Many-body physics with ultra-cold atoms in disorder

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Introduction.

Many-body localization-delocalization transition

MBLDT for 1D disordered bosons

MBLDT in the AAH model

Phase diagram

Conclusions

Collaborations B.L. Altshuler/I.L. Aleiner (Columbia Univ.), V. Michal (LPTMS, Orsay)
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Many-body system in disorder

Many-particle system in disorder ⇒ Transport and localization properties

Anderson localization (P.W. Anderson, 1958)
Destructive interference in the scattering of a particle from random defects

Old question. How does the interparticle interaction influence localization?
Long standing problem. Crucial for charge transport in electronic systems
Appears in a new light for disordered ultracold bosons

Palaiseau, LENS, Rice, Urbana experiments. More underway
What was known and expected?

What was done?

Anderson localization of

Light

Microwaves

Sound waves

Electrons in solids

What is expected?

Anderson localization of neutral atoms
Experiments with cold atoms

BEC

BEC in a harmonic + weak random potential $|V(z)| \ll n g$ ⇒ small density modulations of the static BEC. Switch off the harmonic trap, but keep the disorder ⇒ What happens? (Orsay, LENS, Rice)

Orsay experiment
Quantum gases in disorder. What was not expected?

One-dimensional disordered bosons at finite temperature

DOGMA → No finite temperature phase transitions in 1D as all spatial correlations decay exponentially

There is a non-conventional phase transition between two distinct states

Fluid and Insulator

Interaction-induced transition

I.L. Aleiner, B.L. Altshuler, GS, (2010)
Many-body localization-delocalization transition

(Aleiner, Altshuler, Basko 2006-2007)

How different states of two particles $|\alpha, \beta\rangle$ hybridize due to the interaction?

The probability $P(\varepsilon_\alpha)$ that for a given state $|\alpha\rangle$ there exist $|\beta\rangle, |\alpha'\rangle, |\beta'\rangle$

such that $|\alpha, \beta\rangle$ and $|\alpha', \beta'\rangle$ are in resonance:

\[
\langle \alpha, \beta | H_{int} | \alpha', \beta' \rangle \text{ exceeds } \Delta_{\alpha\beta}^{\alpha'\beta'} \equiv |\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_{\alpha'} - \varepsilon_{\beta'}|
\]

MBLDT criterion \[P(\varepsilon_\alpha) \sim 1\]

Mismatch \[\Delta_{\alpha\beta}^{\alpha'\beta'} = |\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_{\alpha'} - \varepsilon_{\beta'}| \approx \left| \frac{1}{\zeta_\alpha \rho(\varepsilon_\alpha)} + \frac{1}{\zeta_\beta \rho(\varepsilon_\beta)} \right| \approx \frac{1}{(\zeta \rho)_{\text{min}}}
\]
MBLDT criterion

The probability that $\langle \alpha, \beta | H_{int} | \alpha', \beta' \rangle$ exceeds $\Delta_{\alpha \beta}^{\alpha' \beta'}$

$$P_{\alpha \beta}^{\alpha' \beta'} \approx UN_{\beta} \frac{a(\zeta \rho)_{\text{min}}}{\zeta_{\text{max}}}$$

$$P(\varepsilon_{\alpha}) = \sum_{\beta, \alpha', \beta'} P_{\alpha \beta}^{\alpha' \beta'} = U \int d\varepsilon_{\beta} \rho(\varepsilon_{\beta}) \zeta_{\beta} N_{\beta} \frac{a(\zeta \rho)_{\text{min}}}{\zeta_{\text{max}}}$$

Critical coupling strength $U_c \approx \left[ \int d\varepsilon_{\beta} \rho(\varepsilon_{\beta}) \zeta_{\beta} N_{\beta} \frac{a(\zeta \rho)_{\text{min}}}{\zeta_{\text{max}}} \right]^{-1}$
1D bosons

Interacting 1D Bose gas. No disorder $\Rightarrow$ Fluid phase

QuasiBEC

Degenerate thermal gas

Classical gas

$T_d \sqrt{\gamma}$

$T_d = \frac{\hbar^2 n^2}{m}$

$\gamma = \frac{mg}{\hbar^2 n} = \frac{ng}{T_d} \ll 1 \rightarrow$ weakly interacting regime

Disordered non-interacting 1D bosons

All single-particle states are localized at any energy $\rightarrow$ Anderson insulator
1D Bose gas in disorder
I.L. Aleiner, B.L. Altshuler, G.S., 2010

\[ \rho(\varepsilon) \approx \frac{m}{2\pi\hbar^2\varepsilon}; \quad \zeta(\varepsilon) \approx \frac{\hbar\varepsilon}{m^{1/2}\varepsilon_*^{3/2}} \quad \varepsilon > \varepsilon_* = U_0 \left( \frac{U_0\sigma^2 m}{\hbar^2} \right)^{1/3} \]

