Early history dependence in stat-phys models: from $2d$ coarsening to mean-field disordered models

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KITP, January, 2018

Plan

- $2d$ coarsening systems: early approach to critical percolation determines the ground state statistics.

Very brief presentation, open to discussions

- Mean-field disordered (spin) models

  — Dynamics of isolated systems:

  classical vs. quantum, integrable *vs.* non-integrable

Main interest

  — Memory effects in spin-glasses & Kovacs effects in glasses.

No time, open to discussions

1st part

$2d$ coarsening systems: early approach to critical percolation determines the (zero temperature) ground state statistics.

Series of works (**06-18**) in collaboration with

Jeferson Arenzon, Thibault Blanchard, Alan Bray, Federico Corberi, Ingo Dierking, Michikazu Kobayashi, Ferdinando Insalata, Marcos-Paulo Loureiro, Marco Picco, Yoann Sarrazin, Alberto Sicilia, Hugo Ricateau and Alessandro Tartaglia.

Framework

A very well-known problem

The stochastic dynamics of the $2d$ Ising model after an instantaneous quench from high to low temperature

- There is a 2nd order phase transition, and

  the **equilibrium phases** are the paramagnet at high T and

  the (degenerate) ferromagnet at low T.

- Standard knowledge (non-conserved order parameter dynamics)

  The **dynamic mechanism** is curvature-driven domain growth.

$2d$ **Ising Model (IM)**

Archetypical example for classical magnetic systems

$$H = -J \sum_{\langle ij \rangle} s_i s_j$$

$s_i = \pm 1$ Ising spins.

$\langle ij \rangle$ sum over nearest-neighbours on the lattice.

$J > 0$ ferromagnetic coupling constant.

critical temperature $T_c > 0$ for $d > 1$.

Monte Carlo rule $s_i \to -s_i$ accepted with

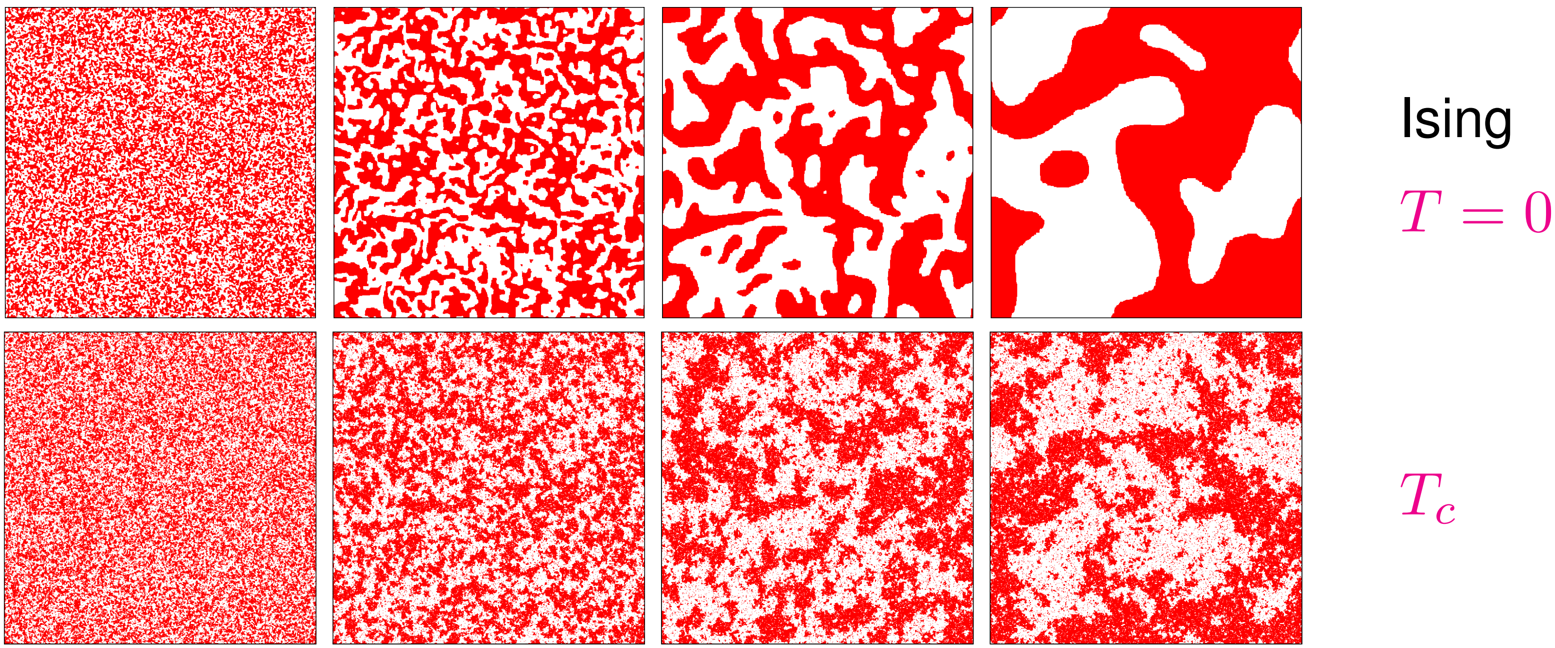
$$p = 1 \quad \text{if } \Delta E < 0$$
$$p = e^{-\beta \Delta E} \quad \text{if } \Delta E > 0$$
$$p = 1/2 \quad \text{if } \Delta E = 0$$

Non-conserved order parameter dynamics [$\uparrow\downarrow$ towards $\uparrow\uparrow$] etc. allowed.

[$m = 0$ to $m = 2$]

Phase ordering kinetics

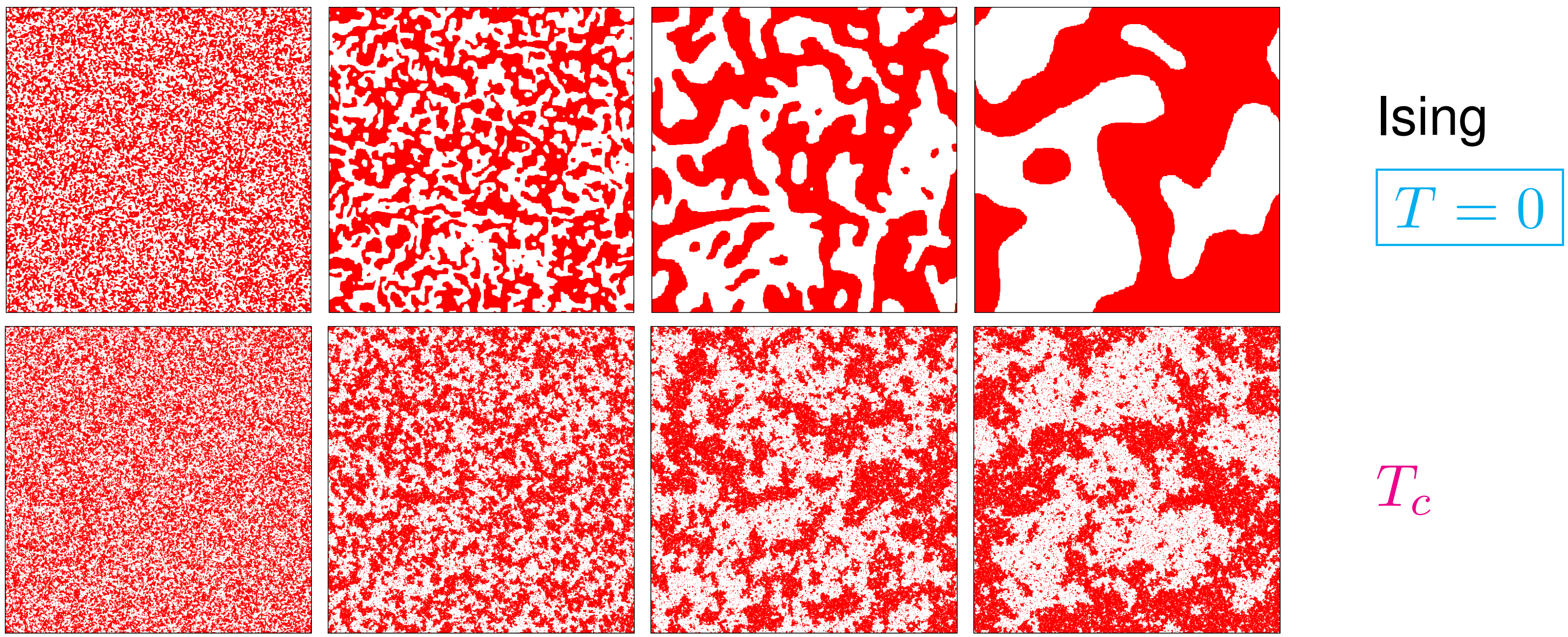
$s_i = \pm 1$ **at** $t = 0$ **MCs, snapshots at** $t = 4,\ 64,\ 512, 4096$ **MCs**



$T \to \infty$ initial condition in both cases and periodic boundary conditions

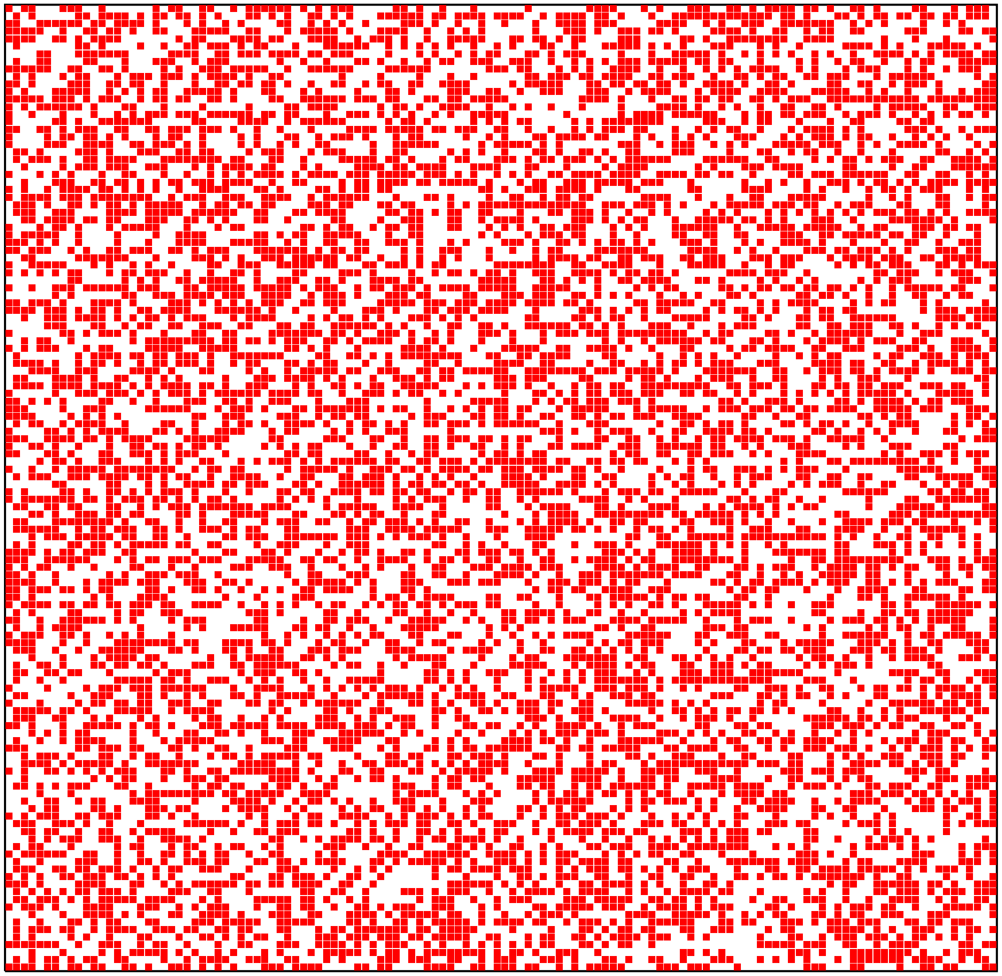
Phase ordering kinetics

$s_i = \pm 1$ **at** $t = 0$ **MCs, snapshots at** $t = 4,\ 64,\ 512, 4096$ **MCs**



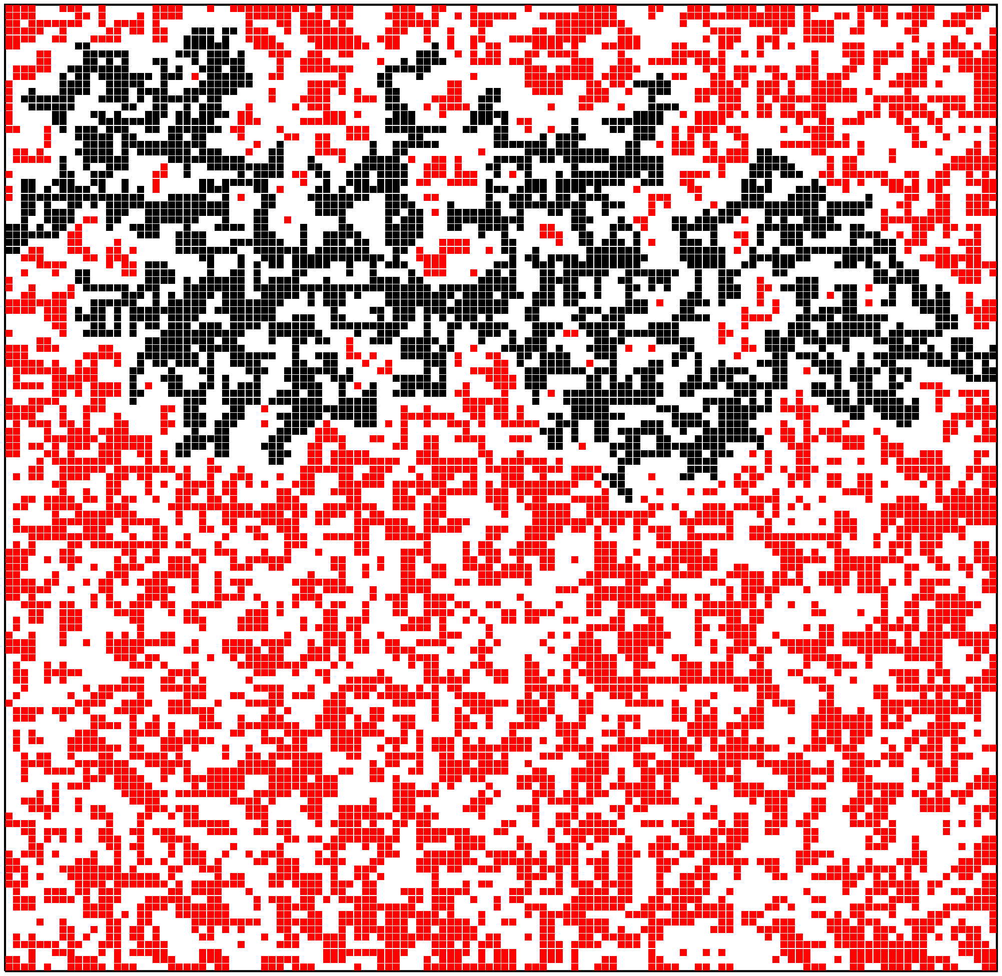
What happens in the very early stages of the $T = 0$ evolution ?

