

QCD Calculations at NLO: adding showers

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Some references

M. Krämer and D. E. Soper, Phys. Rev. D **66**, 054017 (2002) [arXiv:hep-ph/0204113].

M. Krämer and D. E. Soper, Phys. Rev. D **69**, 054019 (2004) [arXiv:hep-ph/0306222].

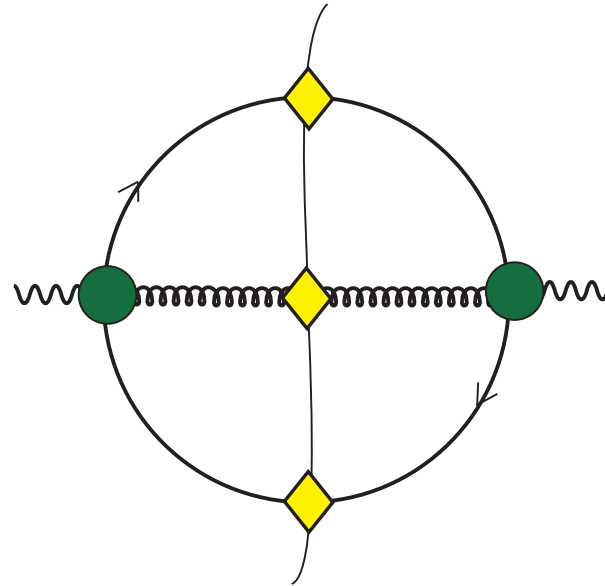
D. E. Soper, Phys. Rev. D **69**, 054020 (2004) [arXiv:hep-ph/0306268].

S. Frixione and B. R. Webber, JHEP **2606**, 029 (2002) [arXiv:hep-ph/0204244].

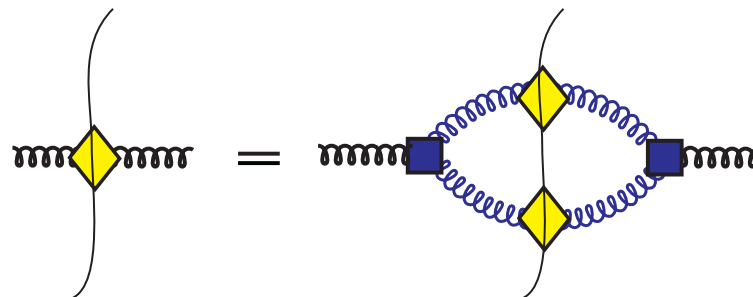
S. Frixione, P. Nason, and B. R. Webber, JHEP **0308**, 007 (2003) [arXiv:hep-ph/0305252].

The idea of showers

From α_s^B diagrams.
Each parton splits.
Partons \rightarrow showers.



In a shower the parton splits.
Daughters make showers.



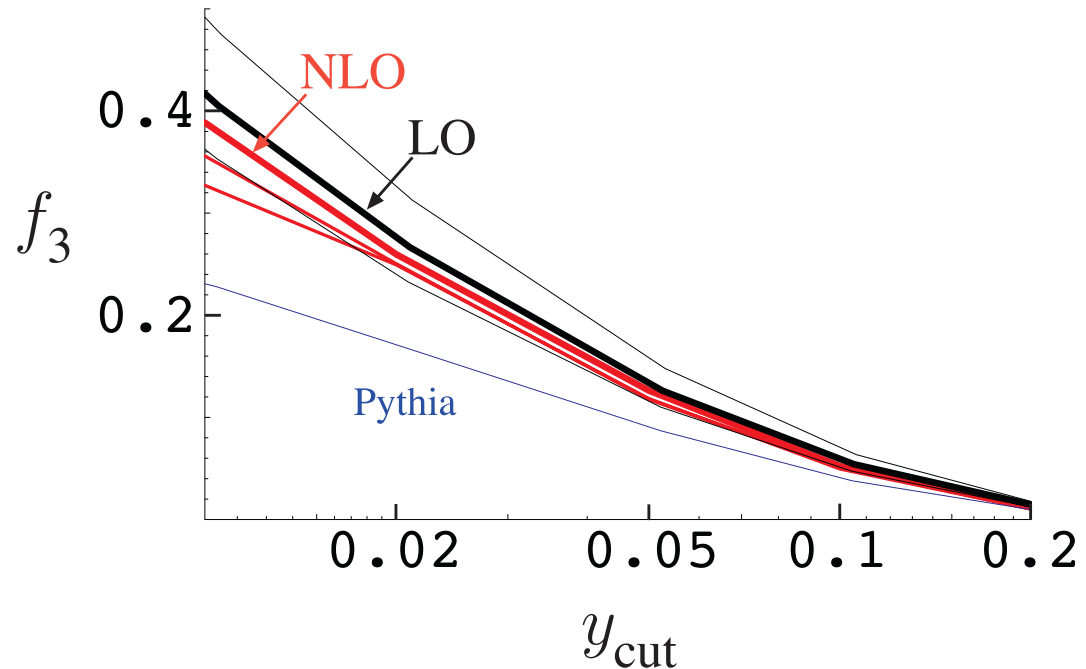
- After showers, there is a model for partons \rightarrow hadrons.

- Parton shower Monte Carlo programs produce a list of events.
- Each event is a list of the final state particles and their momenta, together, possibly, with a weight for the event.
- The user analyses the simulated events to get a prediction.
- These programs originated in the mid 1980s. Among the inventors are Thomas Gottschalk, Bryan Webber, and Torbjörn Sjöstrand.
- Without them, our ability to learn about nature from experiments would be substantially degraded.

What is good about NLO calculations?

