Interaction effects in a system with localized and delocalized single-electron states

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Many-body localization

Disordered (localized) 1d system + weak interactions

\[ H = \hat{T} + W \hat{H}_{\text{dis}} + \lambda \hat{H}_{\text{int}} \]

\( W > 0, \lambda = 0 \): Anderson insulator
\( W > 0, \lambda > 0 \): MBL?

Localization \quad \leftrightarrow \quad Eigenstate thermalization

Gornyi, Mirlin & Polyakov 2005; Basko, Aleiner & Altshuler
Localization in Fock space

\[ H = \hat{T} + W \hat{H}_{\text{dis}} + \lambda \hat{H}_{\text{int}} \]

Non-interacting Anderson insulator

Weakly interacting system

Eigenstate = 1 Slater determinant

Eigenstate = “a few” Slater determinants

Gornyi, Mirlin & Polyakov 2005; Basko, Aleiner & Altshuler
Adiabatic continuity

- Phase 1
- Phase 2
- Phase 3

Many-body ground state of (trivial) gapped system

Adiabatic path

Product states

Energy spectrum

\[ \Delta_0 \]
Localized eigenstates

Many-body ground state of (trivial) gapped system

Product states

Many-body eigenstates of MBL system

Eigenstates of Anderson insulator

Energy spectrum

$\lambda$

$\Delta_0$
Finite-depth local unitary
Localized eigenstates

Many-body ground state of (trivial) gapped system

Product states dressed with local fluctuations

Adiabatic path

Many-body eigenstates of MBL system

Eigenstates of Anderson insulator dressed with local fluctuations

Finite-depth local unitary

BB & C. Nayak, 2013
Localized starting point

\[ H = \hat{T} + W \hat{H}_{\text{dis}} + \lambda \hat{H}_{\text{int}} \]

- \( W > 0, \lambda = 0 \): Anderson insulator
- \( W > 0, \lambda > 0 \): MBL?

What if the non-interacting limit is not fully localized?

- Li, Ganeshan, Pixley & Das Sarma, PRL 2015 and Modak & Mukerjee 2015: Incommensurate potential in \( d = 1 \) with single-particle mobility edge

- Generic situation in higher dimensions
Ladder model

\[ H = - \sum_{\alpha} t_\alpha \sum_{i=1}^{L} \left( \hat{c}_{\alpha,i}^\dagger \hat{c}_{\alpha,i+1} + \text{h.c.} \right) + \sum_{i=1}^{L} w_i \hat{n}_{1,i} + V \sum_{i=1}^{L} \hat{n}_{1,i} \hat{n}_{2,i} \]

- No intra-chain hopping:
  - Particle number preserved on each chain
  - Equivalent to bosons/spins (for OBC)
- Also equivalent: Two-component system

\[ W > 0, V = 0: \text{Particles in upper layer localized, lower layer delocalized} \]
\[ W > 0, V > 0: \text{???} \]

K. Hyatt, J. Garrison, BB, to appear; Nandkisho
Ladder model

\[ H = - \sum_{\alpha} t_{\alpha} \sum_{i=1}^{L} \left( \hat{c}_{\alpha,i}^\dagger \hat{c}_{\alpha,i+1} + \text{h.c.} \right) \]
\[ + \sum_{i=1}^{L} w_i \hat{n}_{1,i} + V \sum_{i=1}^{L} \hat{n}_{1,i} \hat{n}_{2,i} \]

**Localization destroyed**

- Delocalized electrons act as bath for localized electrons
- Energy transport through lower layer leads to delocalization in upper layer

**Localization survives**

- Localized layer acts as effective disorder on other layer:
  \[ V \sum \hat{n}_{1,i} \hat{n}_{2,i} \rightarrow V \sum \langle \hat{n}_{1,i} \rangle \hat{n}_{2,i} \]

*K. Hyatt, J. Garrison, BB, to appear; Nandkisho*
MBL coupled to a bath

Weak coupling to bath: Spectral features of MBL phase are broadened (Nandkishore, Gopalakrishnan & Huse 2014; Johri, Nandkishore & Bhatt 2014)

\[ V_{\text{bath}} \gg V_{\text{sys}} \]
MBL coupled to a “small bath”

- Potential for back-action: System can localize bath!
- Explore numerically: exact eigenstates using shift-and-invert algorithm (Luitz et al, PRB 2014)

\[ V_{\text{bath}} \approx V_{\text{sys}} \]

Huse et al 2014; Nandkishore 2015; K. Hyatt, J. Garrison, BB,
Entanglement

Volume law: $S(\rho_A) \sim \text{vol}(A)$
(generic quantum state, highly excited states, thermal states)

Area law: $S(\rho_A) \sim \partial A$
(ground states of local Hamiltonians, MBL eigenstates)

$S(l) = s_{th}(\epsilon) L$

$S(l) \rightarrow \text{const}$

BB & C. Nayak, 2013
Entanglement in MBL

\[ S(l) = s_{th}(\epsilon)L \]

\[ S(l) \rightarrow \text{const} \]

\[ l = \frac{L}{2} \]

BB & C. Nayak, 2013
Entanglement in MBL

$S_c$ vs $L$

$W = 4 V = 1.2$

$W = 6 V = 1.2$

$H(S)$ vs $S$

$W = 4 V = 2$

$W = 6 V = 1.2$

BB & C. Nayak, 2013

Rare cuts

Typical cuts
Ladder entropies

- Decoupled layers: $V = 0$
- One layer localized: $W > 0$

- Coupled layers: $V > 0$
- One layer s.p. localized: $W > 0$
Entropy cuts
Entropy scaling

Delocalization of both layers?
Tune into MBL regime?

\[ H = - \sum_{\alpha} t_{\alpha} \sum_{i=1}^{L} \left( \hat{c}^{\dagger}_{\alpha,i} \hat{c}_{\alpha,i+1} + \text{h.c.} \right) + \sum_{i=1}^{L} w_i \hat{n}_{1,i} + V \sum_{i=1}^{L} \hat{n}_{1,i} \hat{n}_{2,i} \]

- Several ways to make bath less effective:
  - Reduce particle density in bath – finite-size corrections?
  - Reduce bandwidth: \( t_{\alpha} \ll t_d \)
Reduced hopping
Entropy scaling
Entropy scaling

Block Size $b$ by $t_c$
Entropy scaling
Entropy scaling

Graph showing the relationship between entropy scaling and various parameters.
Area law & finite size

T. Grover 2014: if $l \ll L$, then $\partial^2 S / \partial l^2 \leq 0$

Would like to achieve $\xi \ll l \ll L$

but in practice $\xi_{sp} < l, l = L/2$
Entropy scaling
Conclusions

• Ladder model for
  – Many-body localization where non-interacting limit is not fully localized
  – MBL coupled to a small “bath”
Thank you!