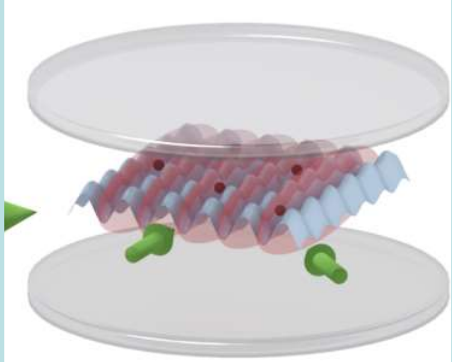
Der Wissenschaftsfonds.

Innsbruck Physics Research Center

Quantum Gas Cavity QED



Helmut Ritsch
Universität Innsbruck

New Perspectives in Many-body Physics with Quantum Optical Systems

KITP - MBQOptics24 - Nov.. 5, 24

CRYST

quantA

universität innsbruck

https://www.uibk.ac.at/th-physik/cqed/

Innsbruck Physics Research Center

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Collaborations (Theory):
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Claudiu Genes, Maria Moreno-Cardoner, Darrick Chang, Susanne Yelin

Open source quantum dynamics software

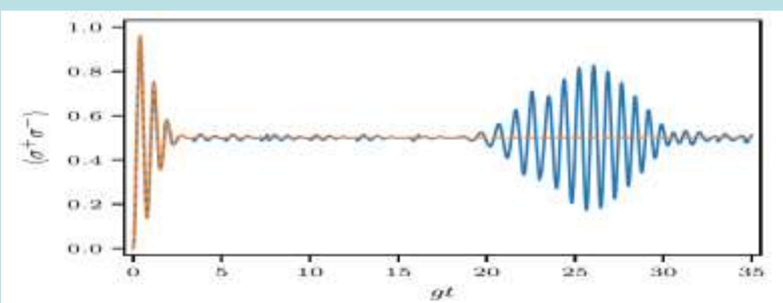
www.qojulia.org

QuantumOptics.jl

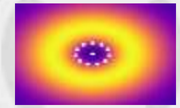
Quantum Optics toolbox in *Julia*

```
a = destroy(bc) ⊗ one(ba)
σ⁻ = one(bc) ⊗ sigmam(ba)

# Construct Hamiltonian
H = Δ*dagger(a)*a + g*(dagger(a)*σ⁻ + a*dagger(σ⁻))

# Define initial state
ψ₀ = coherentstate(bc, α) ⊗ spindown(ba)

# Define list of time steps
T = [0:0.01:35;]

# Evolve in time according to Schrödinger's equation
tout, ψₜ = timeevolution.schroedinger(T, ψ₀, H)

# Calculate atomic excitation
excitation = expect(dagger(σ⁻)*σ⁻, ψₜ)
```

Code sample 1: *Jaynes-Cummings model.*



dipole-dipole package



CollectiveSpins.jl

Simulate Dipole-Dipole Coupled Spin Systems

https://qojulia.github.io/CollectiveSpins.jl/

QuantumCumulants.jl

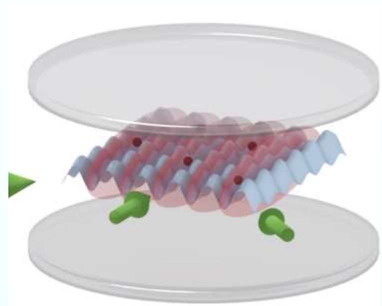
symbolic & numerical package for automated cumulant expansions

https://github.com/qojulia/QuantumCumulants.jl

Basic Idea

Quantum optics:
quantized light modes
&
classical point particles
with quantized energy levels

Jaynes Cummings model



Ultracold gases:
quantum particle motion
in
classical optical potentials

BEC / Bose-Hubbard model

Quantum optics with quantum particles:

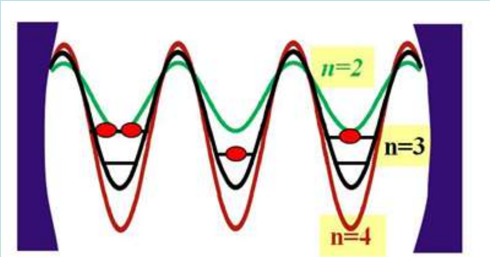
full quantum *dynamics* of light and matter waves

dispersive (non-resonant) regime

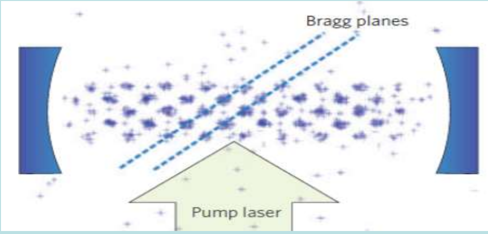
light induces
dynamical optical potentials & forces
+
atoms generate
a nonlinear dynamical refractive index

Review: F. Mivehvar, T. Donner, F. Piazza, H.R., Advances in Physics, 2021

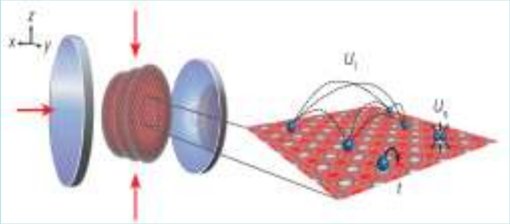
- *Atomic dynamics in a cavity generated potential*



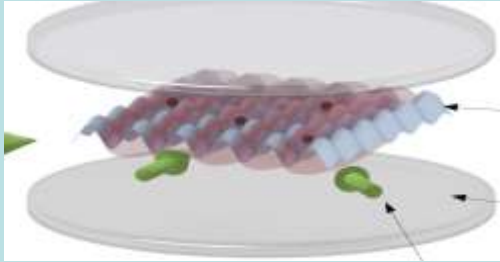
- *Atom light crystallization of laser illuminated gases*



- *Quantum simulation of exotic quantum phases*



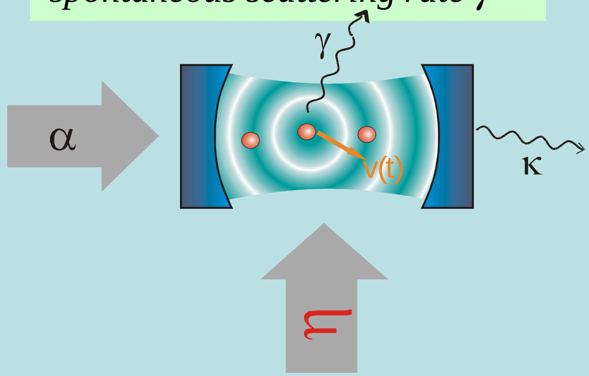
- *Cavity based optimization: the N-Queens problem*



Cavity QED: Jaynes/Tavis Cummings coupling + open system dynamics

$$H = \hbar\omega_f a^\dagger a + \tfrac{1}{2}\hbar\omega_a\sigma^z + \hbar \mathbf{g}(a^\dagger\sigma^- + a\sigma^+)$$

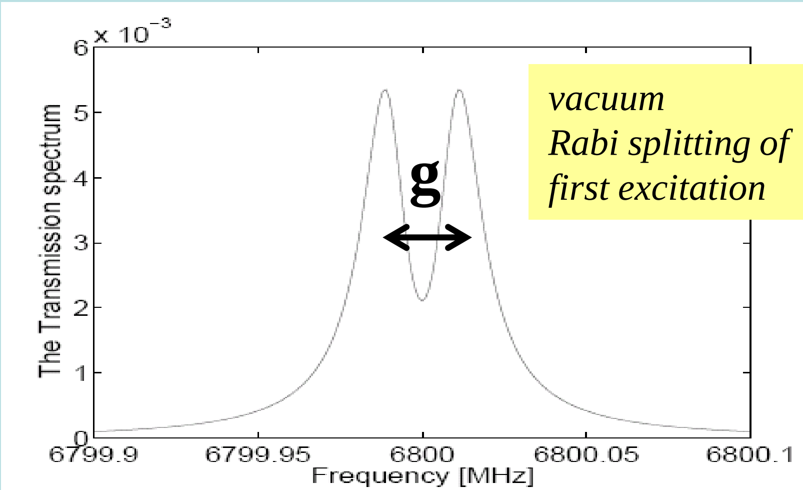
spontaneous scattering rate γ



Photon decay rate κ

strong coupling

$$\omega_f\,,\ \omega_a \gg \mathbf{g} \gg (\kappa,\gamma)$$



input and output channels :

☺ **measurement + feedback control**

☹ **damping + fluctuations + decoherence**

- Nonlinear atomic response at less than a single photon
- Single atom splits cavity resonance by more than a line width

Gedankenexperiments of Quantum Mechanics + Quantum Information Processing realized , …
‚Classic CQED‘: Haroche, Walther, Kimble, Rempe, …
Circuit QED more recently: Schoelkopf, Wallraff, Majer, ….

dispersive **Cavity-QED at** large detuning

g_0 … coupling strength
γ … atomic width
κ … cavity linewidth

$$\Delta = \omega_a - \omega_L >> \gamma, \kappa$$

eliminate upper atomic state => effective atom-field interaction potential

$$U(x) := \frac{\Delta_a}{\Delta_a^2 + {\gamma_0}^2} g_0^2 \cos^2(kx)$$

$$\gamma(x) := \frac{\gamma_0}{\Delta_a^2 + {\gamma_0}^2} g_0^2 \cos^2(kx)$$

U(x) = optical potential per photon = cavity frequency shift per atom based on dipole force

>

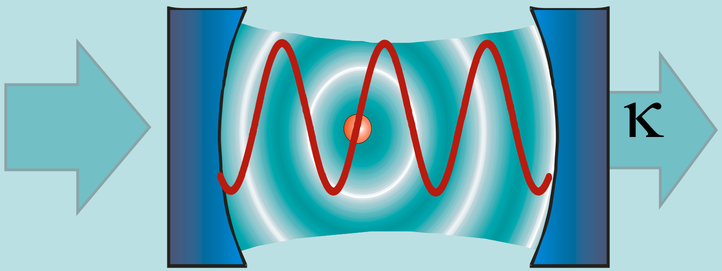
*$\gamma(x)$ = **photon loss** per particle << κ*
radiation pressure / photon

Strong dispersive coupling limit :

$$U >> (\omega_{rec}, \kappa) >> \gamma$$

$\Rightarrow$ single atom shifts cavity in & out of of resonance
$\Rightarrow$ single photon creates an optical trap for an atom

Lightforces on polarizable particles in optical resonators



***Light forces** of resonatorfield determine atomic motion*

interaction

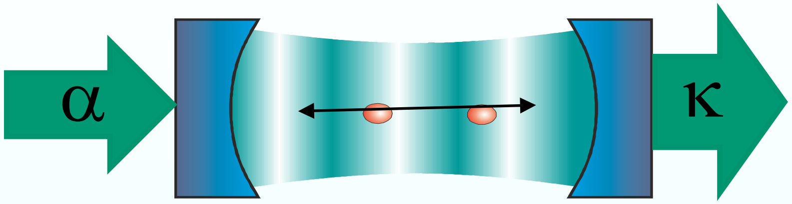
self - trapping

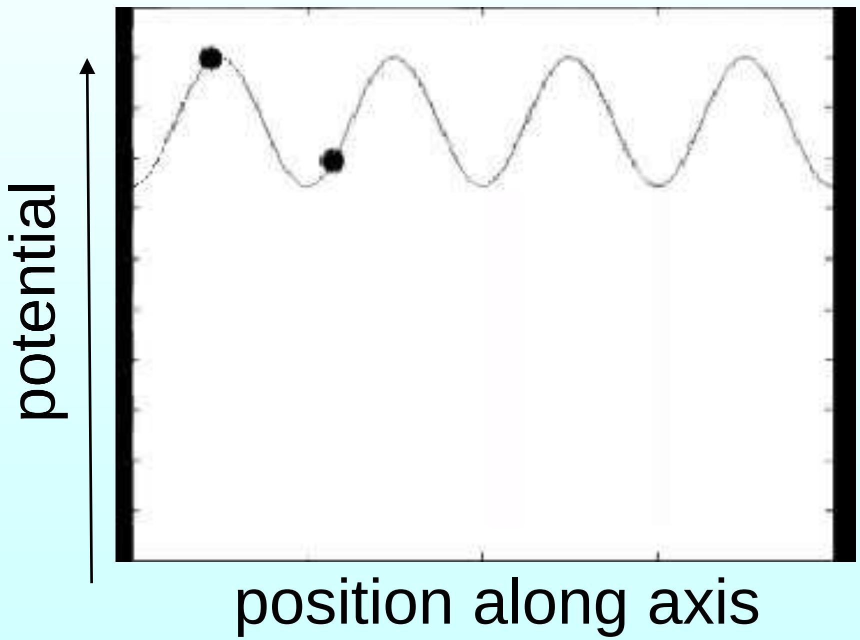
friction + diffusion

correlation and entanglement

***Cavity QED** : atoms change resonator field dynamics*

„long“ range interaction:
two particles in a driven optical resonator





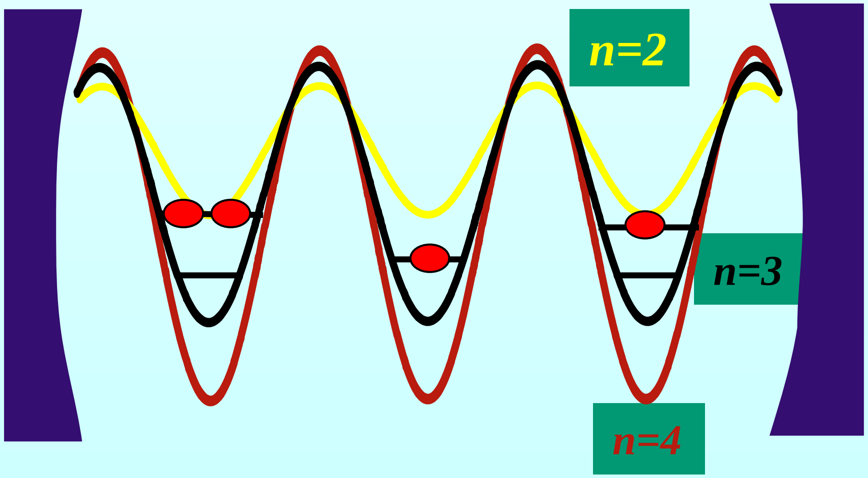
friction and „long“ range interaction

H. R., P Domokos, F. Brennecke, T. Esslinger, Rev. Mod. Phys., 2013

Dynamics at T ~ 0
*=> **quantum** optical lattice potential*

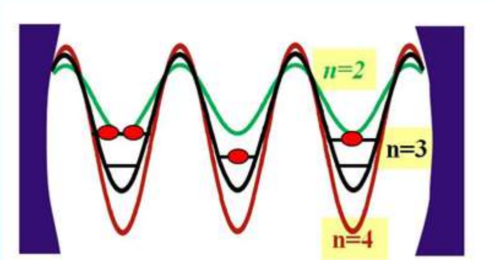
Effective single particle Hamiltonian at large atomic detuning

$$H = \frac{p^2}{2m} + \underbrace{\hbar U_0 a^\dagger a \cos^2(kx)}_{\text{quantum potential}} - \hbar\Delta_c a^\dagger a - i\hbar\eta\left(a - a^\dagger\right)$$



Quantum limit of cavity cooling in sub-recoil regime :

$T \sim \kappa < \omega_{rec}$



atomic motion + field quantized:

$$H = \frac{p^2}{2m} + \underbrace{\hbar U_0 a^\dagger a \cos^2(kx)}_{\text{quantum potential}} - \hbar\Delta_c a^\dagger a - i\hbar\eta\left(a - a^\dagger\right)$$

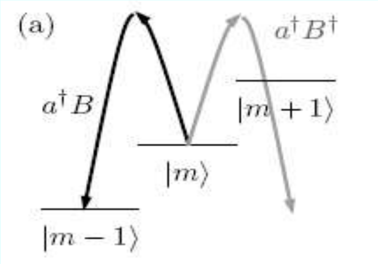
strong pump:
deep lattice $U > \kappa, \omega_{rec}$

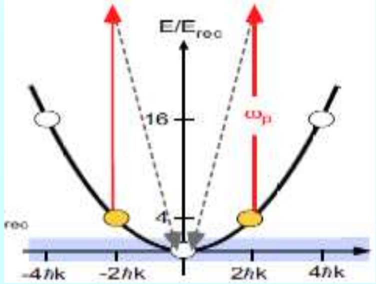
Cavity tuned to

weak pump:
free motion $\kappa < \omega_{rec}$

„blue“ vibrational sideband of trapped atoms

higher momentum states for a free gas





vibrational trap states in cavity field
=> cavity sideband cooling

free space momentum states
=> cavity Doppler cooling

cavity cooling towards zero temperature: $\kappa < \omega_r$

several cooling steps in time
for optimized cooling sequence
to ground state = ‘ BEC ’

bosons

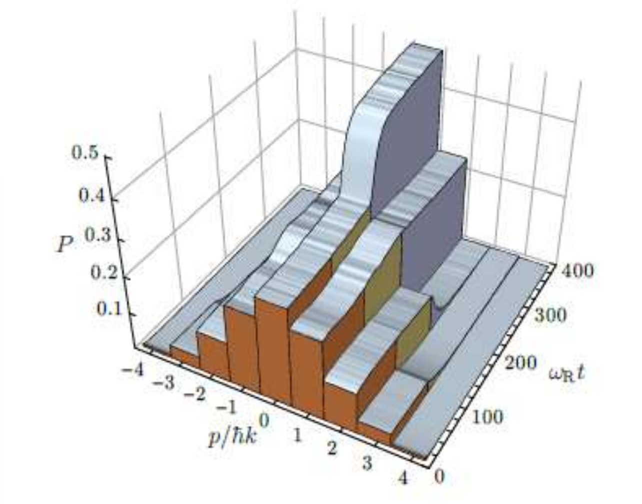
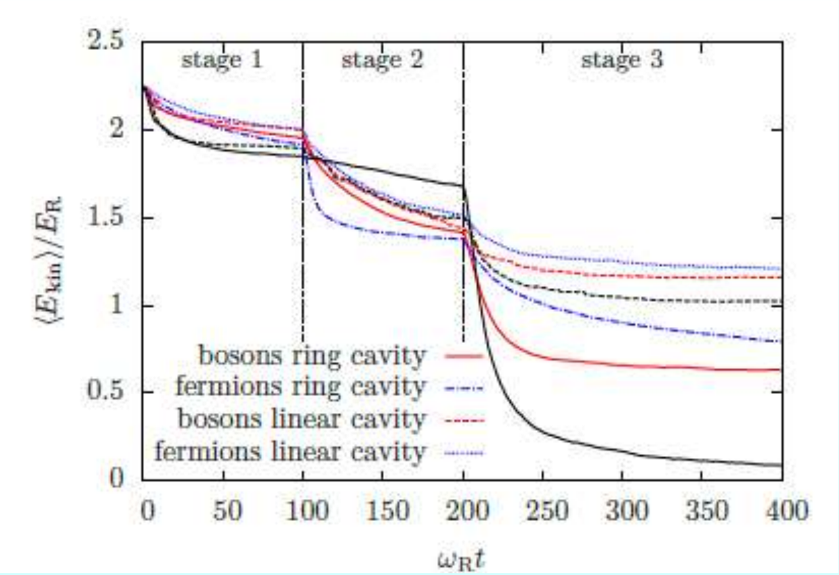


Fig. 4: (Colour on-line) Single-particle momentum distribution for $\Delta_c/\omega_R = -14.75|-12|-7$ (ring cavity bosons).

Cooling dynamics
for different particle quantum statistics



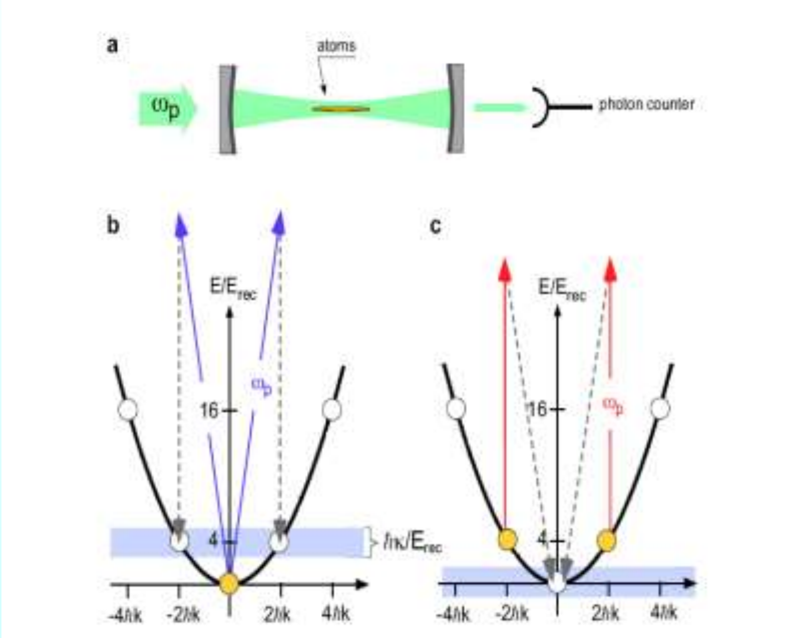
Cavity cooling has no principle temperature limit
=> one can reach degeneracy !

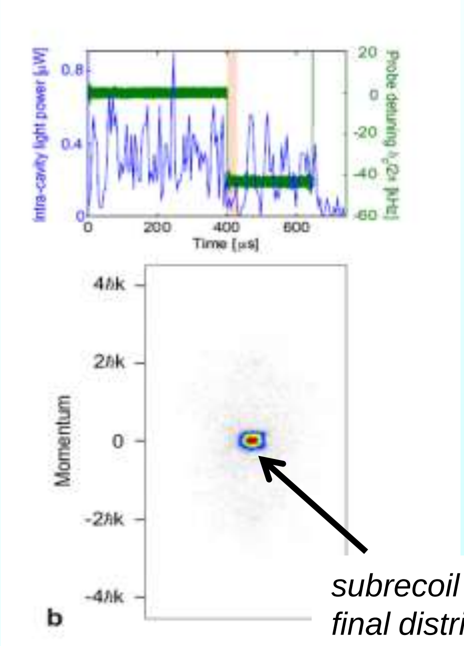
Experiment: cavity ground state cooling of „free“ atoms

A. Hemmerich, Hamburg (Science 2012)

„sub-recoil“ regime:
$\kappa < \omega_r$

final momentum distribution
smaller than
single photon recoil ?





