KITP – LoopFest III — 3rd April 2004

Automated resummation of QCD final state observables

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Fermilab

☛ *In collaboration with*

A. Banfi (Amsterdam) and G. Salam (Paris)

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- ✗ The most famous example: the Thrust

$$T \equiv \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T| = \frac{1}{Q} \sum_i |p_{iz}|$$

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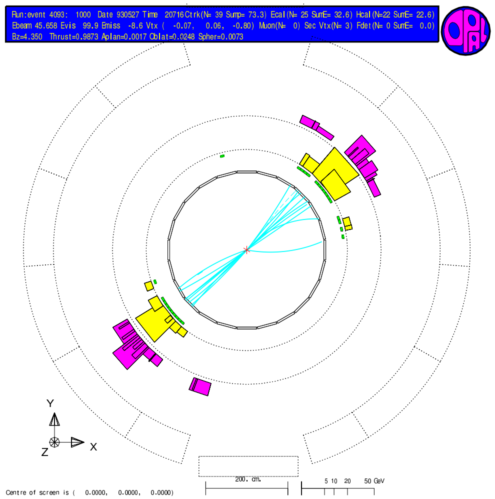
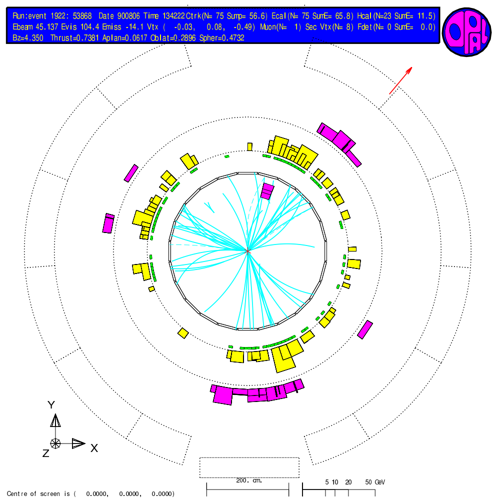
Pencil-like event: $\tau \equiv 1 - T \ll 1$



Planar event: $T \simeq 2/3$



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Leading order (LO) $\equiv \mathcal{O}(\alpha_s)$

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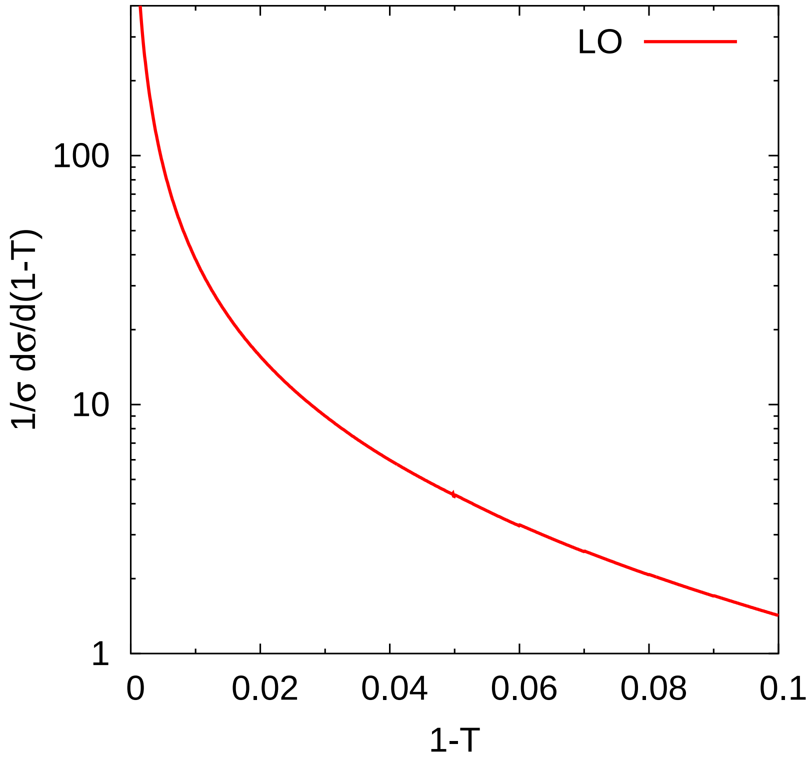

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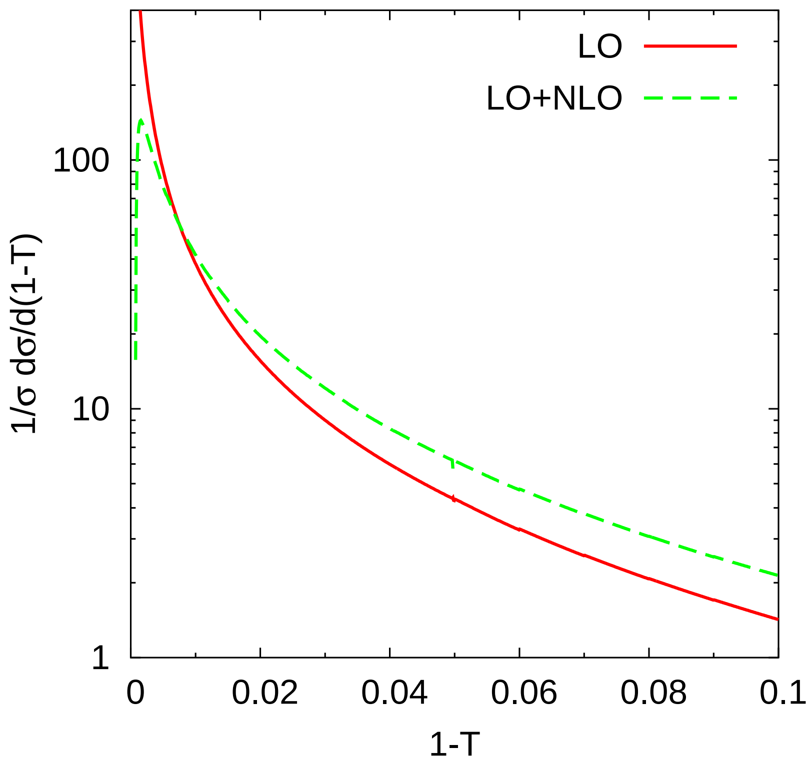