Classical gas \( \Rightarrow T > T_d \sim \hbar^2 n^2 / m; \quad \mu = T \ln n \Lambda_T \)

\[ ng_c \sim \varepsilon_* \left( \frac{\varepsilon_*}{T} \right)^{1/2} \ll \varepsilon_* \]

Quantum decoherent gas \( \Rightarrow T_d \sqrt{\gamma} < T < T_d; \quad \mu \sim T^2 / T_d \)

\[ ng_c \sim \varepsilon_* \left( \frac{\varepsilon_* T_d}{T^2} \right)^{1/2} \sim \frac{1}{T} \ll \varepsilon_* \]

QuasiBEC \( \Rightarrow T < T_d \sqrt{\gamma}; \quad ng_c \sim \varepsilon_* \)
1D Bose gas in disorder
LENS experiment. What is expected?

1D quasiperiodic potential

$$J(\psi_{n+1} + \psi_{n-1}) + V \cos(2\pi \kappa n) \psi_n = \varepsilon \psi_n$$

$$V > 2J \rightarrow \text{all single-particle states are localized}$$

Aubry/Andre (1980)
LENS experiment

Feshbach modification of the interaction for $^{39}$K

Observation of the fluid-insulator transition

\[
\Delta/J \quad nU/J
\]

- insulator
- fluid
AAH model

Localization length for all eigenstates is $\zeta = a \ln^{-1}[V/2J]$ (Aubry/Andre, 1980); $\zeta \simeq V a/(V - 2J) \gg a$ for $V$ close to $2J$

Single-particle energy states for $\kappa \ll 1$ ($\kappa = \sqrt{2}/20$ and $V = 2.05J$)

Interacting bosons $H_{int} = U \sum_j n_j(n_j - 1)/2$
MBLDT in the AAH model

The number of clusters $N_1 \simeq 1/\kappa$ for $\kappa \ll 1$

The width of a cluster $\Gamma$ grows exponentially with energy

For $N_1 < \zeta$

$\zeta/N_1 \Rightarrow$ number of states of a given cluster participating in MBLDT

$T \ll 8J \rightarrow$ lowest energy cluster

MBLDT criterion

$$\int_0^{\Gamma_0} d\varepsilon \rho^2(\varepsilon) \zeta n_\varepsilon U_c = 1$$

Occupation number of particle states

$$n_\varepsilon = \left[\exp(\varepsilon + Un_\varepsilon/\zeta - \mu)/T - 1\right]^{-1}$$

Chemical potential

$$\int \rho(\varepsilon)n_\varepsilon d\varepsilon = \nu$$
Critical coupling at $T = 0$

$$T = 0 \Rightarrow \varepsilon + Un_\varepsilon/\zeta(\varepsilon) = \mu$$

$$n_\varepsilon = \zeta(\mu_0 - \varepsilon)/U; \quad \varepsilon < \mu_0$$

$$n_\varepsilon = 0; \quad \varepsilon > \mu_0$$

$$U_{c\nu} \simeq \frac{2\Gamma_0}{\kappa\zeta}$$

Valid also at $T \ll \omega$
Critical coupling at finite temperatures

\[ n_\varepsilon = \frac{\zeta}{2U} \left\{ (\mu - \varepsilon) + \sqrt{(\mu - \varepsilon)^2 + 4TU/\zeta} \right\} \text{ if } n_\varepsilon \gg 1 \]

\[ n_\varepsilon = \exp - (\varepsilon - \mu)/T \text{ if } n_\varepsilon \lesssim 1 \ (\varepsilon > \mu) \]

\[ \frac{U_c(T)}{U_c(0)} \simeq \left[ 1 + \frac{T}{8\nu J} \ln \left( \frac{T}{\omega} \right) \right] ; \quad \omega \ll T \ll 8J \]

Ab initio not expected. Anomalous temperature dependence!

\[ T \to \infty \implies n_\varepsilon \simeq \nu ; \mu \simeq -T/\nu \]

\[ U_c \nu \simeq \frac{\Gamma_0}{\kappa^2 \zeta} ; \quad \frac{U_c(\infty)}{U_c(0)} = \frac{1}{\kappa} \]
Critical coupling

$\kappa$ close to $1/8$ and $V = 2.05J$

Increase in temperature favors the insulator state. "Freezing with heating"
Critical coupling

Golden ratio \( \kappa = (\sqrt{5} - 1)/2 \) and \( V = 2.1J \)
Conclusions

1D bosons is a promising system to study the many-body localization-delocalization transition

Atoms in quasiperiodic potentials ⇒ Increasing temperature may favor localization

Thank you for attention!