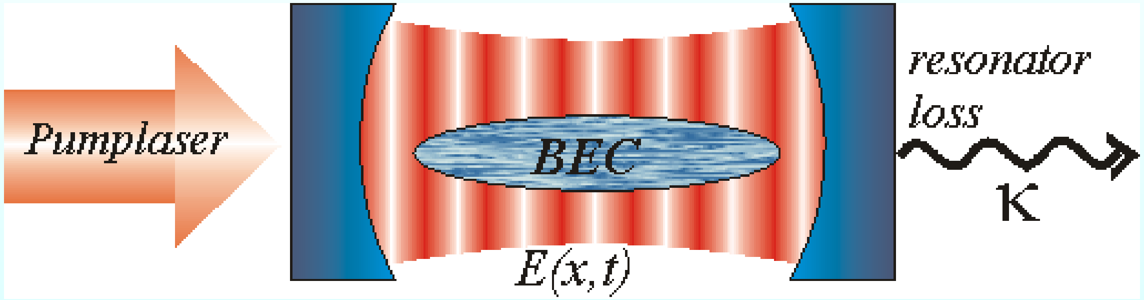
subrecoil width
final distribution !

Cavity cooling with Bose stimulation to replace evaporation :

⇒ BEC formation without particle loss

*Quantum **dynamics** of many particles and field near T~0*

**** BEC** in **optical lattice with dynamic (quantum) properties***



coupled nonlinear and nonlocal equations with a wealth of dynamic effects

Refs: ***Horak, Barnett, Zoller, Meystre, Liu, Bhattacherjee…***

Experiments: ***Esslinger, Reichl, Zimmermann, Hemmerich, Vuletic, Treutlein ...***

Mean field description of many particles and field

mean field approximation for particles and field

$$\frac{d}{dt}\alpha(t) = [i\Delta_c - iN\langle U(\hat{x})\rangle - \kappa]\alpha(t) + \eta, \qquad (1a)$$

$$i\frac{d}{dt}\psi(x,t) = \left\{\frac{\hat{p}^2}{2m} + |\alpha(t)|^2 U(x) + N g_{coll}|\psi(x,t)|^2\right\}\psi(x,t).$$

simple effective theory :
two state expansion of BEC

$$\psi(x,t) = c_0(t) + c_2(t)\sqrt{2}\cos(2kx)$$

get two position coupled oscillators

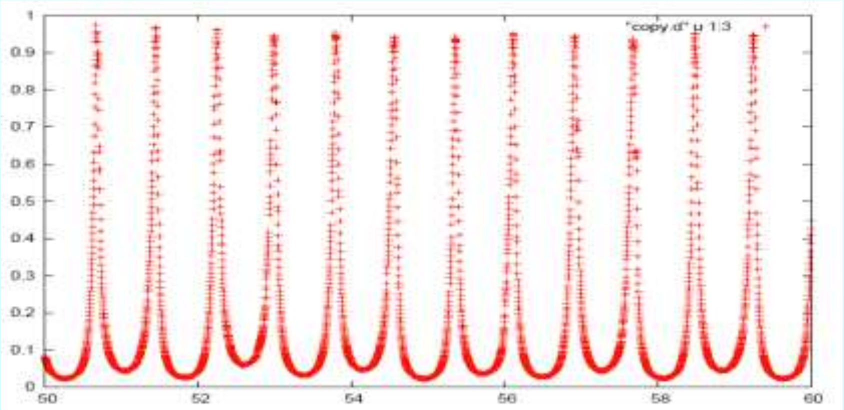
$$X = 2\sqrt{1/N}\ \mathrm{Re}(c_0{}^* c_2)$$

$\Rightarrow$ ***optomechanics – Hamiltonian***
at T=0 with strong coupling

$$\ddot{X} + (4\omega_{rec})^2 X = -\omega_{rec} U_0 \sqrt{8N}\langle\hat{a}^\dagger\hat{a}\rangle$$

e.g.: operation in unstable regime
=> self-sustained oscillations at 4 ω_r

cavity field

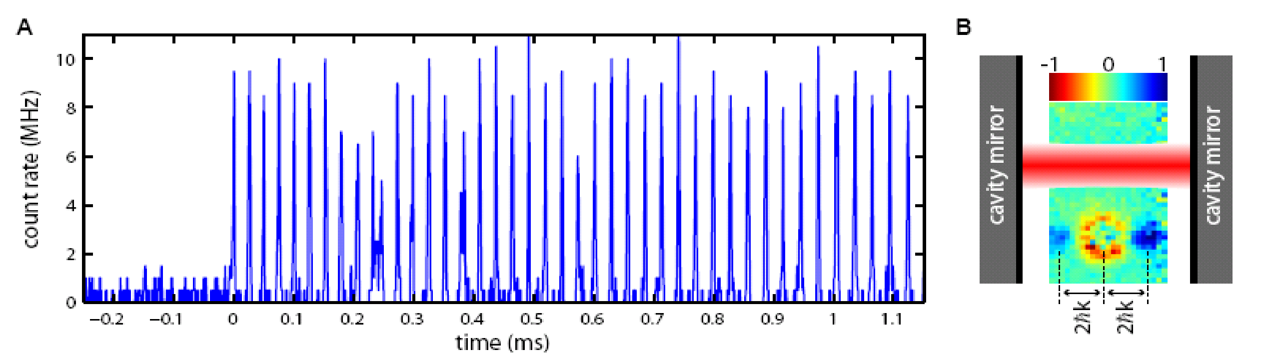


time

Experiment ETH Zürich
+
several theoretical descriptions

Dynamical Coupling between a Bose-Einstein Condensate and a Cavity Optical Lattice

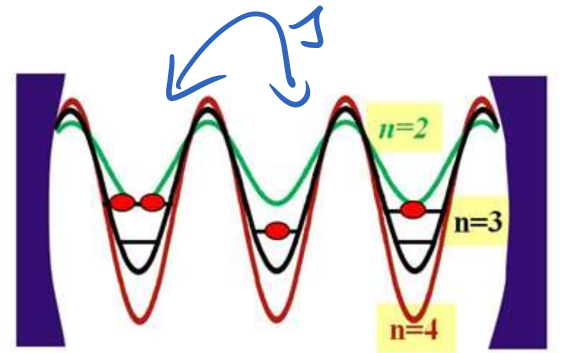
Stephan Ritter[1,2], Ferdinand Brennecke[1], Christine Guerlin[1],
Kristian Baumann[1], Tobias Donner[1,3], Tilman Esslinger[1]*
[1] *Institute for Quantum Electronics, ETH Zürich, CH-8093 Zürich, Switzerland*
[2] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
[3] *JILA, University of Colorado and National Institute of Standards and Technology, Boulder CO 80309, USA*
(Dated: November 24, 2008)



study of zero T optomechanics, atom field entanglement

Beyond mean field

Hubbard model for a quantized single mode



$$H = E_0\hat{N} + E\hat{B} + \left(\hbar U_0 a^\dagger a + V_{\mathrm{cl}}\right)\left(J_0\hat{N} + J\hat{B}\right) - \hbar\Delta_c a^\dagger a - i\hbar\eta\left(a - a^\dagger\right) + \frac{U}{2}\hat{C}.$$

$$\hat{N} = \sum_k \hat{n_k} = \sum_k b_k^\dagger b_k$$

$$\hat{B} = \sum_k \left(b_{k+1}^\dagger b_k + h.c.\right)$$

Looks similar to standard Bose Hubbard model
but
"parameters" for lattice dynamics are field operators

single field mode as observable for atomic quantum statistics

*Heisenberg equation for field amplitude operator **a** :*

$$\dot{a} = \left\{ i \left[\Delta_c - U_0 \left(J_0 \hat{N} + J \hat{B} \right) \right] - \kappa \right\} a + \eta$$

$$\hat{N} = \sum_k \hat{n_k} = \sum_k b_k^\dagger b_k$$

$$\hat{B} = \sum_k \left(b_{k+1}^\dagger b_k + h.c. \right)$$

atom number in cavity

local atom-atom coherence

field amplitude depends of quantum statistics and gets entangled with atomic distribution

long range interaction effects of a quantum potential:

bad cavity limit :

$$a_0^\dagger a_0 = \frac{|\eta_0|^2}{(\Delta_p - \delta_0 \hat{D}_{00})^2 + \kappa^2}$$

$$H = \left[E + J\left(V_{cl} - \hbar U_0 \eta^2 \frac{\kappa^2 - {\Delta'_c}^2}{\left(\kappa^2 + {\Delta'_c}^2\right)^2}\right)\right]\hat{B} \qquad (13)$$

$$+ \; 3\hbar U_0^2 \eta^2 \Delta'_c \frac{3\kappa^2 - {\Delta'_c}^2}{\left(\kappa^2 + {\Delta'_c}^2\right)^4} J^2 \hat{B}^2 + \frac{U}{2}\sum_k \hat{n}_k\left(\hat{n}_k - 1\right)$$

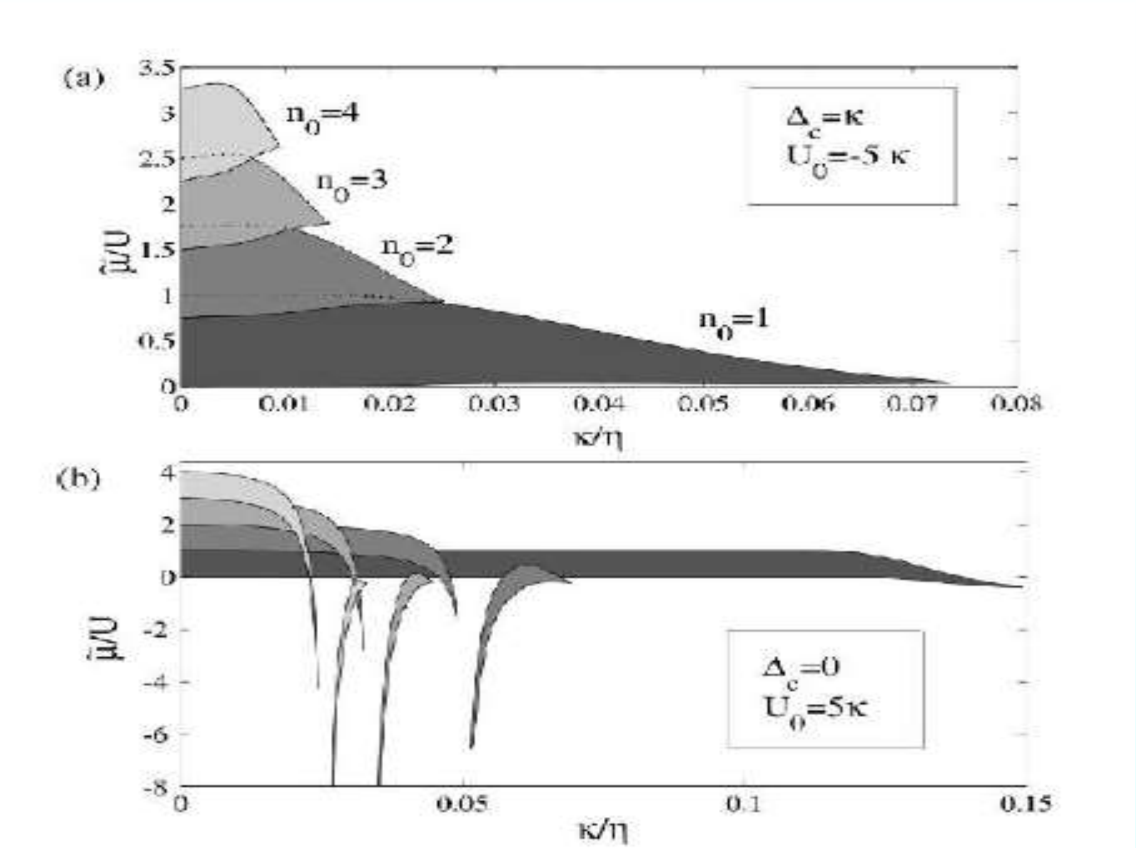
rescaled hopping terms
(sign change possible)

Nonlocal atom-atom interaction
via nonlocal correlated hopping

Cavity parameters can be used to effectively tune size and type of interactions !

phases of cavity generated lattices in T = 0 limit

Cavity creates extra effective long range attraction or repulsion

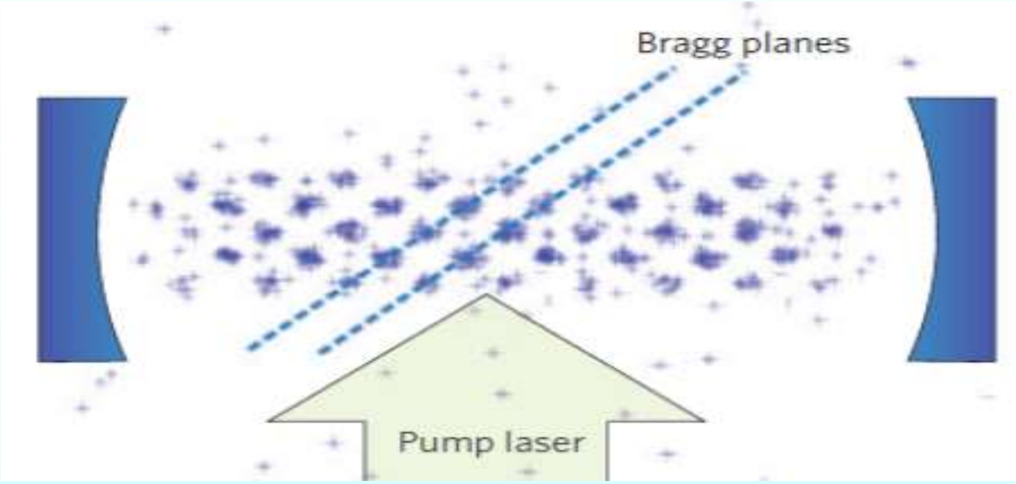


=> **parameter regions with two stable phases**

=> phase „superpositions“ of Mott insulator + superfluid ?

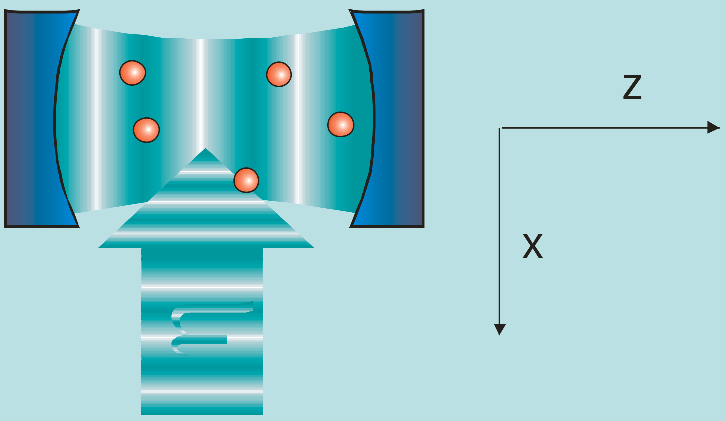
Bosons in thermodynamic limit: *M. Lewenstein, G. Morigi et. al. (PRL 2007,2008)*

generalization to fermions: *Morigi PRA 2008, Piazza,* Part II : transverse illumination



Crystallisation of atoms via collective light scattering

***New geometry:* illumination of atoms from side**



phase of scattered light depends on position x,z

$$\dot{\sigma}_i = (i\Delta_A - \gamma)\sigma_i - g(z_i)a + \boldsymbol{\eta}_x + \xi_A$$

$$\dot{a} = (i\Delta_C - \kappa)a + \sum_{i=1}^{N} g^*(z_i)\sigma_i + \xi_i$$

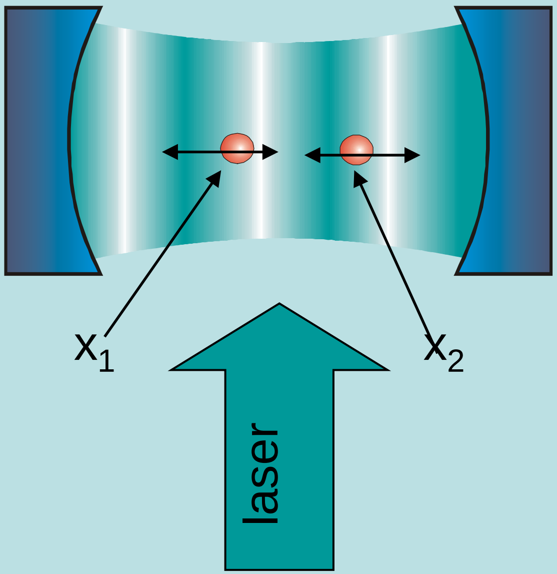
collective pump strength ***R***

Cavity field generated by collective scattering by atoms:

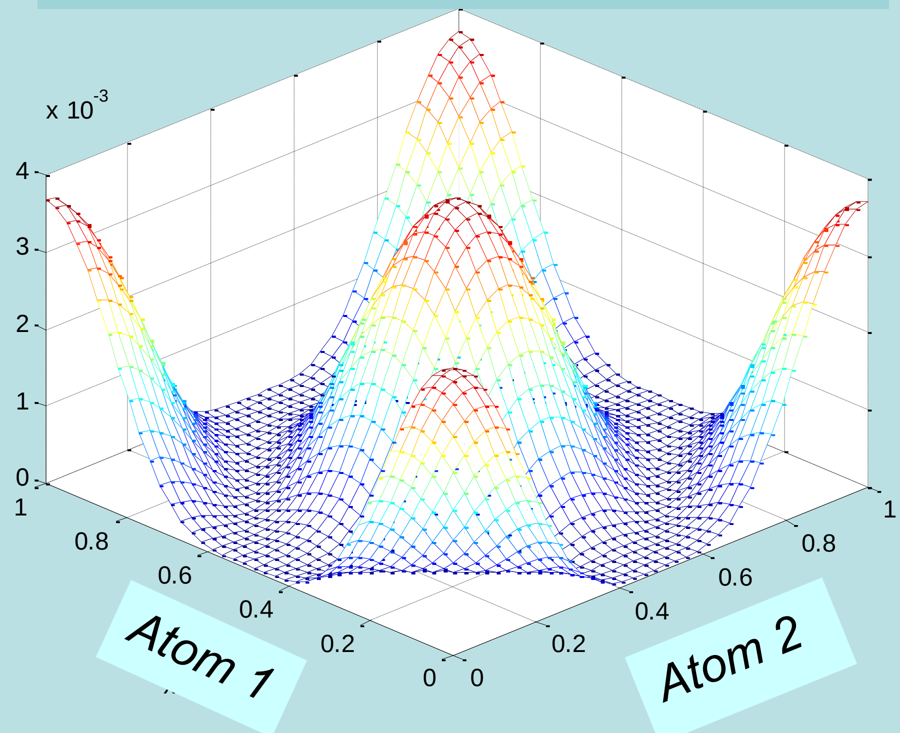
R = 0 *for random atomic distribution*
R ~ Ng *for regular lattice (superradiance = n ~ N^2)*

Two scatterers placed along cavity axis

(talk by Dan Stamper-Kurn)



Cavity field as a function of atom position

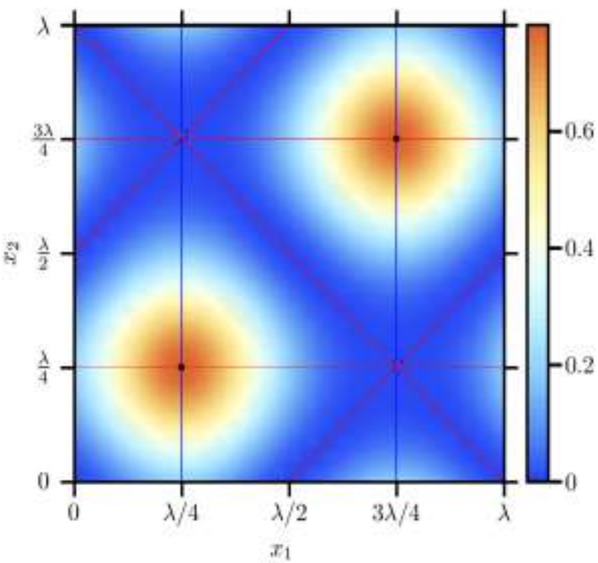


Maximum photon number for 0 and λ distance
Minimum photon number for $\lambda/2$ distance

$\Rightarrow$ ***λ distance gives maximal field***
$\Rightarrow$ ***for high field seekers***
ordering is energetically favorable

optical potential and force field on high field seeking atoms:

Two atoms - Stable configurations



Single frequency: $n = 1$

Black dots: Stable points

Density plot: $\sum_n |\alpha_n|^2$

Contours: $F_i = 0$

Z2 symmetry !