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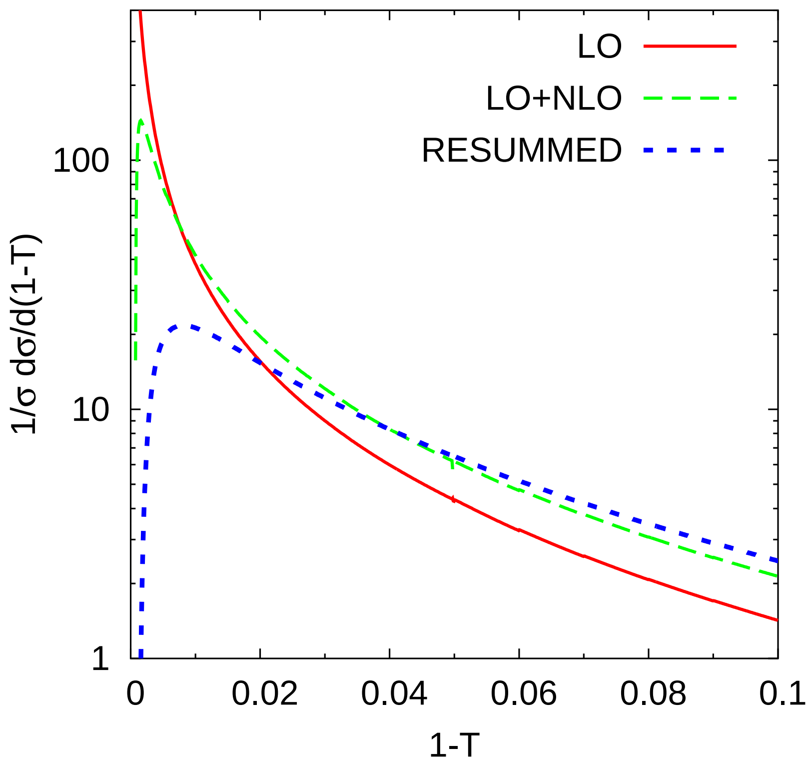

LO, NLO, . . . all *diverge* in two-jet region $(1 - T \to 0)$

Subject of this seminar is

FINAL-STATE RESUMMATION

i. e. *all-orders* description of the “exclusive” 2-jet limit.

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- sensitivity to properties of QCD radiation

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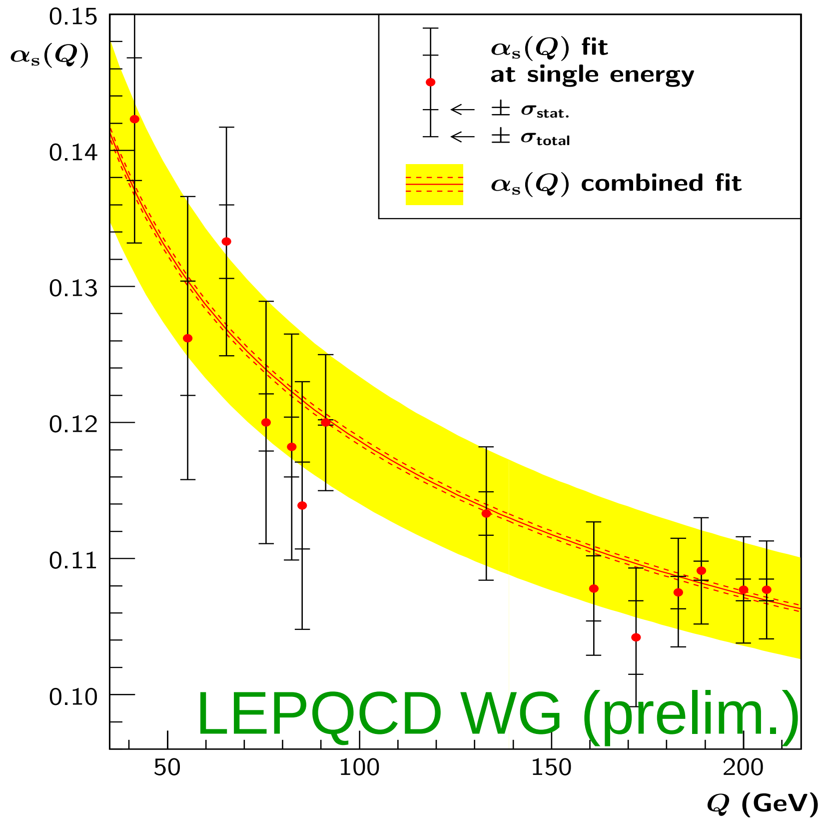
Provide a wealth of information, e.g.:

- Measurements of the coupling α_s and its renormalization group running



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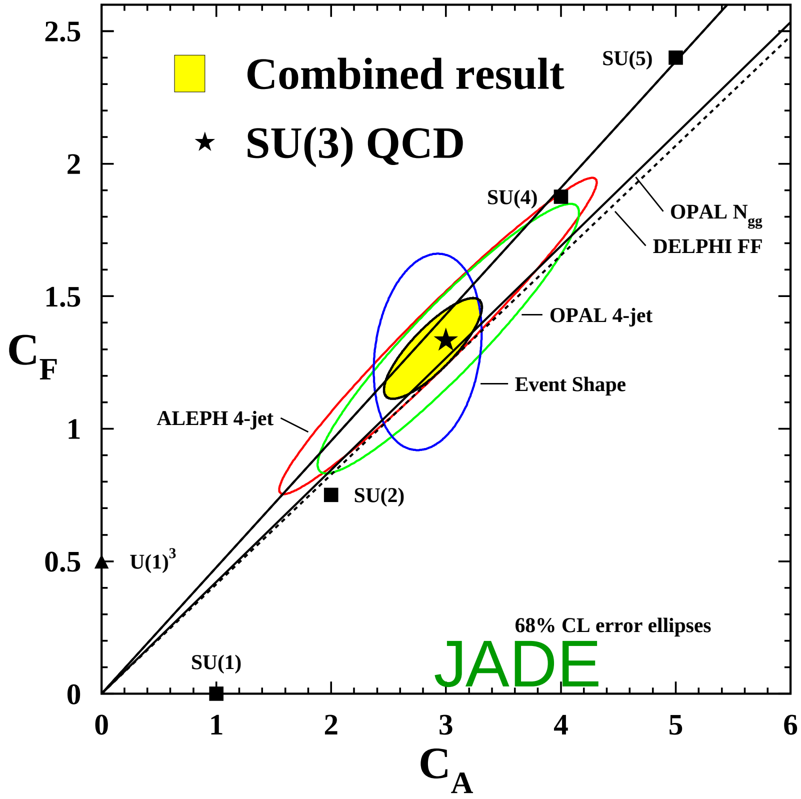