2d square IM at T=0



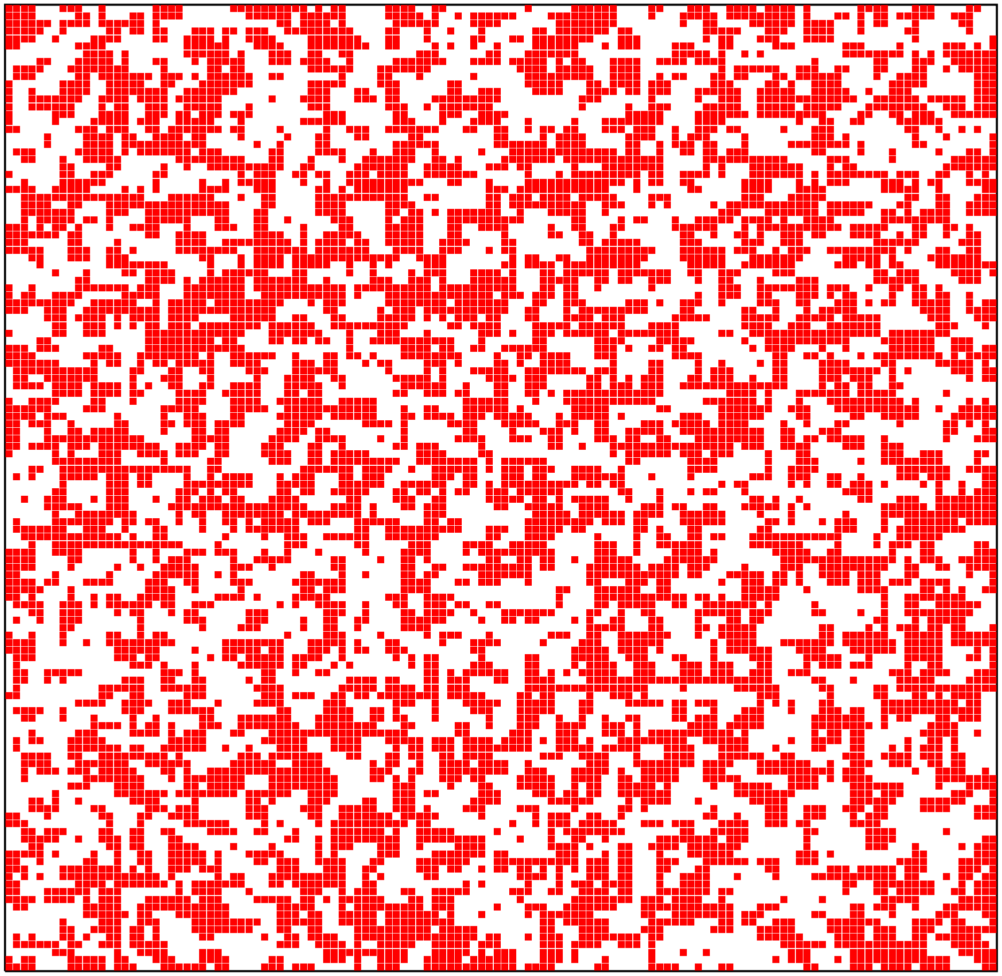
t=0.0

2d square IM at T=0



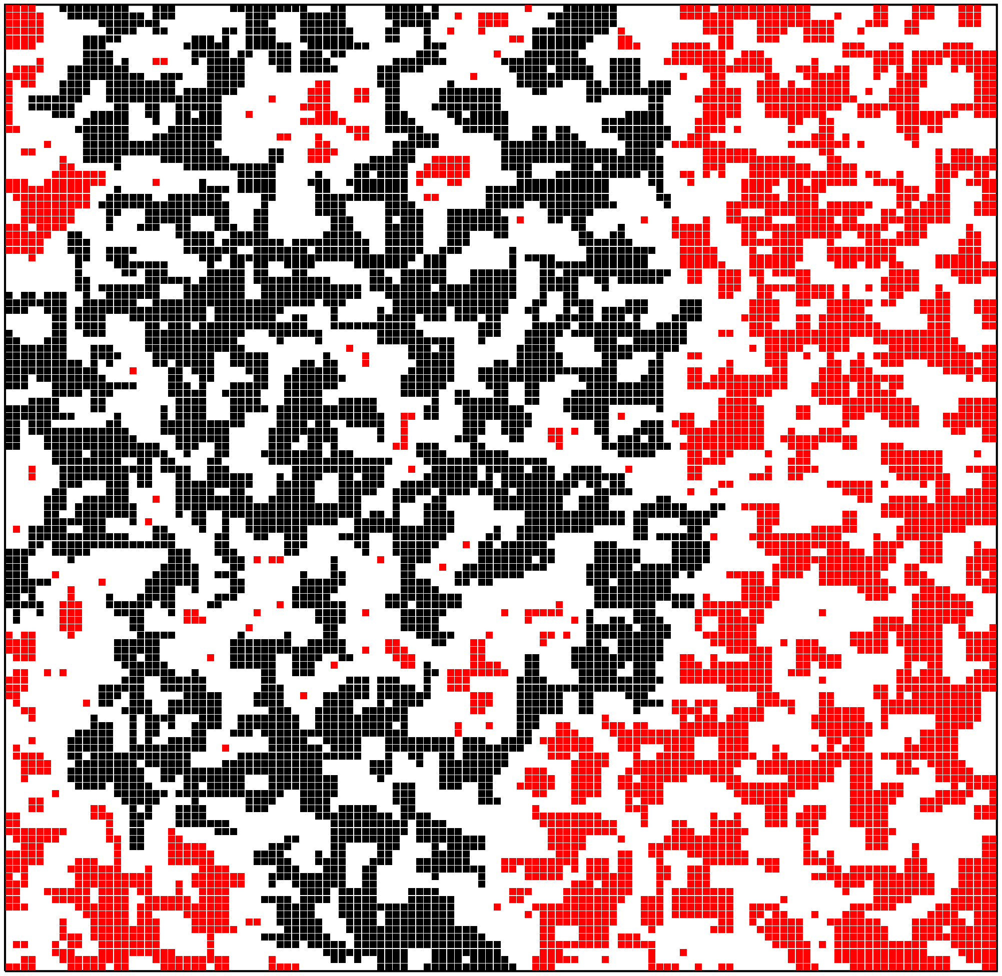
t=0.57533

2d square IM at T=0



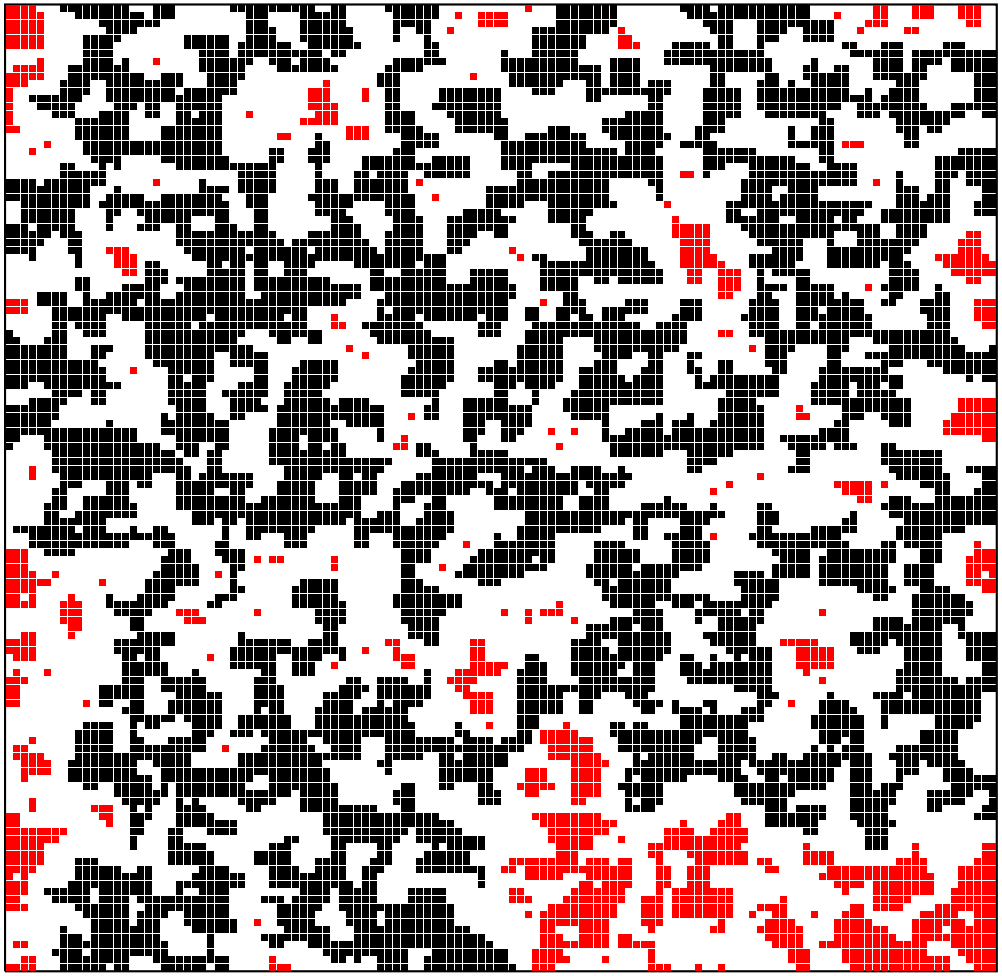
t=0.94844

2d square IM at T=0



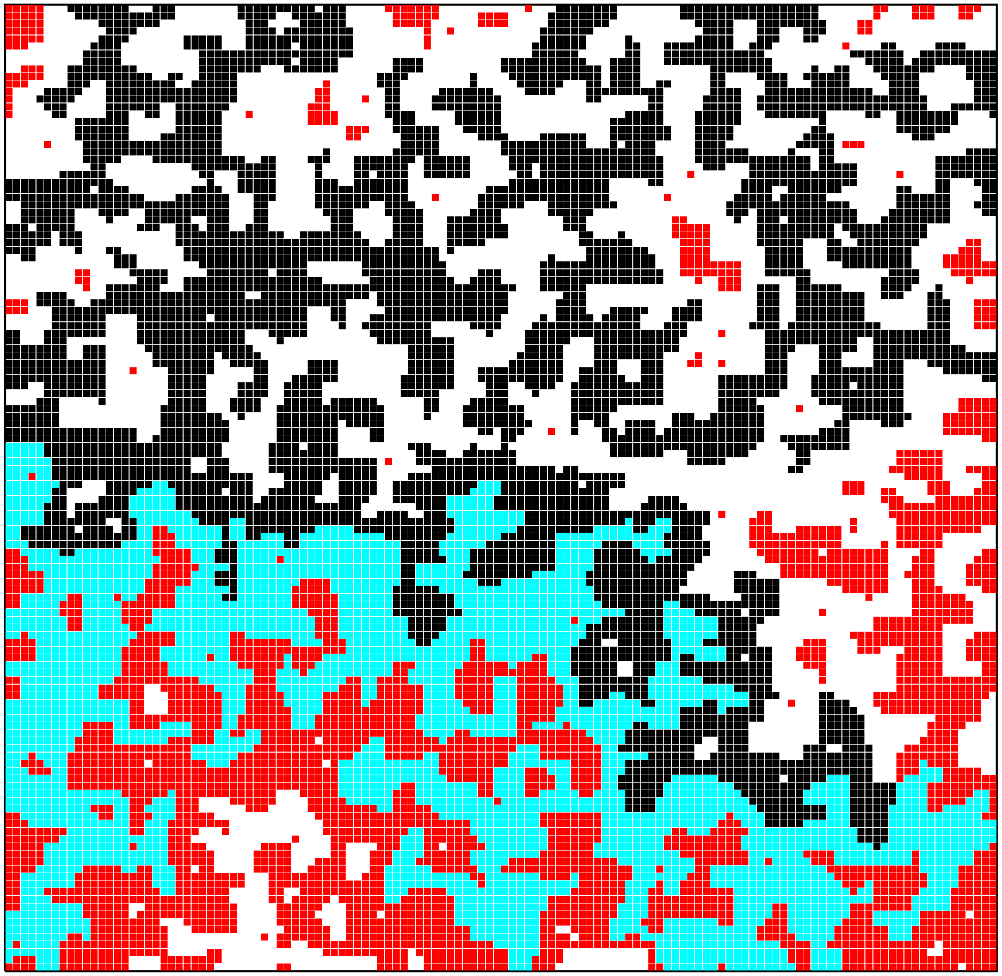
t=2.00847

2d square IM at T=0



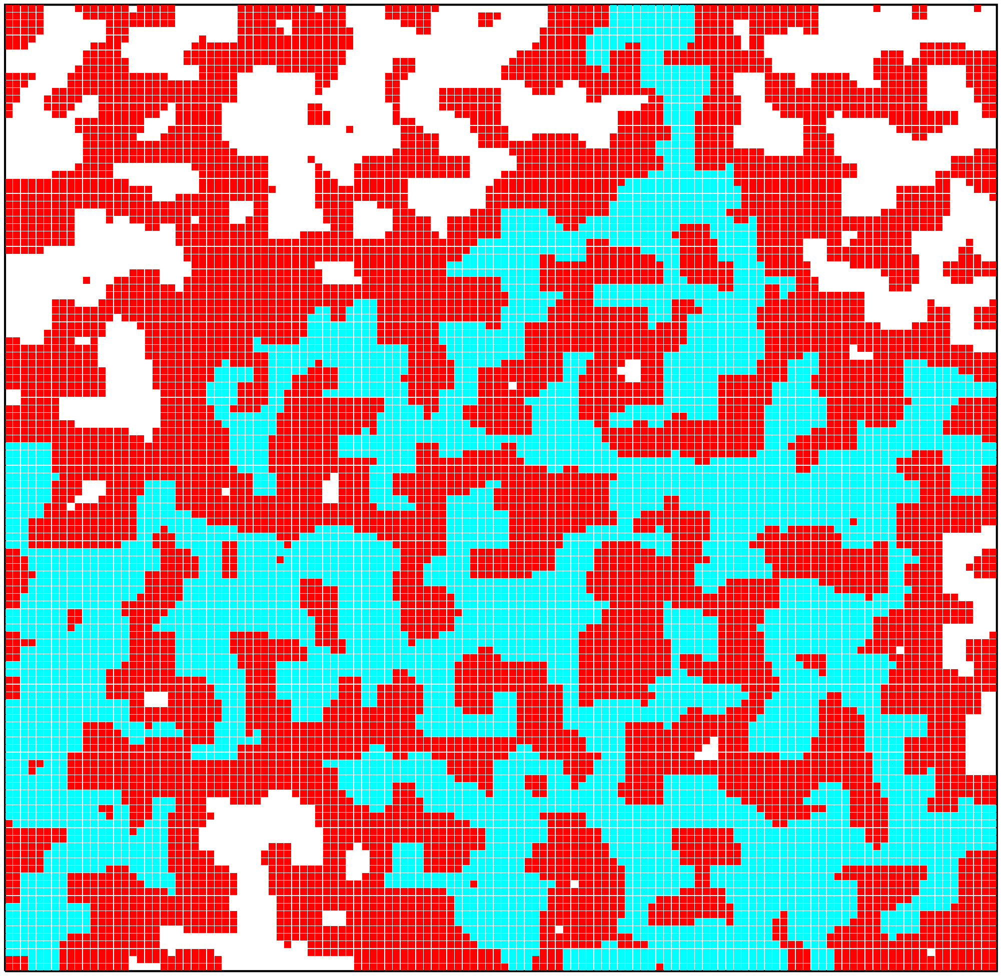
t=2.57898

2d square IM at T=0



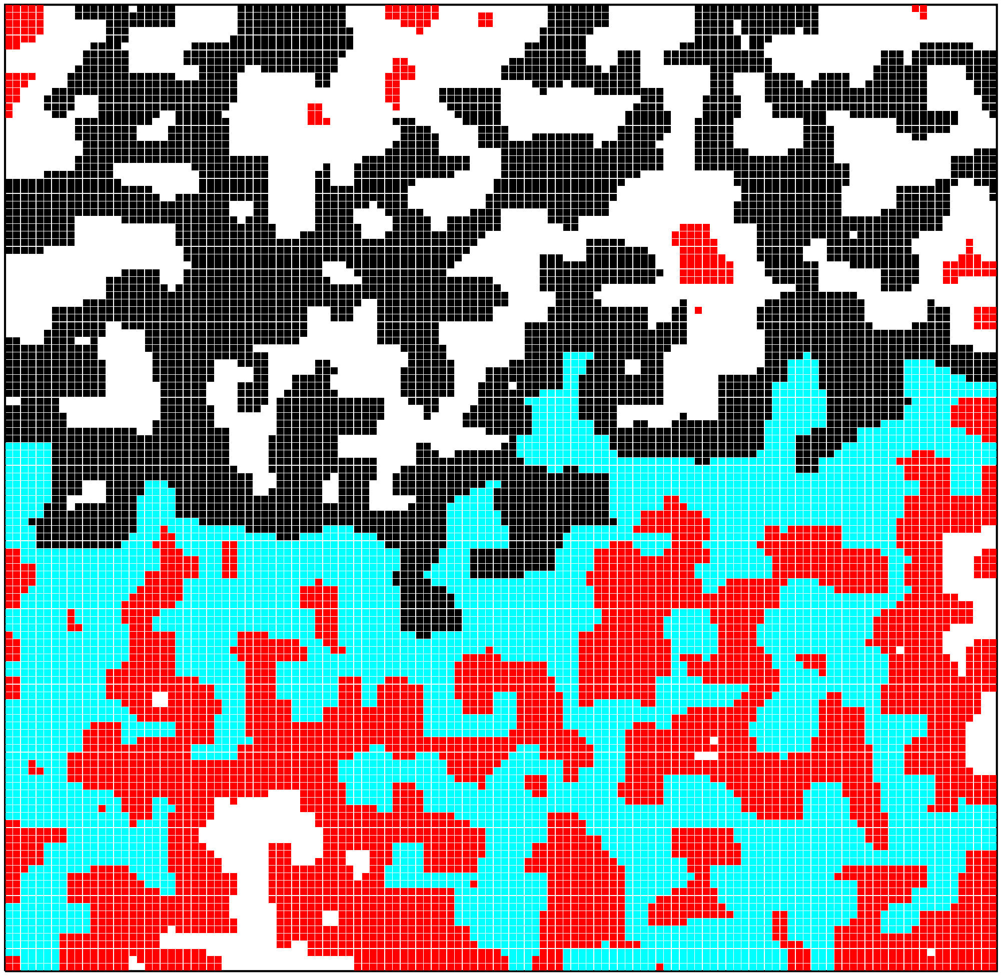
t=3.99211

2d square IM at T=0



t=6.58423

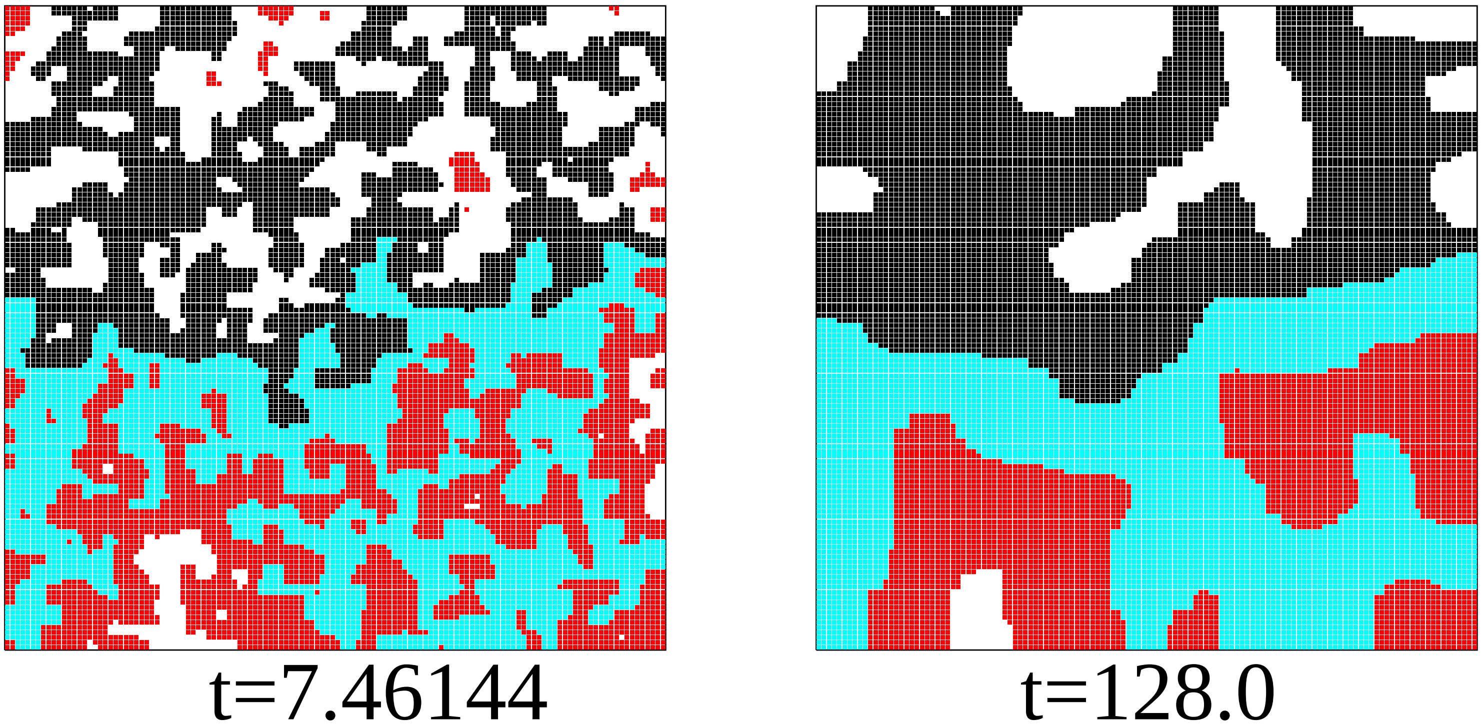
2d square IM at T=0



t=7.46144

2d square IM at T=0

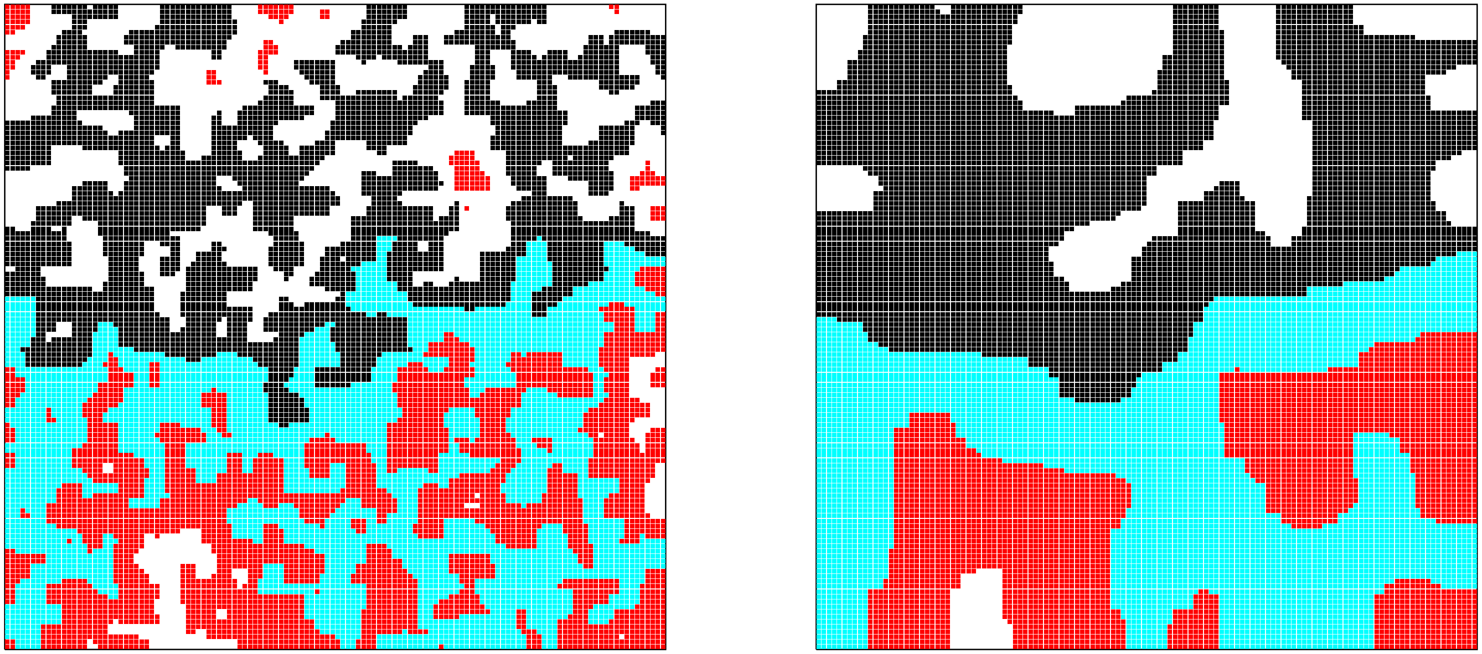
The percolating structure was decided at $t_p \simeq 8$ MCs



Arenzon, Bray, LFC & Sicilia 07, Blanchard, Corberi, LFC & Picco 14

2d square IM at T=0

The final configuration will be one with two horizontal stripes and a flat interface in between



t=7.46144   t=128.0

Is the state at $t \simeq 8$ MCs one of $2d$ critical percolation ?

If so, how does the system reach it ?

Does it influence the subsequent dynamics ?

Do their effects last for long ?

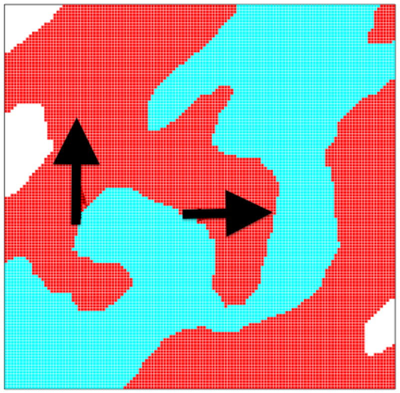
Is it critical percolation ?

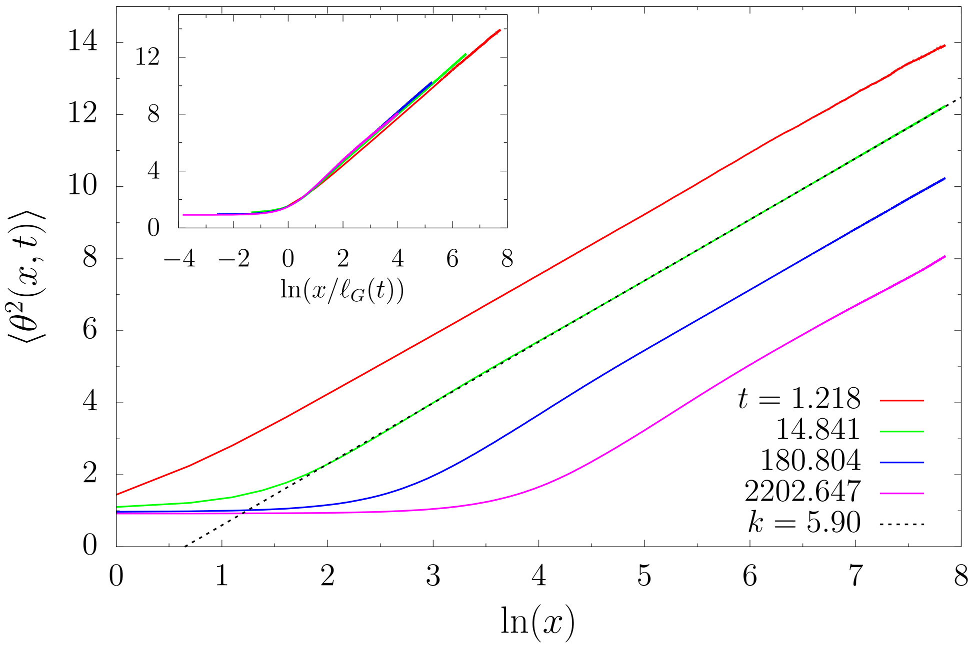
The winding angle

Winding angle *vs.* wall curvilinear length

