Depinning, Coagulation & Hysteresis

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Depinning and Coagulation:
DCK & MM, EPL, **103** (2013) 46002,
DCK & MM, Ann. H. Poinc., **16** (2015) 2837,
MI, DCK & MM EPL, **115** (2016) 46003.

Memory and RPM:
MMT & MM, arXiv-preprint

MEMFORM18, Goleta

Depinning, Dynamical Critical Phenomena & Hysteresis

Depinning as paradigm for models of friction, earthquakes, jamming transitions?, ...

- Interface pinned by disorder
- Interested in response of elastic interface to loading.
- Evolution through abrupt changes: Avalanches.
- With increased loading, transition from a pinned phase to a macroscopically sliding phase.
- Characterization of the subthreshold evolution towards the depinning transition.
- **This talk:** Look at hysteretic behavior

Revisiting old problems – Old wine in new bottles?

Questions & Goals:

- Obtain exact results from first principles.
- Are dynamical critical phenomena just plain old critical phenomena?

Our results for a 1d CDW type model:

- Threshold configuration as solution of a variational problem.
- Explicit construction of threshold configuration ⇒ characterization of scaling behavior.
- Generic subthreshold evolution: coagulation process, mechanism for growth of correlation length.

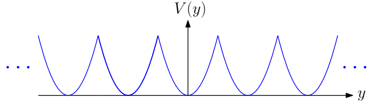
This talk: Hysteresis, return-point-memory, structure of state-transition graphs, response to periodic forcing.

The Fukuyama-Lee-Rice CDW Hamiltonian

A 1d chain of particles connected by springs:

$$\mathcal{H}(\mathbf{y}) = \sum_i \frac{1}{2}(y_i - y_{i-1})^2 + \lambda V(y_i - \alpha_i) - F y_i, \quad \text{PBC: } y_{i+L} = y_i.$$

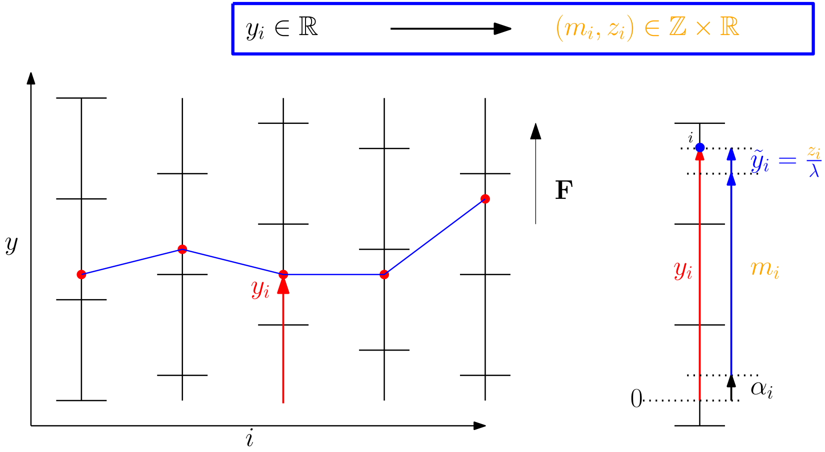
- Each particle rests on a 1-periodic substrate: $V = \frac{1}{2}(y - [\![y]\!])^2$:



- $[\![y]\!]$ is the nearest integer to y,
- λ controls potential strength
- Substrates translated by **random** α_i, i.i.d. uniform on $(-\frac{1}{2}, +\frac{1}{2})$.
- A force F is applied uniformly to all particles.
- Impose relaxational dynamics:

$$\frac{\mathrm{d}y_i}{\mathrm{d}t} = -\frac{\partial \mathcal{H}}{\partial y_i}.$$

Particle configurations and coarse-graining: $y_i \to m_i$:



- well number $m_i = [\![y_i - \alpha_i]\!] \in \mathbb{Z}$, and
- well coordinate $\tilde{y}_i = \frac{z_i}{\lambda} = y_i - \alpha_i - m_i \in (-1/2, 1/2)$.

The 1d toy model in the AQS regime

1. We consider L particles indexed as $i = 0, 1, 2, \dots L - 1$.
2. The neighbours of particles i are $i \pm 1 \mod L$ (periodic BC).
3. To each site i we assign a
   - well-coordinate: $z_i \in \mathbb{R}$, ("local position" inside pinning well)
   - well-number: $m_i \in \mathbb{Z}$, ("interface height")
   - quenched disorder: $\rho_i \sim \mathrm{Uniform}[-1, 1)$, drawn *i.i.d.*
4. We have a constitutive equation (condition for static equilibrium)

$$z_i = \rho_i + m_{i+1} - 2m_i + m_{i-1} = \rho_i + \Delta m_i$$

5. Initially we set

$$m_i = 0,$$
$$z_i = \rho_i,$$

so we start with a set of L random numbers z_i uniformly distributed between $[-1, 1]$.

Evolution to Depinning

Given a configuration $(\mathbf{z}, \mathbf{m})$, produce a new configuration $(\mathbf{z}', \mathbf{m}')$ as follows:

(A1) Record $h_c = \max_i z_i$.

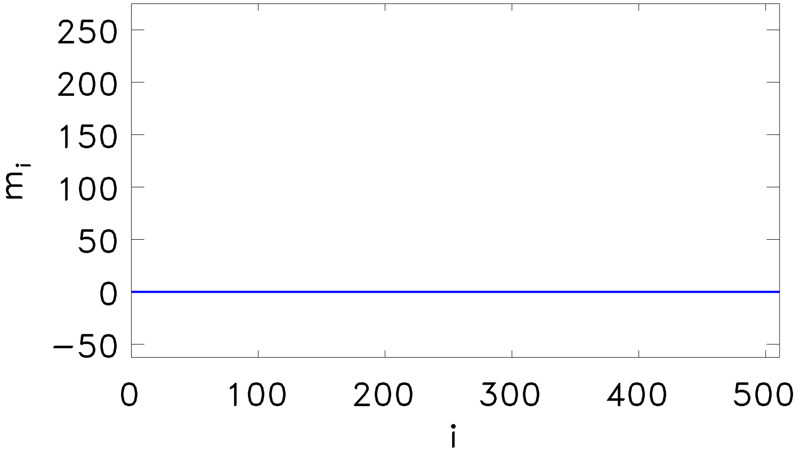
(A2) For any j with $z_j \geq h_c$ do

$$\begin{aligned} m_j &\to m_j + 1 \\ z_{j\pm1} &\to z_{j\pm1} + 1 \\ z_j &\to z_j - 2. \end{aligned}$$

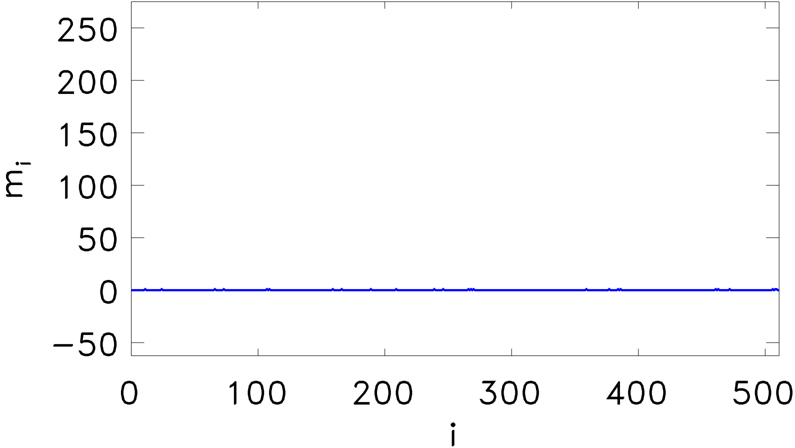
(A3) Repeat (A2), until $\max_i z_i < h_c$.

- The termination by (A3) forms a step of the algorithm. Next step, start with (A1), *etc.*
- The integer m_i counts the number of times site i has "jumped", (height of interface).