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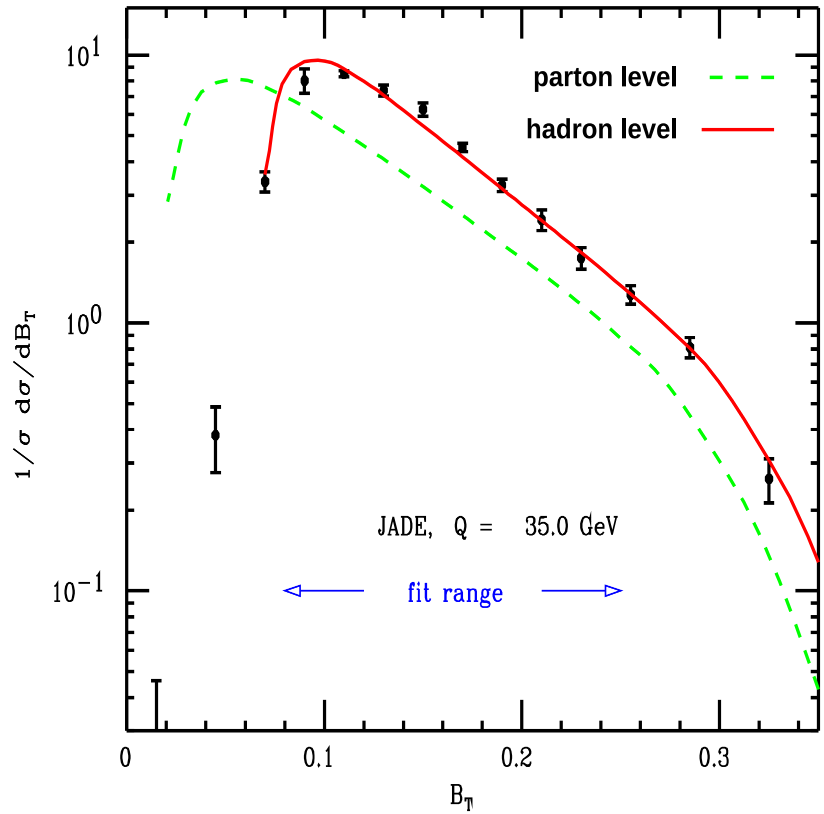
Provide a wealth of information, e.g.:

- Measurements of the coupling α_s and its renormalization group running
- Measurements/cross checks of the values of the colour factors of QCD
- Studies of connection between parton-level (perturbative description of quarks and gluons) and hadron-level (the real)



Large Logarithms to all orders

Probability of "constrained" events, i. e. $V(k_1 \dots k_n) < v$, has a *divergent* PT expansion

$$\Sigma(v) \equiv \mathrm{Prob}(V < v) = 1 + \sum_{m \le 2n} R_{n,m} {\alpha_s}^n Log^m v + \dots$$

i. e. there is a soft & collinear divergence [$\rightsquigarrow$ Log] for each emitted gluon

Today's state-of-the art accuracy

- accounts for all Leading (LL) and Next-to-Leading Logs (NLL)

$$\Sigma(v) = \exp\{\underbrace{L g_1(\alpha_s L)}_{LL} + \underbrace{g_2(\alpha_s L)}_{NLL} + \dots\}$$

☞ NB:

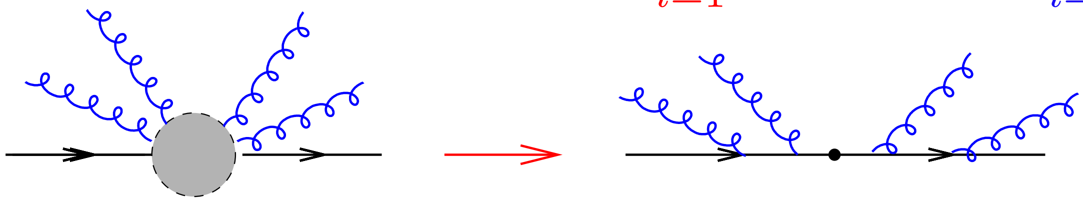
- LL means ${\alpha_s}^n L^{n+1}$ in $\ln \Sigma$, not just ${\alpha_s}^n L^{2n}$ in Σ
- NLL means $(\alpha_s L)^n$ in $\ln \Sigma$, not just ${\alpha_s}^n L^{2n-1}$ in Σ

- furthermore resummed results are matched to Fixed Order at NLO

First half of the history: Matrix elements and phase space

exploit *angular ordering* $\Rightarrow$ soft *independent emissions* ($\Rightarrow$ QED)

e.g. $e^+e^- \to 2$ jets $\Rightarrow w_{p\bar{p}}(k_1,\dots,k_n) = \frac{1}{n!}\prod_{i=1}^{n} w_{p\bar{p}}(k_i) \sim \frac{1}{n!}\prod_{i=1}^{n}\frac{\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$



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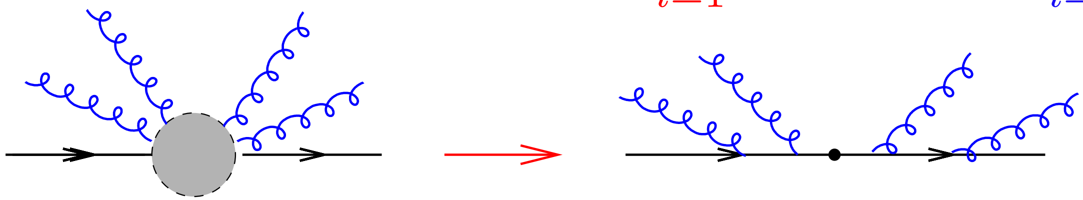

Second half of the history: The observable definition

analyze the observable & use Mellin transforms

$$1 - T \simeq \frac{1}{Q}\sum_{i=1}^{n}\frac{E_i\theta_i^2}{2} \qquad \longrightarrow \qquad \Theta(1 - T < \tau) = \int \frac{d\nu}{2\pi i\nu} e^{\nu\tau}\prod_{i=1}^{n} e^{-\nu\frac{E_i\theta_i^2}{2Q}}$$

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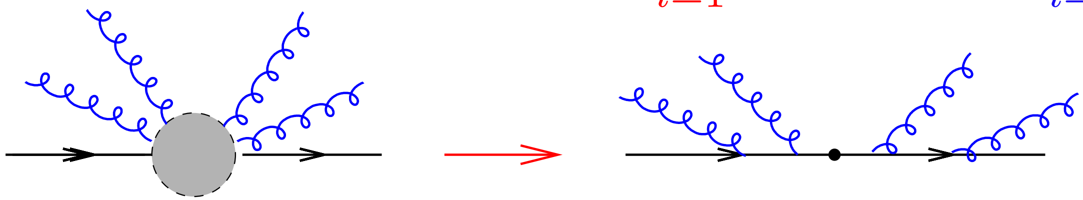