$$\langle \theta^2(x) \rangle = \text{ct} + \frac{4\kappa}{8+\kappa} \ln x$$

from SLE & CFT





Fit $\kappa \simeq 5.9$

At $2d$ critical percolation

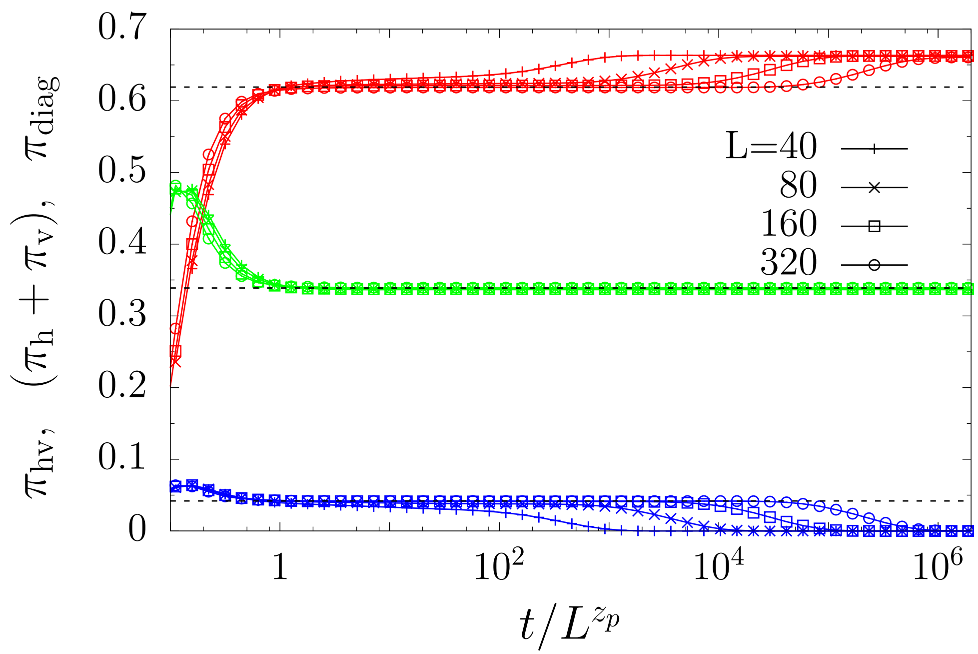
$\kappa = 6$

Saleur & Duplantier 87

Wieland & Wilson 03

Is it critical percolation ?

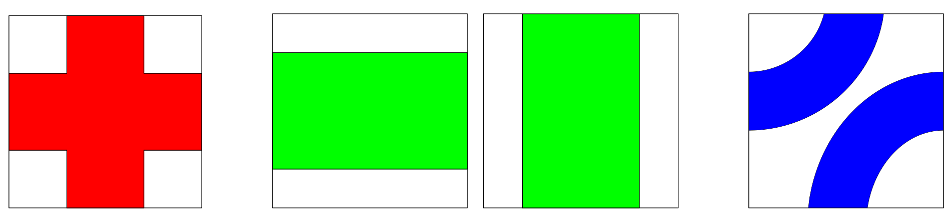
The probabilities of wrapping in different directions



$\pi_{hv} \approx 0.62 \;\; (0.6190)$

$\pi_h + \pi_v \approx 0.34 \;\; (0.3388)$

$\pi_d \approx 0.03 \;\; (0.0418)$

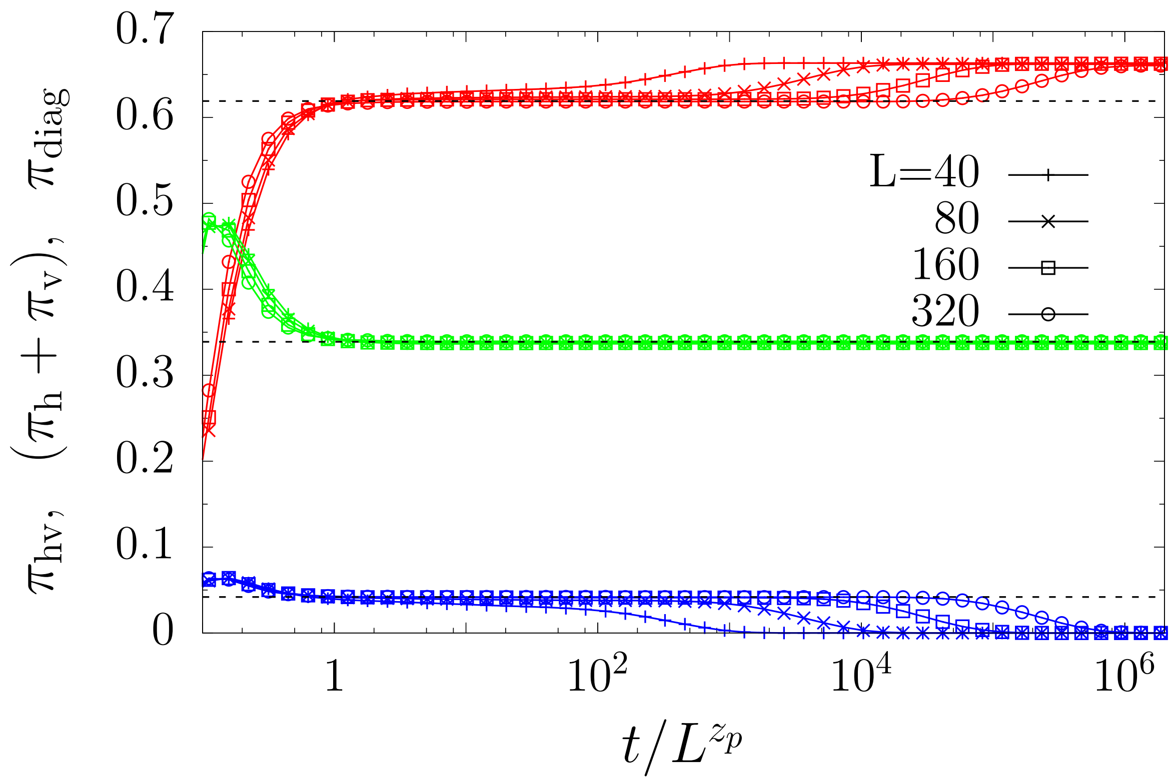


hv $\quad h + v \quad$ d

Exact values for $\pi_{hv}^{(p)}$, $\pi_h^{(p)} + \pi_v^{(p)}$, $\pi_d^{(p)}$ between parenthesis from **Pinson 94**

2d square IM at T=0

The final configuration was decided at t_p



$\pi_{hv} \approx 0.62$

$\pi_h + \pi_v \approx 0.34$

$\pi_d \approx 0.03$

stripe final states with the probabilities of $2d$ critical percolation

Barros, Olejarz, Krapivsky & Redner 09-12, Blanchard & Picco 13

First conclusion

Approach to critical percolation

The zero-temperature NCOP dynamics of the $2d$ IM starting from a totally uncorrelated $T_0 \to \infty$ paramagnetic initial state, approach uncorrelated critical percolation with a growing length $\ell_p(t)$.

