2$

$\theta_B \approx q_T/Q$



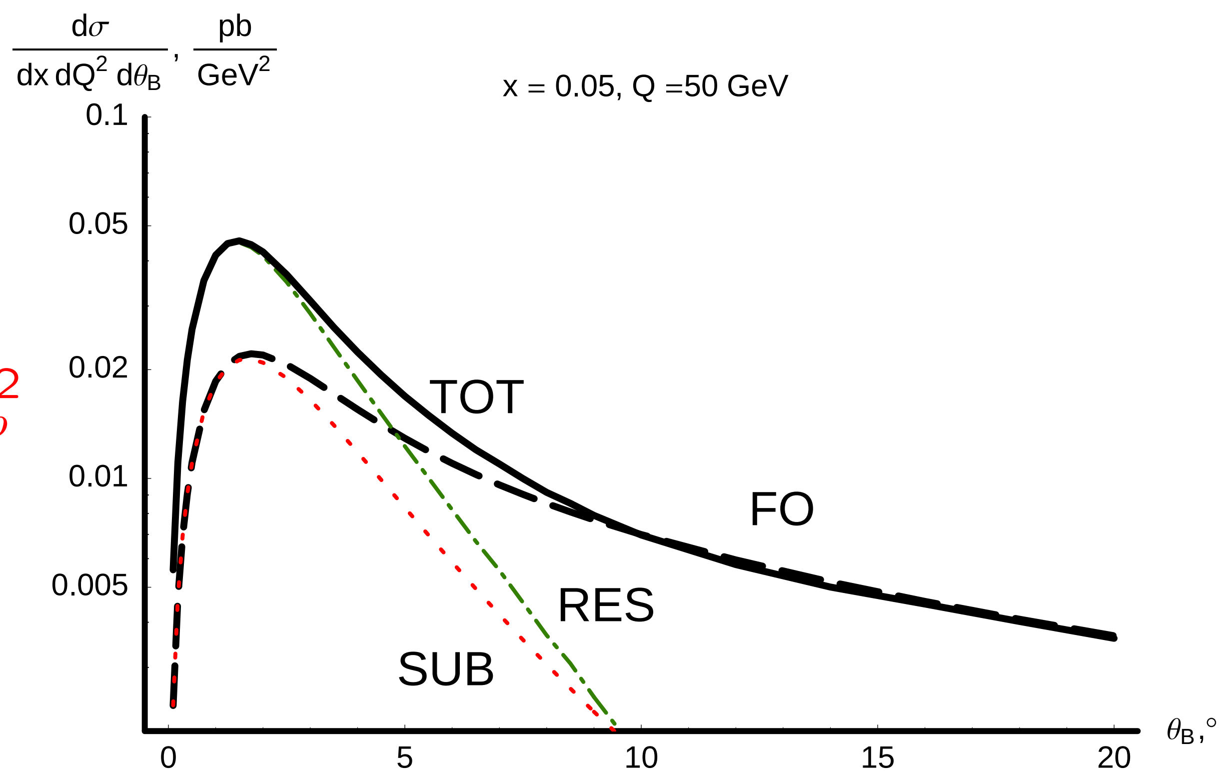
$$\sigma_{TOT} = \sigma_{RES} + \sigma_{FO} - \sigma_{SUB} \approx \sigma_{FO}$$

$\sqrt{S_{ep}} = 300$ GeV; $M_b = 4.5$ GeV; CTEQ5HQ PDFs; Peterson FF's; $S^{NP}(b,Q) = 0$; $b_{max} = 1.123$ GeV^{-1}