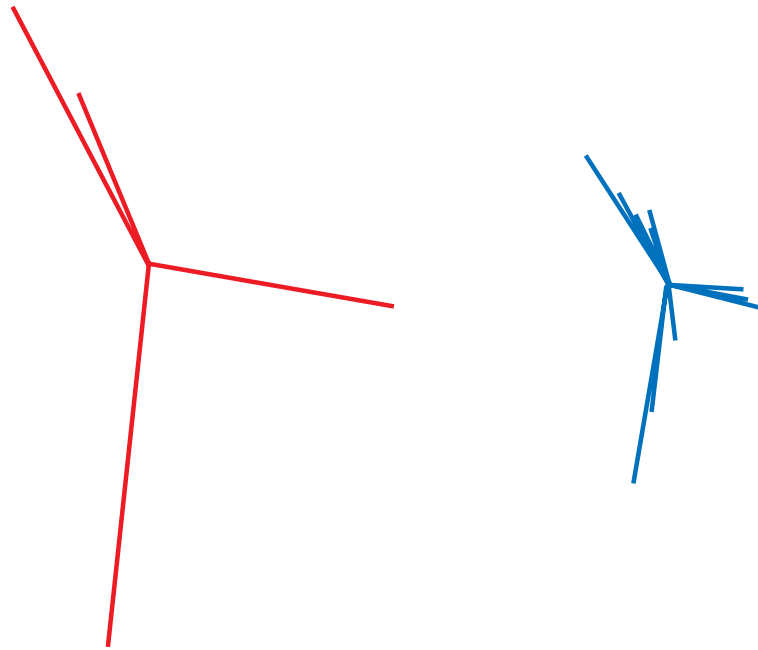
Precision (estimated) is better than with LO calculations; new effects can appear at NLO.

Three jet fraction in e^+e^- annihilation vs. y_{cut} at LO and NLO. (Durham or k_T algorithm; $\mu = \sqrt{s}/6$; μ varied by a factor 2.)



- Nb. **Need an infrared safe observable.**

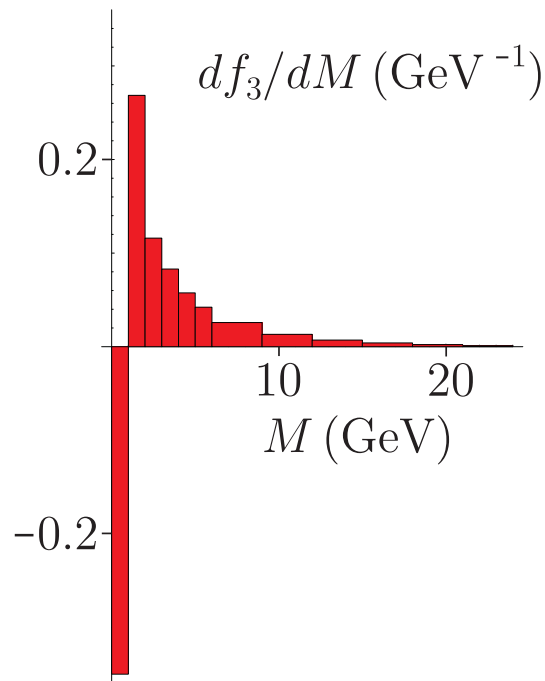
What is the problem with NLO calculations?



NLO: Few parton states

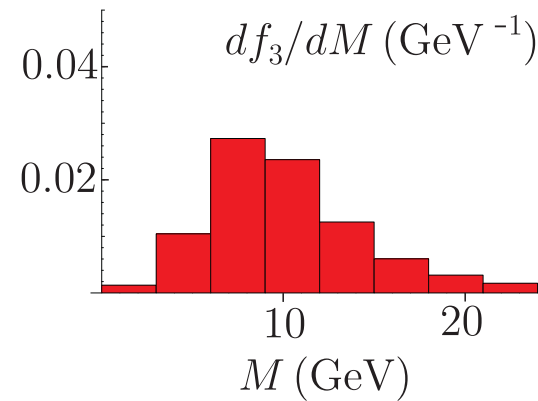
Pythia (LO) : Many hadron states

Examine df_3/dM where f_3 is the fraction of events that have three jets (Durham algorithm, $y_{\text{cut}} = 0.05$) and M is the mass of a jet (so each event contributes three jets).



NLO

Wrong jet structure



Pythia (LO)

Realistic jet structure

Adding showers

- Produce realistic final states f . (Hadrons, or just a parton shower down to some minimum virtuality.)

$$\mathcal{I} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N w_n \mathcal{S}(f_n).$$

- For an infrared safe observable, the result \mathcal{I} should have a perturbative expansion,

$$\mathcal{I} = C_0 \alpha_s^B + C_1 \alpha_s^{B+1} + C_2 \alpha_s^{B+2} + \dots$$

- Keep C_0 and C_1 exactly the same.
- Cf. Frixione, Nason, & Webber.

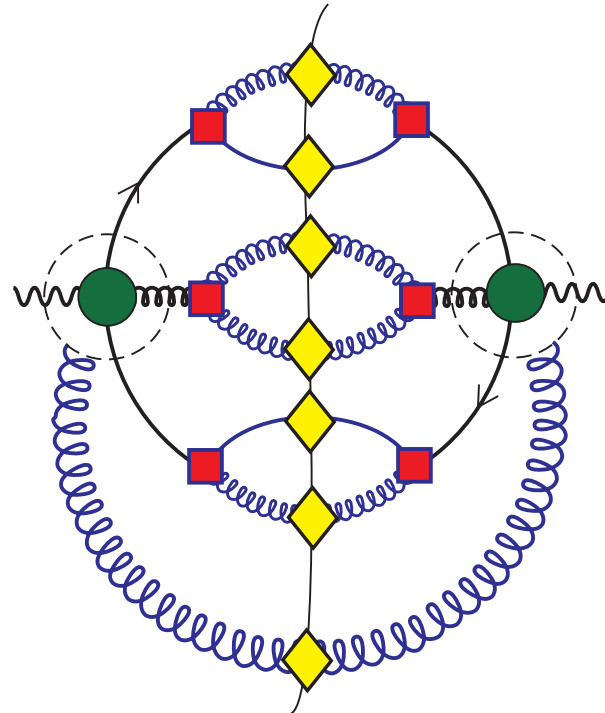
Structure of the calculation

From α_s^B diagrams.

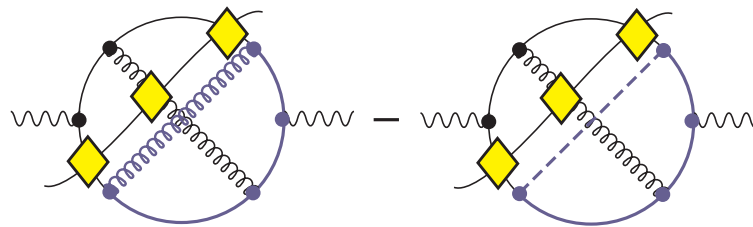
Each parton splits.

