

Non-Local

More to come from:

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Rheology



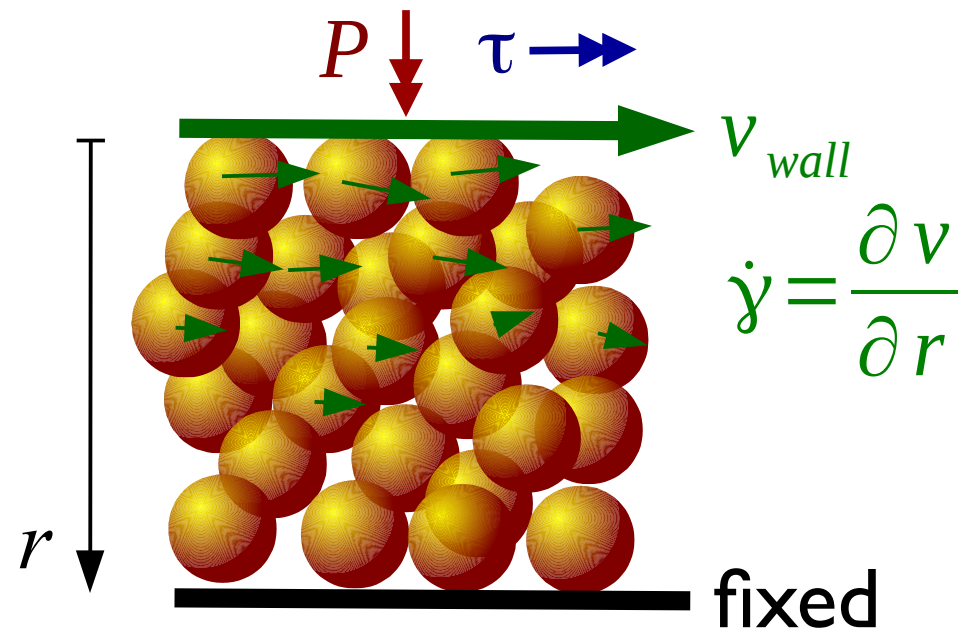
IFPRI

International Fine Particle Research Institute

Local Granular Rheology

- inertial number: ratio $I = \frac{\dot{\gamma} d}{\sqrt{P/\rho}}$ between
 - micro timescale (T) to squeeze a particle into a hole
 - macro timescale of deformation
 - large I corresponds to rapid flow

- stress ratio: ratio $\mu = \frac{\tau}{P}$ between
 - shear stress
 - normal pressure



Key Failures of Local Rheology

- cannot quantitatively capture the transition from inertial to quasistatic (but still creeping) flow

Koval, Roux, Corfdir, Chevoir. PRE (2009)

- boundary-driven flows form shear bands whose dimensions depend on both the geometry and the grain size

GDR MiDi. EPJE (2004)

Fenistein & van Hecke. Nature (2003)

Cheng, Lechman, ... Nagel. PRL (2006)

- shear/vibration in one region of a granular material can fluidize regions far from the perturbation

Nichol, Zanin, Bastien, Wandersman, van Hecke. PRL (2010)

Reddy, Forterre, Pouliquen. PRL (2011)

Wandersman & van Hecke. PRL (2014)

Local vs. Nonlocal Rheology

Local

- the local shear rate is determined by only the local shear stress
- resistance to flow is a function of only the local shear rate

Nonlocal

- particle rearrangements in one part of a flow trigger rearrangements elsewhere
- resistance to flow is a function of both the local shear rate and these nonlocal events

cooperative model

Kamrin & Koval (PRL 2012)

gradient model

Bouzid et al. (PRL 2013)

$$g \equiv \frac{\dot{\gamma}}{\mu}$$

granular fluidity

$$f = \frac{\dot{\gamma}}{Y} = \frac{\mu_s}{\mu(I)} \dot{\gamma}$$

$$I(\mu) = \frac{(\mu - \mu_s) H(\mu - \mu_s)}{b}$$

$$I_{\text{loc}}(f) = \frac{Tf}{1 - aTf}$$

$$g_{\text{loc}}(\mu, P) = H(\mu - \mu_s) \frac{\mu - \mu_s}{b\mu T}$$

$$\xi^2 \nabla^2 g = (g - g_{\text{loc}})$$

$$\dot{\gamma} = \frac{I_{\text{loc}}(f)}{T} - \ell^2 \nabla^2 f$$

Laplacian term accounts for nonlocal effects

$$\frac{\xi}{d} = A \sqrt{\frac{1}{|\mu - \mu_s|}}$$

- based on extending a local Bagnold-type granular flow law
- length scale ξ diverges at μ_s
- 3 fit parameters: A, b, μ_s

$$Y = \frac{\mu(I)}{\mu_s} \left(1 - \nu_\ell \frac{d^2(\nabla^2 I)}{I} \right)$$

- based on gradient expansion
- solutions have a divergent lengthscale $L(Y, \nu_\ell)$ at μ_s
- 4 fit parameters: l, ν_ℓ, a, μ_s

cooperative model

Kamrin & Koval. PRL (2012); Henann & Kamrin. PNAS (2013), PRL (2014), Soft Matter (2014)

- $b \sim 1.0 \pm 0.1$ controls the steepness of rise of $\mu(l)$
- $A \sim 0.8 \pm 0.3$ controls of strength of cooperativity (divergence at μ_s)
- $\mu_s \sim 0.25$ is the yield ratio – can be obtained independently

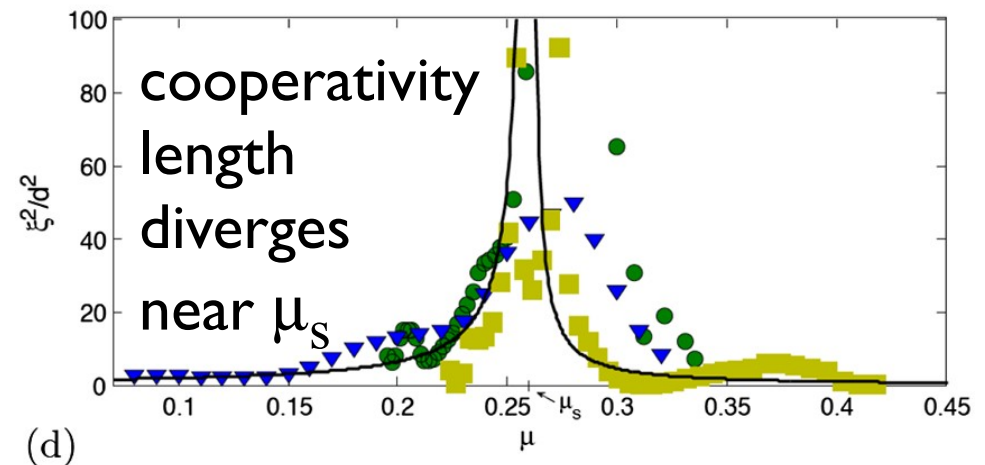
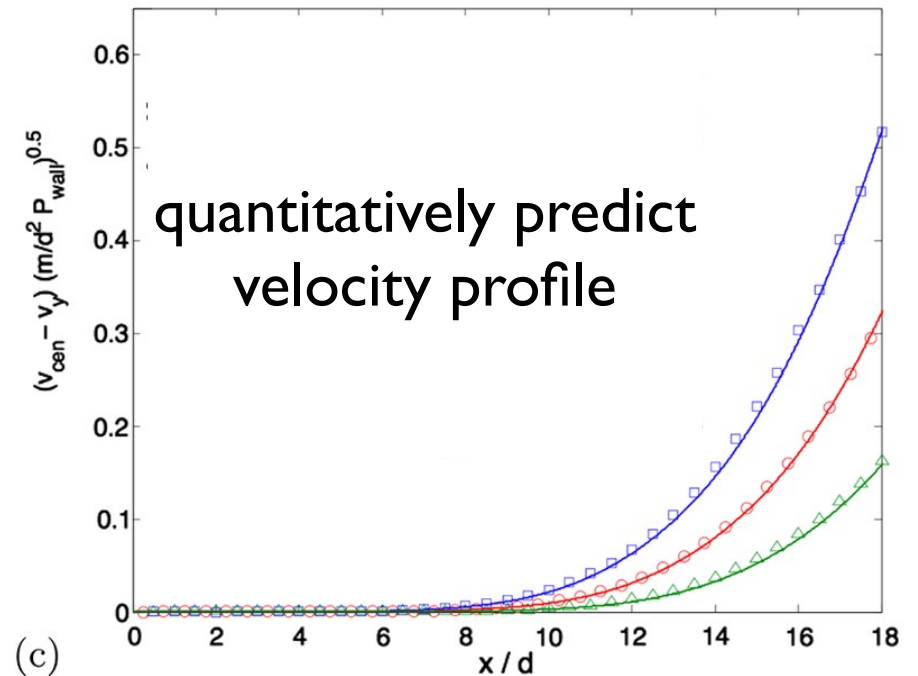
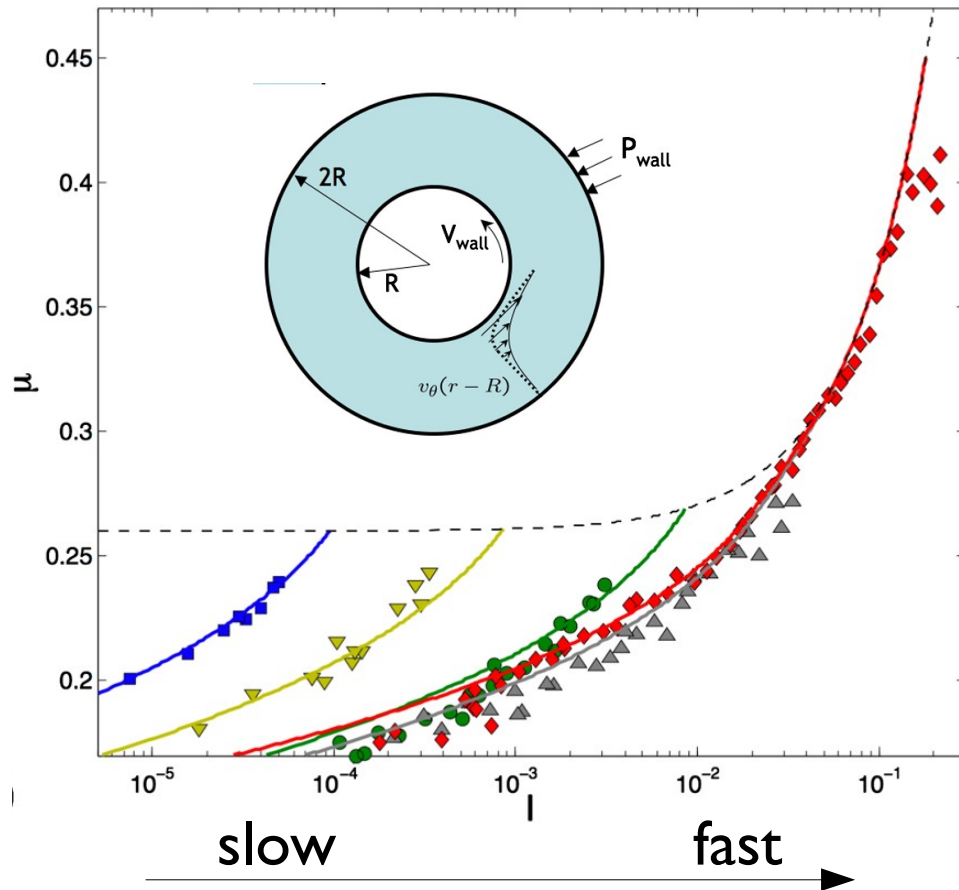
gradient model

Bouzid et al PRL (2013), EPJE (2015)

- $a \sim 4.3$ in constitutive relation
$$Y = 1 + a I_{\text{loc}}$$
- $\nu_l \sim 8$ controls magnitude of higher-order approximation
- $l \sim$ fluidity contributes over a few grain diameters
- $\mu_s \sim 0.25$ is the yield ratio – can be obtained independently

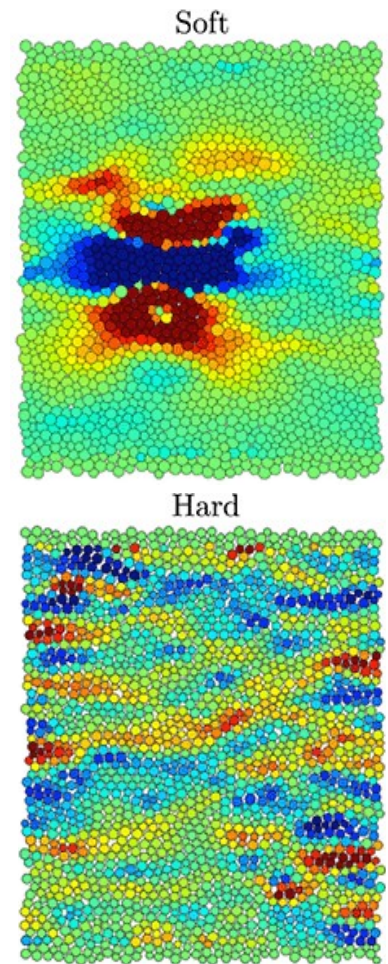
- Test: determine parameters in one flow geometry \rightarrow reuse those values in another
- Ultimately: predict parameters for given shape/size/roughness/stiffness

Success of Nonlocal Rheology in Sims

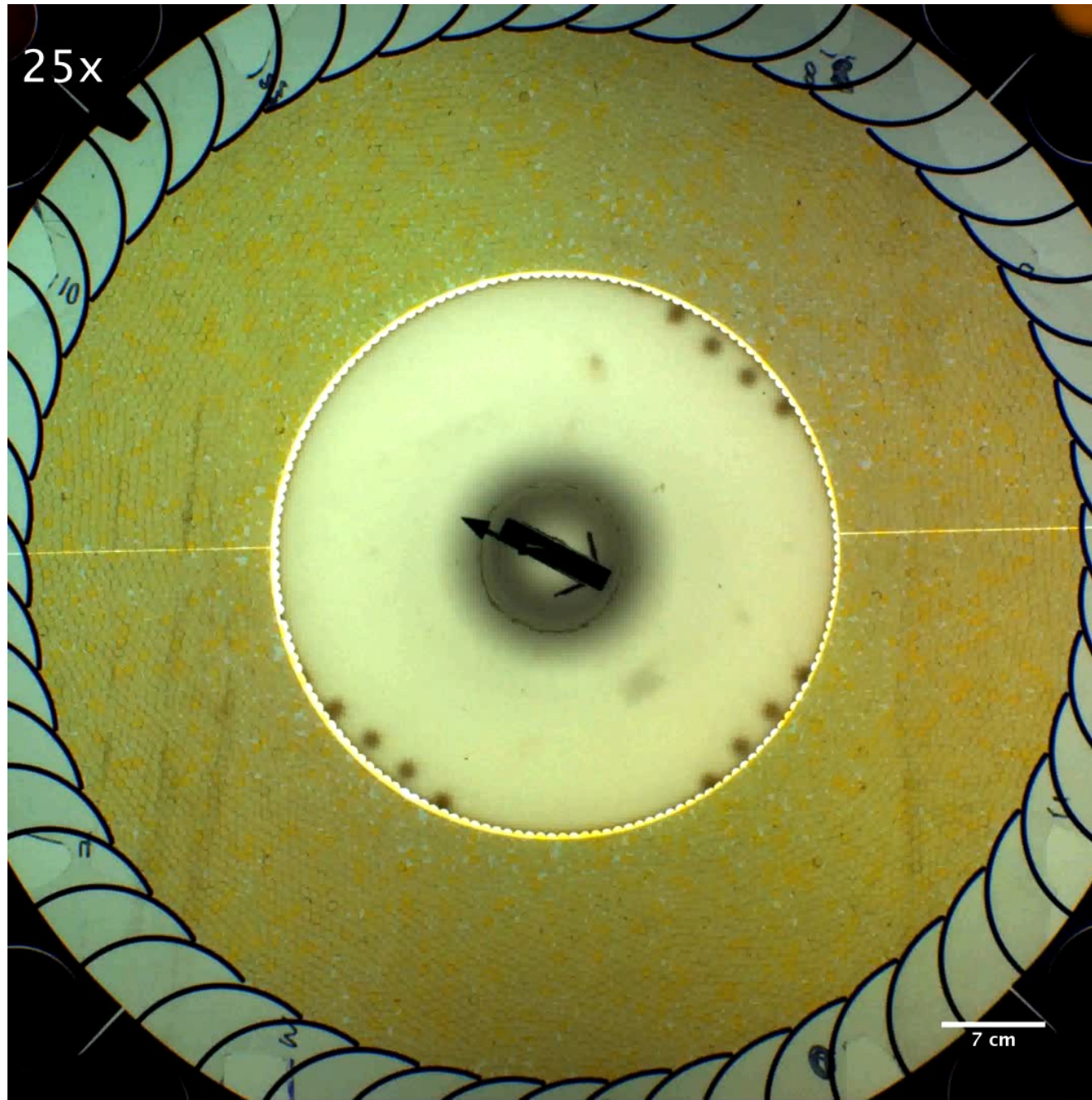


Key Challenges

- need to directly test underlying assumptions: e.g. do force chains distinguish local vs. nonlocal effects?
- do experiments show diverging lengthscale?
- what sets parameters? (friction, particle shape, stiffness)
 - hard vs. soft particles matter
Bouzig et al. EPJE, 2015 →
- transient behaviors
- memory?