Biophysics: Hopfield

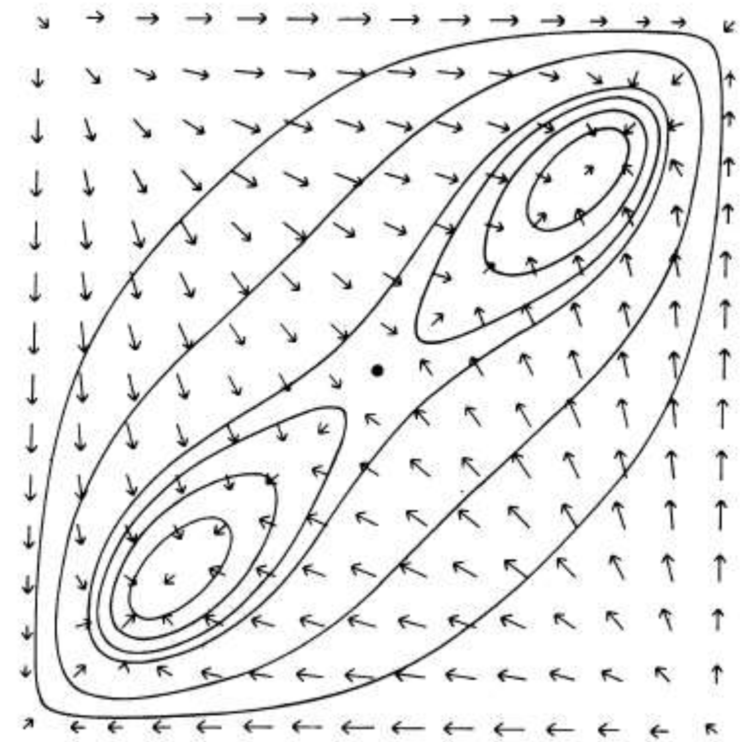
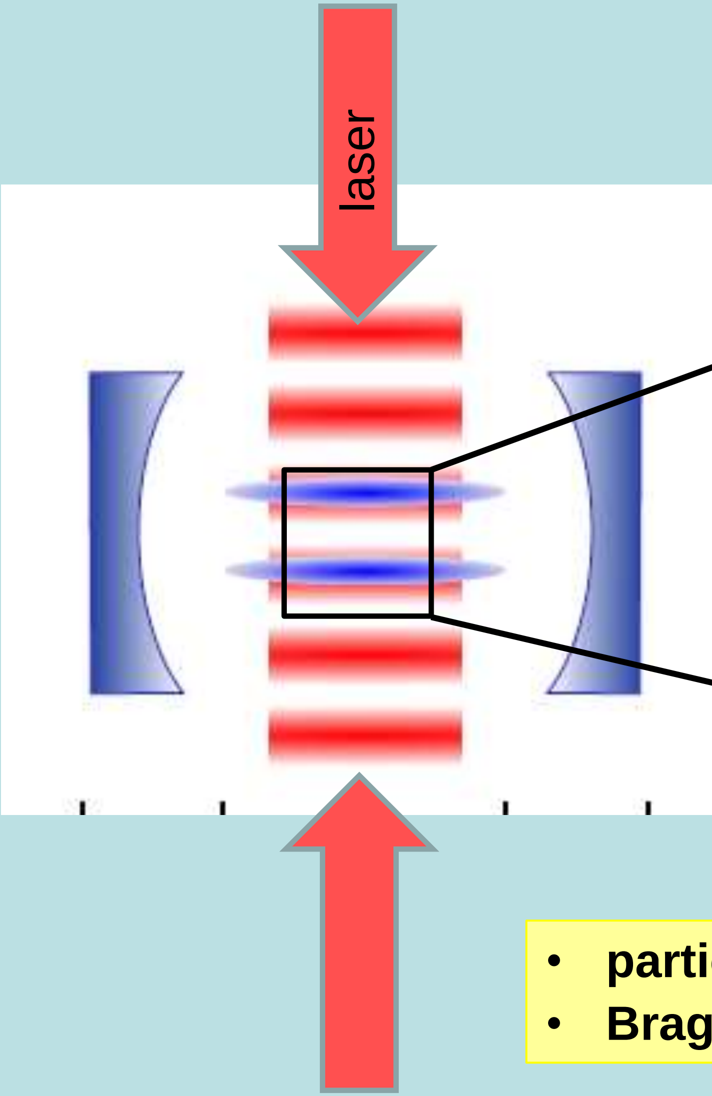


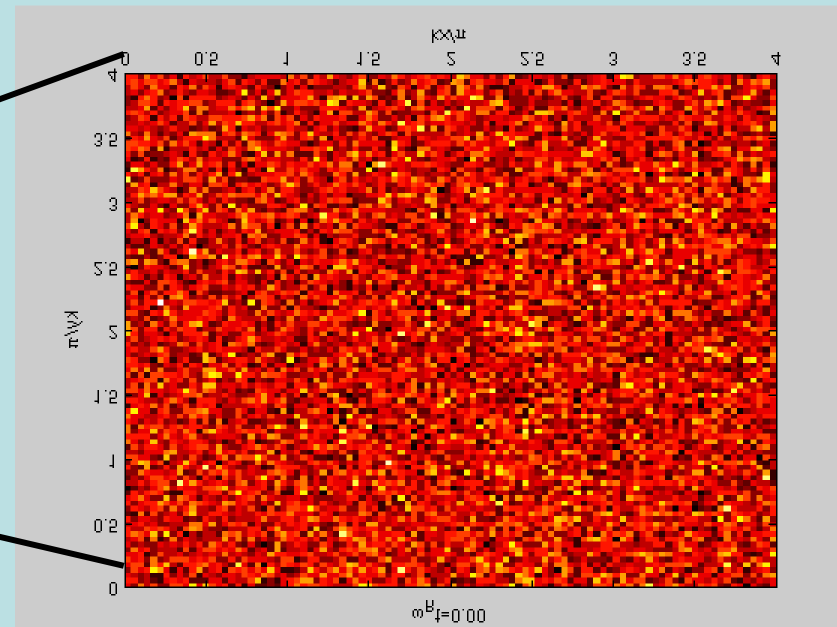
FIG. 3. An energy contour map for a two-neuron, two-stable-state system. The ordinate and abscissa are the outputs of the two neurons. Stable states are located near the lower left and upper right corners, and unstable extrema at the other two corners. The arrows show the motion of the state from Eq. 5. This motion is not in general perpendicular to the energy contours. The system parameters are $T_{12} = T_{21} = 1$, $\lambda = 1.4$, and $g(u) = (2/\pi)\tan^{-1}(\pi\lambda u/2)$. Energy contours are 0.449, 0.156, 0.017, −0.003, −0.023, and −0.041.

picture taken from “old” Hopfield paper

coupled atom field dynamics for high field seekers



particle motion
starting from
random distribution

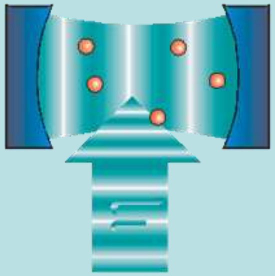


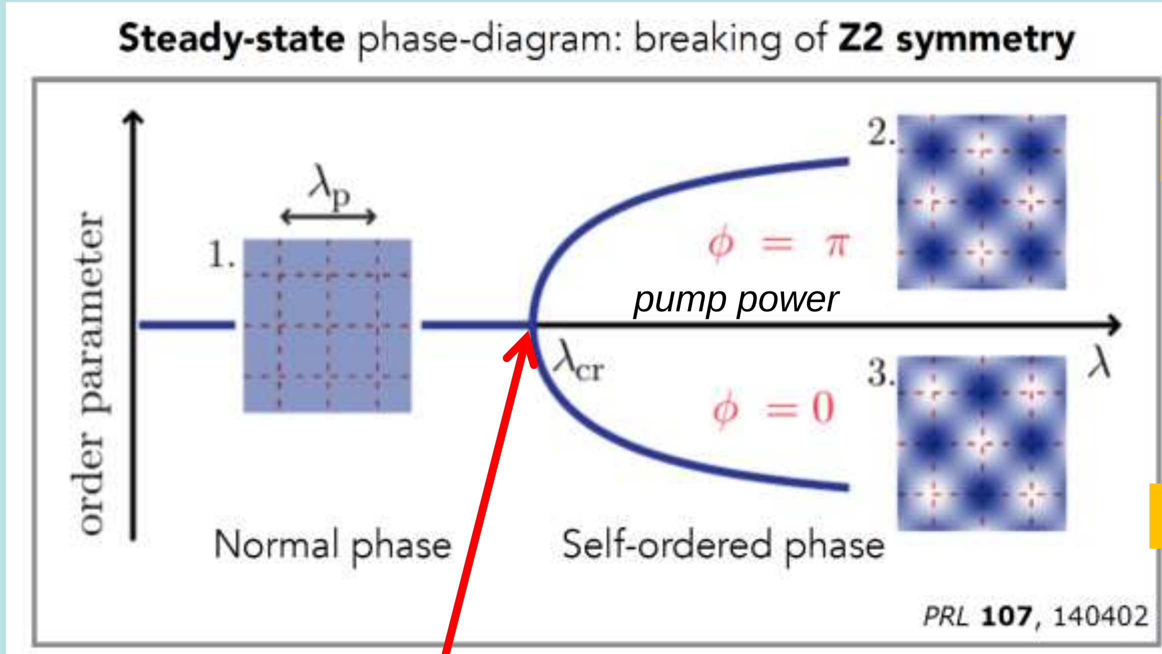
- **particles spontaneously form crystalline order**
- **Bragg scattering creates N^2 `superradiance'**

Atoms minimize their potential energy by creating maximal field

P Domokos, H R., PRL 89 , 253003, (2002)

Selfconsistent atom-field distribution





odd sites

even sites

critical point: Z2 – symmetry breaking

$$U_0 N V_{opt} > \kappa \, k_B T$$

frequency shift of cavity

pump laser opt. potential

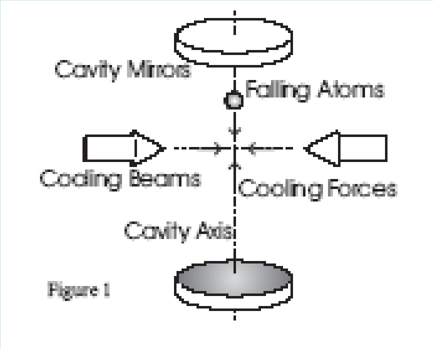
cavity damping

temperature

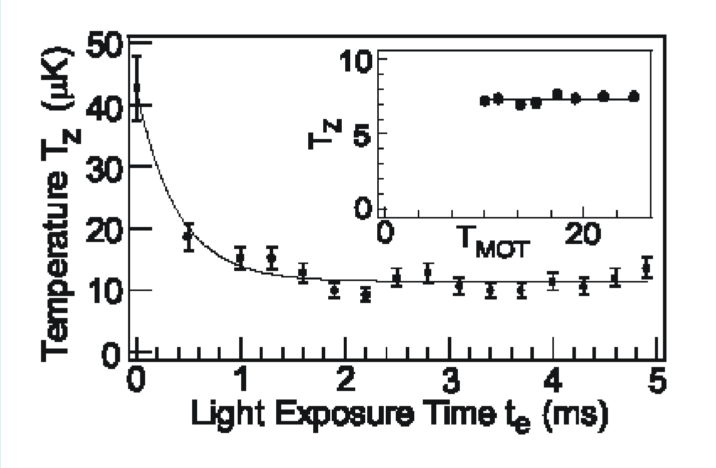
Selforganisation is an open system phase transition

experiments at finite T

Vuletic group : Stanford /MIT (talk to James)



10^6 Caesium atoms in resonator with transverse coherent pump field



- *N >10^6 Atoms trapped and cooled to*
- *T ~ mK with coherent light emission*

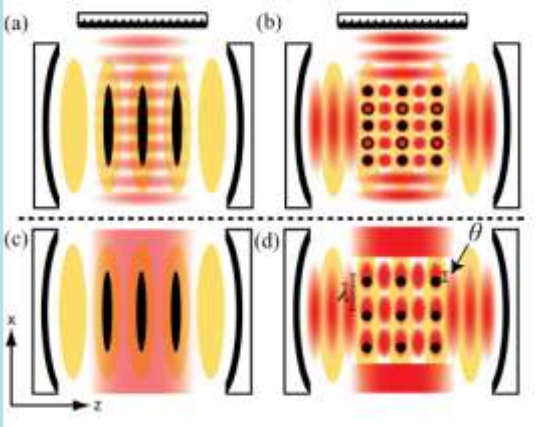
** Experiment works better than predictions*
** accelerations of >10^6 g at low saturation*

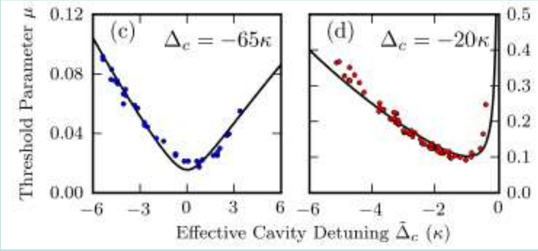
Barret-group Signapore

Self-Organization Threshold Scaling For Thermal Atoms Coupled to a Cavity

K. J. Arnold, M. P. Baden, M. D. Barrett
Centre for Quantum Technologies and Department of Physics, National University of Singapore, 3 Science Drive 2, 117543 Singapore

(Dated: November 6, 2012)





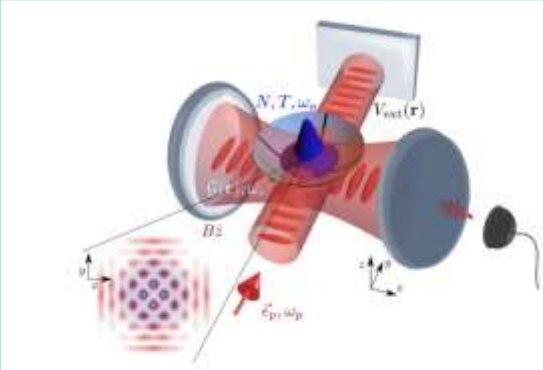
threshhold law verified – very little cooling

recent new tweezer experiments:
Vuletic, Stamper-Kurn, …

Selforganisation is an open system phase transition

Is it a quantum phase transition at T = 0 ?

experiment at ETH: selforganization of a BEC at T ~ 0

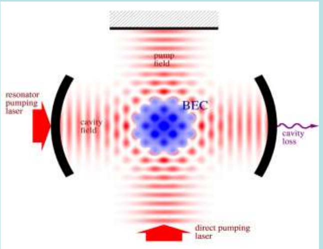


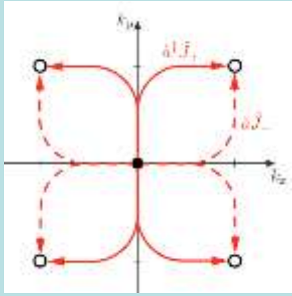
$$H = -\Delta_C\, a^\dagger a + \int_0^L \Psi^\dagger(x)\Big[-\frac{\hbar}{2\,m}\frac{d^2}{dx^2} + U_0\, a^\dagger a \cos^2(kx) + i\eta_t \cos kx(a^\dagger - a)\Big]\Psi(x)dx,$$

Effective Tavis-Cummings Hamiltonian

Two-mode BEC approximation
=> Tavis-Cummings model

$$H = -\delta_C\, a^\dagger a + \omega_R \hat{S}_z + iy(a^\dagger - a)\hat{S}_x/\sqrt{N} + u a^\dagger a\left(\tfrac{1}{2} + \hat{S}_z/N\right)$$





$$\Psi(x) = \frac{1}{\sqrt{L}}c_0 + \sqrt{\frac{2}{L}}c_1 \cos kx$$

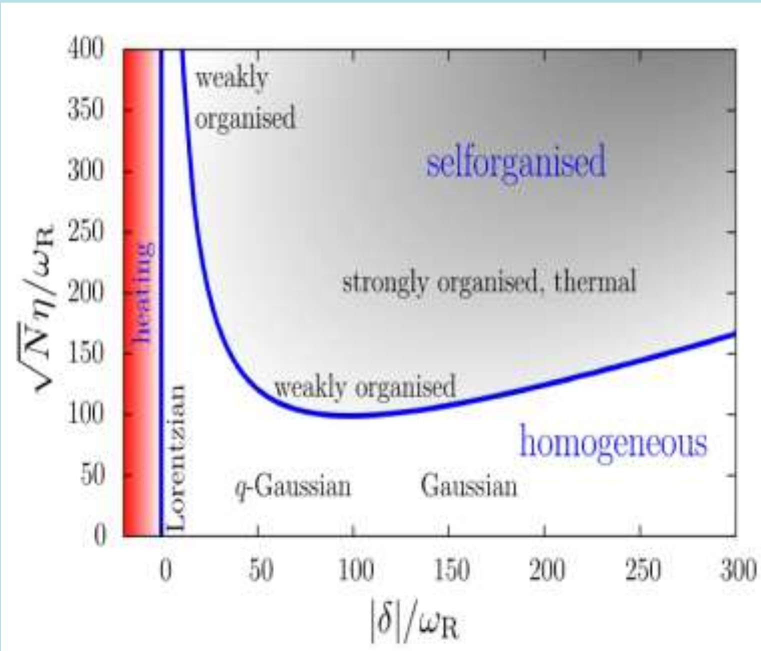
large N => Quantum Simulation
of
„*Dicke Superradiant Phase Transition*"
(predicted by Hepp+Lieb 1973)

Domokos: Phys. Rev. Lett. 104, 130401

compare classical vs quantum phase diagram

classical gas T>>0

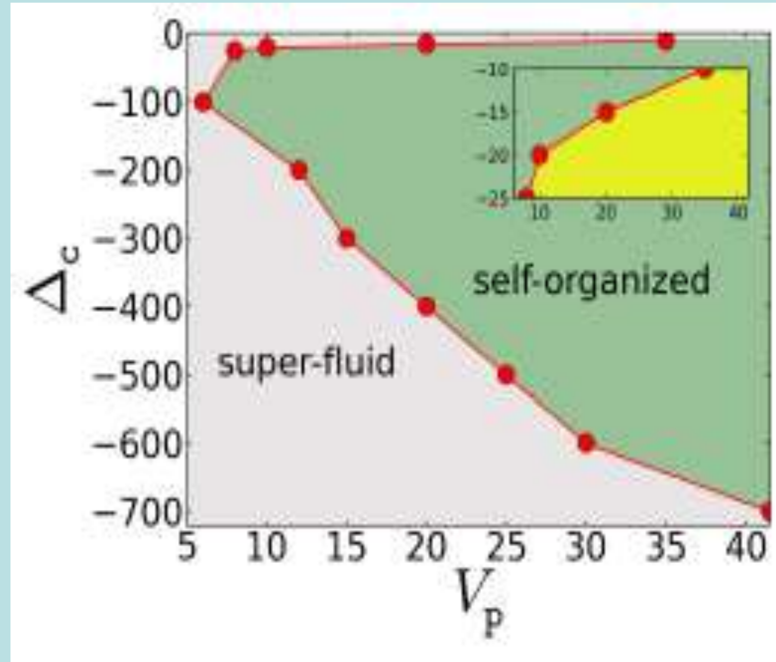
Power



detuning

quantum gas T=0

detuning

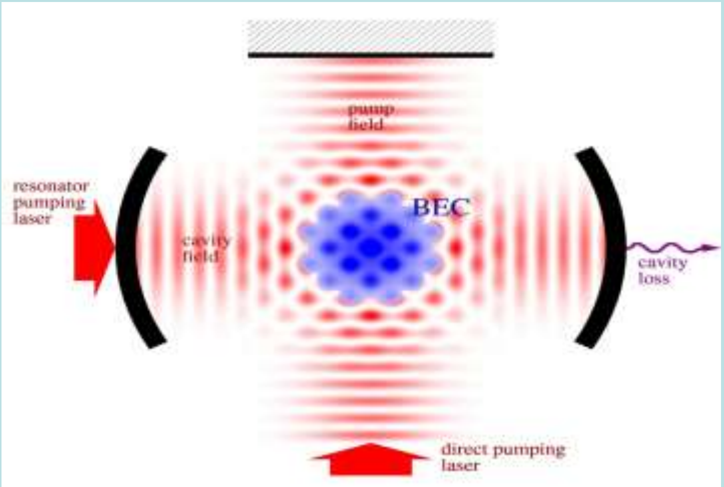


Power

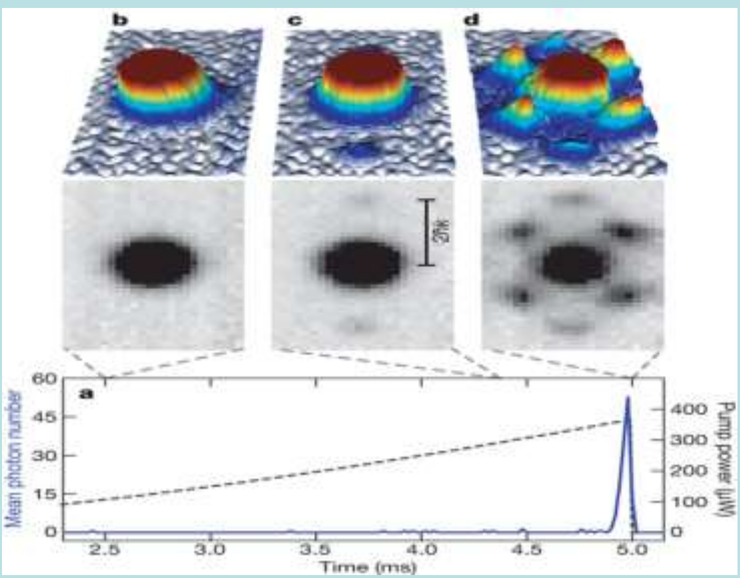
G. Morigi, H.R. 2004
W. Niedenzu, H.R., 2011

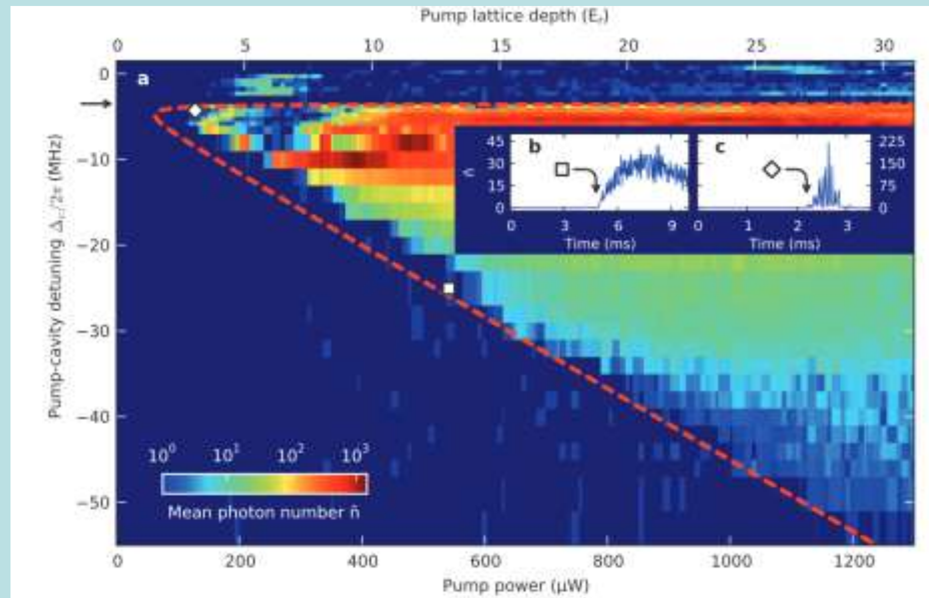
Domokos, Esslinger, Donner 2010
Hofstetter 2013, Thorwart 2014
Piazza, Zwerger, 2013 (fermions)

Experiment ETH 2010: Baumann et. al. (Esslinger)