A soft gluon is radiated.

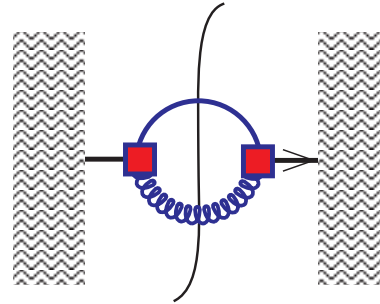
Partons \rightarrow showers.



From α_s^{B+1} diagrams with subtractions.



Shower splitting.



$$\mathcal{I}[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^\pi \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) \right. \\ \left. \times \exp \left(- \int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2} \int_0^1 dz \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{l}^2, z) \right) \right\}.$$

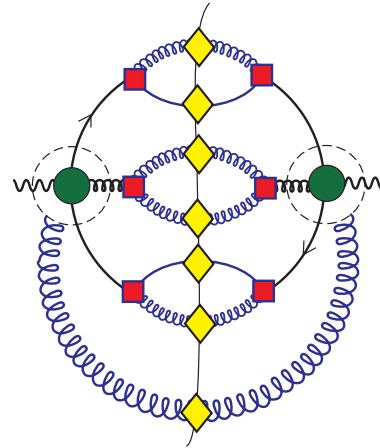
- This is similar to what shower MC programs use.
- The **Sudakov exponential** suppresses splitting with small \bar{q}^2 .
- It represents the probability that the parton did not split with virtuality \bar{l}^2 above \bar{q}^2 .

Soft radiation

Three jets make an antenna.
Radiate one gluon.
Use eikonal approximation,

$$F_{ij}(\hat{l}; \hat{q}_1, \hat{q}_2, \hat{q}_3).$$

Part of F_{ii} in jets.



$$\begin{aligned} \mathcal{I}[\text{radiate}] &= \int d\vec{q}_1 \int d\vec{q}_2 \int d\vec{q}_3 \delta(\sum \vec{q}_i) \int_0^{M_{\text{soft}}} \frac{d|\vec{l}|}{|\vec{l}|} \int \frac{d^2\hat{l}}{4\pi} \\ &\times \sum_{ij} \frac{\alpha_s}{\pi} F_{ij}(\hat{l}; \hat{q}_1, \hat{q}_2, \hat{q}_3) R_{ij}(\vec{l}; \vec{q}_1, \vec{q}_2, \vec{q}_3) \\ &\times \exp\left(-\int_{|\vec{l}|}^{M_{\text{soft}}} \frac{d|\vec{l}'|}{|\vec{l}'|} \int \frac{d^2\hat{l}'}{4\pi} \sum_{i'j'} \frac{\alpha_s}{\pi} F_{i'j'}(\hat{l}'; \hat{q}_1, \hat{q}_2, \hat{q}_3)\right). \end{aligned}$$

A test

Calculate $d\sigma/dt$ at thrust $t = 0.86$ ($\sqrt{s} = M_Z$, $\mu = \sqrt{s}/6$).

$$R = \frac{(\text{NLO-shower}) - \text{NLO}}{\text{NLO}},$$

where “NLO-shower” includes a parton shower but not hadronization. If calculation is correct,

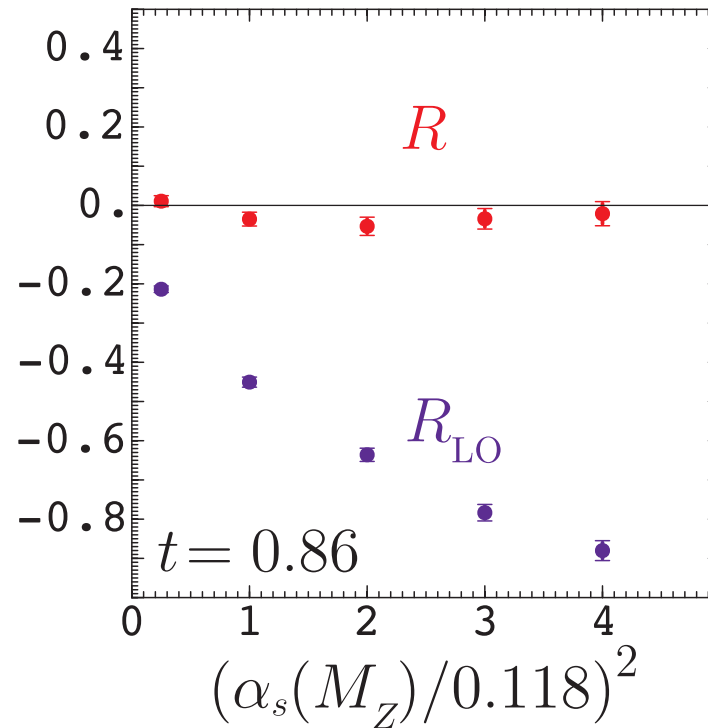
$$R = \frac{(C_0\alpha_s^B + C_1\alpha_s^{B+1} + C_2\alpha_s^{B+2} + \dots) - (C_0\alpha_s^B + C_1\alpha_s^{B+1})}{C_0\alpha_s^B + C_1\alpha_s^{B+1}}$$

so

$$R = \frac{C_2}{C_0} \alpha_s^2 + \dots$$

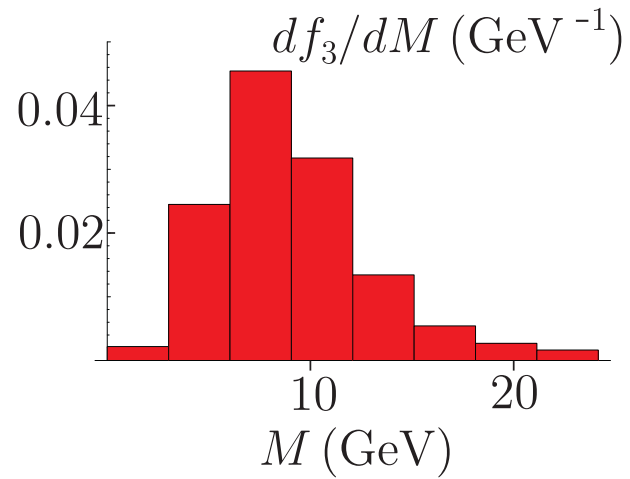
So plot R versus $\alpha_s(M_Z)^2$.