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THE ANSWER

$$\Sigma(\tau) \int \frac{d\nu}{2\pi i\nu} e^{\nu\tau} \exp\left[\int \frac{d\theta}{\theta}\frac{dE}{E}\frac{\alpha_s(E\theta)C_F}{\pi}\left(e^{-\nu\frac{E_i\theta_i^2}{2Q}} - 1\right)\right]$$

An incomplete list of analytical NLL predictions

$e^+e^- \to$ 2 jets

- S. Catani, G. Turnock, B. R. Webber and L. Trentadue, *Thrust distribution in e^+e^- annihilation,* Phys. Lett. B **263** (1991) 491.
- S. Catani, G. Turnock and B. R. Webber, *Heavy jet mass distribution in e^+e^- annihilation,* Phys. Lett. B **272** (1991) 368.
- S. Catani, Yu. L. Dokshitzer, M. Olsson, G. Turnock and B. R. Webber, *New clustering algorithm for multi-jet cross-sections in e^+e^- annihilation,* Phys. Lett. B **269** (1991) 432.
- S. Catani, L. Trentadue, G. Turnock and B. R. Webber, *Resummation of large logarithms in e^+e^- event shape distributions,* Nucl. Phys. B **407** (1993) 3.
- S. Catani, G. Turnock and B. R. Webber, *Jet broadening measures in e^+e^- annihilation,* Phys. Lett. B **295** (1992) 269.
- G. Dissertori and M. Schmelling, *An Improved theoretical prediction for the two jet rate in e^+e^- annihilation,* Phys. Lett. B **361** (1995) 167.
- Y. L. Dokshitzer, A. Lucenti, G. Marchesini and G. Salam, *On the QCD analysis of jet broadening,* JHEP **9801** (1998) 011
- S. Catani and B. R. Webber, *Resummed C-parameter distribution in e^+e^- annihilation,* Phys. Lett. B **427** (1998) 377
- S. J. Burby and E. W. Glover, *Resumming the light hemisphere mass and narrow jet broadening distributions in e^+e^- annihilation,* JHEP **0104** (2001) 029
- M. Dasgupta and G. Salam, *Resummation of non-global QCD observables,* Phys. Lett. B **512** (2001) 323
- C. F. Berger, T. Kucs and G. Sterman, *Event shape / energy flow correlations,* Phys. Rev. D **68** (2003) 014012

DIS 1+1 jet

- V. Antonelli, M. Dasgupta and G. Salam, *Resummation of thrust distributions in DIS,* JHEP **0002** (2000) 001
- M. Dasgupta and G. Salam, *Resummation of the jet broadening in DIS,* Eur. Phys. J. C **24** (2002) 213
- M. Dasgupta and G. Salam, *Resummed event-shape variables in DIS,* JHEP **0208** (2002) 032

e^+e^-, DY, DIS 3 jets

- A. Banfi , G. Marchesini, Y. L. Dokshitzer and GZ, *QCD analysis of near-to-planar 3-jet events,* JHEP **0007** (2000) 002
- A. Banfi , Y. L. Dokshitzer, G. Marchesini and GZ, *Near-to-planar 3-jet events in and beyond QCD perturbation theory,* Phys. Lett. B **508** (2001) 269
- A. Banfi , Y. L. Dokshitzer, G. Marchesini and GZ, *QCD analysis of D-parameter in near-to-planar three-jet events,* JHEP **0105** (2001) 040
- A. Banfi , G. Marchesini, G. Smye and GZ, *Out-of-plane QCD radiation in hadronic Z0 production,* JHEP **0108** (2001) 047
- A. Banfi , G. Marchesini, G. Smye and GZ, *Out-of-plane QCD radiation in DIS with high p(t) jets,* JHEP **0111** (2001) 066
- A. Banfi , G. Marchesini and G. Smye, *Azimuthal correlation in DIS,* JHEP **0204** (2002) 024
- C. F. Berger, T. Kucs and G. Sterman, *Energy flow in interjet radiation*, Phys. Rev. D **65**, 094031 (2002)

$\sim$ 1 observable per article

The current situation can be summarized as follows

- experimental studies limited by availability of theoretical calculations
- error-prone business, many subtle effects understood on the way

  On the previous slide, *only 4 authors*, out of 21, can say that their results were always correct to the accuracy claimed [three of them quit physics. . .]
- there are many phenomenological applications

➥ need to automate resummations (as for fixed order)

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➥ need to automate resummations (as for fixed order)

On the other hand

- resummations exploit always the same standard factorization techniques (for matrix element and observable)
- the origin of logarithms is clearly the *SAME* for all observables

➥ automating the job seems feasible

IDEA: Define a simpler observable

$$V(k_1,\dots k_n) \quad \Longrightarrow \quad V_s(k_1,\dots k_n) \equiv \max\{V(k_1),\dots,V(k_n)\}$$

e. g.

$$B(k_1,\dots k_n) \equiv \sum_i \frac{k_{ti}}{Q} \quad \Longrightarrow \quad B_s(k_1,\dots k_n) \equiv \max\{\frac{k_{ti}}{Q}\}$$

- With just one soft-collinear emission

$$V(k_1,\dots k_n) = V_s(k_1,\dots k_n)$$

$\Rightarrow$ same double logs and most of the single logs

- Simple factorization (no Mellin integrals)

$$\Theta(V_s - v) = \prod_i \Theta(V_i - v)$$

$\Rightarrow$ analytical resummation straightforward!

Fix a Born event and emit a soft gluon k collinear to a given hard leg ℓ.
We parametrize

$$V_s(k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)$$

k_t $\Rightarrow$ transverse momentum wrt the leg

η $\Rightarrow$ rapidity wrt the leg

ϕ $\Rightarrow$ azimuthal angle

☛ Σ_s known given the (automatically determined) quantities $a_\ell, b_\ell, d_\ell, g_\ell(\phi)$, just exponentiating naively the one-gluon result

This account for *all double logs* and *single-logs due to*

✓ hard collinear effects

✓ soft, large angle emission

✓ inclusive gluon splitting

The computation of Σ_s is based on a veto on single-emissions

$$V(k_1,\dots k_n) < v \quad \Longrightarrow \quad V_s \equiv \max[V(k_1),\dots,V(k_n)] < v$$

One then needs to relate the observable to *all secondary emissions*, i.e. account for the observable specific mismatch between $V(k_1,\dots k_n)$ and V_s

- ☛ Physically one needs accurate understanding of the kinematics
- ☛ Mathematically this translates into performing Mellin integrals

We call these multiple emission effects.