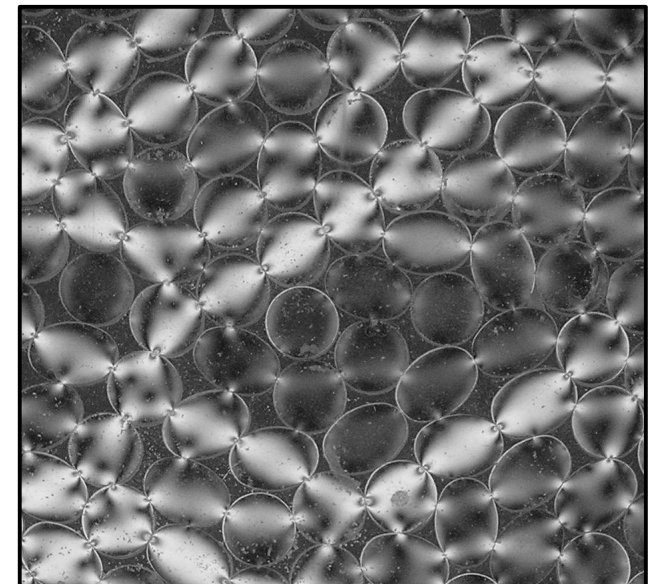
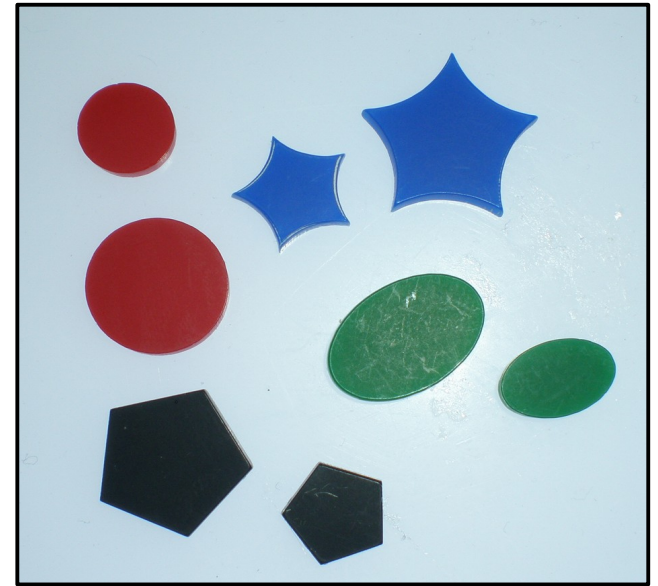


Annular Granular Rheometer



Idealized (2D) Experiments

- controlled shearing of particles
- instrumented at both boundaries
- bespoke particles & boundaries
- all particles visible: track their individual positions and forces
- allows for first-principles determination of material parameters
- allows for the isolation of local vs. nonlocal parameters



$$\tau(r) = S \frac{R^2}{r^2}$$

$$\mu(r) = \frac{\tau(r)}{P}$$

P, τ



$2R$

$R = 15 \text{ cm}$

$N \approx 10^4$
disks and ellipses
 $d = 5 \text{ mm}, 7 \text{ mm}$

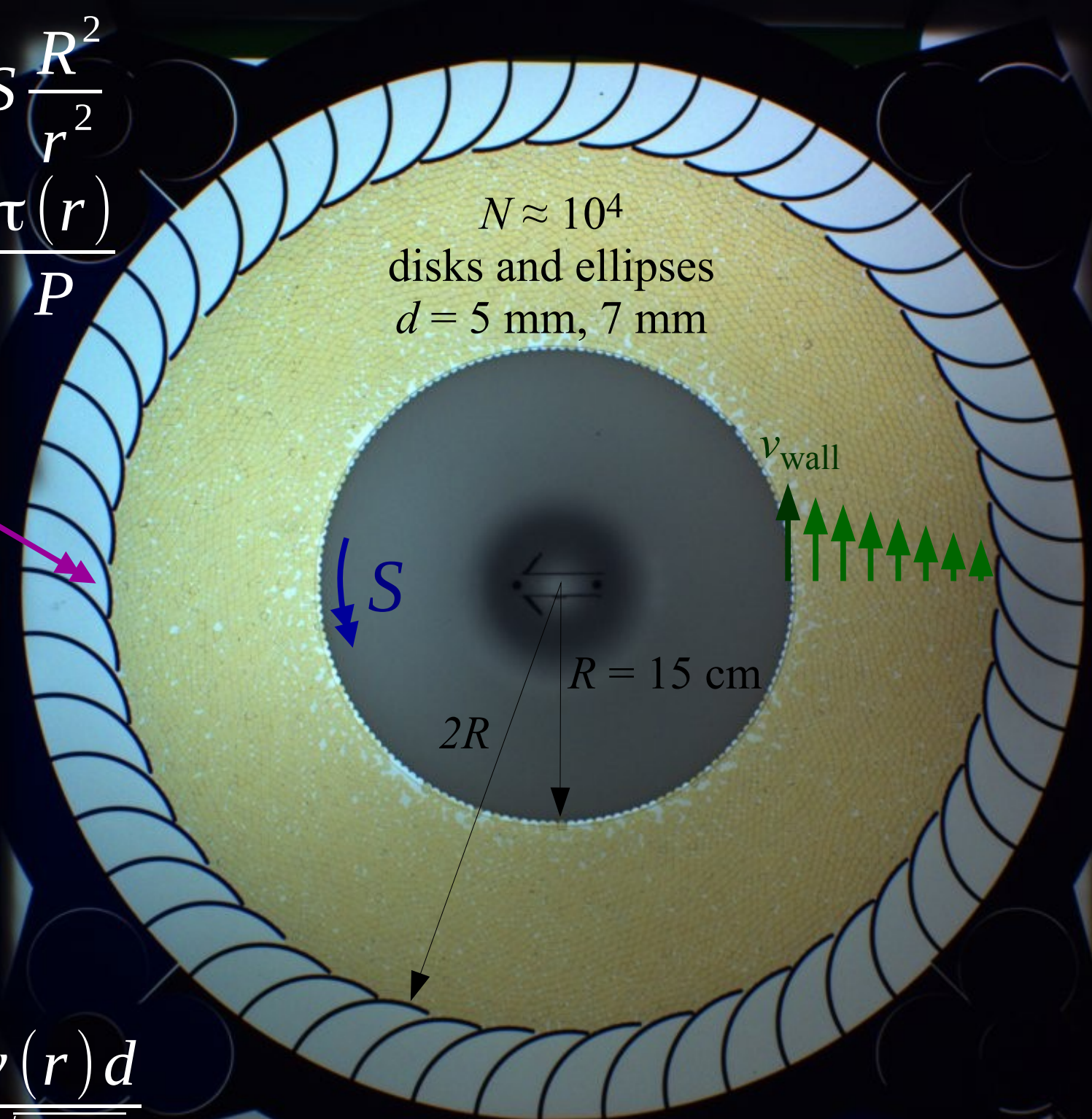
v_{wall}



$v(r)$

$$\dot{\gamma} = \frac{\partial v}{\partial r}$$

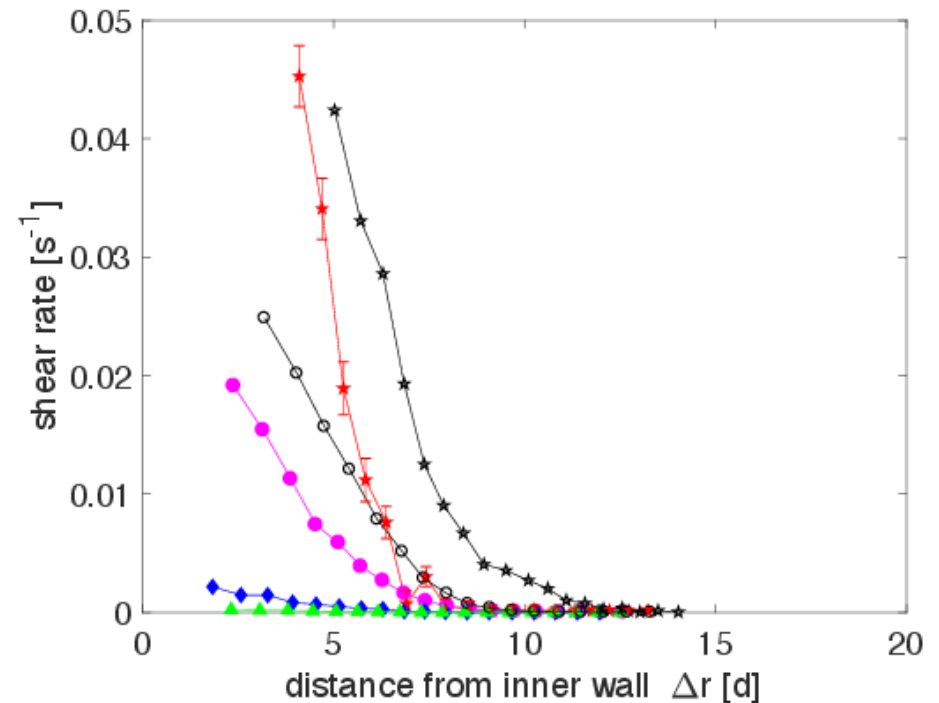
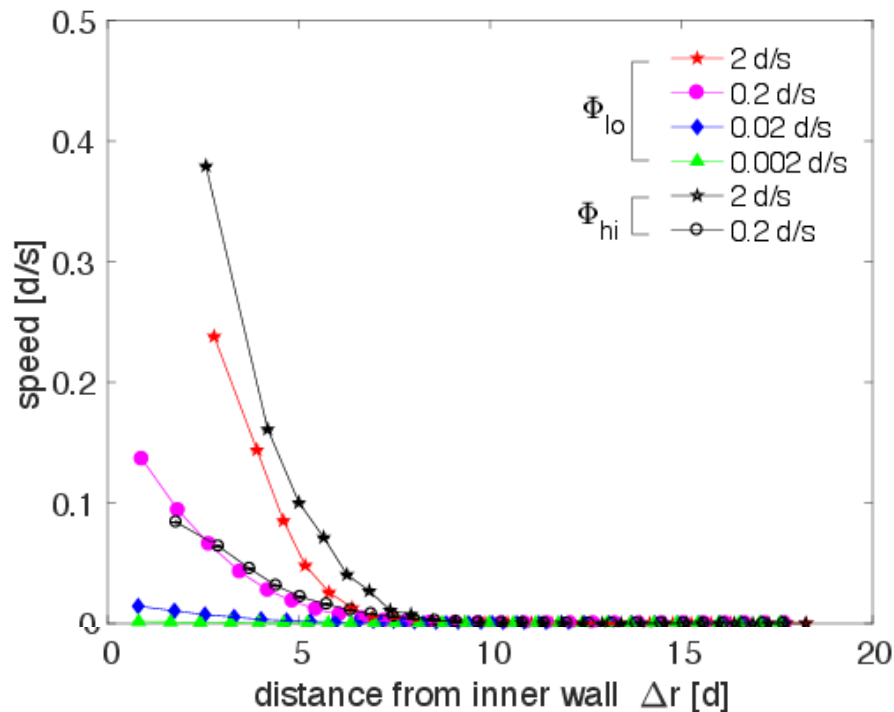
$$I(r) = \frac{\dot{\gamma}(r) d}{\sqrt{P/\rho}}$$



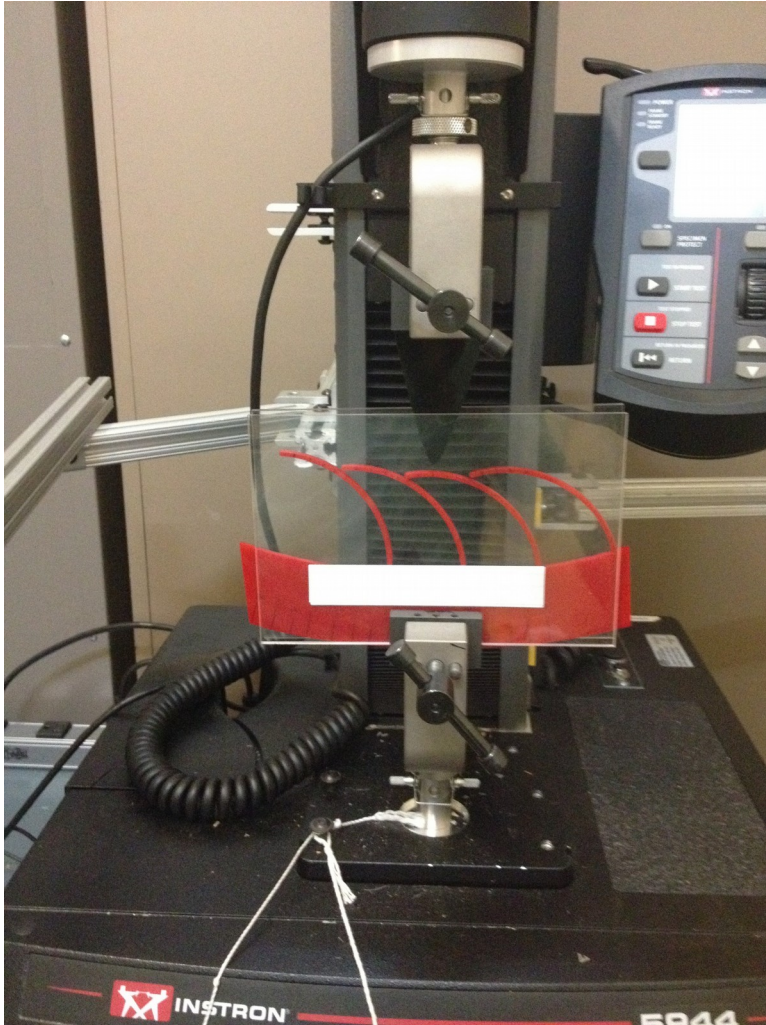
6 Experimental Runs:

4 speeds: 0.02, 0.02, 0.2, 2 d/s

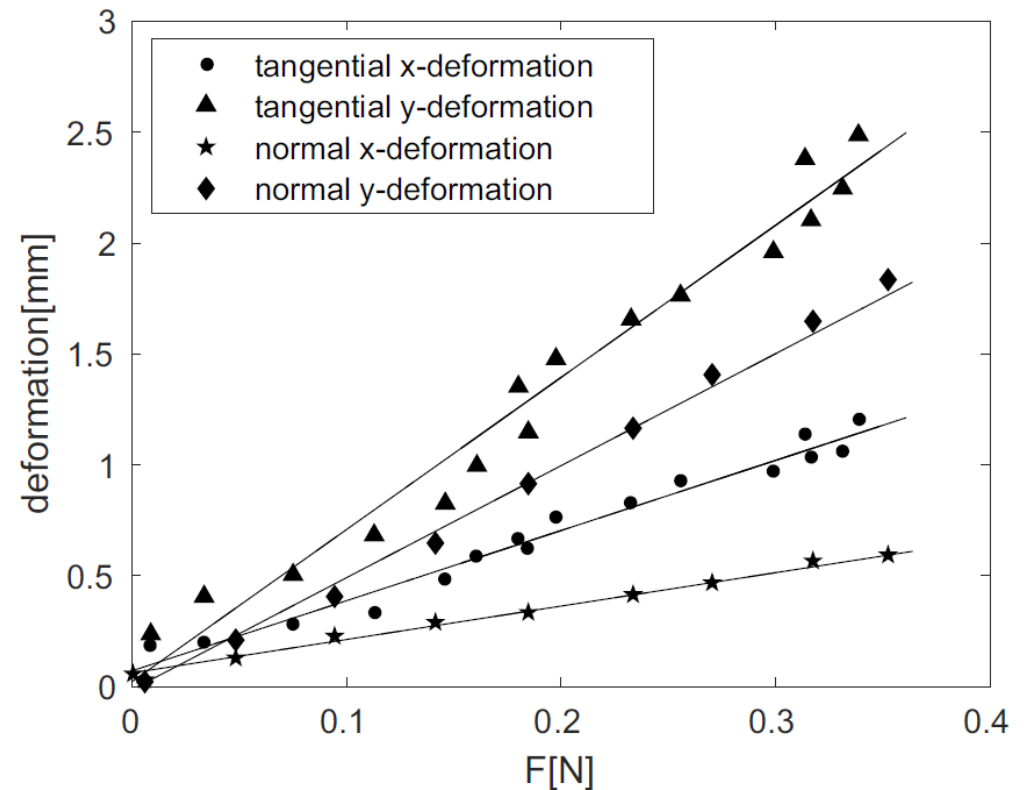
2 packing fractions: $\phi \sim 0.82, 0.84$



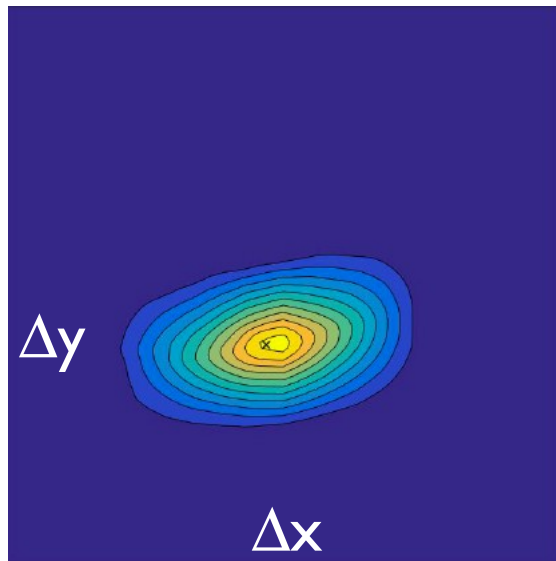
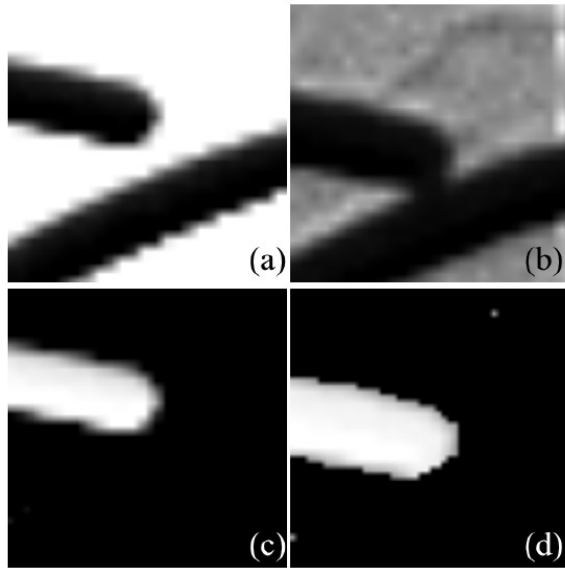
Calibration of Leaf-Spring Walls



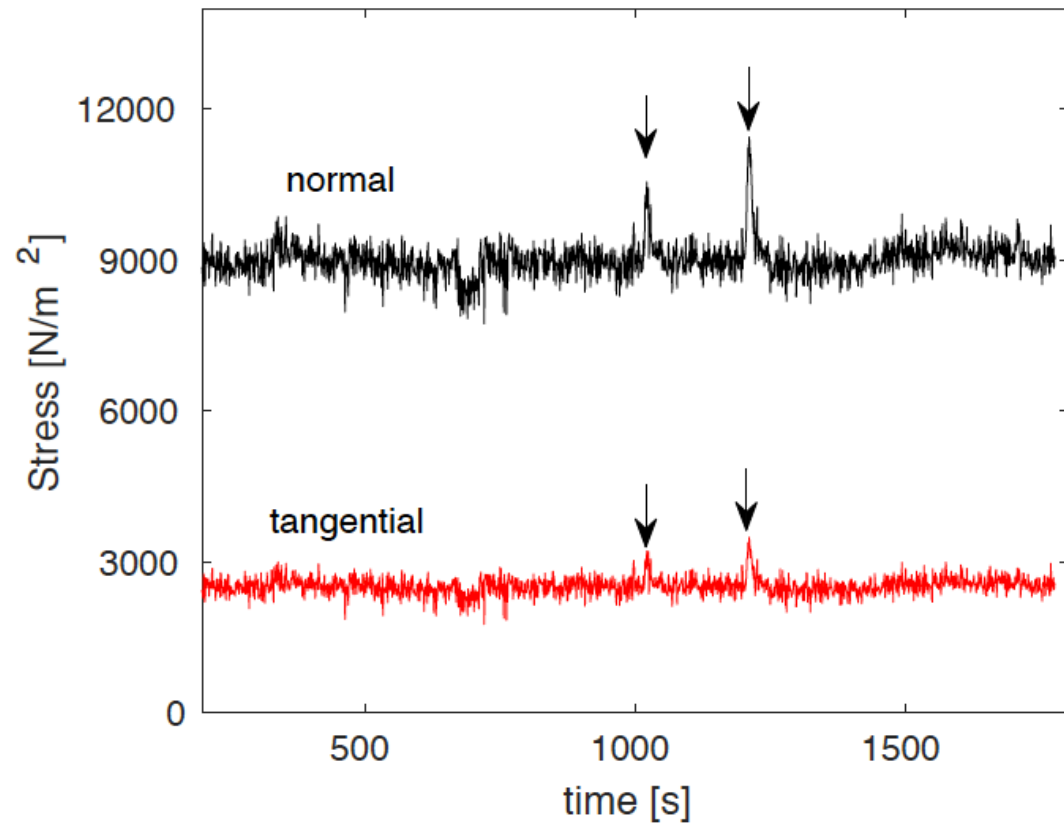
outer boundary is
composed of laser-cut
leaf springs



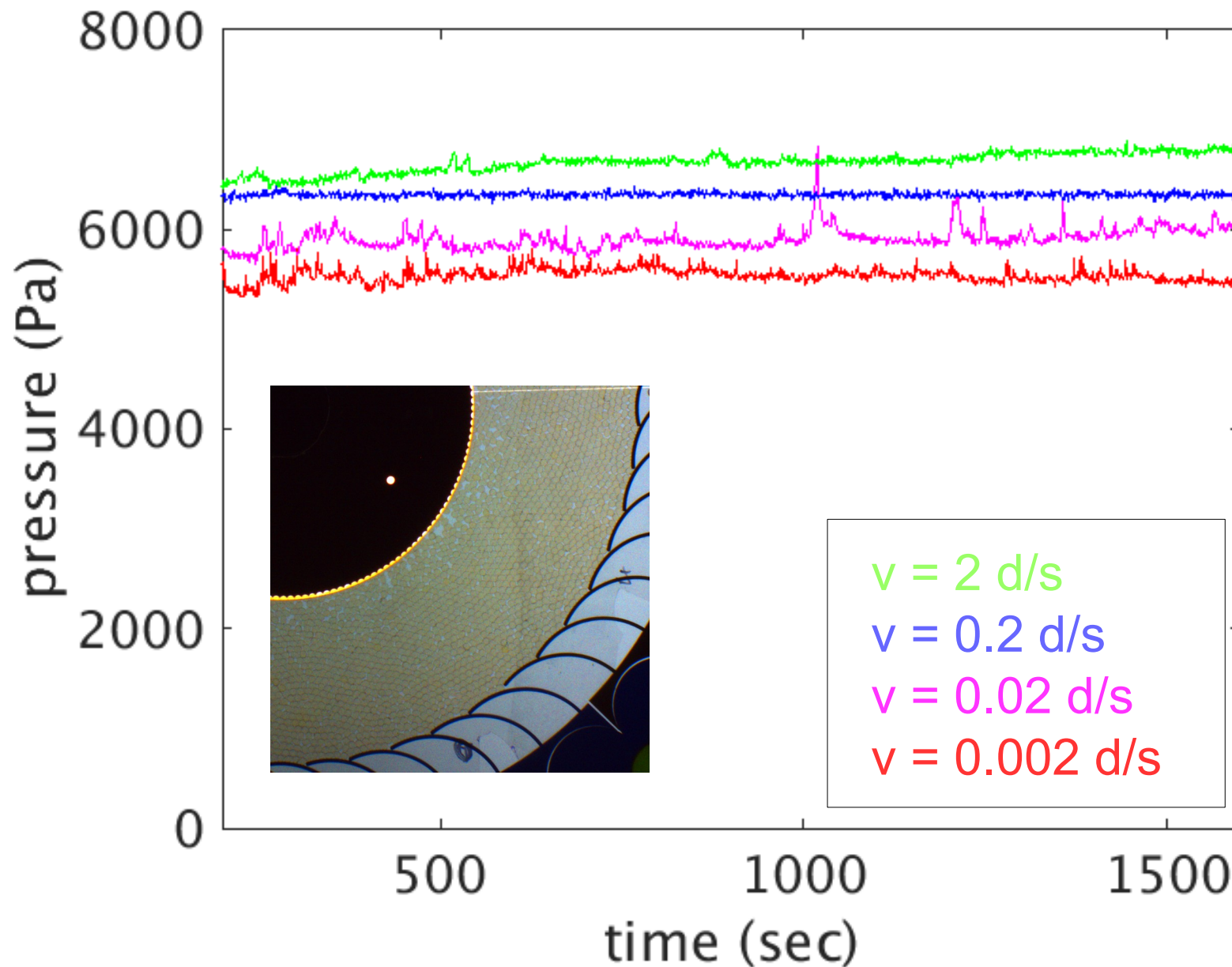
Spring tips → normal & tangential force



measure spring wall deformation in experiment by cross-correlation



Example: Pressure & Dilatancy



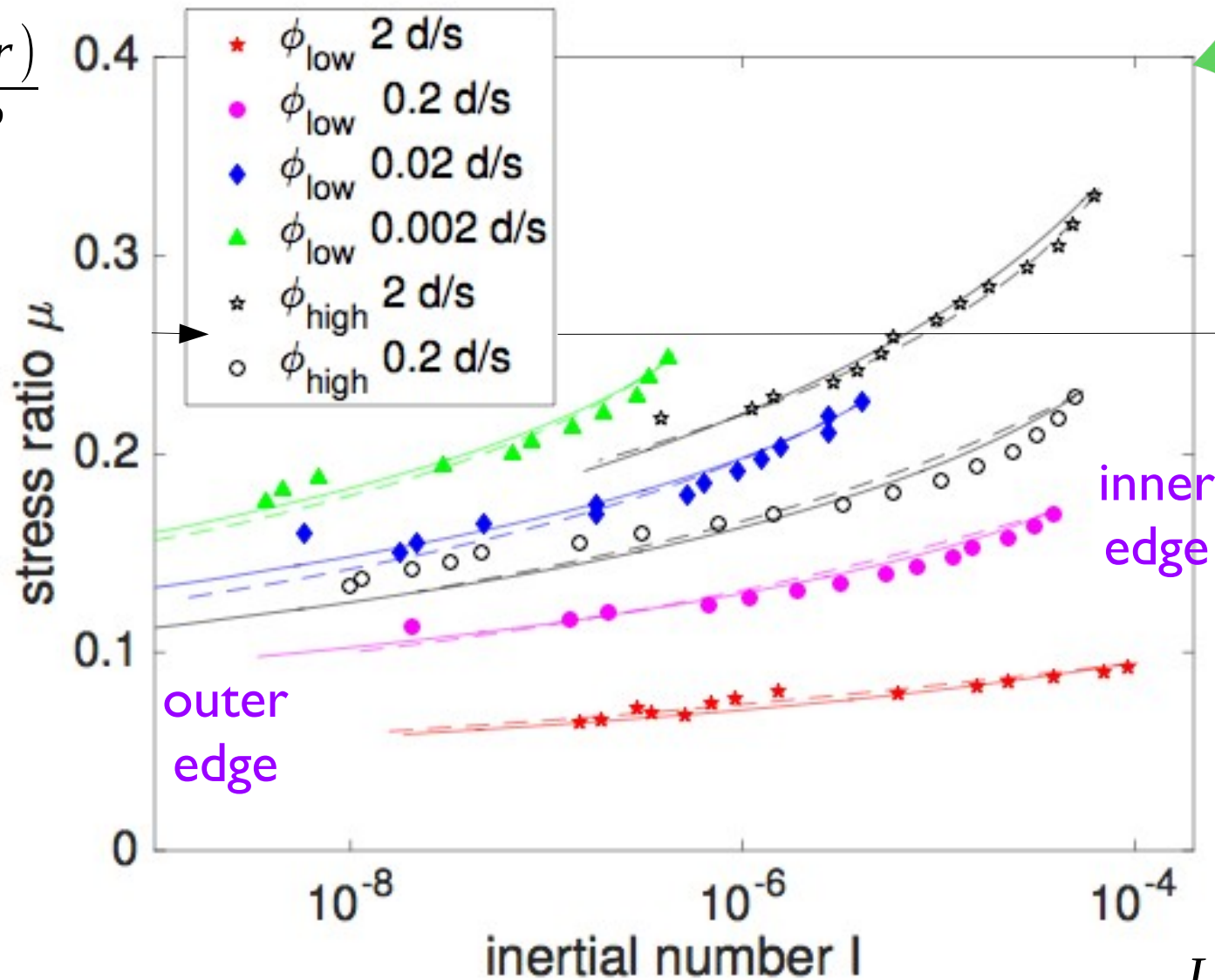
Testing the Two Nonlocal Models

- Can we capture the shape of $\mu(I)$?
- Can we use $\mu(I)$ to capture the shape of $v(r)$?
- Does a lengthscale diverge at μ_s ?
- Can one set of parameters capture all 6 datasets?