The result for thrust = 0.86

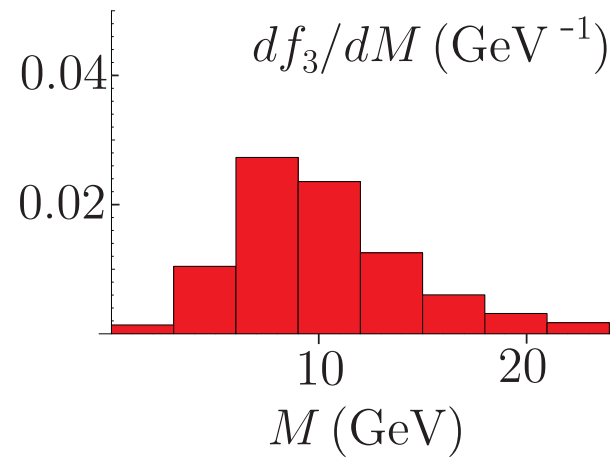


- R_{LO} is the same without the order α_s^{B+1} corrections.

Jet mass distribution

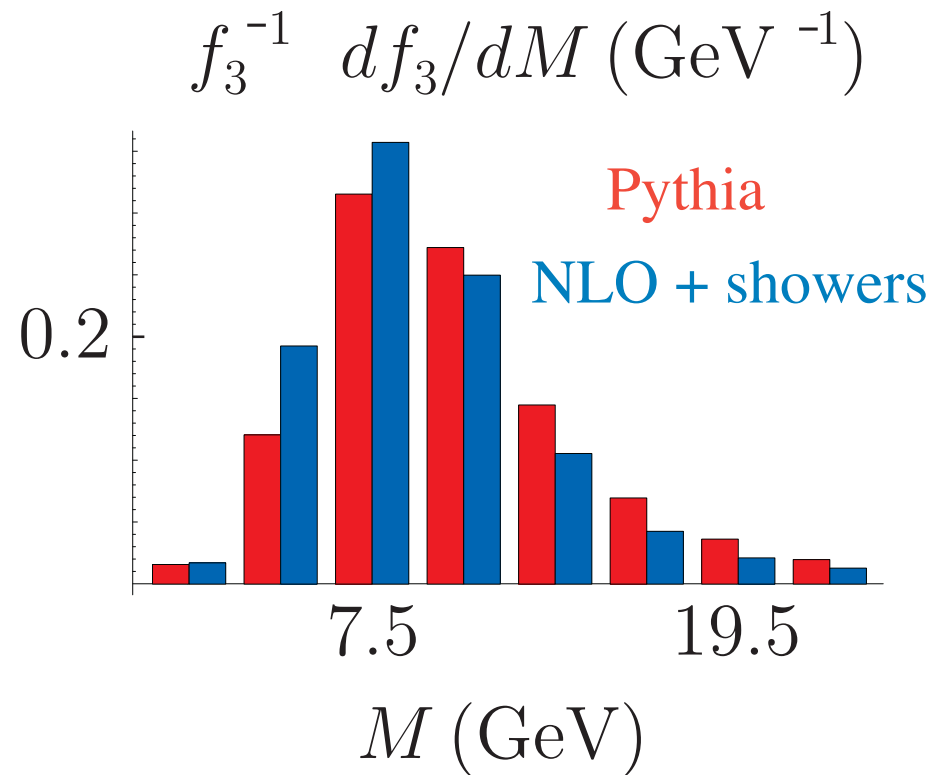


NLO with showers
and Pythia hadronization



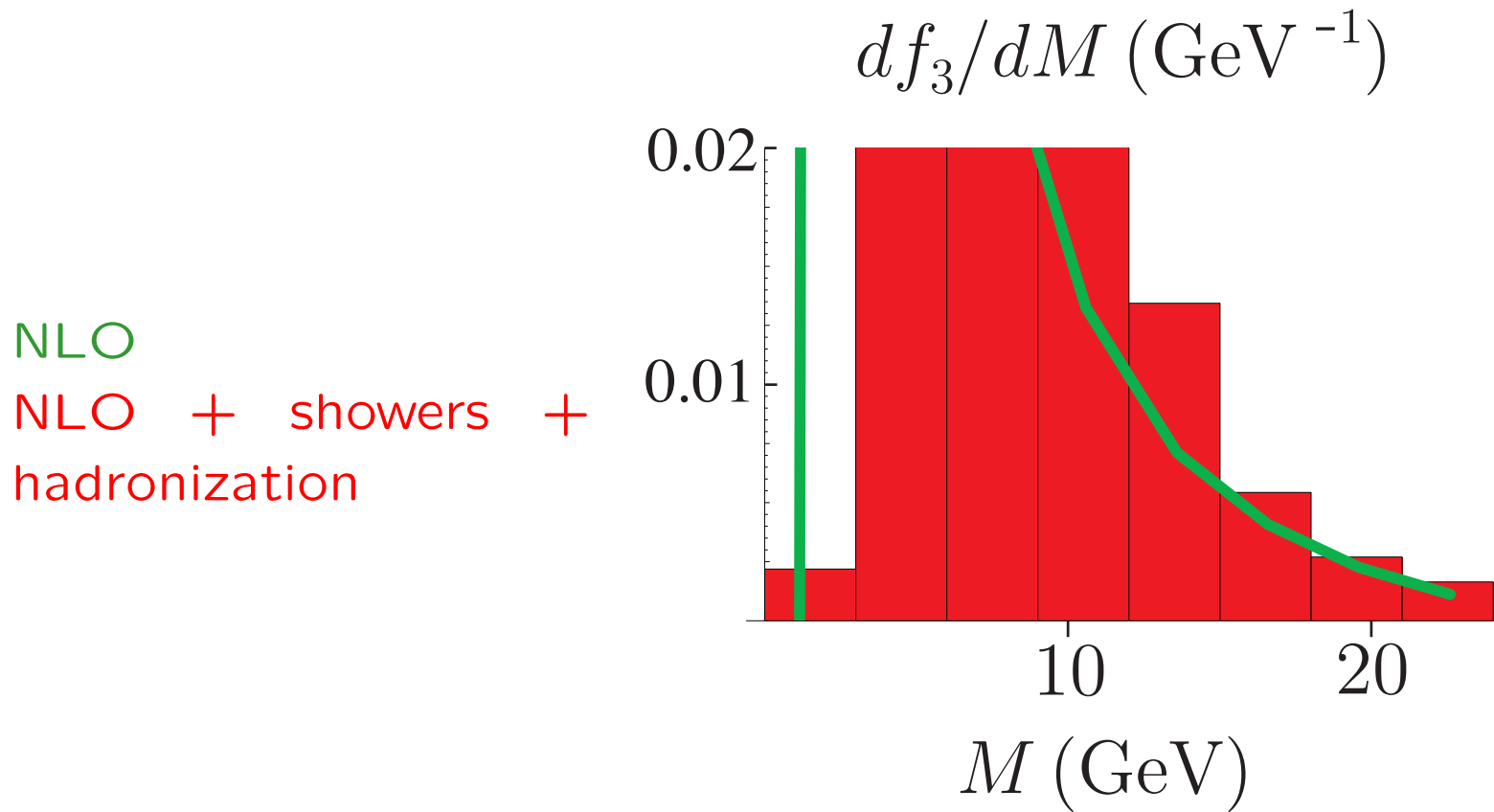
Pythia (LO)

