

# Fermi node, Topological Phases and Domain Walls of Pyrochlore Iridates

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# Part 1

## Fermi node and Topological Phases of Pyrochlore Iridates

UCSB

Leon Balents



Eun-Gook Moon  
(now U. Chicago)



Japan

Takeshi Kondo



Satoru Nakatsuji

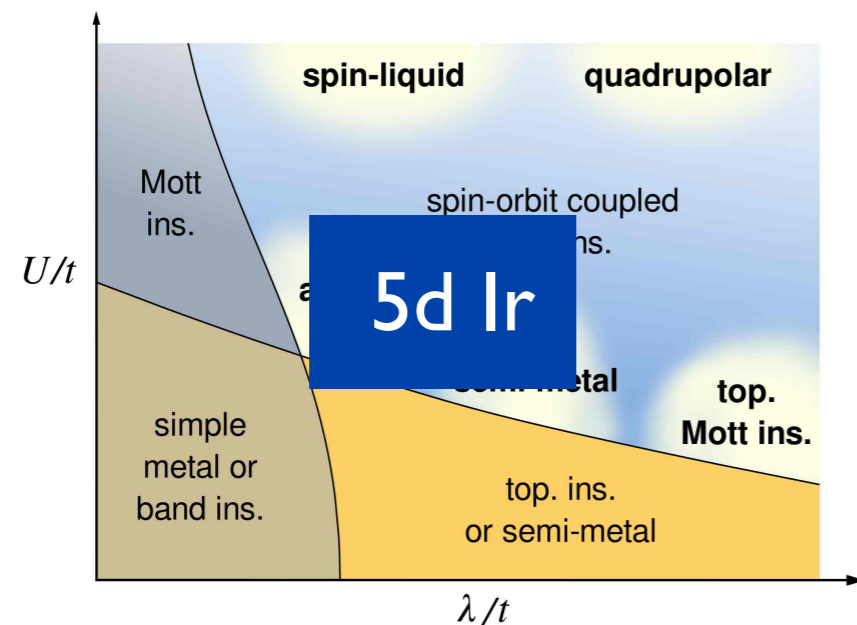
Shik Shin

...

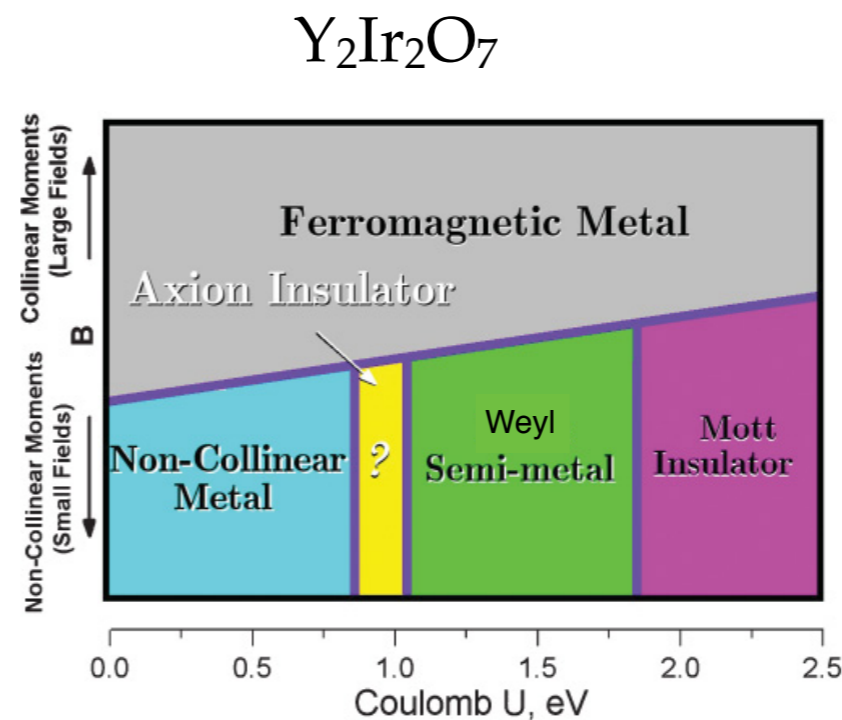


# Motivations

- Correlation physics: Mott insulator, superconductivity...
- Spin-orbit coupling: topological insulators, Majorana fermions...
- Coulomb interaction+spin-orbit coupling  $\rightarrow$  new physics?
- Iridates:  $U(\text{correlation}) \sim W(\text{band width}) \sim \lambda(\text{spin-orbit})$



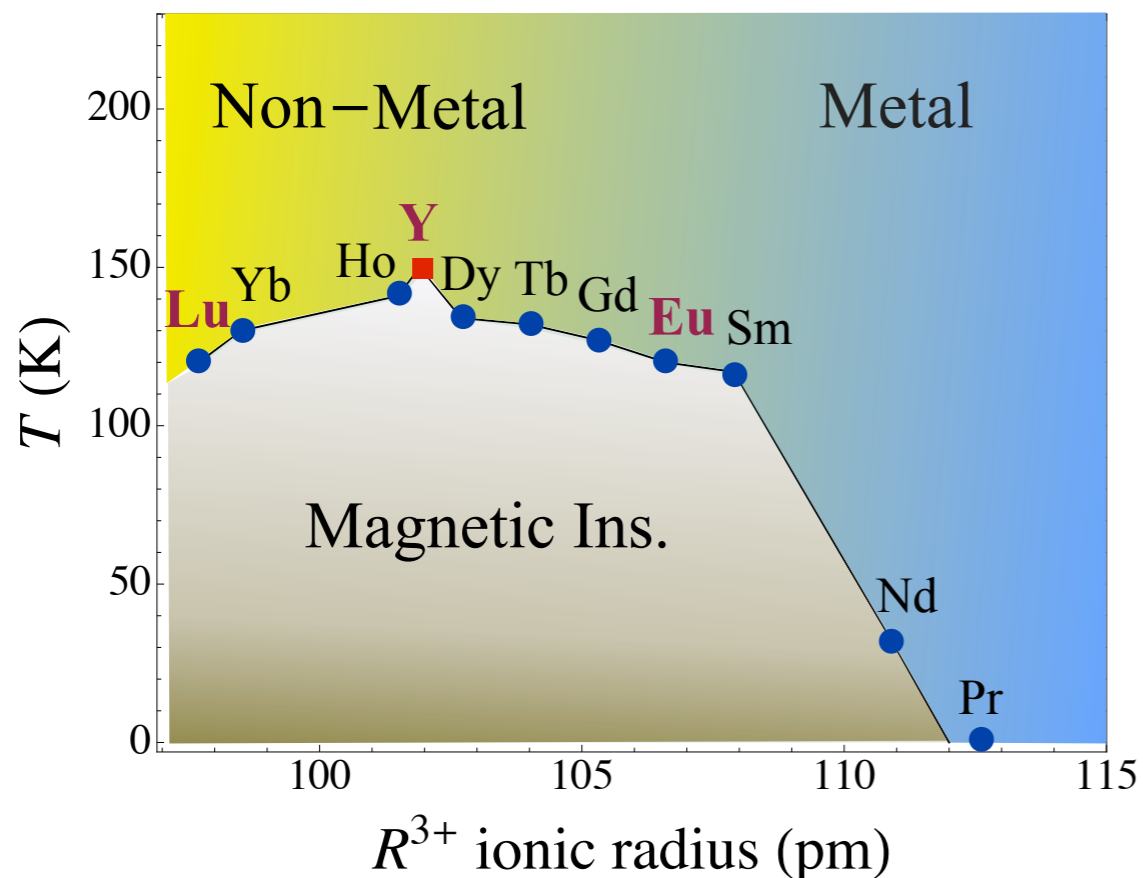
W. Witczak-Krempa et al, ARCMP 2013



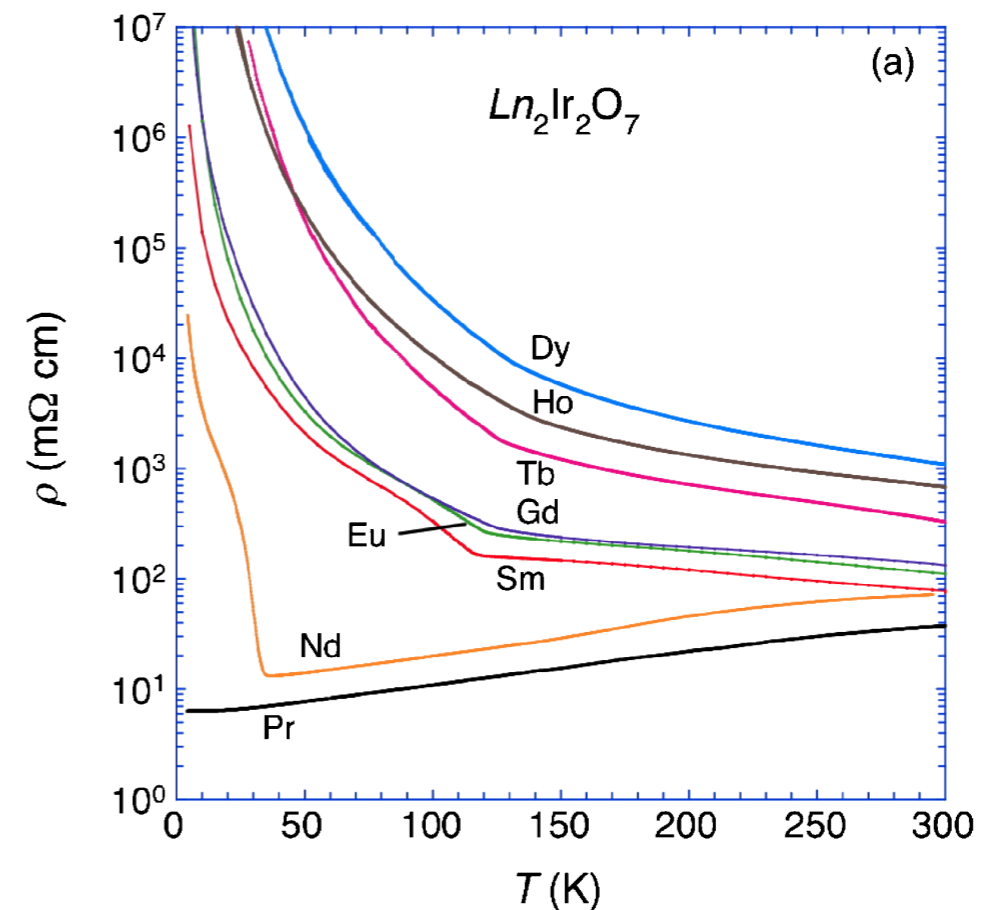
X. Wan et al, PRB 2011

# Review on Pyrochlore Iridates $R_2Ir_2O_7$

- Magnetic ordering transition, 2nd order transition
- Magnetic transition and metal-insulator transition coincides for relatively larger rare earth radius compound
- As rare earth ionic radius decreases,  $T_{MI}$  increases and gap increases



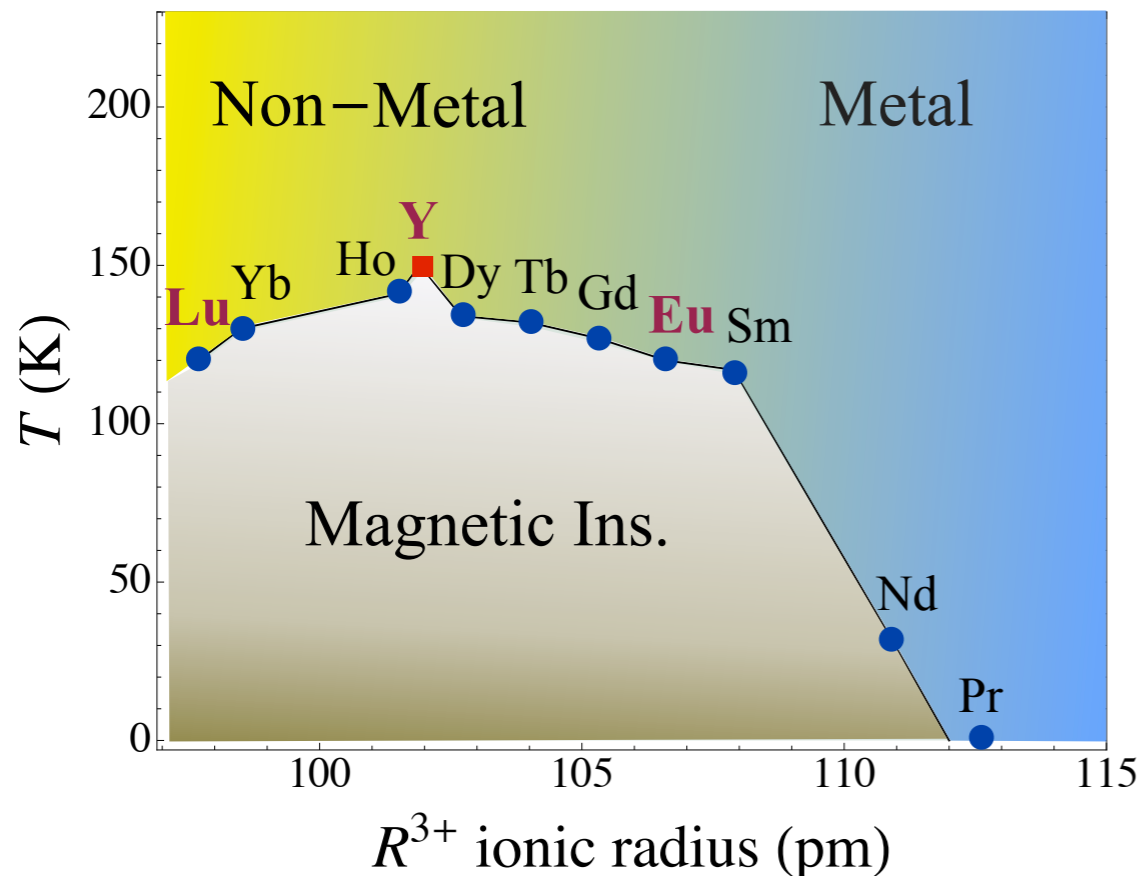
K. Matsuhira et al, JPSJ 2011  
W. Witczak-Krempa et al, ARCOMP 2013



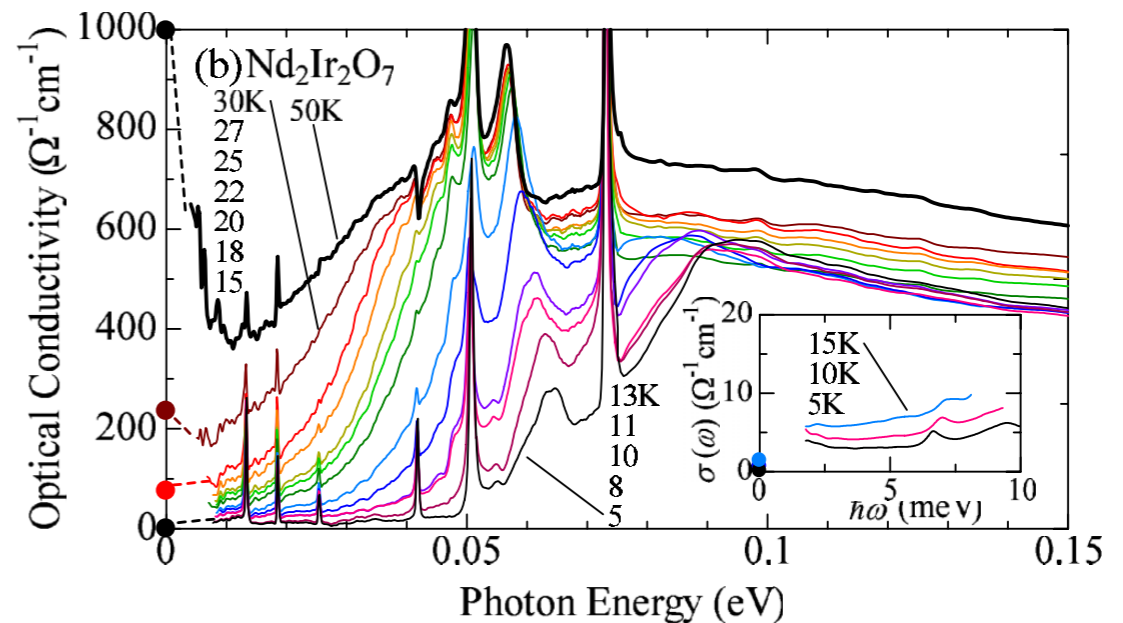
K. Ueda et al, PRL 2012

# Review on Pyrochlore Iridates $R_2Ir_2O_7$

- Magnetic ordering transition, 2nd order transition
- Magnetic transition and metal-insulator transition coincides for relatively larger rare earth radius compound
- As rare earth ionic radius decreases,  $T_{MI}$  increases and gap increases
- Energy gap is small comparing to 3d materials



K. Matsuhira et al, JPSJ 2011  
W. Witczak-Krempa et al, ARCOMP 2013

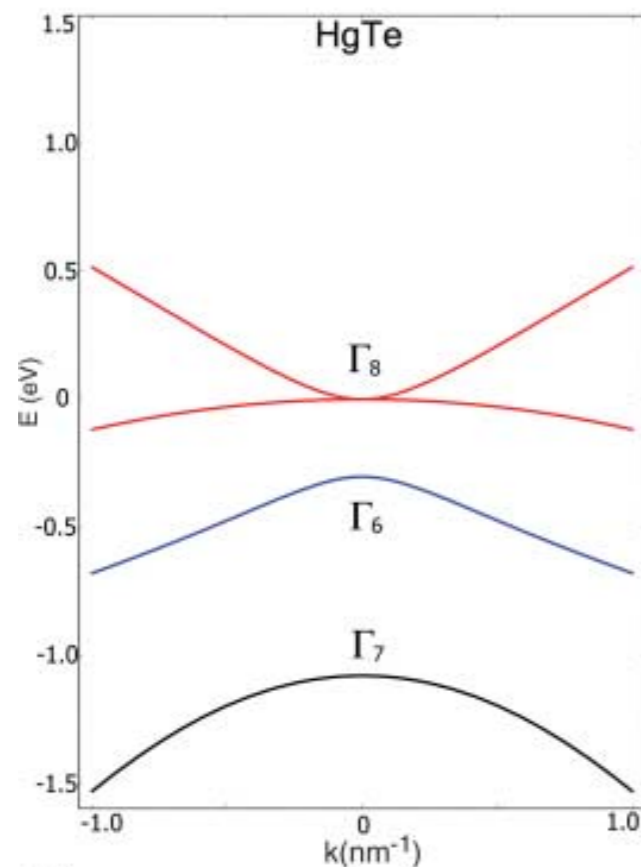


Optical conductivity  
gap  $\sim 0.045 \text{ eV}$

K. Ueda et al, PRL 2012

# Motivations

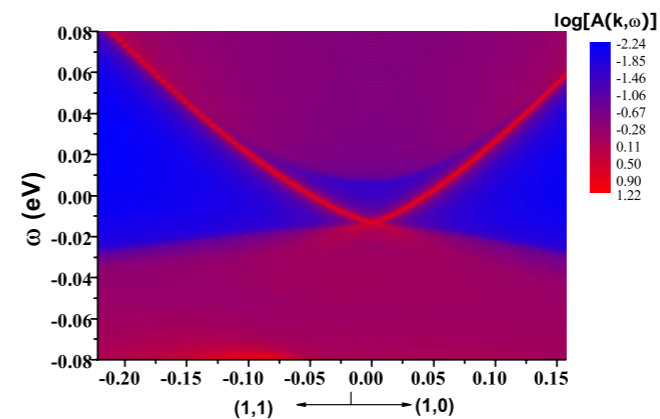
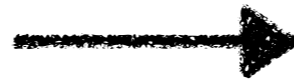
- What is the minimum low energy electronic structure of  $R_2Ir_2O_7$ ? Especially for more metallic ones.
- Quadratic Fermi node in  $Pr_2Ir_2O_7$  (*ab initio* electronic structure calculation and APRES data)



Quadratic band touching:  
HgTe

HgTe quantum wells:  
Analogy with negligible correlation.

Strain/Pressure



With pressure/strain:  
Gapped bulk phase  
Gapless HgTe/CdTe interfacial state

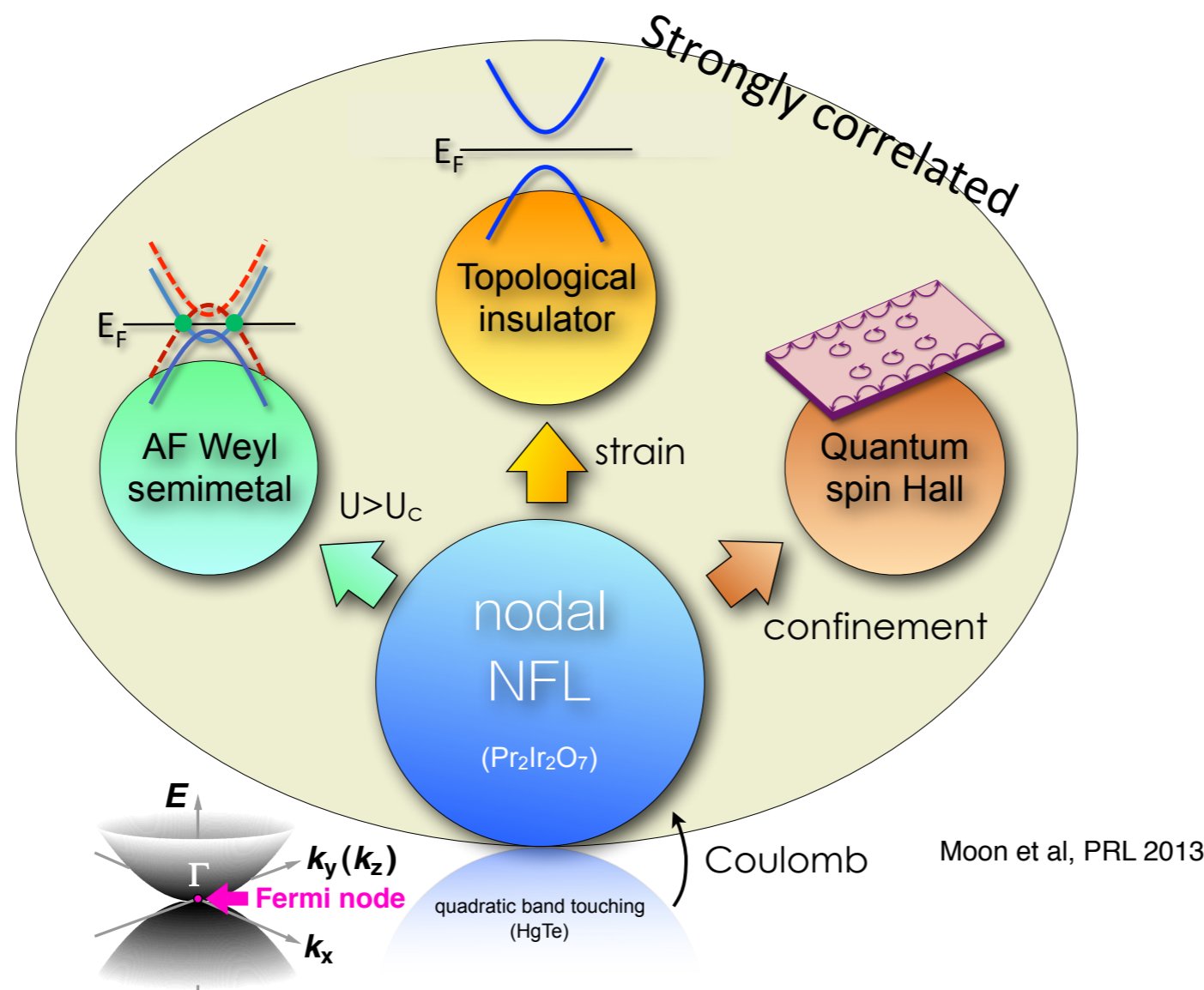
B.A. Bernevig, et al. Science 2006

# Motivations

- Tune the ground state ? If so, how?

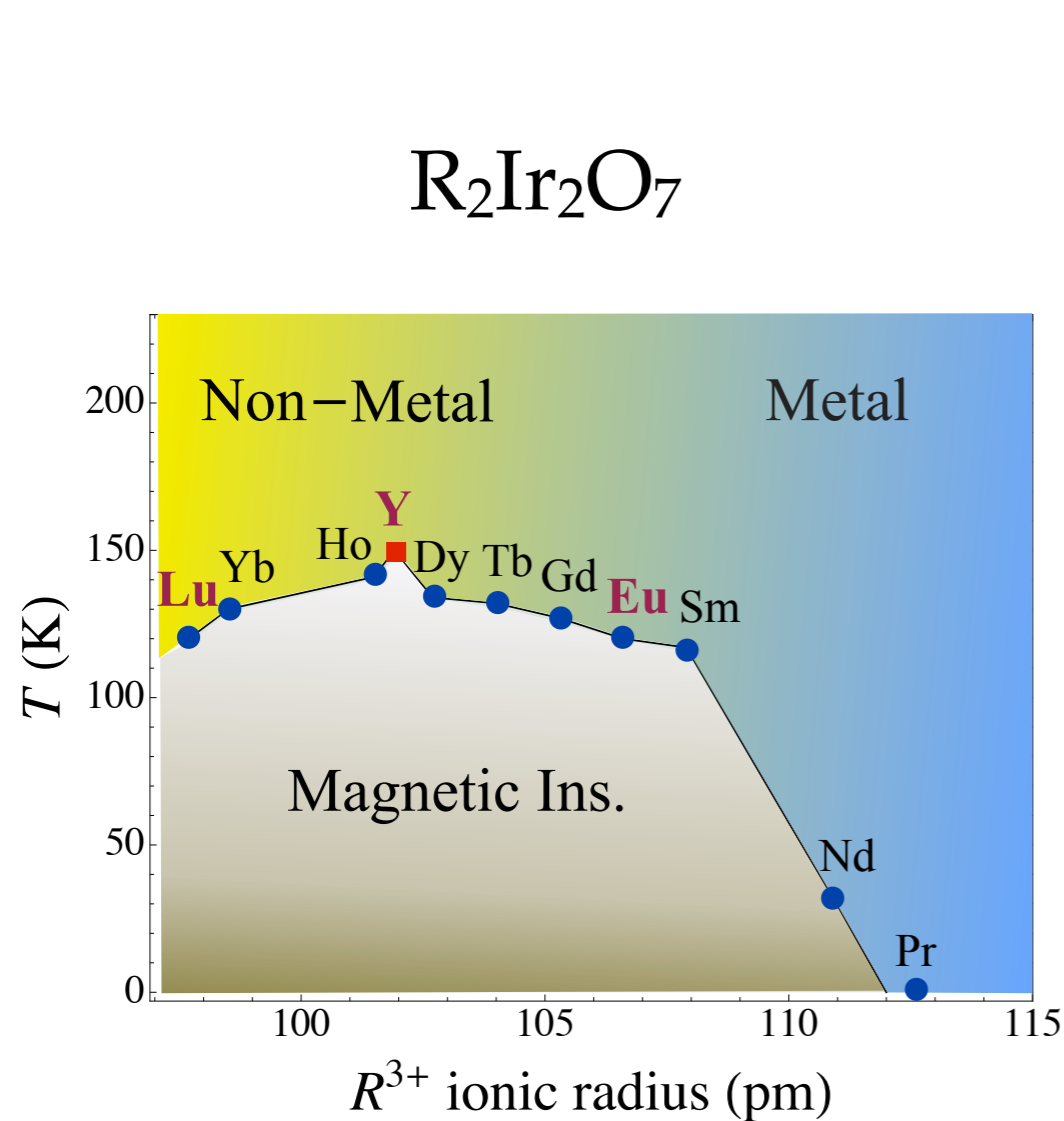
Goal: Topological insulators, quantum anomalous Hall effect, Weyl semimetal

- Growing techniques such as MBE and PLD, magnetic and pressure measurement have made tuning more accessible.