How can these observable-specific effects be computed generally?

Aim: compute the mismatch between $\Sigma_s(v_s)$ and $\Sigma(v)$

The two distributions are related by a simple convolution

$$\frac{D(v)}{v} = \int \frac{dv_s}{v_s} D_s(v_s) P(v|v_s) \qquad D(v) \equiv \frac{d\Sigma}{dL} \qquad L = \mathrm{Lnv}$$

☛ $P(v|v_s)$ is the probability to have v given v_s

Since ☛ $D_s(v_s) = e^{-R(v_s)}$ $\Rightarrow$ known analytically

☛ $v \sim v_s$ $\Rightarrow$ same LL structure

➥ expand and get $D_s(v_s) =_{NLL} D_s(v) e^{-R' \ln(v/v_s)}$ $\qquad R' \equiv dR/dL$

➥ $D(v) =_{NLL} D_s(v) \mathcal{F}(R') \qquad \mathcal{F}(R') = \int \frac{dv_s}{v_s} e^{-R' \ln(v/v_s)} v P(v|v_s)$

How to compute $\mathcal{F} \Leftrightarrow P(v|v_s)$ generally?

Fix a Born configuration and generate *decreasing soft-collinear (SC) emissions* according to phase space

❶ set $v(k_1) = v_s$ [START FROM: $V_s = v_s$]

❷ generate a formally infinite number of SC emissions

according to an independent emission pattern uniform in $\ln k_t, \eta, \phi$ such that on average the density of emissions per unit $\ln V$ from leg ℓ is R'_ℓ

➥ Finally compute $V(k_1, k_2, \ldots k_n) \equiv v$

☞ This gives the weighted probability of having $V = v$ given $V_s = v_s$ and allows so the computation of $\mathcal{F}$ in a completely general way

Banfi , Salam, GZ *JHEP* 0201 (2002) 018

http://www.ippp.dur.ac.uk/˜zander/numsum.html

The master formula

$$\Sigma(v) =_{NLL} \sum_{\text{sub.}} \int [d\Phi]_{\text{hard}} \Sigma_s(v) \cdot \mathcal{F}(R')$$

Banfi , Salam, GZ hep-ph/0304148

✓ Analytical resummation for the “easy” Σ_s: *pure LL and NLL terms*

$$\Sigma_s(v) = \prod_{\ell=1}^{n_{inc}} \underbrace{f_\ell(v^{\frac{2}{a+b_\ell}} \mu_F^2)}_{\text{pdfs}} \otimes \prod_{\ell=1}^{N} \underbrace{J_\ell(L)}_{\text{jet function}} \cdot \underbrace{S\,(T(L/a))}_{\text{soft}}$$

- soft and collinear emission $\Rightarrow$ jet function $J_\ell(L)$
  (all LL Sudakov suppression and some NLL terms)
- hard collinear splitting $\Rightarrow$ evolution of the pdfs
- soft large angle $\Rightarrow$ QCD coherence and geometry dependence in S

✓ the observable-dependent “difficult” $\mathcal{F}$ is computed numerically but is by construction a pure NLL function

Requirements on the observable

For the observable to be resummed automatically it should

- ✗ vanish in the Born limit and be positive defined
- ✗ behave as $V(k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)$ for 1 SC gluon along leg ℓ
- ✗ be infrared and collinear safe
- ✗ be continuously global ($a_\ell = a \ \forall$ hard legs ℓ)
- ✗ exponentiate (no JADE)

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  [all other conditions are satisfi ed by all observables resummed so far]

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- ☛ the essential feature of the program is the ability to perform all checks automatically

  [☛ use arbitrary precision to take asymptotic limits]

Bailey, RNR Technical Report RNR-94-013

Some observables have exponentiating double (and single) logs

$$\mathrm{P}(v) = 1 - X\frac{\alpha_s C_F}{\pi}\ln^2 v + \frac{1}{2}X^2\left(\frac{\alpha_s C_F}{\pi}\right)^2 \ln^4 v + \cdots \Rightarrow e^{-X\frac{\alpha_s C_F}{\pi}\ln^2 v}$$

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others do not, e.g. Jade-algorithm jet rates:

$$\mathrm{P}_{\mathrm{Jade2-jet}}(y_{\mathrm{cut}}) = 1 - \frac{\alpha_s C_F}{\pi}\ln^2 y_{\mathrm{cut}} + \frac{1}{2}\cdot\frac{5}{6}\left(\frac{\alpha_s C_F}{\pi}\right)^2 \ln^4 y_{\mathrm{cut}} + \ldots$$

Brown and Stirling, Phys.Lett.B 252 (1990)

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- No one jet knows how to resum Double Logs, let alone what matrix-element ingredients are needed to achieve NLL accuracy!

> Any automated approach to NLL resummation has better be able to establish whether an observables exponentiates

Consider n emissions $k_1(\lambda_1), \ldots k_n(\lambda_n)$ such that the soft-collinear limit corresponds to $\lambda_i \to 0$ and $V(k_i) = \lambda_i$. Then

Normal IRC safety implies

$$\lim_{\epsilon \to 0} V(k_1(\lambda_1), \ldots k_n(\lambda_n), k_{n+1}(\epsilon \lambda_{n+1})) = V(k_1(\lambda_1), \ldots k_n(\lambda_n))$$

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Recursive IRC safety adds two conditions

(a) $\lim_{\epsilon' \to 0} V(k_1(\epsilon'\lambda_1), \dots k_n(\epsilon'\lambda_n))/\epsilon' = \text{const.}(\neq 0)$

the SC scaling properties of V should be the same with just one or many emissions

(b) $\lim_{\epsilon \to 0} \lim_{\epsilon' \to 0} V(k_1(\epsilon'\lambda_1), \dots k_n(\epsilon'\lambda_n), k_{n+1}(\epsilon\epsilon'\lambda_{n+1}))/\epsilon' = \text{same const.}$

i. e. the addition of a relatively much softer/more collinear parton should not change asymptotically the limit

This condition is *the formal requirement for exponentiation*

The condition of IRS safety allows one to translate

- a restriction on an ensemble of emissions $\Rightarrow V(k_1, \dots k_n) < v$ into
- a restriction on individual emissions $\Rightarrow V(k_i) < v$ (modulo NLL terms in $\mathcal{F}$)

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- recursive IRC safety for exponentiation of infrared logarithms

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- IRC safety needed for fixed order predictions to be well-defined