Experiment follows prediction
of
Dicke Superradiant Phase Transition

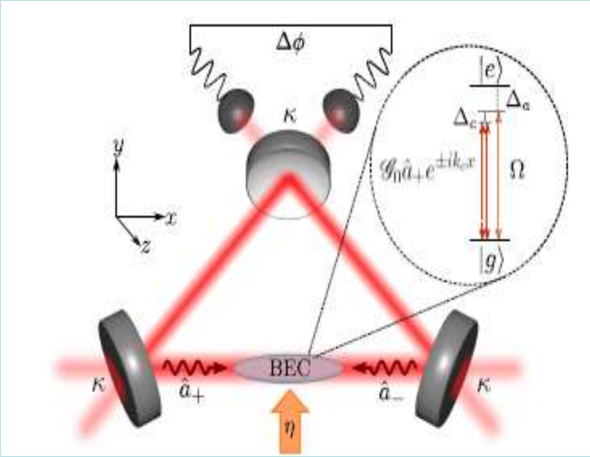


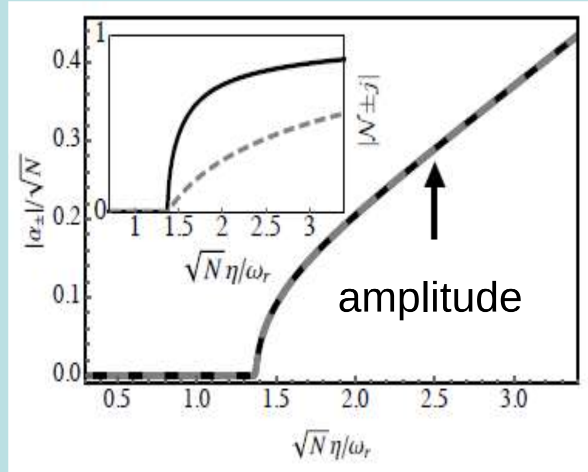


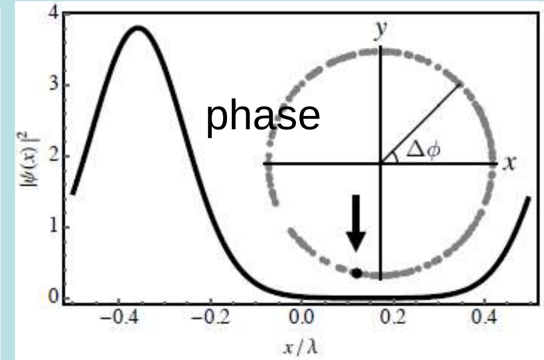
„quantum simulation" of T=0 phase transition
with Z2 symmetry breaking
predicted by Hepp+Lieb 1973

K Baumann *et al. Nature* 464, (2010)

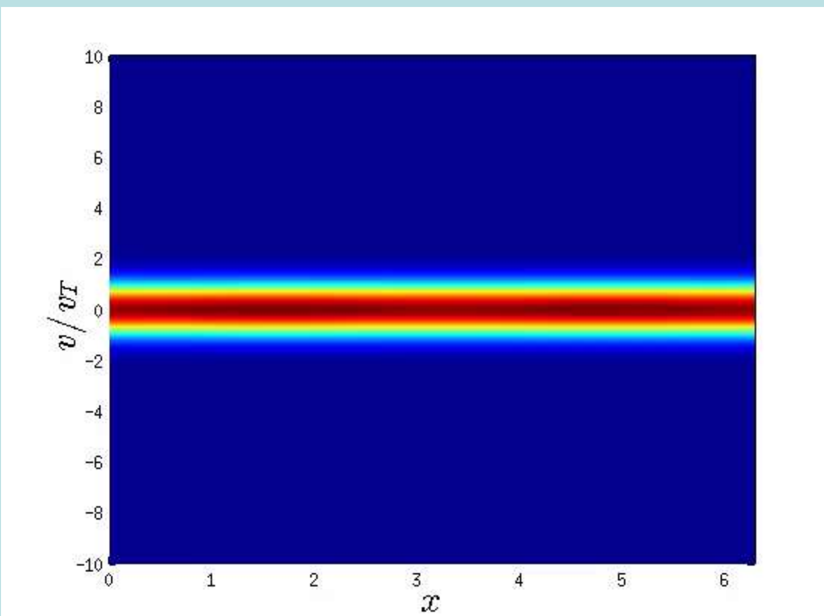
ring cavity with transverse pump: translation inavriance



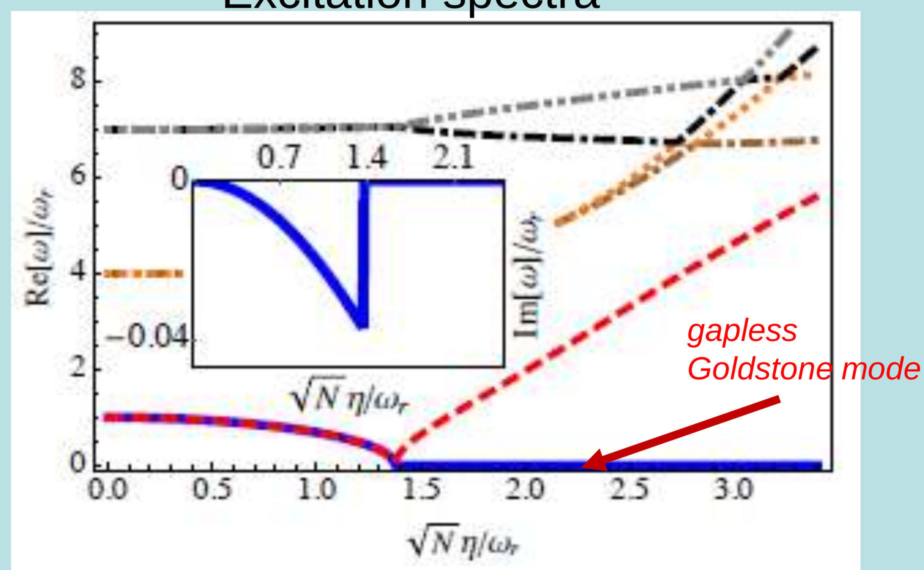




spontaneous breaking of a continous (U_1) symmetry



Excitation spectra

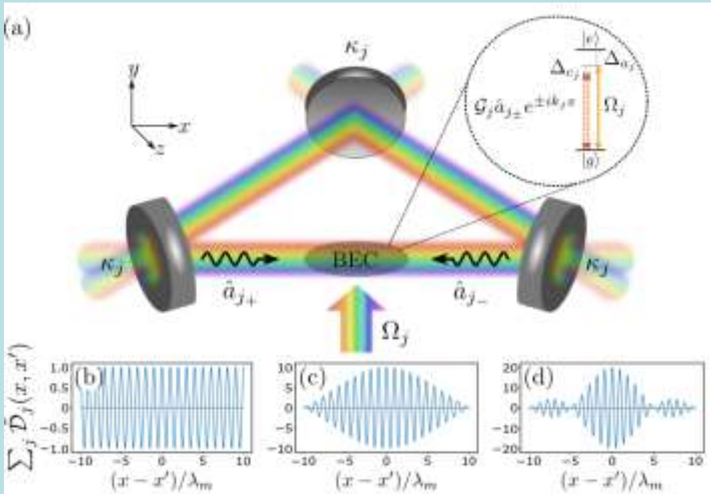


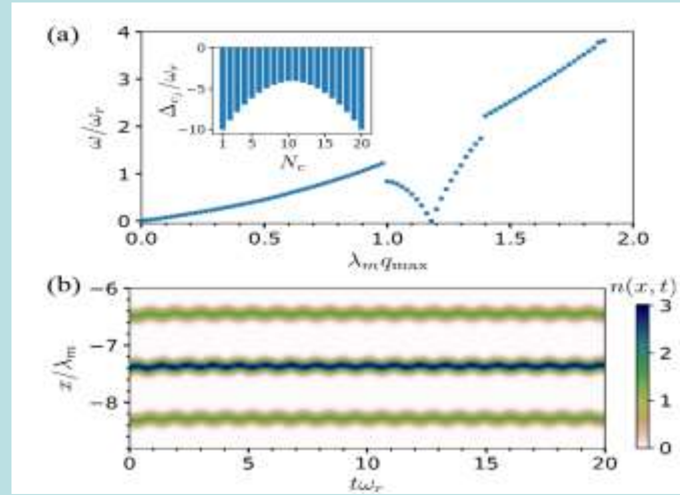
Experiment in Tübingen:
PRL124.14 (2020) -143602

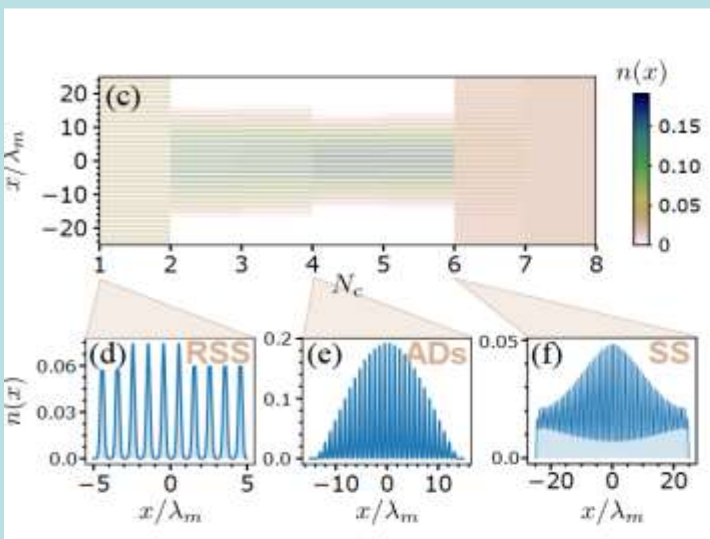
~ frictionless motion in a dissipative cavity

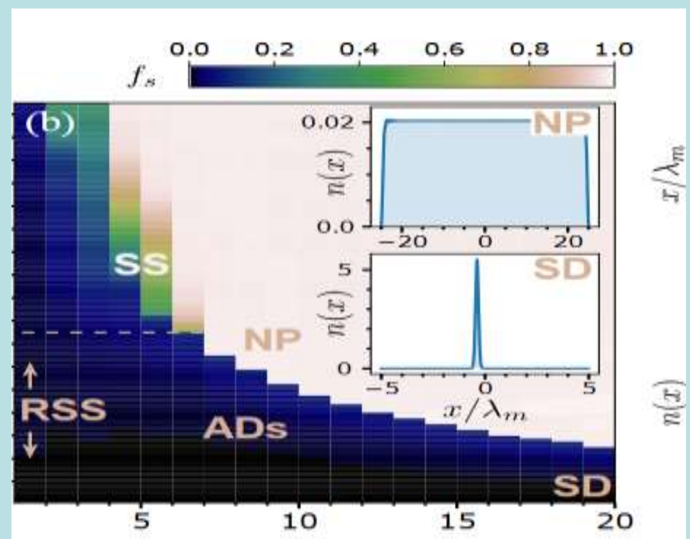
Tuning photon-mediated interactions in a multimode cavity: from supersolid to insulating droplets hosting phononic excitations

Natalia Masalaeva,* Helmut Ritsch, and Farokh Mivehvar





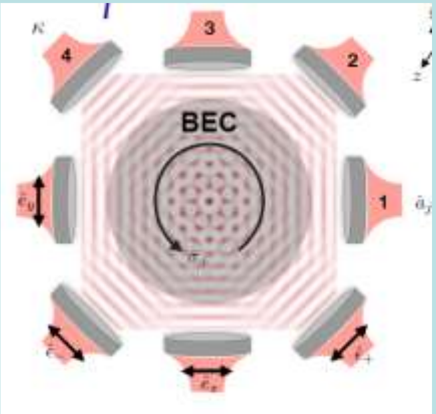


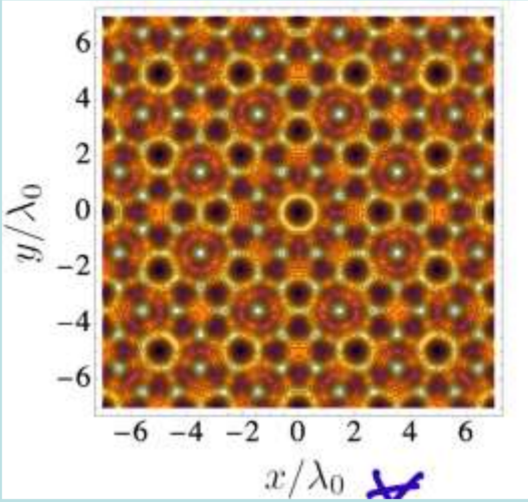


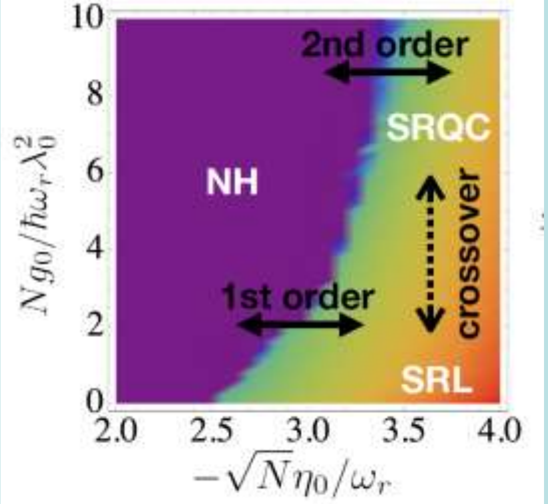
Masalaeva, N., R. H., & Mivehvar, F. (PRL 2023)

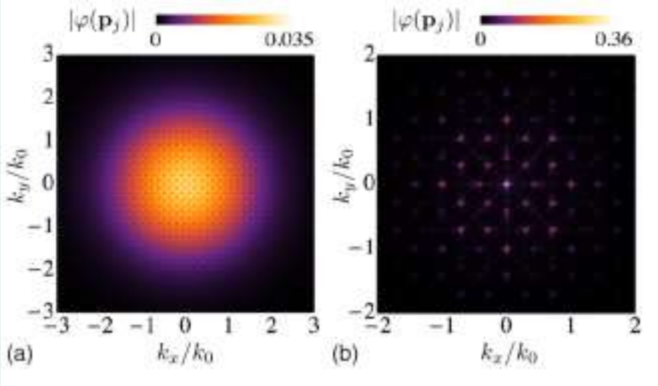
collective scattering in more complex symmetries

4-fold rotation symmetry: self ordering in 4 cavities









Systems above threshold attains C-8 symmetry

=>

quasi-crystal formation

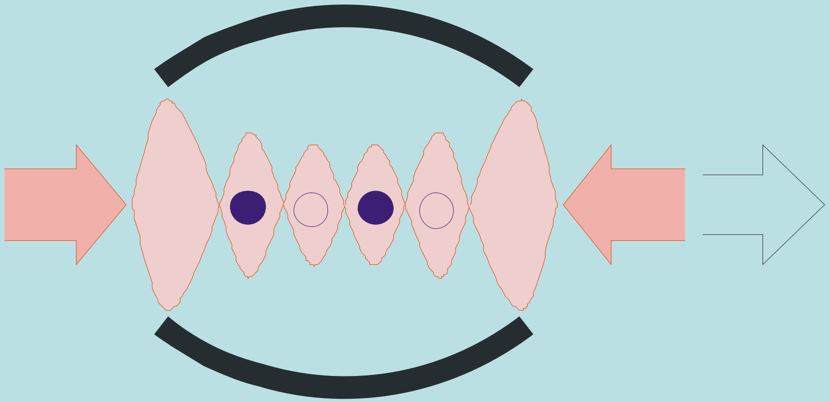
=>

emerging new symmetry

F. Mivehvar, PRL 2019

Quantum-gas CQED => Cavity BH

Beyond mean field:
Quantum description of selforganization of atoms in a lattice

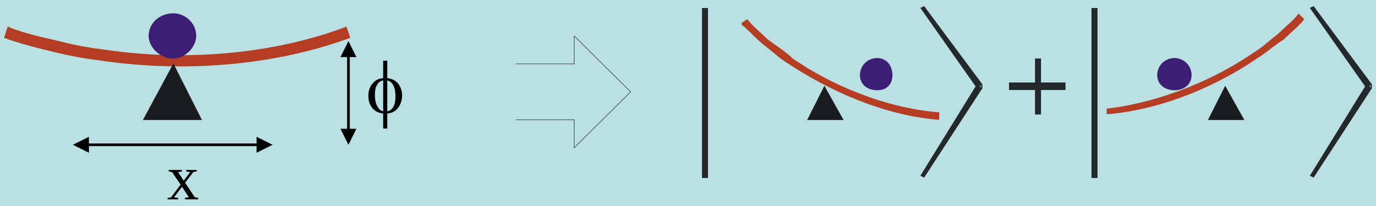


Atoms close to T=0 in standing wave
(e.g. perpendicular to cavity)

Lattice generates order while cavity mediates interactions

How will selforganization happen here ?
(dynamics of analogous to quantum phase transition)

Very simple toy model: “decay of a quantum seesaw “

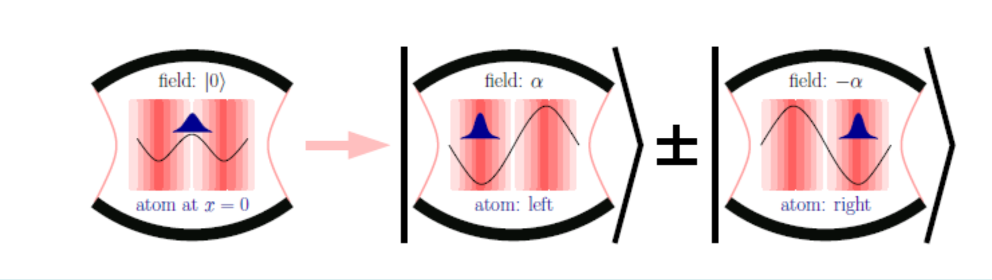


classical

quantum

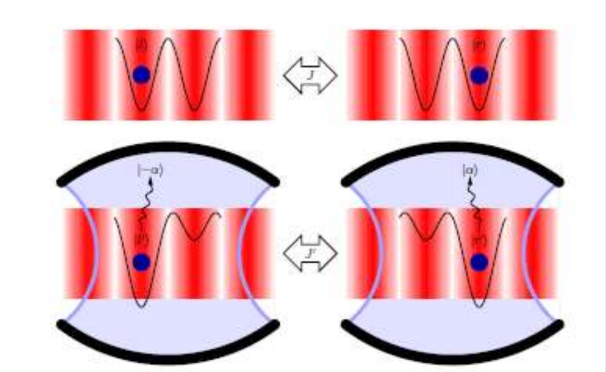
Two degrees of freedom: tilt angle ϕ and particle position x

Note: classical equilibrium point at x=ϕ=0
(Z2 – Symmetry)
product state of oscillator ground states is not stationary



field phase replaces tilt angle <> occupation difference replaces position

Two degenerate states for single atom at two sites ...



Lowest energy eigenstates of double well

$$a = -i \frac{\eta'}{\kappa - i(\Delta_c - U_0)} \tilde{J}_0 \left(b_1^\dagger b_1 - b_2^\dagger b_2 \right)$$

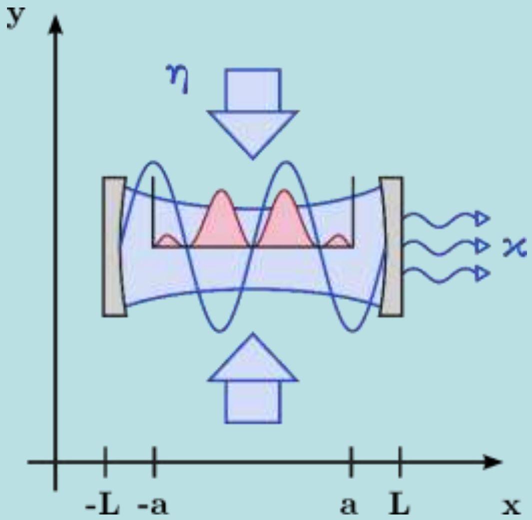
$$a^\dagger a \sim \left(b_1^\dagger b_1 - b_2^\dagger b_2 \right)^2$$

$$\frac{1}{\sqrt{2}} \left(|\text{left}\rangle \, |\alpha\rangle \pm |\text{right}\rangle \, |-\alpha\rangle \right)$$

... show atom field entanglement

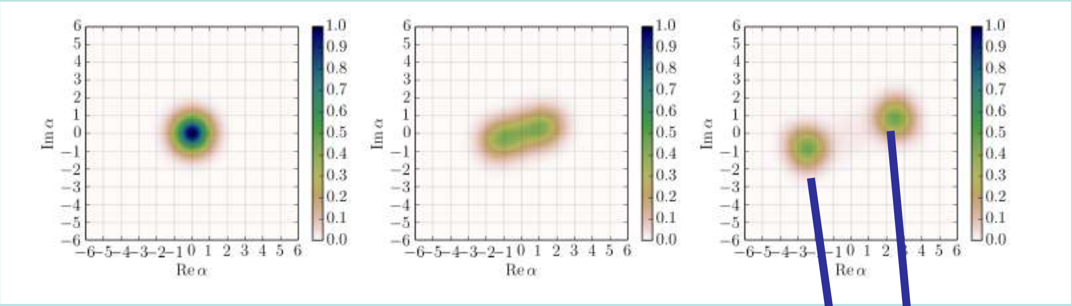
- strongly entangled ground state
- atom tunneling suppressed as it needs field phase flip (stabilization)
- Symmetry leads to zero field amplitude but nonzero intensity (photons)

Selfordering of a trapped **quantum** particle within a cavity: a numerical study