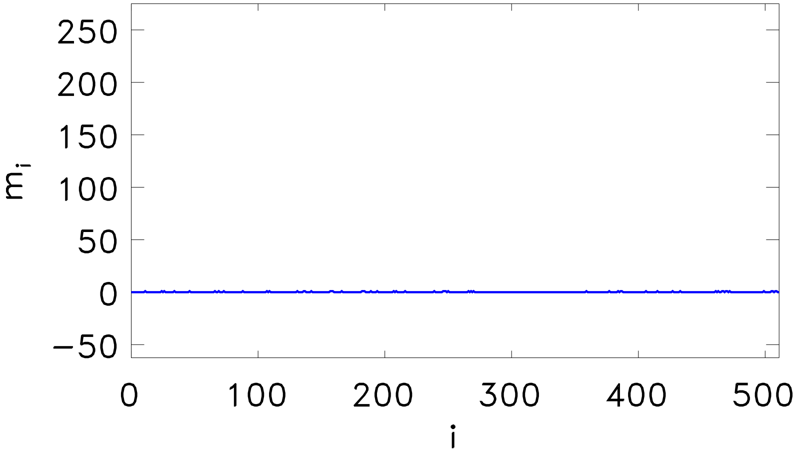
Avalanche Algorithm in Action – Step 0



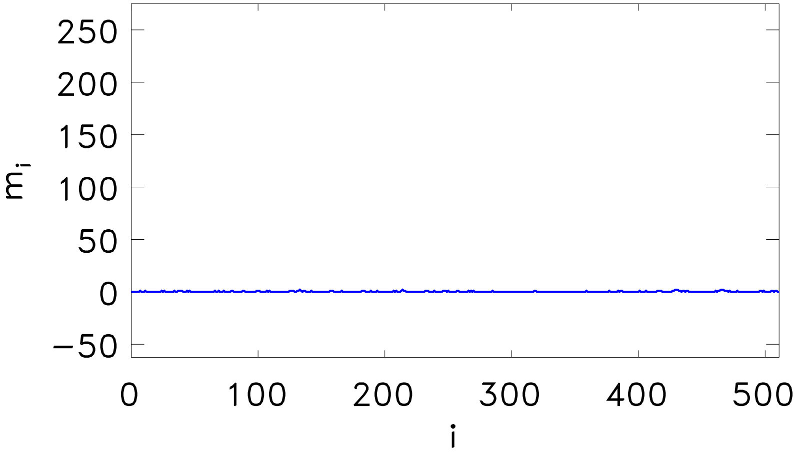
Avalanche Algorithm in Action – Step 25



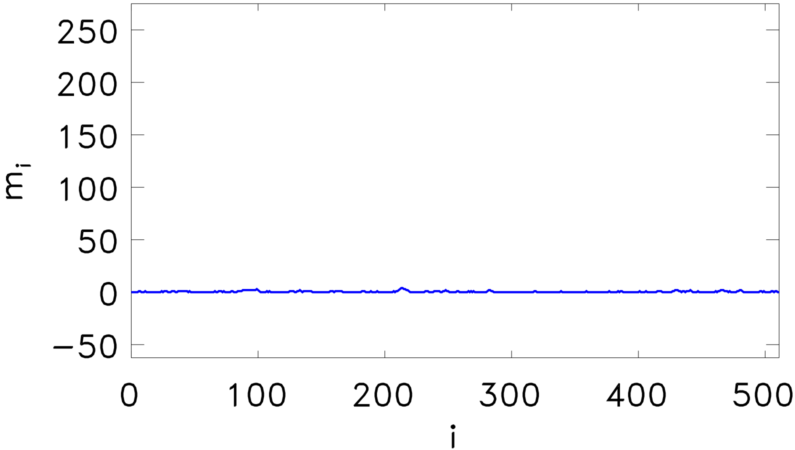
Avalanche Algorithm in Action – Step 50



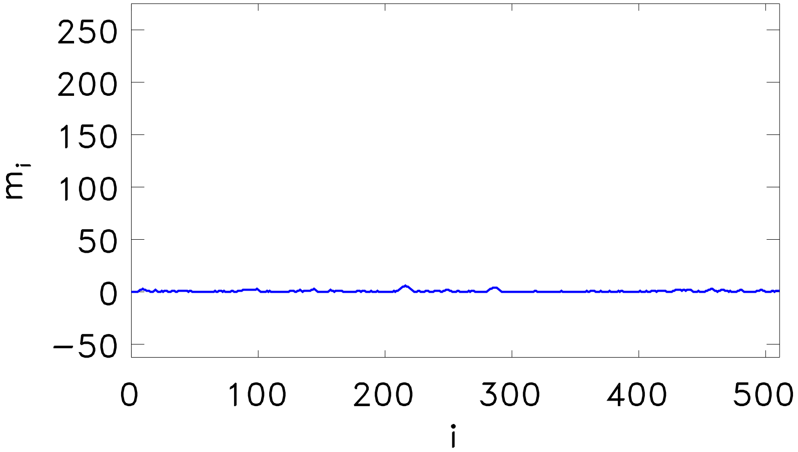
Avalanche Algorithm in Action – Step 75



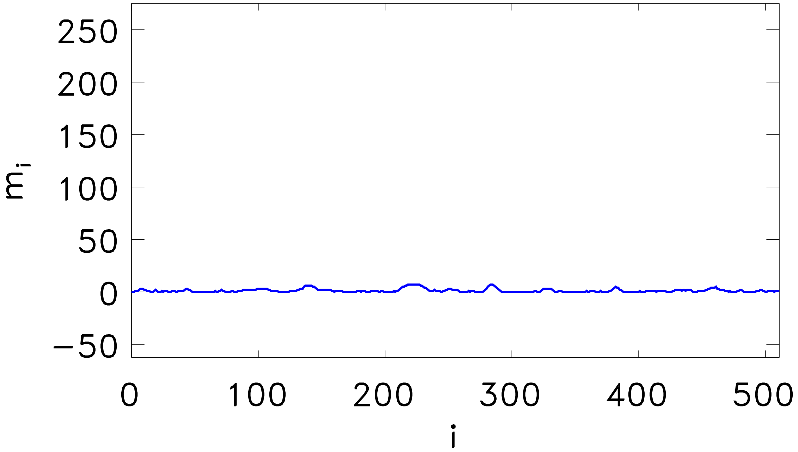
Avalanche Algorithm in Action – Step 100



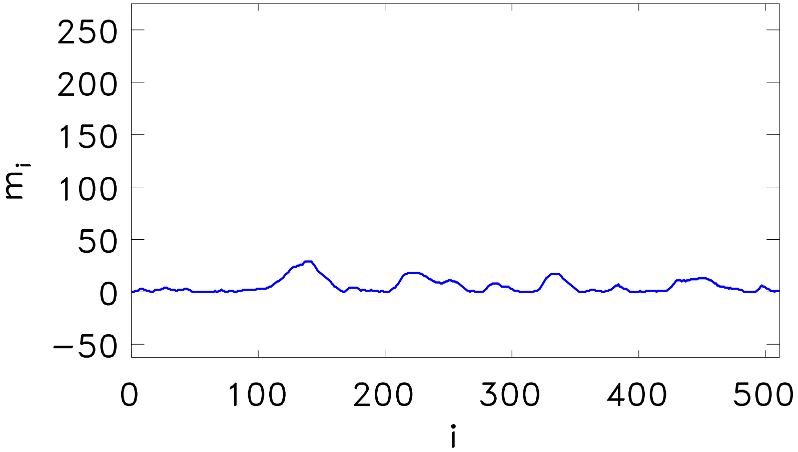
Avalanche Algorithm in Action – Step 125



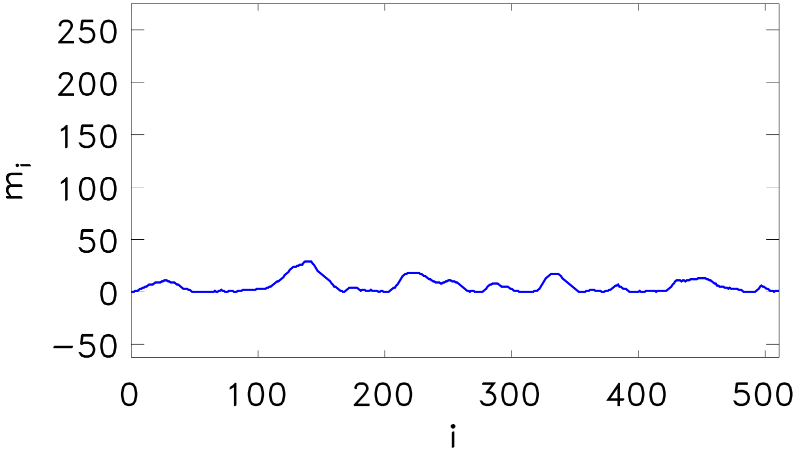
Avalanche Algorithm in Action – Step 150



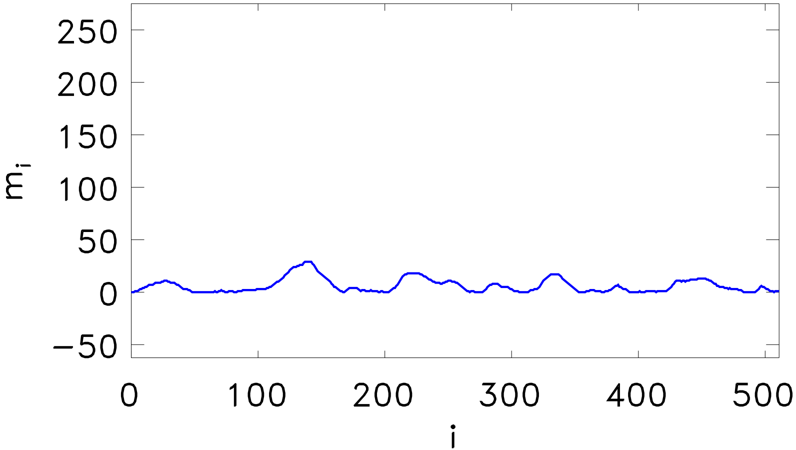
Avalanche Algorithm in Action – Step 175



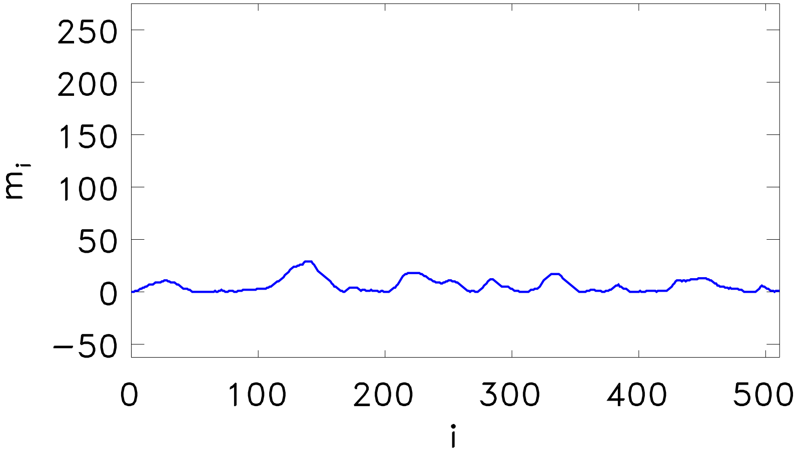
Avalanche Algorithm in Action – Step 176



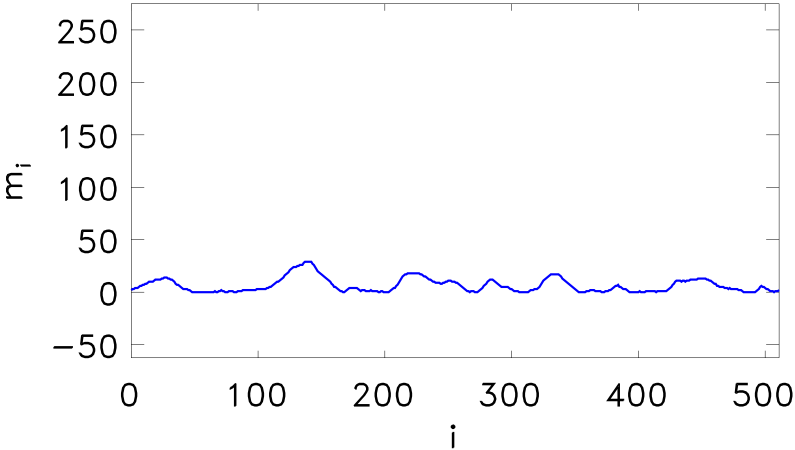
Avalanche Algorithm in Action – Step 177



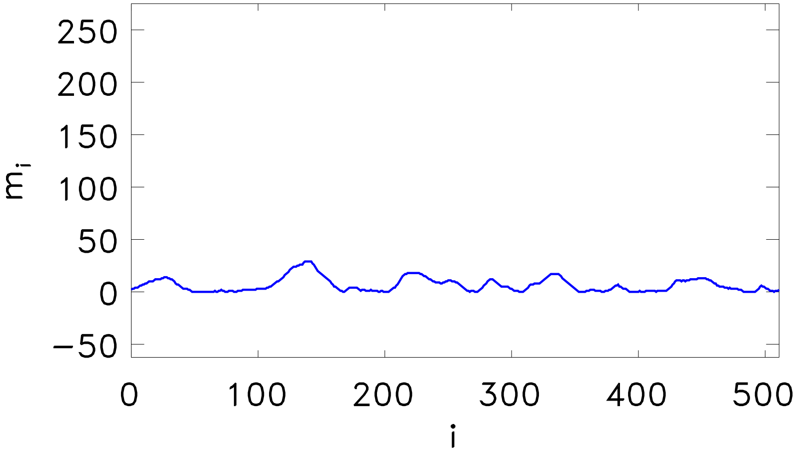
Avalanche Algorithm in Action – Step 178