Test #1: Fit the $\mu(I)$ data

cooperative model – gradient model - - -

$$\mu(r) = \frac{\tau(r)}{P}$$



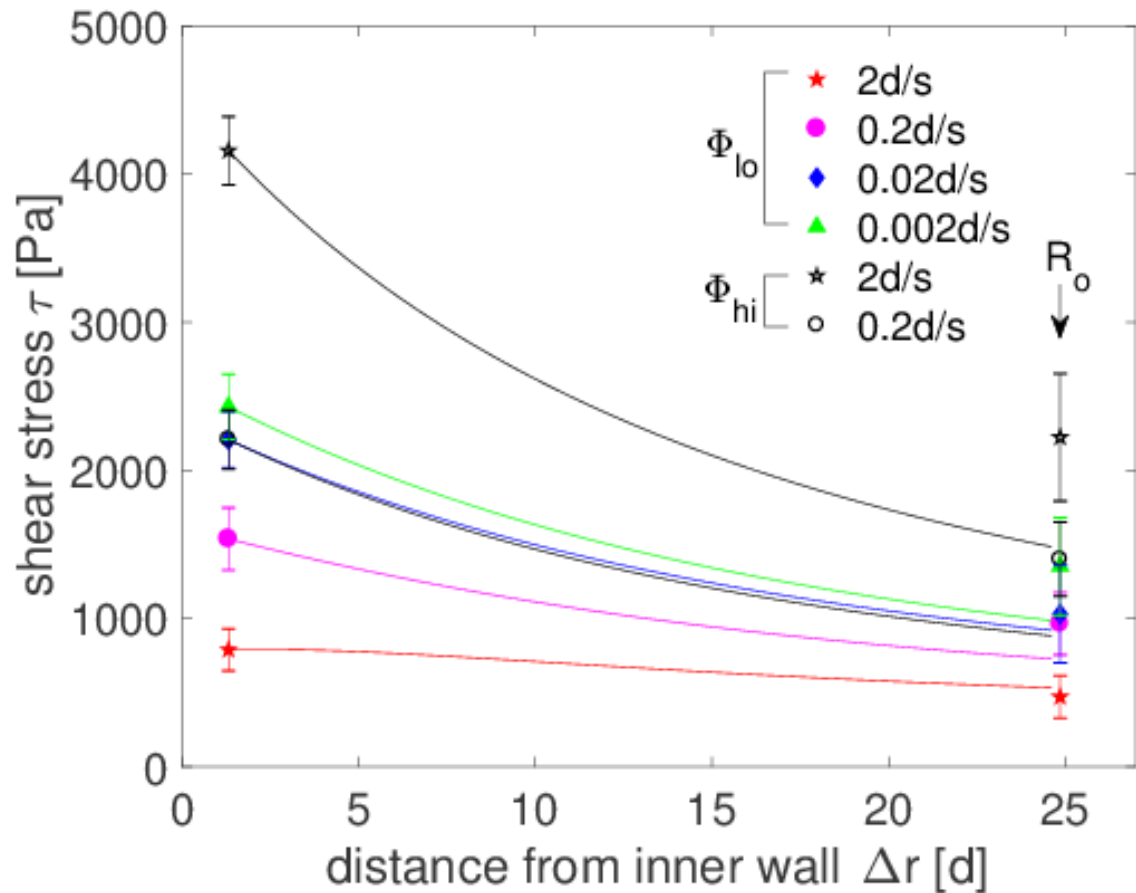
$$I(r) = \frac{\dot{\gamma}(r)}{\sqrt{P/\rho}} d$$

Experimental Detail #1: $\mu = \tau/P$

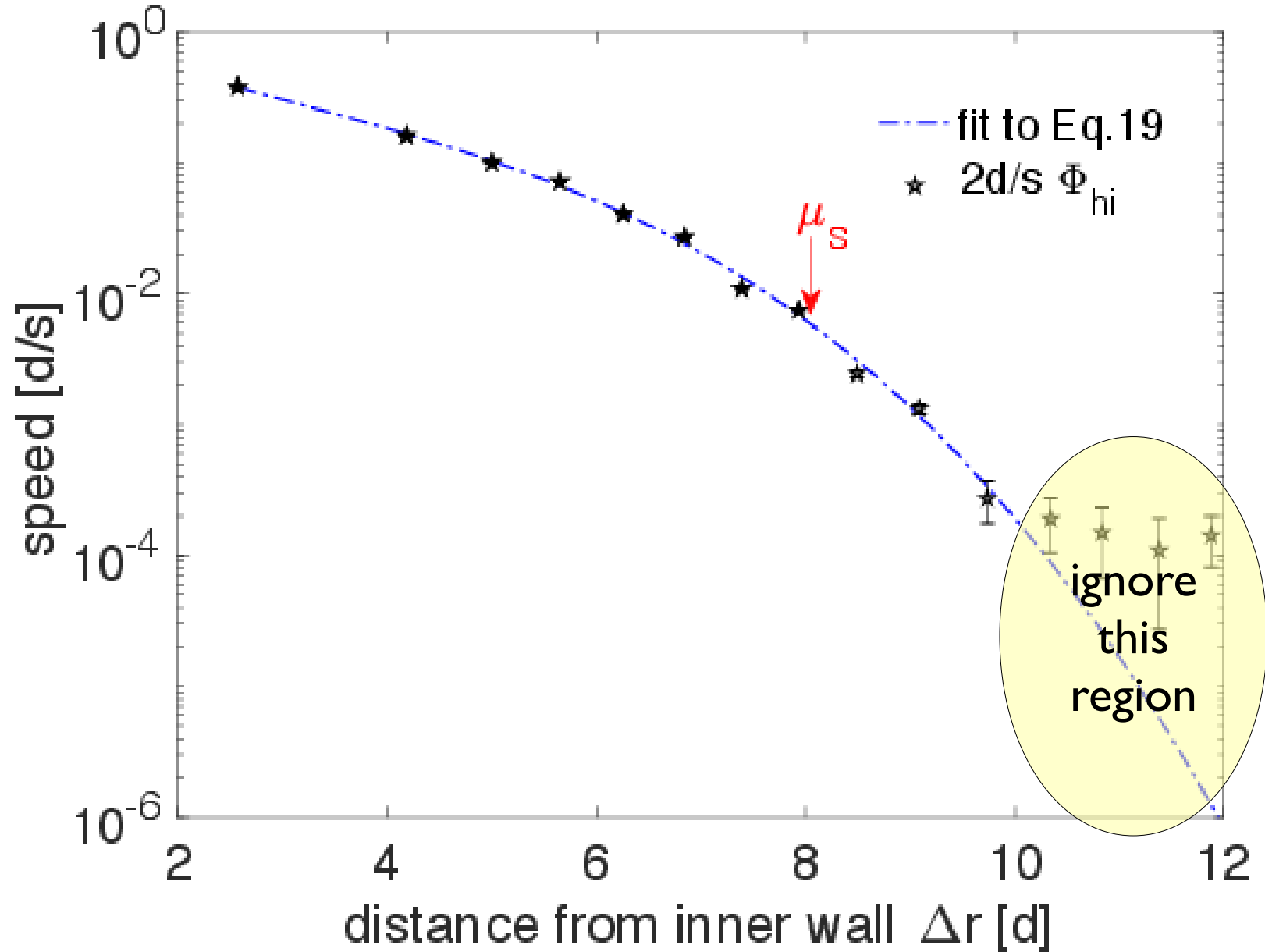
$$\tau(r) = S \left(\frac{R_i}{r} \right)^2 + \tau_0 \left[1 - e^{-(r-R_i)/r_0} \right]$$

geometry

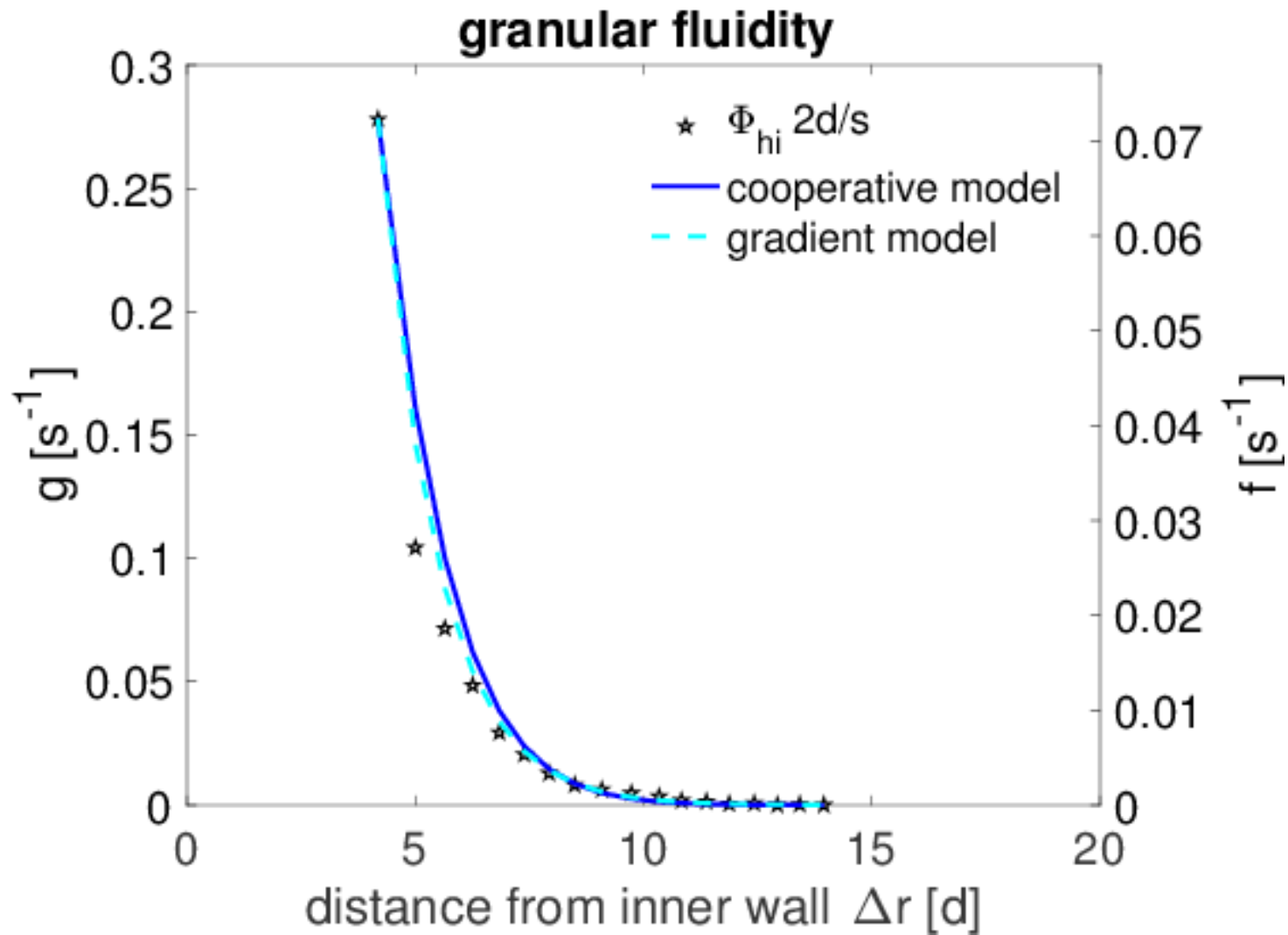
basal friction



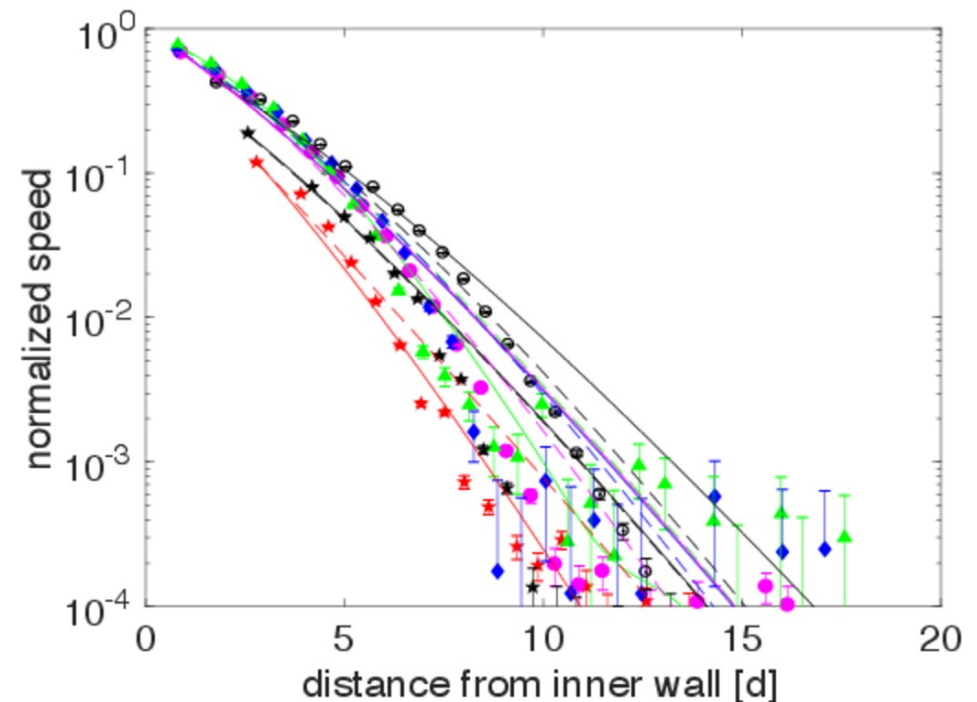
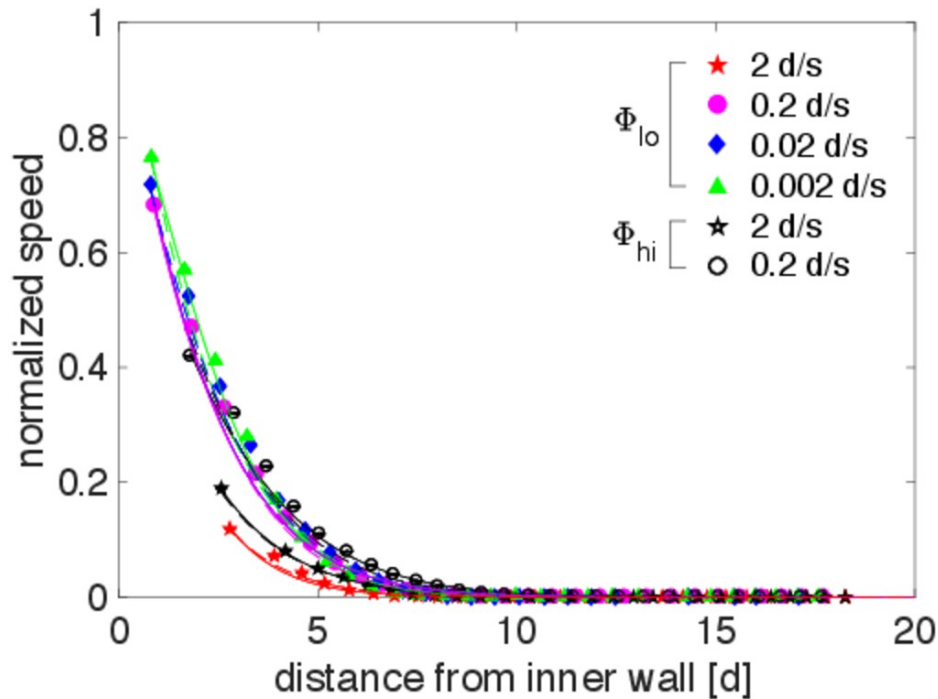
Experimental Detail #2: $v(r)$ derivatives