The growing length $\ell_p(t)$ depends upon the *effective connectivity* of the lattice and the *microscopic dynamics*.

For example, $\ell_p(t) \simeq t^{1/z_p}$ with $z_p \approx 1/2$ for NCOP on the square lattice ($z_p < z_d = 2$).

Phenomenon also observed for $T_c < T_0 < \infty$ and $0 < T < T_c$, local and non-local COP dynamics, under weak-disorder, *etc.*

Arenzon, Bray, LFC & Sicilia 07-09

Blanchard, Corberi, LFC, Insalata, Picco, Ricateau & Tartaglia 12-17

First conclusion

Approach to critical percolation : why is this feature interesting ?

A mechanism that went unnoticed in this context so-far.

Seems to be universal.

In RG language, it suggests the first approach to a fixed point that is not fully attractive (critical percolation), that acts as the 'true' initial state, and the subsequent departure *via* curvature driven coarsening from it.

Analytical challenge : how can one prove this claim ? SLE, CFT ?

Manifold consequences :

metastability, blocked striped states at zero temperature

corrections to dynamic scaling, $\xi_p(t) \simeq t^{1/z_p}, \ \xi_d(t) \simeq t^{1/z_d}$.

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Very brief presentation, open to discussions

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Main interest

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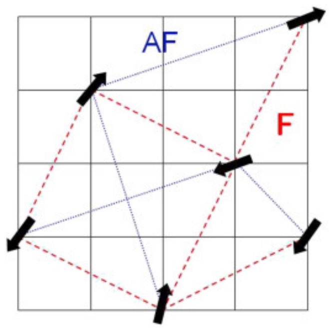
No time, open to discussions

Quenched disorder

Disordered Spin interactions



Instead

$V = -\sum_{ij} J_{ij} s_i s_j - \sum_{ijk} J_{ijk} s_i s_j s_k + \dots$

the exchanges $J_{ij}, J_{ijk},$ etc. taken from

a probability distribution (details later)

Continuous variables $s_i \in \mathbb{R}$

Spherical constraint $\sum_{i=1}^{N} s_i^2 = N$

Connection with the following problem

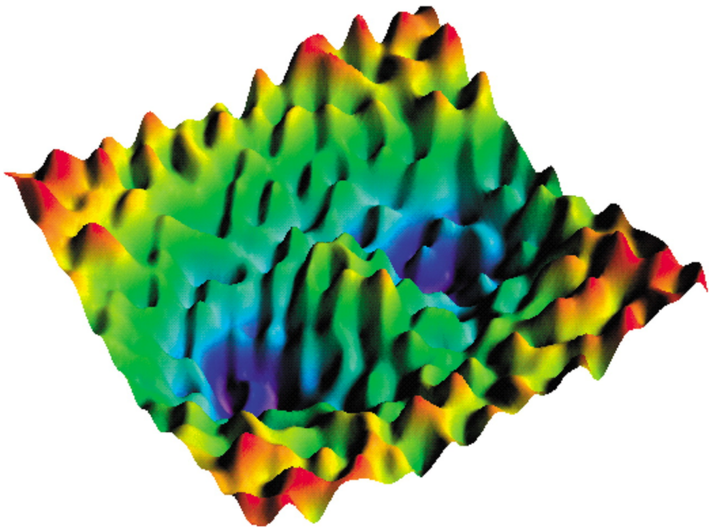
A particle

position $\vec{s} = (s_1, \dots, s_N)$

in an N dimensional space

under a random potential $V(\vec{s})$

Sketch for $N = 2$



but wrapped on the sphere

General setting

Classical mechanics

In spin models, no inertia. But, in the **particle in a random potential** interpretation, one can add kinetic energy.

Coordinate-momentum pair $\{\vec{s}, \vec{p}\}$ and Hamiltonian

$$H(\vec{p}, \vec{s}) = K(\vec{p}) + V(\vec{s})$$

with the kinetic energy $K(\vec{p}) = \frac{1}{2m} \sum_{i=1}^{N} p_i^2$

Newton-Hamilton equations

$$\boxed{\dot{s}_i = \frac{p_i}{m} \qquad\qquad \dot{p}_i = -\frac{\partial V(\vec{s})}{\partial s_i}}$$

of the isolated system

General setting

System coupled to a bath

Statistical equilibrium: partition function

$$\mathcal{Z} = \int \mathcal{D}\vec{p}\,\mathcal{D}\vec{s}\; e^{-\beta H(\vec{p},\vec{s})}$$

Relaxation dynamics: Langevin equation

$$\dot{s}_i = \frac{p_i}{m} \qquad\qquad \dot{p}_i - \frac{\gamma}{m}\,p_i = -\frac{\partial V(\vec{s})}{\partial s_i} + \xi_i$$

of the open system

$\langle \xi_i(t) \rangle = 0$ and $\langle \xi_i(t)\xi_j(t') \rangle = 2\gamma k_B T\,\delta_{ij}\,\delta(t-t')$

General setting

System coupled to a bath

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of the open system

$\langle \xi_i(t) \rangle = 0$ and $\langle \xi_i(t)\xi_j(t') \rangle = 2\gamma k_B T(t)\ \delta_{ij}\ \delta(t-t')$

Focus on :

Switching off the bath $\gamma = 0$ (dynamics of isolated systems)

Time-dependent temperature protocols $T(t)$ (cycles in spin-glasses, Kovacs effects, etc.) in the overdamped limit

Quenched disorder

Classes according to $V_J(\vec{s})$

From replica theory, Thouless-Anderson-Palmer equations, cavity methods, and relaxation dynamics:

two body	higher monomial	tuned polynomial
$\sum_{i\neq j} J_{ij} s_i s_j$	$\sum_{i\neq j\neq k} J_{ijk} s_i s_j s_k$	$a_1 \sum_{i\neq j} J_{ij} s_i s_j + a_2 \sum_{i\neq j\neq k} J_{ijk} s_i s_j s_k$
N saddles	$\exp(N\Sigma)$ saddles	$\exp(N\Sigma)$ saddles
Finite barriers	Barriers scale with N	with N^α and $\alpha < 1$
Domain growth	Fragile glasses	Spin-glasses

Quenched disorder

According to $[V_J(\vec{s})V_J(\vec{s}\,')] = N f(\vec{s}\cdot\vec{s}\,'/N) = N f(C)$

From relaxation dynamics:

two body	higher monomial	tuned polynomial
$\sum_{i\neq j} J_{ij} s_i s_j$	$\sum_{i\neq j\neq k} J_{ijk} s_i s_j s_k$	$a_1 \sum_{i\neq j} J_{ij} s_i s_j + a_2 \sum_{i\neq j\neq k} J_{ijk} s_i s_j s_k$

$$\eta(C) \equiv \frac{f'''(C)}{(f''(C))^{3/2}} = \begin{cases} 0 & \text{domain growth} \\ \uparrow \text{ for } C\downarrow & \text{fragile glass} \\ \downarrow \text{ for } C\uparrow & \text{spin-glass} \end{cases}$$

LFC & Le Doussal 96

Quenched disorder

According to $[V_J(\vec{s})V_J(\vec{s}')] = N f(\vec{s}\cdot\vec{s}'/N) = N f(C)$

From relaxation dynamics:

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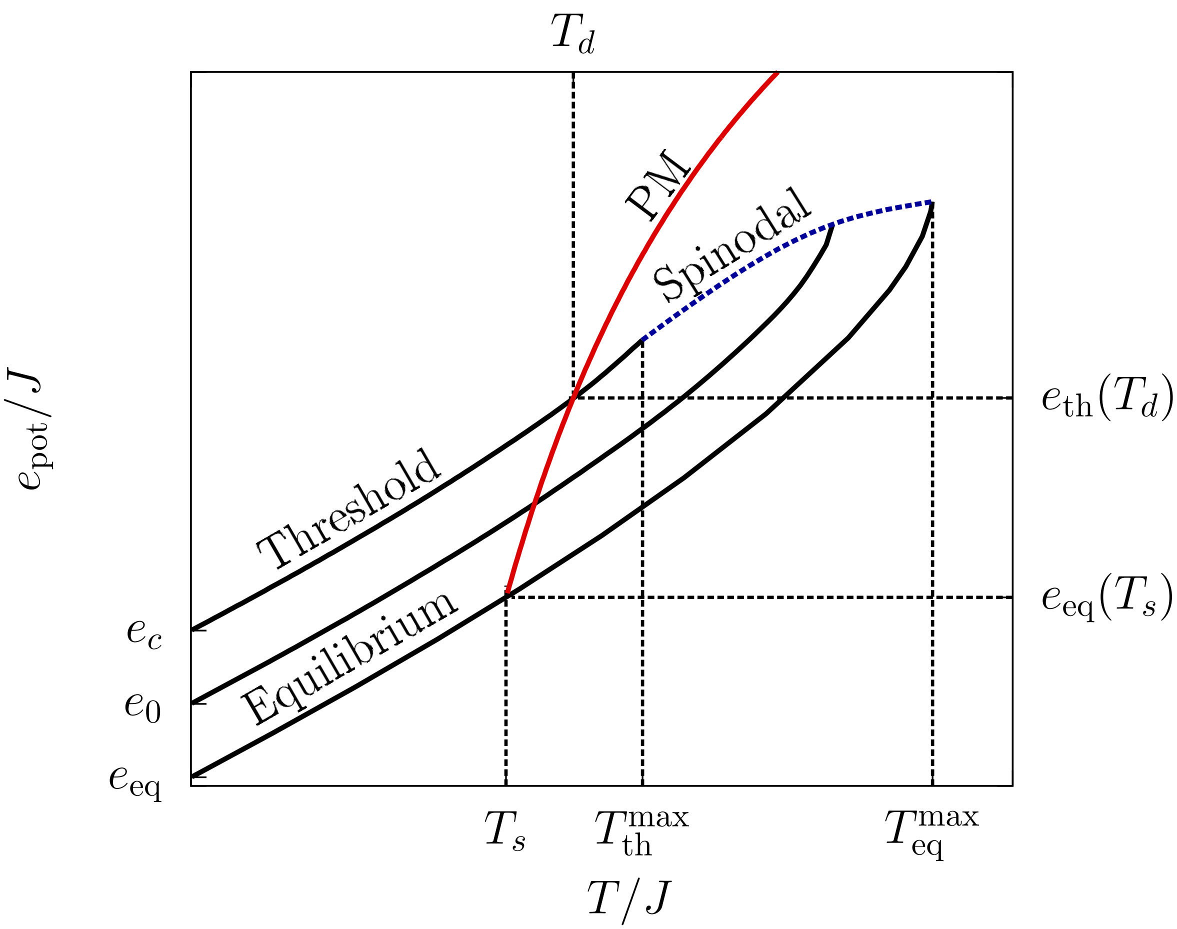
Three body interactions

Potential energy landscape: a guideline



Kurchan, Parisi & Virasoro 92, LFC & Kurchan 93, Cavagna, Giardina & Parisi 97-04

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Questions

Does an isolated system reach equilibrium ?

Boosted by recent interest in

— the dynamics after quantum quenches of cold atomic systems

rôle of interactions (integrable *vs.* non-integrable)

— many-body localisation

novel effects of quenched disorder

Foini, Gambassi, Konik & LFC 17. de Nardis, Panfil *et al.* 17

And, an isolated classical systems ?

The (old) ergodicity question revisited

LFC, Lozano & Nessi 17. LFC, Lozano, Nessi, Picco & Tartaglia 17

Quantum quenches

Definition & questions

- Take an isolated quantum system with Hamiltonian $\hat{H}_0$
- Initialize it in, say, $|\psi_0\rangle$ the ground-state of $\hat{H}_0$ (or any $\hat{\rho}(t_0)$)
- Unitary time-evolution $\hat{U} = e^{-\frac{i}{\hbar}\hat{H}t}$ with a Hamiltonian $\hat{H}$.

Does the system reach a steady state ?

Is it described by a thermal equilibrium density matrix $e^{-\beta_f \hat{H}}$?

Do at least some observables behave as thermal ones?

Does the evolution occur as in equilibrium ?

Other kinds of density matrices ?

Classical quenches

Definition & questions

- Take an isolated classical system with Hamiltonian H_0, evolve with H
- Initialize it in, say, ψ_0 a configuration, *e.g.* $\{\vec{s}_i, \vec{p}_i\}$ for a particle system.

  ψ_0 could be drawn from a probability distribution, *e.g.* $\mathcal{Z}^{-1} e^{-\beta' H_0(\psi_0)}$

Does the system reach a steady state ?

Is it described by a thermal equilibrium density matrix $e^{-\beta_f H}$?

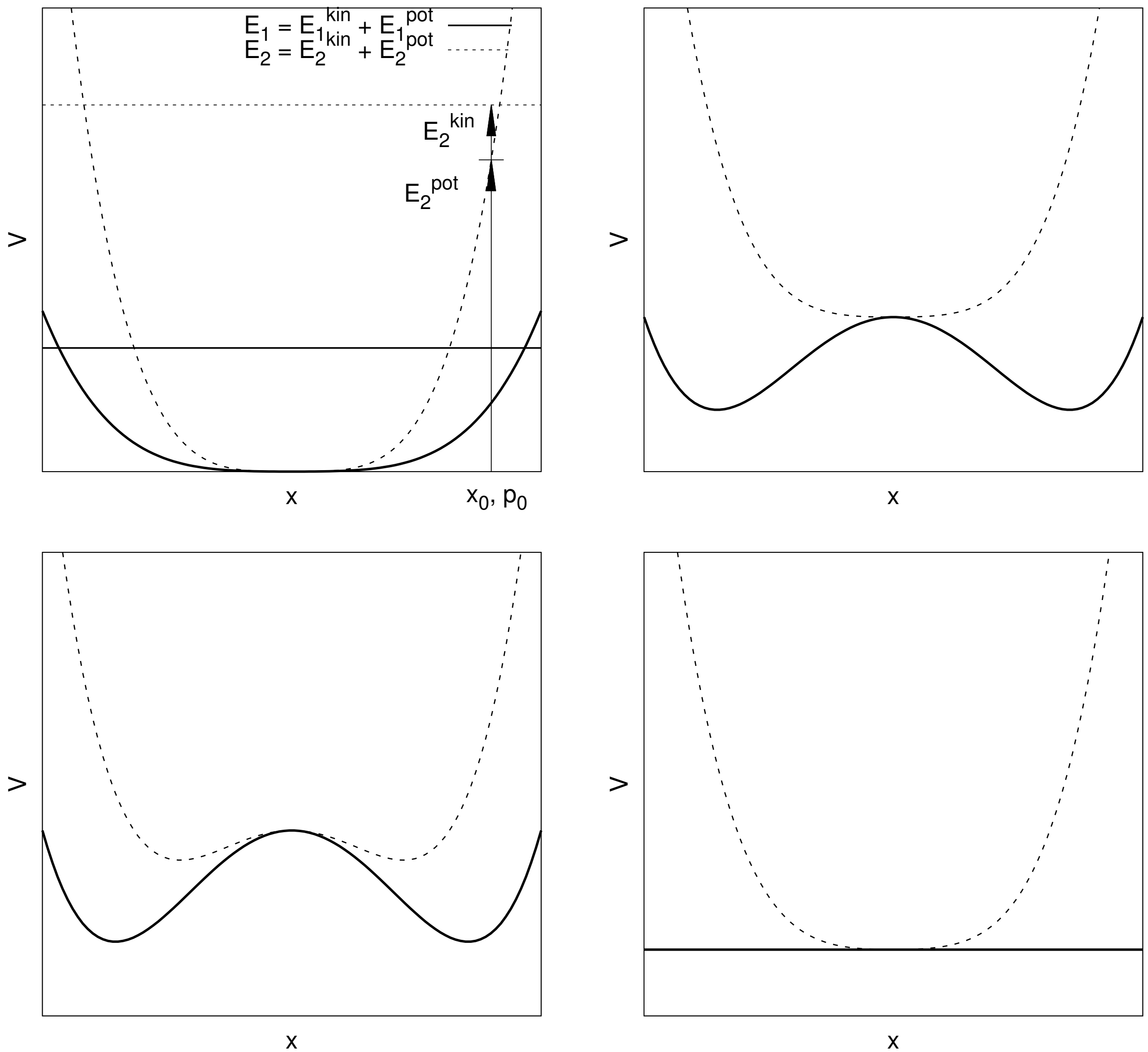
Do at least some observables behave as thermal ones?

Does the evolution occur as in equilibrium ?

Other kinds of probability distributions ?