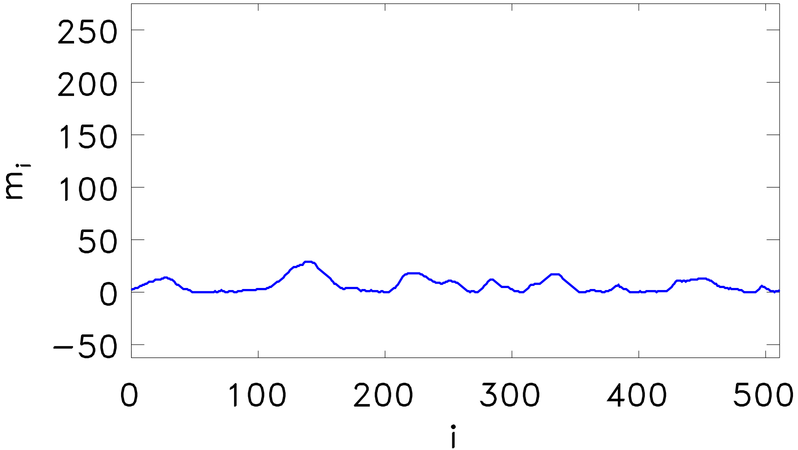
Avalanche Algorithm in Action – Step 179



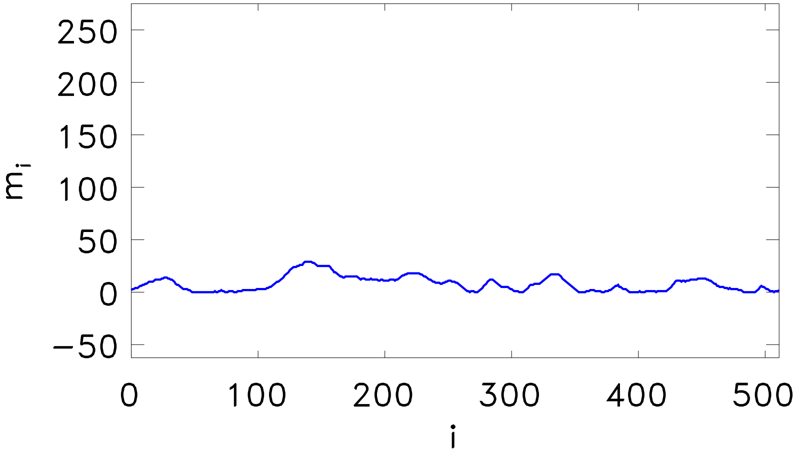
Avalanche Algorithm in Action – Step 180



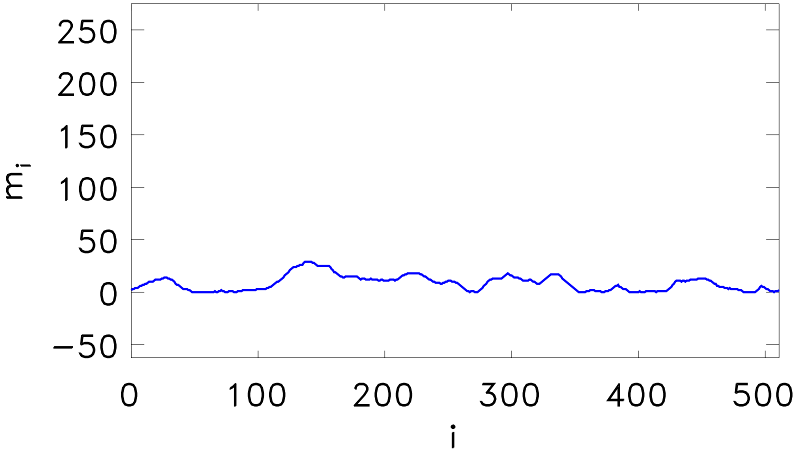
Avalanche Algorithm in Action – Step 181



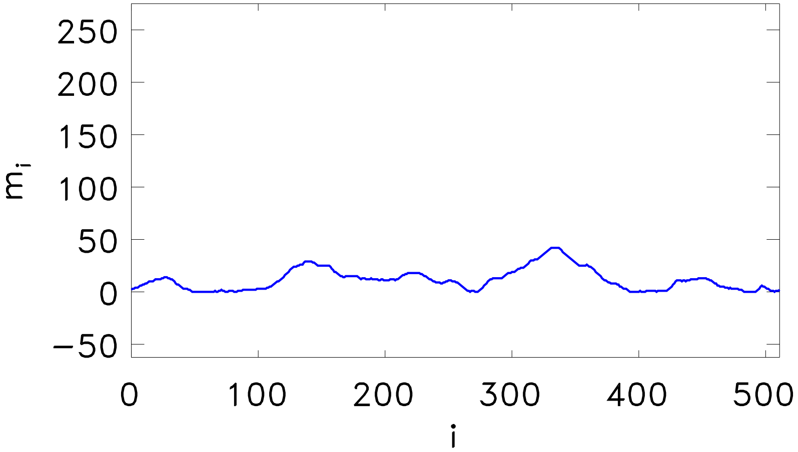
Avalanche Algorithm in Action – Step 182



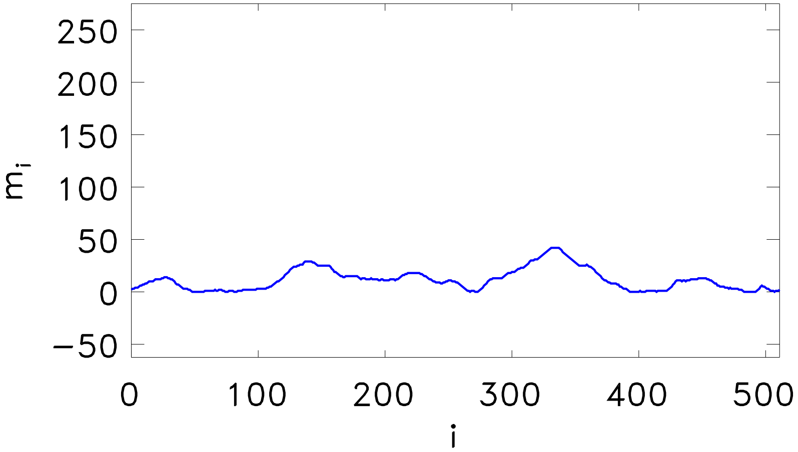
Avalanche Algorithm in Action – Step 183



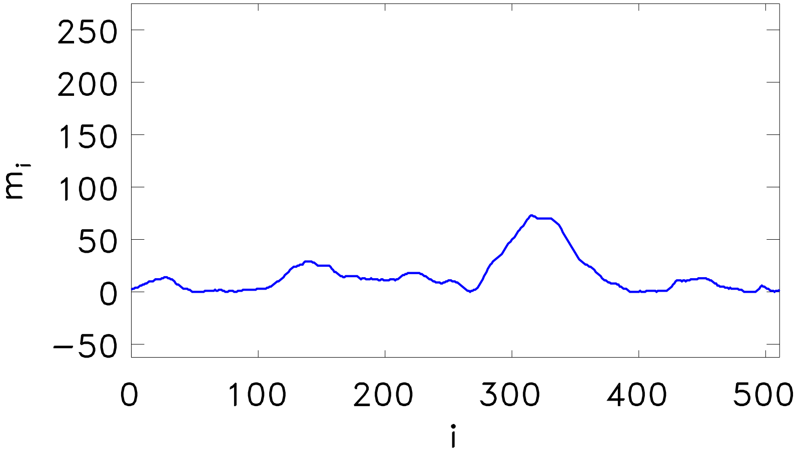
Avalanche Algorithm in Action – Step 184



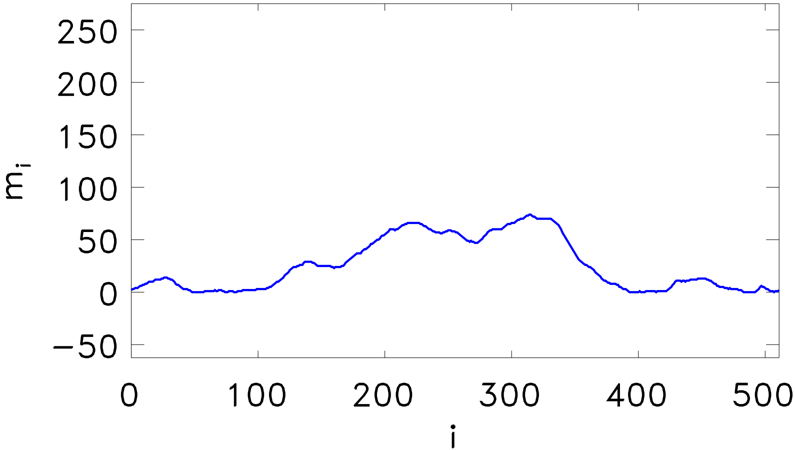
Avalanche Algorithm in Action – Step 185



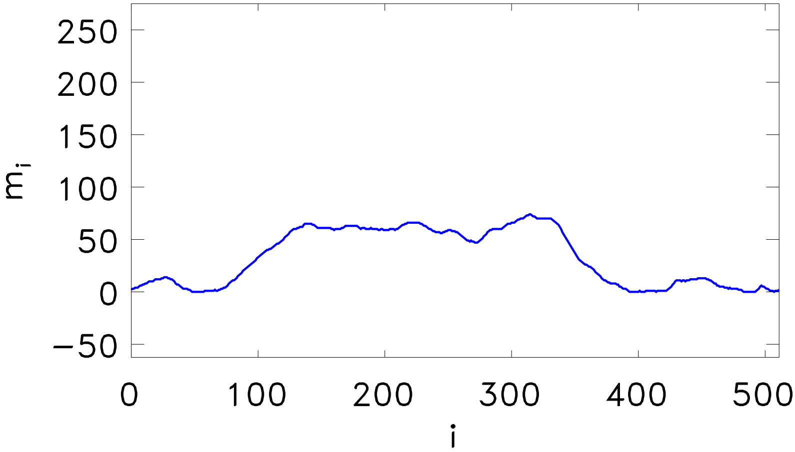
Avalanche Algorithm in Action – Step 186