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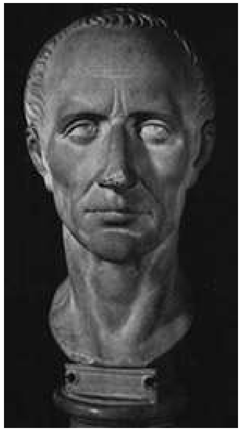
Example of observables NOT satisfying the condition

- Jet rates in Jade-algorithm
- Combinations of "usual" event shapes $\tau \cdot B_T, B_T^3/(1-\tau), y_{3D} \cdot C \dots$

CAESAR: conquering resummations



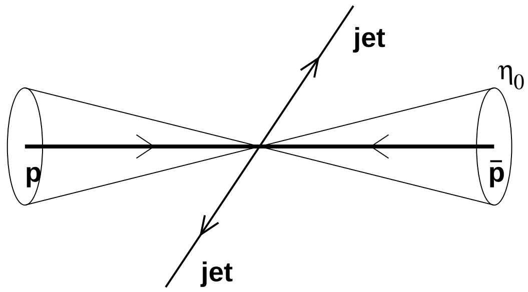
Computer **A**utomated **E**xpert
Semi-**A**nalytical **R**esummer

- currently limited to global observables
- tested against all known global, exponentiable event shapes
- results from an early version used by the LEP-QCD-WG for fits of α_s
- can be applied to
  - 2 & 3 jets in e^+e^-
  - [1+1] & [1+2] jets in DIS
  - Drell-Yan + 1 jet
  - hadron-hadron dijet events [$\Leftarrow$ first resummations]

Observables in hadronic dijet production

Cut around the beam $|\eta| < \eta_0$

➥ Problems with globalness ⚠



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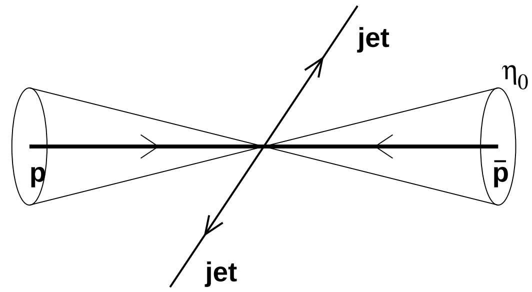

Directly global observables: $\eta_0 > 1$

✗ Transverse thrust

$$T_T = \frac{1}{E_T} \max_{\vec{n}_T} \sum_i |\vec{p}_{ti} \cdot \vec{n}_T|$$

✗ Thrust minor

$$T_m = \frac{1}{E_T} \sum_i |p_i^{out}|$$

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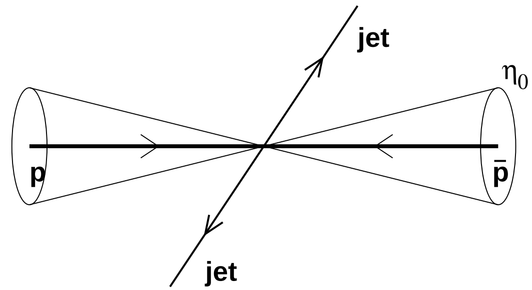

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Indirectly global observables: $\eta_0 = \mathcal{O}(1)$

✗ Transverse thrust

$$T_T = \frac{1}{E_{T,\eta_0}} \left(\max_{\vec{n}_T} \sum_{|\eta_i|<\eta_0} |\vec{p}_{ti} \cdot \vec{n}_T| - \left| \sum_{|\eta_i|<\eta_0} \vec{p}_{ti} \right| \right)$$

✗ Thrust minor

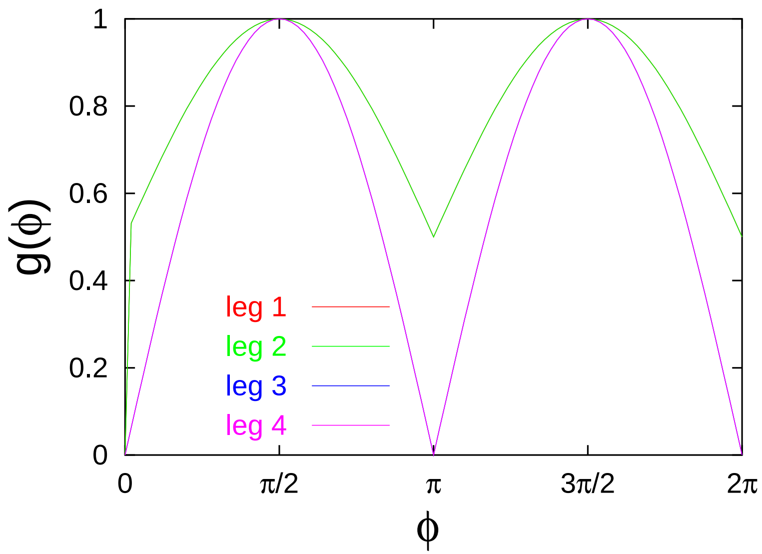
$$T_m = \frac{1}{E_{T,\eta_0}} \left(\sum_{|\eta_i|<\eta_0} |p_i^{out}| + \left| \sum_{|\eta_i|<\eta_0} \vec{p}_{ti} \right| \right)$$

Predictions valid as usual,
but $\mathcal{F}$ diverges at $R' = R'_c$

Sample output: the indirectly global thrust minor

✗ Tests on the observable

Test	result
check number of jets	T
observable positive	T
global	T
continuously global	T
additive	F
exponentiate	T
eliminate subleading effects	T
opt. probe region exists	T