Quenches

Simple examples (kind of building blocks)



At $t = 0$ change in V

Continuity in variables

$x(0^-) = x(0^+) = x_0$

$p(0^-) = p(0^+) = p_0$

Jump in (potential) energy

dashed to solid:

energy extraction

solid to dashed:

energy injection

The p spin models

$p \geq 3$ clearly non-integrable

Gibbs-Boltzmann equilibrium expected (β_f)

unless the system is set on the threshold

$p = 2$ integrable !

Neumann's 1850 model of classical mechanics (thanks to **Olivier Babelon**)

N constants of motion in involution **K. Uhlenbeck 82**

No Gibbs-Boltzmann equilibrium expected

Generalized Gibbs Ensemble :

$$P(\vec{s},\vec{p}) = \mathcal{Z}^{-1} \; e^{-\sum_{\mu=1}^{N} \beta_\mu I_\mu(\vec{s},\vec{p})} \quad ?$$

Quantum : **Rigol, Dunjko, Olshanii, Muramatsu 07-09**

Cardy, Caux, Calabrese, Essler, etc.

The initial conditions

- We chose initial states drawn from canonical equilibrium with Hamiltonian H_0 at inverse temperature β'
- We tune β' to choose the initial states from
  - — the high temperature disordered paramagnetic (PM) phase
  - — the low temperature equilibrium phase
  - — a metastable state

  The two last ones are different in the $p = 2$ and $p = 3$ cases.

More later

The quench

Disordered Spin Interactions

$V = -\sum_{ij} J_{ij} s_i s_j - \sum_{ijk} J_{ijk} s_i s_j s_k + \dots$

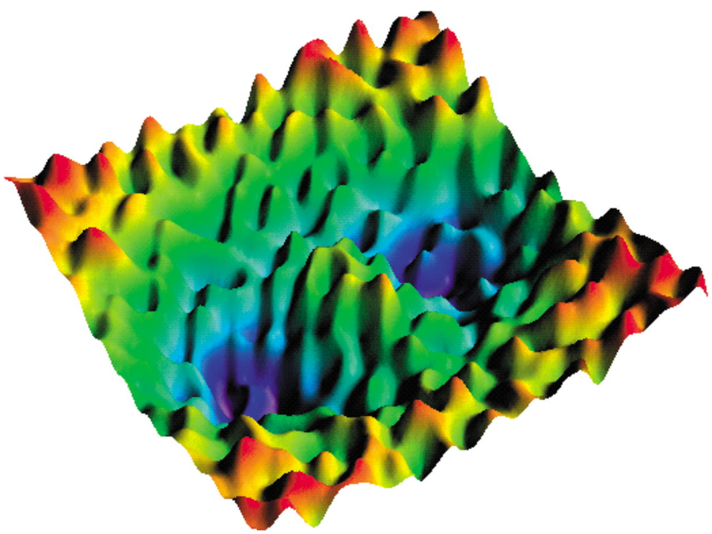
with exchanges J_{ij}, J_{ijk}, etc. taken

from a Gaussian pdf with

zero mean $[J_{ij}] = 0$ and

$[J^2_{i_1\dots i_p}] = p! J_0^2/(2N^{p-1})$

Initial energy scale J_0

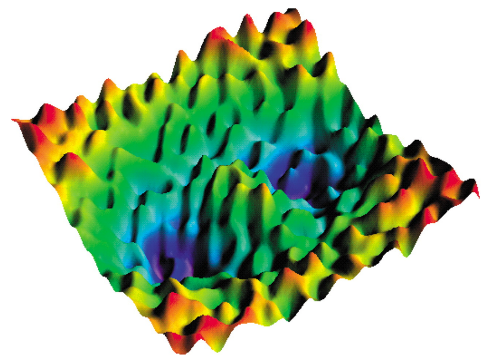


At time $t = 0$

Same configuration $p_i(0), s_i(0)$

quench $J^0_{i_1\dots i_p} \mapsto J_{i_1\dots i_p}$

Final energy scale J



The rugged landscape is

stretched/contracted and pulled up/down

On the sphere

$\gamma = 0$ bath switched off

Dynamic equations

Conservative dynamics

In the $N \to \infty$ limit exact causal Schwinger-Dyson equations

$$(m\partial_t^2 - z_t)R(t,t_w) = \int dt' \, \Sigma(t,t')R(t',t_w) + \delta(t-t_w)$$

$$(m\partial_t^2 - z_t)C(t,t_w) = \int dt' \, \left[\Sigma(t,t')C(t',t_w) + D(t,t')R(t_w,t')\right]$$

$$\boxed{+\frac{\beta' J_0}{J}\sum_{a=1}^{n} D_a(t,0)C_a(t_w,0)}$$

$$\boxed{(m\partial_t^2 - z_t)C_a(t,0) = \int dt' \, \Sigma(t,t')C_a(t',0) + \frac{\beta' J_0}{J}\sum_{a=1}^{n} D_b(t,0)Q_{ab}}$$

$a = 1, \ldots, n \to 0$, replica method to deal with $e^{-\beta' H_0(\vec{s}(0), \, \vec{p}(0))}$

Dynamic equations

Conservative dynamics

In the $N \to \infty$ limit exact causal Schwinger-Dyson equations

with the post-quench self-energy and vertex

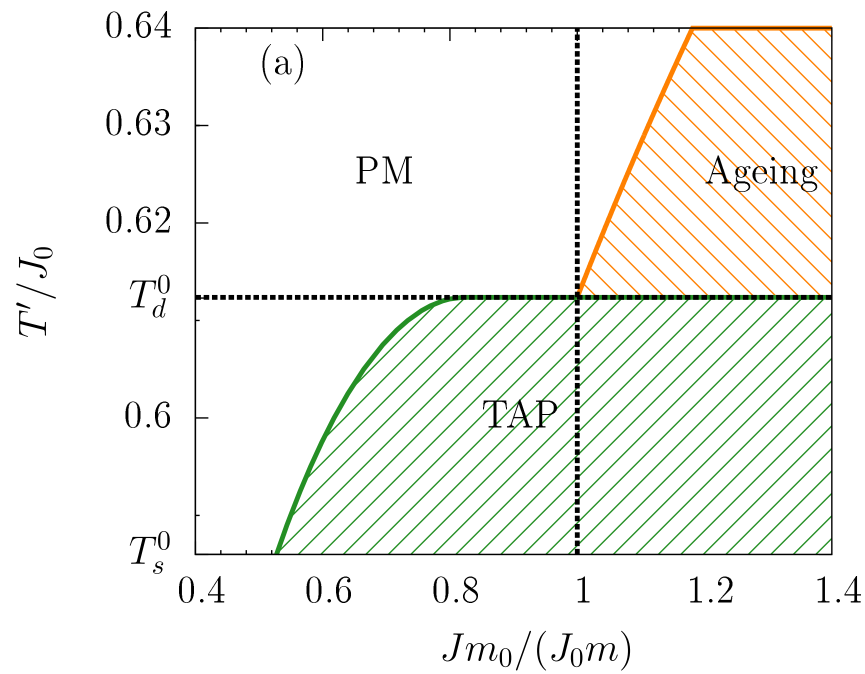
$$
\begin{aligned}
D(t, t_w) &= \frac{J^2 p}{2} \, C^{p-1}(t, t_w) \\
D_a(t, 0) &= \frac{J^2 p}{2} \, C_a^{p-1}(t, 0) \\
\Sigma(t, t_w) &= \frac{J^2 p(p-1)}{2} \, C^{p-2}(t, t_w) \, R(t, t_w)
\end{aligned}
$$

and the Lagrange multiplier z_t fixed by $C(t, t) = 1$

Solvable numerically & analytically at long times

Three body model

Dynamic phase diagram



Initial conditions

Energy injection

Energy extraction

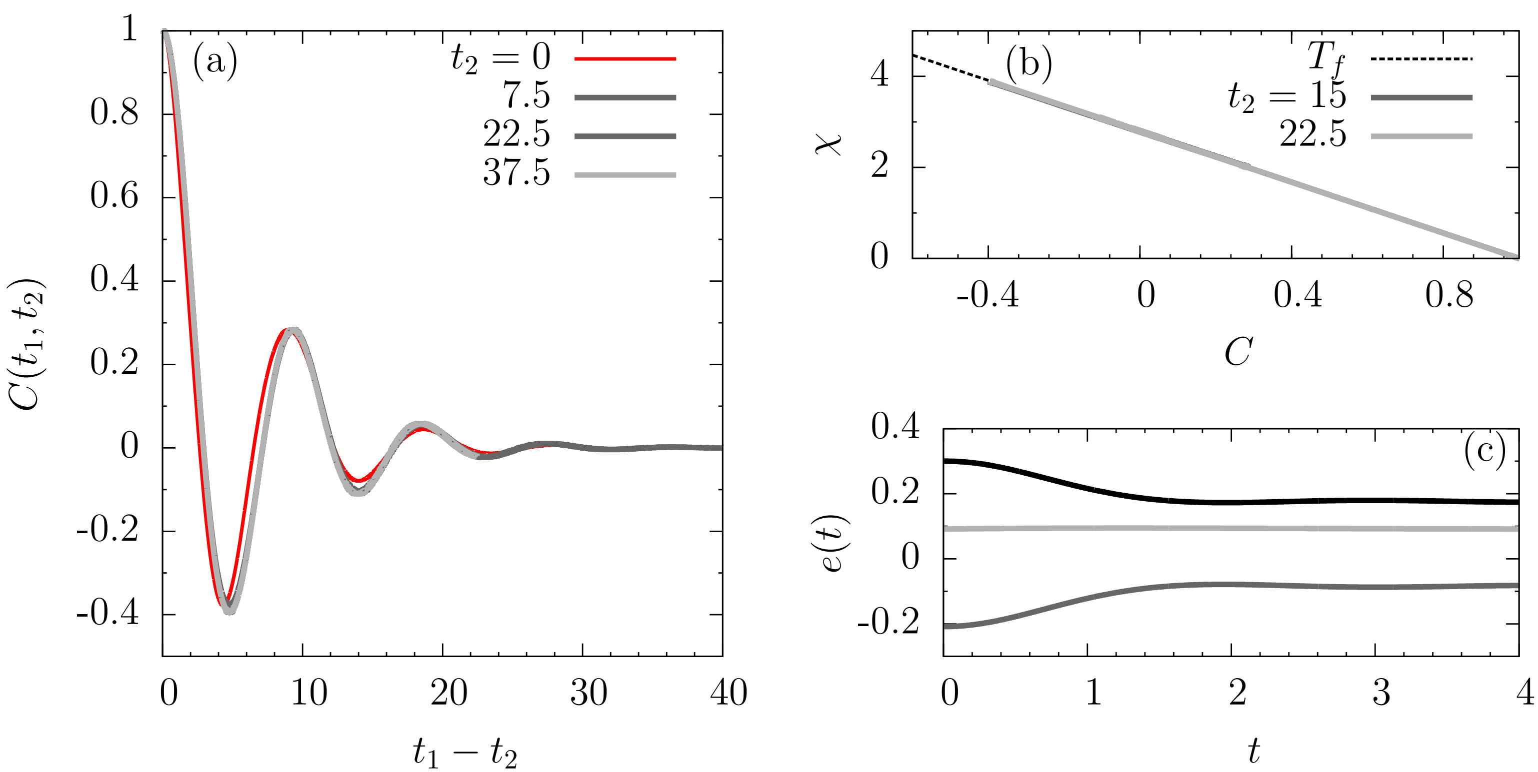
In PM, quenches go to GB equilibrium at $\beta_f(e_f)$ with e_f the final energy

Following metastable states, GB-like equilibration at β_f determined by e_f

Out of equilibrium relaxation with ageing effects when $e_f = e_{\rm th}$

Three body model

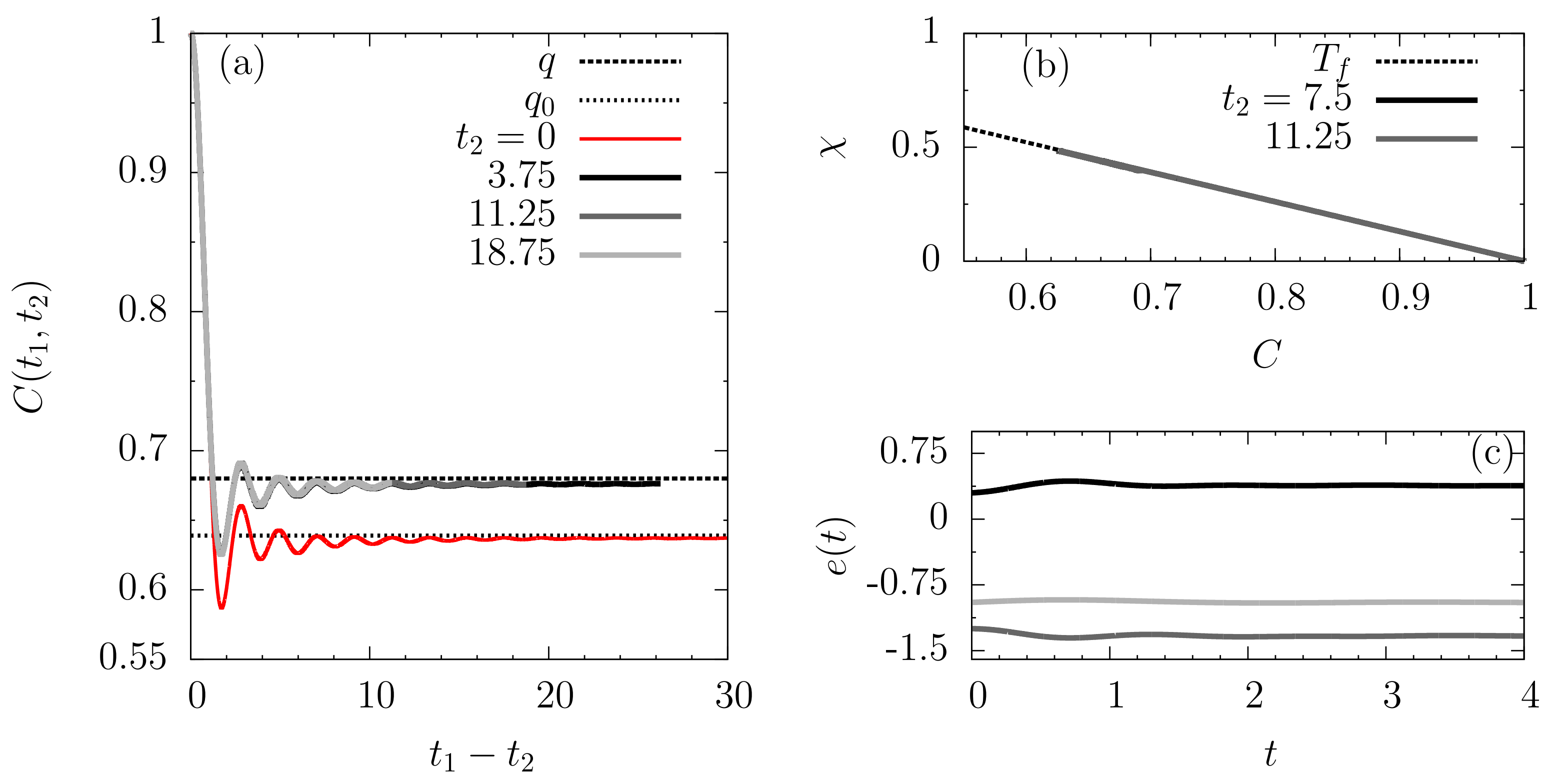
From equilibrium within a TAP state ($T' < T_d^0$) to the PM



GB equilibration at the temperature of a PM $\quad T_f = e_f + \sqrt{J^2 + e_f^2}$