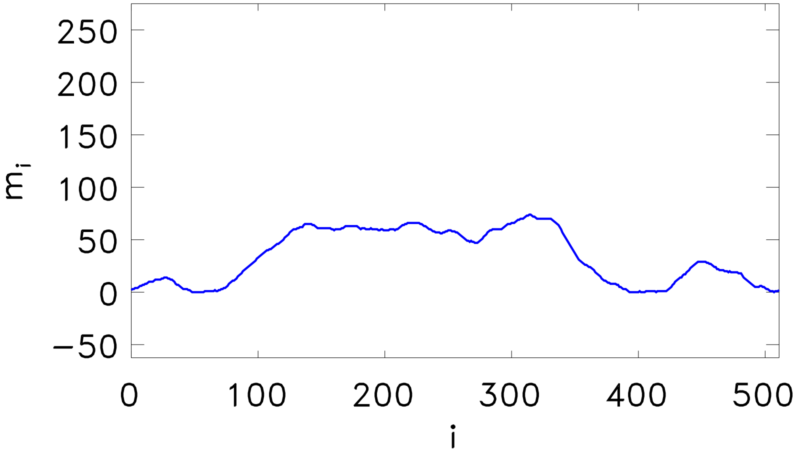
Avalanche Algorithm in Action – Step 187



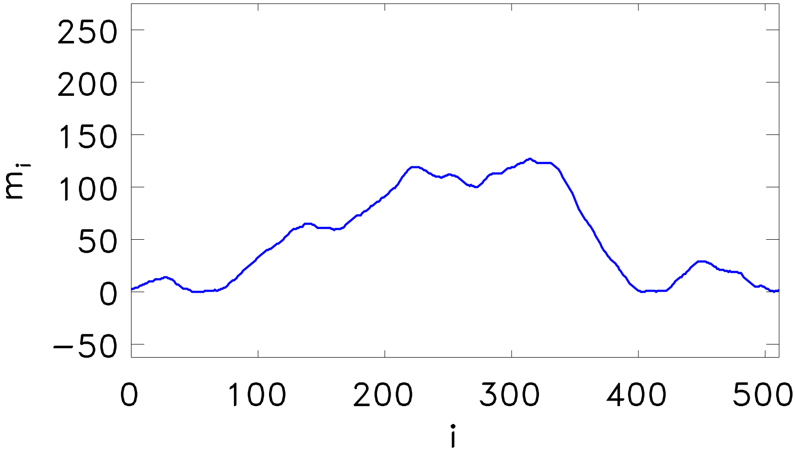
Avalanche Algorithm in Action – Step 188



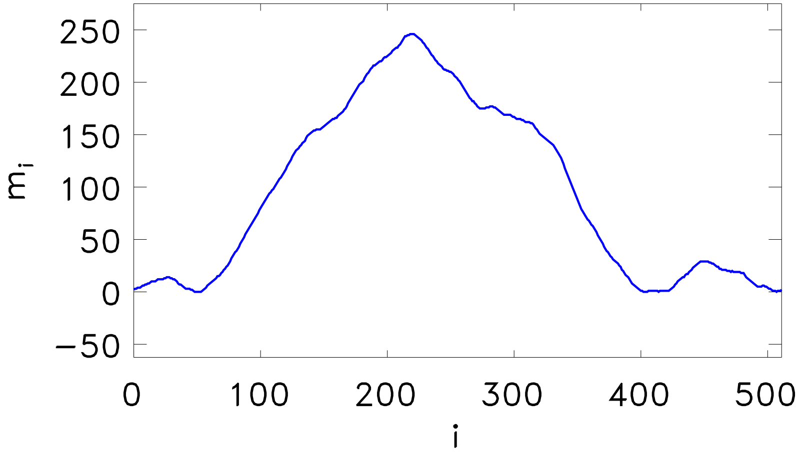
Avalanche Algorithm in Action – Step 189



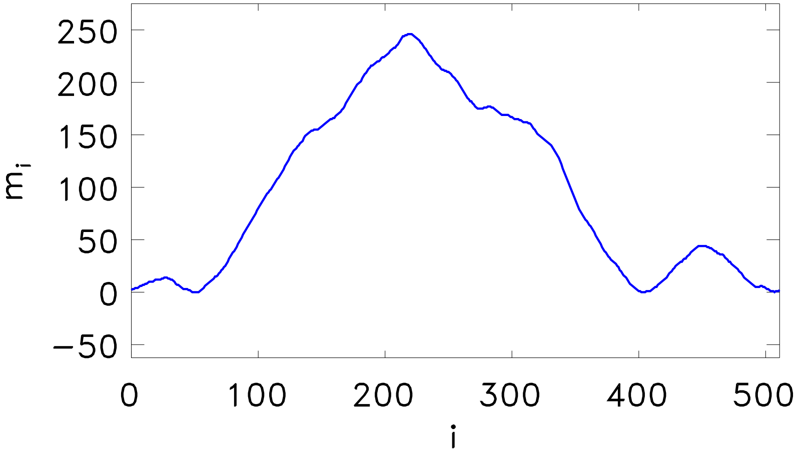
Avalanche Algorithm in Action – Step 190



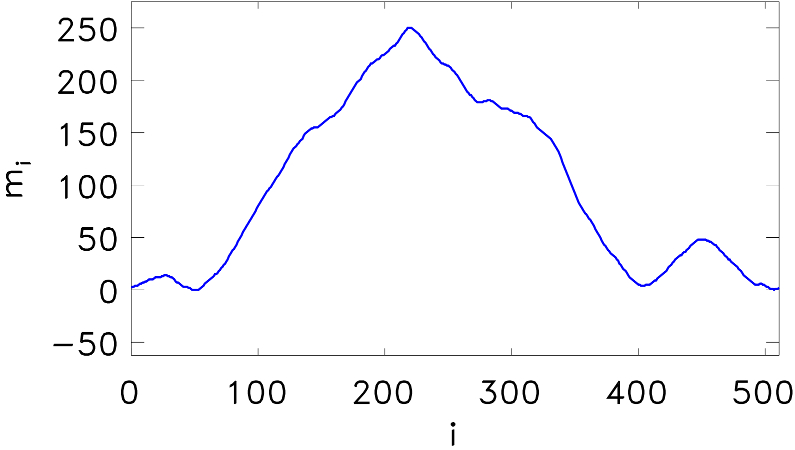
Avalanche Algorithm in Action – Step 191



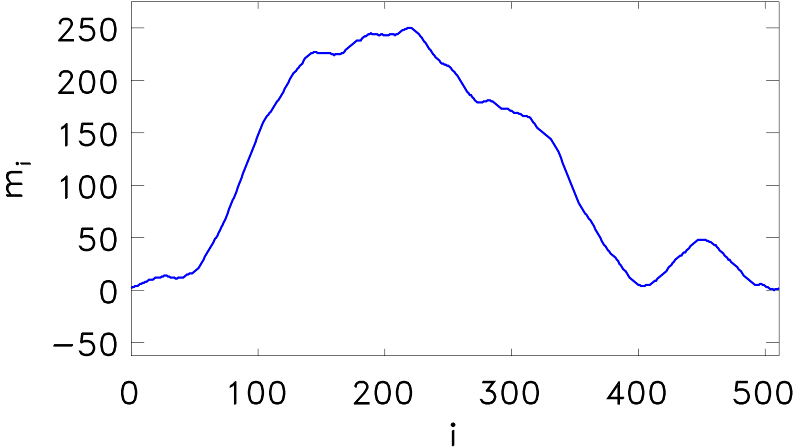
Avalanche Algorithm in Action – Step 192



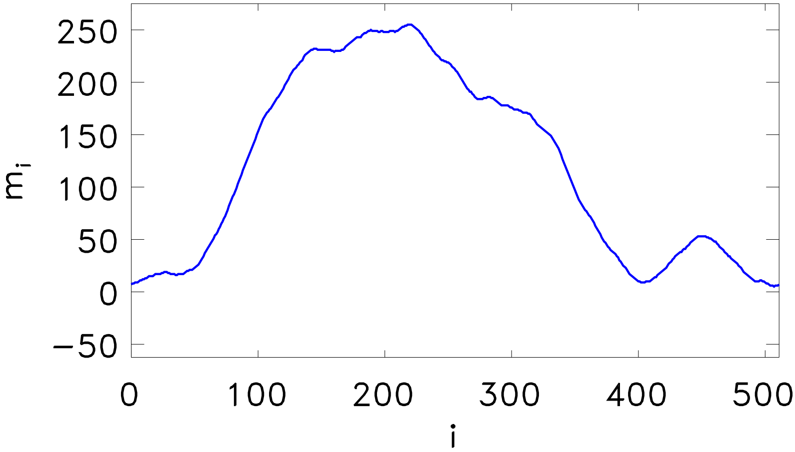
Avalanche Algorithm in Action – Step 193



Avalanche Algorithm in Action – Step 194



Avalanche Algorithm in Action - Depinned!



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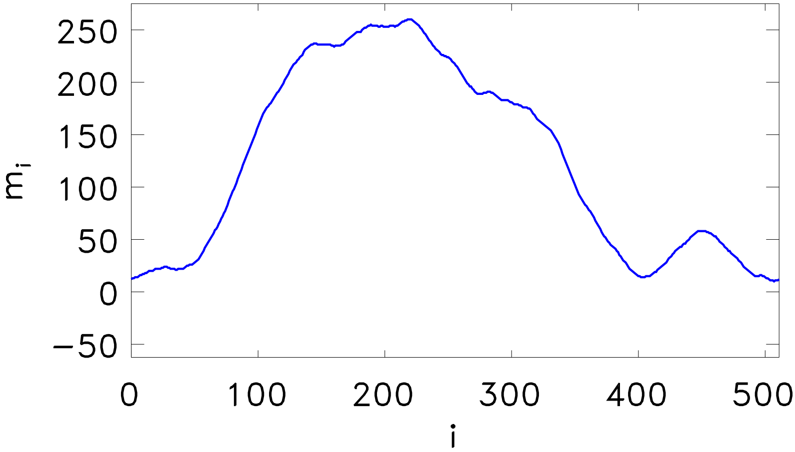

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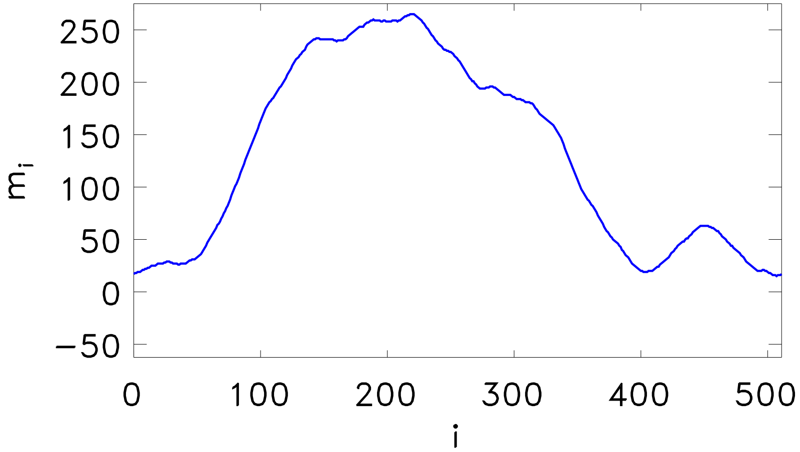