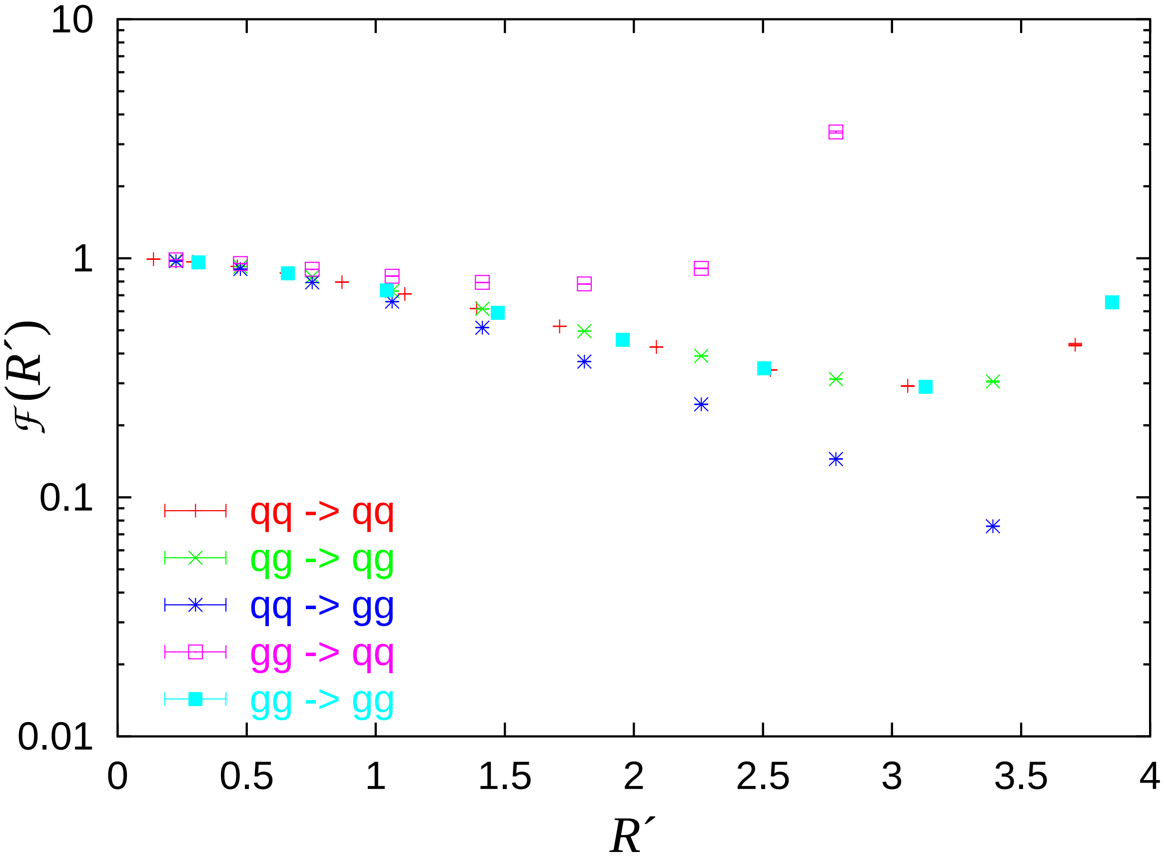
✗ Single emission properties

leg ℓ	a_ℓ	b_ℓ	$g_\ell(\phi)$	d_ℓ	$\langle \ln g_\ell(\phi) \rangle$
1	1	0	tabulated	2.0000	-0.2201
2	1	0	tabulated	2.0000	-0.2201
3	1	0	$\sin(\phi)$	2.0000	-Ln(2)
4	1	0	$\sin(\phi)$	2.0000	-Ln(2)

☛ Tables and plots generated automatically by CAESAR

$\mathcal{F}(R')$ *for the indirectly global thrust minor*

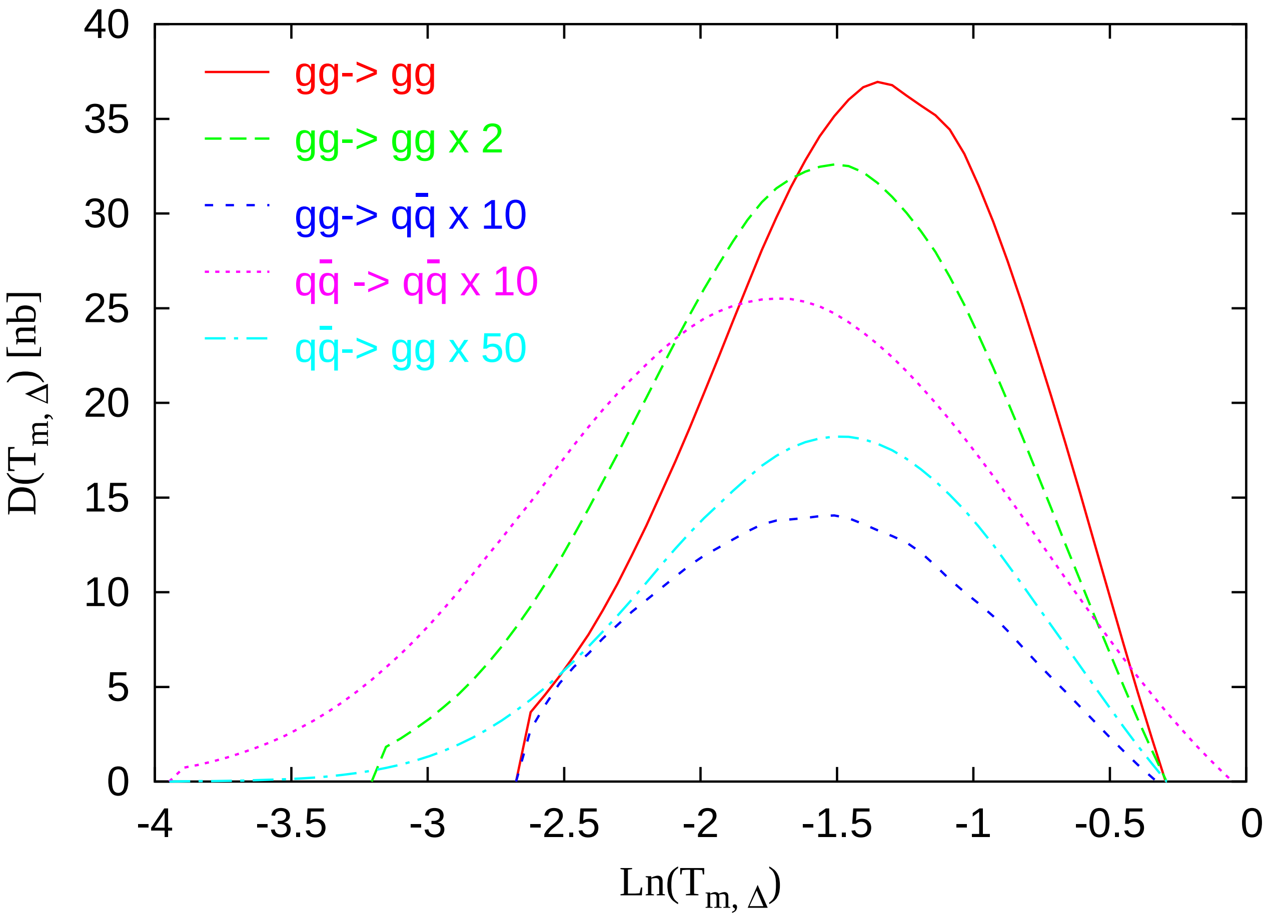
The multiple emission function $\mathcal{F}(R')$