Comparison with same normalization

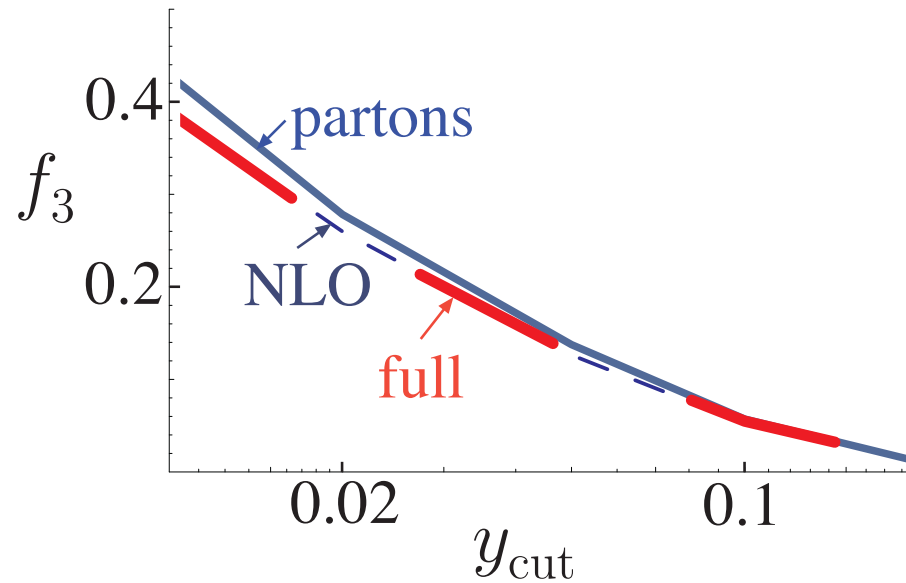


Large mass tail

Showering and hadronization have little effect on at high M .

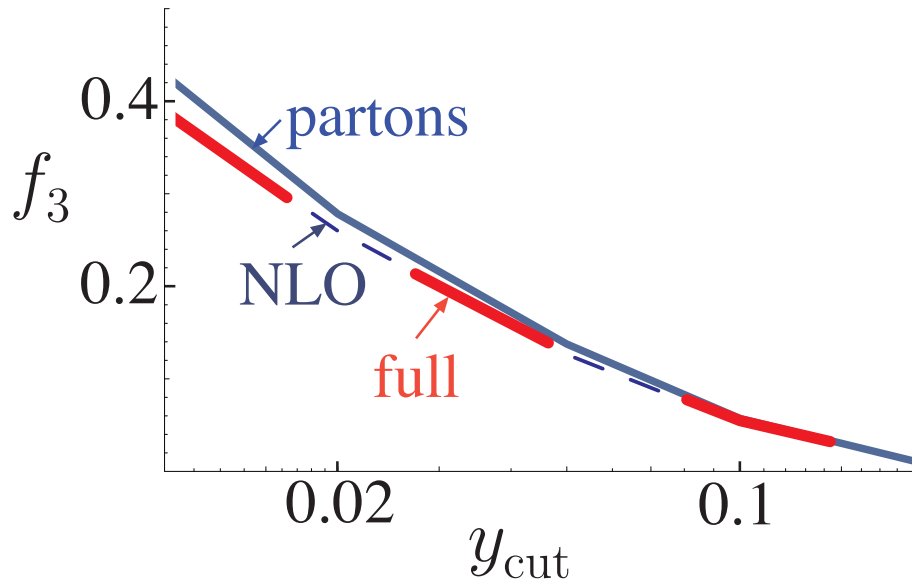


Effect of showers and hadronization on f_3

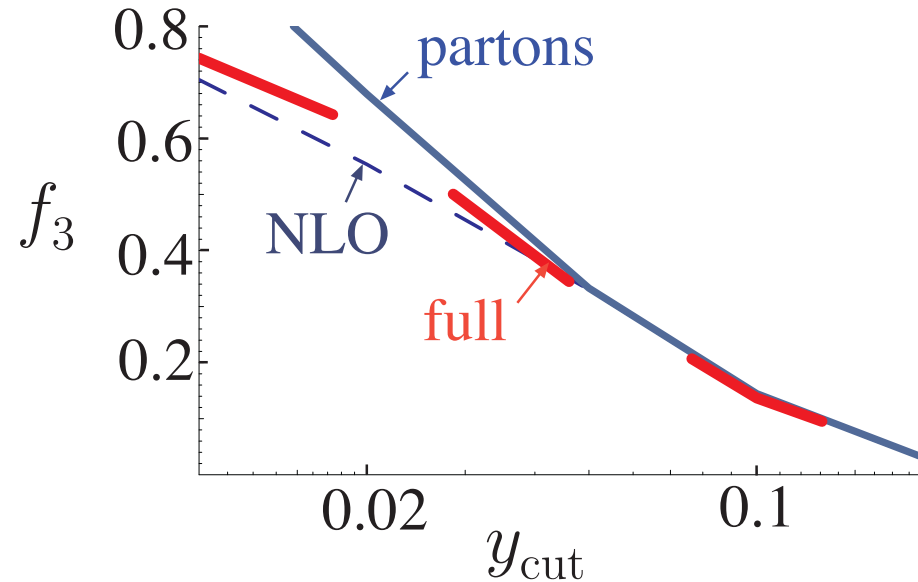


The effect of going from a pure NLO calculation to NLO + parton showers to NLO + parton showers + hadronization is not large.