Three body model

Initial and final configurations in a metastable (TAP) state



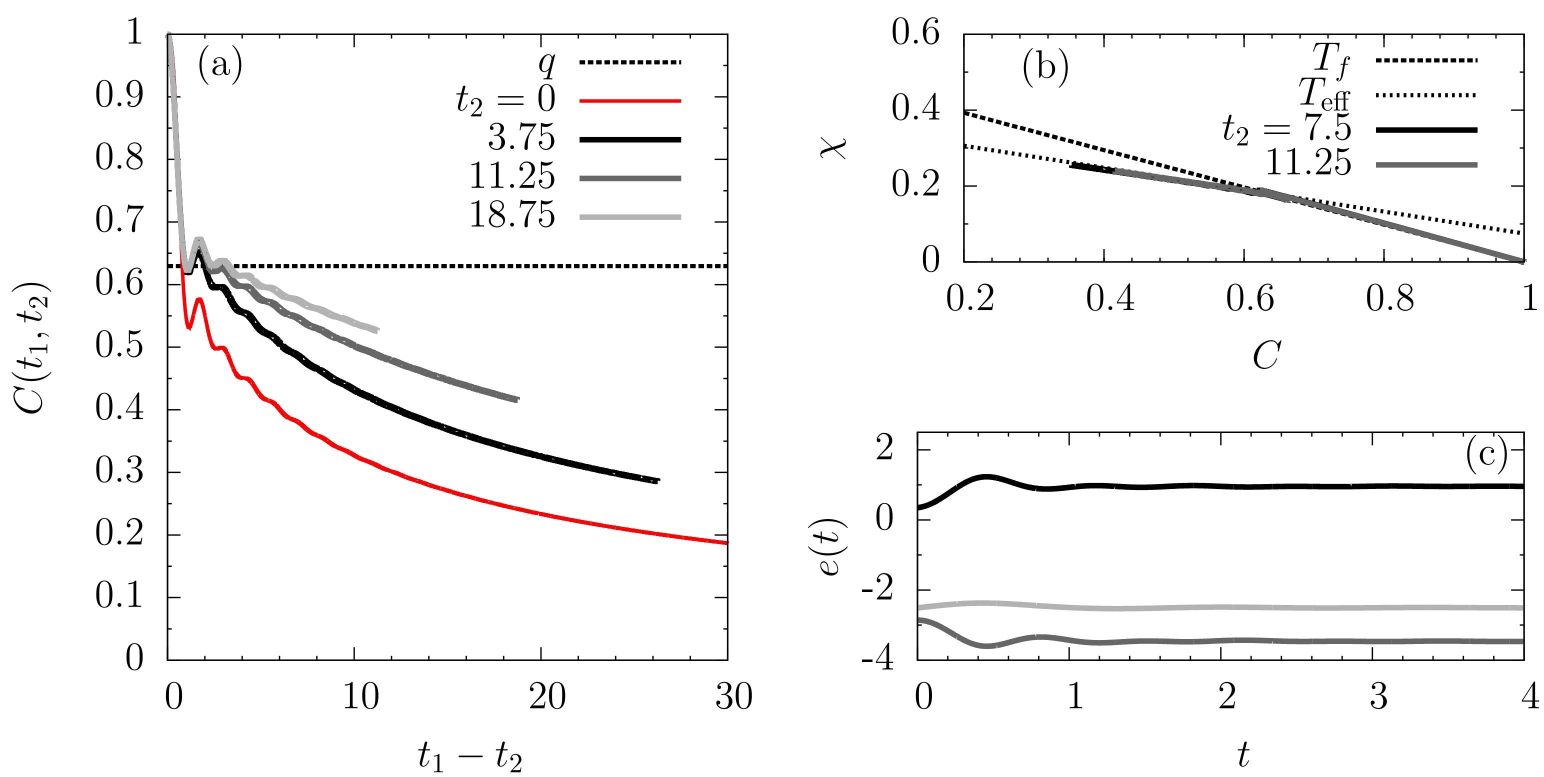
$C(t_1, 0) \to q_0$ Fidelity

$\lim_{t_1 - t_2 \gg t_0} \lim_{t_2 \gg t_0} C(t_1, t_2) = q$ Decorrelation

Following metastable states, equilibration at β_f fixed by $e_f = e_f^{\rm kin} + e_f^{\rm pot}$

Three body model

Energy extraction from PM to threshold

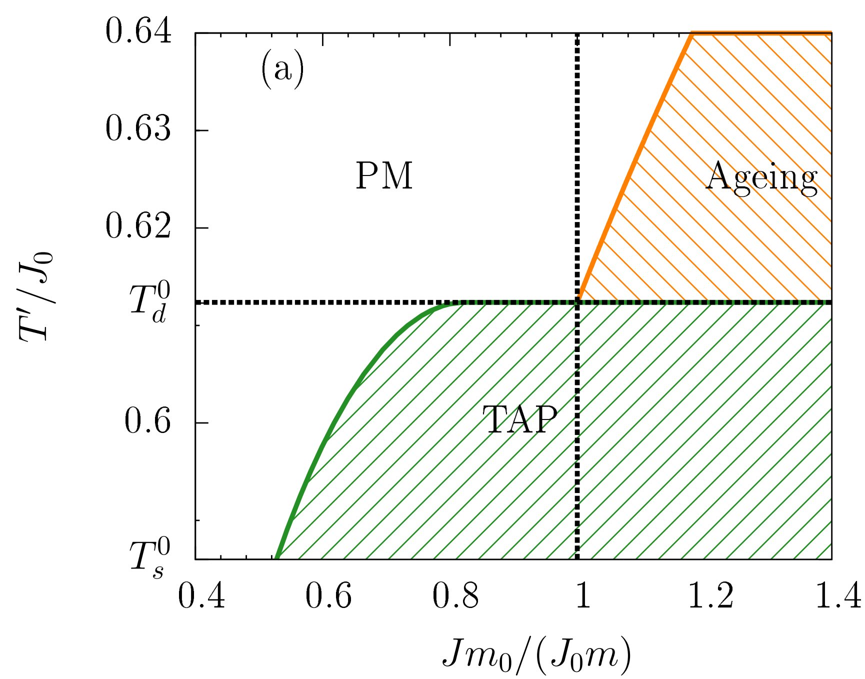


Similar to the relaxational case. Two temperature behaviour, fast and slow decay.

Out of equilibrium relaxation when quench parameters tuned so that $e_f = e_{\rm th}$

Three body model

Dynamic phase diagram - recap



Initial conditions

Energy injection Energy extraction

In PM, quenches go to GB equilibrium at $\beta_f(e_f)$ with e_f the final energy

Following metastable states, GB-like equilibration at β_f determined by e_f

Out of equilibrium relaxation with ageing effects (β_f and $\beta_{\rm eff}$) when $e_f = e_{\rm th}$.

The p spin models

$p \geq 3$ clearly non-integrable

Gibbs-Boltzmann equilibrium expected (β_f)

unless the system is set on the threshold

$p = 2$ integrable !

Neumann's 1850 model of classical mechanics (thanks to **Olivier Babelon**)

N constants of motion in involution **K. Uhlenbeck 82**

No Gibbs-Boltzmann equilibrium expected

Generalized Gibbs Ensemble :

$$P(\vec{s}, \vec{p}) = \mathcal{Z}^{-1} \; e^{-\sum_{\mu=1}^{N} \beta_\mu I_\mu(\vec{s},\vec{p})} \quad ?$$

Quantum : Rigol, Dunjko, Olshanii, Muramatsu 07-09

Cardy, Caux, Calabrese, Essler, etc.

Two body model

Non-linear coupling through the Lagrange multiplier only

Stat-phys notions: Potential energy landscape

The N eigenvectors of the J_{ij} matrix are saddles, the barriers between them are $\mathcal{O}(1)$, the absolute minimum is the alignment of $\vec{s}$ on the eigenvector $\vec{v}_N$ with eigenvalue λ_N at the edge of the spectrum.

Kosterlitz, Thouless & Jones 76 … LFC & Dean 96 … Fyodorov 12-17 …

Mehta, Hauenstein, Niemerg, Simm & Stariolo 14

Classical mechanics/integrable systems **K. Uhlenbeck 82**

Motion of a particle on S_{N-1}, enforced by $\sum_\mu s_\mu^2 = N$.

The integrals of motion are $I_\mu = s_\mu^2 + \frac{1}{N} \sum_{\nu(\neq\mu)} \frac{s_\mu^2 p_\nu^2 + s_\nu^2 p_\mu^2 - 2 s_\mu p_\mu s_\nu p_\nu}{\lambda_\nu - \lambda_\mu}$

Two body model

Non-linear coupling through the Lagrange multiplier only

Diagonal in the basis of eigenvectors $\vec{v}_\mu$ of the interaction matrix J_{ij}

Projection of the coordinate (spin) vector on the eigenvectors $s_\mu = \vec{s} \cdot \vec{v}_\mu$

with $\mu = 1, \dots, N$. Newton equations are almost quadratic

$$\boxed{m\ddot{s}_\mu(t) = [z(t) - \lambda_\mu] s_\mu(t)}$$

with $z(t)$ the Lagrange multiplier that enforces the spherical constraint and λ_μ the eigenvalues (semi-circle law, with support in $[-2J, 2J]$)

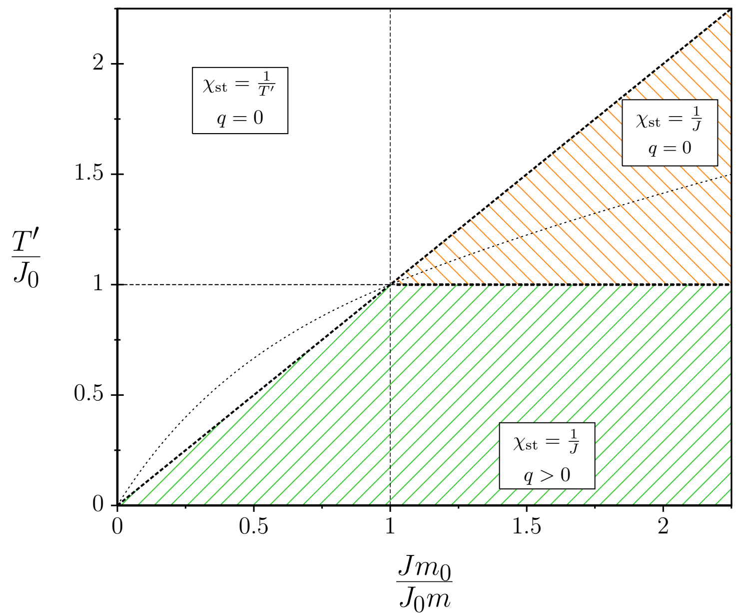
Two methods to solve:

— for $N \to \infty$, closed Schwinger-Dyson equations on $C(t, t_w)$ and $R(t, t_w)$, the global self-correlation and linear response (already shown for general p)

— for finite N, solve Newton equations under the spherical constraint. Similar to **Sotiriadis & Cardy 10** for quantum O(N) model

Two body model

Richer results !



Initial conditions

Three Sectors

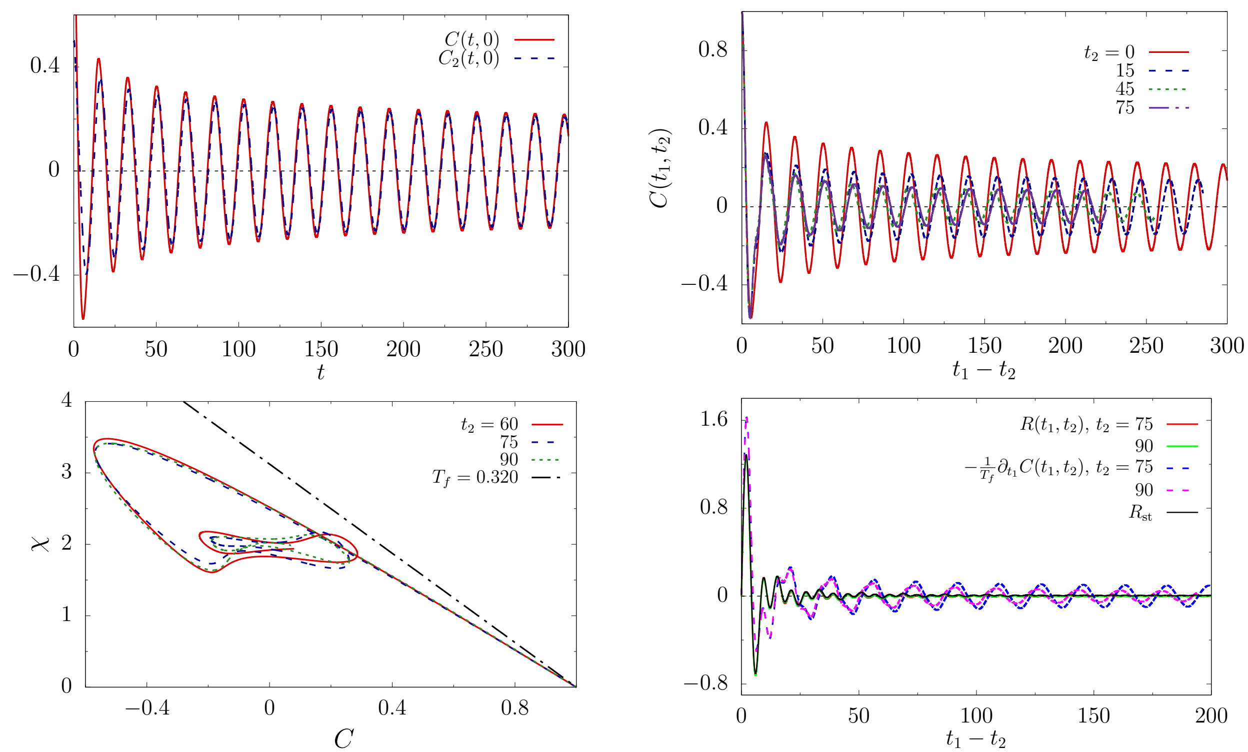
I $\chi_{st} = 1/T'$ and $\lim_{t \gg t_w} C(t, t_w) = 0$

II $\chi_{st} = 1/J$ and $\lim_{t \gg t_w} C(t, t_w) = 0$

III $\chi_{st} = 1/J$ and $\lim_{t \gg t_w} C(t, t_w) > 0$

Two body model

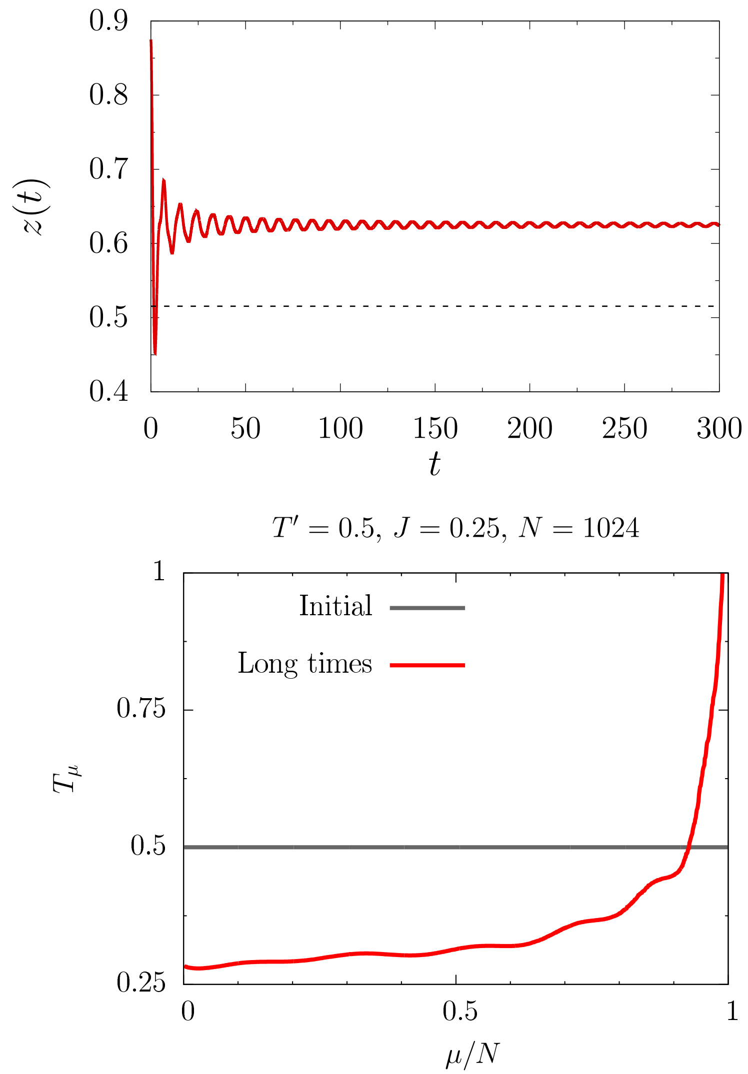
I Large energy injection on a condensed state