☞ Different result for different colour configurations

Dijets events at Tevatron run II regime

- run II regime $\sqrt{s} = 1.96$ TeV
- cut on jet transverse energy $E_T > 50$GeV and on rapidity $|\eta| < 1$



$\alpha_s(M_Z) = 0.118$

$\mu_F = \mu_R = P_T$

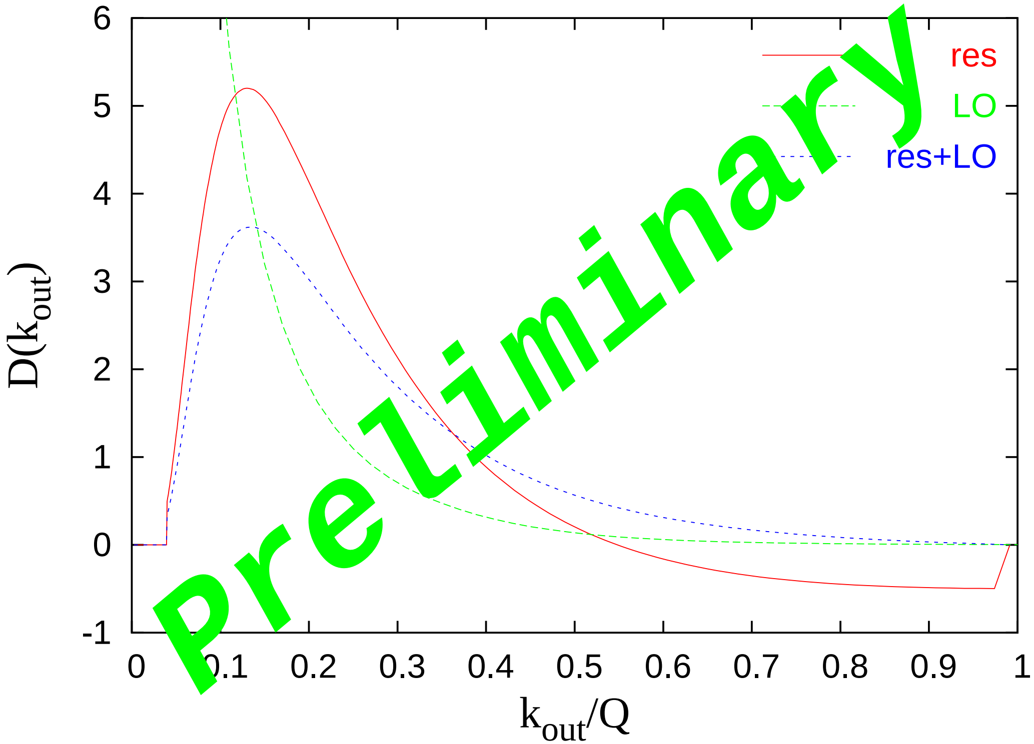
$X_c = 1$

PDFS: CTEQ6M

Out-of plane radiation in DIS [1+2] jet events

Dijets events at Hera

- Kinematical variables: $\sqrt{s} = 300$ GeV $Q = 36.7$ GeV $x_B = 0.056$
- Cuts: $y_{cut} = 0.1$ $\eta_{max} = 3$
- Scale choice and PDFs: $\alpha_s(M_Z) = 0.118$ $\mu_F = \mu_R = P_T$ PDFS: CTEQ6M



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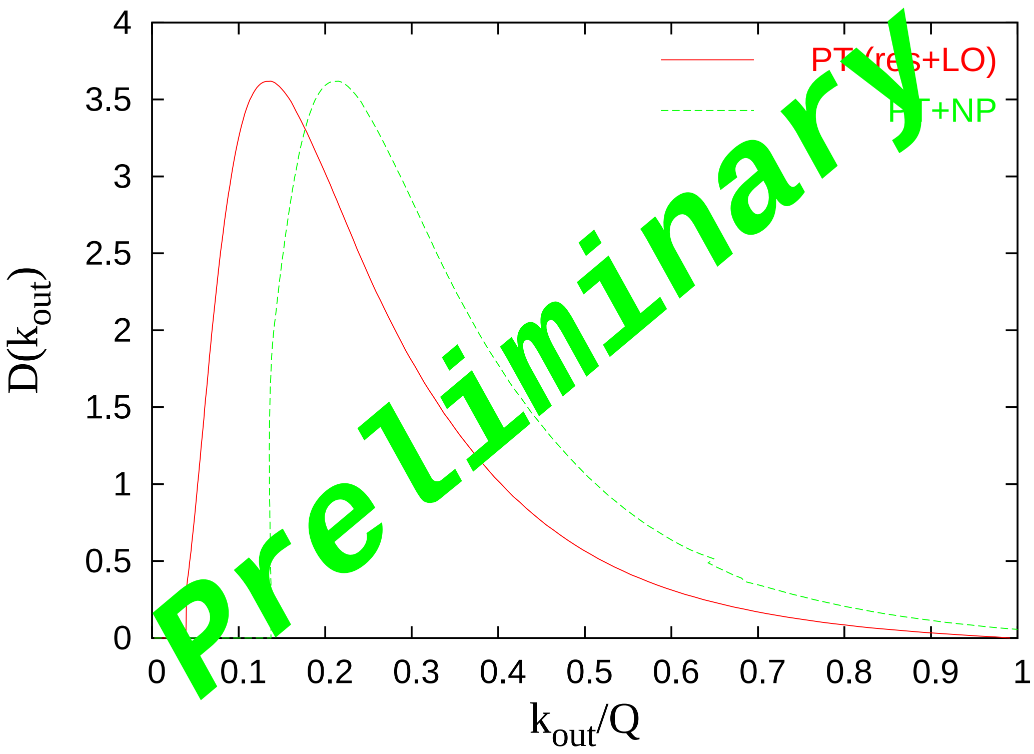

NP-shift: Banfi , Dokshitzer, Marchesini, GZ, hep-ph/0111157

Comparison with usual MCs

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Our predictions

- do not contain subleading Logs $\Rightarrow$ matching feasible
- purely perturbative, any hadronization model can be apply on top
- allow studies of factorization, renormalization scale dependencies
- are limited to a precise, well-defined class of observables

Main result: rigorous procedure to perform resummation semi-analytically

Banfi , Salam, GZ hep-ph/0304148

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✗ Most relevant applications

- *Theoretical:* criterion of recursive infrared and collinear safety
- *Experimental:* first NLL predictions in hadronic dijet events

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✗ To-do list and wish list

- automated matching of NLL with NLO(JET++)
- extension non-global observables and inclusion of mass effects