We can examine effect of jet algorithm



Durham algorithm

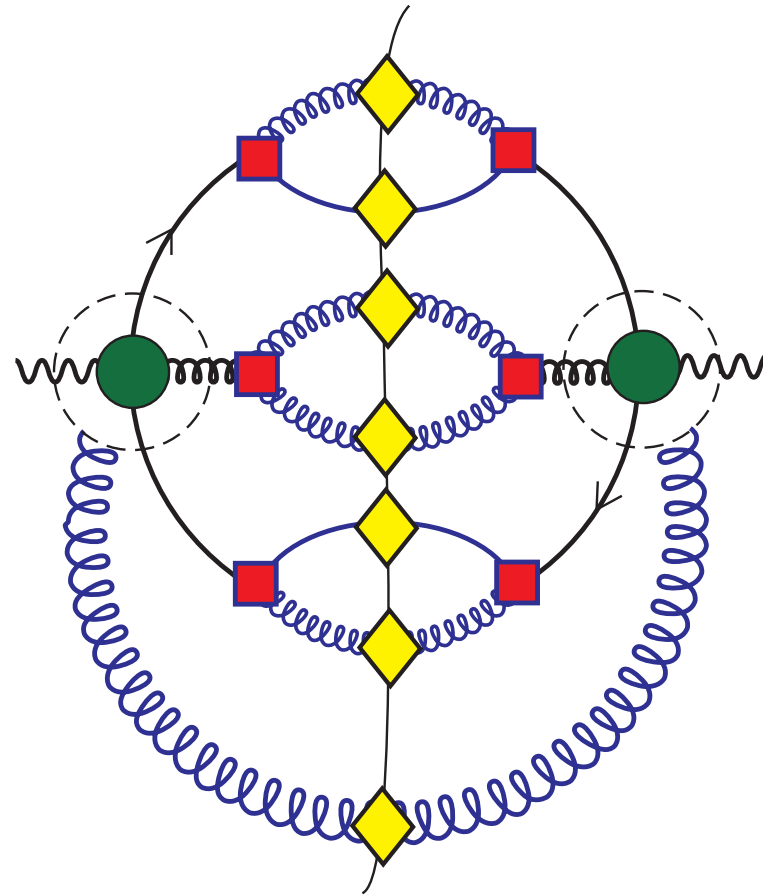


Jade algorithm

Problems arise for the Jade algorithm when $y_{\text{cut}} < 0.03$ and f_3 becomes non-perturbative.

My guess about the future

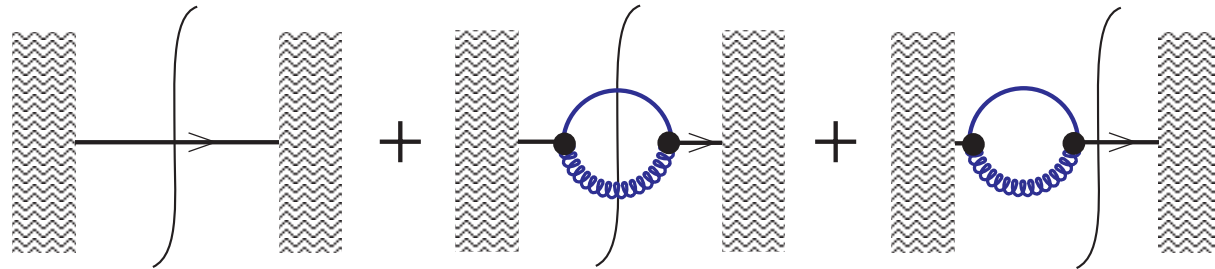
- NLO calculations will include showers.
 - NLO authors will be responsible for primary splittings (including soft gluon effects).
 - The rest will come from Pythia, Herwig, . . .
 - S. Mrenna is helping with this.
-
- Z. Nagy and I are working on generalizing this within Catani-Seymour dipole scheme.



Extra slides

Collinear singularities in perturbation theory

Perturbative calculation in the Coulomb gauge.



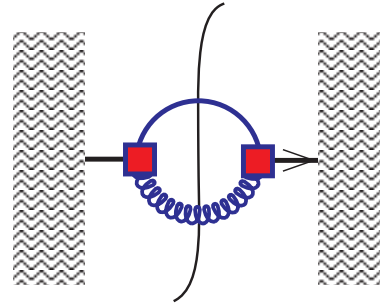
$$\mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] =$$

$$\int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \not{q} R_0 + \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^\pi \frac{d\phi}{2\pi} \right.$$

$$\left. \times \left[\frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^2, x) \not{q} R_0 \right] \right\}.$$

- $R(\bar{q}^2, x, \phi)$ and R_0 represent the rest of the graph, including the measurement function. $R(\bar{q}^2, x, \phi) \rightarrow R_0$ for $\bar{q}^2 \rightarrow 0$.
- $\bar{q}^2 \rightarrow 0$ singularities cancel.

Shower splitting.



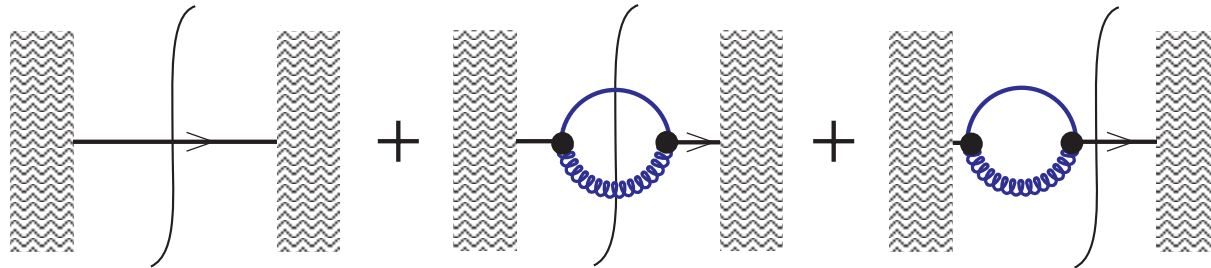
$$\mathcal{I}[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^\pi \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) \right. \\ \left. \times \exp \left(- \int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2} \int_0^1 dz \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{l}^2, z) \right) \right\}.$$

- Splitting with the exact Coulomb gauge \mathcal{M} .
- Sudakov suppression with the exact Coulomb gauge \mathcal{P} .