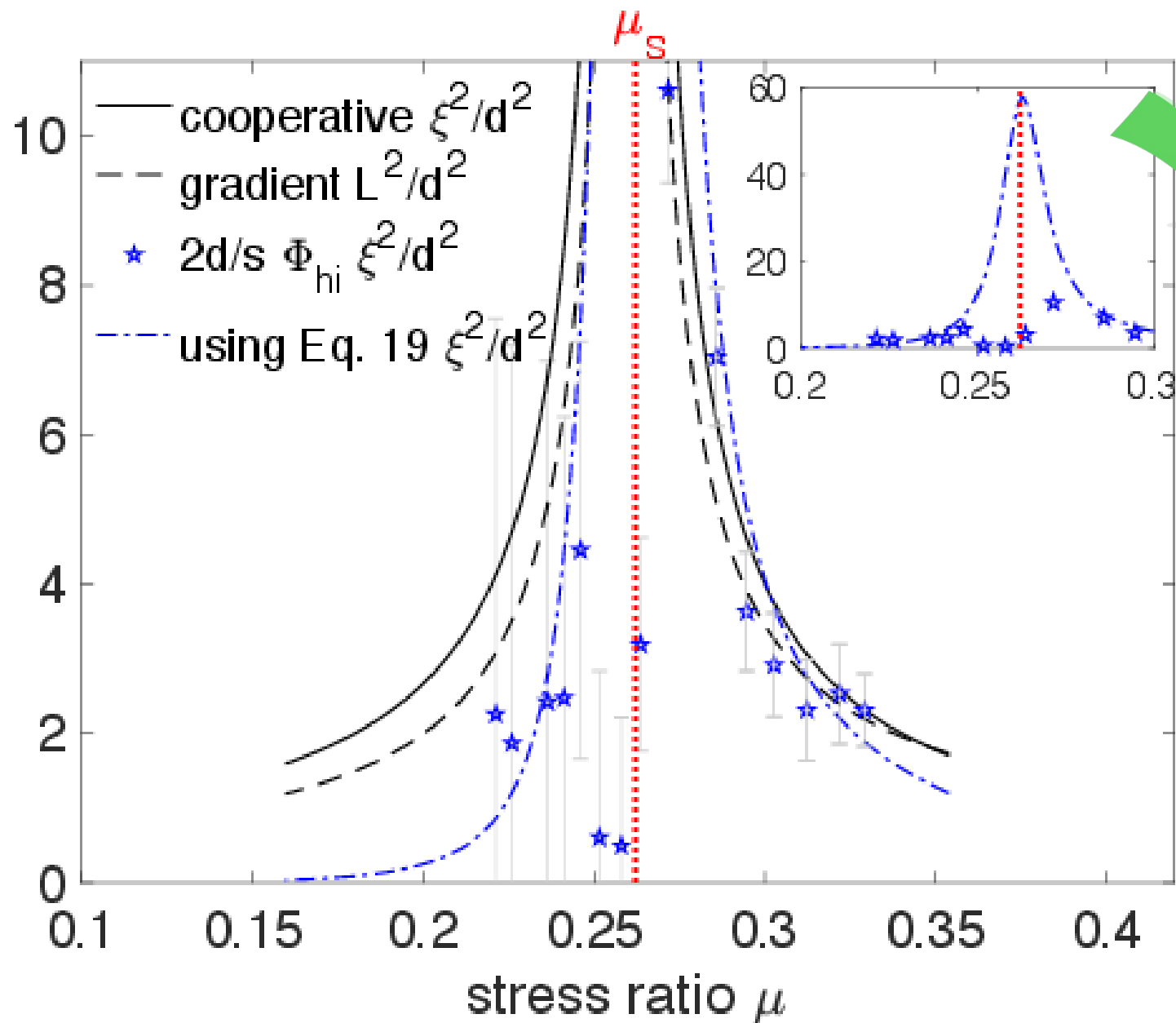
Granular Fluidity Profile



Test #2: Fit the Speed Profiles



Test #3: Nonlocal Lengthscale Diverges?



Test #4: All 6 runs, same parameters?

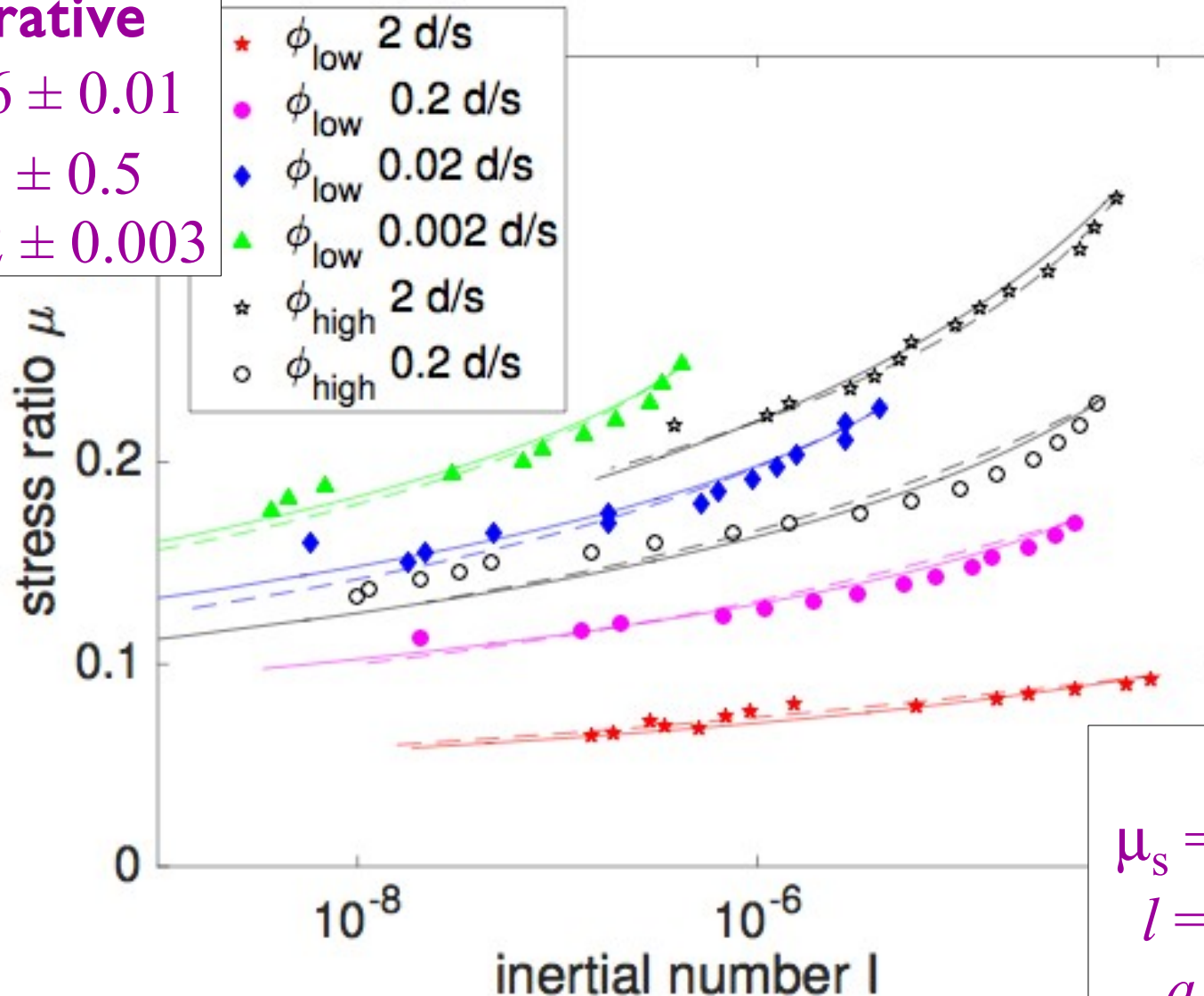
cooperative model – gradient model - -

cooperative

$$\mu_s = 0.26 \pm 0.01$$

$$b = 1.1 \pm 0.5$$

$$A = 0.402 \pm 0.003$$



gradient

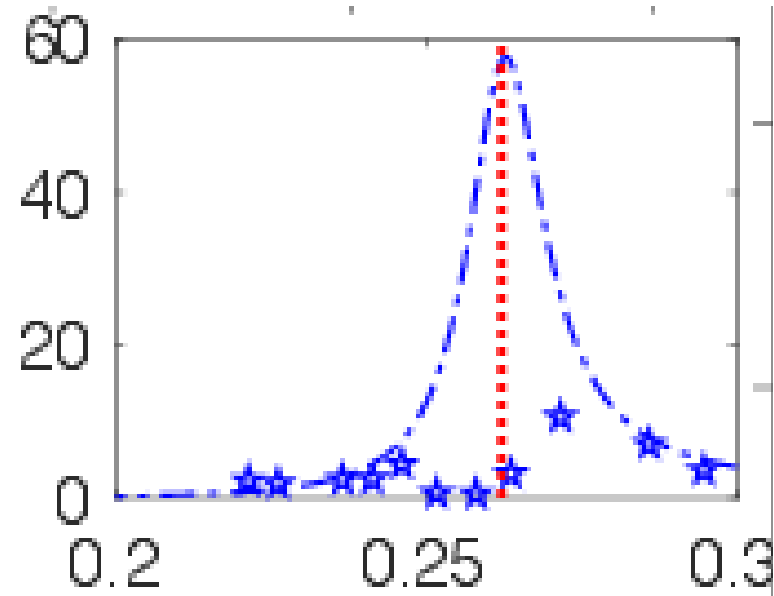
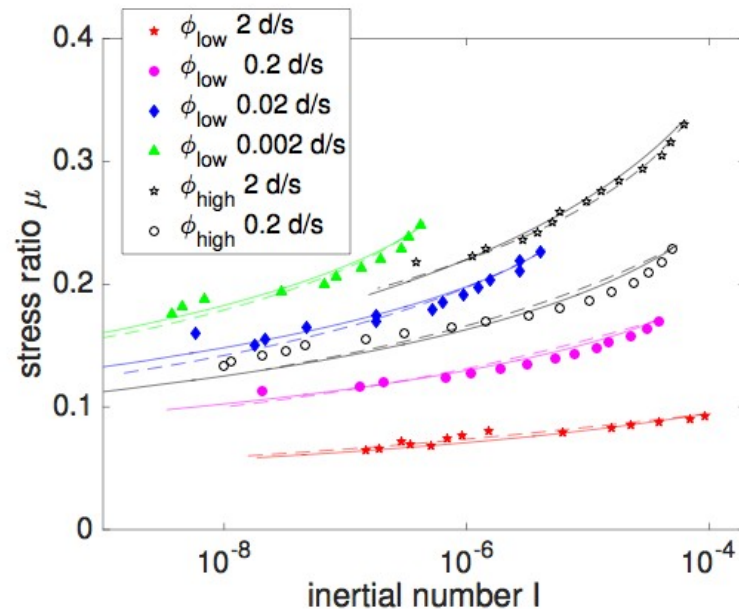
$$\mu_s = 0.26 \pm 0.01$$