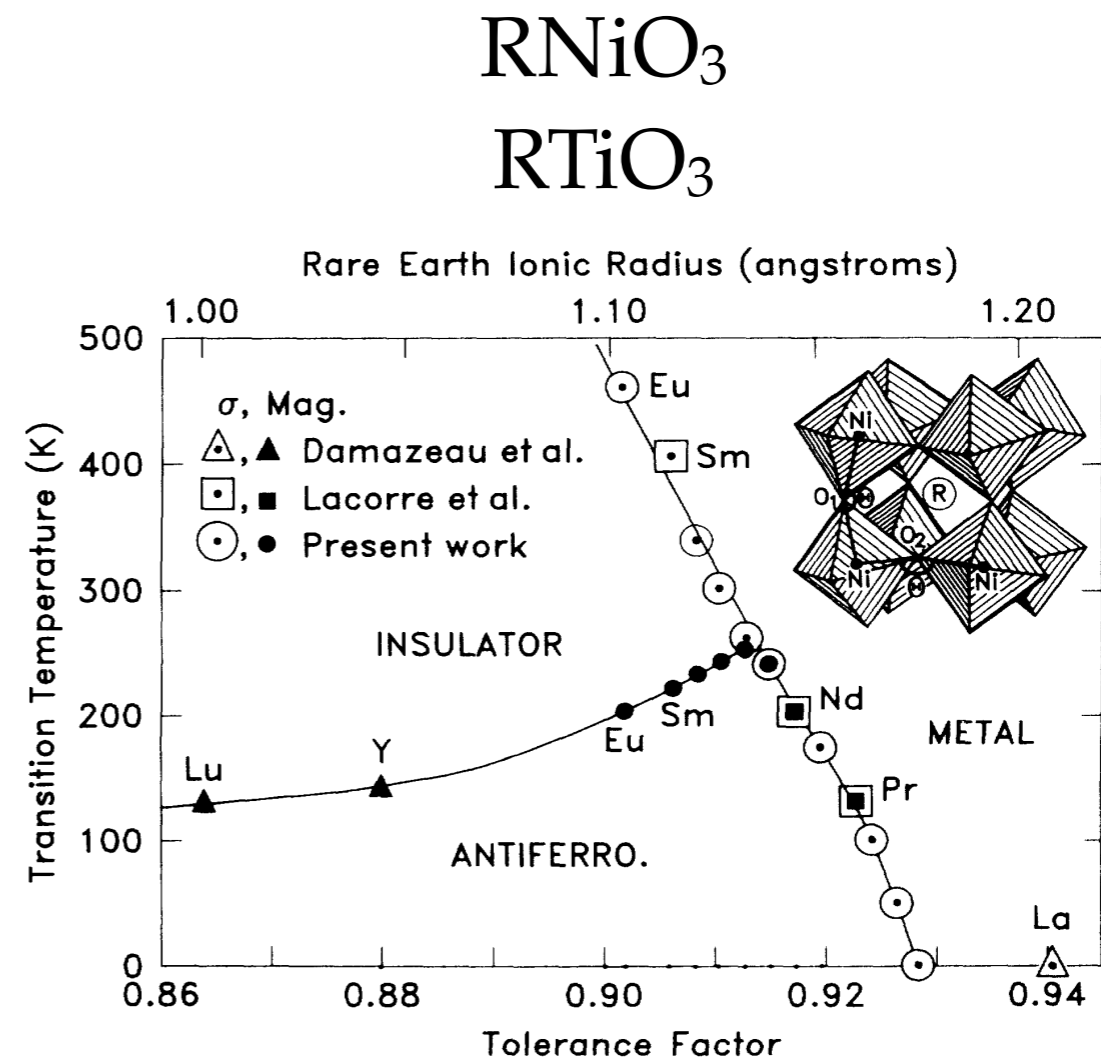


# Motivations

- What are the important microscopic parameters which drive the metal-insulator transition? What drives MIT?



K. Matsuhira et al, JPSJ 2011  
W. Witczak-Krempa et al, ARCOMP 2013

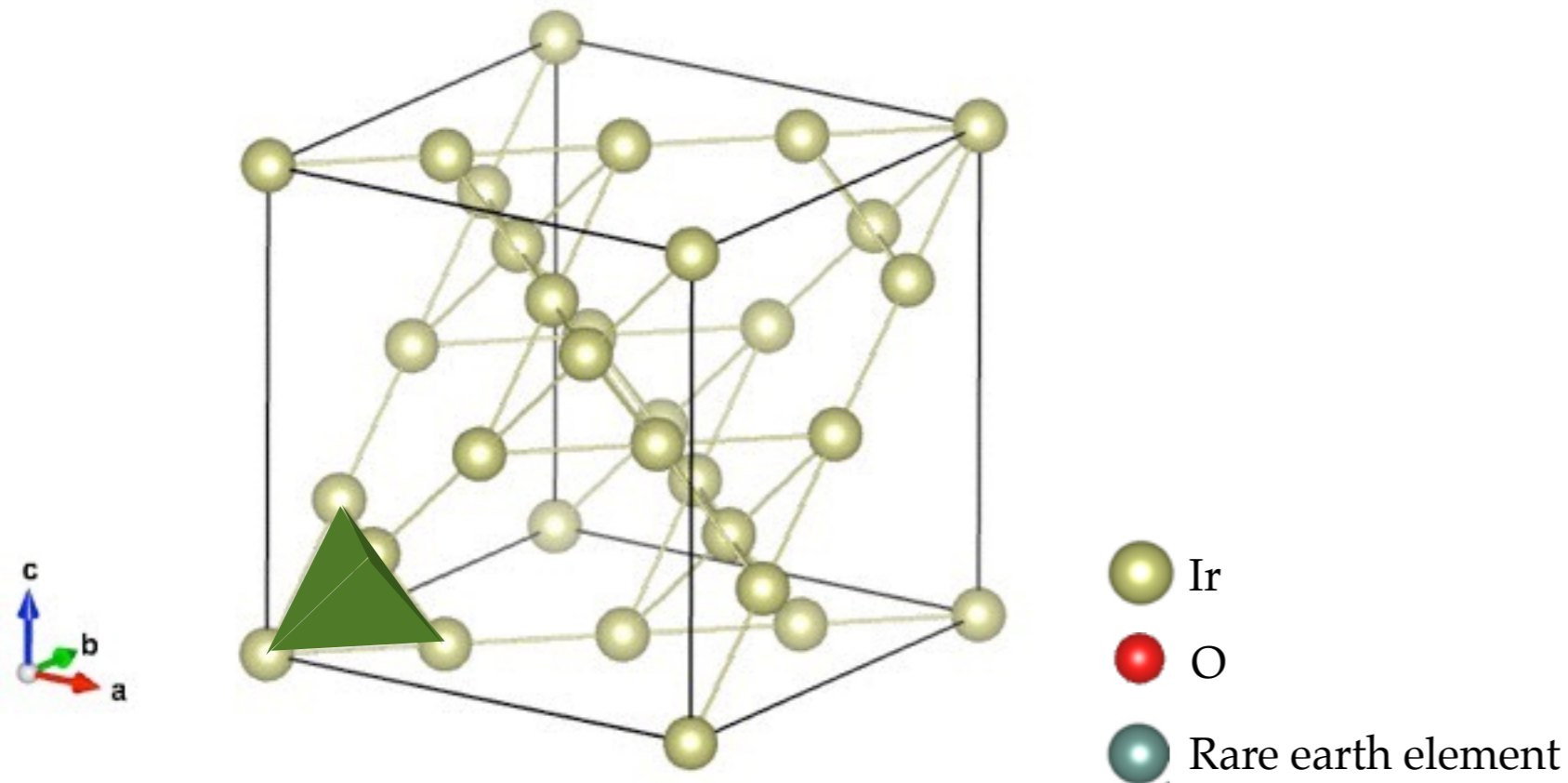


Lacorre et al, PRB 1992



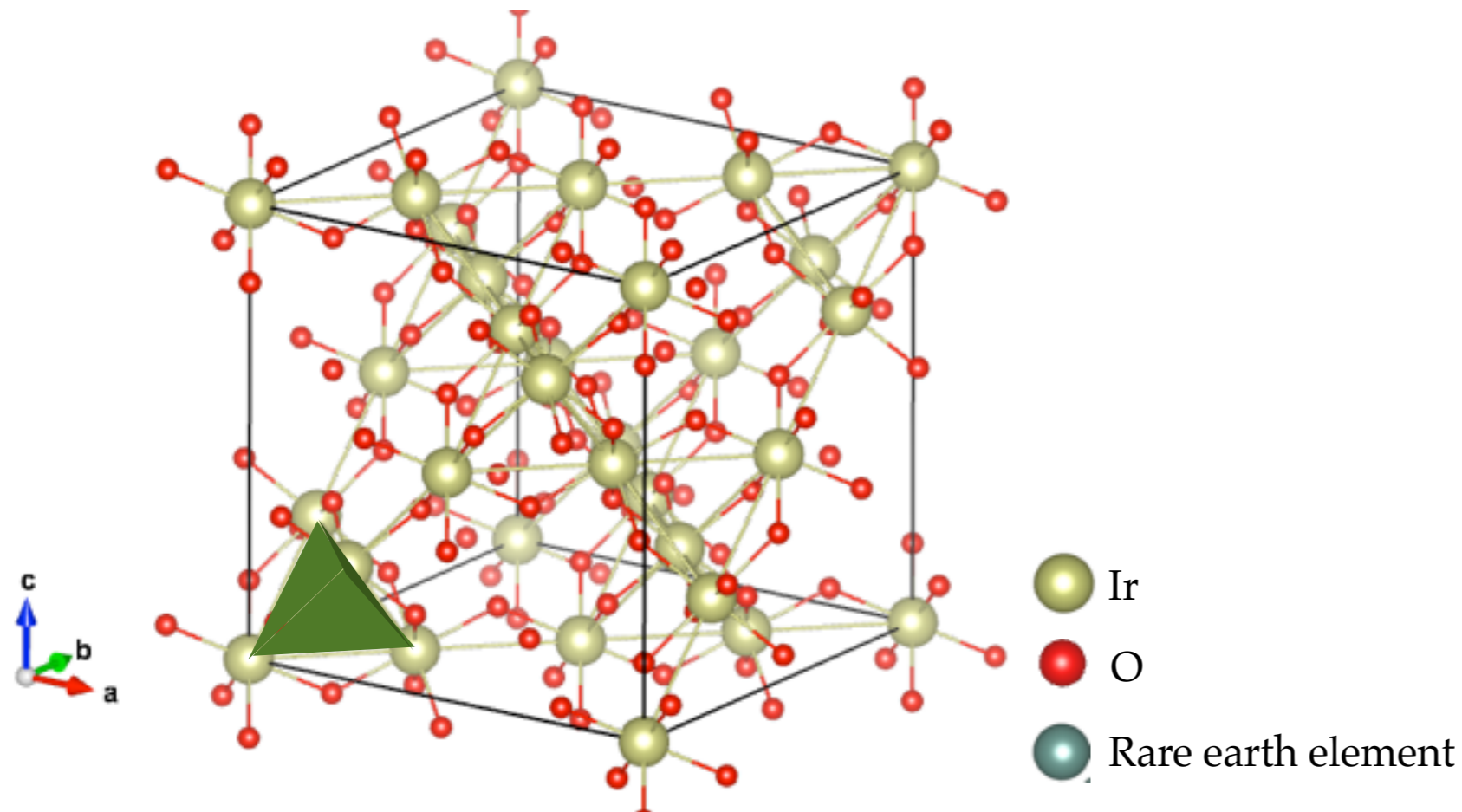
# Review on Pyrochlore Iridates $R_2Ir_2O_7$

- Structure of pyrochlore iridate  $R_2Ir_2O_7$  :  
FCC corner sharing tetrahedra
- Ir-O-Ir bonds are distorted, compressed along 111 direction



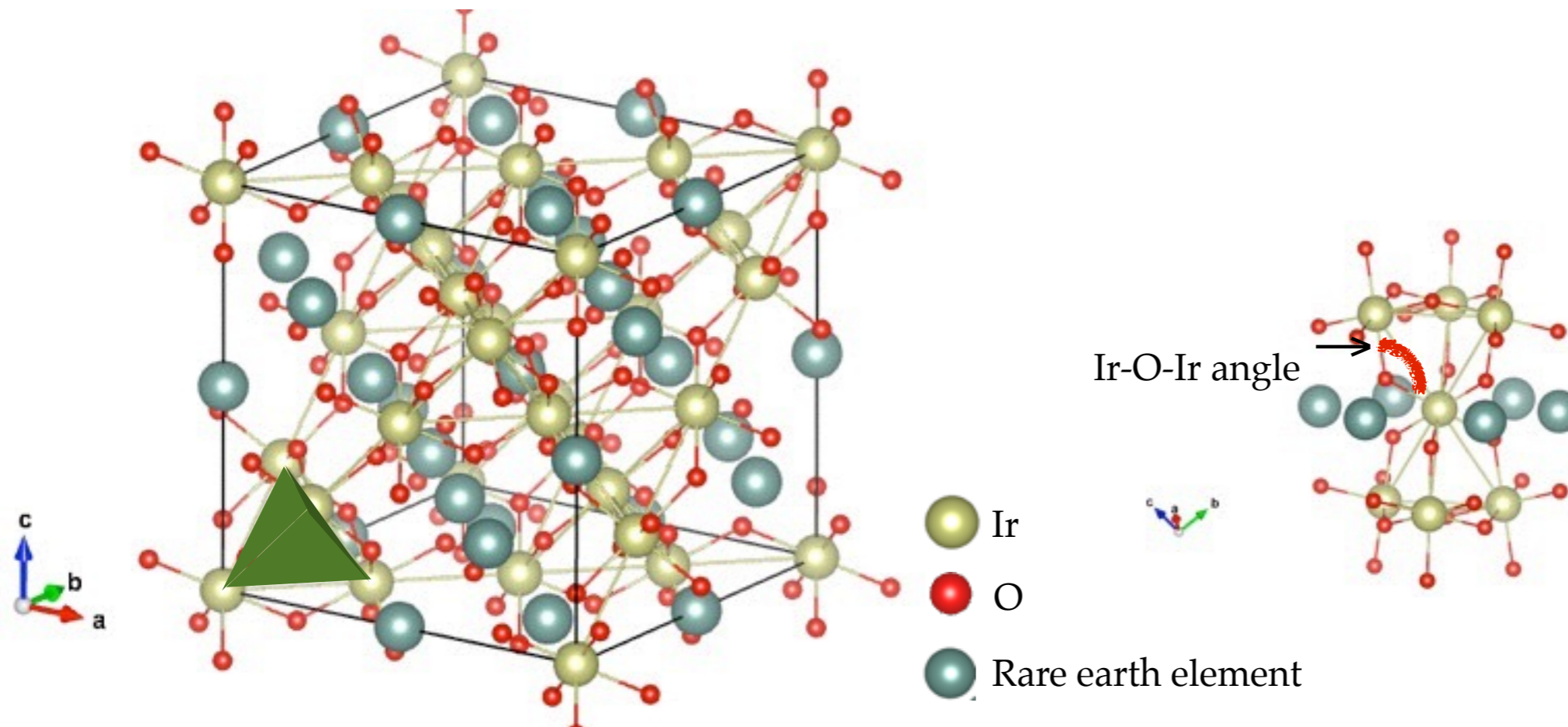
# *Review on Pyrochlore Iridates $R_2Ir_2O_7$*

- Structure of pyrochlore(correct all!) iridate  $R_2Ir_2O_7$  :  
FCC corner sharing tetrahedra
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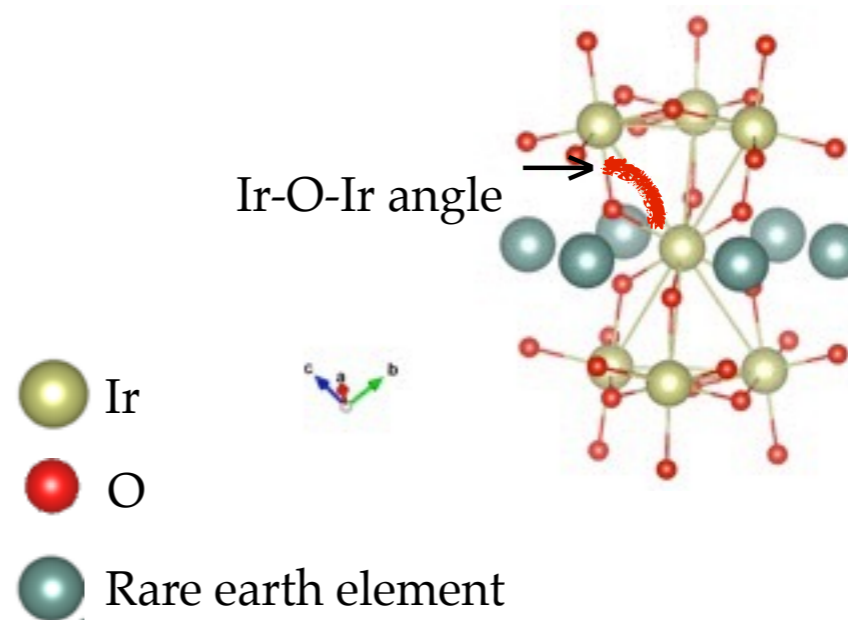
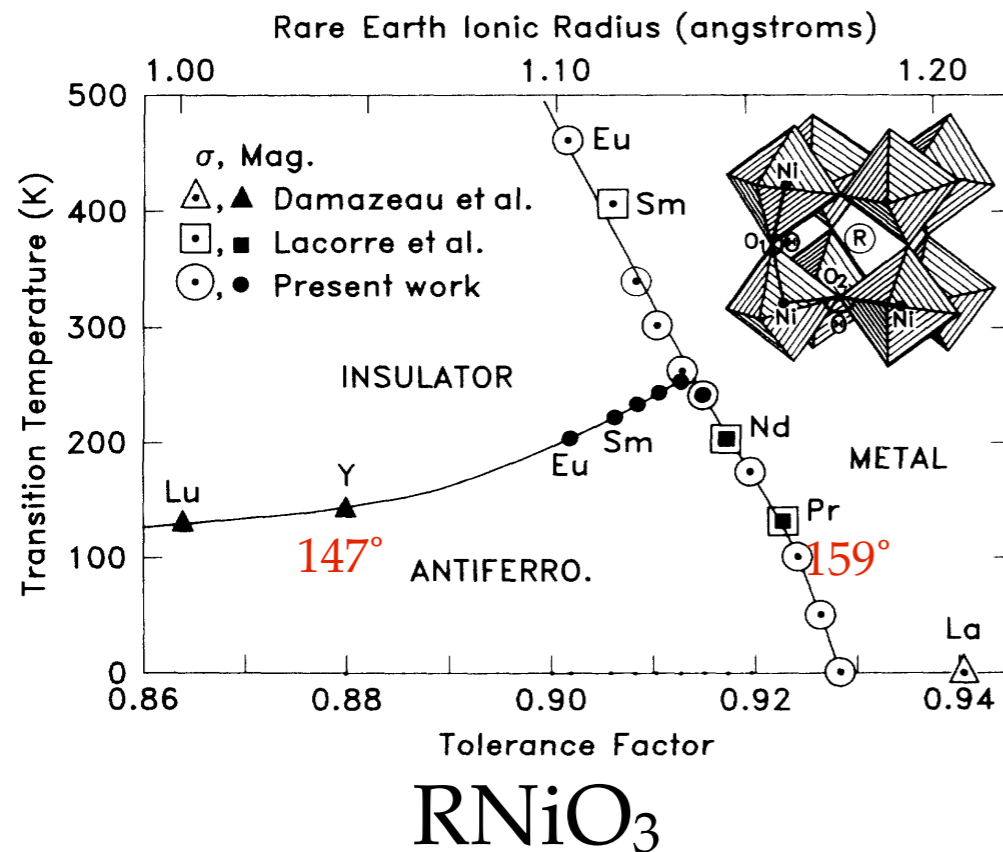
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- Ir-O-Ir bond angle Y:  $\sim 129^\circ$ ; Pr:  $\sim 132^\circ$



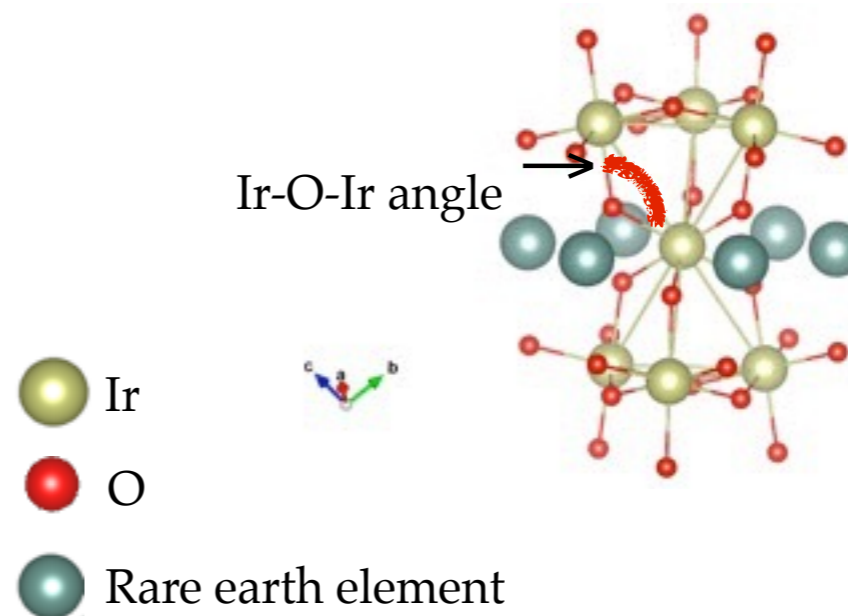
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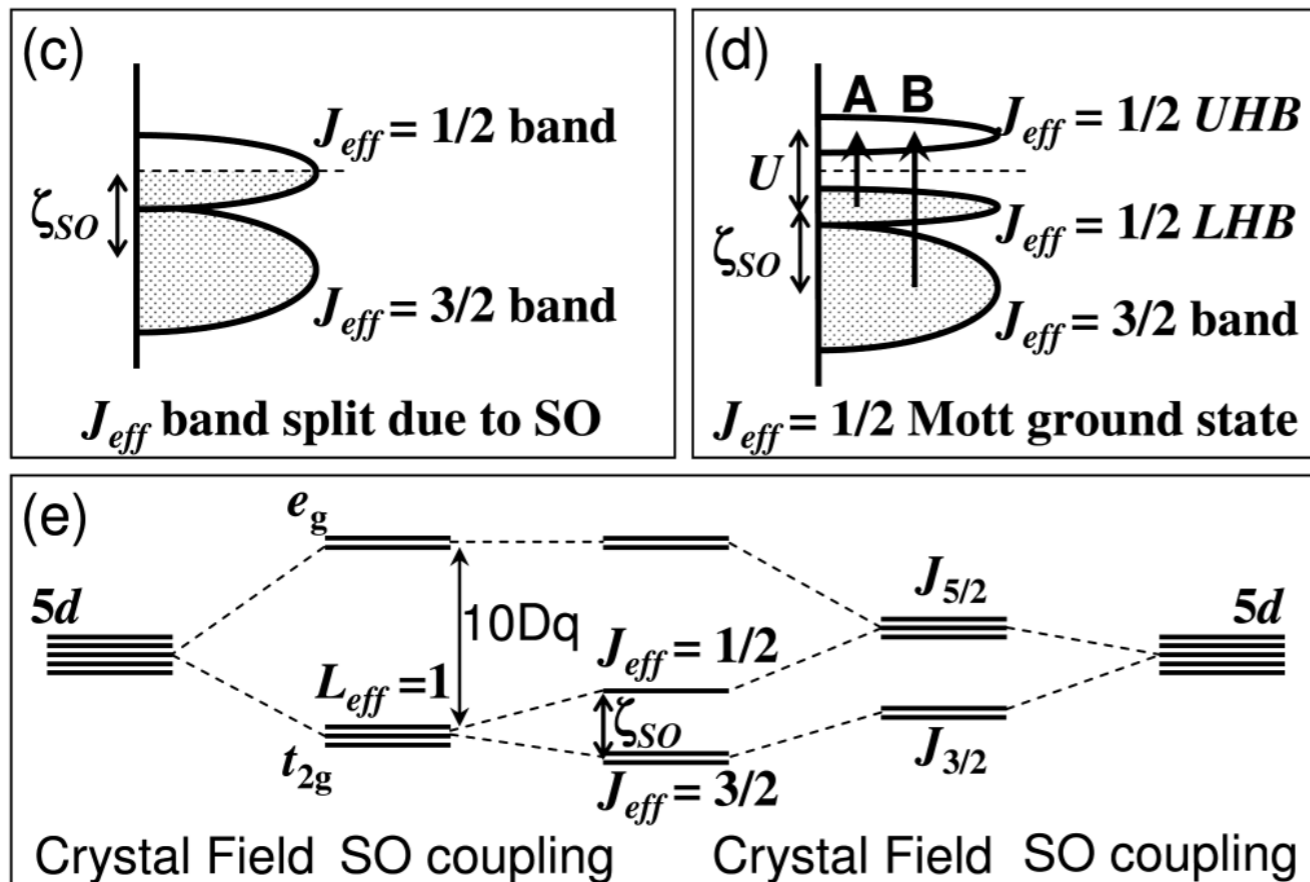
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- Ir-O-Ir bond are distorted, compressed along 111 direction
- Ir-O-Ir bond angle Y:  $\sim 129^\circ$ ; Pr:  $\sim 132^\circ$
- Ir-O bond length Y:  $1.997\text{\AA}$  ; Pr:  $2.014\text{\AA}$
- Ir-Ir bond length Y:  $3.599\text{\AA}$  ; Pr:  $3.677\text{\AA}$



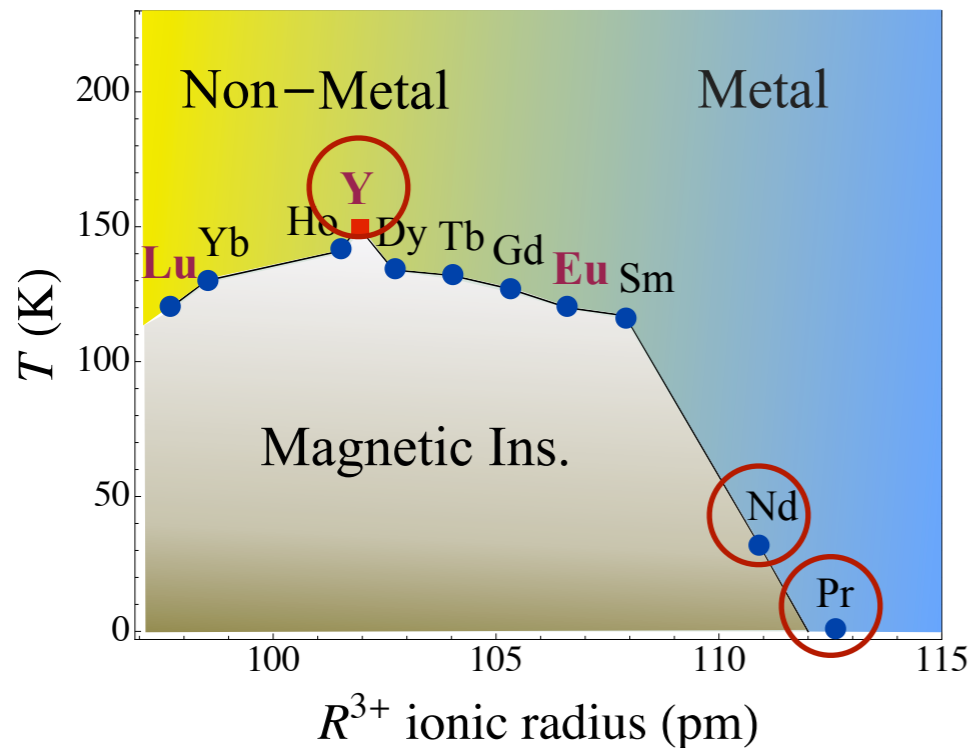
# Review on Pyrochlore Iridates $R_2Ir_2O_7$