Distributions of bottom quarks in the $\gamma^* p$ c.m. frame

$Q = 50$ GeV

$Q^2 \gg M_b^2$



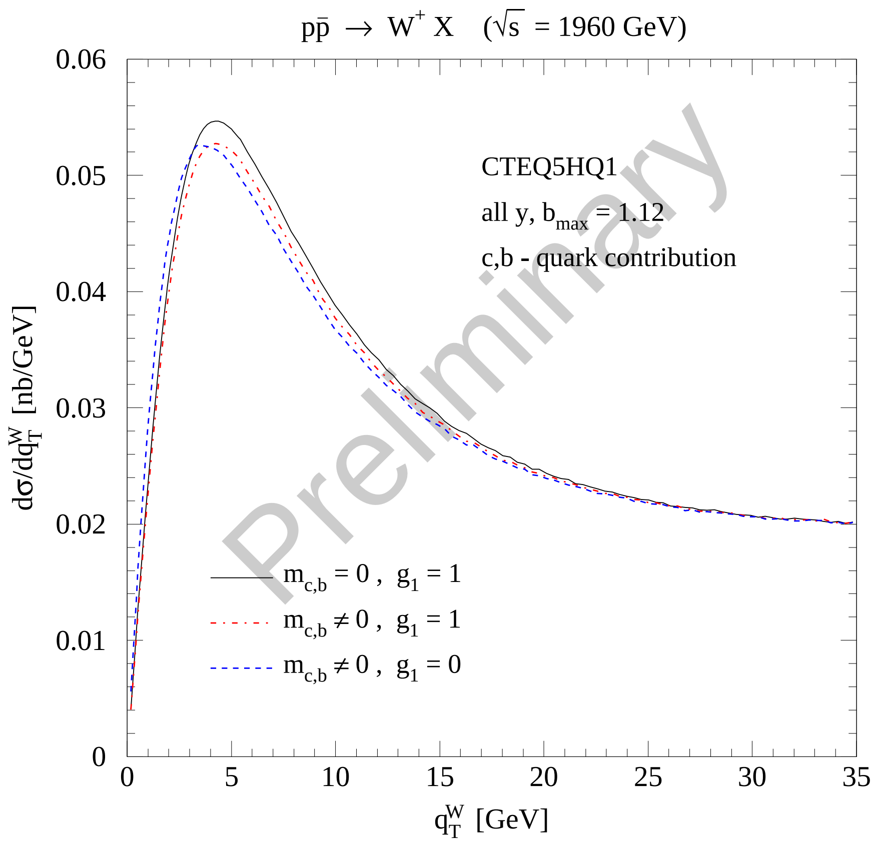
$\theta_B \to 0$: σ_{TOT}(solid) $\approx \sigma_{RES}$(dot-dashed)

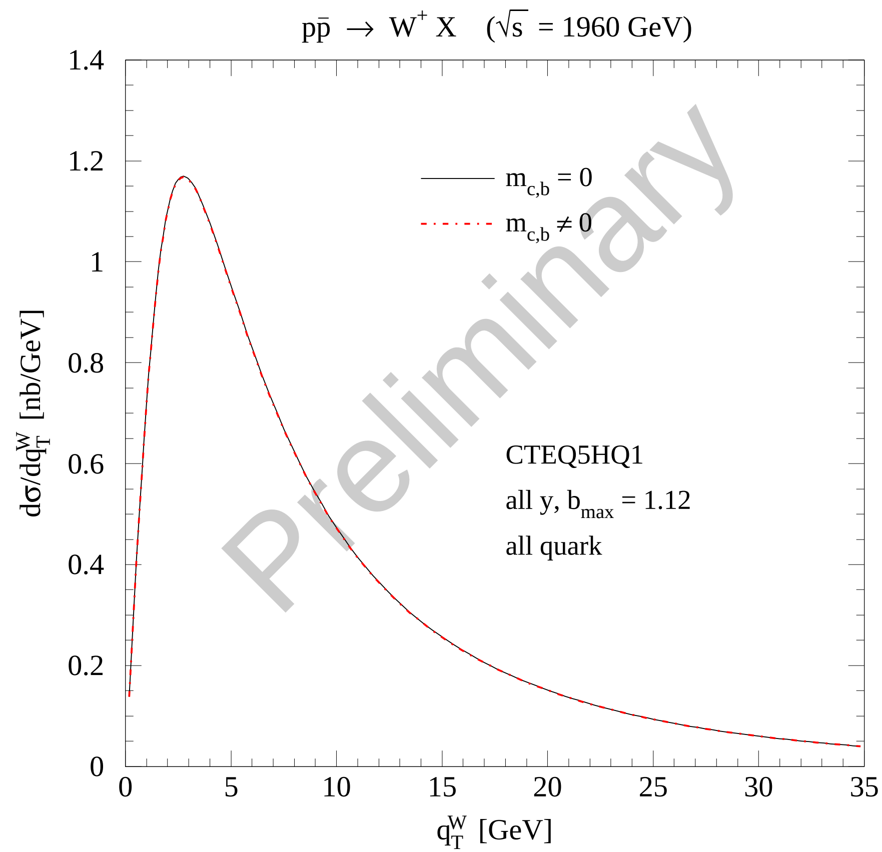
$\theta_B \gg 0$: σ_{TOT}(solid) $\approx \sigma_{FO}$(dashed)

Enhancement of 25% in $F_2^b(x, Q^2)$ as compared to FFN

Back to W boson production...