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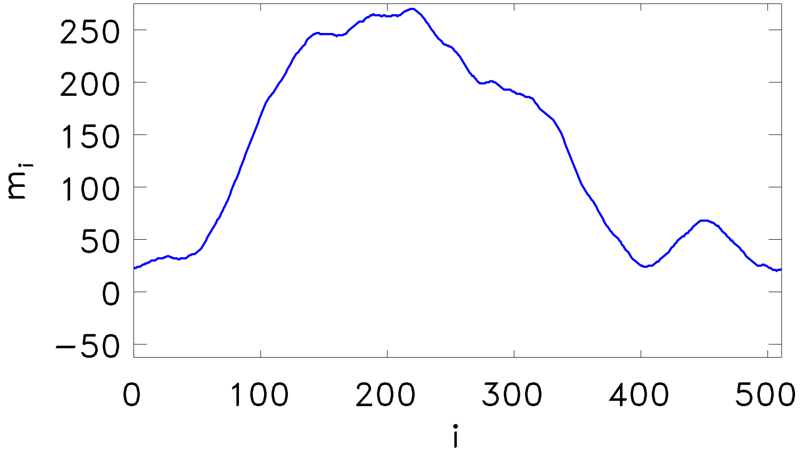

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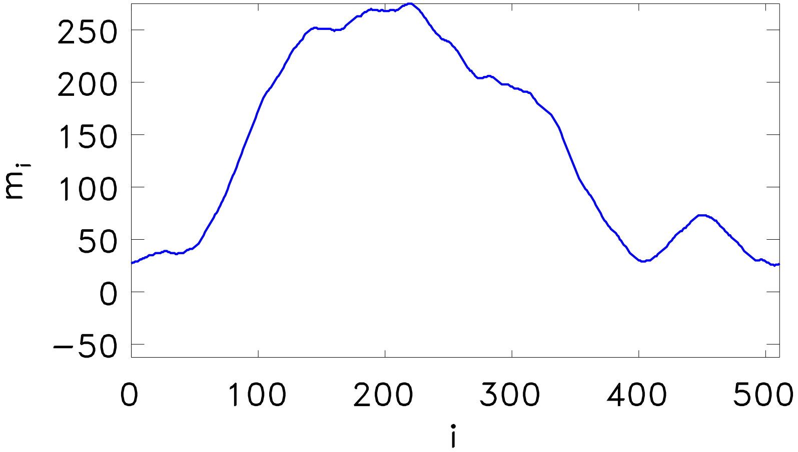
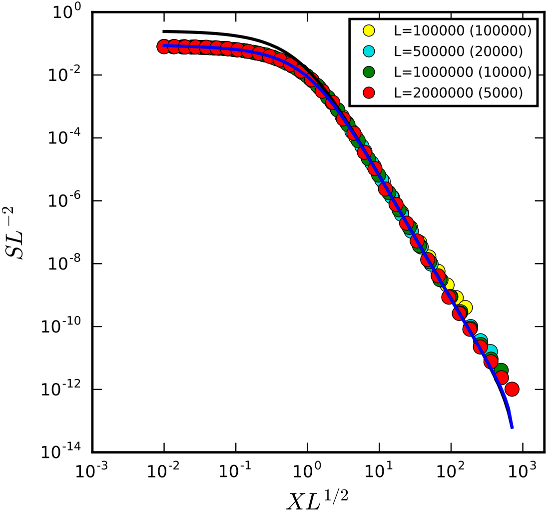


Parametrization of the Evolution

Critical behavior of the **evolution to threshold**.

- Label the steps as $\tau = 0, 1, 2, \ldots$
- Index the configurations using $X(\tau) = \max_i z_i(\tau) - \max_i z_i^+$.
- Evolution to threshold: $X \searrow 0$.
- We are interested in the behavior of the disorder-averaged correlation length and avalanche size, $\xi(X)$ and $\mathcal{S}(X)$.

Criticality of Avalanche Size and Correlation Length



From the finite size scaling:

$$\xi \sim X^{-\nu},$$
$$\mathcal{S} \sim X^{-\gamma},$$

with

$$\nu = 2 \quad \gamma = 4.$$

(Narayan Middleton 1994).

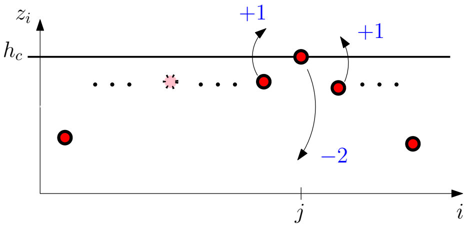
Start of Avalanche Step:

Avalanche Step:

1. $h_c = \max_i z_i$
2. For any $z_j \geq h_c$:

$$\begin{aligned} m_j &\rightarrow m_j + \mathbf{1}, \\ z_j &\rightarrow z_j \mathbf{-2}, \\ z_{j\pm1} &\rightarrow z_{j\pm1} \mathbf{+1} \end{aligned}$$

3. Repeat **2.** until $z_i < h_c$ for all sites i.



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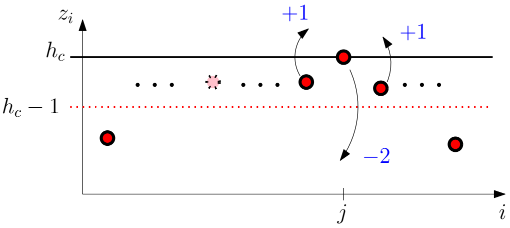

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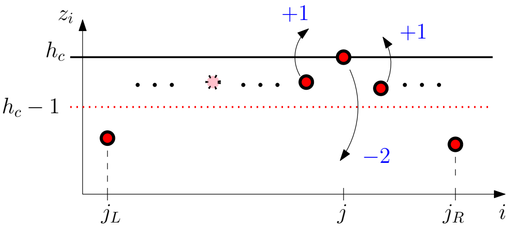

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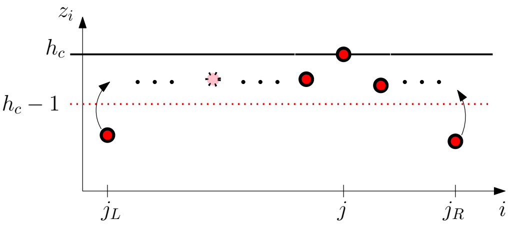

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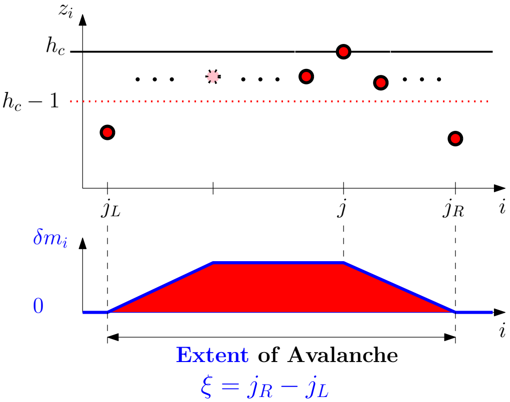

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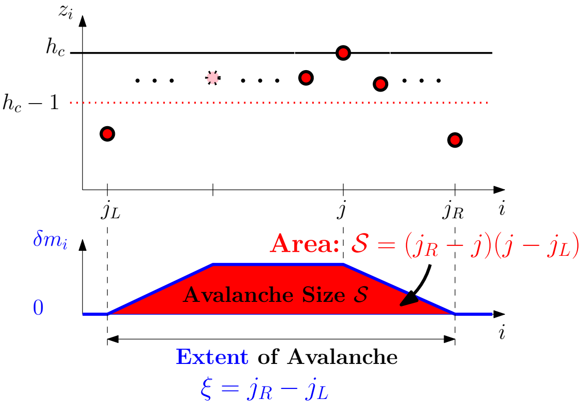

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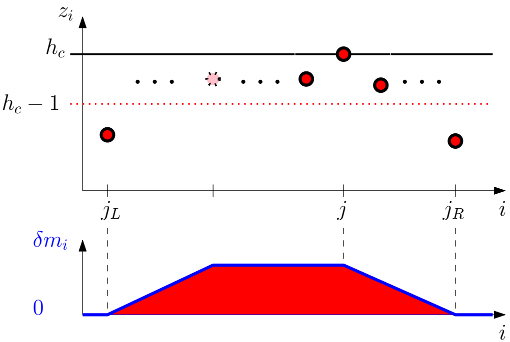

End of Avalanche Step:

Avalanche Step:

1. $h_c = \max_i z_i$
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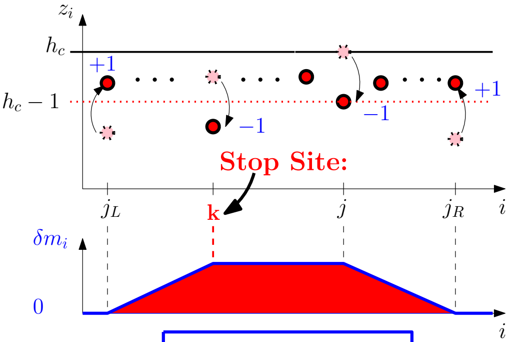


$$z_i = \rho_i + \Delta m_i$$

Toy Model: The Importance of Active Regions

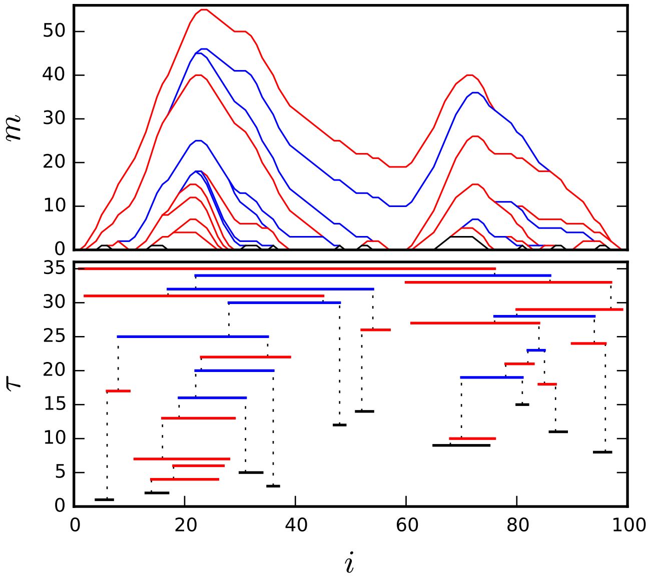
In order to understand the evolution, we need to distinguish:

- Pristine Regions Sites where no avalanche activity occurred so far.
- Active Regions Sites where avalanche activity has occurred already.