- Ionic picture:  $Ir^{4+} 5d^5$
- $J_{eff}=1/2$  doublet

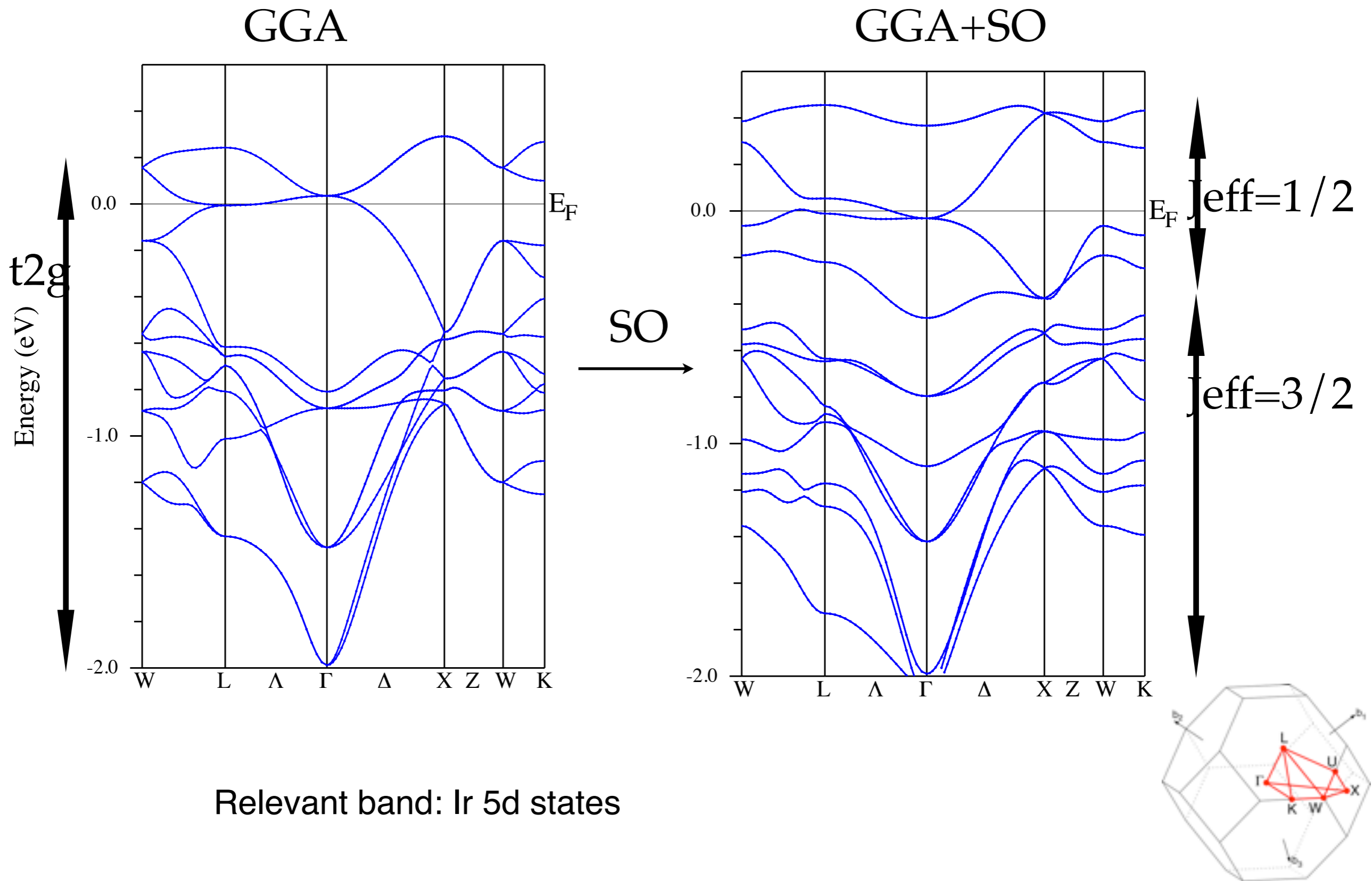


# Paramagnetic band structure calculation

- Focus on Pr, Nd, Y
- Various functional to explore the trend: GGA+SO, LDA+SO, mBJ+SO
- All electronic potential Wien2k



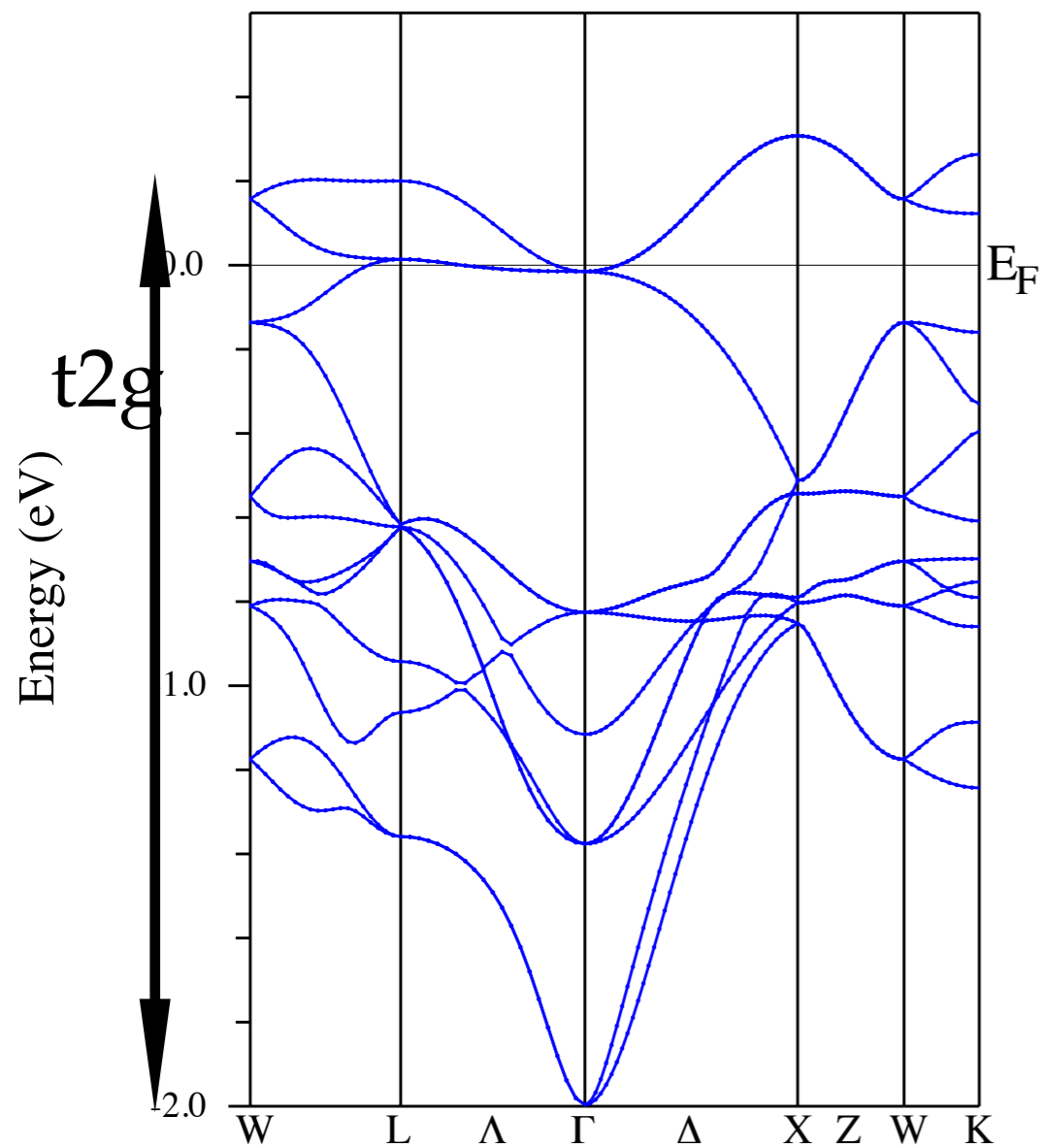
# $Pr_2Ir_2O_7$





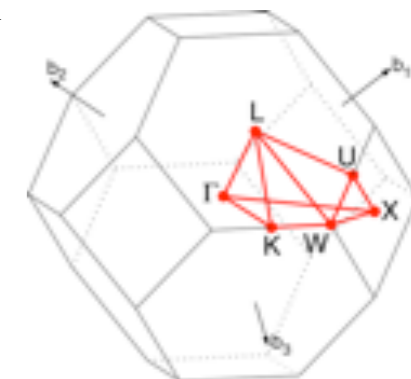
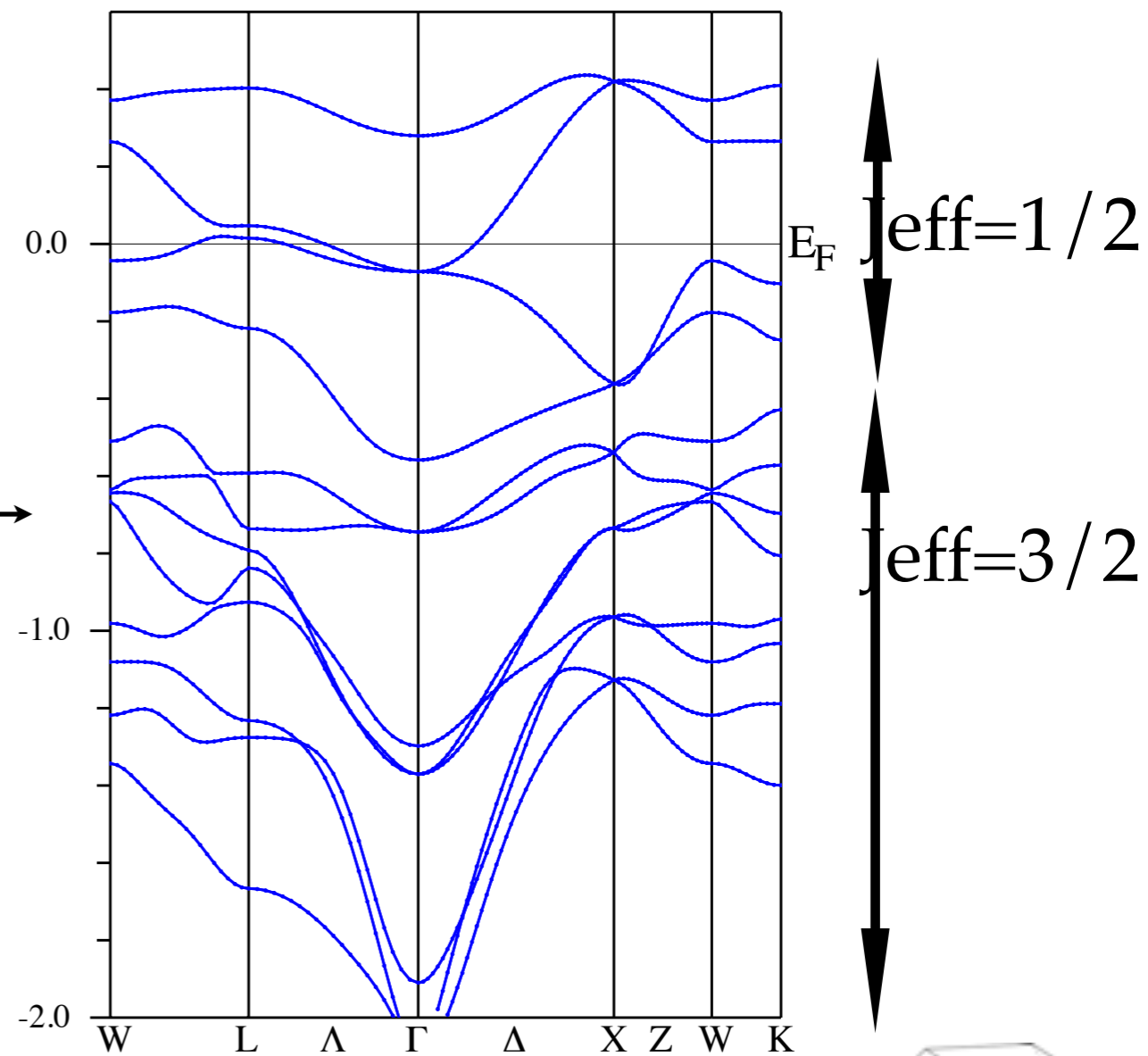
# $Y_2Ir_2O_7$

GGA

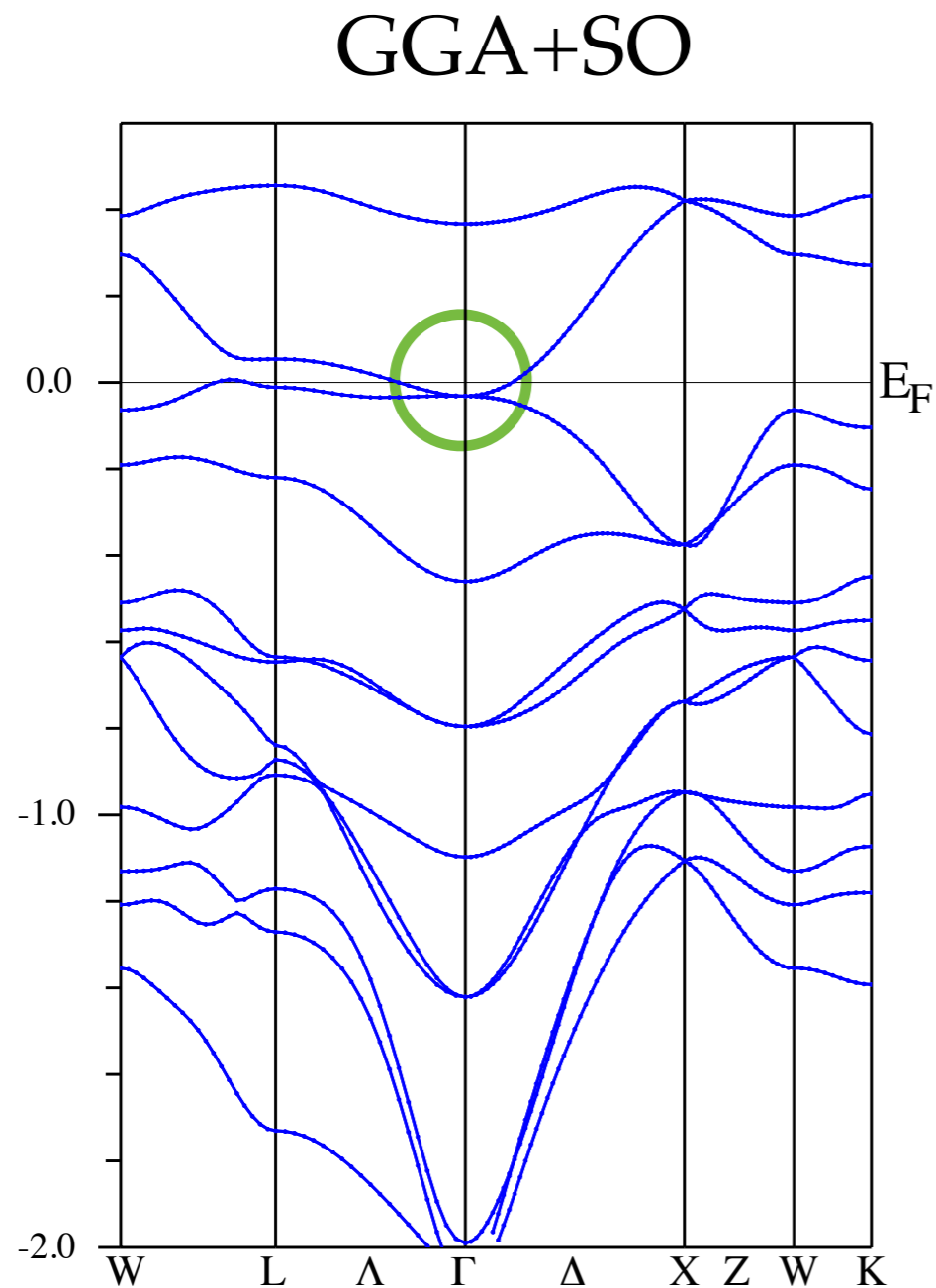


SO

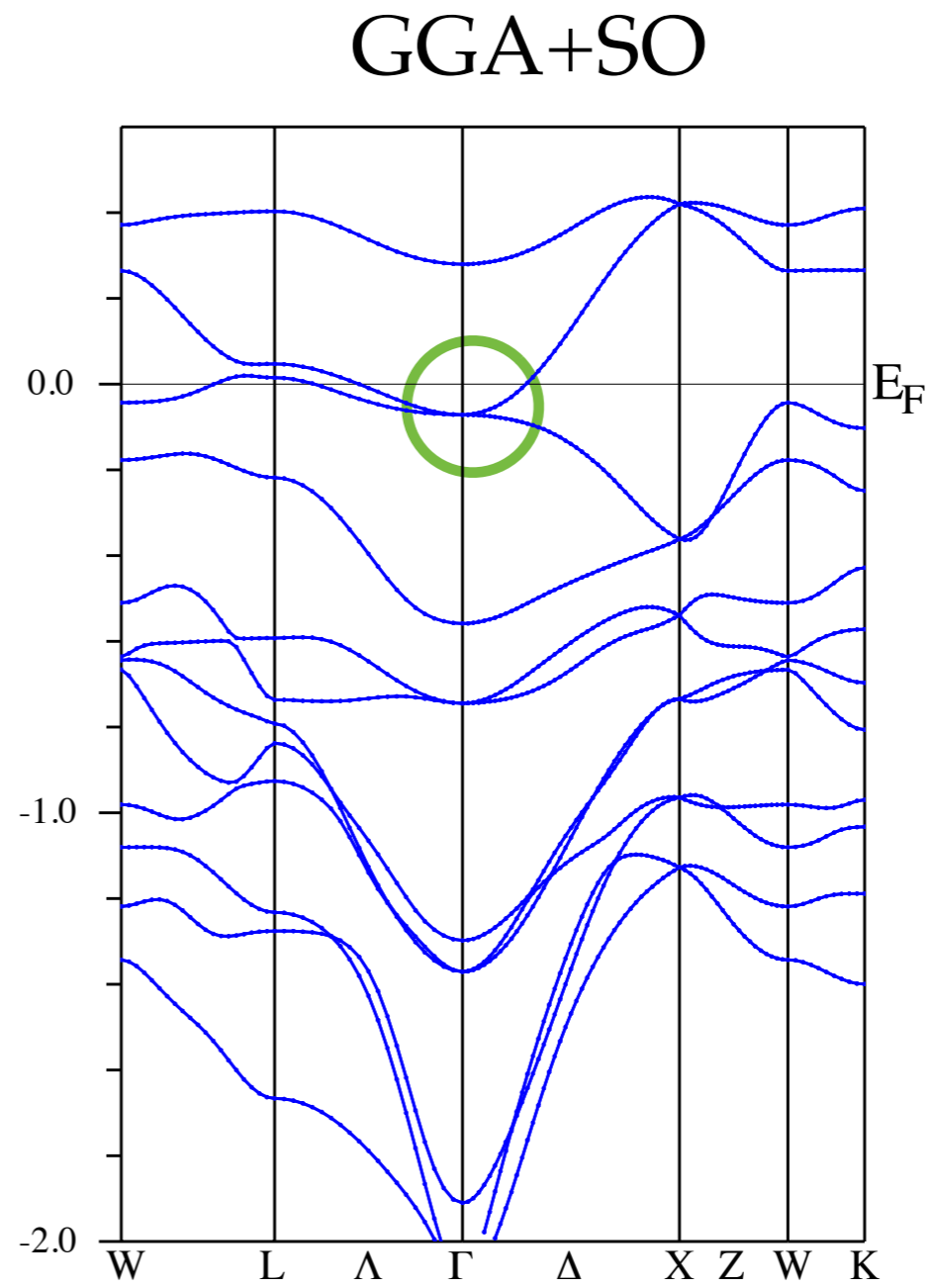
GGA+SO



# Comparison: $Pr_2Ir_2O_7$ and $Y_2Ir_2O_7$

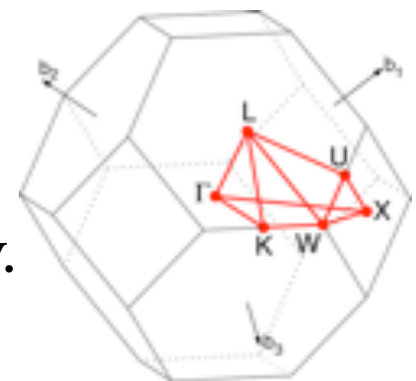


$Pr_2Ir_2O_7$



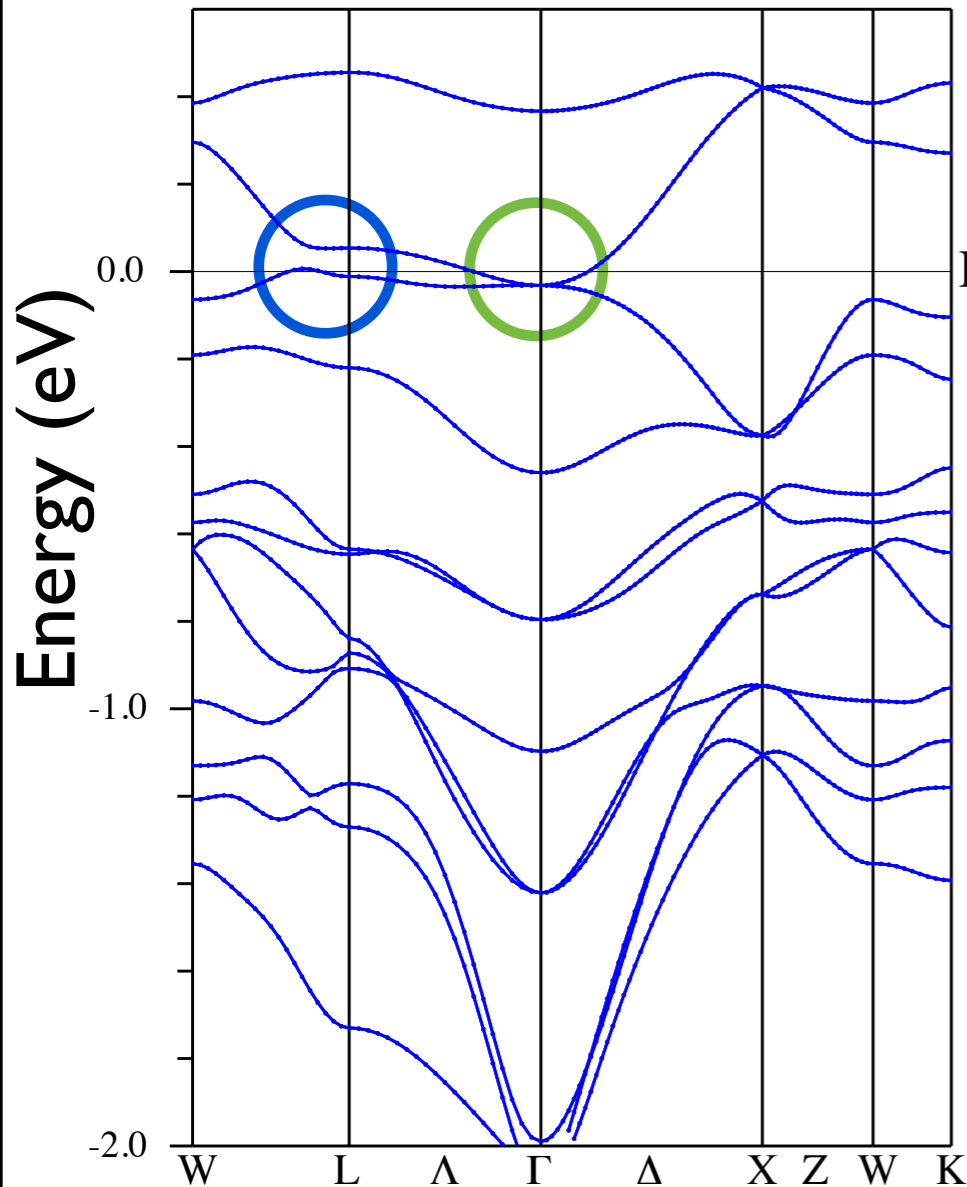
$Y_2Ir_2O_7$

4-fold quadratic band touching is protected by TR and cubic symmetry.



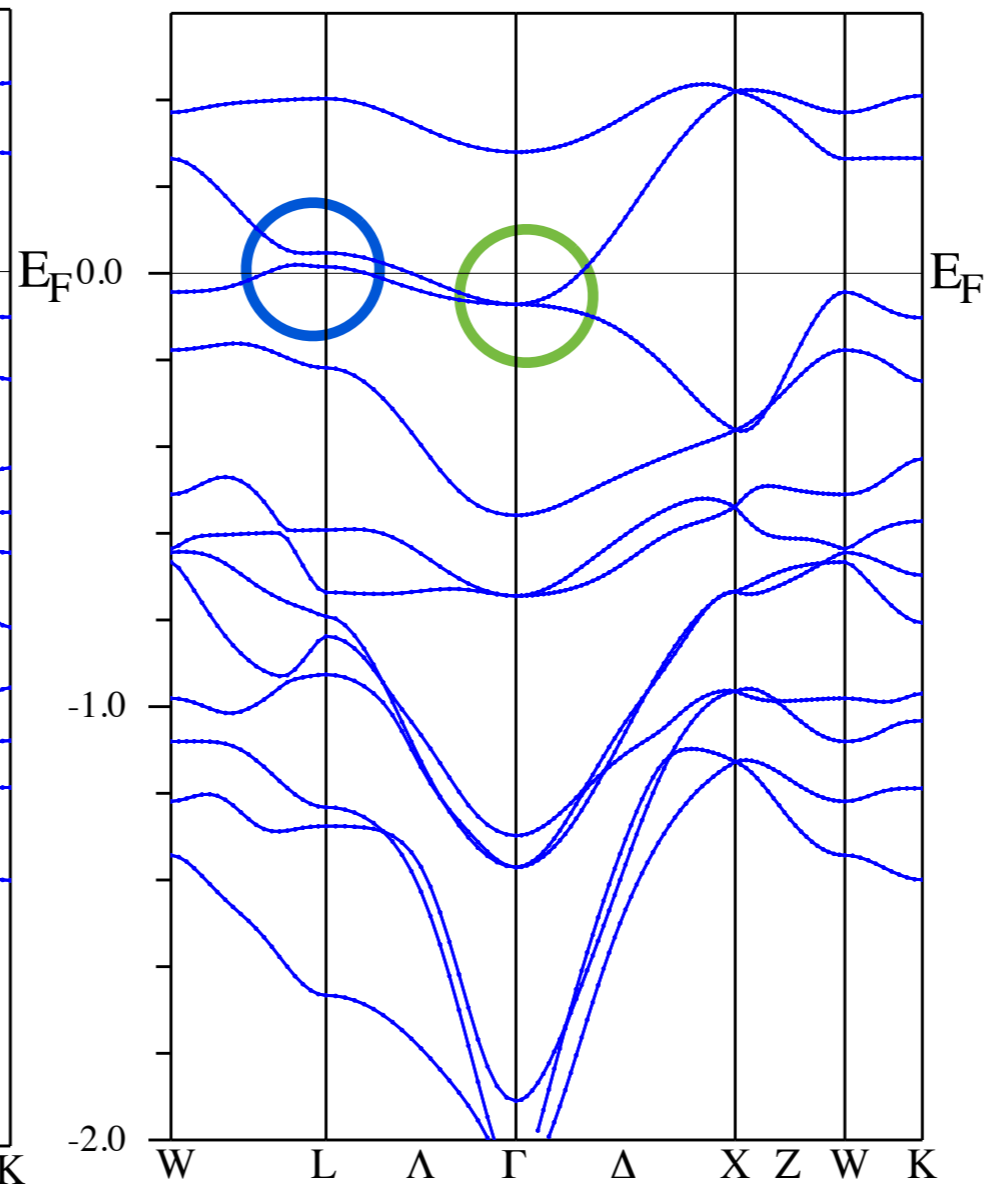
# Comparison: $\text{Pr}_2\text{Ir}_2\text{O}_7$ and $\text{Y}_2\text{Ir}_2\text{O}_7$