$$l = 1.02 \pm 0.02$$

$$a = 7.2 \pm 4.3$$

$$v_l = 0.46 \pm 0.03$$

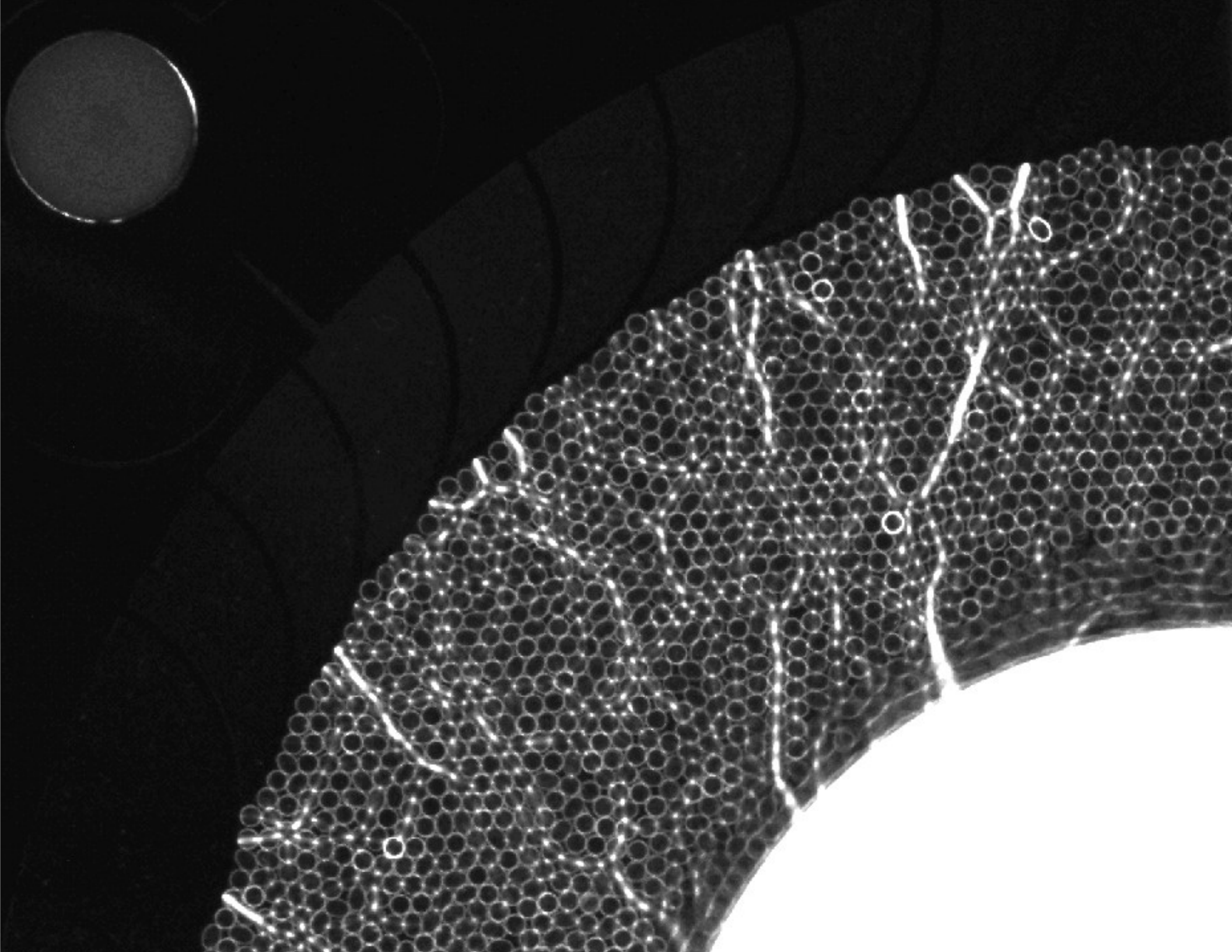
Determination of μ_s



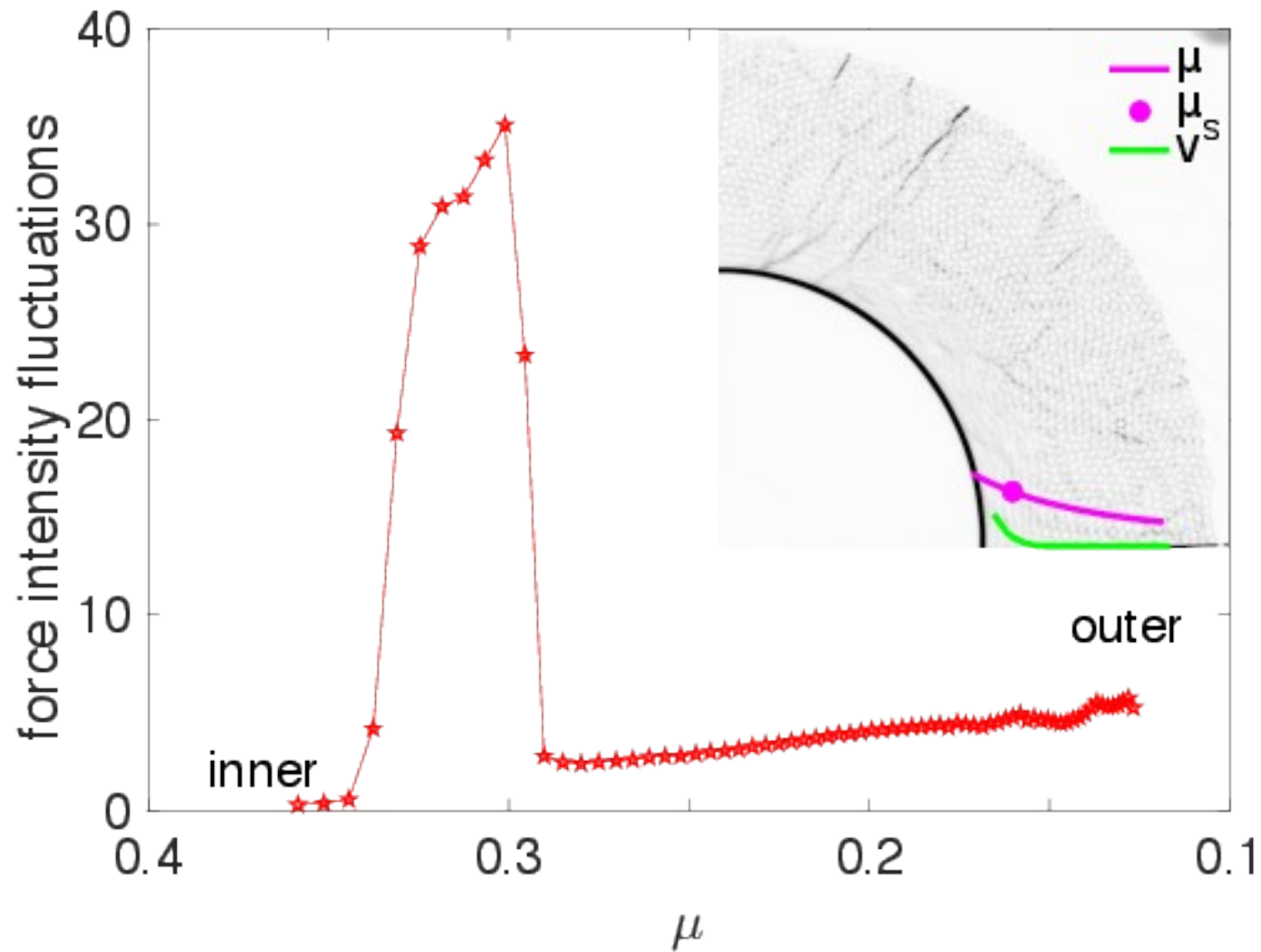
(1) upper limit of slowest $\mu(I)$ curve: $\mu_s > 0.26$

(2) maximum of $\xi(\mu)$: $\mu_s \sim 0.26$

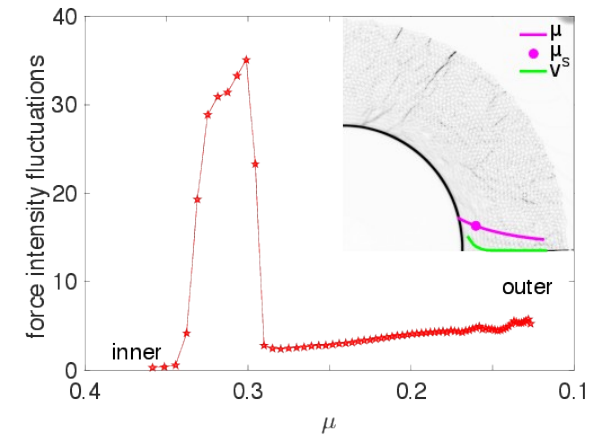
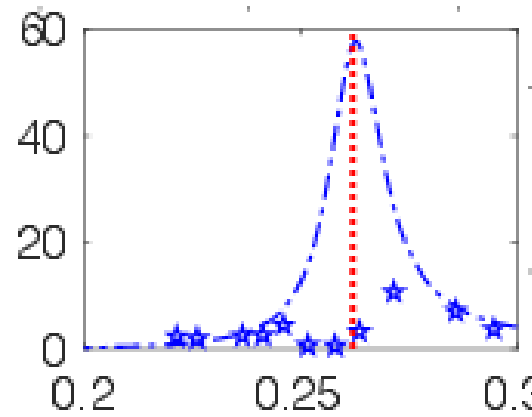
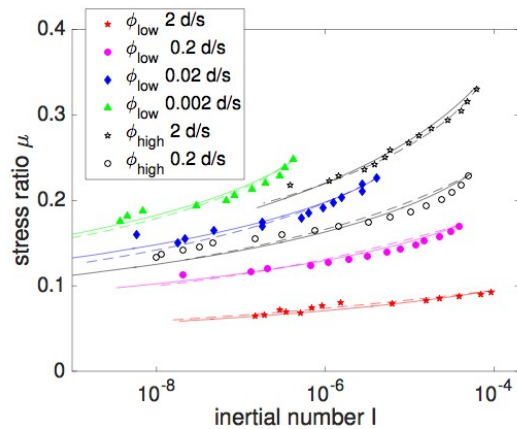
... but shouldn't it have to do with forces?



Is μ_s a susceptibility?



Determination of μ_s

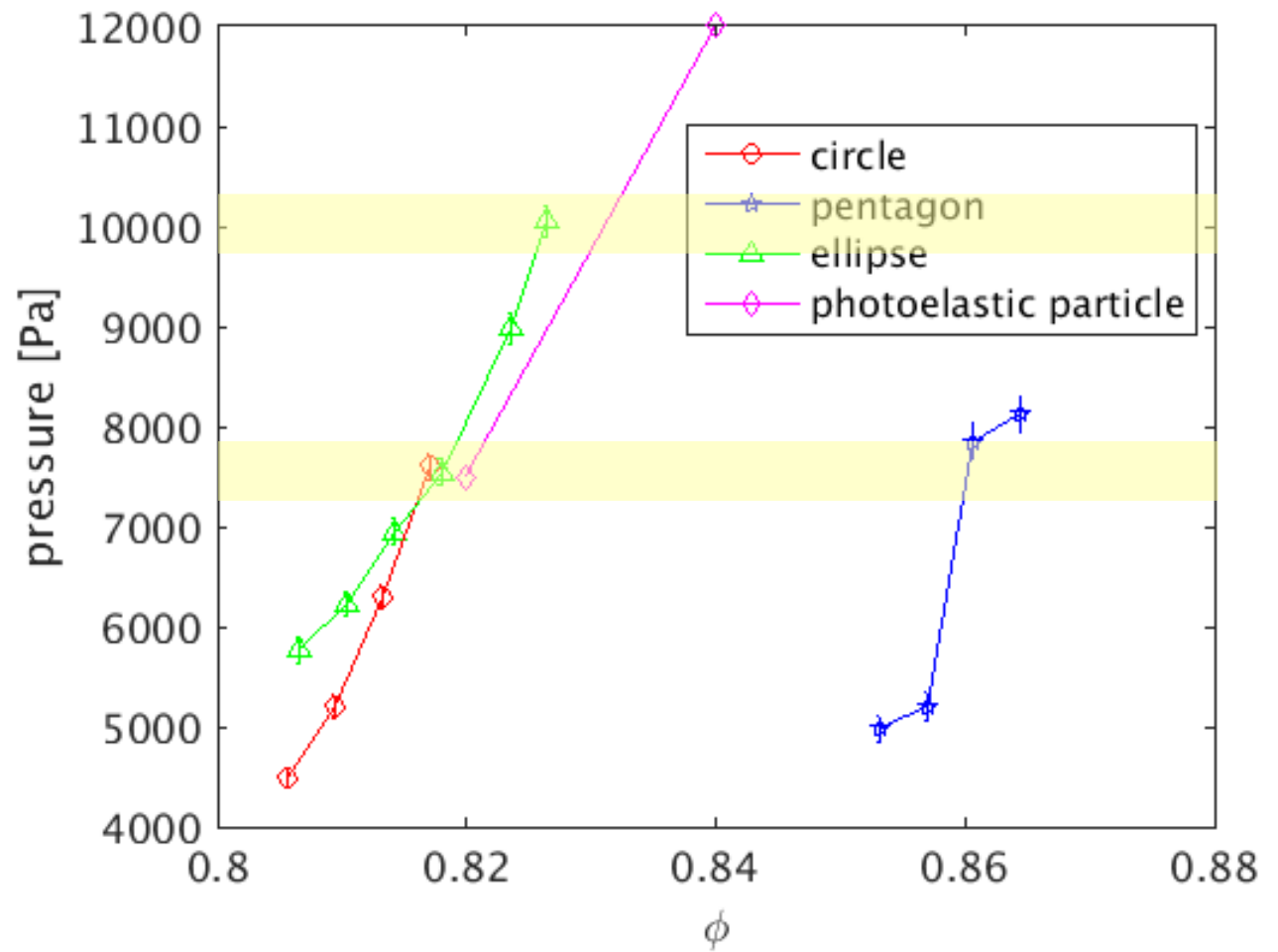
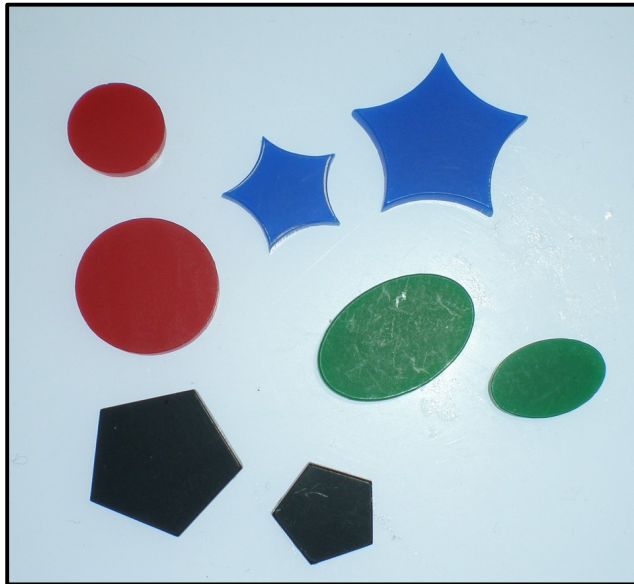


(1) upper limit of slowest $\mu(I)$ curve: $\mu_s > 0.26$

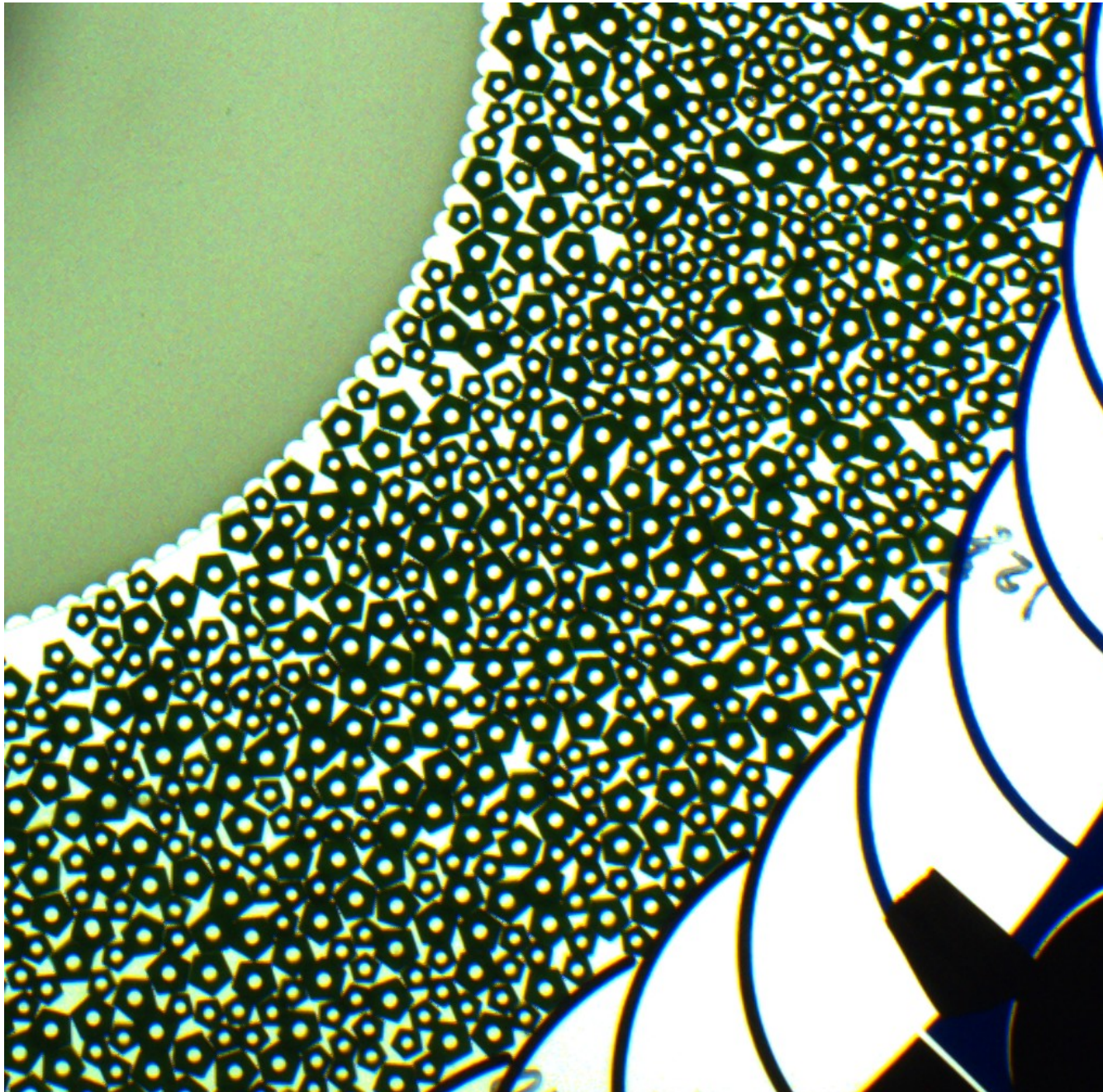
(2) maximum of $\xi(\mu)$: $\mu_s \sim 0.26$