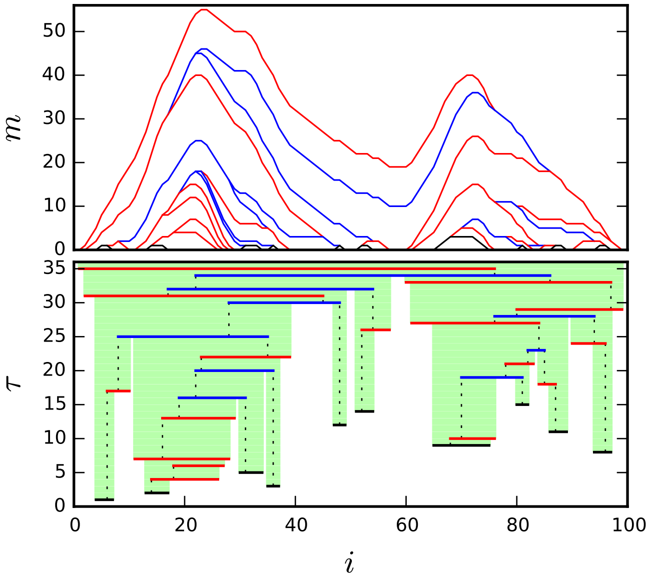
Avalanches propagate differently in Pristine and Active Regions!

- Pristine Region: Propagation depth remains at fixed length scale, independent of system size,
  $\Rightarrow$ **microscopic length scale, as $L \to \infty$**.
- Active Region: Like a highway, only stop sites can stop propagation,
  $\Rightarrow$ **macroscopic growth process**.

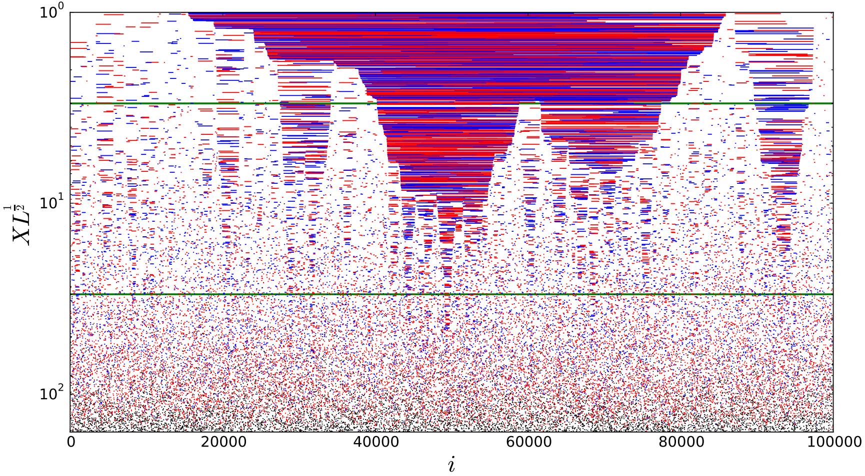
Active Regions



Active Regions



Active Regions



Merger of ARs as a Coagulation Process

- The merger of two ARs is facilitated by an avalanche.
- Basic process:
  - Nucleation: Avalanches start at the site with largest z_i.
  - Location: Overwhelmingly, this happens in an AR.
  - Preferrentiality: The larger the AR, the larger the probability that avalanche will start there.
  - Spreading: The nucleated avalanche extends beyond the initial AR into a neighbouring AR where it is stopped at the AR's stop-site.
  - Merging: The two ARs merge and a new stop site is created.
  - Overall: We have the following coagulation process of ARs

AR of size ℓ_1 merges with AR of size ℓ_2 $\longrightarrow$ AR of size $\ell_1 + \ell_2$

Merger of ARs as a Coagulation Process

- Overall: Coagulation process for ARs

AR of size ℓ_1 merges with AR of size ℓ_2 $\longrightarrow$ AR of size $\ell_1 + \ell_2$

- $N(\tau, \ell)$: Number of ARs of length ℓ at step τ of evolution
- Leads to a Smoluchowski Coagulation Equation with known solution.

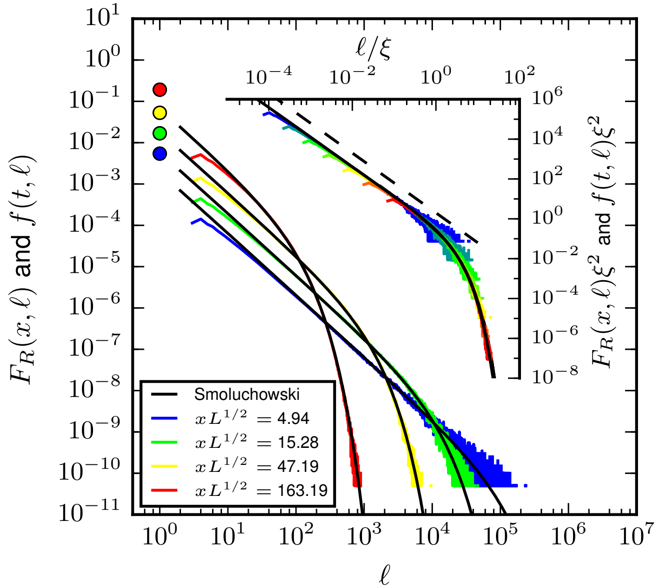
$$N(\tau+1, \ell) - N(\tau, \ell) \sim \sum_{\ell'}^{\ell-1} \frac{\ell' N(\tau, \ell')}{L} \frac{N(\tau, \ell - \ell')}{M_0(\tau)} - \frac{\ell N(\tau, \ell)}{L} - \frac{N(\tau, \ell)}{M_0(\tau)},$$

where

$$M_n = \sum_{\ell} \ell^n N(\tau, \ell), \quad n = 0, 1, 2, \ldots \tag{1}$$

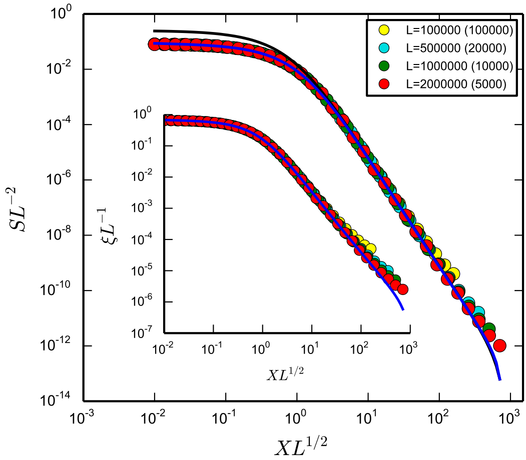
are the moments of the AR distribution ($M_1 = L$).

Comparison with Smoluchowski Solution



Predictions for ξ and $\mathcal{S}$

$$\xi = \frac{2}{3}\frac{M_2}{M_1} + \frac{1}{2}\frac{M_1}{M_0} \quad \text{and} \quad \mathcal{S} = \frac{1}{12}\frac{M_3}{M_1} + \frac{1}{6}\frac{M_2}{M_0},$$



Memory Part Starts HERE!

Plan for the rest of this talk:

- Start from considering hysteresis in the toy model
- Generalize to a class of minimal models (DAMA)
- Consider response to periodic forcing
- Discuss possible extensions.

Hysteresis: Two directions of change: U and D

- In the toy model for depinning we have a choice of direction for the transverse force, **U**p and **D**own.
- In the **U** case the particle with **largest** z will jump first:
  1. $m_j \to m_j + 1$,
  2. $z_j \to z_j - 2$,
  3. $z_{j\pm1} \to z_{j\pm1} + 1$.

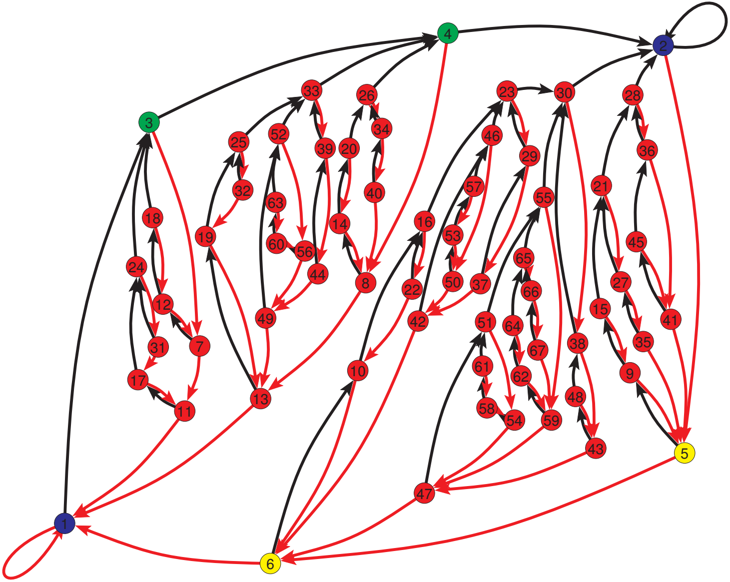
Hysteresis: Two directions of change: U and D

- In the toy model for depinning we have a choice of direction for the transverse force, **U**p and **D**own.
- In the **D** case the particle with **smallest** z will jump first:
  1. $m_j \to m_j - 1$,
  2. $z_j \to z_j + 2$,
  3. $z_{j\pm1} \to z_{j\pm1} - 1$.

Two choices for an Avalanche Algorithm: U and D

- Given a configuration, we can transit to a new configuration by the U or D operation.
- The lower and upper threshold configurations are the absorbing states for the U and D operation.
- All configurations evolve into these absorbing states under monotonous U or D applications.
- Like a magnet, the threshold configuratins are the two saturated states (all spins up or down).
- U, D correspond to increasing/decreasing external magnetic field until you trigger spin flips.
- Threshold reachable states: The set of states reachable from threshold via arbitrary sequences of U or D.