GGA+SO



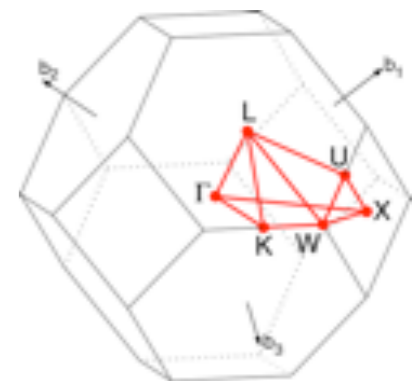
$\text{Pr}_2\text{Ir}_2\text{O}_7$

GGA+SO



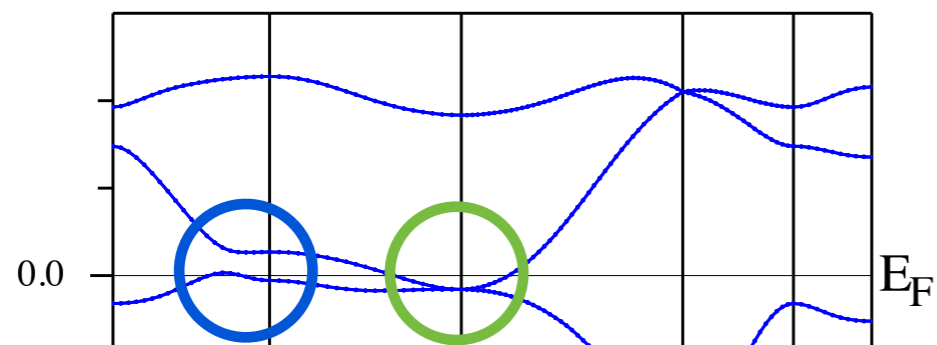
$\text{Y}_2\text{Ir}_2\text{O}_7$

- Not band width tuned (hopping Ir-O-Ir, Ir-Ir competing)
- Very subtle changes near  $E_F$
- Larger R is closer to quadratic band touching at  $E_F$  (R=Pr, Nd, Y)

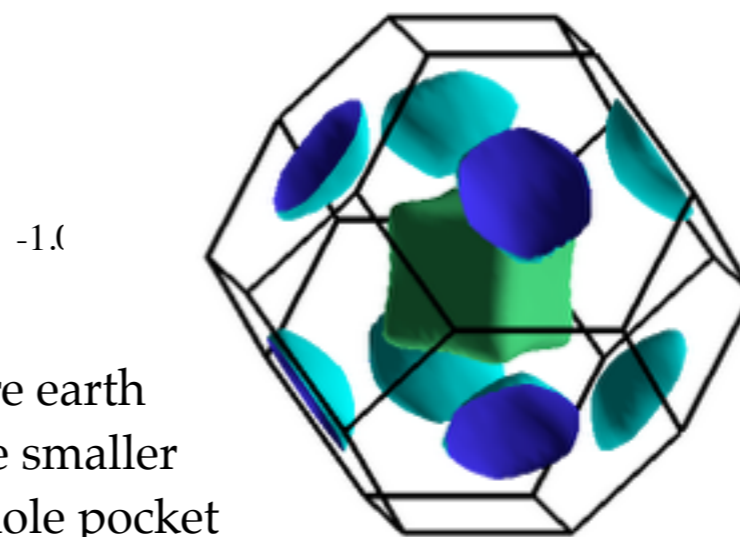
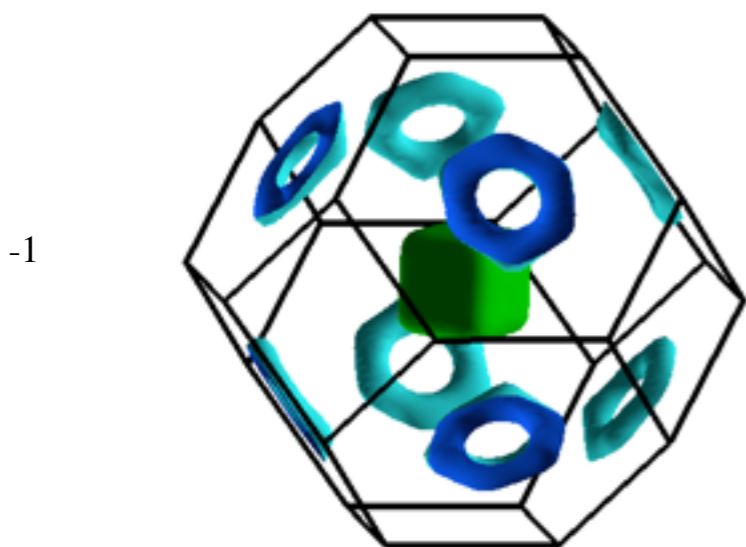
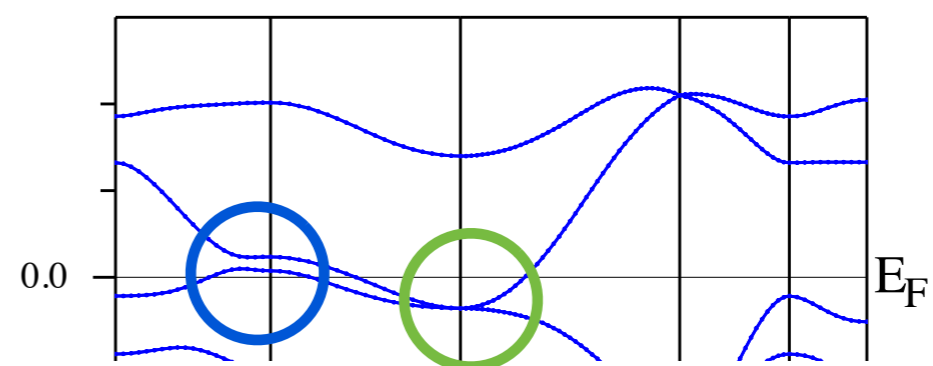


# Comparison: $Pr_2Ir_2O_7$ and $Y_2Ir_2O_7$

GGA+SO



GGA+SO



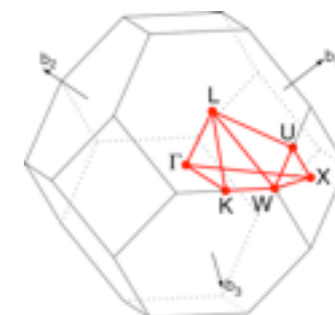
The bigger the rare earth ionic radius is, the smaller the electron and hole pocket Fermi surface is.

-2.0  $\overline{W}$   $\overline{L}$   $\overline{\Delta}$   $\overline{\Gamma}$   $\overline{\Delta}$   $\overline{X}$   $\overline{Z}$   $\overline{W}$   $\overline{K}$

$Pr_2Ir_2O_7$

-2.0  $\overline{W}$   $\overline{L}$   $\overline{\Delta}$   $\overline{\Gamma}$   $\overline{\Delta}$   $\overline{X}$   $\overline{Z}$   $\overline{W}$   $\overline{K}$

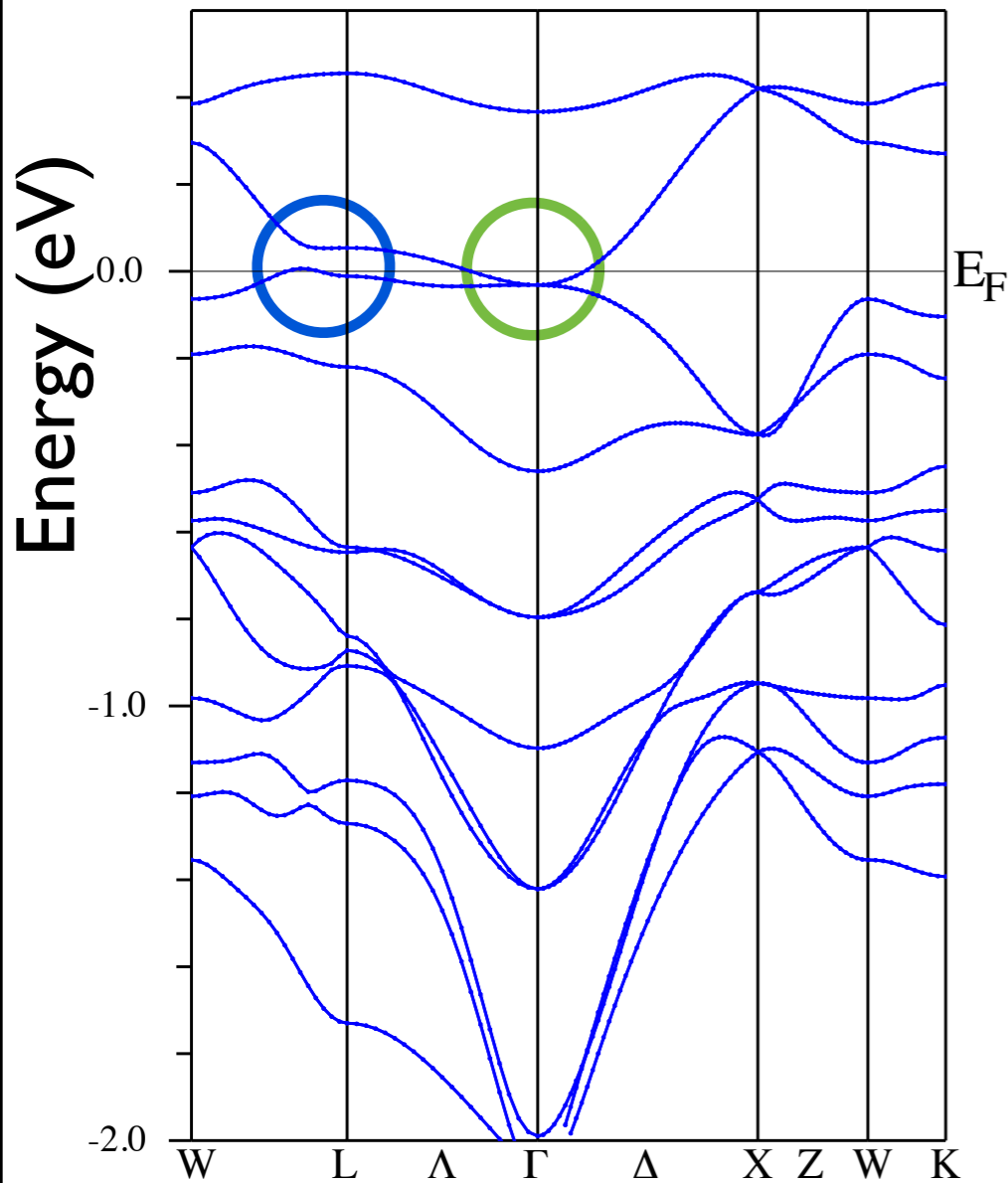
$Y_2Ir_2O_7$



Why is Pr metallic and Y not?

# Quadratic Band Touching State

GGA+SO



Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>

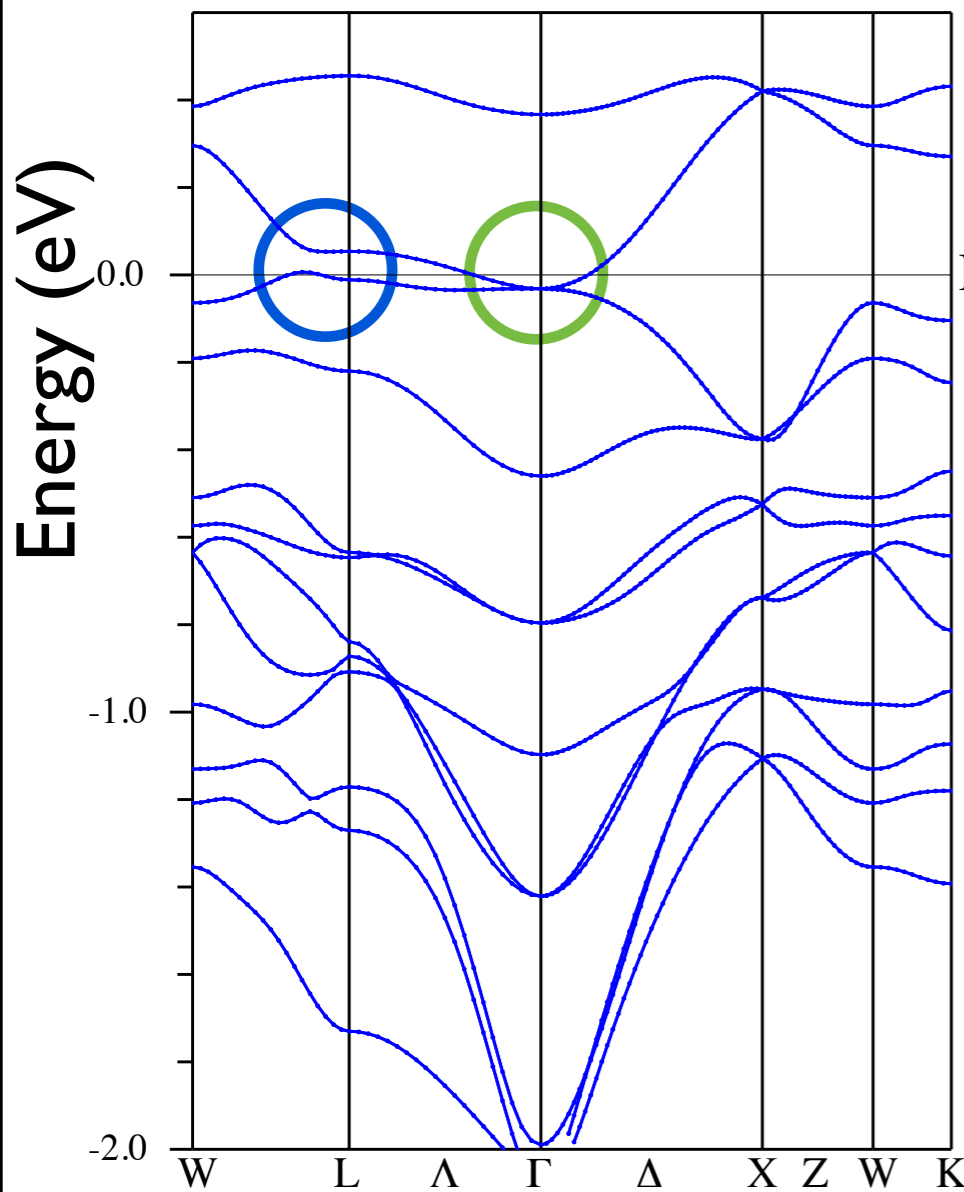
- GGA+SO calculations show Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub> is very close to quadratic band touching at the Fermi level.
- 4-fold degeneracy at the  $\Gamma$  point is protected by the cubic lattice symmetry, everywhere else gapped.
- GGA/LDA is well known for underestimating gaps.
- Modified Becke-Johnson potential gives band gap very close to experiments/GW calculation.
- Widely used for small gap semiconductor, topological insulator.

# Quadratic Band Touching State

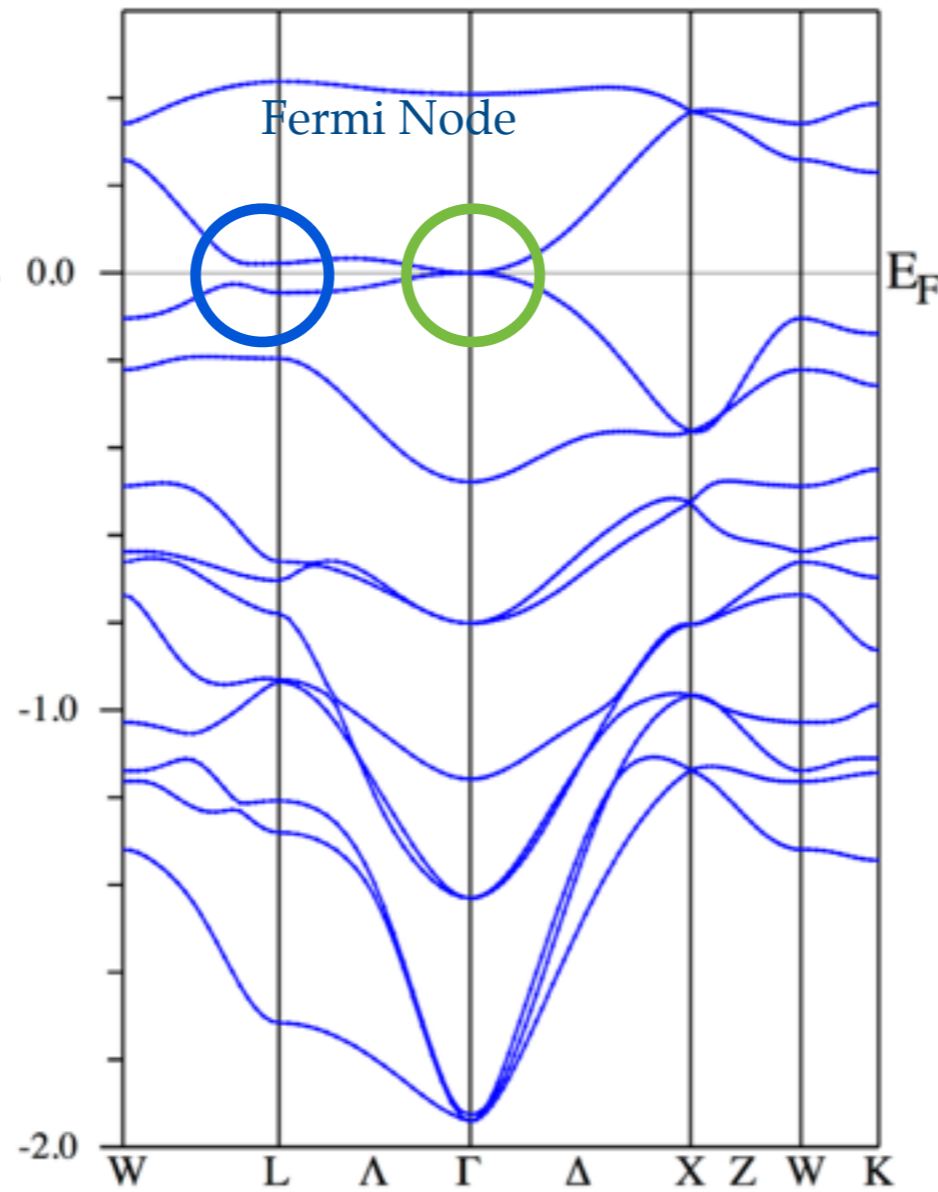
GGA+SO

MBJ+SO

(modified Becke-Johnson potential)



Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>



Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>

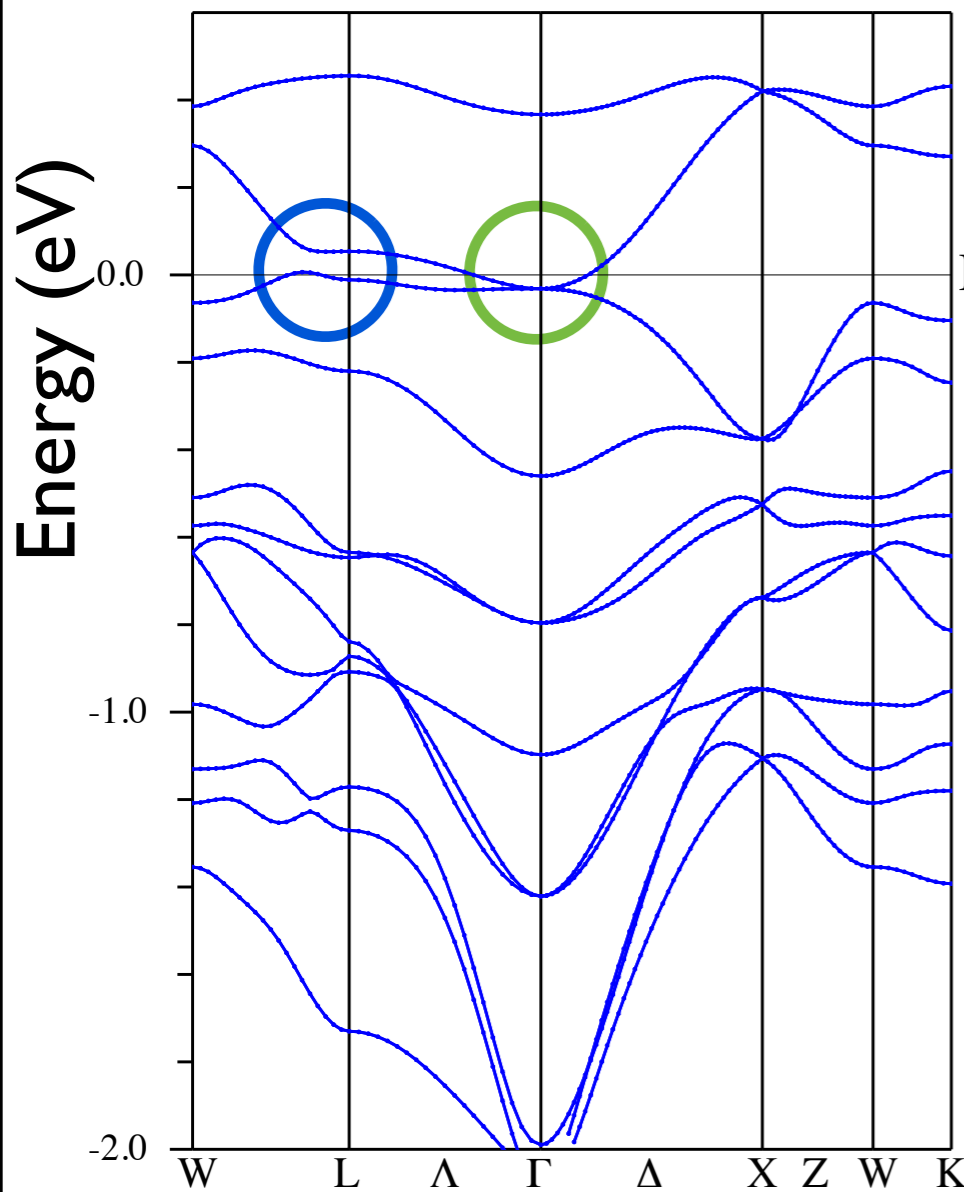
- Quadratic band touching at the Fermi level!
- Universal trend: larger R corresponds to smaller FS and closer to quadratic band touching at  $E_F$  regardless of methods

# Quadratic Band Touching State

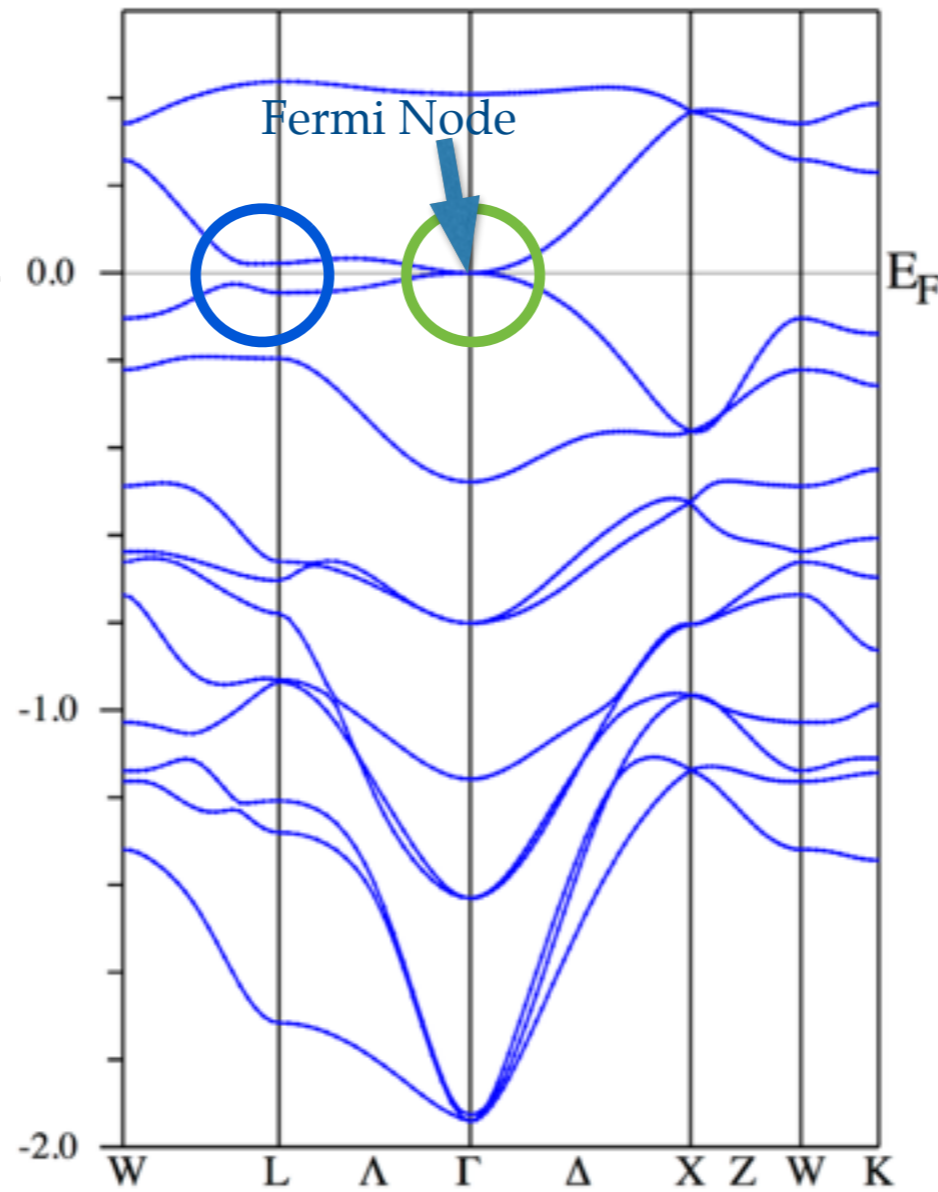
GGA+SO

MBJ+SO

(modified Becke-Johnson potential)



Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>



Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>

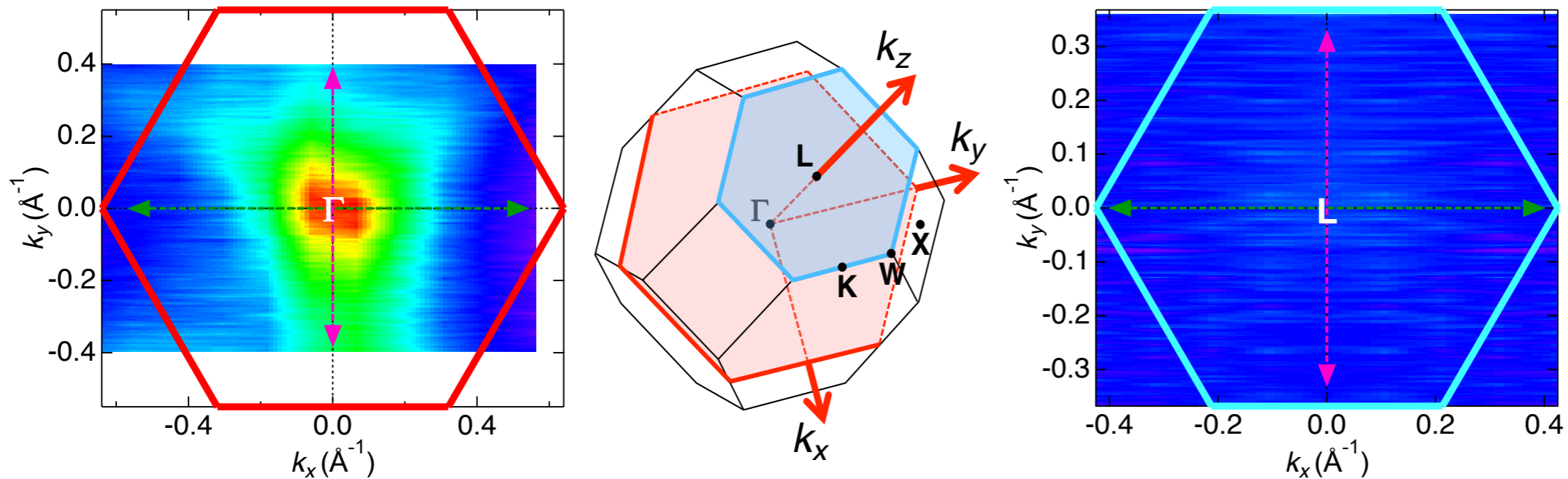
- Quadratic band touching at the Fermi level!
- Universal trend: larger R corresponds to smaller FS and closer to quadratic band touching at  $E_F$  regardless of methods





# Quadratic Band Touching State

- Direct experimental evidence is strongly desired.
- ARPES result confirms the quadratic Fermi node!