(3) force chain fluctuations: $\mu_s < 0.29$

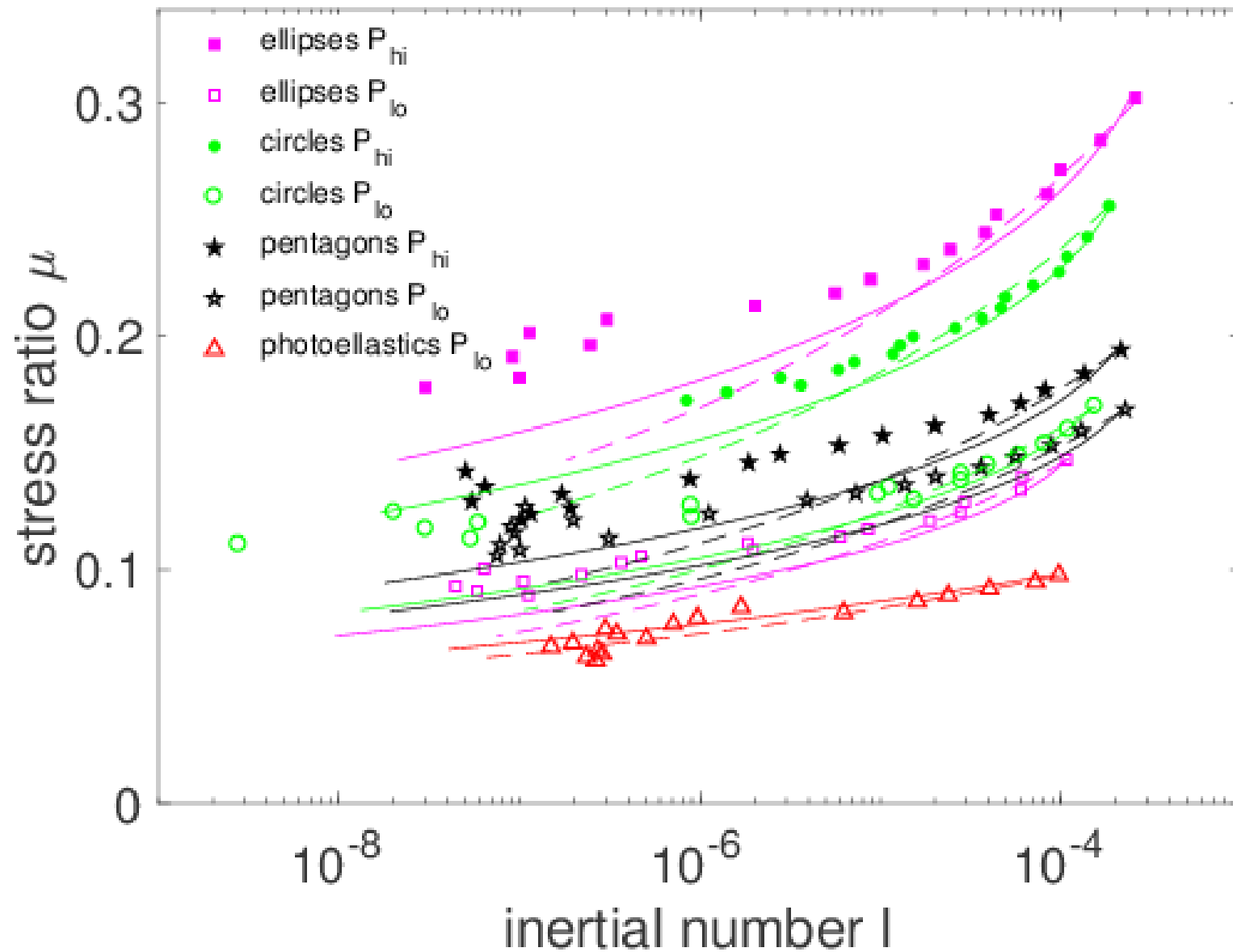
What about shape?



Sample Image: Pentagons



Unsurprisingly, $\mu(I)$ rheology changes



Goal: take simple inputs

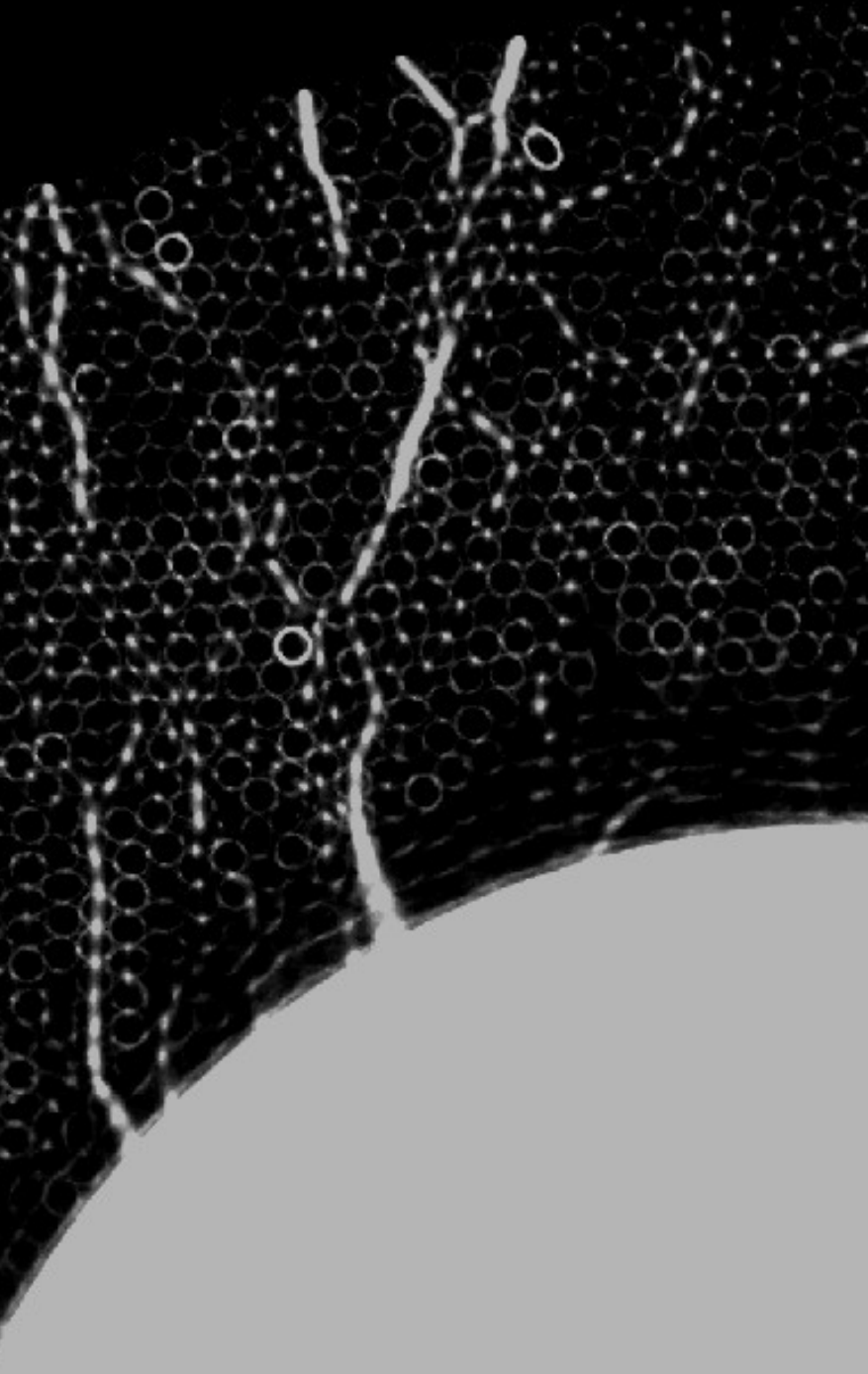


- time taken for grains to flow through an orifice
- force required to shear a prepared sample
- angle of repose

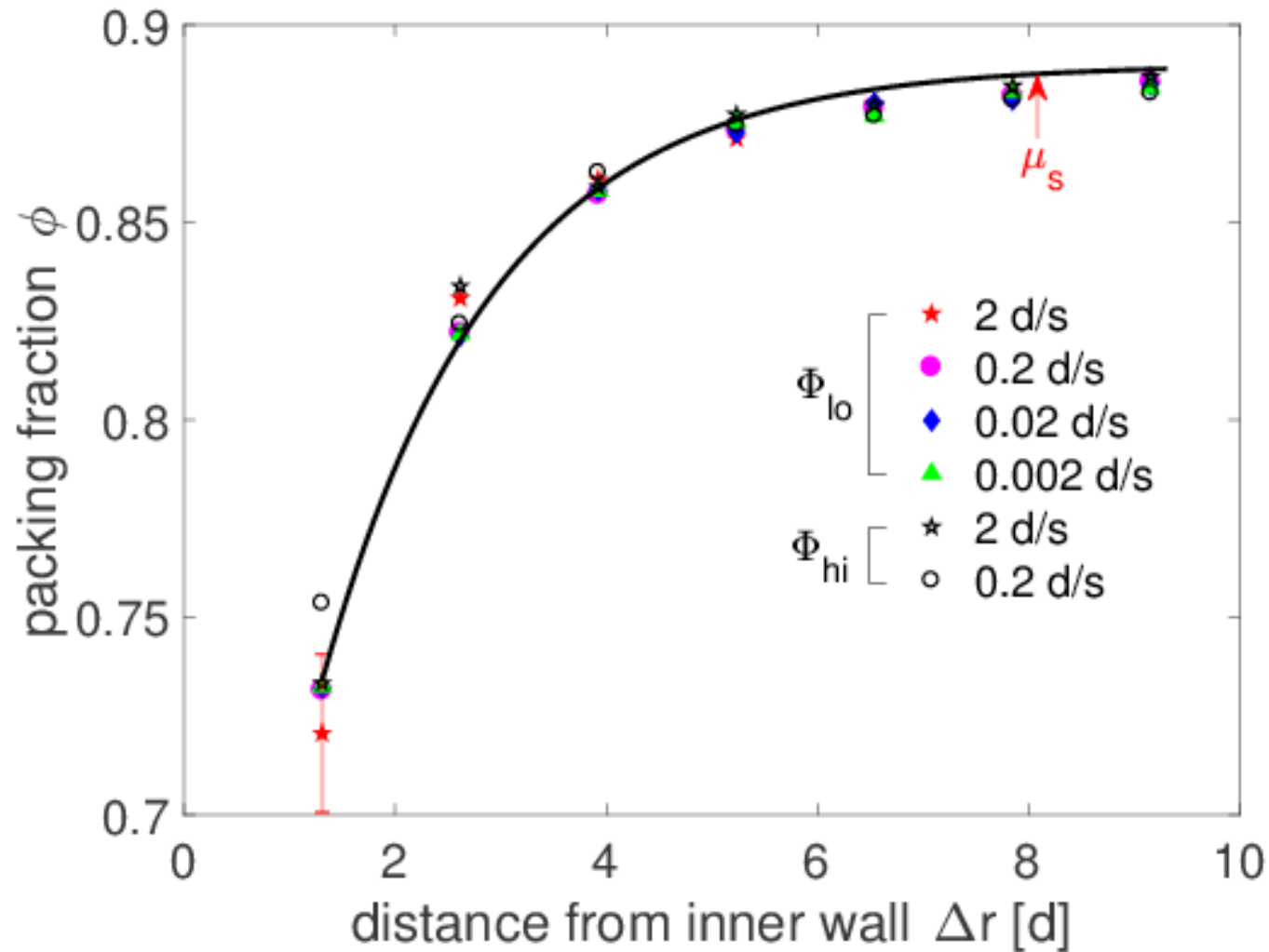
... output the 3-4 parameters needed for the nonlocal model

Conclusions

- New leaf-spring design allows for measurement of boundary stresses
- Tested two nonlocal rheologies:
 - ✓ single set of parameters works to capture $\mu(l)$ and $v(r)$
 - ✓ growing lengthscale at μ_s
- Material parameters are consistent with previous work using DEM simulations
- Newly associate μ_s with a drop in susceptibility to force chain fluctuations



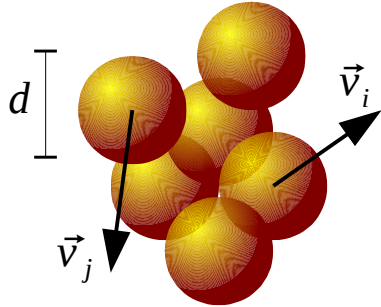
Exponential Decay of Packing Fraction



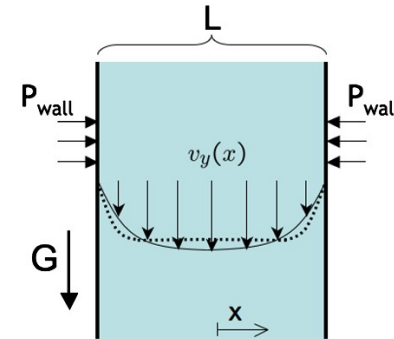
Discrete

vs.