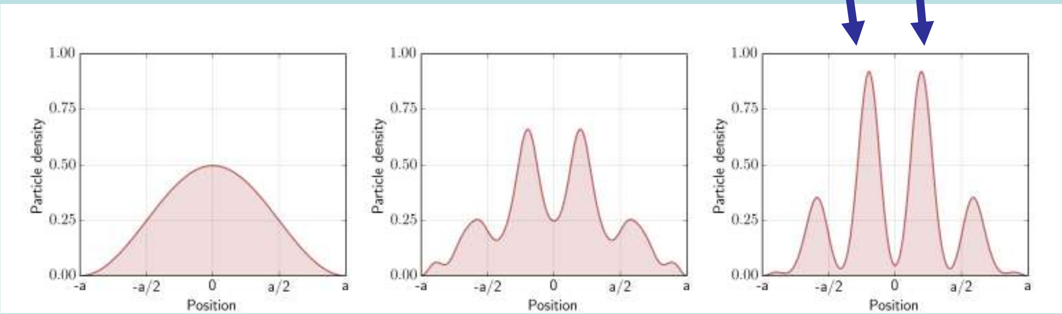
Flat box trap
(Theorist - tweezer)

cavity field Q-function



no pump   weak pump   strong pump

particle density



several particle modes excited bi-modal Q-function of field

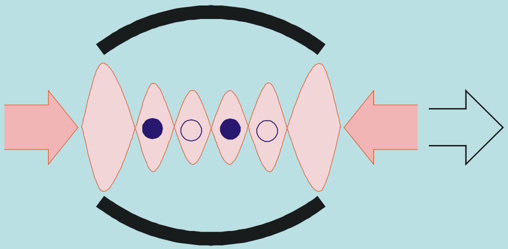
=> entangled ground state:

$$\frac{1}{\sqrt{2}}\left(|\mathrm{left}\rangle\,|\alpha\rangle \pm |\mathrm{right}\rangle\,|-\alpha\rangle\right)$$

Krämer, S., & R. H. , 2014 *PRA*, *90*(3), 033833

Beyond mean field

multiparticle quantum description of selforganization in a lattice

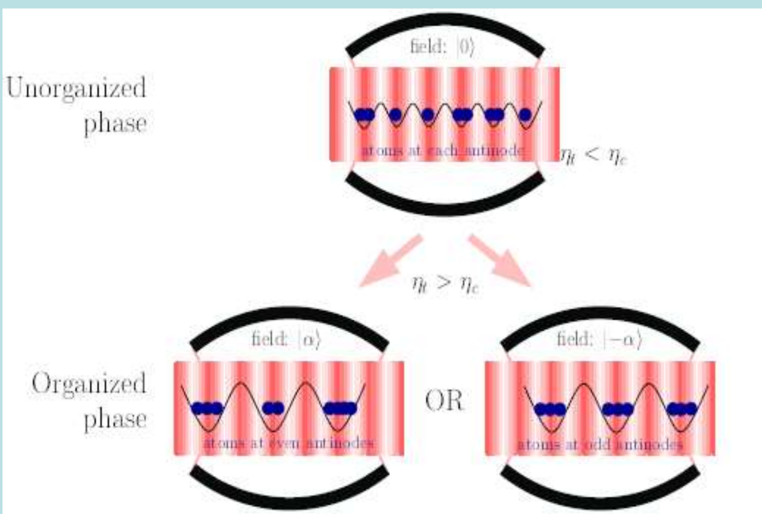


- pump creates optical lattice with
- atoms in lowest band
- cavity field from scattered lattice light

Effective Hamiltonian:

$$H = \sum_{k,l} E_{k,l} b_k^\dagger b_l + \hbar U_0 \eta' g \sum_{k,l} J_{k,l} b_k^\dagger b_l + \hbar \eta' \left(a + a^\dagger\right) \sum_{k,l} \tilde{J}_{k,l} b_k^\dagger b_l - \hbar \left(\Delta_c - U_0\right) a^\dagger a$$

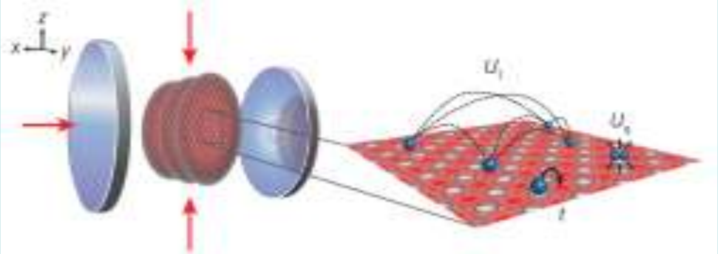
pump amplitude determined by atomic distribution operator



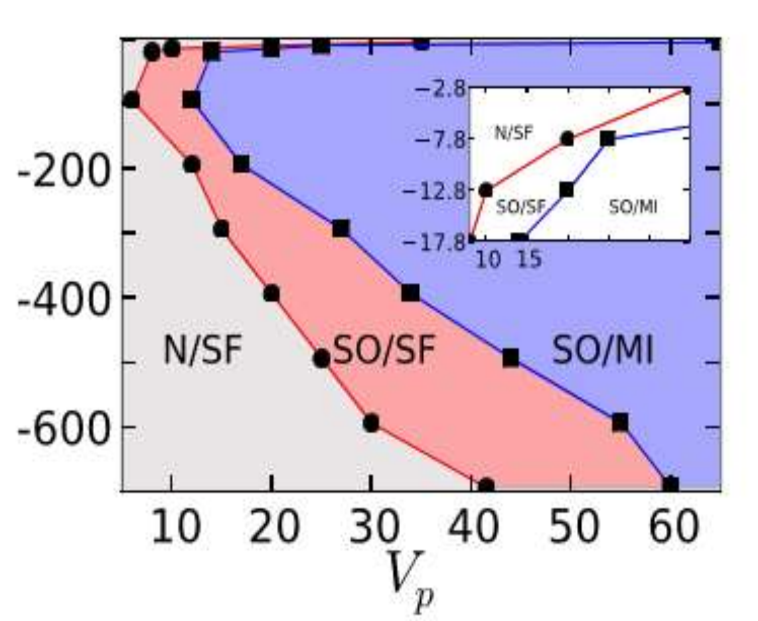
Mekhov, I.B. & H.R.,
JOPB: AMO 45.10 (2012), 102001

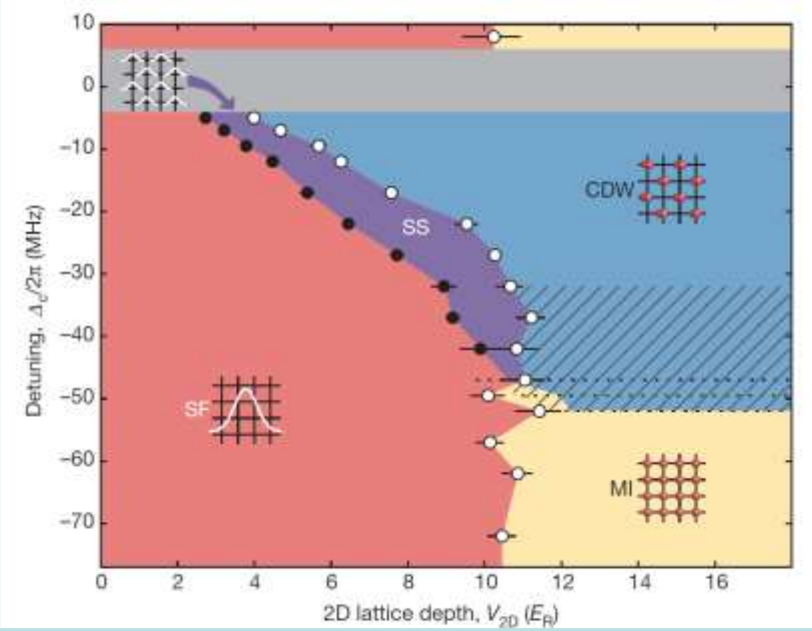
quantum regime (BH): self ordering phase transition in optical lattice

Theory



Experiments: ETH (Hamburg, Stanford)





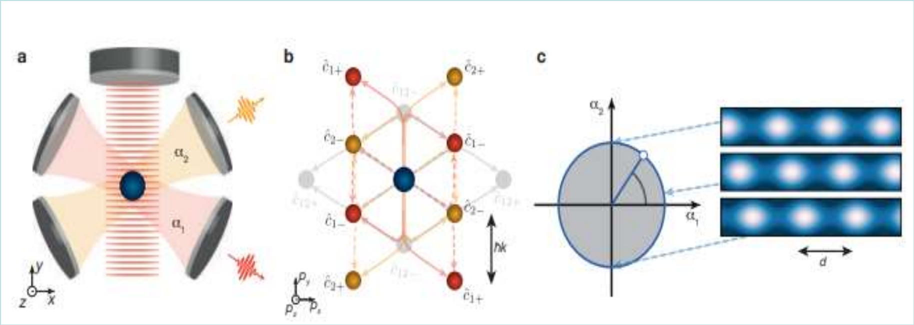
intermediate phase with coherence + diagonal order => „**supersolid**“

W. Hofstetter (2010), F. Piazza ,
R. Bakhtiari, M. Thorwart, HR (2014)
G. Morigi 2016, A. Lode 2017,
C. Kollath 2019, 2020

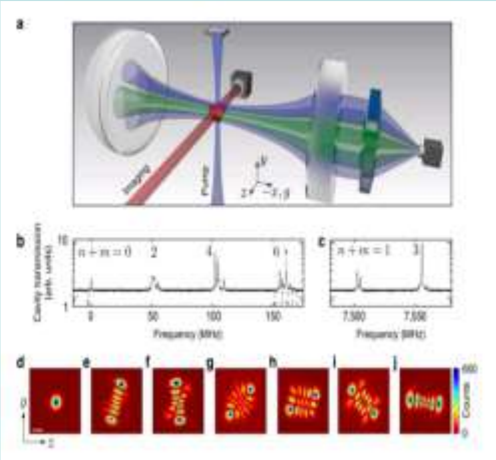
ETH: T. Donner, Nature 533, 2016
Hamburg: A. Hemmerich, PRL 2016
Stanford: B. Lev, PRL 2016
EPFL: J.P. Brantut, Nature 2021
Tübingen: Zimmermann, PRL 2021

crytallization in complex symmetries: experiments

Experiment ETH : Nature 2017 : two modes

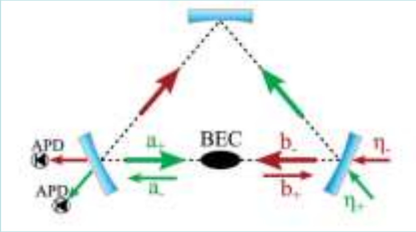


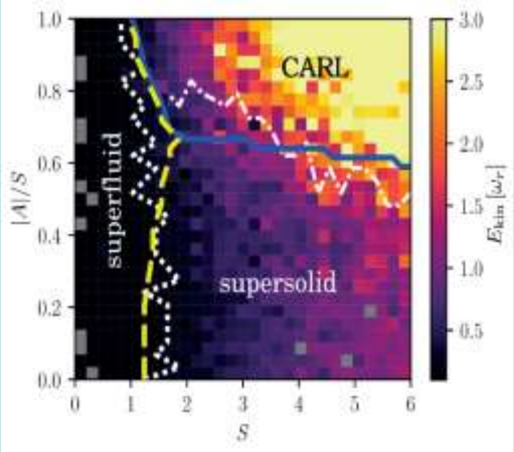
full rotation symmetry: transverse modes in
~ degenerate cavity



B. Lev – group, PRL
(talk to Alicia)

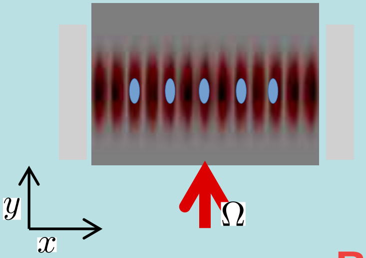
Experiment in Tübingen: PRL124.14 (2020) -143602: Ring





systems above threshold break cavity symmetry choosing higher order modes

Effects of quantum statistics on collective scattering, self-ordering

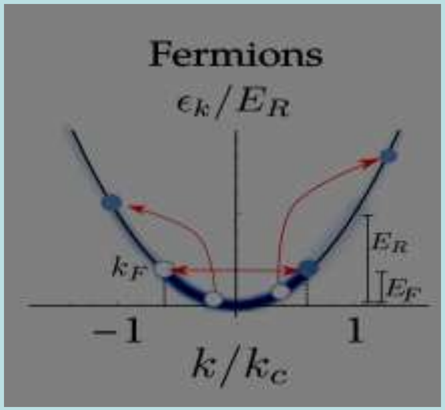


Effects of quantum statistics

(Piazza, Mivehvar, Strack …)

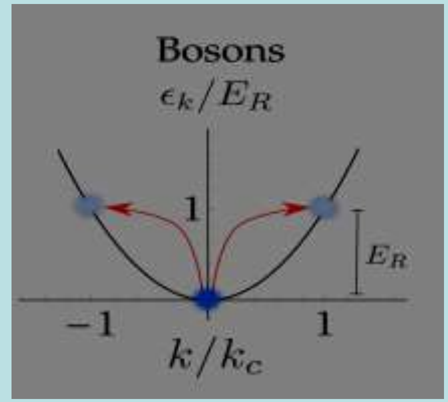
Superradiant phase transition

Bragg scattering

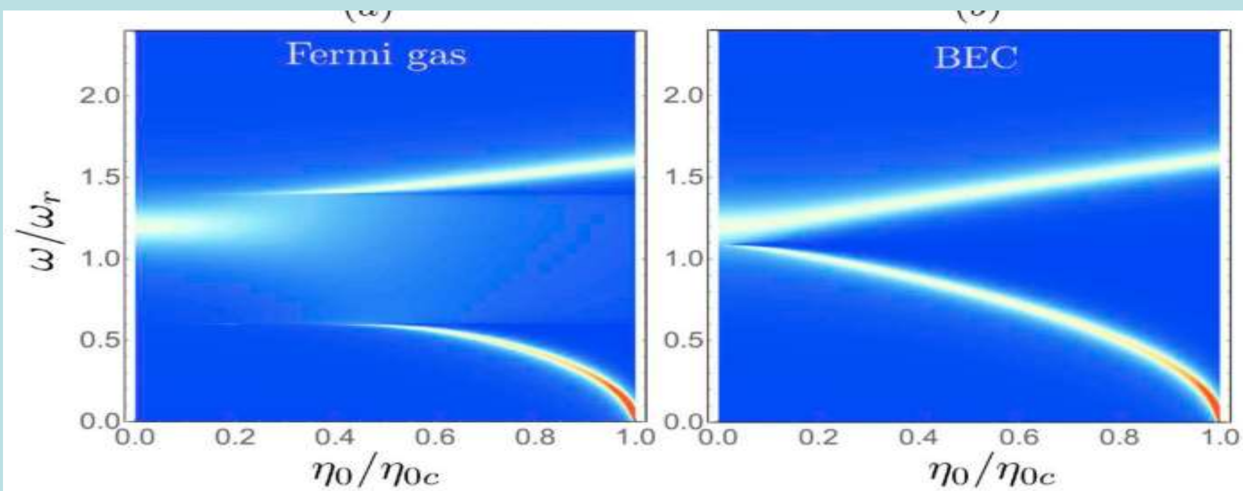


≠

Bragg scattering

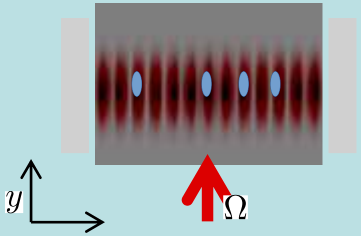


weak excitation probing



Effects of quantum statistics

superradiant phase transition



Fermions

Bosons

$k_F > k_c$

$k_F \sim k_c$

$k_F = k_c/2$

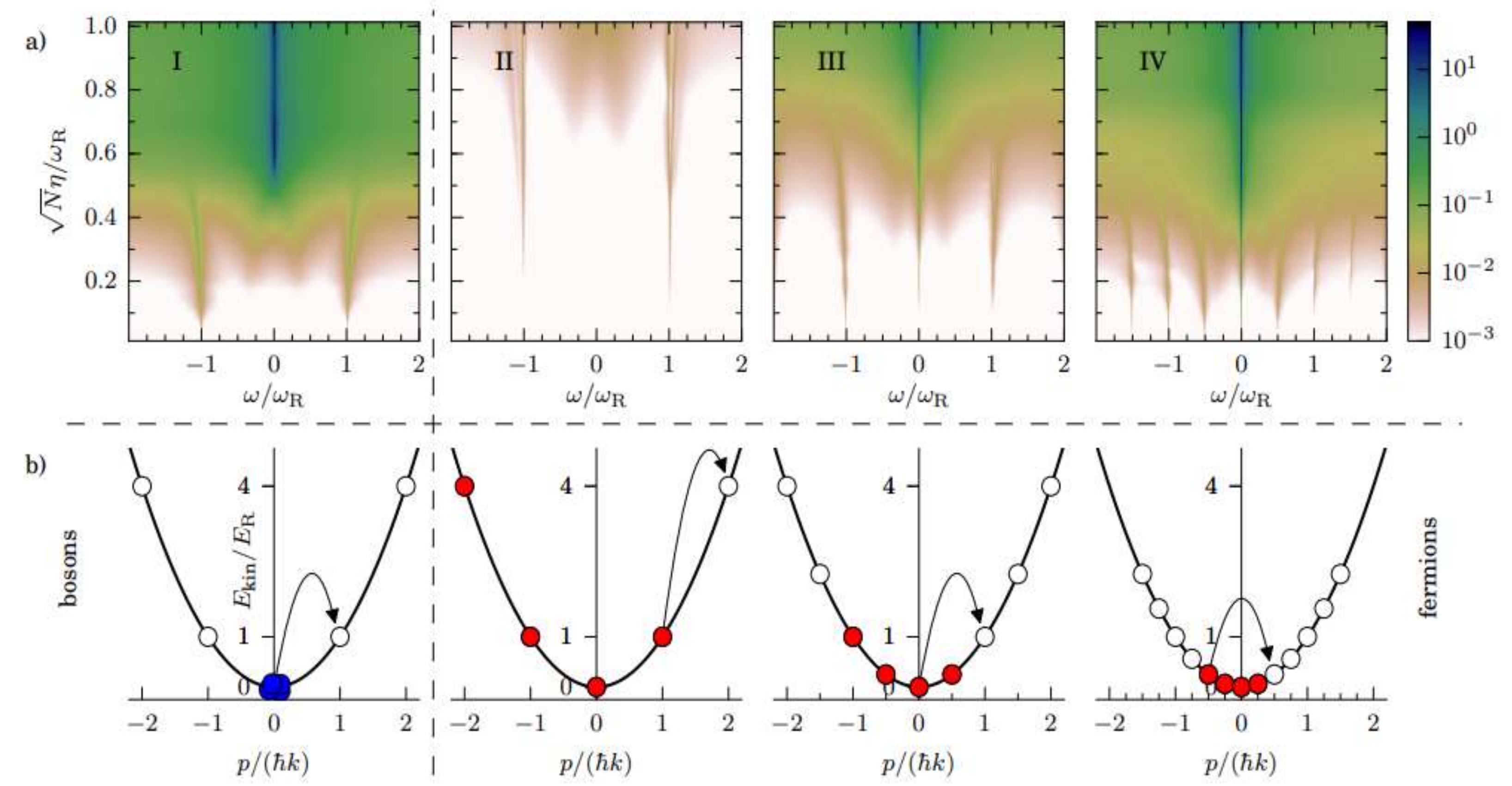
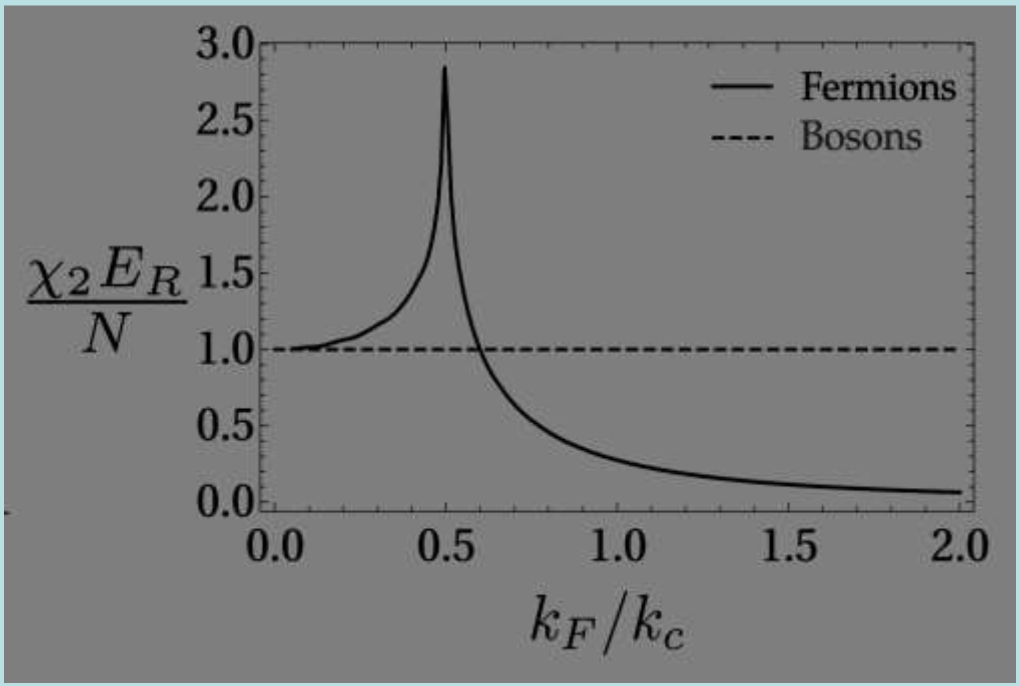


Figure 5. a) Cavity field spectrum and b) initial conditions with lowest-energy particle excitations for bosons (I) and fermions

Fermionic self-ordering

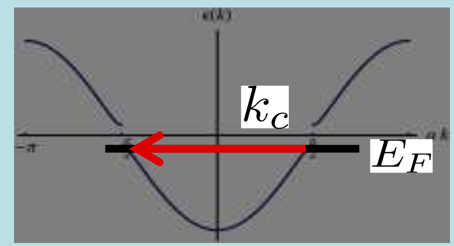
Critical Threshold $\eta_c = \sqrt{\frac{\Delta_c^2 + \kappa^2}{2\Delta_c \chi_2}}$

susceptibility χ



Fermi gases
Divergence at the **nesting** condition

$$k_c = 2k_F$$



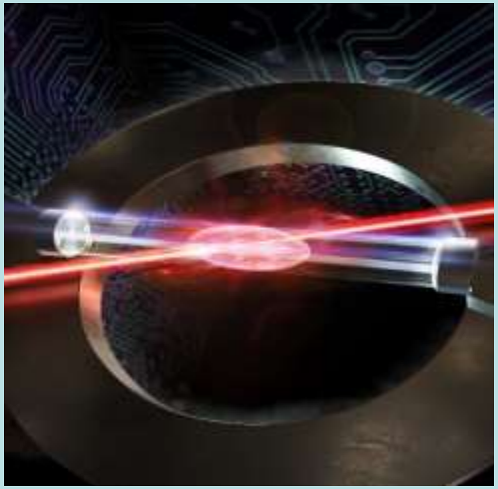
$$\Delta E = 0$$

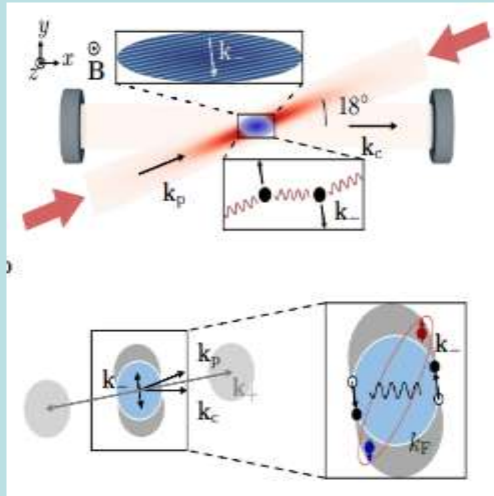
The scattering process has no energy cost!