A quadratic band touch only at gamma,  
and there is no other FS crossings around L.

# Quadratic Band Touching State

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- k.p theory, Luttinger Hamiltonian around  $\Gamma$  point with cubic symmetry

$$\mathcal{H}_0(k) = \frac{k^2}{2\tilde{M}_0} + \frac{\frac{5}{4}k^2 - (\vec{k} \cdot \vec{J})^2}{2m} - \frac{(k_x^2 J_x^2 + k_y^2 J_y^2 + k_z^2 J_z^2)}{2M_c}$$

E.-G. Moon et al, PRL 2013

- $J_{\text{eff}} = 3/2$  matrix
- Similar to electronic structure of semiconductors with diamond structure and zinc blende structure in terms of the k · p perturbation theory
- Effective mass fitting to the band structure:  
 $M_0 = 19.97 m_e$ ,  $m = 6.30 m_e$ ,  $M_c = 7.44 m_e$

# Quadratic Band Touching State

- k.p theory, Luttinger Hamiltonian around  $\Gamma$  point with cubic symmetry

$$\mathcal{H}_0(k) = \frac{k^2}{2\tilde{M}_0} + \frac{\frac{5}{4}k^2 - (\vec{k} \cdot \vec{J})^2}{2m} - \frac{(k_x^2 J_x^2 + k_y^2 J_y^2 + k_z^2 J_z^2)}{2M_c}$$

E.-G. Moon et al, PRL 2013

- Strain/pressure will break the cubic symmetry

$$\mathcal{H}' = -\delta(J_z^2 - \frac{5}{4})$$

- Consider strain/pressure along (111) direction

$$\mathbf{k} = k\hat{n}_{(111)}$$

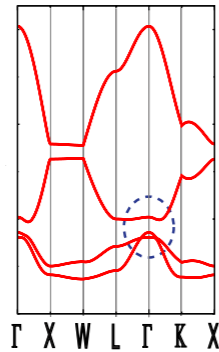
$$E_{\pm}(k) = \frac{k^2}{2M_0} \pm \sqrt{\left(\frac{k^2(m + M_c) + 2\delta m M_c}{2mM_c}\right)^2}$$

- $\delta > 0$ , opens up a gap at  $\Gamma$  point (topological insulator?)

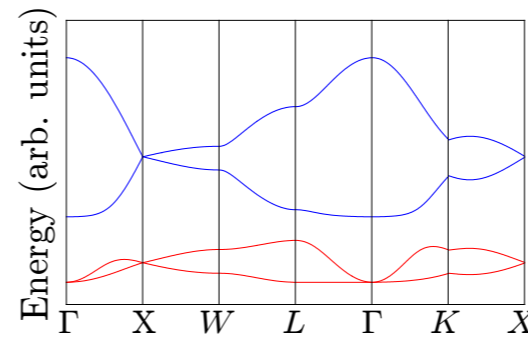
$\delta < 0$ , bands crossing along (111) direction

# Quadratic Band Touching State

- Several tight-binding studies suggest that pressure would produce topological insulator.



B.-J. Yang et al, PRB2010



D.A. Pesin et al, Nat phys (2010)

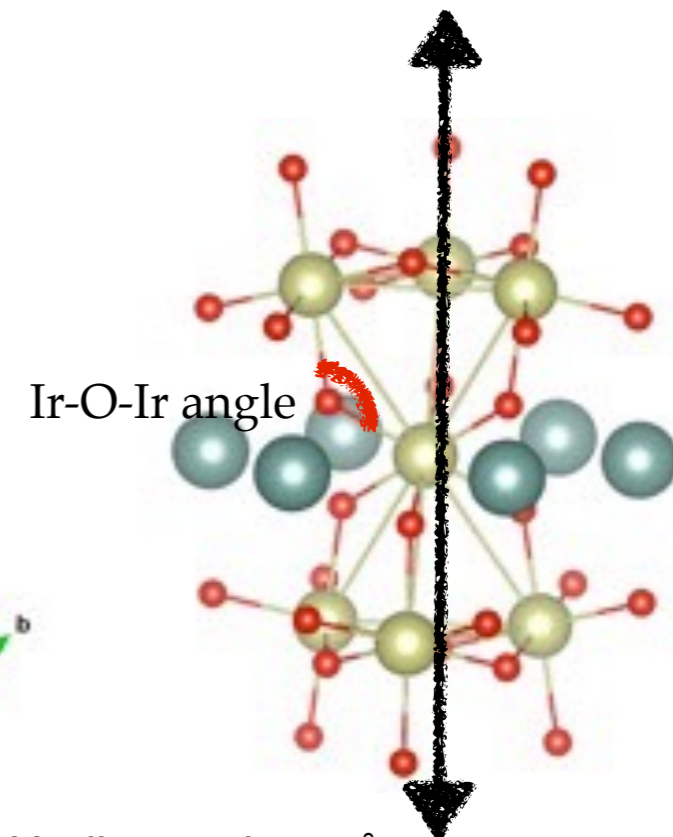
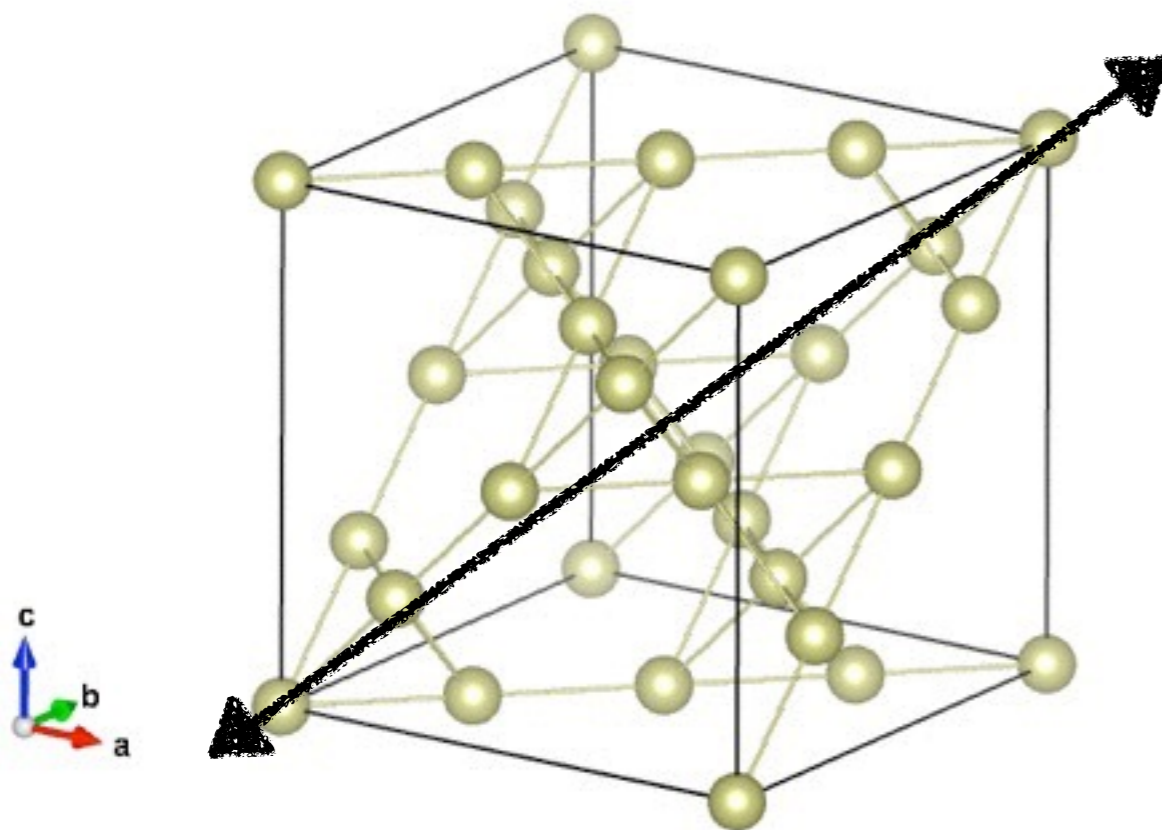
Without pressure, didn't take Ir-Ir direct hopping into account

- Ir 5d states are quite extended and a tight-binding model will require many parameters for a good fit. It will be interesting to see full ab initio calculation!
- Ab initio results show that  $\text{Pr}_2\text{Ir}_2\text{O}_7$  is closest to quadratic band touching at Fermi energy  $\longrightarrow$  it should be very sensitive to strain/pressure!

# Pressure / Strain effect on Pyrochlore

- Uniaxial pressure/Strain along (111) direction
- Cubic  $\rightarrow$  Rhombohedral symmetry
- Full lattice relaxation both on the lattice parameter and the atom positions.

Strain/Pressure

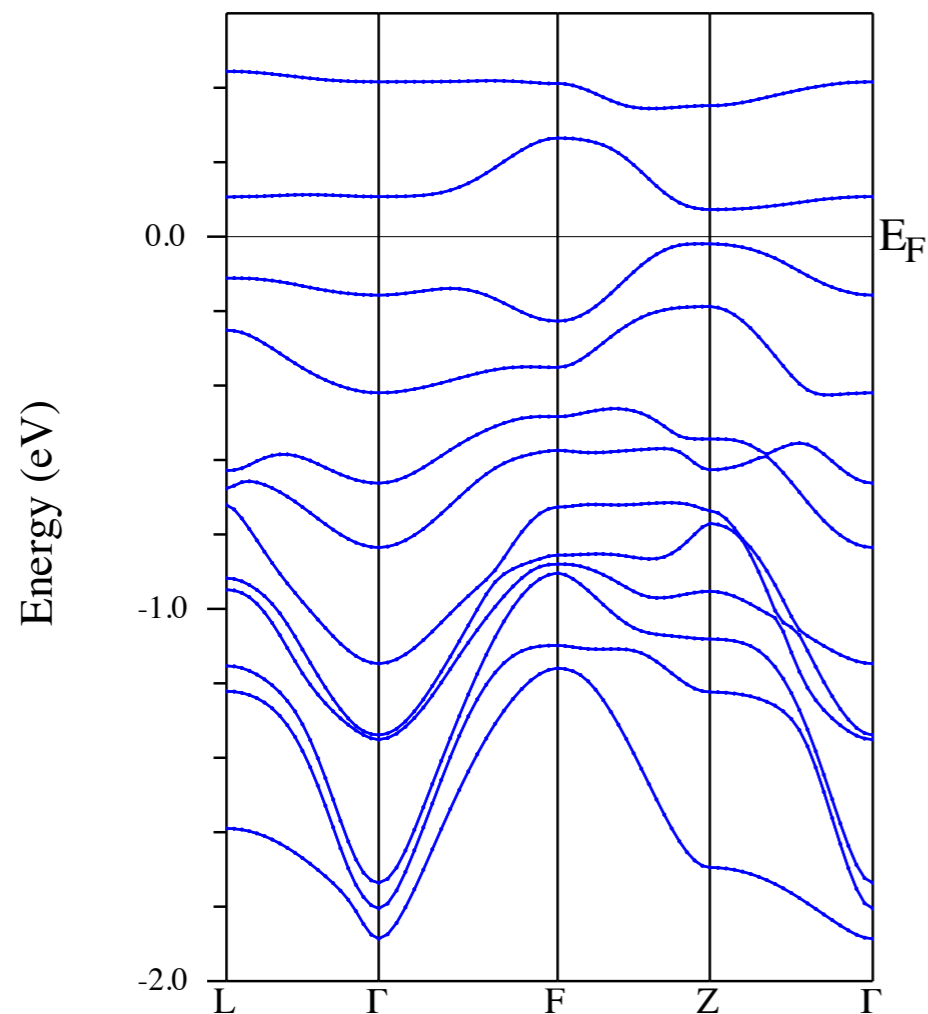


Undistorted:  $132^\circ$   
Increase pressure: bond angle  $< 132^\circ$   
Decrease pressure: bond angle  $> 132^\circ$

# Pressure / Strain effect on Pyrochlore

- Pressure/Strain along (111) direction
- Cubic  $\rightarrow$  Rhombohedral symmetry
- Compressive pressure (in-plane tensile strain)

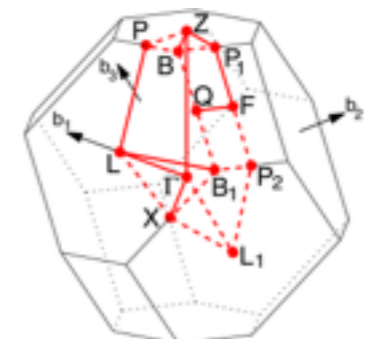
$$c=0.96c_0, a=1.05a_0$$



Full lattice relaxation: GGA

both GGA+SO and mBJ+SO calculation  
Insulator

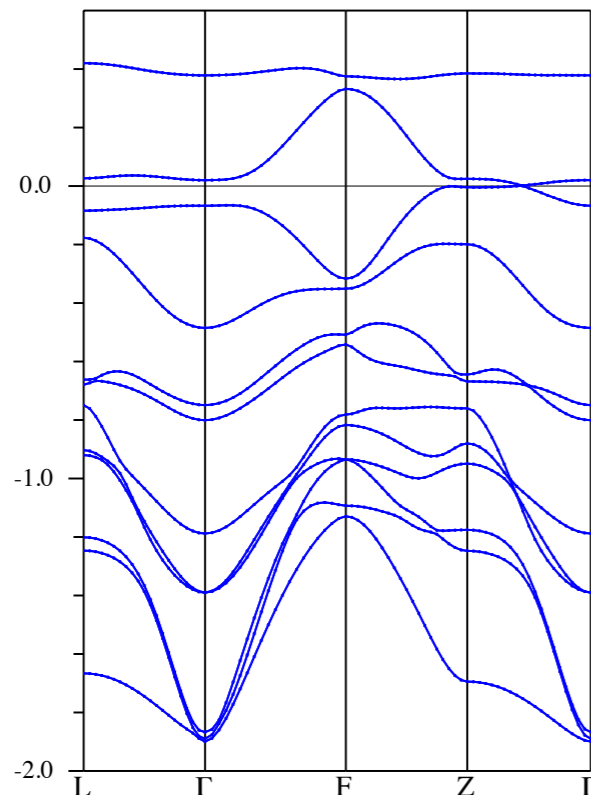
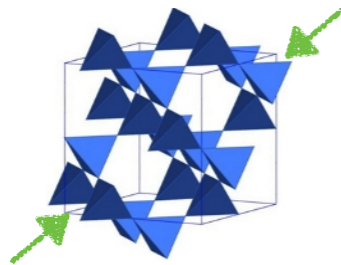
mBJ+SO gap=0.09eV



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- Compressive pressure (in-plane tensile strain)

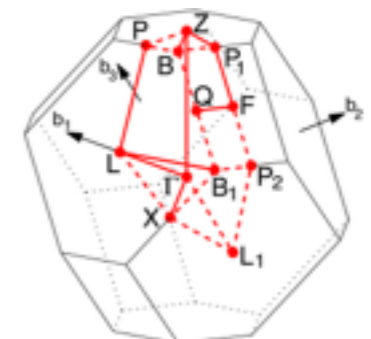
$$c=0.99c_0, a=1.01a_0$$



Full lattice relaxation: GGA

both GGA+SO and mBJ+SO calculation  
Insulator

mBJ+SO gap > 0 eV



# Pressure / Strain effect on Pyrochlore

- Compressive pressure (tensile strain)

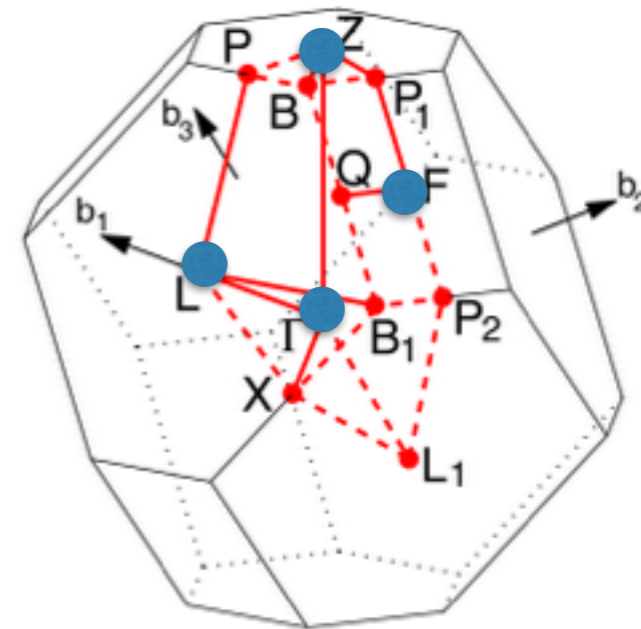
$$c=0.96c_0, a=1.05a_0$$

$$\delta_a = \prod_m \xi_m(\Lambda_a)$$

Parity at time reversal invariant momenta:

$\Gamma$	L	Z	F
+	-	-	-

$Z_2$  invariant (1; 000)



**Compressive pressure/ tensile strain along (111) direction will make  $\text{Pr}_2\text{Ir}_2\text{O}_7$  a topological insulator!**

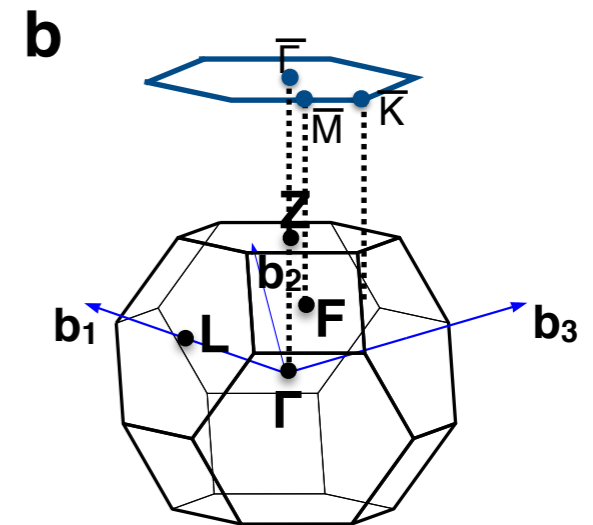
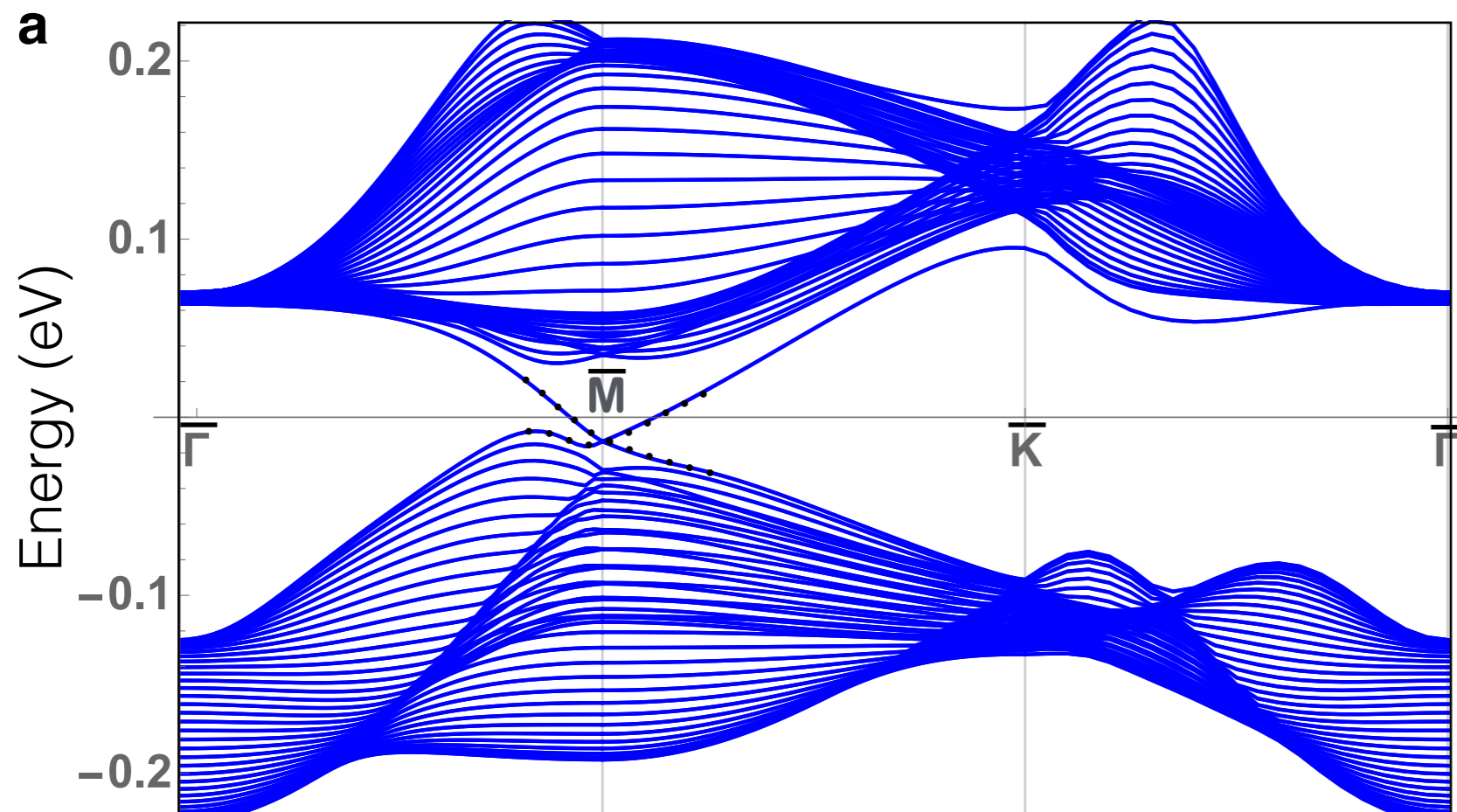


# Pressure / Strain effect on Pyrochlore

- Compressive pressure (tensile strain)

$$c=0.96c_0, a=1.05a_0$$

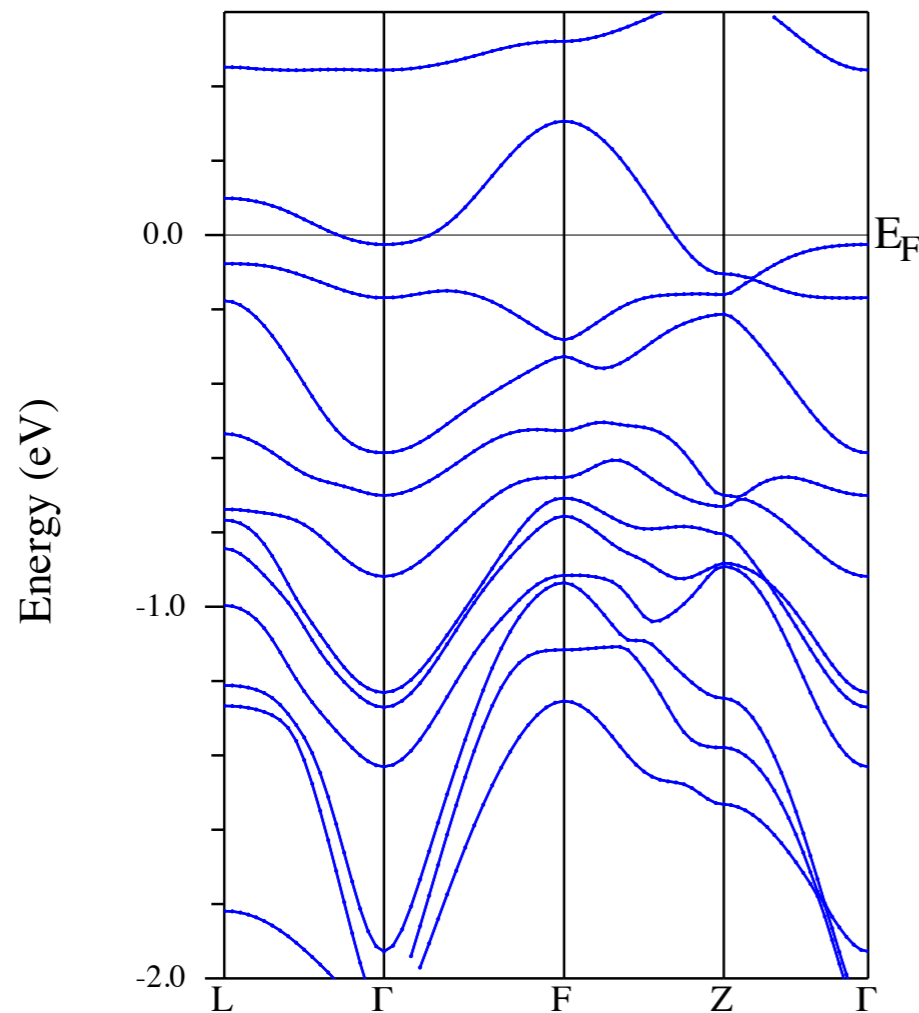
Surface state



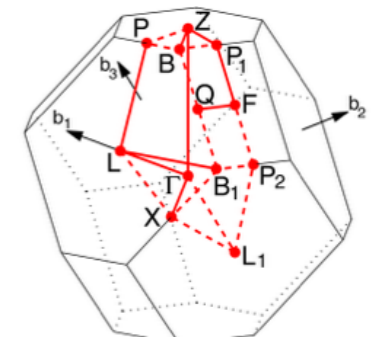
# Pressure / Strain effect on Pyrochlore

- Reducing pressure (in-plane compressive strain)

$$c=1.06c_0, a=0.95a_0$$

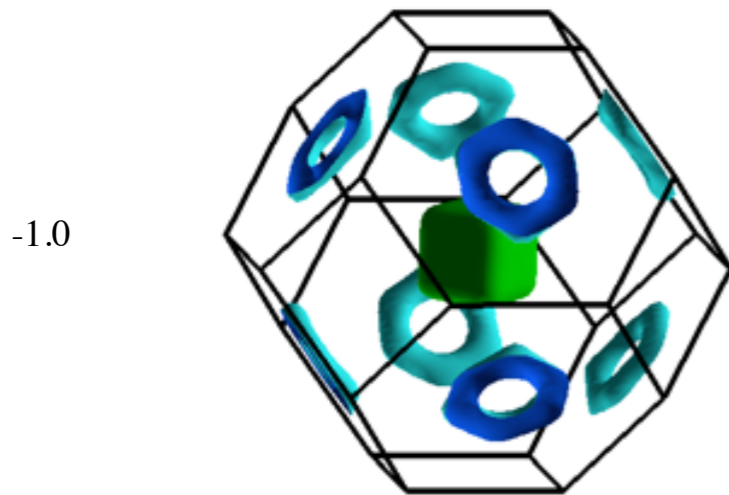
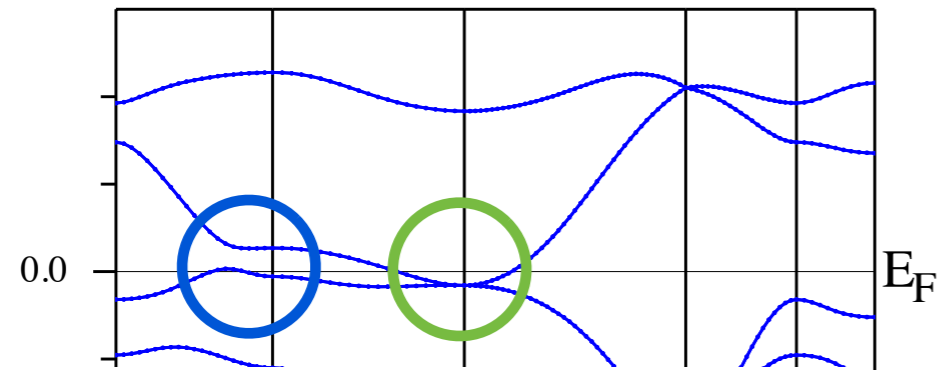


both GGA and mBJ+SO calculation  
Metallic  
Band crossing along (111) direction

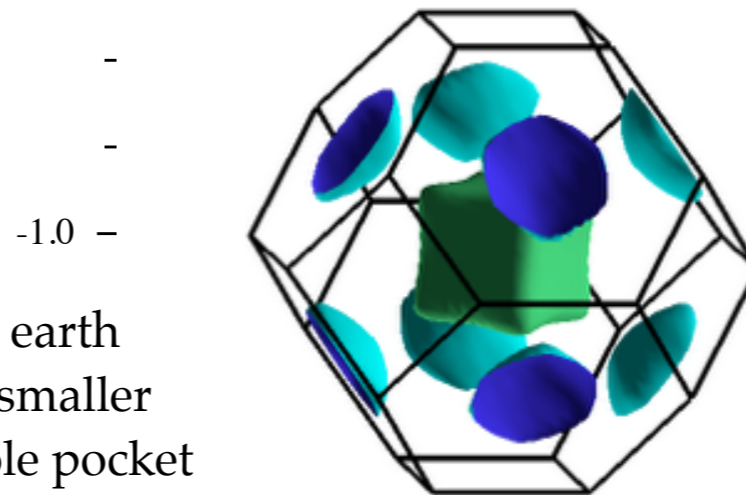
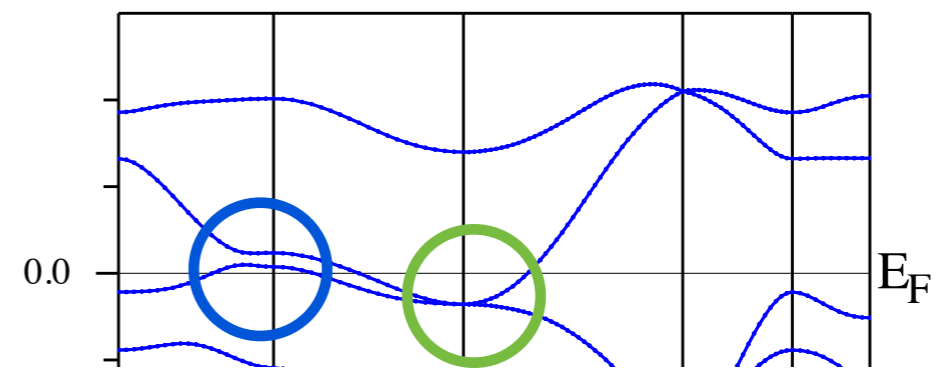


# Paramagnetic calculation: $Pr_2Ir_2O_7$ and $Y_2Ir_2O_7$

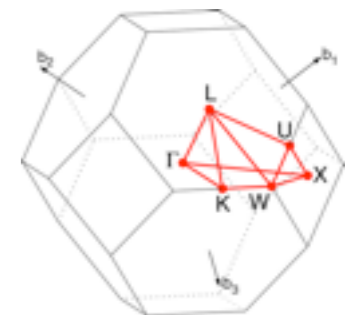
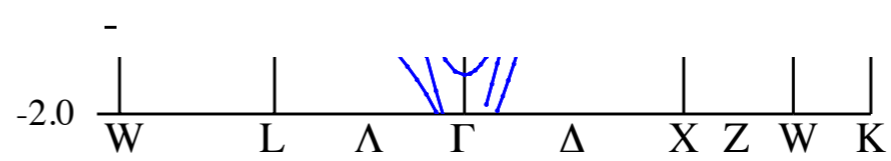
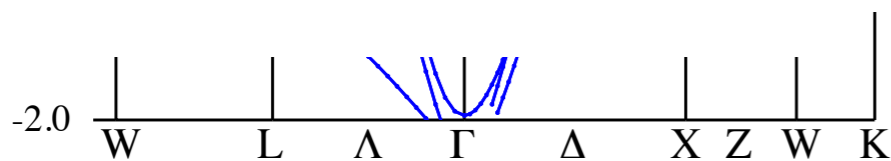
$Pr_2Ir_2O_7$  GGA+SO



$Y_2Ir_2O_7$  GGA+SO



The bigger the rare earth ionic radius is, the smaller the electron and hole pocket Fermi surface is.