Continuum

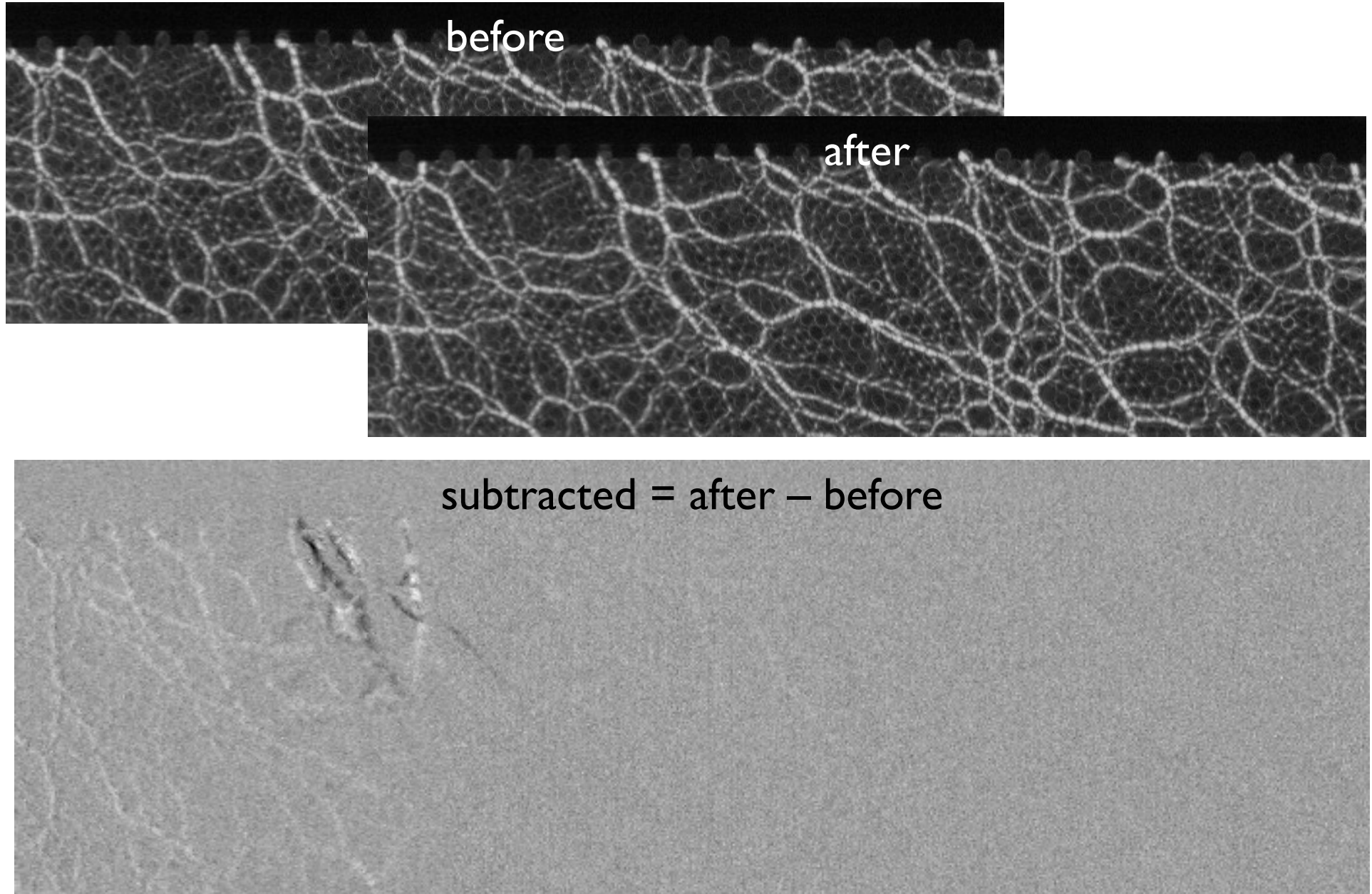


- computer simulation (DEM) solves Newton's Laws for every inter-particle collision
- Advantage: obtain complete trajectories, forces for all particles
- Key Challenges:
 - limited to particles made from sphere/circles
 - provides fictional friction
 - a new simulation (slow) for any new loading geometry or particle properties

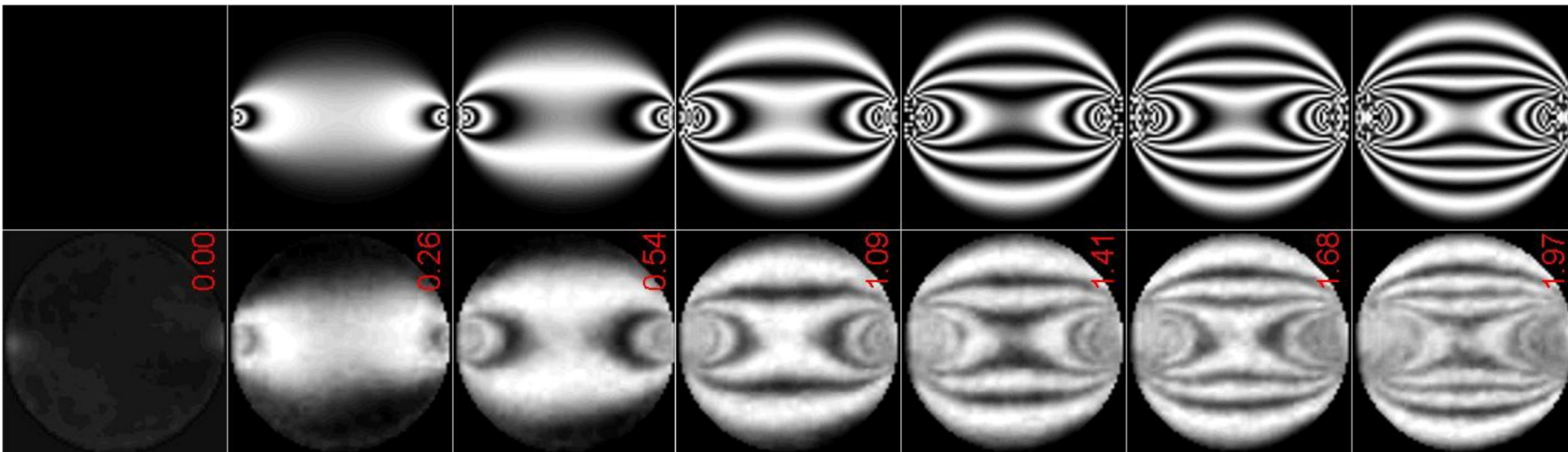
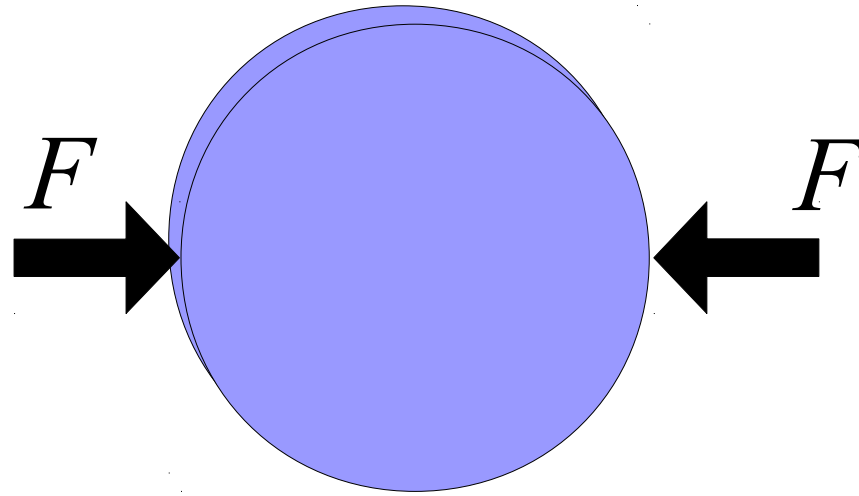


- equation for deformation/flow as a function of a few material parameters
- Advantage: obtain flow field from numerical solution (fast)
- Key Challenges:
 - experiments needed to relate grain-scale parameters to bulk properties
 - same equations, independent of geometry?
 - are there non-local effects?

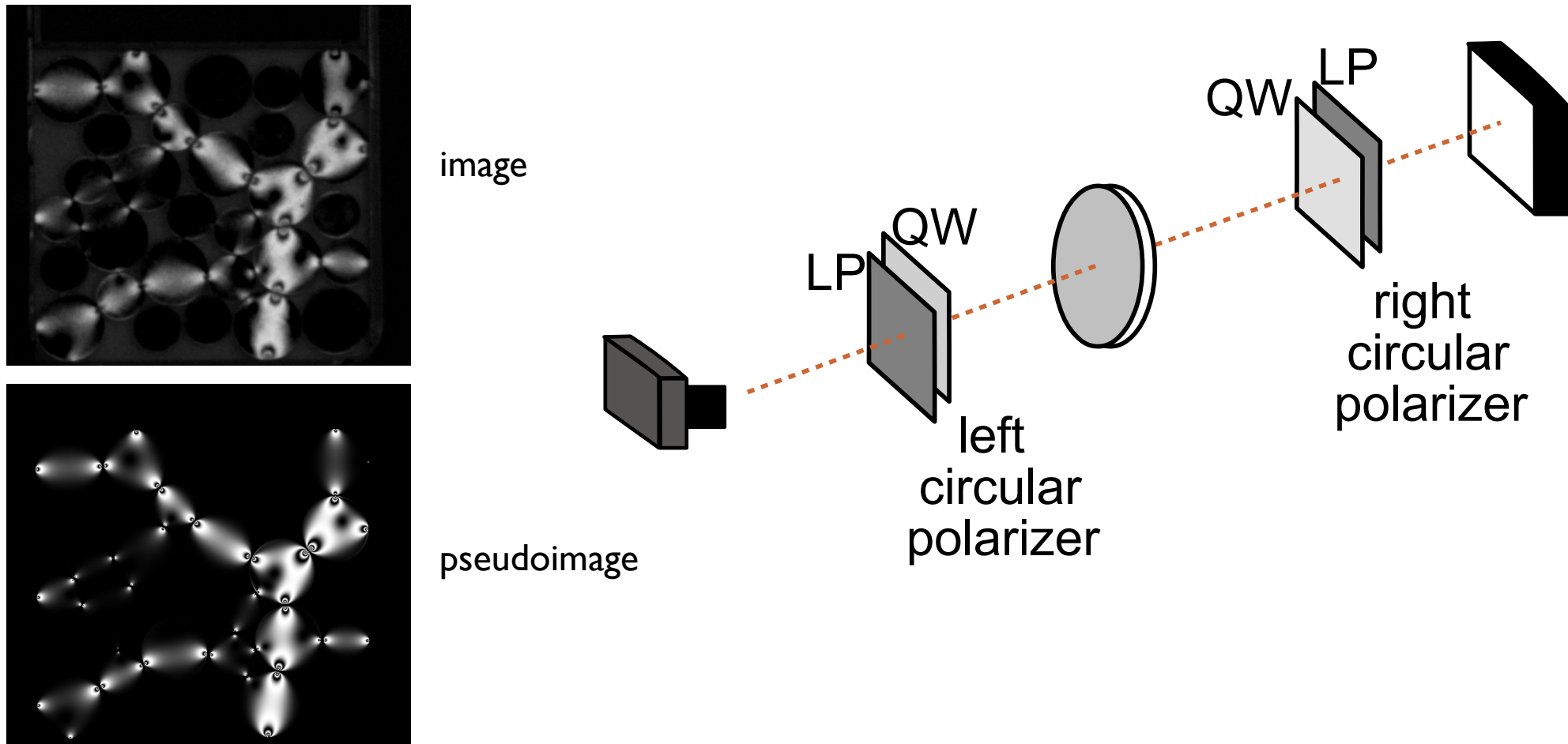
Quick Technique I: Image-Differencing



Increasing force \rightarrow More fringes

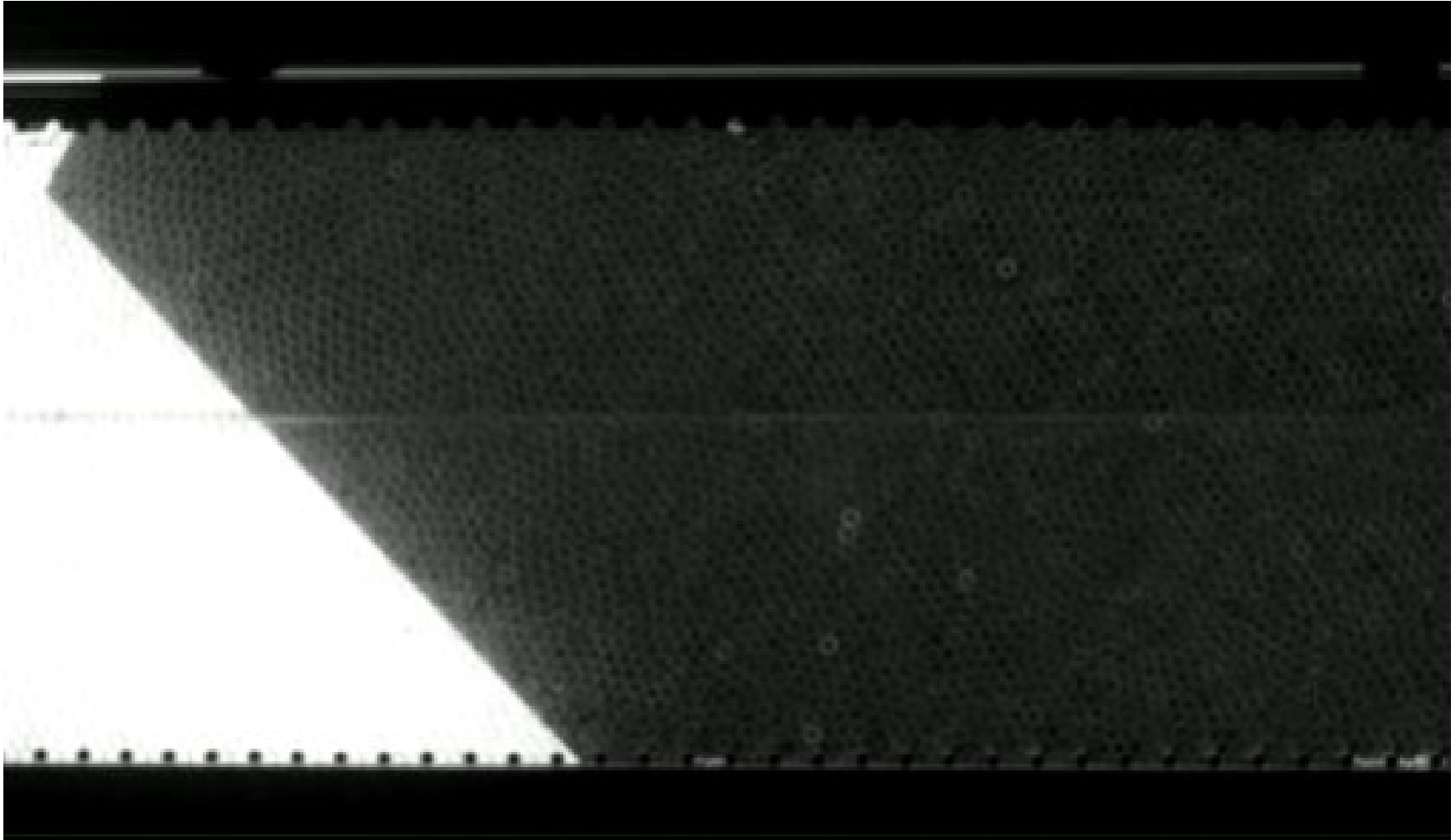


Measuring Vector Forces



Daniels, Kollmer, Puckett. *Rev. Sci. Inst.* (2017)
<https://github.com/jekollmer/PEGS>

Stick-Slip Failure



Daniels & Hayman. *J. Geophys Res.* (2008)