Stationary dynamics but no FDT at a single temperature | no GB equilibrium |

Two body model

- **Large energy injection on a condensed state: T_μ spectrum**



$$z(t) \to z_f = T' + J^2/T'$$

The time-dependent frequencies

$$\Omega^2_\mu(t) \to (z_f - \lambda_\mu)/m \equiv \omega^2_\mu$$

The μ modes $s_\mu(t)$ decouple and

become independent harmonic oscillators

with conserved energy after $t_{\rm st}$

$$e_\mu = e^{\rm kin}_\mu(t) + e^{\rm pot}_\mu(t)$$

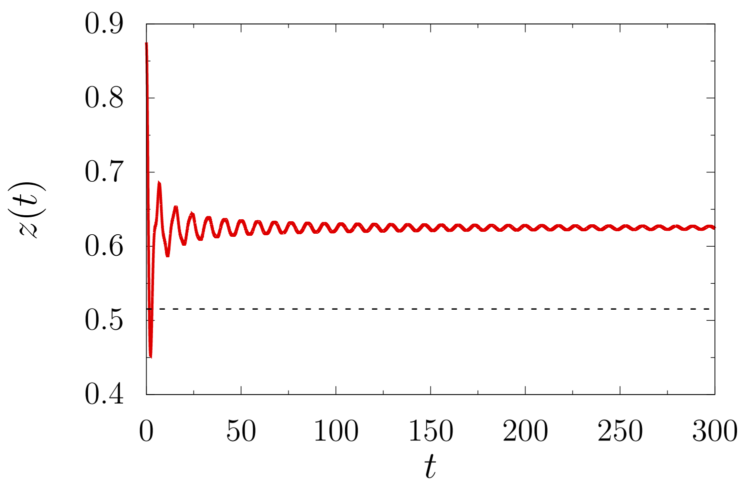
Mode temperatures

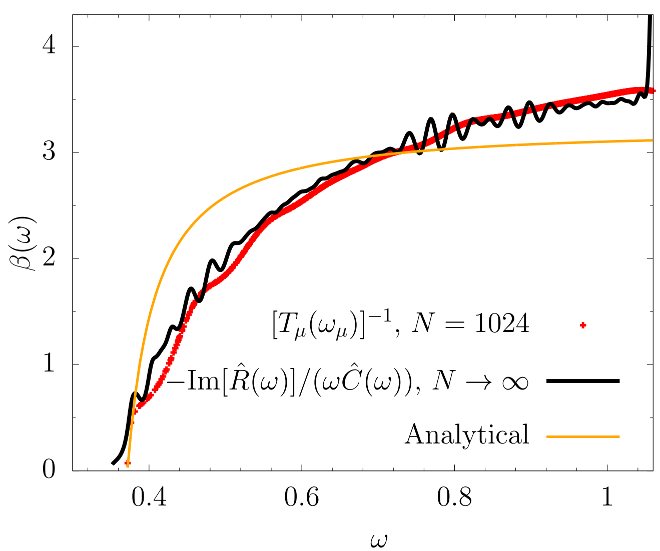
$$\overline{\langle H^{\rm kin}_\mu \rangle} = \overline{\langle H^{\rm pot}_\mu \rangle} = T_\mu$$

where $\overline{\ldots} = \lim_{\tau \gg 1} \frac{1}{\tau} \int_{t_{\rm st}}^{t_{\rm st}+\tau} dt' \ldots$

Two body model

- **Large energy injection on a condensed state:** T_μ **from the FDR**





$$z(t) \to z_f = T' + J^2/T'$$

The time-dependent frequencies too

$$\Omega_\mu^2(t) \to (z_f - \lambda_\mu)/m \equiv \omega_\mu^2$$

The μ modes $s_\mu(t)$ decouple and become independent harmonic oscillators with conserved energy

$$e_\mu = e_\mu^{\mathrm{kin}}(t) + e_\mu^{\mathrm{pot}}(t)$$

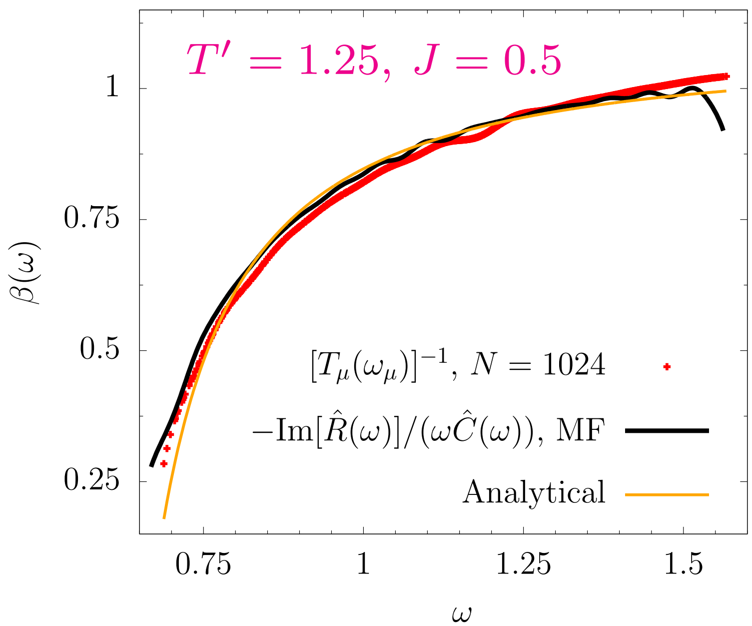
Mode inverse temperatures β_μ *vs* FDR inverse temperature

$$-\mathrm{Im}\hat{R}(\omega)/(\omega\hat{C}(\omega)) = \beta_{\mathrm{eff}}(\omega)$$

Analytical is a very rough approximation

GGE and FDT temperatures

A generic method



FDR

$$-\frac{\mathrm{Im}\hat{R}(\omega)}{\omega\hat{C}(\omega)} = \beta_{\mathrm{eff}}(\omega)$$

The asympt mode freq

$$\omega_\mu^2 = (z_f - \lambda_\mu)/m$$

$$-\frac{\mathrm{Im}\hat{R}(\omega_\mu)}{\omega_\mu\hat{C}(\omega_\mu)} = \beta_\mu$$

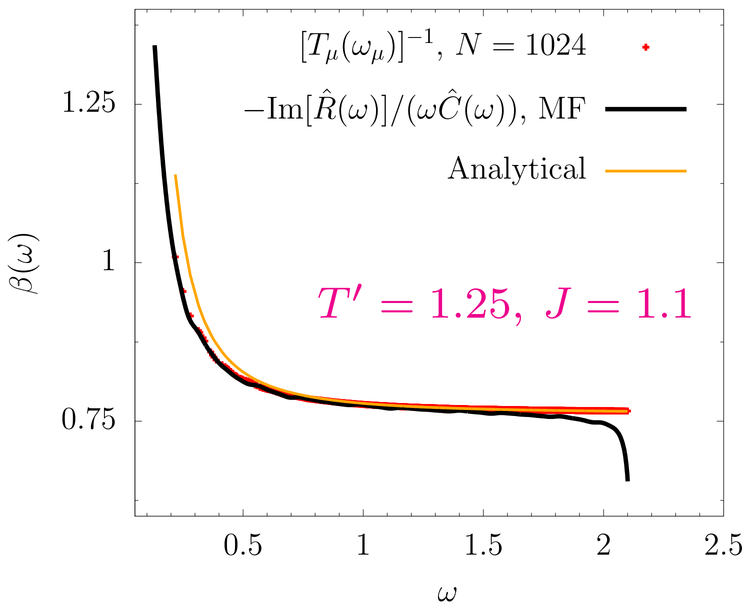
Using the idea in **Foini, Gambassi, Konik & LFC 17, de Nardis, Panfil** ***et al*** **17**

for **quantum integrable systems** now in a **classical integrable** model

LFC, Lozano, Nessi, Picco & Tartaglia 17

GGE and FDT temperatures

A generic method



FDR

$$-\frac{\mathrm{Im}\hat{R}(\omega)}{\omega\hat{C}(\omega)} = \beta_{\mathrm{eff}}(\omega)$$

The asympt mode freq

$$\omega_\mu^2 = (z_f - \lambda_\mu)/m$$

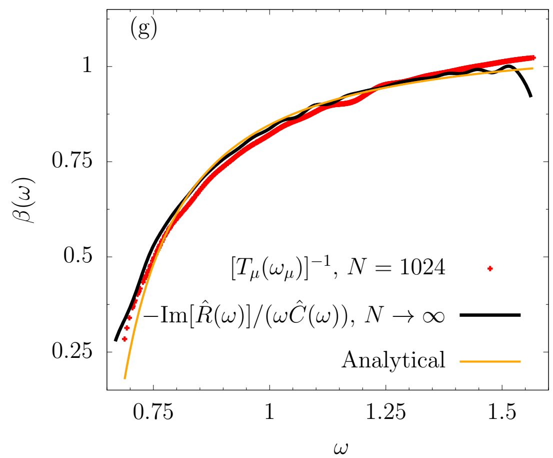
$$-\frac{\mathrm{Im}\hat{R}(\omega_\mu)}{\omega_\mu\hat{C}(\omega_\mu)} = \beta_\mu$$

Using the idea in **Foini, Gambassi, Konik & LFC 17, de Nardis, Panfil, ... 17**

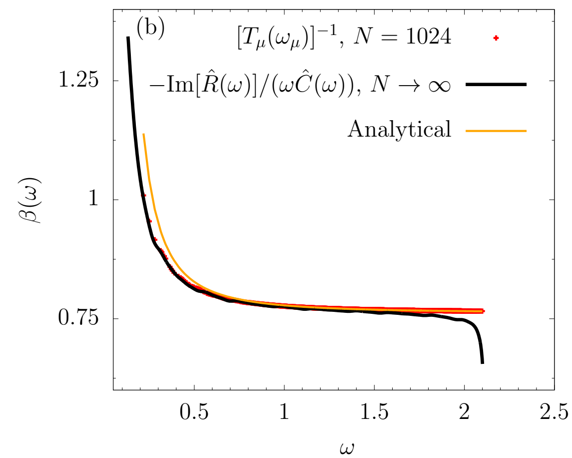
in a **classical** system **LFC, Lozano, Nessi, Picco & Tartaglia 17**

Two body model

The T_μs from the FDR



Injection



Extraction

A way to measure the mode temperatures with a single measurement

$$\beta_{\text{eff}}(\omega_\mu) = -\text{Im}\hat{R}(\omega_\mu)/(\omega_\mu \hat{C}(\omega_\mu)) = \beta_\mu$$

No “partial equilibration” contradiction from the effective temperature perspective. The modes are uncoupled, they do not exchange energy, and can then have different T_μs

Two body model

Two (or more) possibilities : GB, GGE (or none)

- The system **is not** able to act as a bath on itself and equilibrate to

$$\rho \neq \rho_{\rm GB} = \mathcal{Z}^{-1}\, e^{-\beta_f H}$$

as it is an integrable system.

- **Does it approach a Generalised Gibbs Ensemble (GGE)**

$$\boxed{\rho_{\rm GGE} = \mathcal{Z}^{-1}\, e^{-\sum_{\mu=1}^{N} \beta_\mu^{\rm GGE} I_\mu}}$$

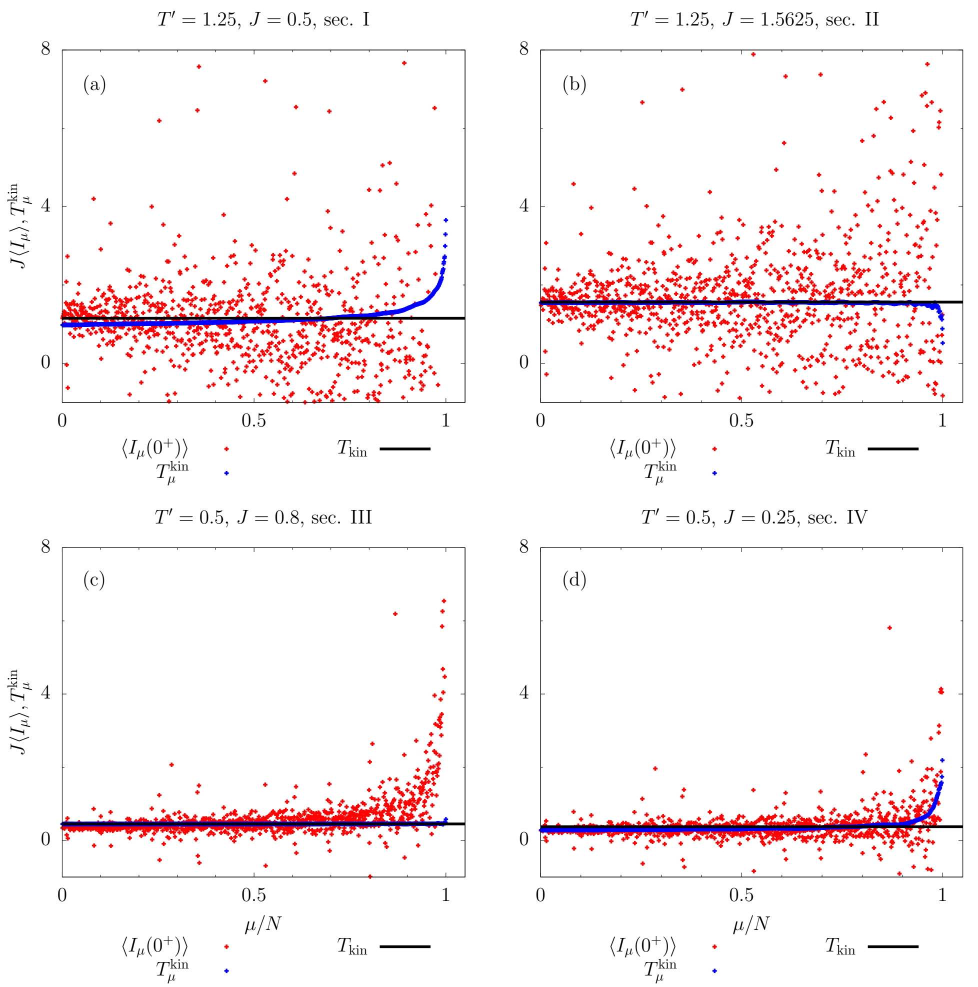
with Uhlenbeck's constants of motion I_μ and $\beta_\mu^{\rm GGE}$ fixed by

$$\boxed{\langle I_\mu \rangle_{\rm GGE} = I_\mu(t=0^+)} \qquad \textbf{?}$$

What are the relations between $\beta_\mu^{\rm GGE}$ and β_μ, and I_μ and e_μ ?

Two body model

Integrals of motion and mode energies



Conclusions

Study of the quenched dynamics of classical isolated disordered models

We showed that they can

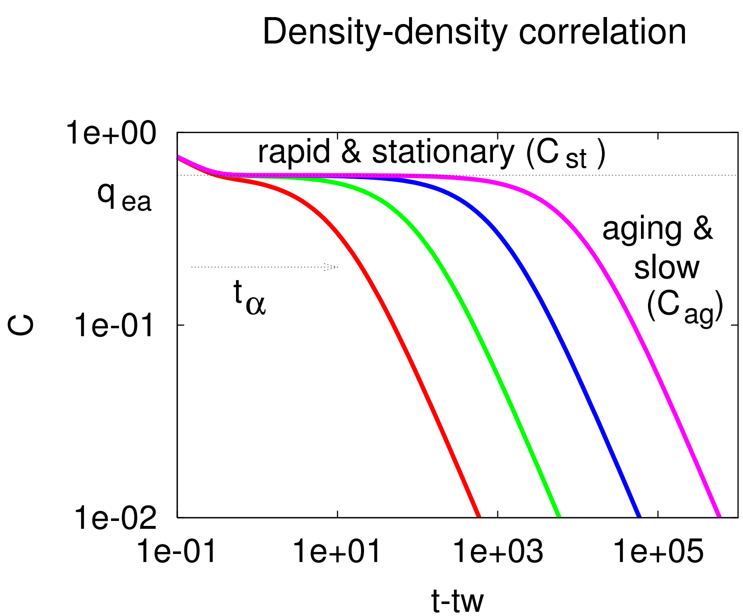
- equilibrate to GB measures
- undergo non-stationary (aging) dynamics
- or (most probably) approach a GGE

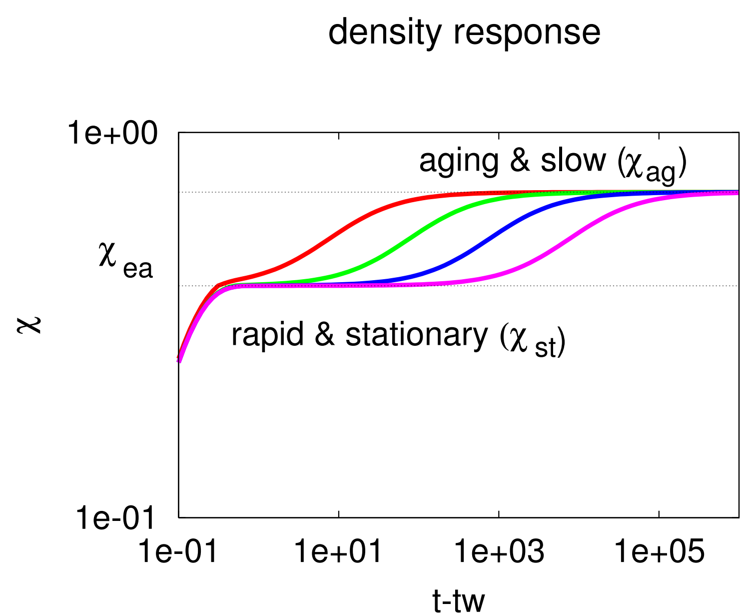
depending on the type of model (highly interacting or quasi quadratic) and the kind of quench performed.