Massive vs. massless results for $d\sigma(p\bar{p} \to W^+X)/dq_T$

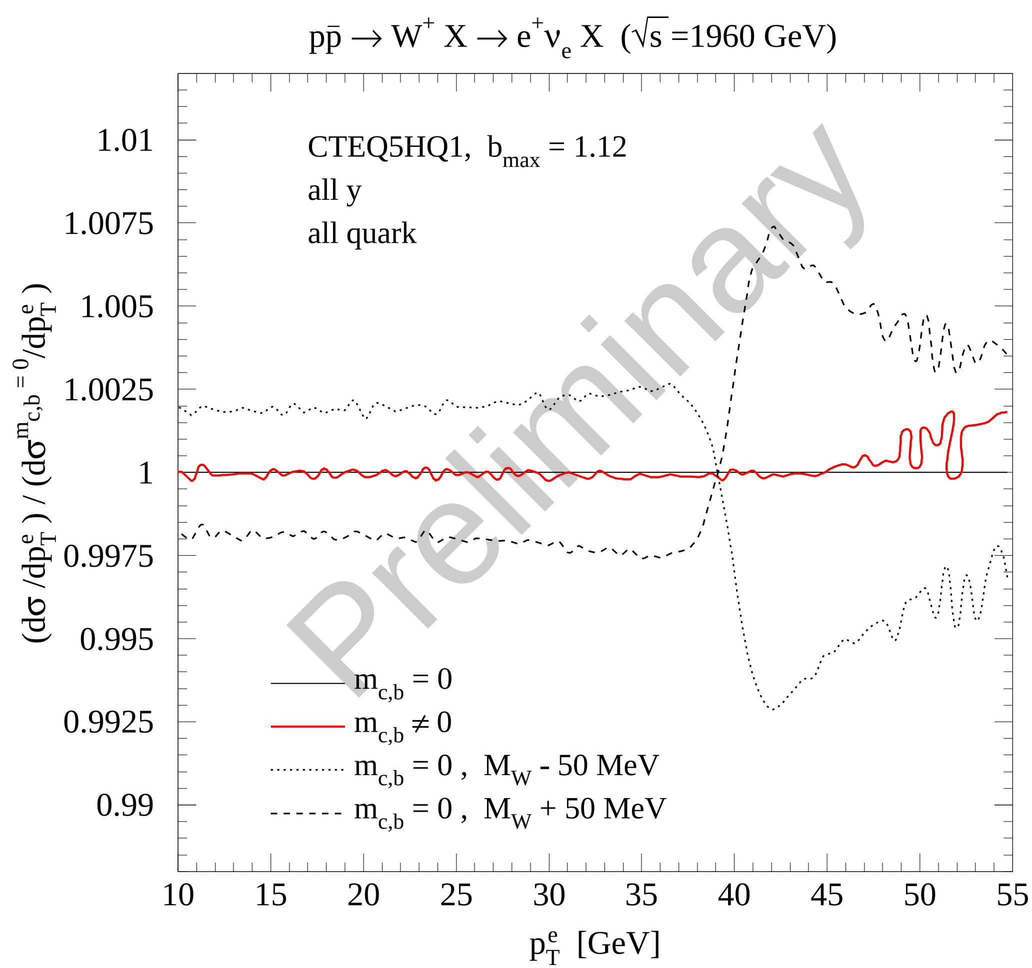




Strong effect of M_c on the charm contribution at $q_T < 15$ GeV...

...but effect is small as compared to other channels

Effect on electron p_T



Effect of $M_c \neq 0$ is larger in W boson production at the LHC

Summary

- ✓ New procedure (CSS+ACOT) for resummation in heavy flavor production
  - ♦ CSS resummation for three momentum scales (q_T, Q, M_H)
  - ♦ Resummation that does not use the dimensional regularization for collinear singularities
  - ♦ Correct behavior at the threshold
  - ♦ Very soft physics suppressed by $M_H^2 \gg \Lambda_{QCD}^2$; nonperturbative Sudakov contributions can be dropped
- ✓ At $Q^2 \gg M_H^2$, event rate enhancement as compared to the fixed-flavor number factorization scheme
  - ♦ Larger reconstructed $F_2^{c,b}(x, Q^2)$ at HERA
- ✓ Effects of $M_c \neq 0$ on the measurement of M_W in the Tevatron Run-2 are not substantial

Future applications

✓ $\mathcal{O}(\alpha_s^2)$ flavor creation contribution to the resummed piece

✓ Higgs boson production in 2 Higgs-doublet model
$b + \bar{b} \rightarrow$ Higgs $+ X$, $b + \bar{b} \rightarrow b +$ Higgs $+ X$

✓ Single-top production at the Tevatron, e.g. $u\bar{b} \rightarrow \bar{d}t$