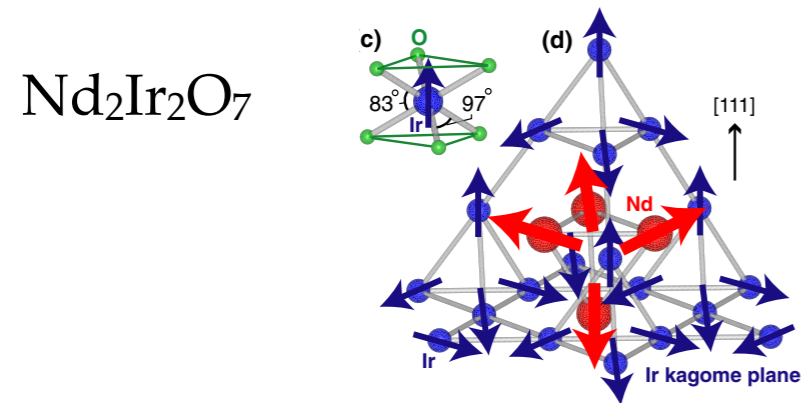


Why is Pr metallic and Y not?

- Ir-O-Ir bond angle Y:  $\sim 129^\circ$ ; Pr:  $\sim 132^\circ$
- Ir-O bond length Y:  $1.997 \text{ \AA}$ ; Pr:  $2.014 \text{ \AA}$
- Ir-Ir bond length Y:  $3.599 \text{ \AA}$ ; Pr:  $3.677 \text{ \AA}$

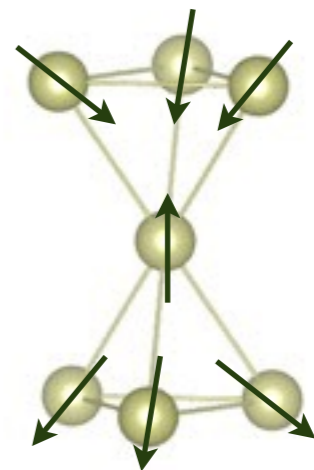
# Magnetic band structure calculation

- Focus on Pr, Nd, Y
- GGA+SO+U,  $U=2\text{eV}$  since 5d are more extended
- Magnetism in R 4f electrons and in Ir 5d electrons  $\rightarrow$  Magnetic ground state can be complicated!
- non-collinear magnetism :Wien2k+WIENNCM



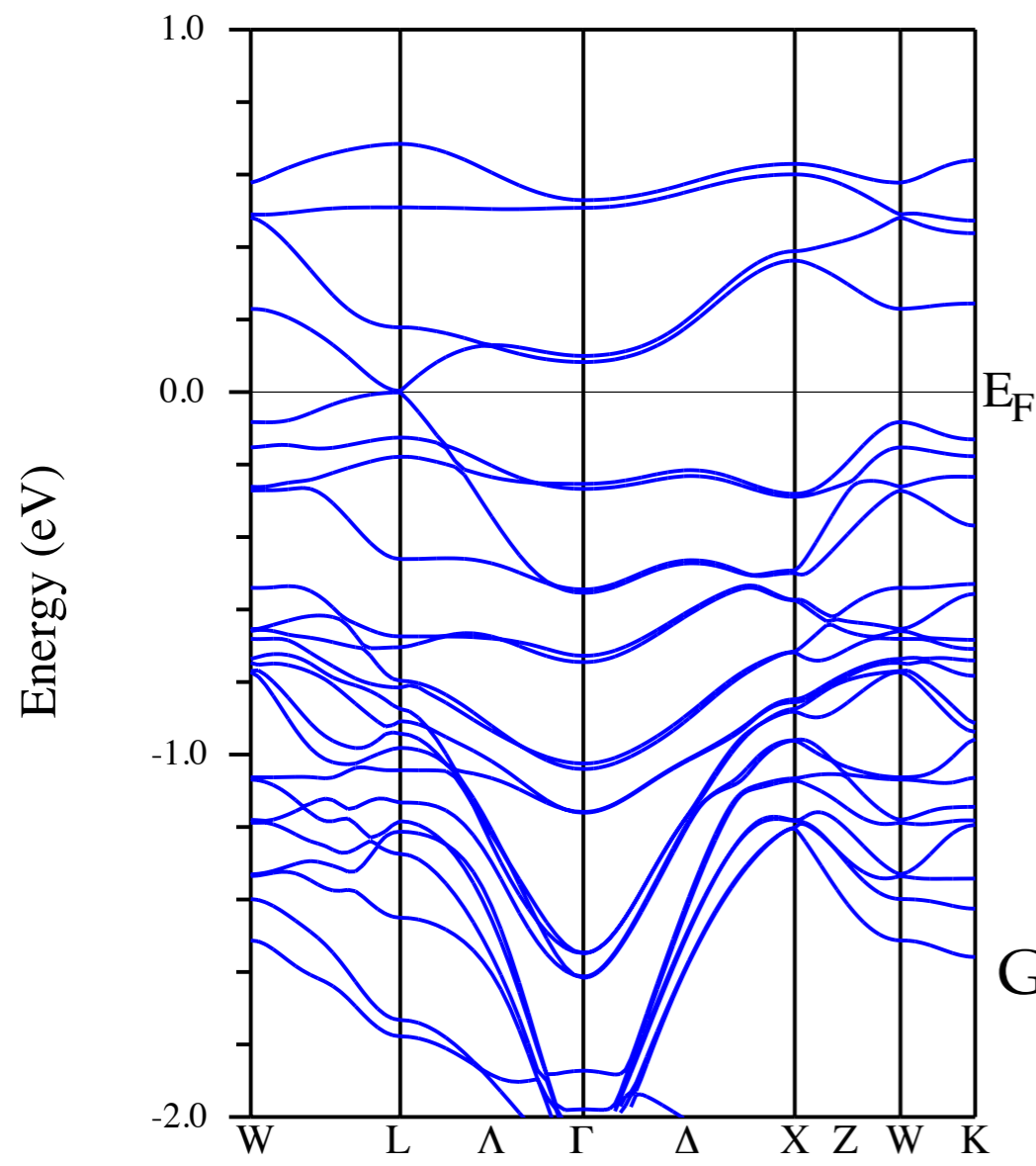
Neutron diffraction and  
inelastic scattering measurement  
 $\mathbf{q}_0=(0,0,0)$

magnetic state:  
all-in all-out state



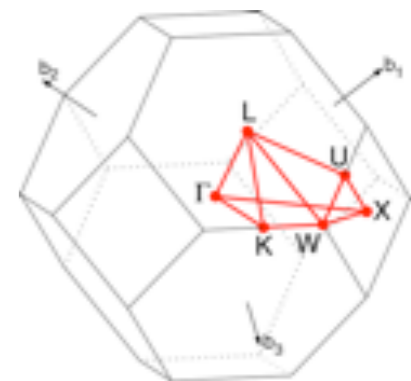
# *Pr2Ir2O7*

- Metallic/paramagnetic down to 0.3K
- Experimental: Pr 4f: 2-in 2-out Ir: paramagnetic
- Try both Pr and Ir all-in all-out order



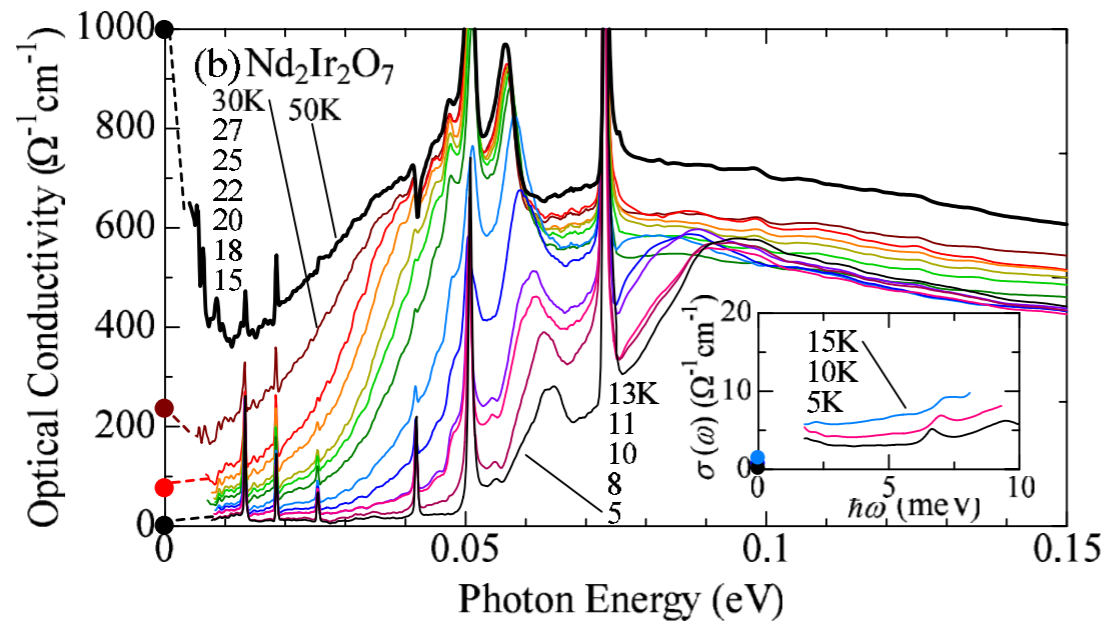
GGA+SO+U  
 $U_{Ir}=2\text{eV}$   
 $U_{Pr}=6\text{eV}$

The gap barely closes at the L point  
In proximity to Weyl semimetal  
GGA/LDA+U may overestimate magnetism



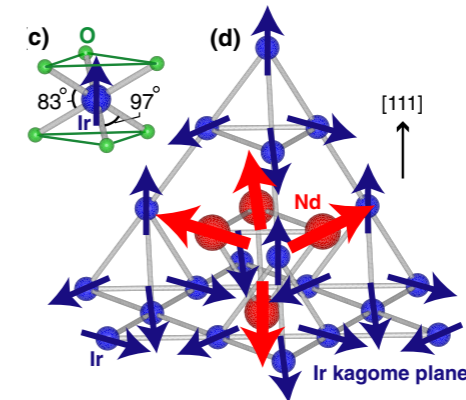
# $Nd_2Ir_2O_7$

- $T_{MI} \sim 33K$ , optical conductivity measurement infers the full opening of the charge gap.
- Experiment suggesting both Nd and Ir are all-in all-out



Optical conductivity  
gap  $\sim 0.045eV$

K.Ueda et al, PRL 2012

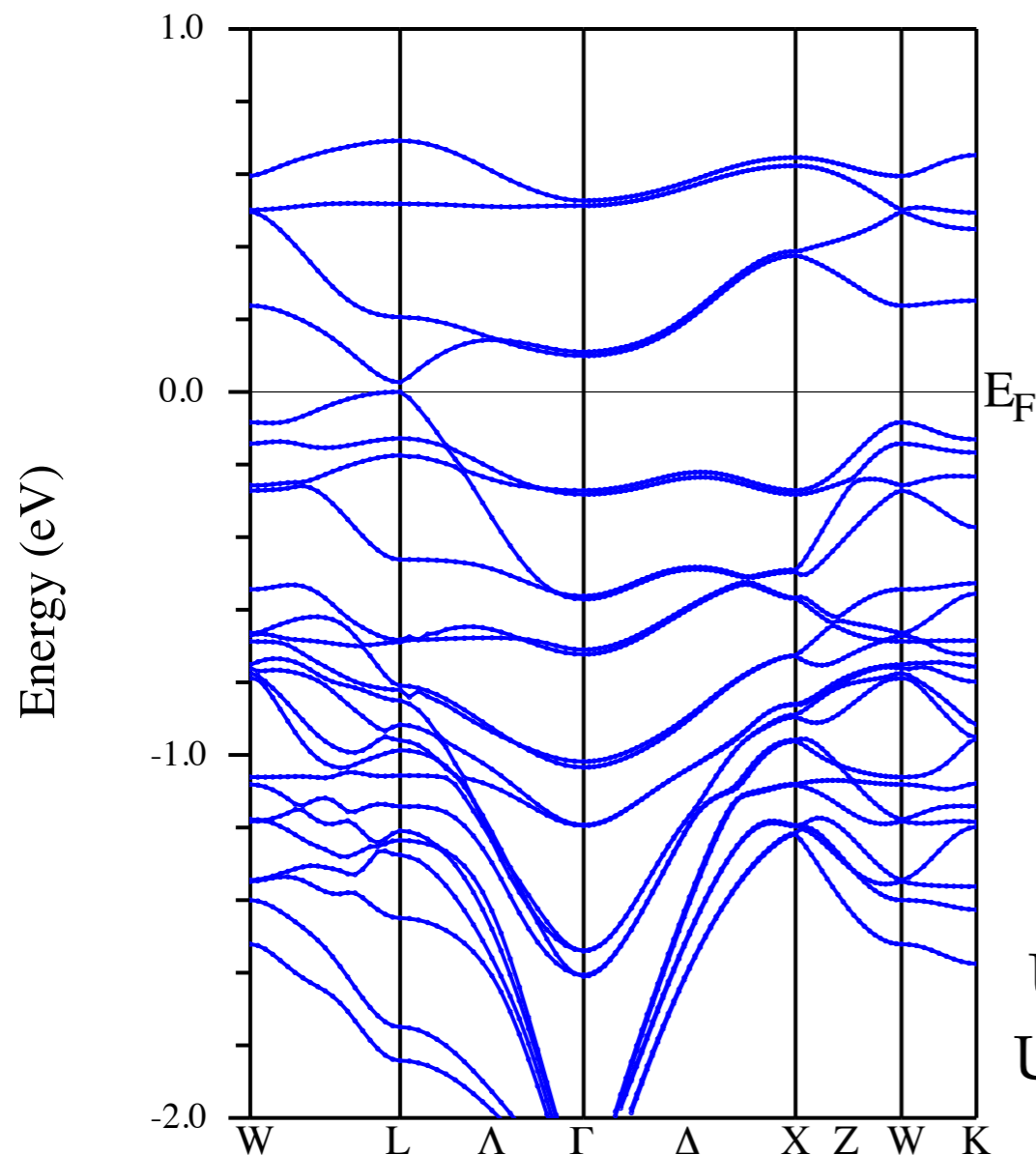


Neutron diffraction and  
inelastic scattering measurement  
 $\mathbf{q}_0 = (0,0,0)$

Keisuke Tomiyasu et al, JPSJ 2012

# $Nd_2Ir_2O_7$

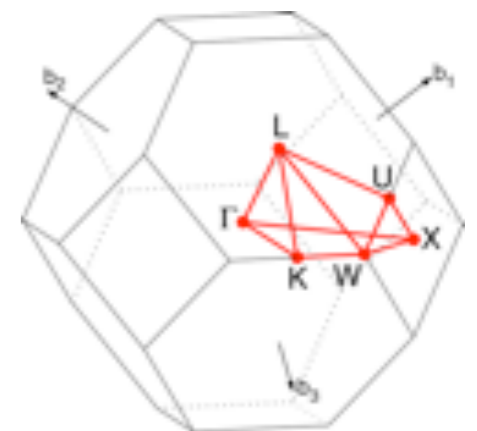
- $T_{MI} \sim 33K$ , optical conductivity measurement infers the full opening of the charge gap.
- Experiment suggesting both Nd and Ir are all-in all-out



GGA+SO+U calculation  
all-in all-out magnetic

Mott insulator  
Gap  $\sim 0.03eV$

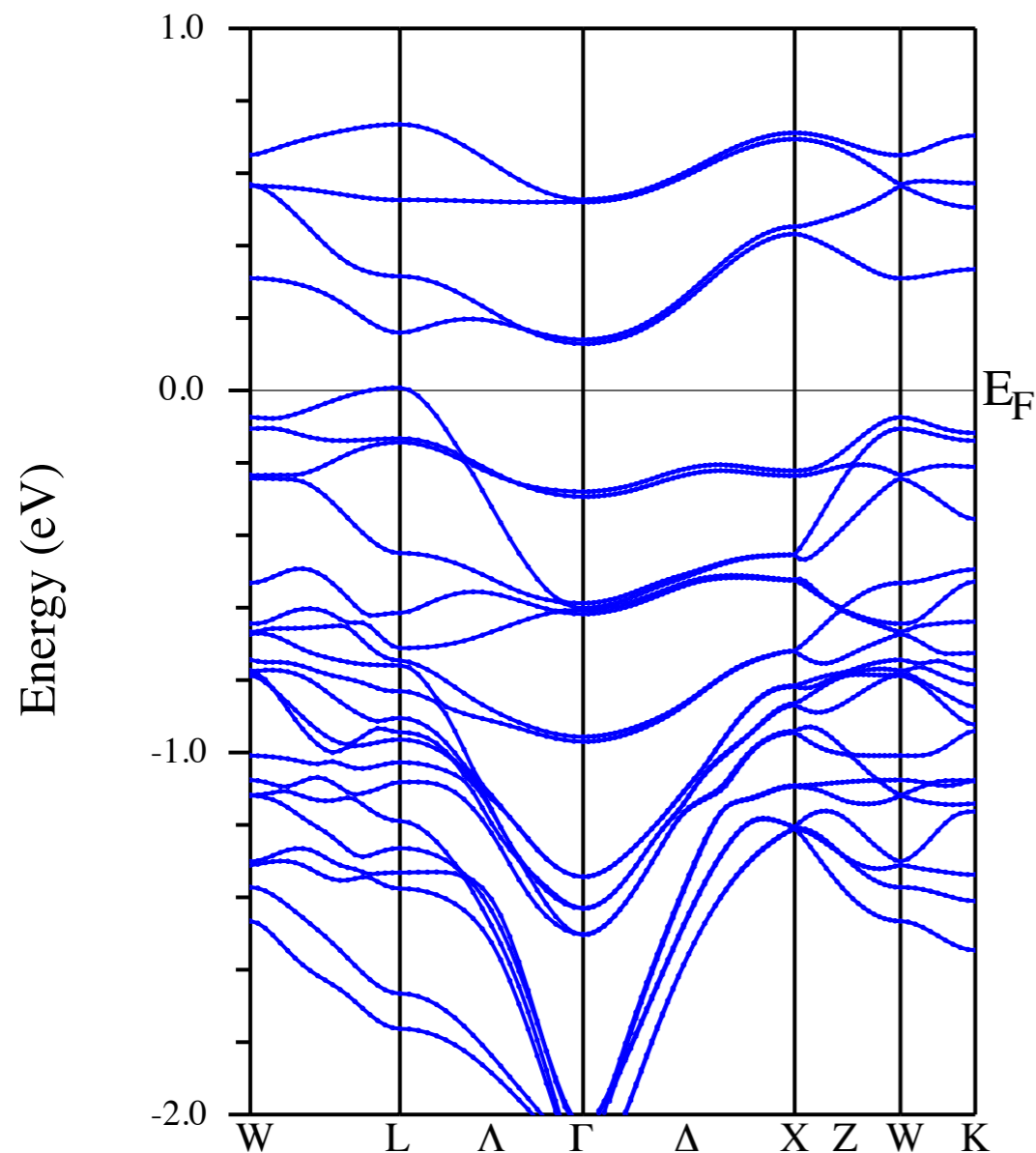
$U_{Ir} = 2eV$   
 $U_{Nd} = 6eV$



# $Y_2Ir_2O_7$

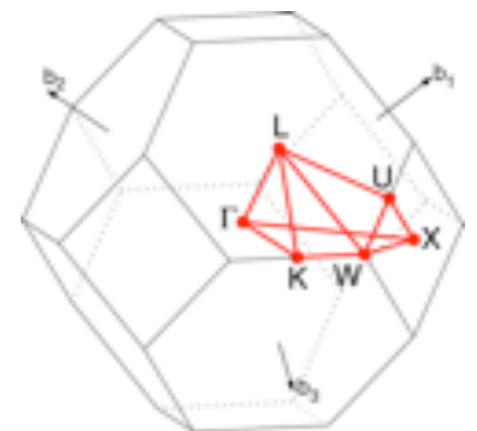
- $T_{MI} \sim 150K$
- Y: no f electron
- Theory and experiment: all-in all-out state

X. Wan et al, PRB 2011



GGA+SO+U  
all-in all-out magnetic  
Mott insulator  
Gap  $\sim 0.12eV$

$$U_{Ir} = 2eV$$





# Summary for Part 1

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- $\text{Pr}_2\text{Ir}_2\text{O}_7$ : Quadratic Fermi node
- Strain/pressure along (111) direction make  $\text{Pr}_2\text{Ir}_2\text{O}_7$  a strong topological insulator with correlation.
- Systematic gap decrease trend as rare earth ionic radius increases.



# Part 2

## Domain walls in the insulating state of Pyrochlore Iridates

UCB & LBNL

Ashvin Vishwanath

Jeffrey Neaton



# Domain wall states in pyrochlore Iridates

Conducting metallic domain wall even at the gapped bulk state!

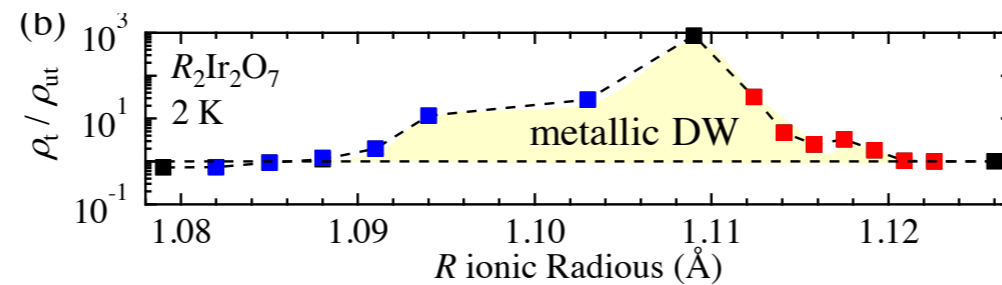
Theory and experiments:

Yamaji & Imada Phys. Rev. X 2014, arXiv: 1507.04153

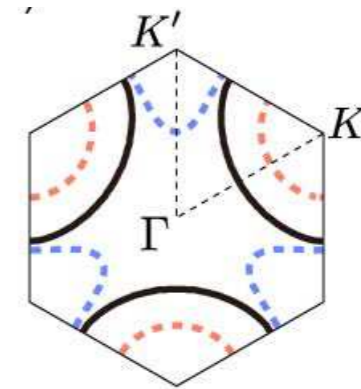
Ueda, Fujioka, Nagaosa, et al. arXiv: 1506.07336, arXiv:1507.04804

Fujita et al. Scientific Reports 2015

many others...