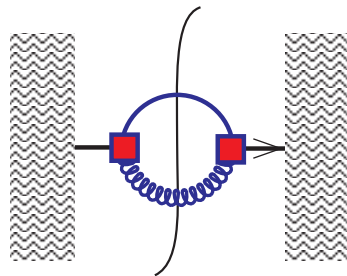
Subtraction to multiplication theorem

$$\mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] = \mathcal{I}[\text{shower}] \times (1 + \mathcal{O}(\alpha_s^2)).$$

That is



is equivalent to



Proof step 0. Add and subtract:

$$\mathcal{I}[\text{shower}] = \mathcal{I}_1[\text{shower}] + \mathcal{I}_2[\text{shower}]$$

with

$$\mathcal{I}_1[\text{shower}]$$

$$= \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^\pi \frac{d\phi}{2\pi} \exp \left(- \int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2} \int_0^1 dz \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{l}^2, z) \right) \right. \\ \left. \times \left[\frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^2, x) \not{R}_0 \right] \right\}$$

and

$$\mathcal{I}_2[\text{shower}]$$

$$= \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \not{R}_0 \int_0^\infty d\bar{q}^2 \right. \\ \left. \times \frac{1}{\bar{q}^2} \int_0^1 dx \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^2, x) \exp \left(- \int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2} \int_0^1 dz \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{l}^2, z) \right) \right\}.$$

Proof step 1. Expand \mathcal{I}_1 [shower]:

$$\begin{aligned}
& \mathcal{I}_1[\text{shower}] \\
&= \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^\pi \frac{d\phi}{2\pi} \exp \left(- \int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2} \int_0^1 dz \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{l}^2, z) \right) \right. \\
&\quad \times \left. \left[\frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^2, x) \not\! R_0 \right] \right\} \\
&= \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^\pi \frac{d\phi}{2\pi} \times \mathbf{1} \right. \\
&\quad \times \left. \left[\frac{\alpha_s}{2\pi} \mathcal{M}_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^2, x) \not\! R_0 \right] \right\} + \mathcal{O}(\alpha_s^2 \times R) \\
&= \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] + \mathcal{O}(\alpha_s^2 \times R)
\end{aligned}$$

Proof step 2. Calculate $\mathcal{I}_2[\text{shower}]$:

$$\begin{aligned}\mathcal{I}_2[\text{shower}] &= \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr}\left\{\not{q} R_0 \int_0^\infty d\bar{q}^2 \right. \\ &\quad \left. \times \frac{1}{\bar{q}^2} \int_0^1 dx \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{q}^2, x) \exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2} \int_0^1 dz \frac{\alpha_s}{2\pi} \mathcal{P}_{g/q}(\bar{l}^2, z)\right)\right\} \\ &= \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr}\{\not{q} R_0\}. \\ &= \mathcal{I}[\text{Born}].\end{aligned}$$

That is

$$\mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] = \mathcal{I}[\text{shower}] \times \left(1 + \mathcal{O}(\alpha_s^2)\right).$$

You could use your favorite \mathcal{M}' & \mathcal{P}'

$$\mathcal{I}'[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^\pi \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} \mathcal{M}'_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) \right. \\ \left. \times \exp \left(- \int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2} \int_0^1 dz \frac{\alpha_s}{2\pi} \mathcal{P}'_{g/q}(\bar{l}^2, z) \right) \right\}.$$

Then

$$\mathcal{I}'[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}|} \text{Tr} \left\{ \not{q} R_0 + \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_{-\pi}^\pi \frac{d\phi}{2\pi} \right. \\ \left. \times \left[\frac{\alpha_s}{2\pi} \mathcal{M}'_{g/q}(\bar{q}^2, x, \phi) R(\bar{q}^2, x, \phi) - \frac{\alpha_s}{2\pi} \mathcal{P}'_{g/q}(\bar{q}^2, x) \not{q} R_0 \right] \right\} + \mathcal{O}(\alpha_s^2 \times R) \\ = \mathcal{I}[\text{Born}] + \mathcal{I}'[\text{real}] + \mathcal{I}'[\text{virtual}].$$

What you get then

$$\mathcal{I}'[\text{shower}] = \mathcal{I}[\text{Born}] + \mathcal{I}'[\text{real}] + \mathcal{I}'[\text{virtual}].$$

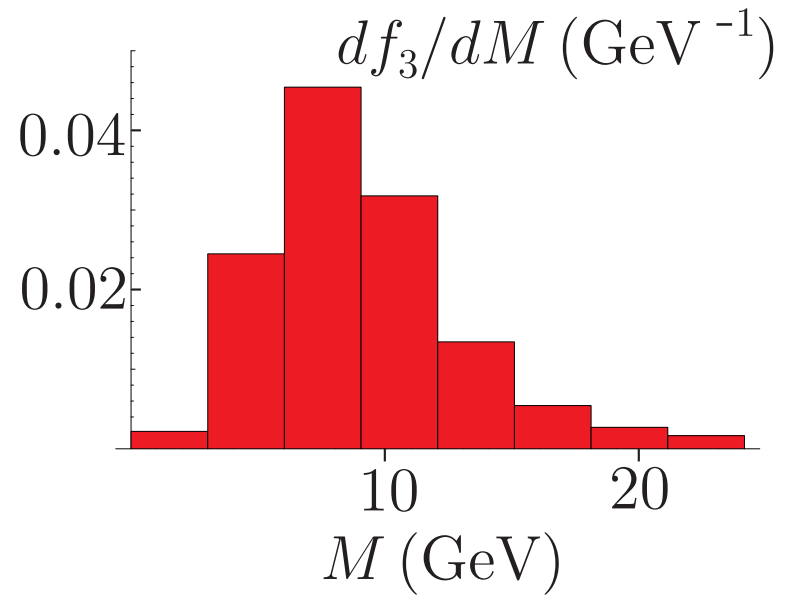
gives

$$\begin{aligned} & \mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] \\ &= \mathcal{I}'[\text{shower}] + (\mathcal{I}[\text{real}] - \mathcal{I}'[\text{real}]) + (\mathcal{I}[\text{virtual}] - \mathcal{I}'[\text{virtual}]). \end{aligned}$$