Works on the extension of these studies to the quantum models and the better understanding of the approach to a GGE are under way

Glassy dynamics

Non stationary relaxation & separation of time-scales





$$C(t, t_w)$$

Correlation

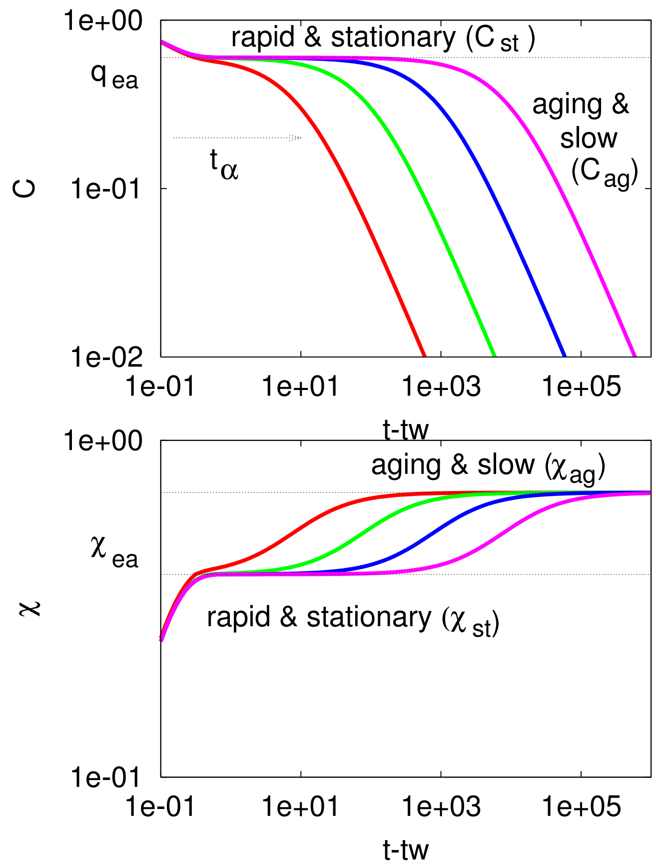
$$\chi(t, t_w) = \int_{t_w}^{t} dt' \; R(t, t')$$

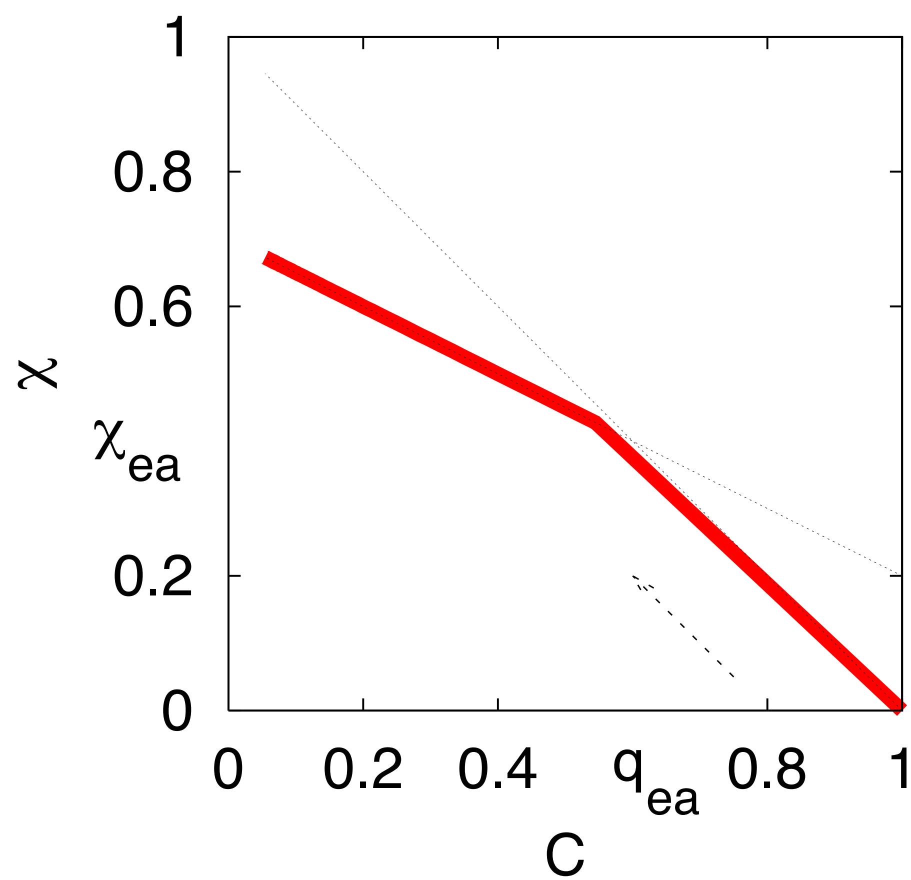
Time-integrated linear response

Analytic solution to a mean-field model **LFC & J. Kurchan 93**

Glassy dynamics

Fluctuation-dissipation relation : parametric plot





Analytic solution to a mean-field model **LFC & J. Kurchan 93**

Harmonic oscillator

β_μ and $\beta_{\rm eff}(\omega_\mu)$ after the quench $m\omega_\mu^0 x^2 \mapsto m\omega_\mu x^2$

Linear response Im $\hat{R}(\omega) = \dfrac{\pi}{2m\omega}\,\delta(\omega-\omega_\mu)$

Time delayed correlation $\omega\overline{\hat{C}}(\omega) = \dfrac{\pi\omega}{m\omega_\mu^2}\,e_{\rm tot}(0^+)\,\delta(\omega-\omega_\mu)$

with $e_{\rm tot}(0^+) = \left(m\omega^2 x_0^2 + p_0^2\right)/2$. (Overline is a long time average.)

Only the internal frequency of the oscillator ω_μ responds, and has a non-trivial contribution to the self-correlation.

The FDR is $$\boxed{\beta_{\rm eff}(\omega) = \frac{2\,\text{Im}\,\hat{R}(\omega)}{\omega\overline{\hat{C}}(\omega)} = e_{\rm tot}^{-1}(0^+)}$$

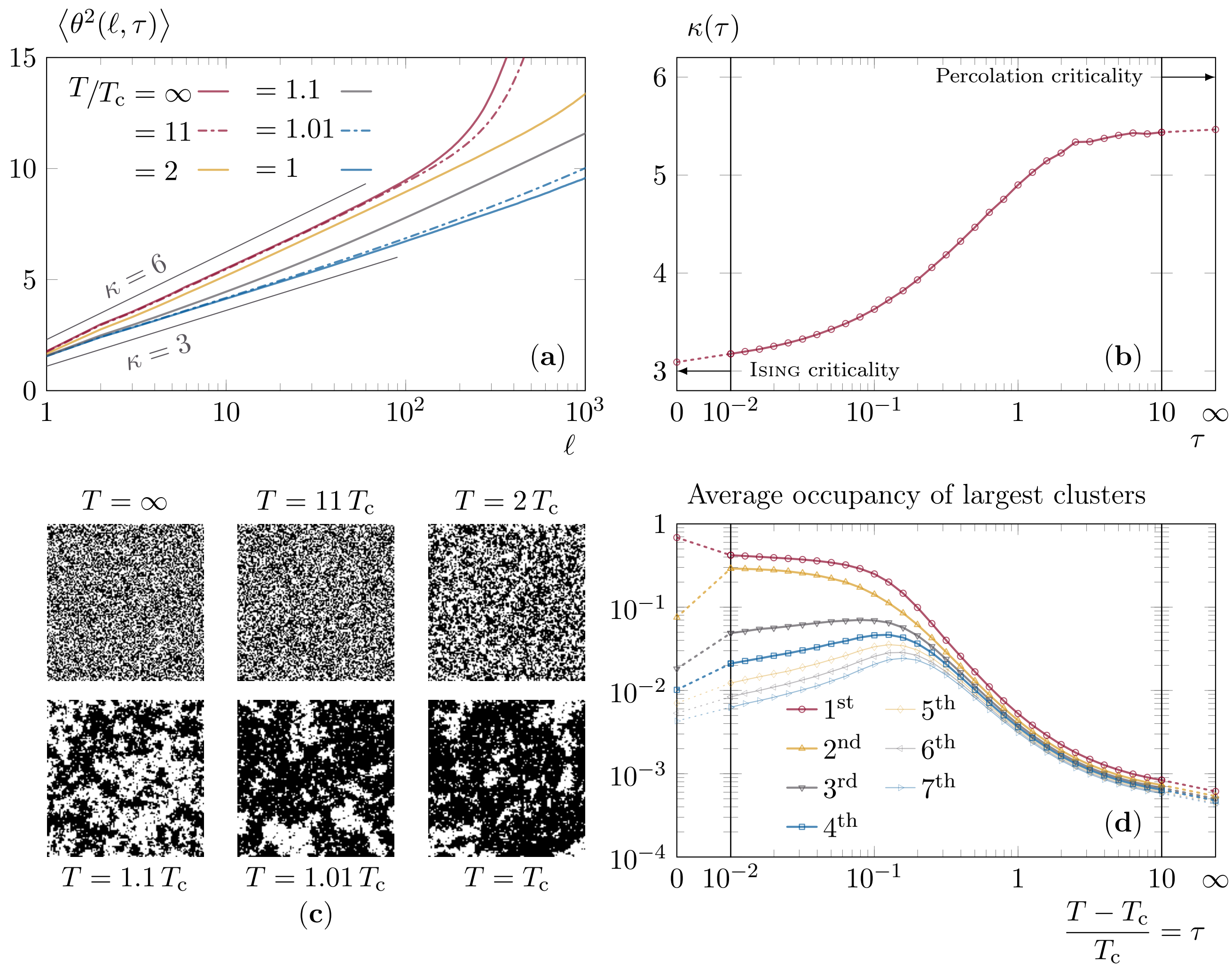
The GGE condition $\langle H\rangle_{\rm GGE} = \mathcal{Z}^{-1}\int dx \int dp\, e^{-\beta_{\rm GGE}H}\; H = e_{\rm tot}(0^+)$

The calculation in the left-hand-side yields $\langle H\rangle_{\rm GGE} = \beta_{\rm GGE}^{-1}$

Therefore $\beta_{\rm eff} = \beta_\mu$ (Different Fourier transform convention from main part of talk)

Initial temperature

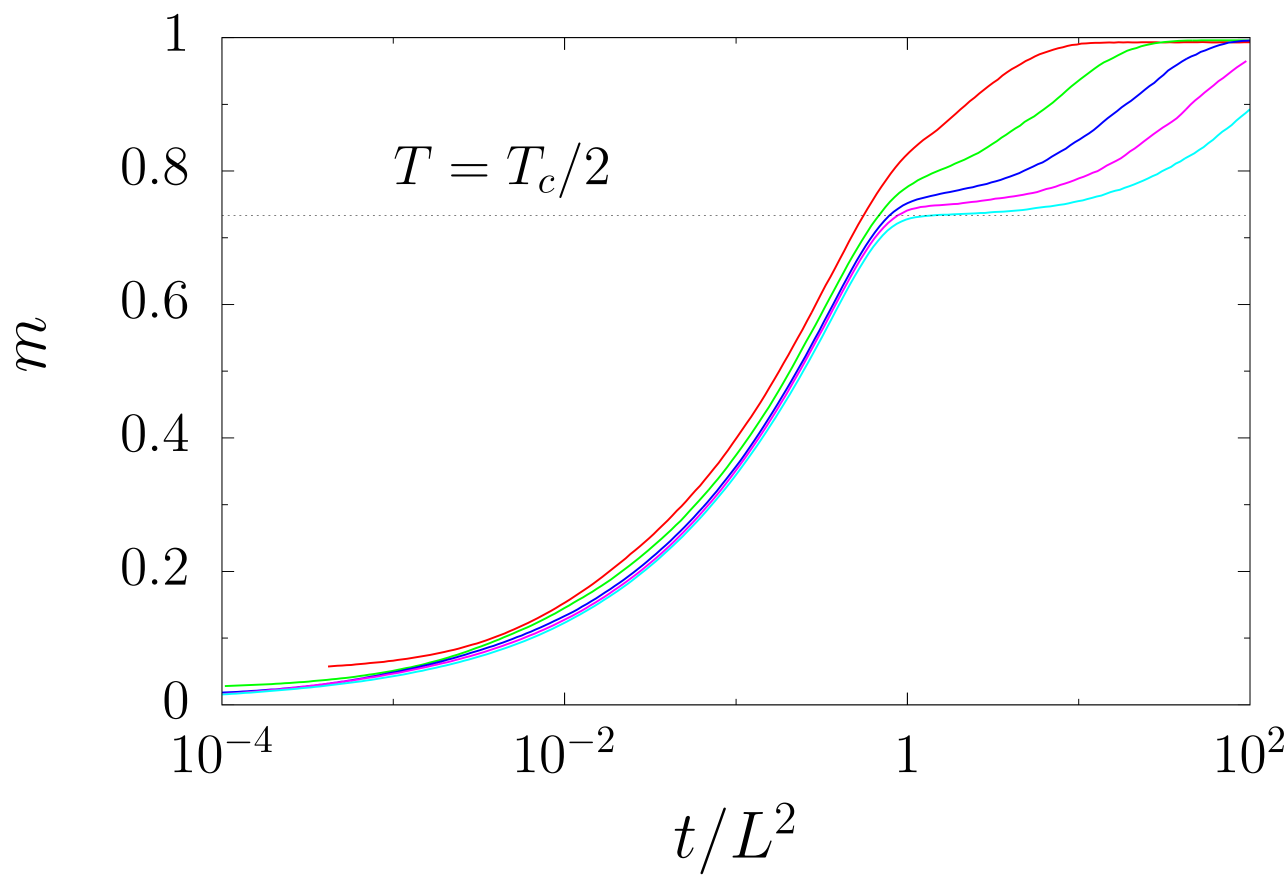
Structure in the paramagnetic phase



Ricateau, LFC & Picco 17

Final temperature

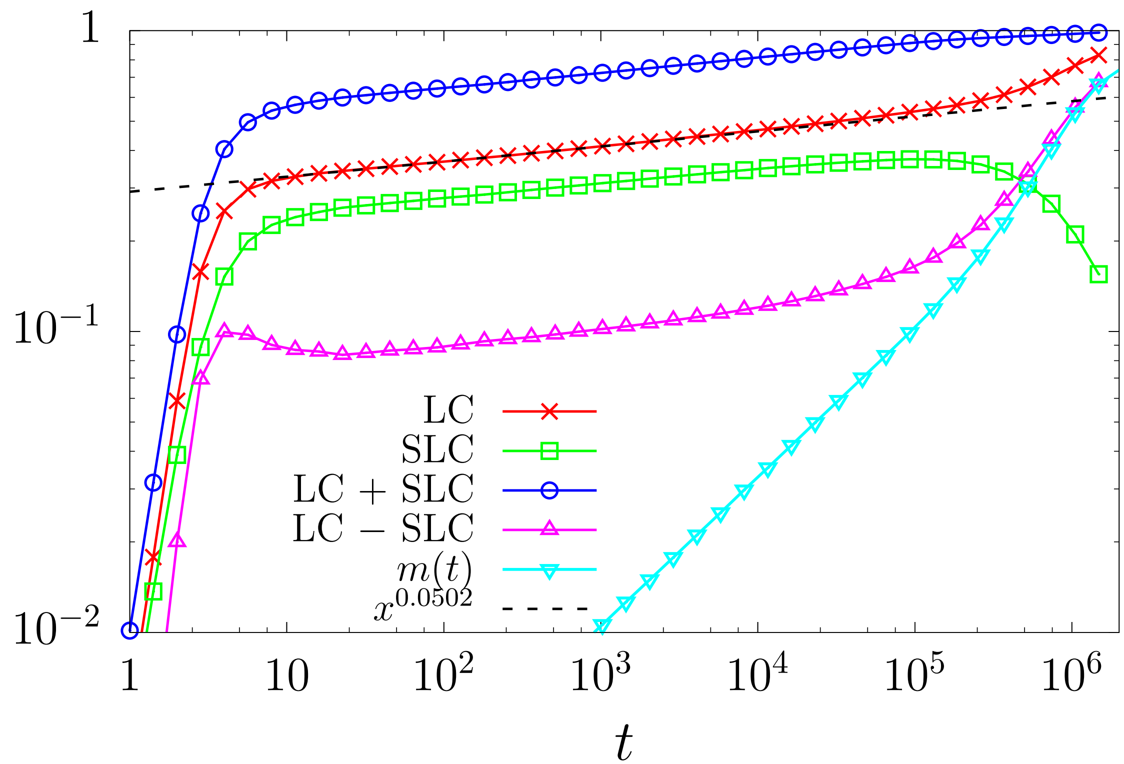
Lifetime of metastability



Blanchard, LFC, Picco & Tartaglia 17

Two largest clusters

Time evolution



Blanchard, LFC, Picco & Tartaglia 17