Ueda, et al. arXiv:1507.04804

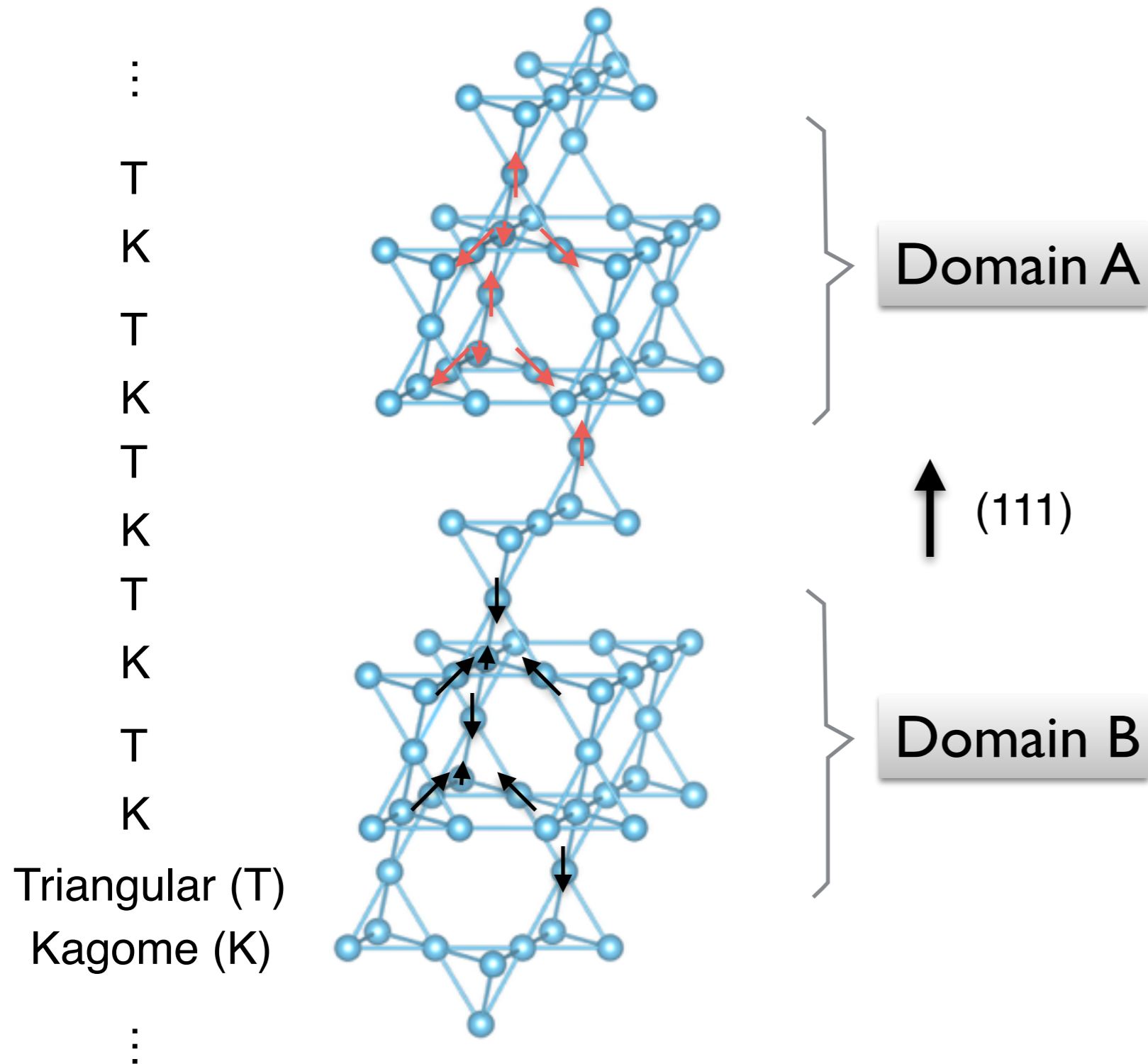


Yamaji & Imada arXiv: 1507.04153

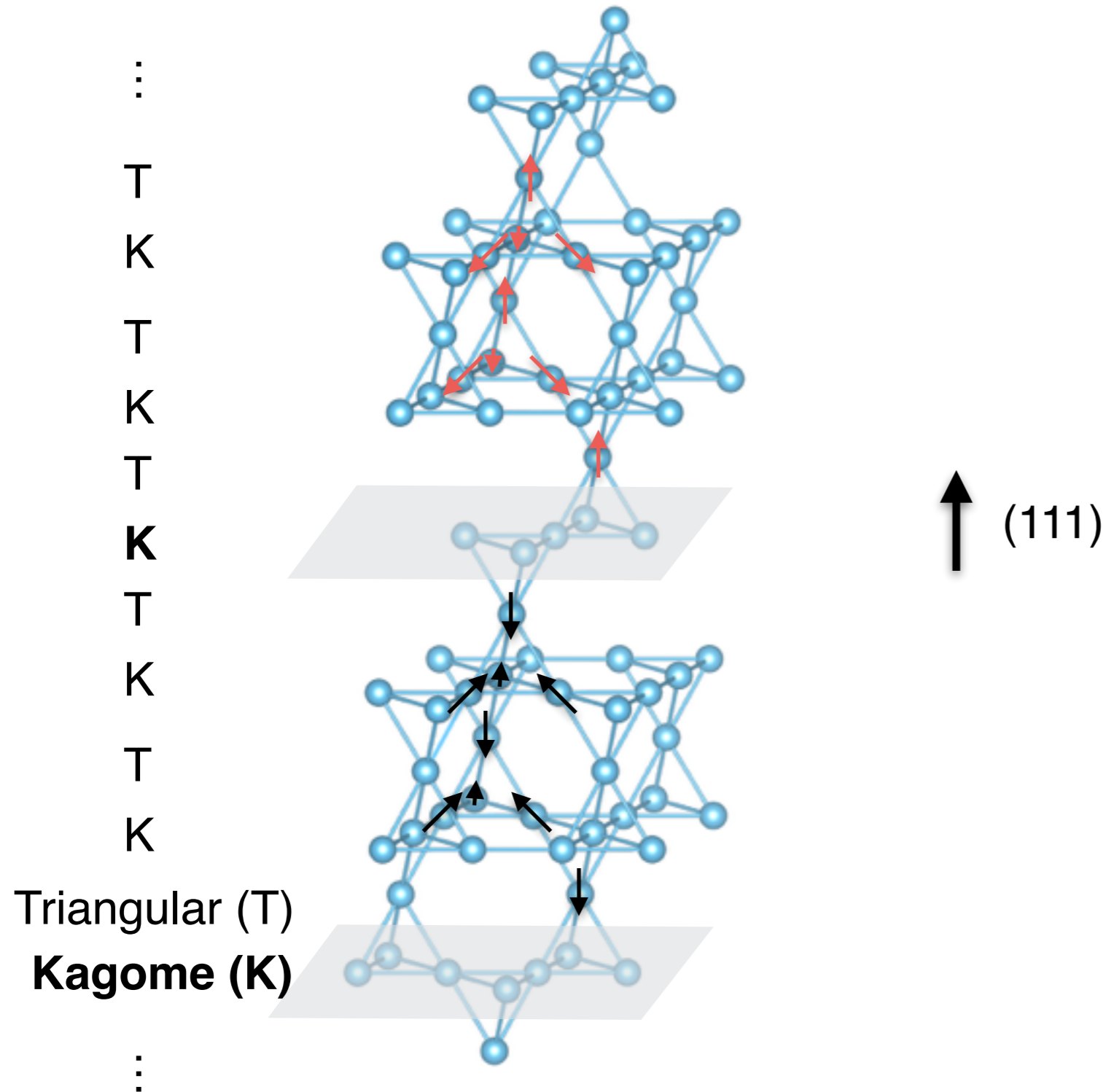
K. Ueda et al., Phys. Rev. B 89, 075127 (2014)

A full ab initio calculation is desirable.

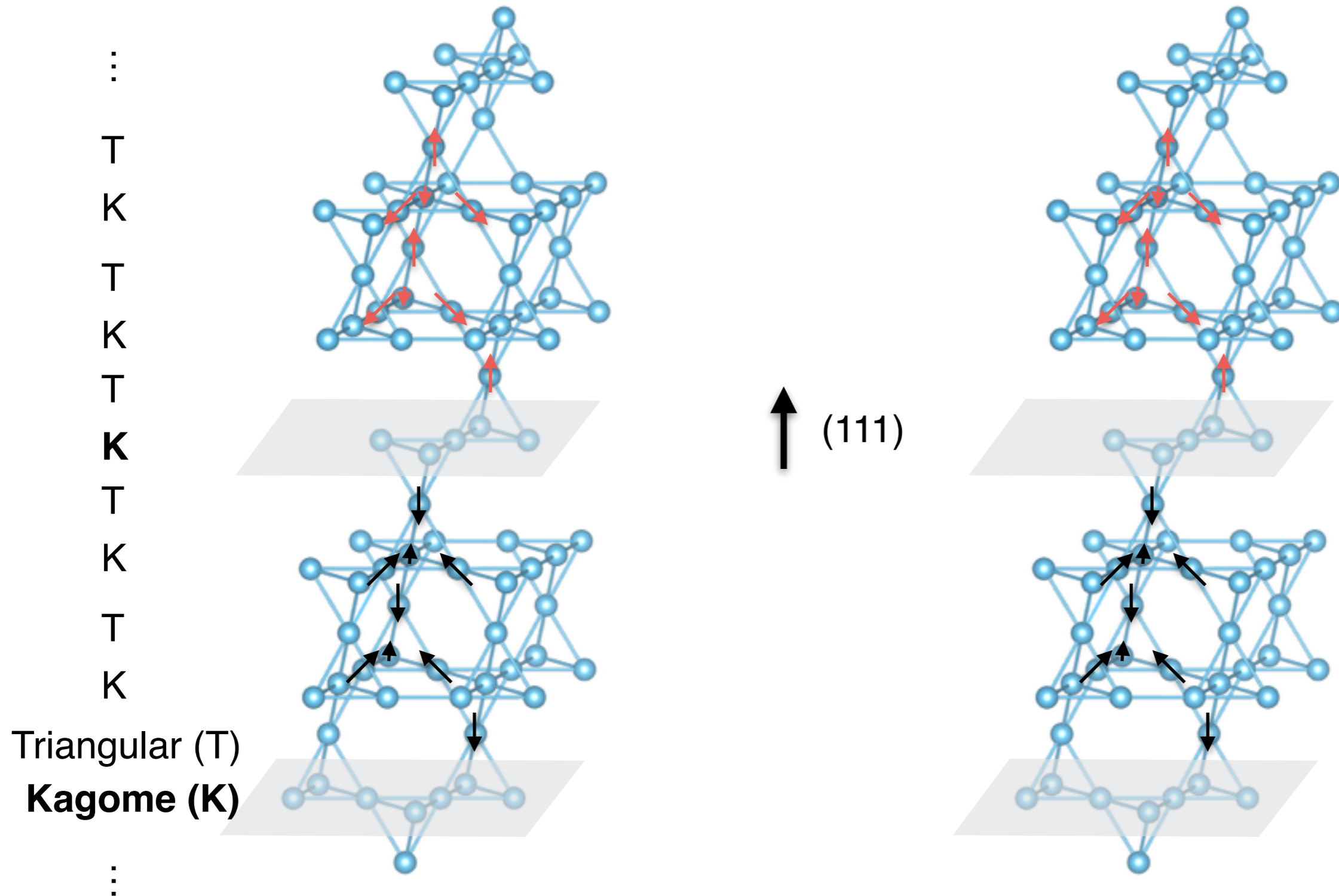
# *Domain wall states in pyrochlore Iridates*



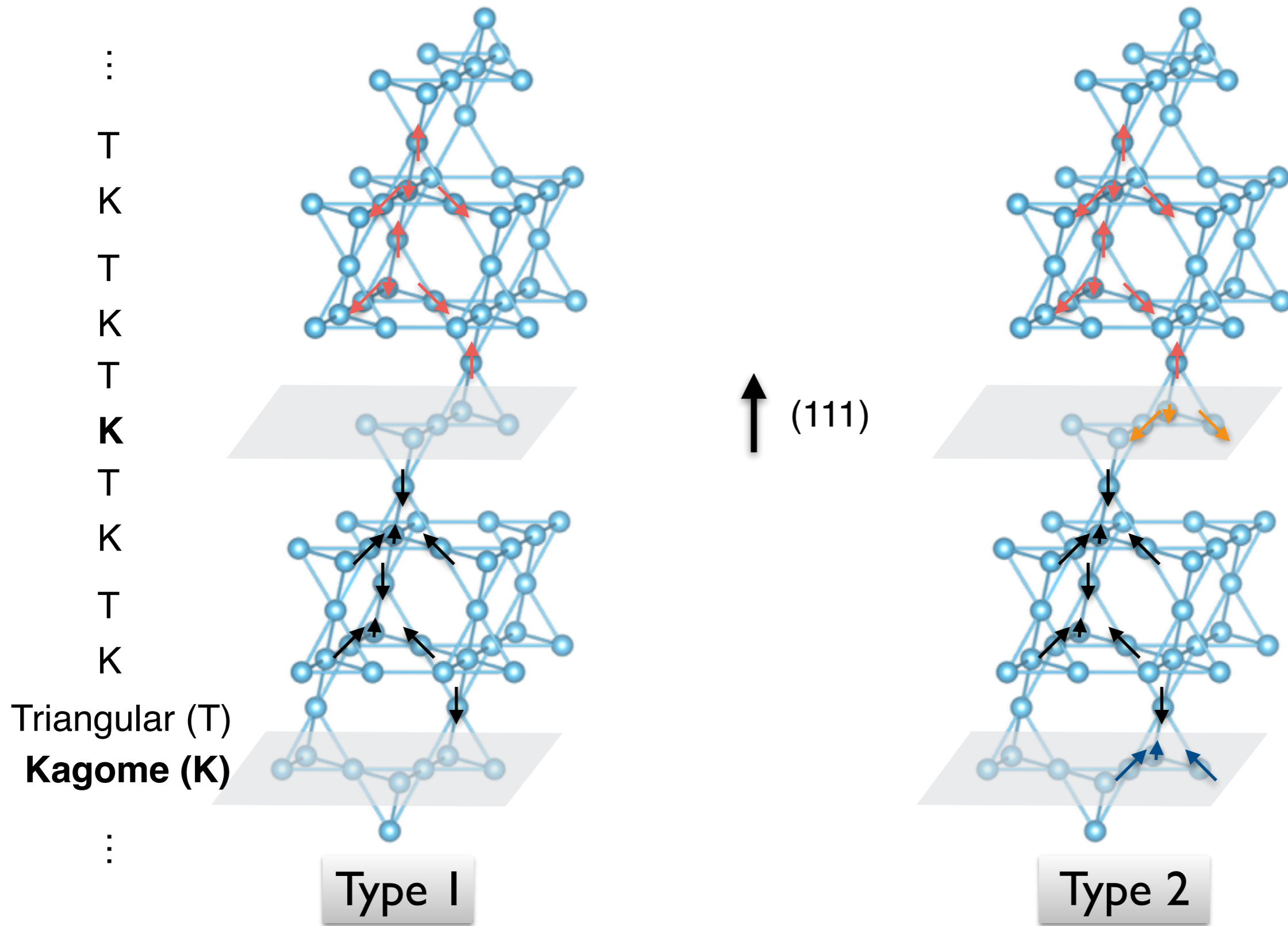
# *Domain wall states in pyrochlore Iridates*



# *Domain wall states in pyrochlore Iridates*



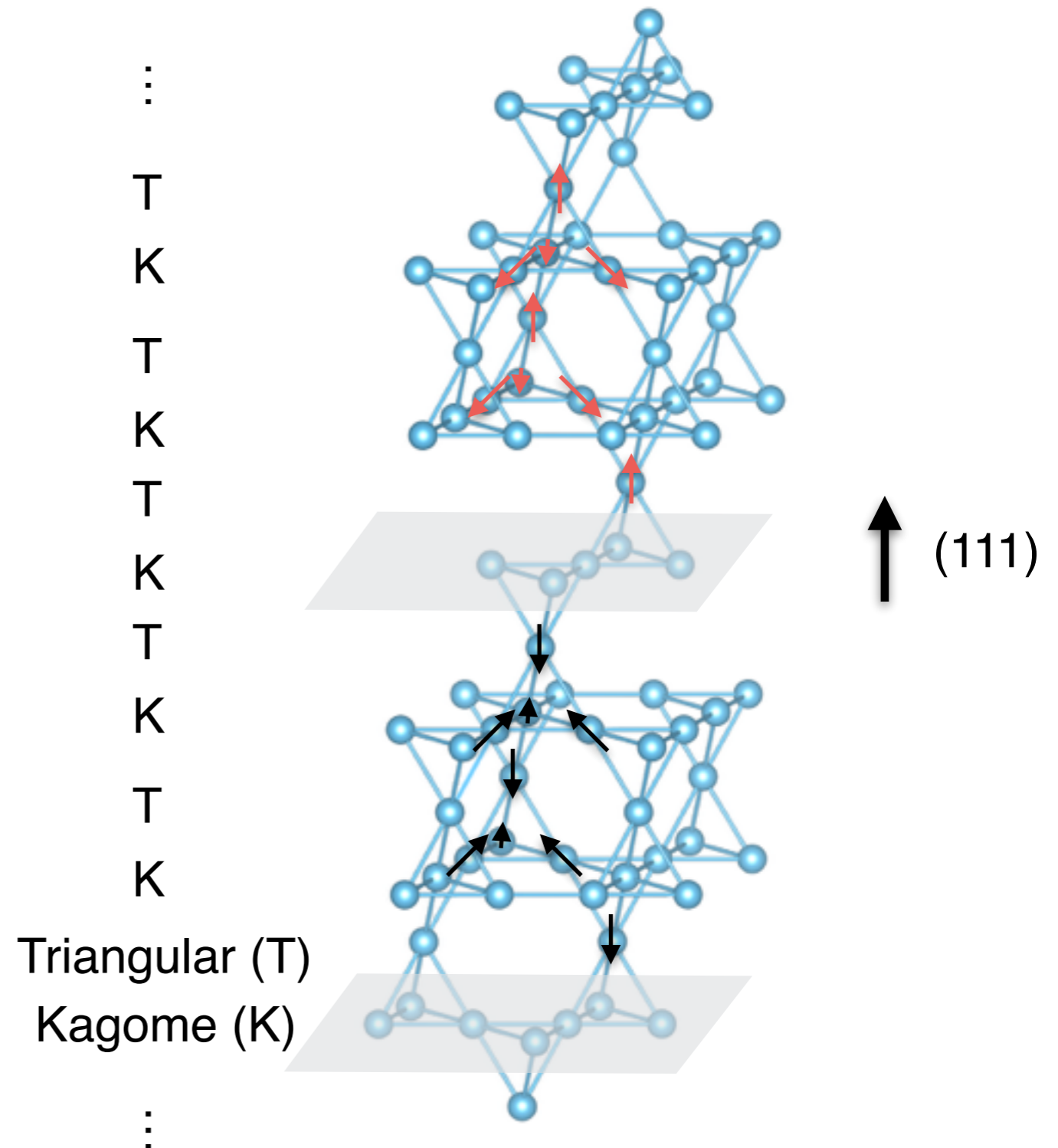
# Domain wall states in pyrochlore Iridates



# Domain wall states in $Y_2Ir_2O_7$

## Method

- Vasp
- GGA+U+SOC
- Periodic domain wall condition
- Magnetic moment is fully relaxed.
- Convergence test:
  - number of k points
  - How big is domain region
- Type 1 and 2 domain wall as local minima, different initial magnetic moments.

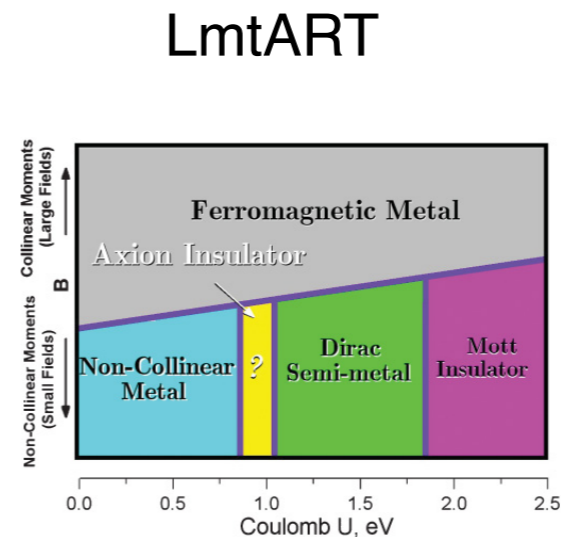
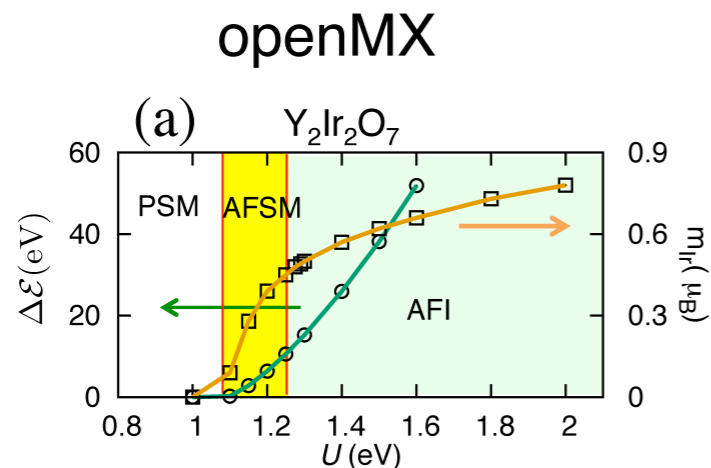
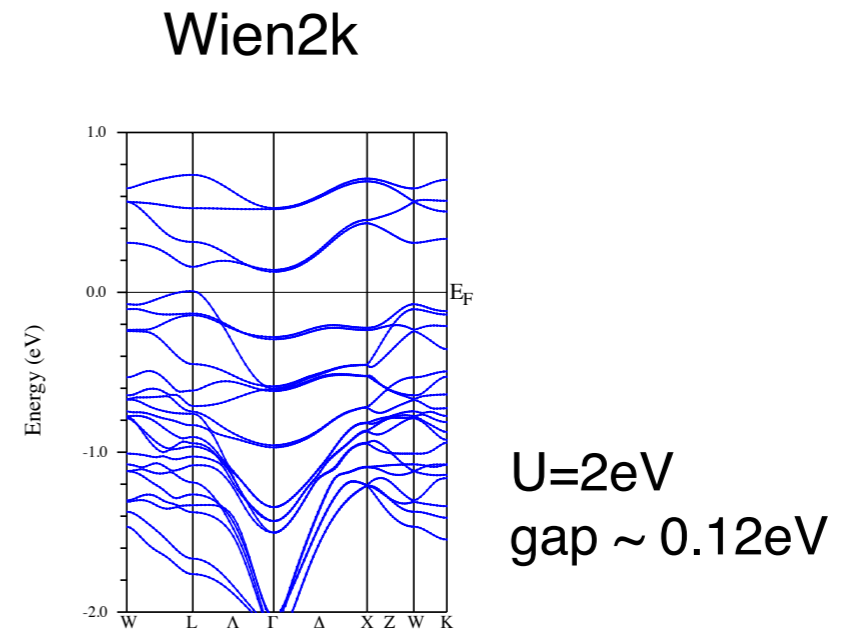
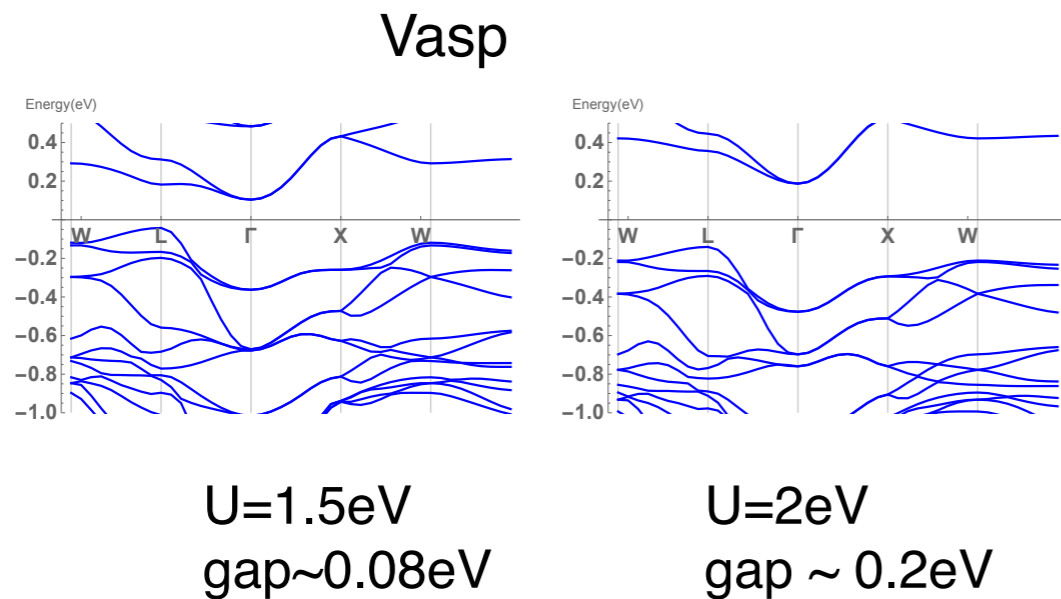




# Domain wall states in $Y_2Ir_2O_7$

Method:

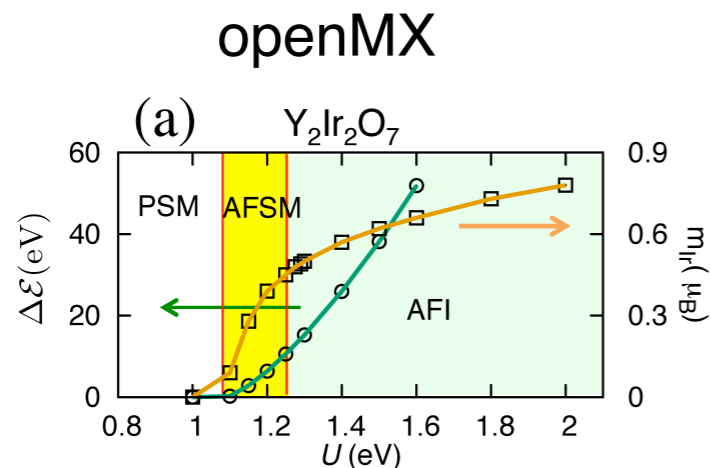
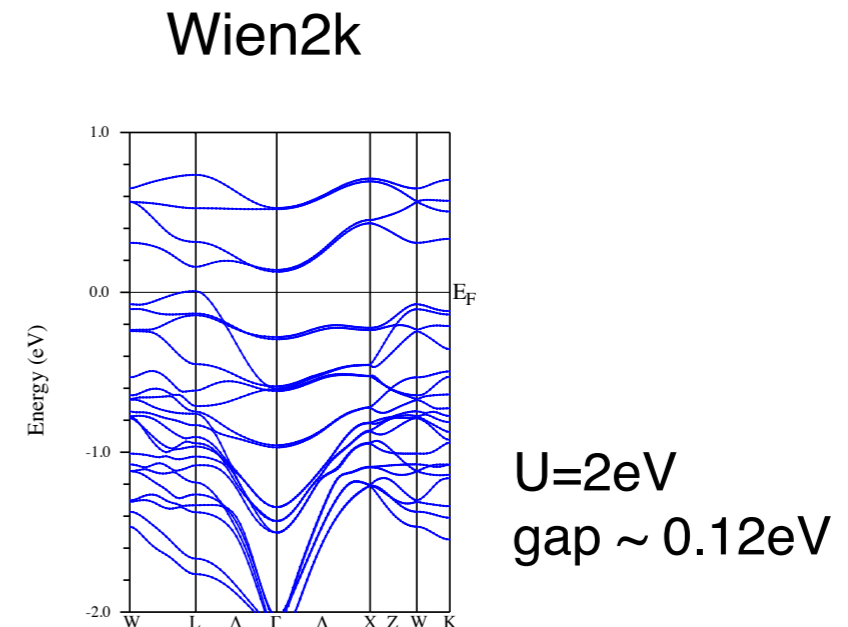
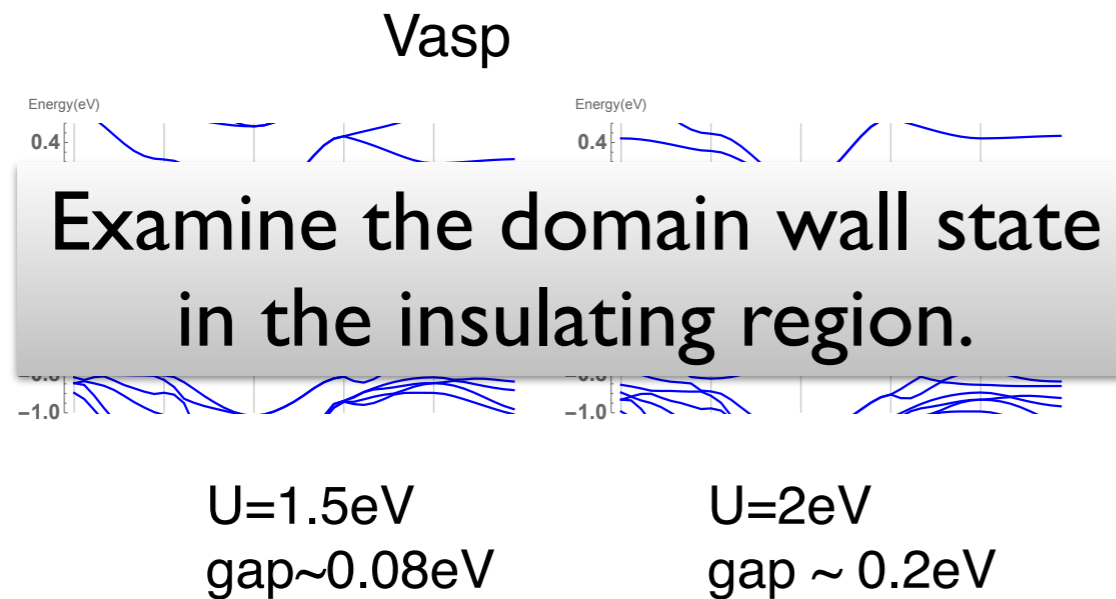
Band structure at different U deviates a little from different program.  
Overall qualitatively agrees.