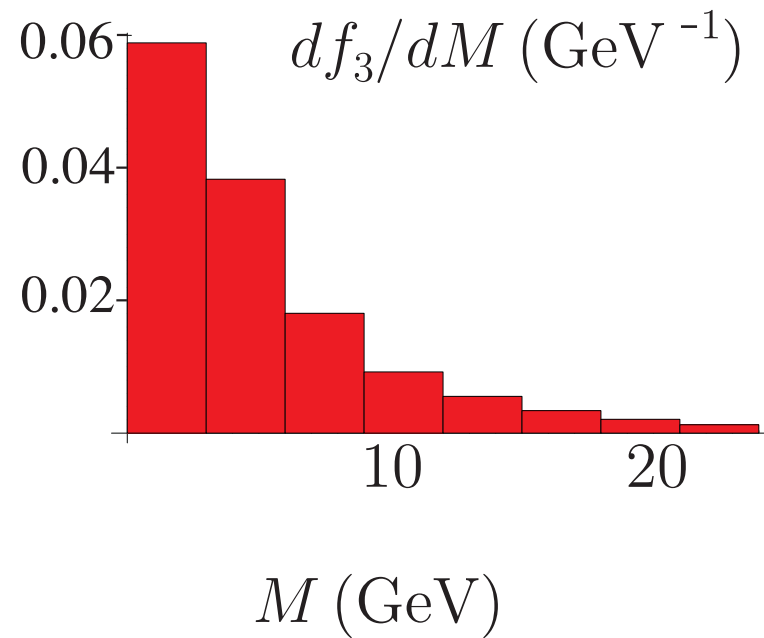
The \mathcal{M}' and \mathcal{P}' functions act as subtractions for \mathcal{M} and \mathcal{P} .

As long as they have the right $\vec{q}^2 \rightarrow 0$ singularities, they cancel the singularities of \mathcal{M} and \mathcal{P} .

Effect of hadronization

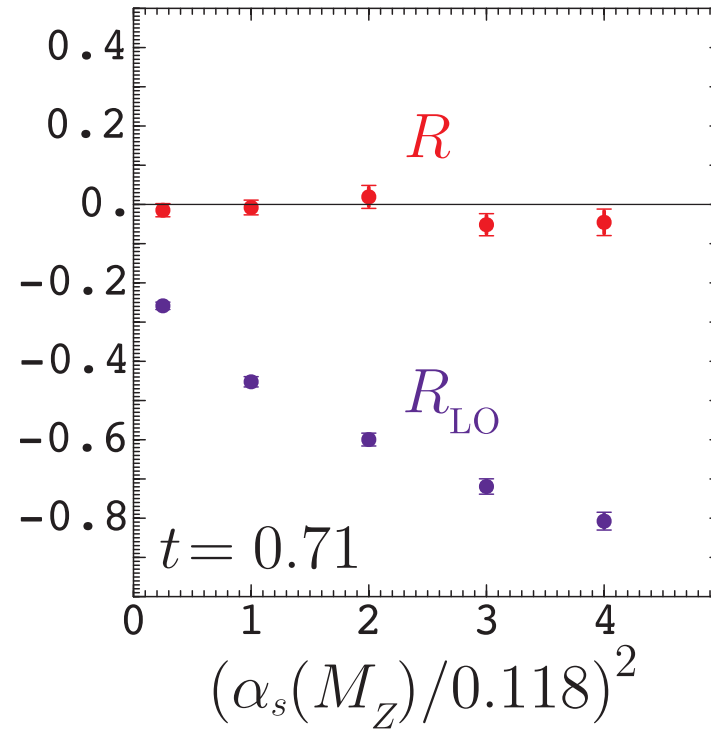


with hadronization

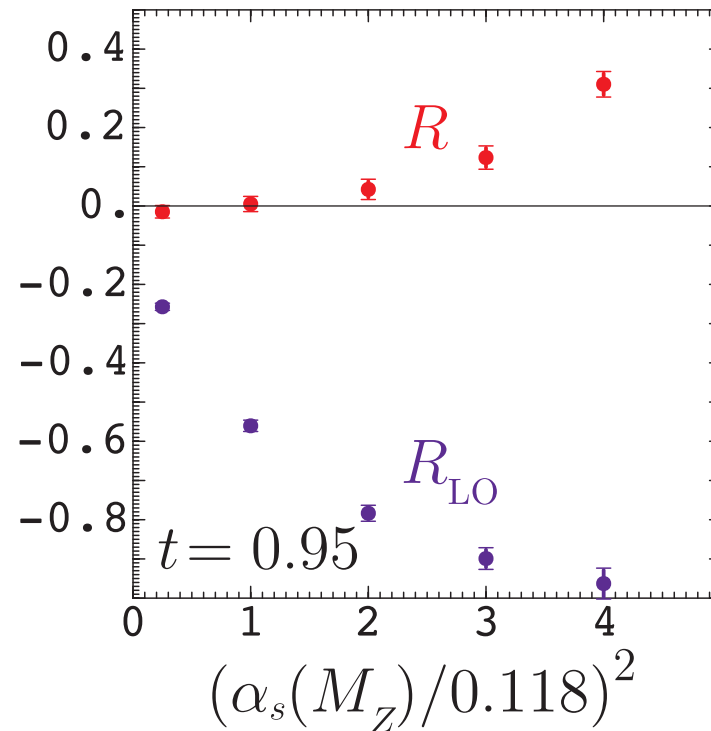


without hadronization

The result for thrust = 0.71



The result for thrust = 0.95



- Note: thrust = 0.95 is out of the three jet region so this result should not be compared to experiment.