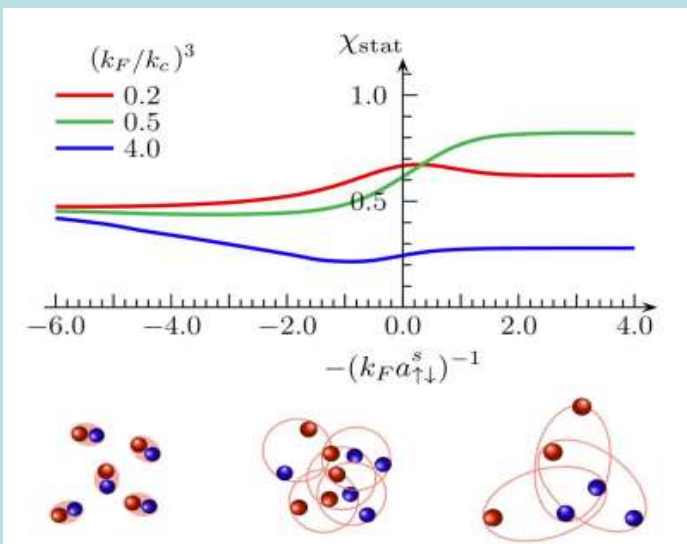
Chen et al, Phys. Rev. Lett. 112 (2014)
Keeling, J., M., et al, Phys. Rev. Lett. 112 (2014)
Piazza, F., et al., Phys. Rev. Lett. 112 (2014)

„Umklapp“-Superradiance !

EPFL 2023 : Density wave ordering of interacting fermions (3D)







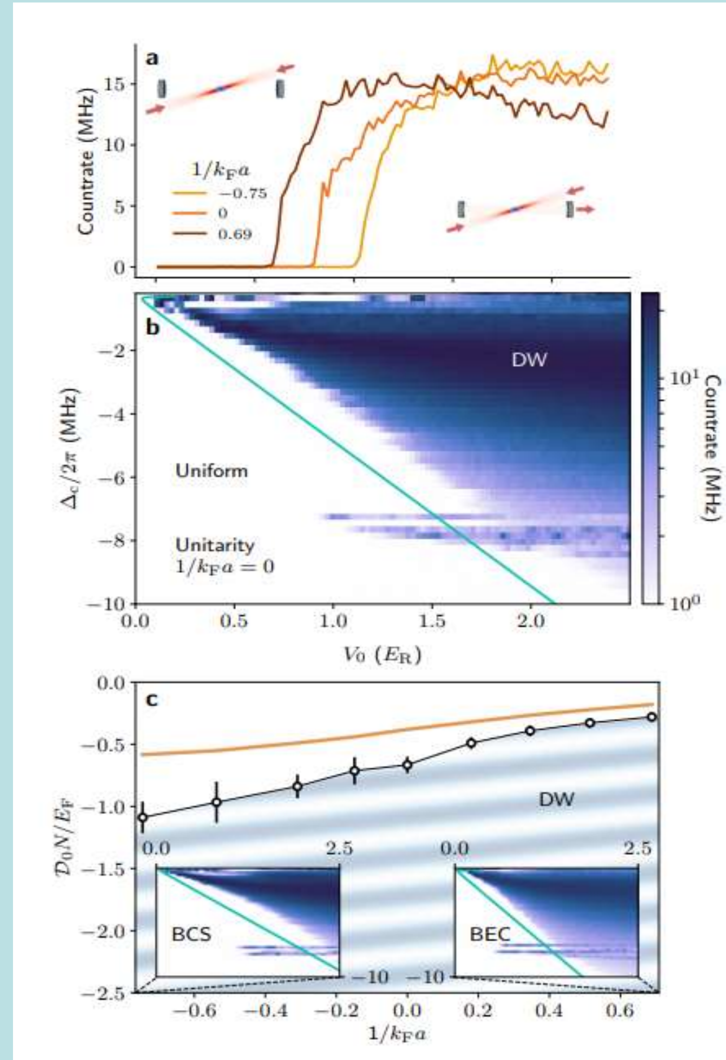


FIG. 2. Phase diagrams of the system. **a**, Photon traces recorded at fixed $\Delta_c = -2\pi \times 2$ MHz as a function of the linearly increasing pump strength V_0, for different values of

Helson et. al., EPFL (Brantut), Nature -2023

Superradiant Topological Peierls Insulator inside an Optical Cavity

Farokh Mivehvar, Helmut Ritsch, and Francesco Piazza*
Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria
(Received 17 November 2016; published 16 February 2017)

repulsive field = blue detuning

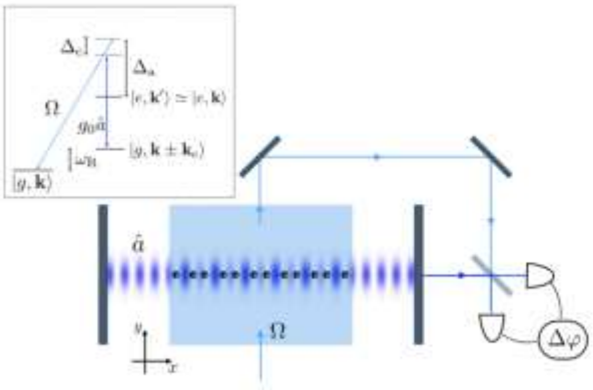


FIG. 1. Schematic view of fermionic atoms trapped in a one-dimensional elongated tube along the axis of an optical resonator

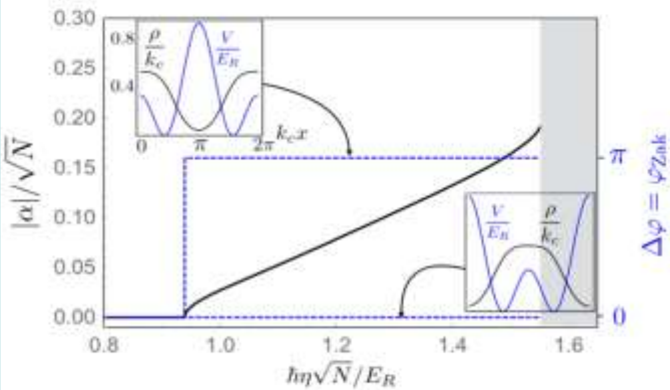


FIG. 2. Spontaneous $\mathbf{Z}_2$-symmetry breaking at the superradiant self-ordering transition in the configuration of Fig. 1 seen via the

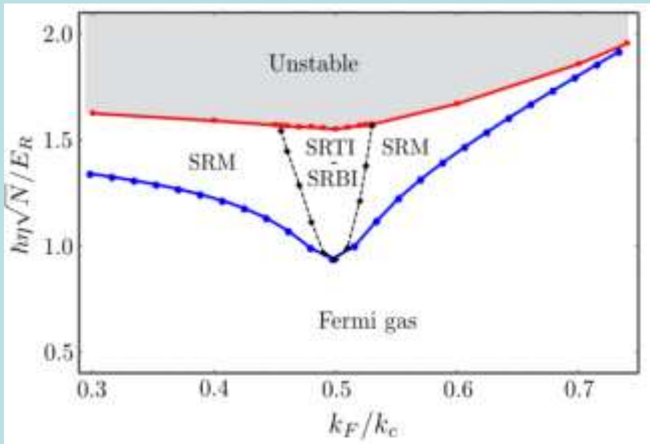
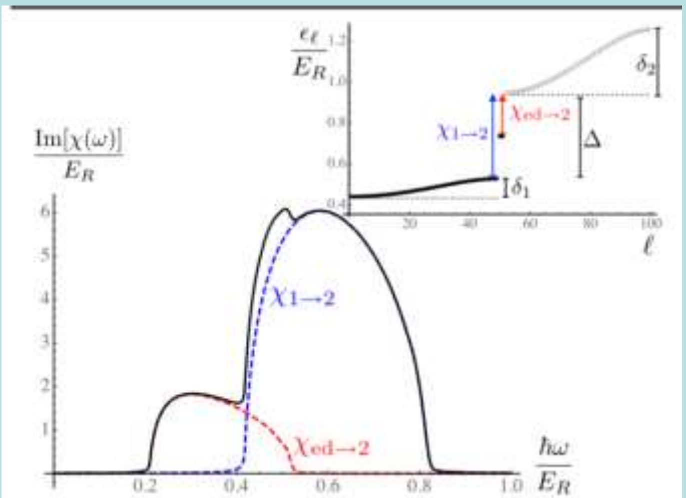
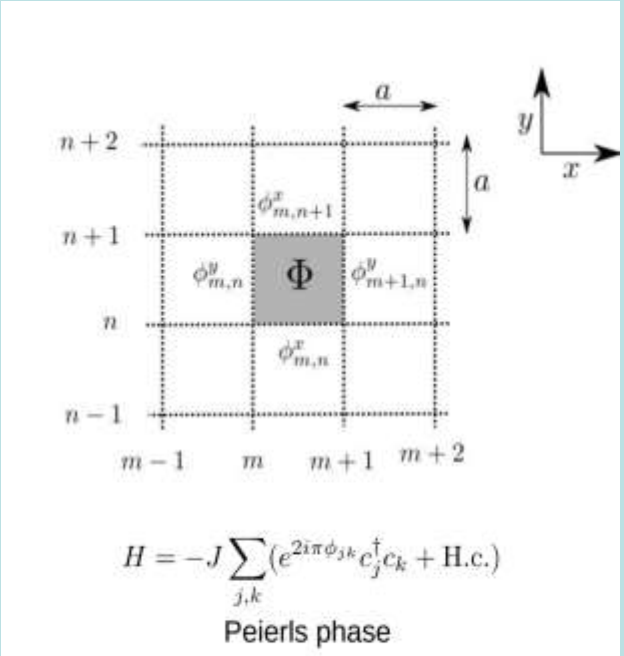
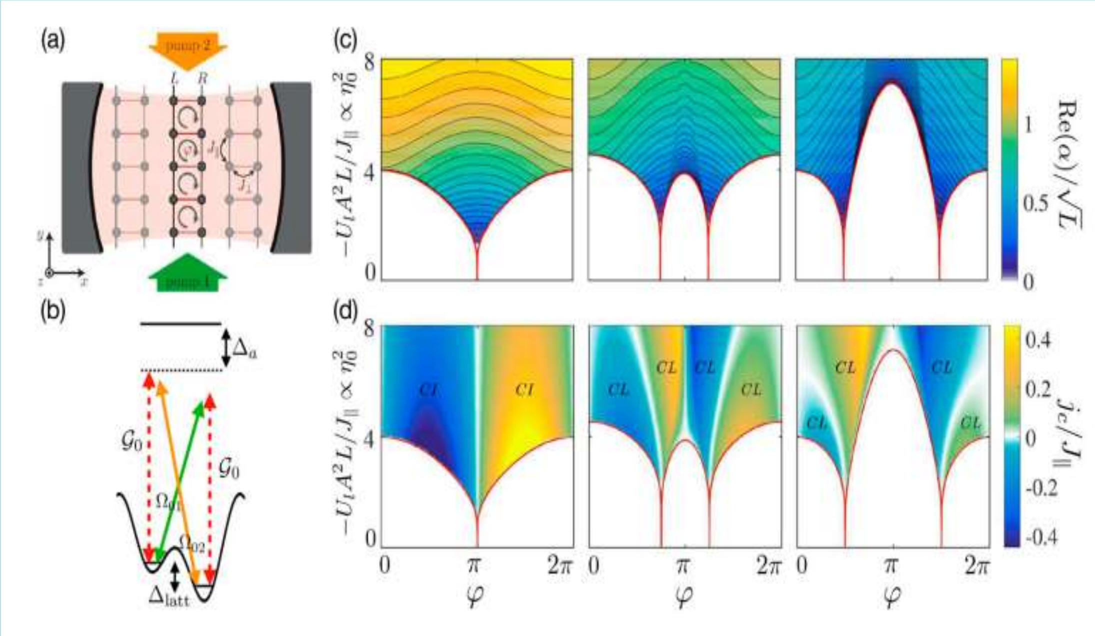


FIG. 3. Phase diagram in the $\eta - k_F$ plane. For a strong enough



Dynamic synthetic gauge fields using a cavity

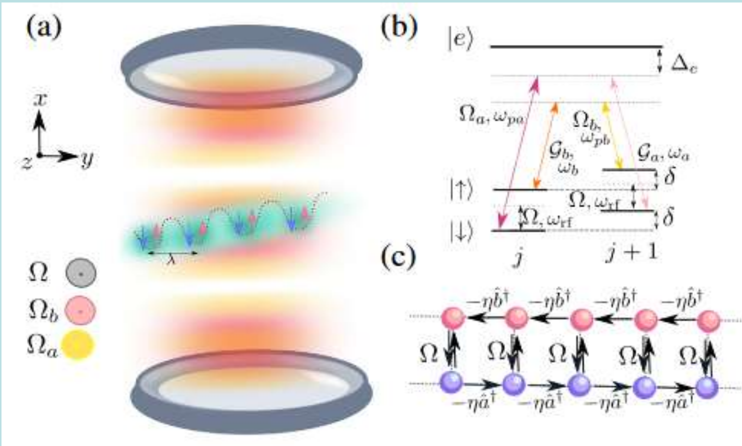


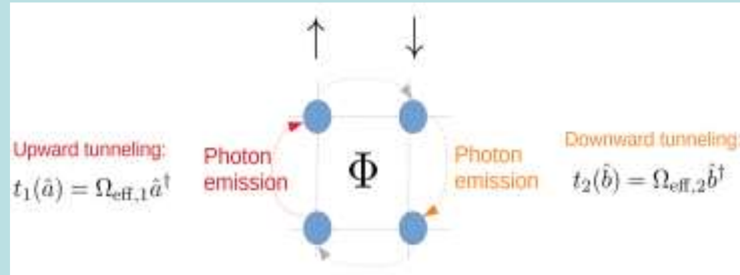


transition to superradiant state is connected
to current and generates a dynamic gauge field

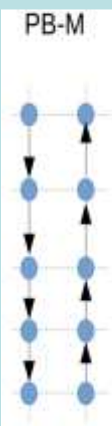
Ultracold Fermions in a Cavity-Induced Artificial Magnetic Field
C. Kollath, PRL116, 060401 (2016), PRA 92 (2016)

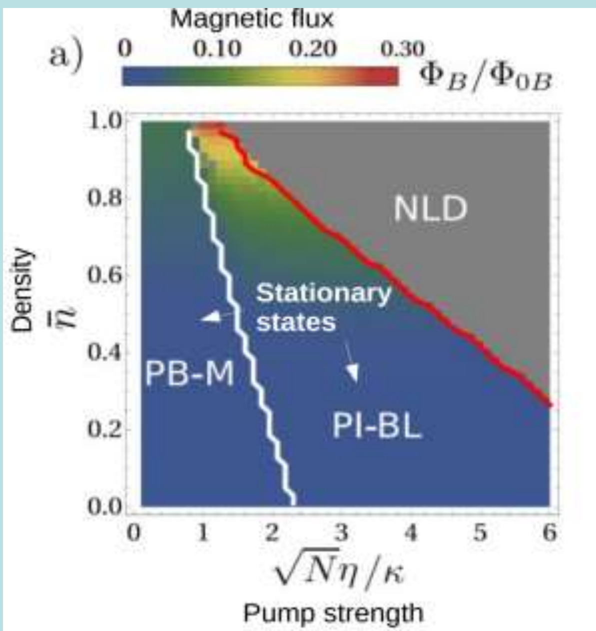
Open Quantum System Simulation of Faraday′s Induction Law via Dynamical Instabilities in ring cavities

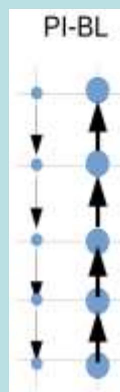


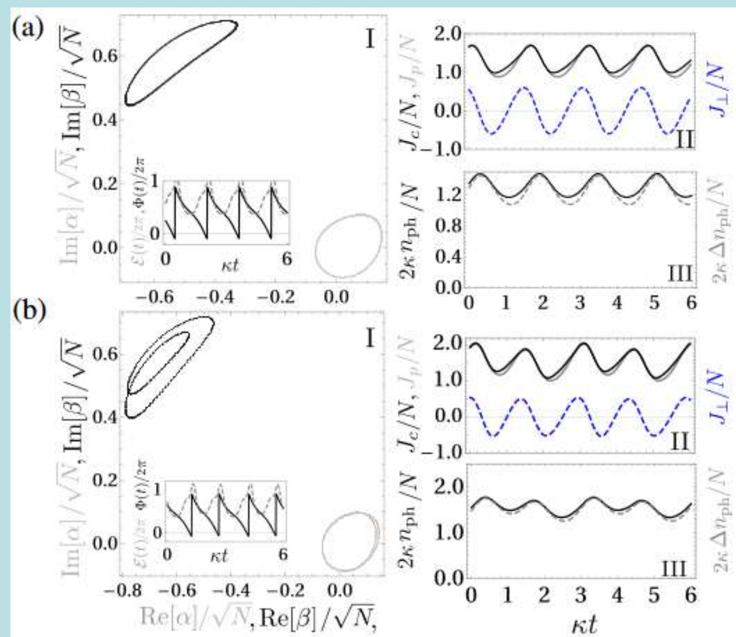


$$\phi_{\alpha(\beta)} = -\arctan\left(\frac{\kappa_{a(b)}}{\Delta_{a(b)} - UN_{\downarrow(\uparrow)}}\right)$$





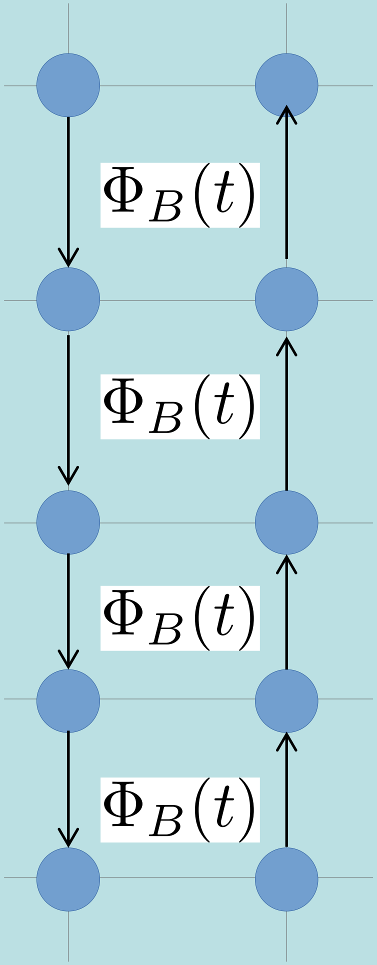




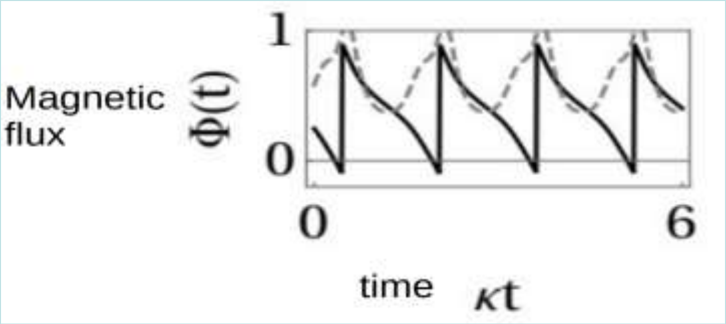
for high density => dynamical instabilities (NLD)

time-dependent fields => time dependent currents
=> time dependent flux

Faraday's law of induction

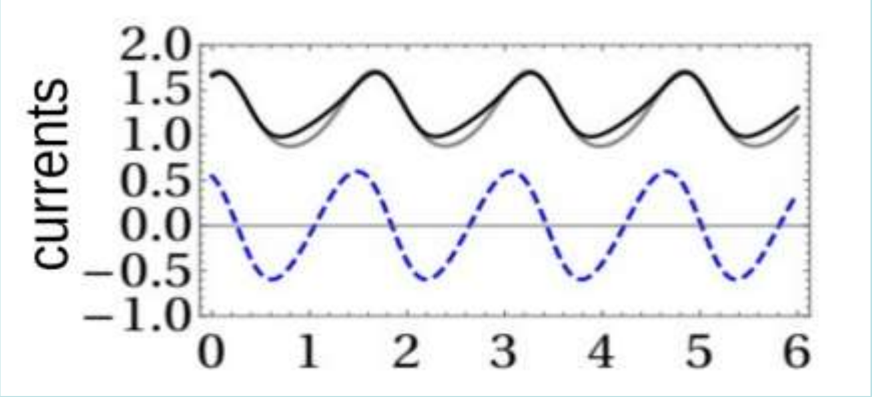


Photon phase oscillations induce an time-dependent effective magnetic field in each plaquette