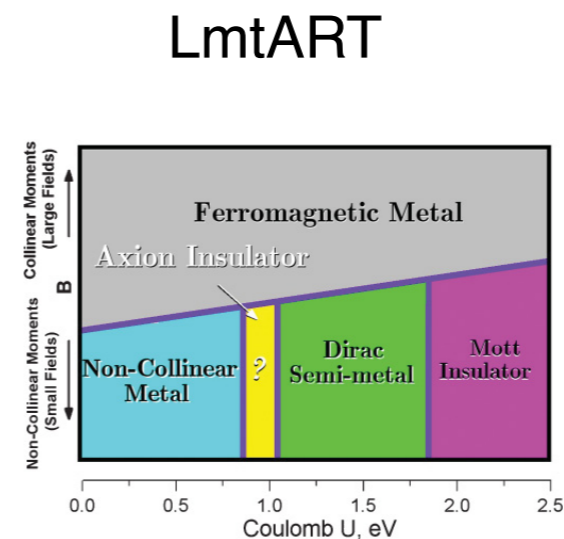
# Domain wall states in $Y_2Ir_2O_7$

Method:

Band structure at different  $U$  deviates a little from different program.  
Overall qualitatively agrees.



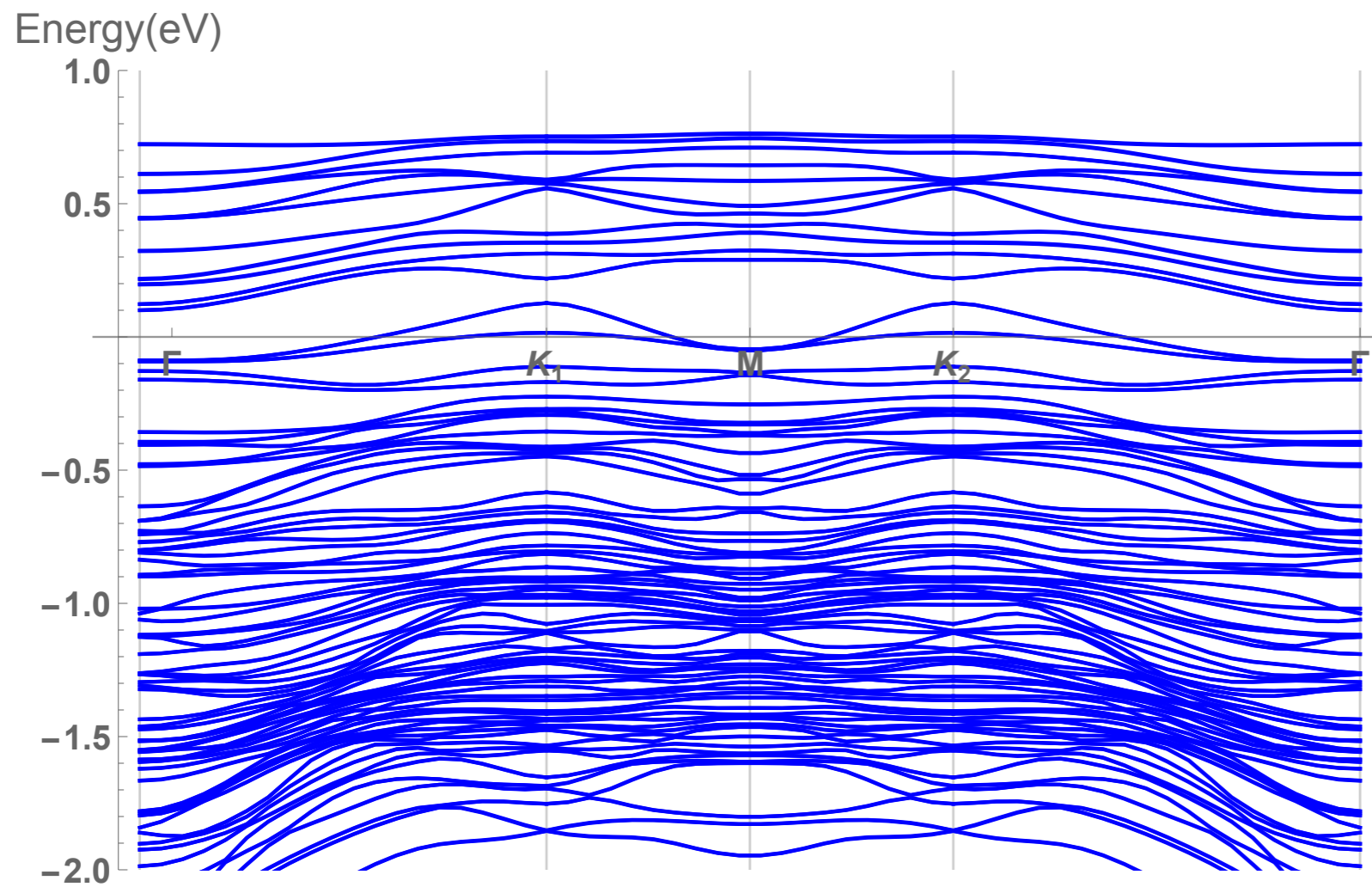
Onoda, et al. J. Phys. Soc. Jpn. 84, 073703 (2015)



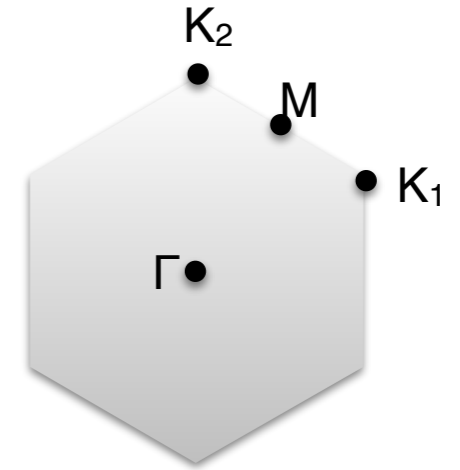
X. Wan et al, PRB 2011

# Domain wall states in $Y_2Ir_2O_7$

## Type 1 Domain walls



6 K & 6 T layers  
GGA+SO+U  
 $U_{Ir}=2\text{eV}$

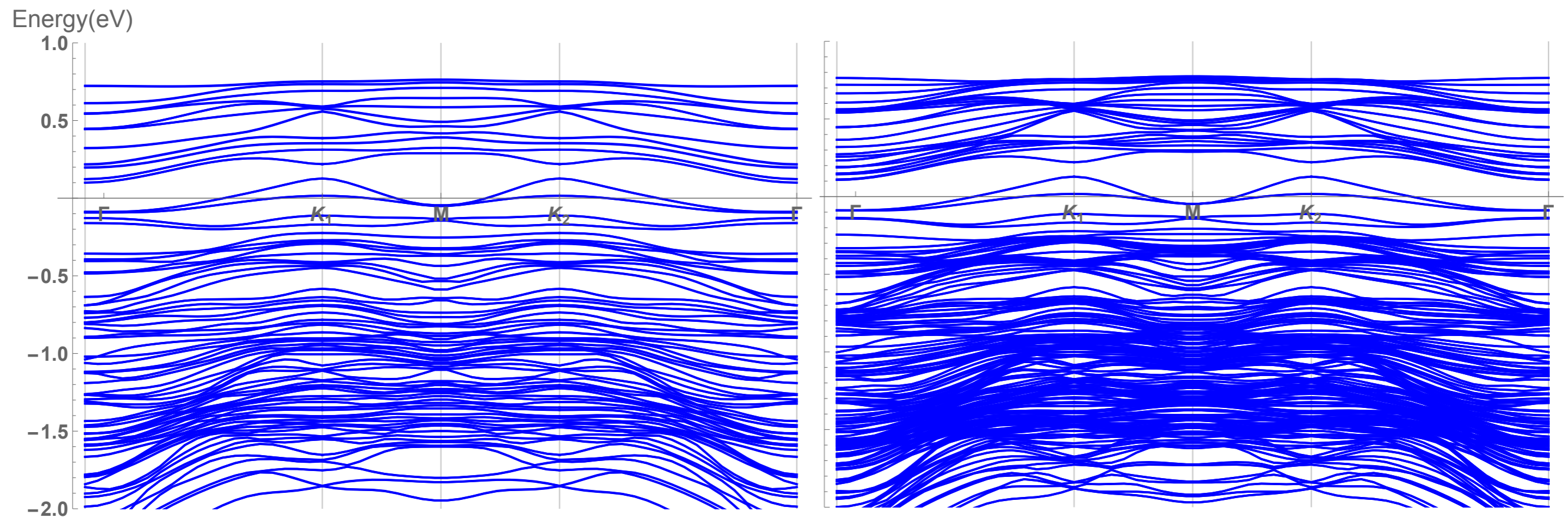


2D Brillouin zone



# *Domain wall states in $Y_2Ir_2O_7$*

Is 6K & 6T superlattice large enough?



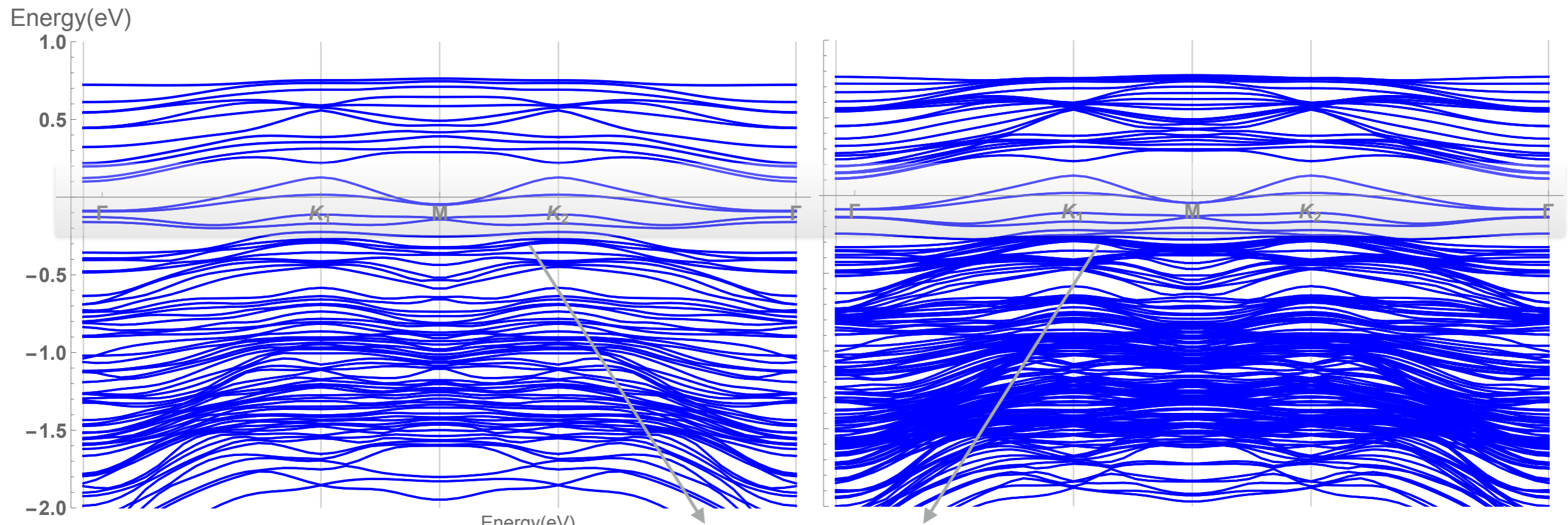
6 K & 6 T layers

12 K & 12 T layers

# Domain wall states in $Y_2Ir_2O_7$

Is 6K & 6T superlattice large enough?

Yes for both type 1 and 2 domain walls.

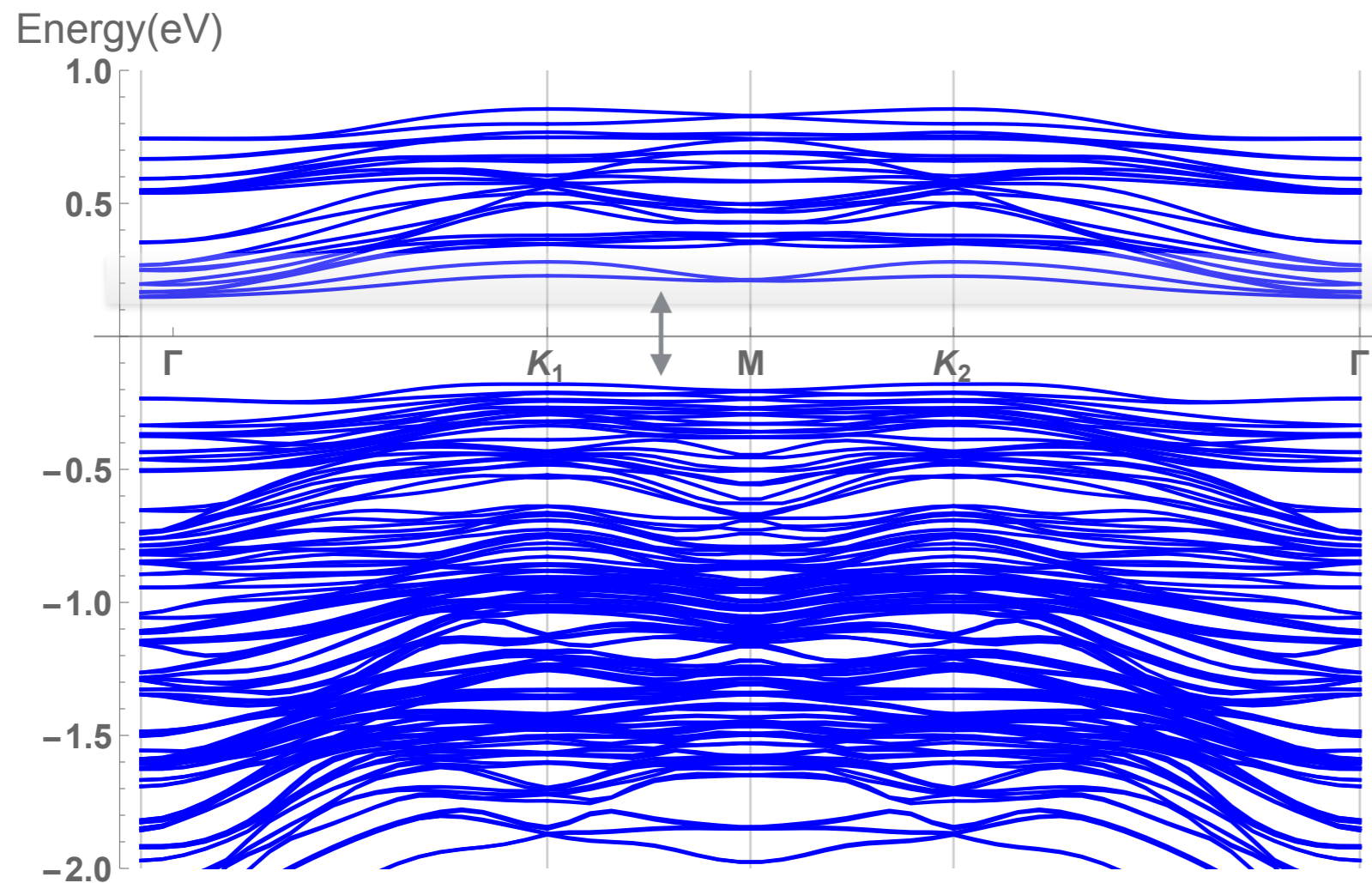


6 K & 6 T layers

12 K & 12 T layers

# Domain wall states in $Y_2Ir_2O_7$

Type 2 Domain walls

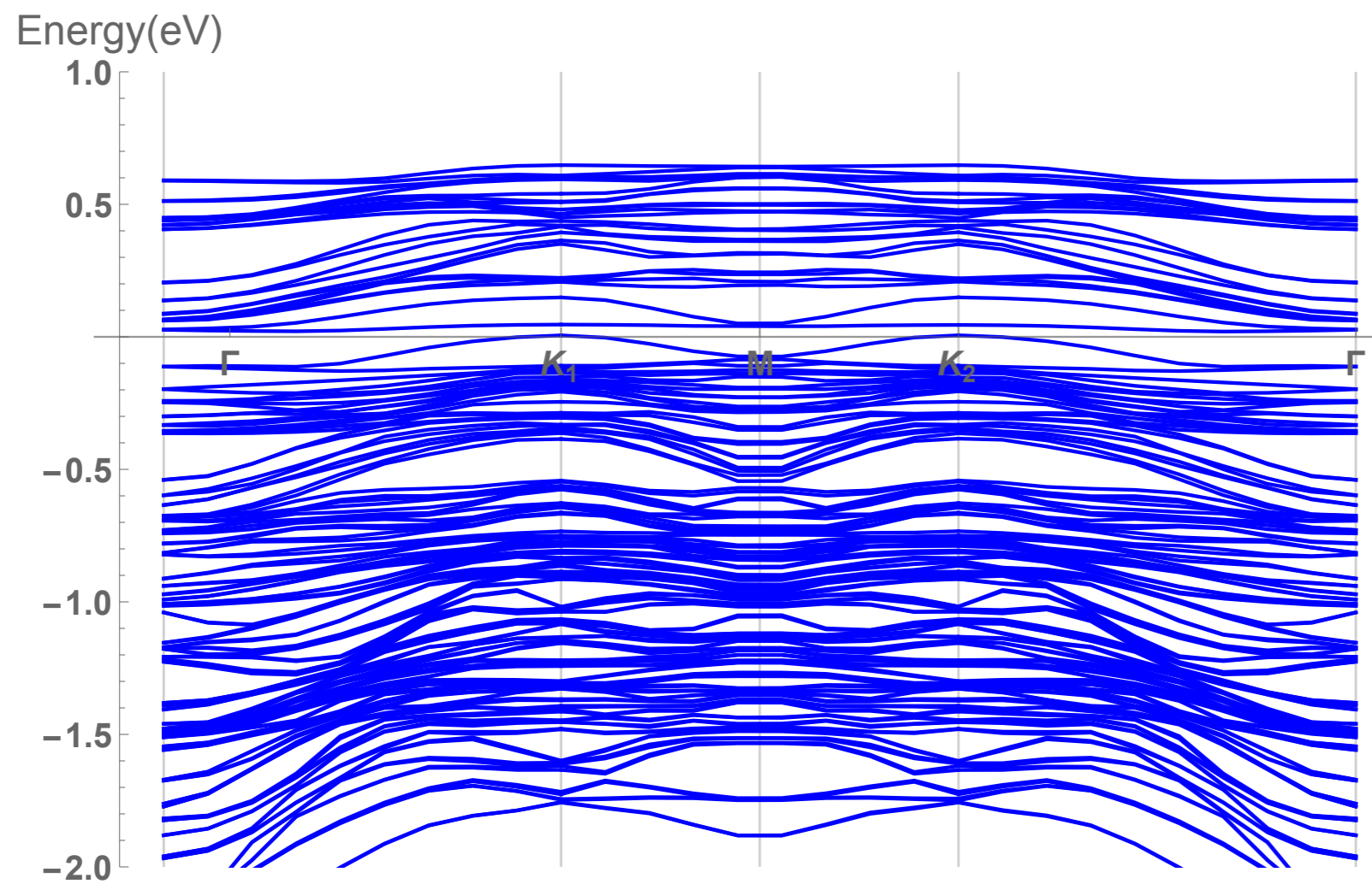


Gapped out domain wall states

6 K & 6 T layers  
GGA+SO+U  
 $U_{Ir}=2\text{eV}$

# *Domain wall states in $Y_2Ir_2O_7$*

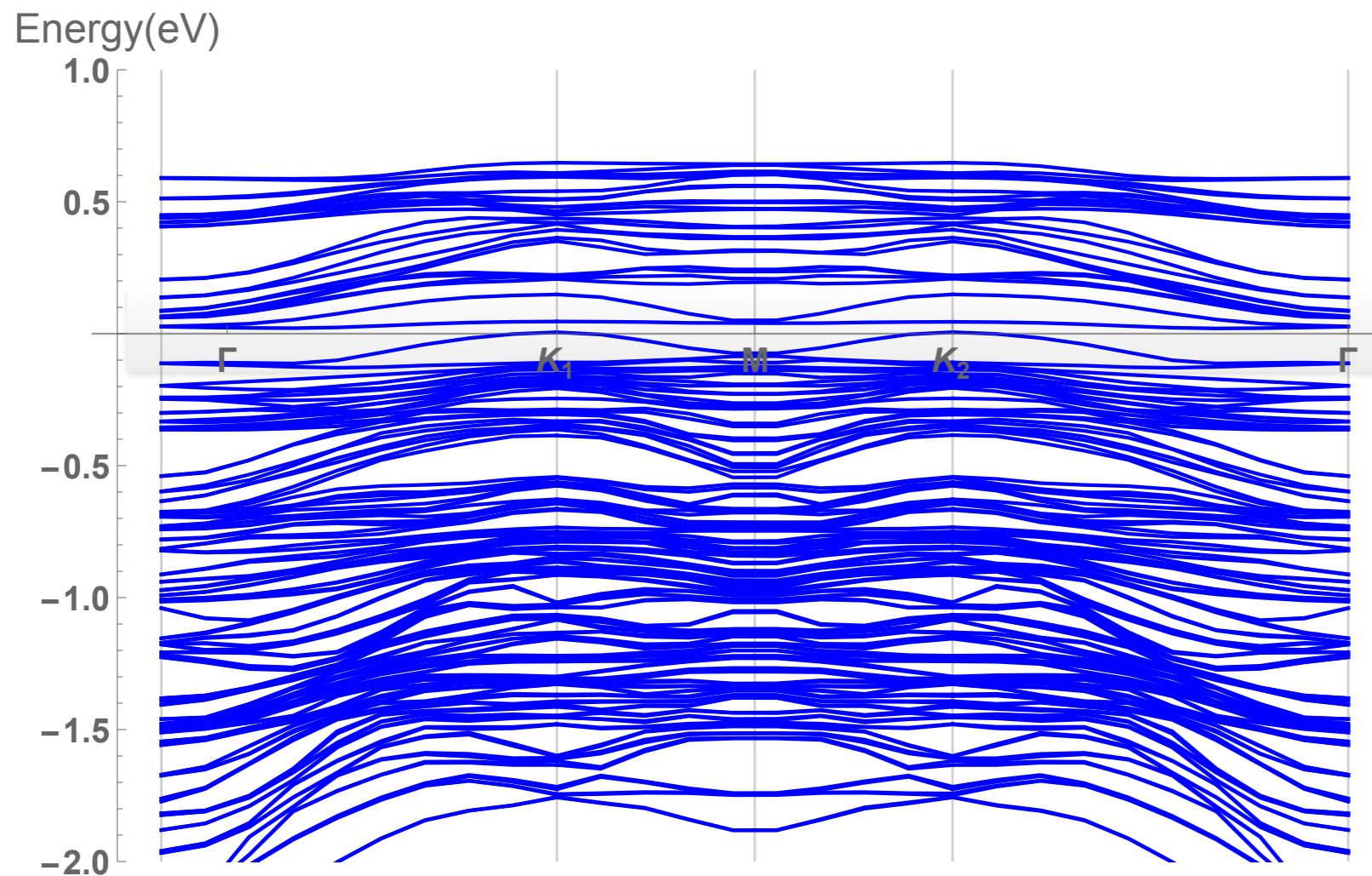
Type 2 Domain walls



6 K & 6 T layers  
GGA+SO+U  
 $U_{Ir}=1.5\text{eV}$

# Domain wall states in $Y_2Ir_2O_7$

## Type 2 Domain walls



Metallic magnetic  
domain wall states

6 K & 6 T layers  
GGA+SO+U  
 $U_{Ir}=1.5\text{eV}$



$\uparrow$  (111)



# *Domain wall states in $Y_2Ir_2O_7$*

---

<b>Total energy per domain wall (2D Kagome plane)</b>	<b><math>U = 1.5 \text{ eV}</math></b>	<b><math>U = 2 \text{ eV}</math></b>
<b>Type 1 Domain walls</b>	<b><math>0.05 \text{ eV}</math></b>	<b><math>0.25 \text{ eV}</math></b>
<b>Type 2 Domain walls</b>	<b><math>0</math></b>	<b><math>0</math></b>

# *Domain wall states in $Y_2Ir_2O_7$*

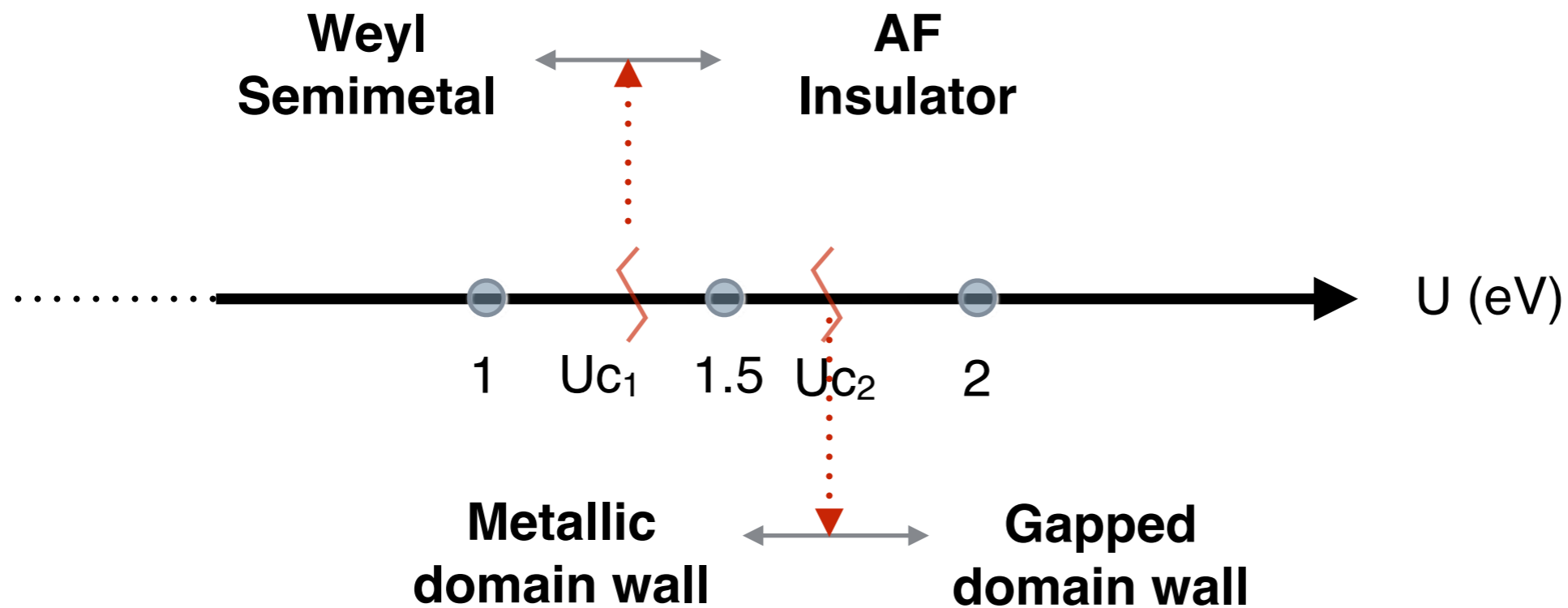
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Total energy per domain wall (2D Kagome plane)	<b><math>U = 1.5 \text{ eV}</math></b>	<b><math>U = 2 \text{ eV}</math></b>
Type 1 Domain walls	<b><math>0.05 \text{ eV}</math></b>	<b><math>0.25 \text{ eV}</math></b>
Type 2 Domain walls	<b>0</b>	<b>0</b>

**energetically favorable!**

# Domain wall states in $Y_2Ir_2O_7$

Tentative DFT phase diagram



**Metallic domain wall**      **Gapped domain wall**

Type 2 Domain walls



↑ (111)

# Summary for Part 2

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- From ab initio calculation, type 2 domain wall (magnetic) is energetically favored
- Within the AF insulating phase of  $Y_2Ir_2O_7$ , there're two phases, one with conducting domain wall, with stronger correlation, the conducting domain wall is gapped out.