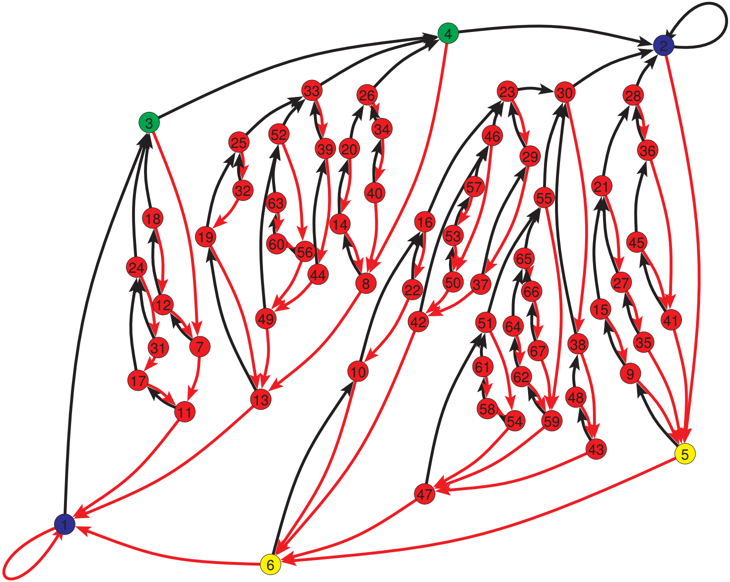
Graph of threshold-reachabable states for the toy model



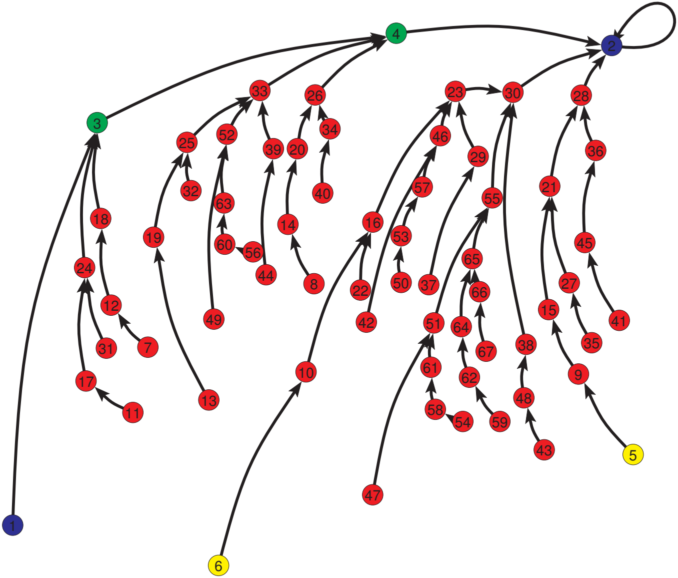
Hystheresis - Features of the reachability graph

- Nested structure: loops within loops.
- Double-tree structure: Transition graph under **black** and **red** arrows are trees.
- Double-tree structure responsible for loops and RPM

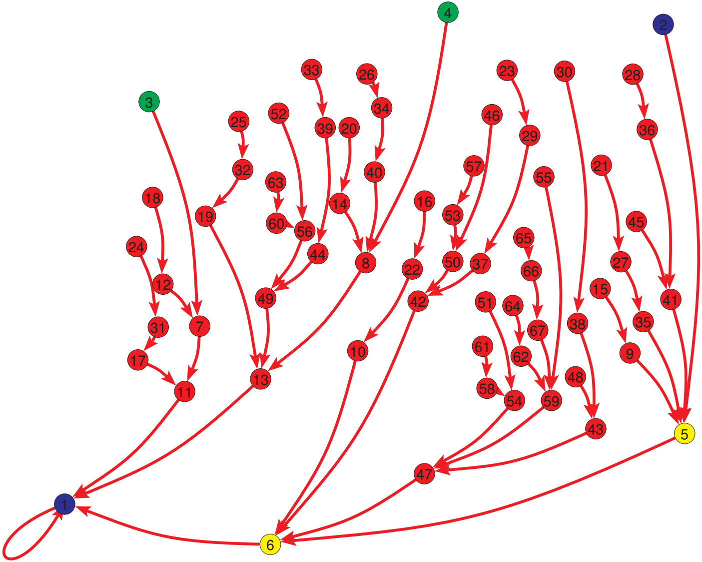
The double-tree structure of a major loop



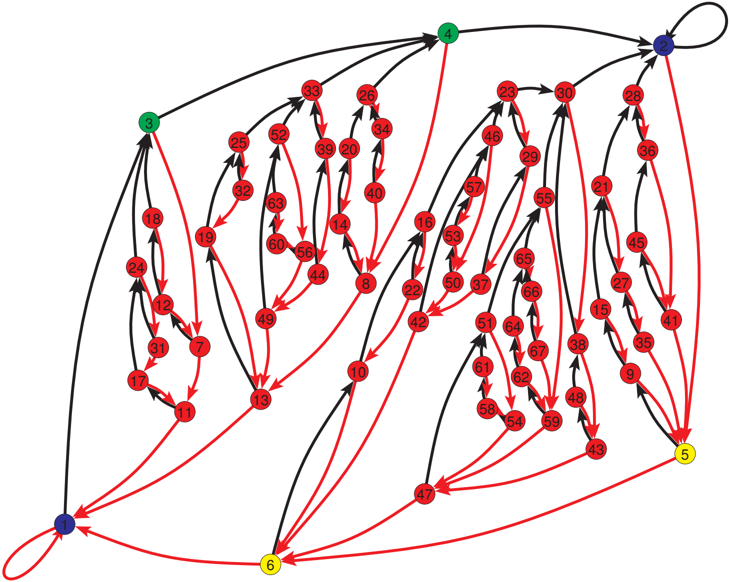
The double-tree structure of a major loop



The double-tree structure of a major loop

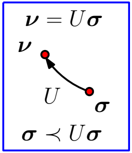


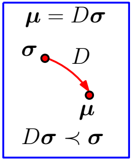
The double-tree structure of a major loop

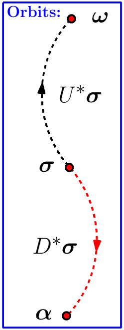


DAMA: Discrete Absorbing Monotonus Automaton:

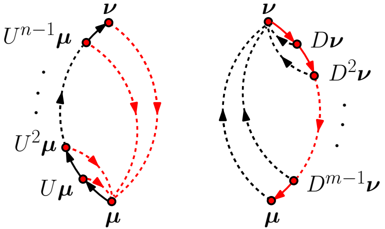
- AQS dynamics.
- Configurations have the structure of a POSET, order relation $\prec$.
- Absorbing states α and ω.
- U, D preserve partial ordering: $\sigma \prec U\sigma$ and $D\sigma \prec \sigma$.
- RPM (return-point-memory):







$\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ endpoints of loop $(\boldsymbol{\mu}, \boldsymbol{\nu})$.
$U^i\boldsymbol{\mu}$ and $D^j\boldsymbol{\nu}$ boundary points of loop.



RPM: $\boldsymbol{\mu} \in D^*(U^i\boldsymbol{\nu})$
$\boldsymbol{\nu} \in U^*(D^j\boldsymbol{\mu})$

DAMA: Global No-passing vs. local RPM

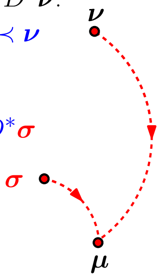
- Sethna, Dahmen *et al.*: Middleton's no passing + AQS implies RPM.
- However Middleton's no passing too strong: global property, involving any pair of ordered states.

No Passing

given $\boldsymbol{\mu} \in D^*\boldsymbol{\nu}$:

IF $\boldsymbol{\mu} \prec \boldsymbol{\sigma} \prec \boldsymbol{\nu}$

$\Rightarrow \boldsymbol{\mu} \in D^*\boldsymbol{\sigma}$



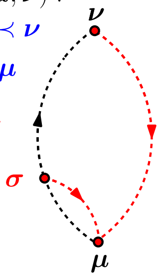
RPM

given loop $(\boldsymbol{\mu}, \boldsymbol{\nu})$:

IF $\boldsymbol{\mu} \prec \boldsymbol{\sigma} \prec \boldsymbol{\nu}$

AND $\boldsymbol{\sigma} \in U^*\boldsymbol{\mu}$

$\Rightarrow \boldsymbol{\mu} \in D^*\boldsymbol{\sigma}$



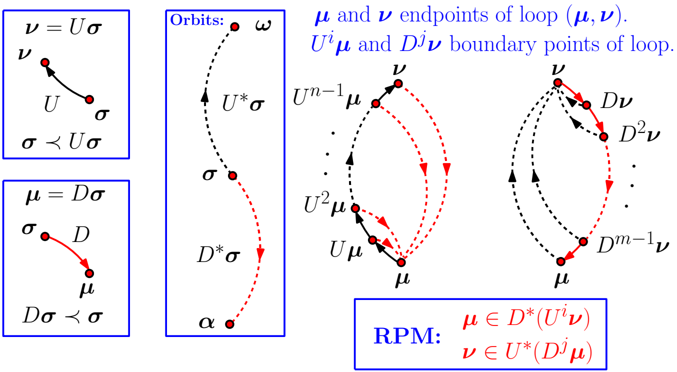
RPM from no-passing: it's a package deal – Preview

- No-passing implies RPM.
- No passing is sufficient, but not necessary for RPM.
- No passing is a global property
- RPM is a property restricted to loops, a local property.
- RPM is about intra-loop structure:
  - Nesting of loops.
  - Tree representation of hierarchy of sub loops.
- No-passing does more: controls inter-loop structure:
  - $\Rightarrow$ transient length of at most one period when subject to periodic forcing. Training cycle at most one-period long.
  - Limits the extend to which one can probe the landscape of meta-stable states by periodic forcing.

DAMA: What do you get if you only have RPM?

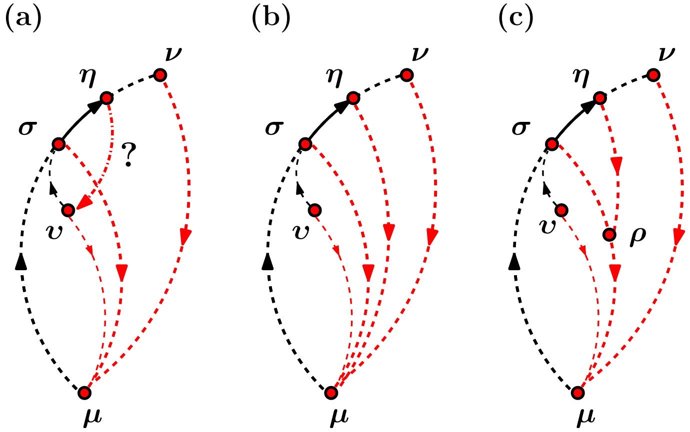
- Why consider RPM **without** No-passing:
  - The inter-loop structure has almost no restrictions.
  - Can have many training cycles before settling into an RPM loop (periodic response).
  - RPM intra-loop structure naturally marginal (ideas of S. Nagel): reducing amplitude after periodic response, still retains periodic response.
  - There are systems exhibiting RPM without having no-passing: Spin ice systems, spin models with AF interactions.
  - RPM can depend on parameters such as disorder strength.
- DAMA is an "honest model": it implements RPM and nothing but RPM.
- DAMA is a useful benchmark/reference model for comparison with real systems exhibiting memory effects.

DAMA RPM and intra-loop structure: Definitions

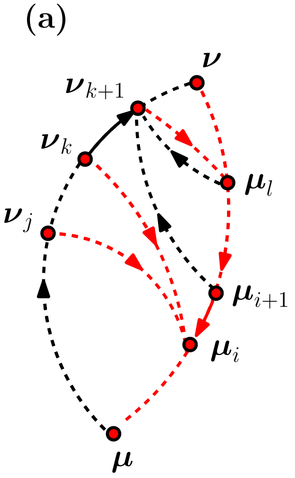


- (μ, ν) endpoints of loop.
- We focus on loops, we don't care right now where $U\nu$ or $D\mu$ lead to.
- States σ associated with a loop: start from μ apply sequence of U and Ds, require $\mu \prec \upsilon \prec \nu$ for all intermediate states
- Loop boundary: states $U^i\mu$ and $D^j\nu$.