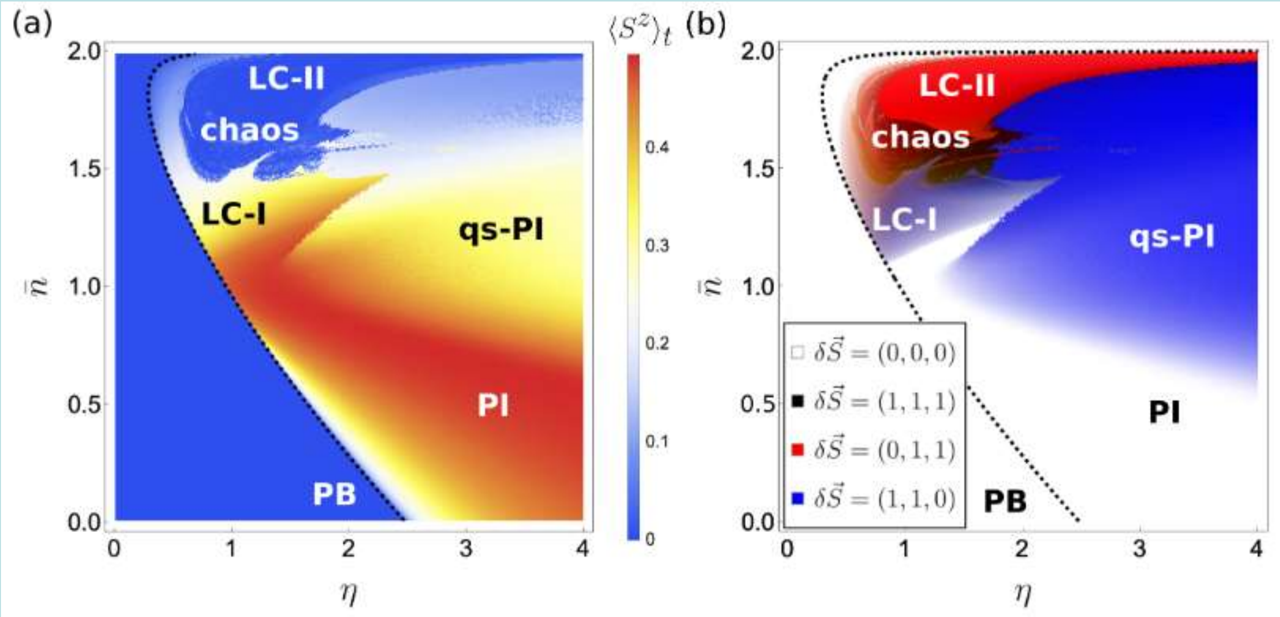
=> $\mathcal{E}(t) = -\frac{\partial \Phi(t)}{\partial t}$

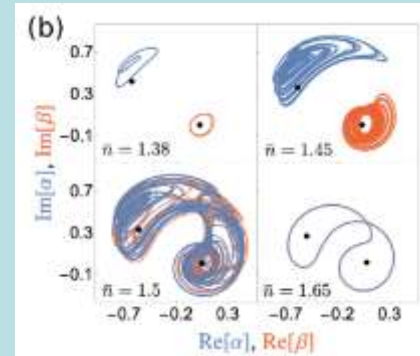
An electromotive force opposes to the variations of the magnetic field by generating periodic currents that



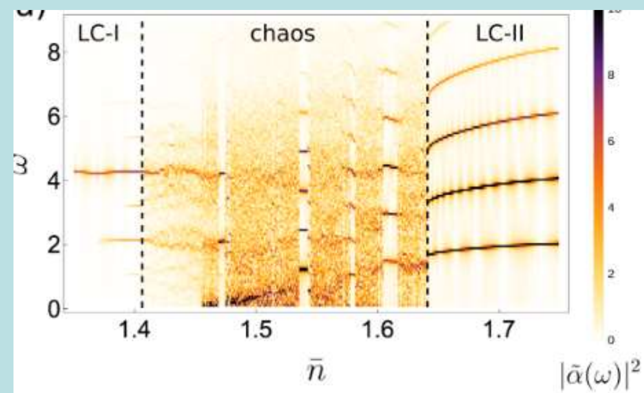
intra-leg currents

large system limit without local interactions:
=> mapping to collective spin problem





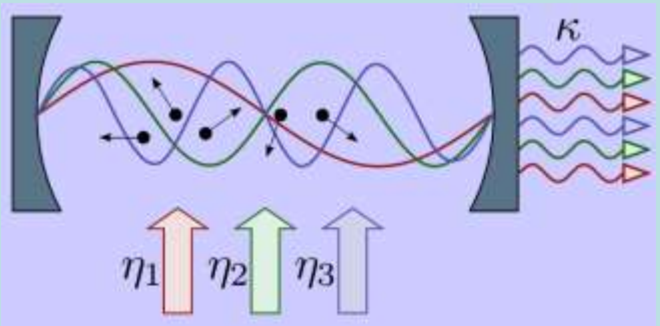
Phase	Acronym	$\langle S^z \rangle_t$	δS^x	δS^y	δS^z	Stationary	$\mathbb{Z}_2$ breaking
photon balanced	PB	0	0	0	0	✓	
photon imbalanced	PI	$\neq 0$	0	0	0	✓	✓
quasi-stationary photon imbalanced	qs-PI	$\neq 0$	≈ 1	≈ 1	≈ 0		✓
limit cycle I	LC-I	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$		✓
limit cycle II	LC-II	≈ 0	≈ 0	≈ 1	≈ 1		
chaos		≈ 0	≈ 1	≈ 1	≈ 1		



A. Kosior et.al, arXiv2208.04602

selfordering
and
simulated annealing
in
optical resonators

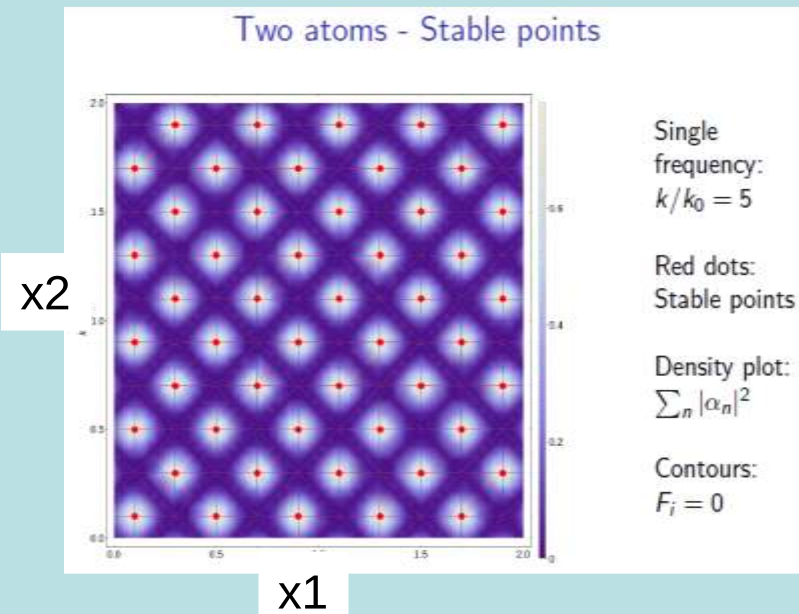
Selfordering in laser fields with several colors



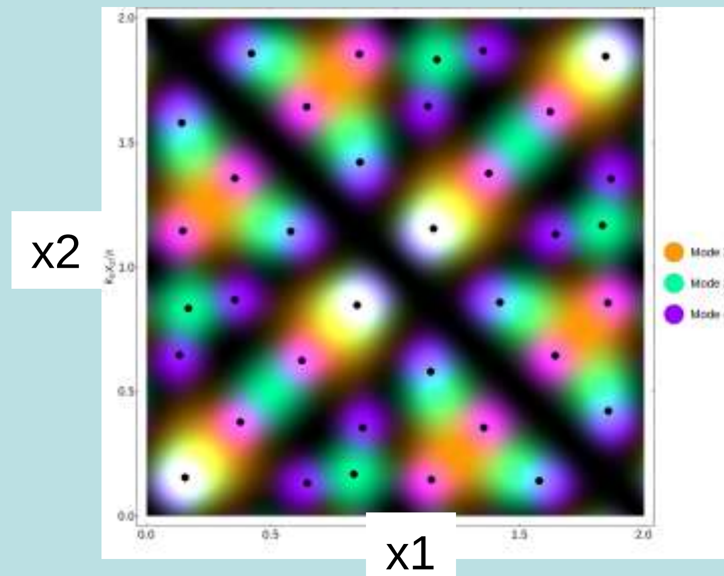
Field amplitudes:

$$\dot{\alpha}_n = i\left(\delta_{c,n} - U_{0,n}\sum_j \sin^2(k_n x_j)\right)\alpha_n - \kappa_n\alpha_n - i\eta_n\sum_j \sin(k_n x_j) + \xi_{\alpha,n}$$

single frequency (mode 5)



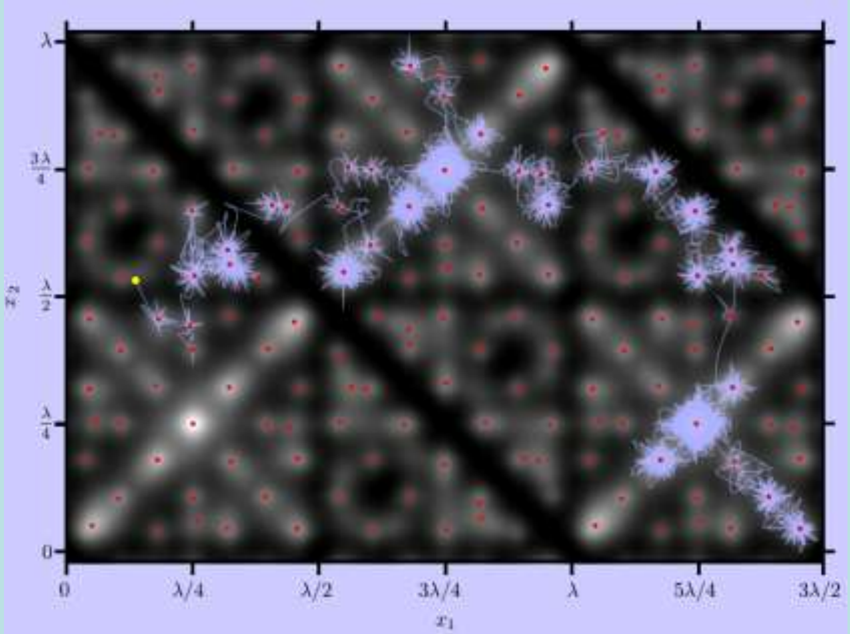
three colors (mode 2+3+4)



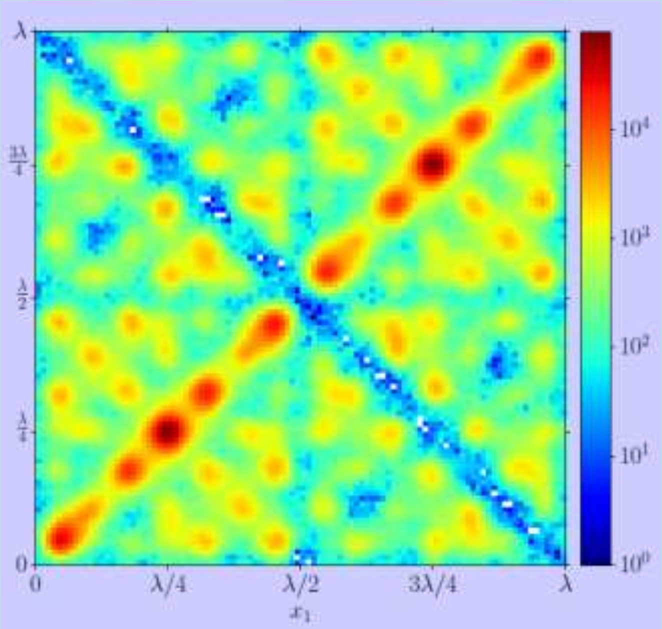
At some positions particles scatter all colors => particles solve an optimization problem

Particle field dynamics with (quantum) noise: guided Brownian motion

Quasi-random walk
between high scattering areas



Time averaged
position distribution

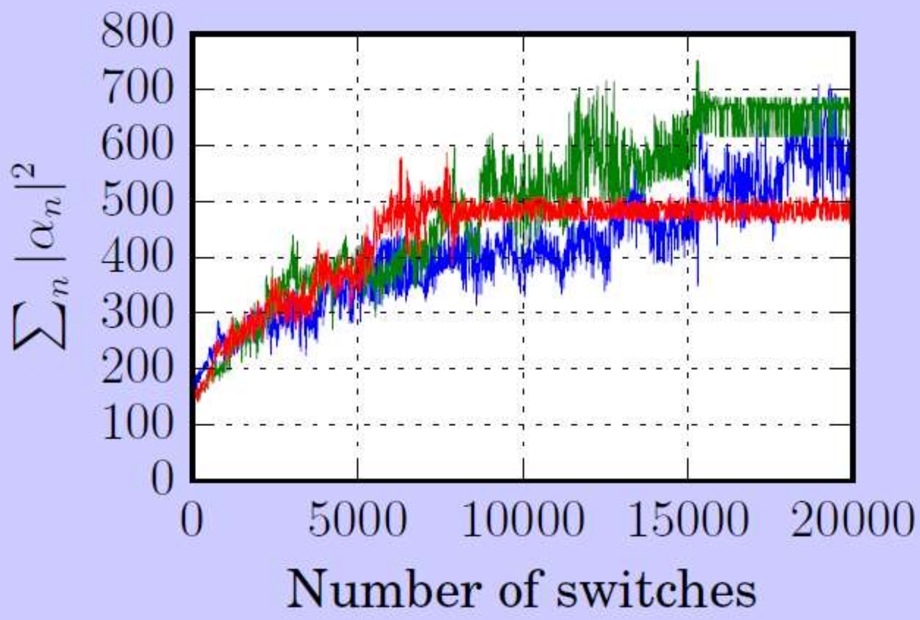


Particles tend to stay close to positions
of optimum scattering and trapping:

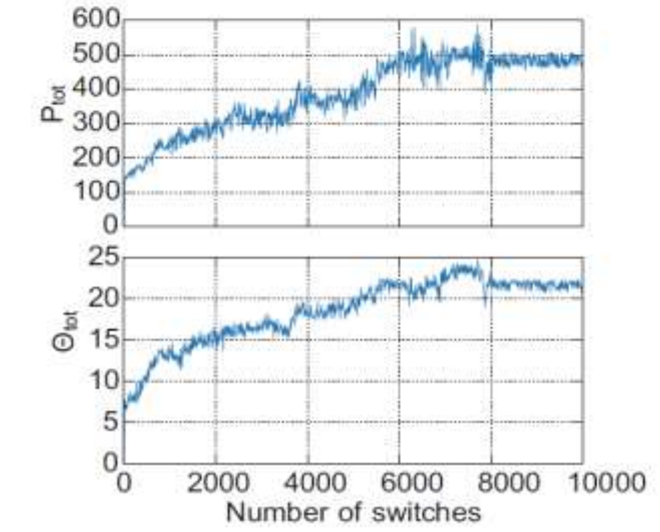
⇒ **adaptive „light collection" system**
⇒ **system „learns" in time**
⇒ **memorizes previous conditions**

Adaptive dynamics in time-varying illumination

Time evolution of 100 particles in time-varying illumination. We choose 5 illuminations, each consisting of about 50 pump lasers close to high order modes ($n > 1000$), which are applied in a **random sequence**.



Sum of order parameters:



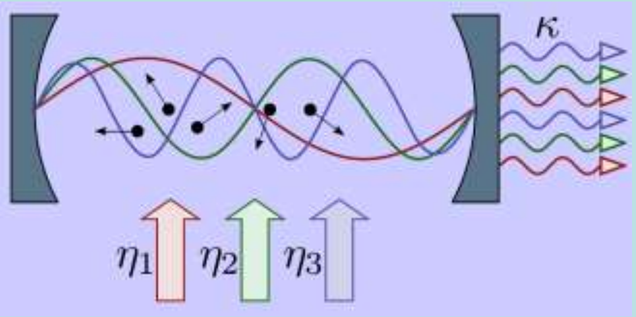
system optimizes scattering and „learns“ from the past

adaptive + learning light collection system

„disspative“ annealing

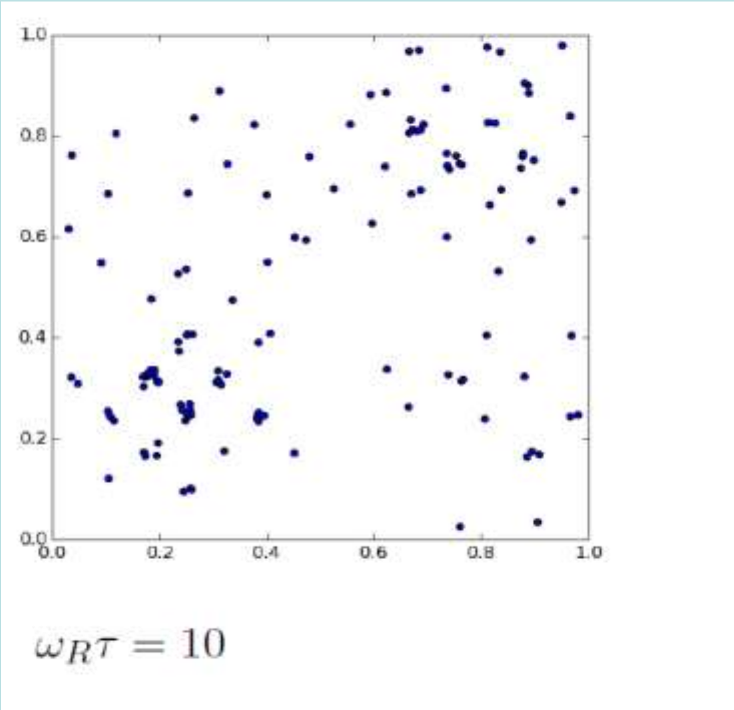
=>

slow turn on 2.+ 9. mode illumination



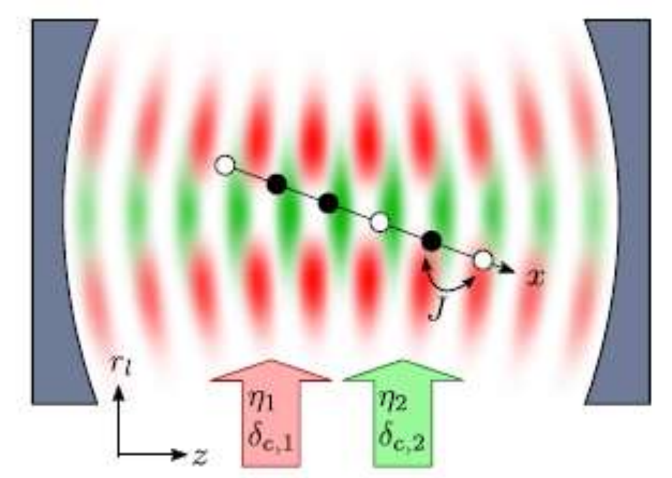
fast switch on

slow switch on



system converges mostly to states optimal for all modes

Quantum annealing in a cavity lattice
T = 0



general coupling matrix A can be constructed via pump mode design

$$H = -J\sum_i (b_{i+1}^\dagger b_i + b_i^\dagger b_{i+1}) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i + 1) - \mu \sum_{i,j} A_{ij}\hat{n}_i\hat{n}_j$$

with the real and symmetric interaction matrix

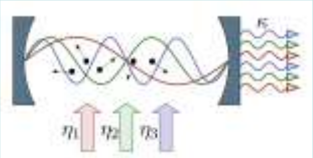
$$A = \sum_m f_m \,\mathrm{Re}(\boldsymbol{v}_m \otimes \boldsymbol{v}_m^*)/\mu$$

See also: Gopalakrishnan, Lev, Goldbart, PRL

Interacting trapped **quantum** particles within a multimode cavity

particle-field Hamiltonian

$$H = -J\sum_i (b^\dagger_{i+1} b_i + b^\dagger_i b_{i+1}) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i + 1)$$
$$-\hbar\sum_m \delta_{c,m} a^\dagger_m a_m + \hbar\sum_m \eta_m (a_m + a^\dagger_m)\sum_i v^i_m \hat{n}_i$$



Effective Hamiltonian after field elimination

$$H = -J\sum_i (b^\dagger_{i+1} b_i + b^\dagger_i b_{i+1}) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i + 1) - \mu\sum_{i,j} A_{ij}\hat{n}_i\hat{n}_j$$

with the real and symmetric interaction matrix

$$A = \sum_m f_m \operatorname{Re}(\boldsymbol{v}_m \otimes \boldsymbol{v}^*_m)/\mu$$

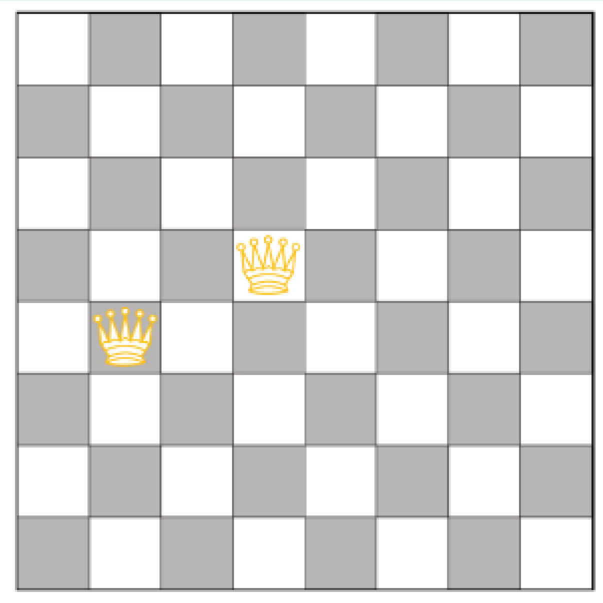
yes, we can **engineer general coupling matrices Aij**

but: **ond needs ~ N^2 pump lasers for N sites**

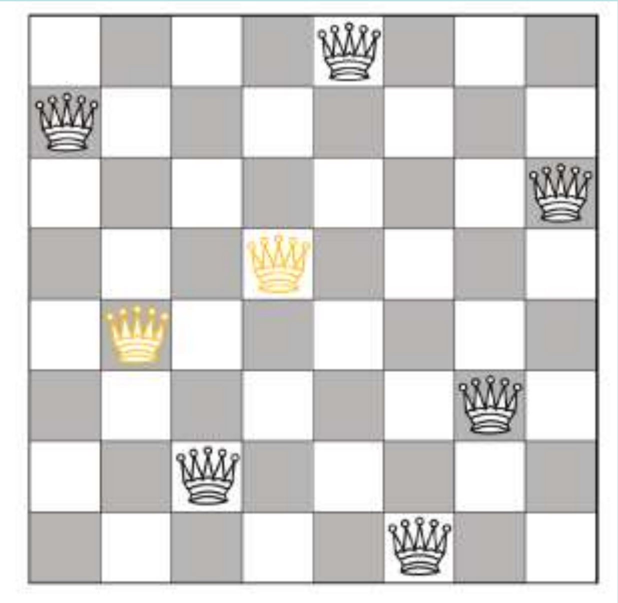
S. Krämer, V. Torggler and H.R., PRA95,3,(2017)

A quantum N-Queens simulator

N Queens problem:



a solution



Is it difficult ?

Total # of queen configurations:

$$\binom{8^2}{8} \approx 4 \times 10^9$$

Placed queens + 1 queen / row:

$$8^6 = 261,144$$

N-Queens completion scales exponentially with hard instances (N>21) and is

=> NP - complete

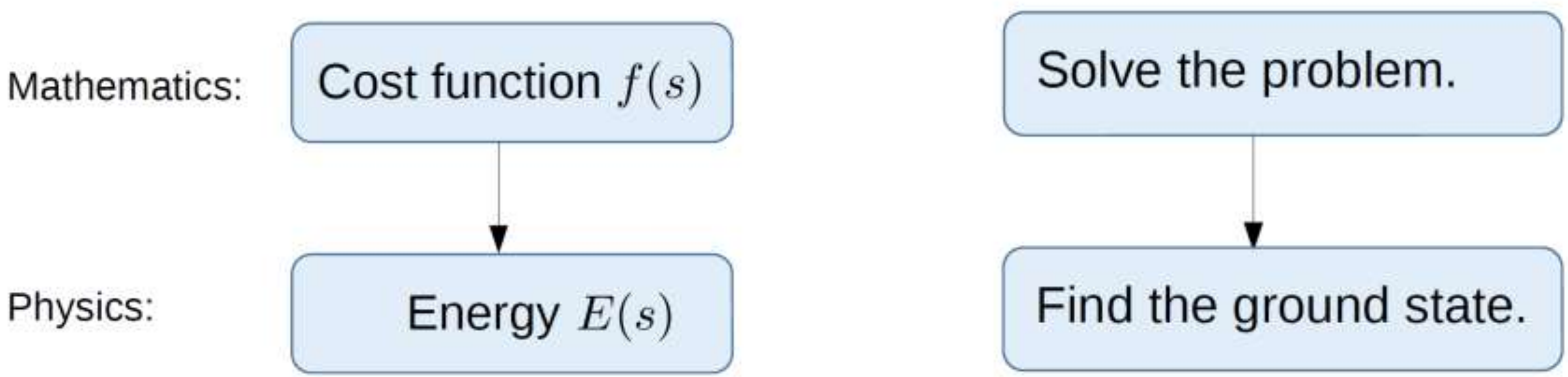
I. Gent, C. Jefferson, and P. Nightingale, *J. Artif. Intell. Res.* **59**, 815 – 848 (2017)

Optimization problem → minimize a cost function.

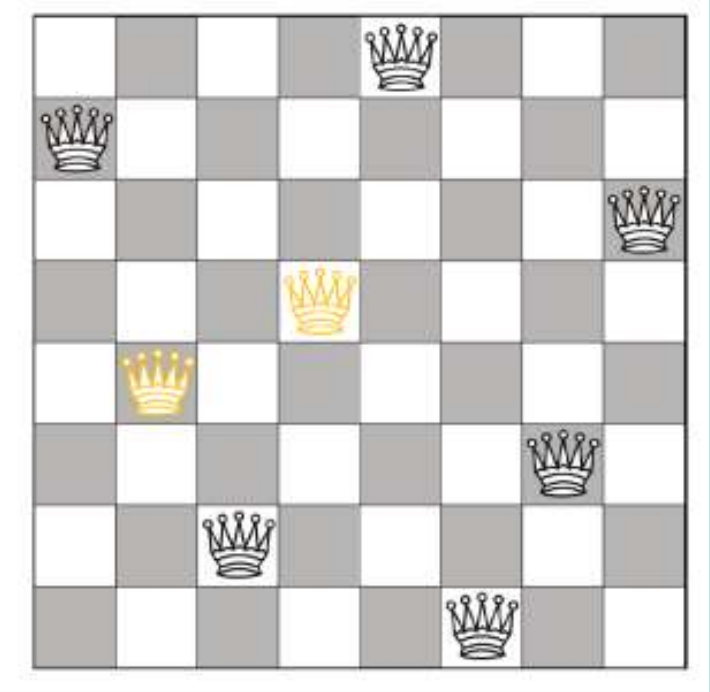
Minimize $f(s)$ subject to $s \in S$

$f(s) : D \to \mathbb{R}$

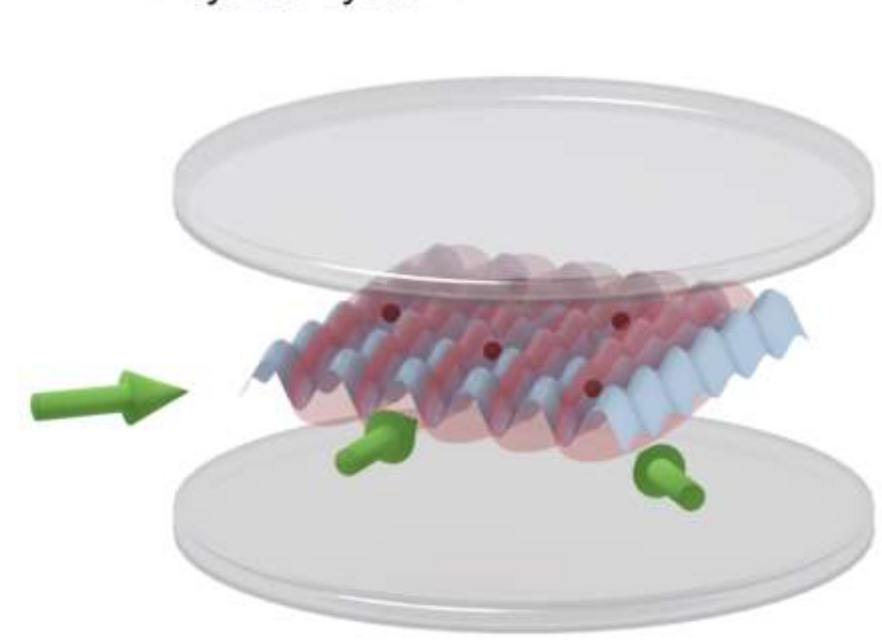
D… configuration space
S… set of feasible solutions (constraints)



Use a quantum mechanical system to solve an optimization problem.

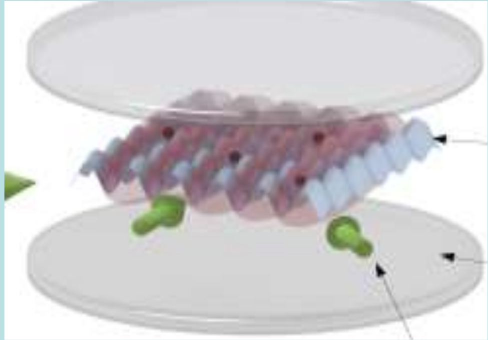


Physical system:



Chess Board => 2D optical lattice in cavity
Queens => ultracold atoms
Chess rules => optical interactions via laser scattering

2D optical lattice of **quantum** particles within a multimode cavity



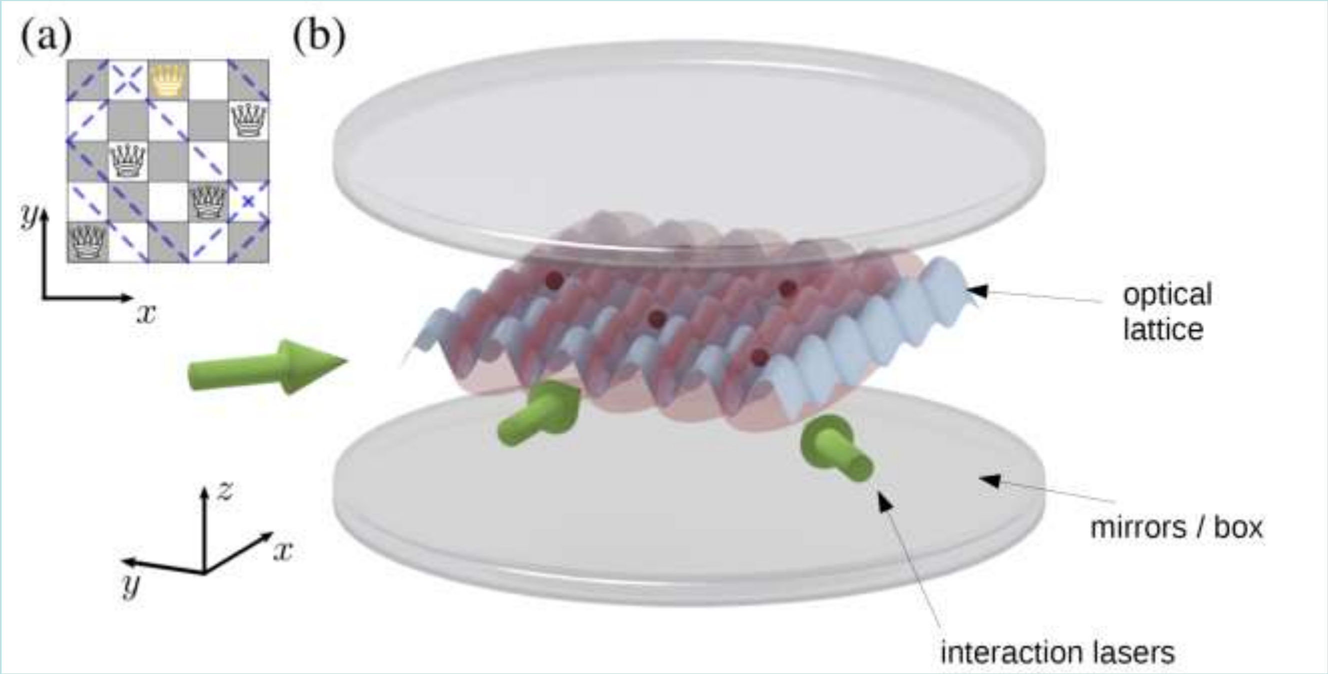
particle-field Hamiltonian

$$H = -J\sum_i (b_{i+1}^\dagger b_i + b_i^\dagger b_{i+1}) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i + 1)$$
$$- \hbar\sum_m \delta_{c,m} a_m^\dagger a_m + \hbar\sum_m \eta_m (a_m + a_m^\dagger)\sum_i v_m^i \hat{n}_i$$

coupling vectors determined by modes and pump geometry

$$v_m^i = \int \mathrm{d}x\, w^2(x - x_i) u_{p,m}(x) u_{c,m}^*(x) \approx u_{p,m}(x_i) u_{c,m}^*(x_i)$$

engineer site to site coupling by suitable choice of modes + pumps !



each laser creates cosine-type interactions:

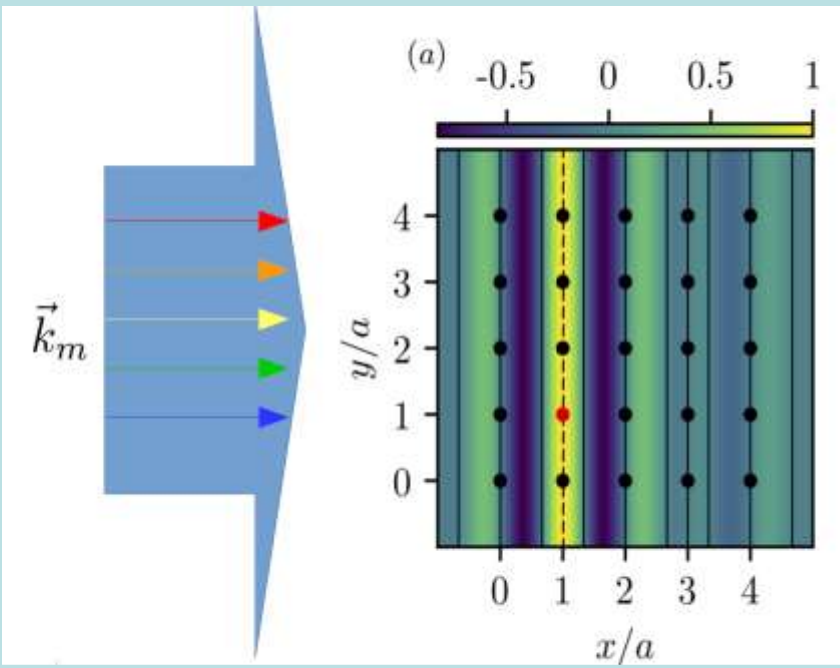
$$V_{\text{interaction}}(x_1, x_2) \propto \Delta_c \sum_{i=1}^{2} \sum_{j=1}^{2} \cos(k(x_i - x_j))$$

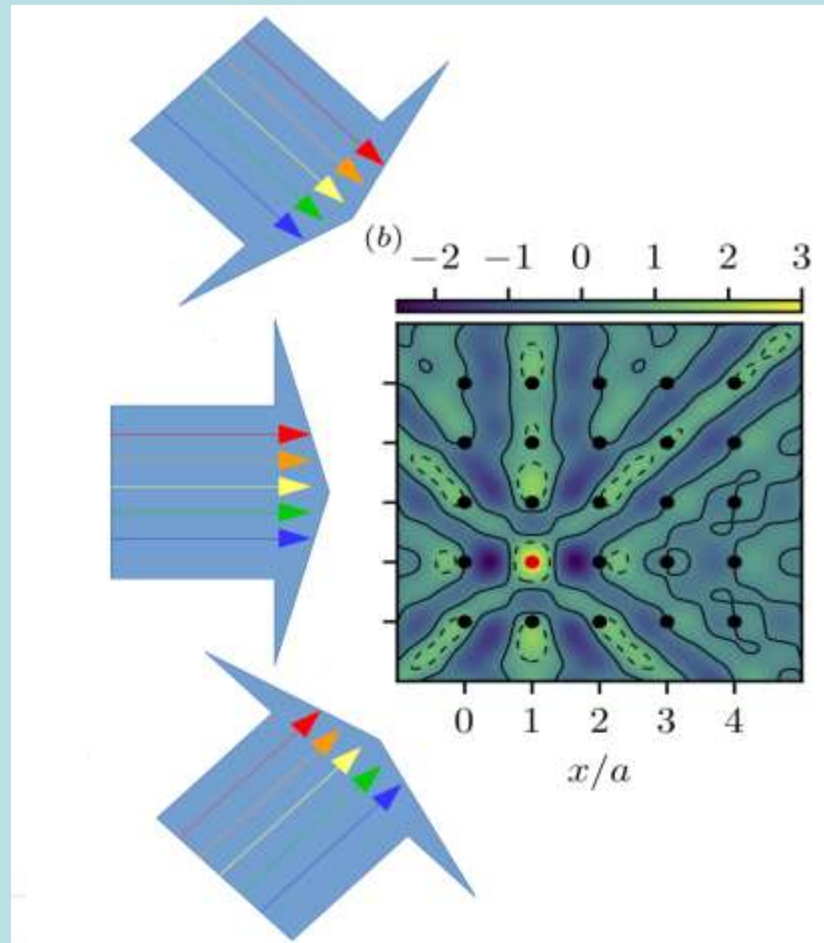
Interaction between lattice site (i,j) and (k,l):

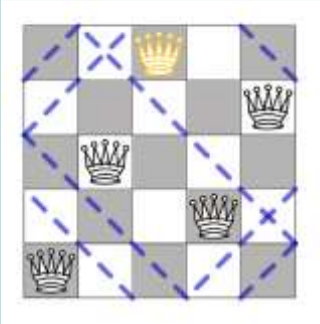
$$A_{ijkl} = \sum_m f_m \cos(\vec{k}_m(\vec{x}_{ij} - \vec{x}_{kl}))$$

three laser directions

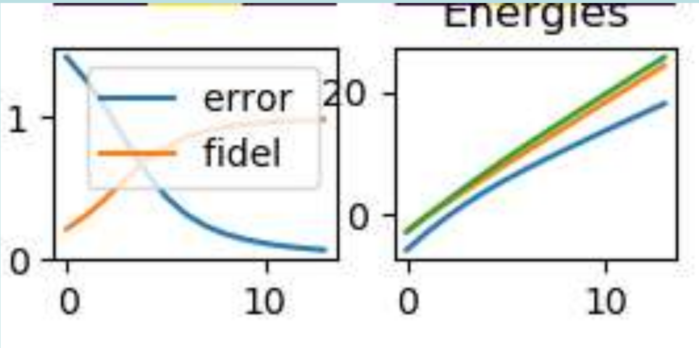
one laser direction

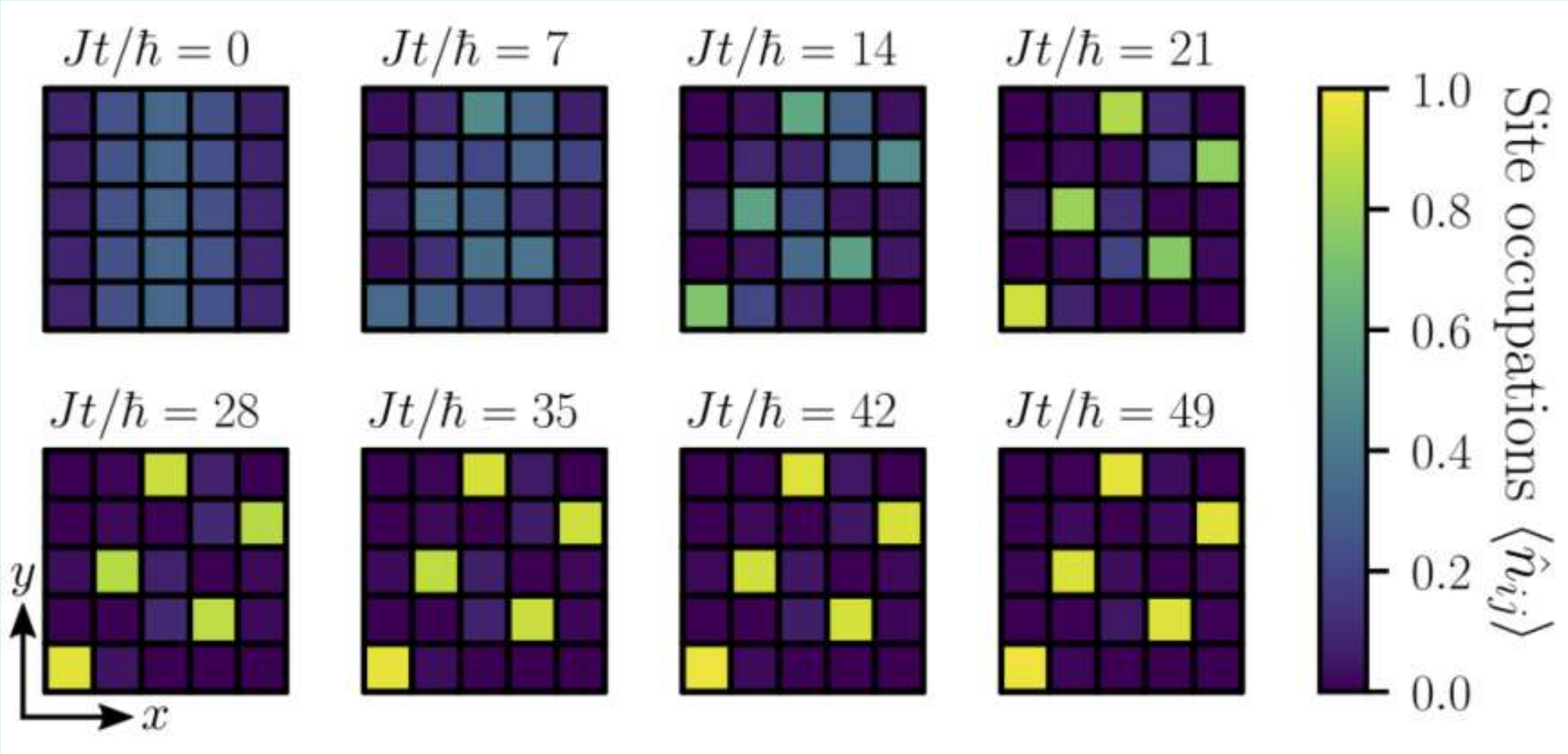






Lasers slowly turned on
=> atoms converge to solution



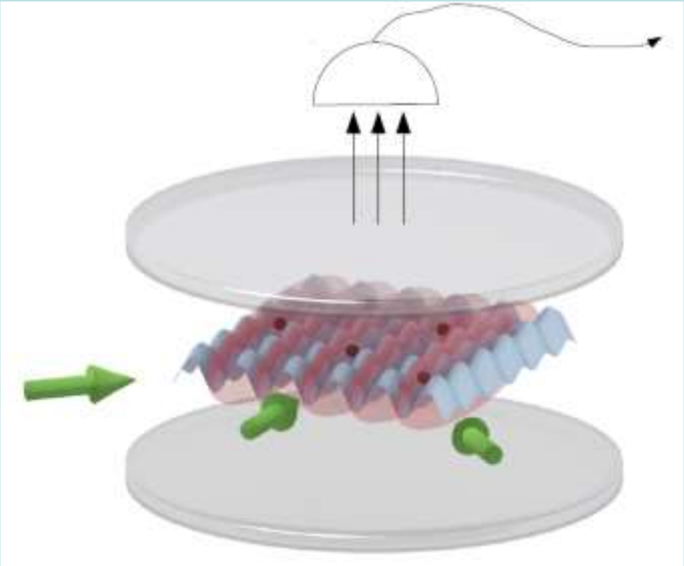


VT, P. Aumann, H. Ritsch, and W. Lechner
"A quantum N-queens solver",
arXiv:1803.00735 [quant-ph] (2018)

N-queens completion problem

Readout of result:

equal cavity output field amplitudes
signal appearance of a solution
but
need an atom microscope to recover
actual solution



Suitable for testing quantum advantage.

N-queens problem:
→ there exist hard instances for N > 21

Implementation fits problem naturally:
→ No qubit overhead – as many atoms as queens
→ Laser resources scale linearly with N


Thanks for listening !
visitors in Innsbruck welcome !