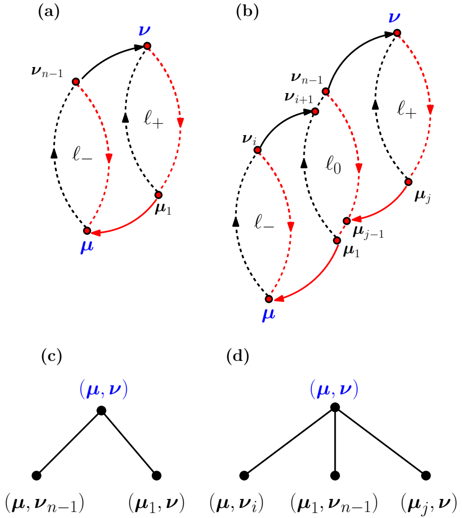
RPM and intra-loop structure: Interior vs. Exterior



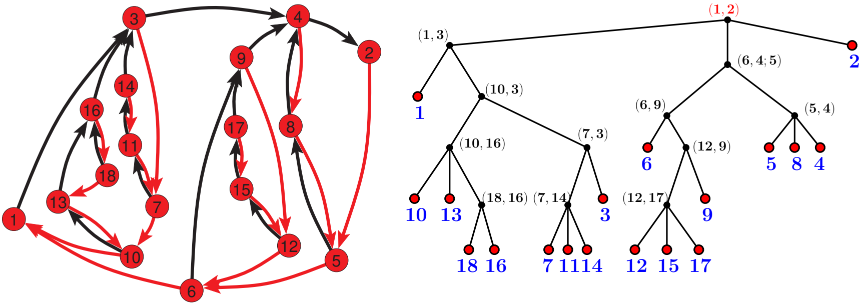
RPM and intra-loop structure: Loop stacking



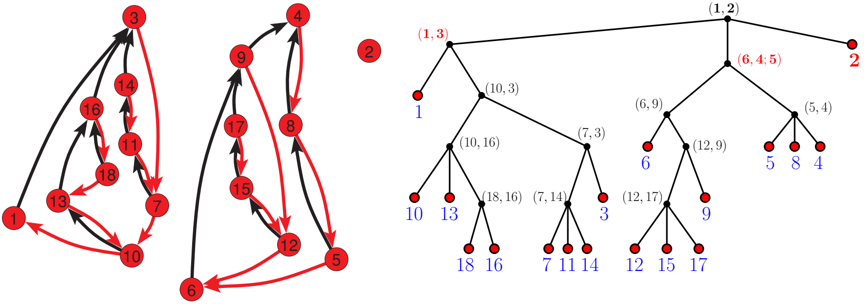
RPM and intra-loop structure: Standard partition



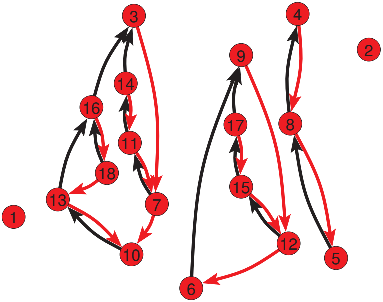
Standard partition of a loop – Root Level

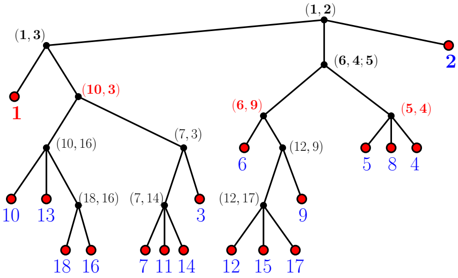


Standard partition of a loop – 1st Generation

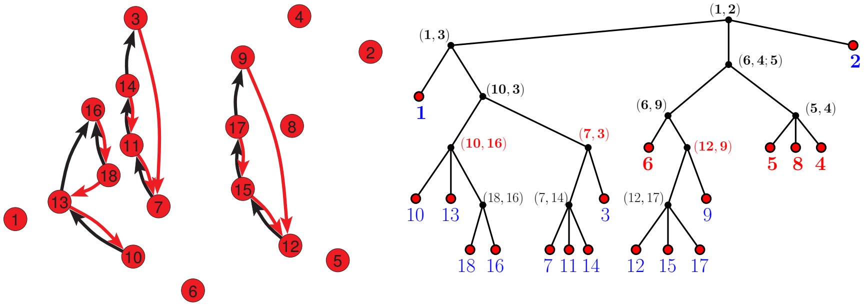


Standard partition of a loop – 2nd Generation

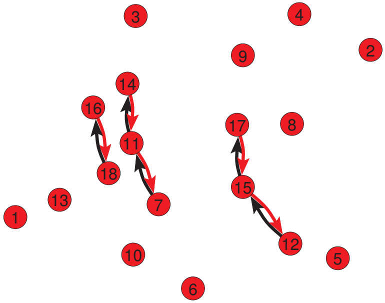


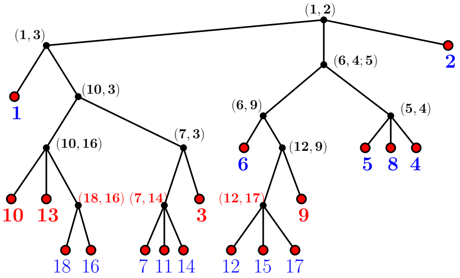


Standard partition of a loop – 3rd Generation

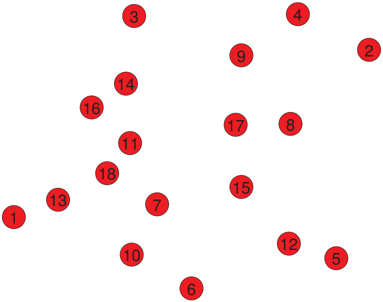


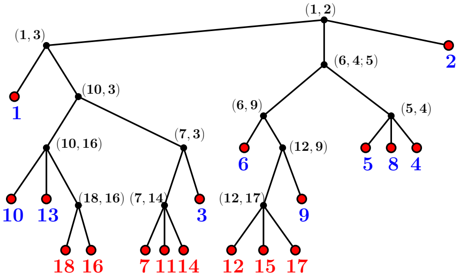
Standard partition of a loop – 4th Generation





Standard partition of a loop – 5th Generation

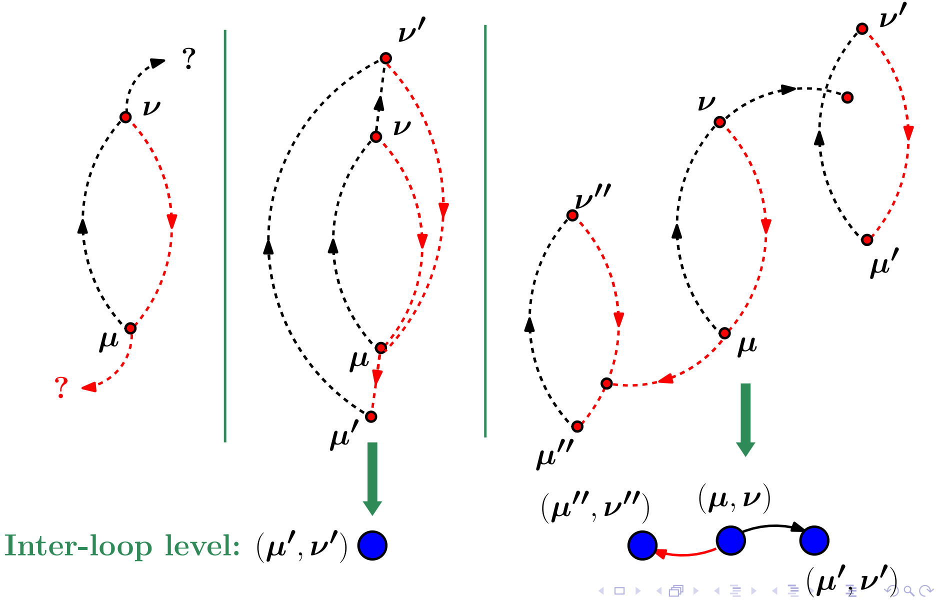




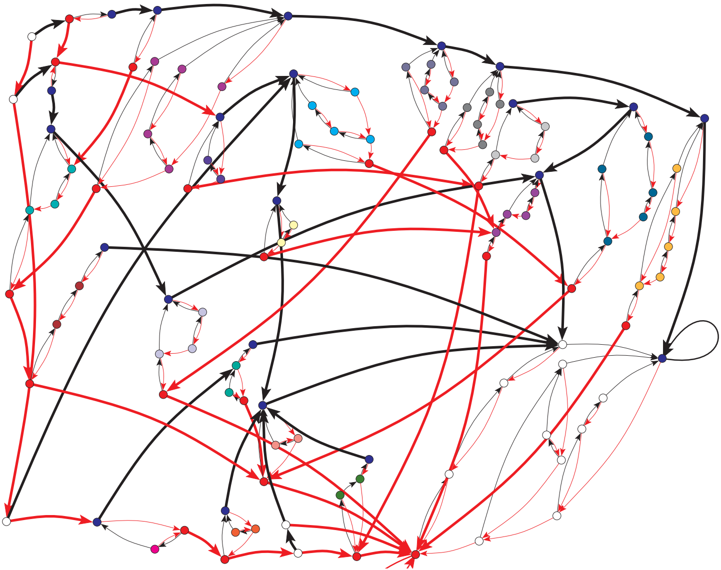
RPM intra-loop structure recap

- Standard partition procedure of decomposing loop into nested sub loops.
- Tree representation of sub loop hierarchy.

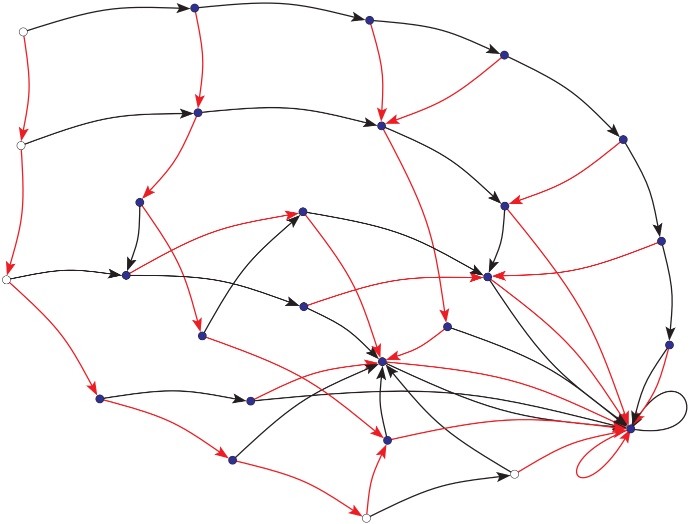
What happens when you leave a loop (μ, ν)?



Sample inter-loop structure: Toy model



Inter-loop graph



DAMA force response: Trapping fields

DAMA ingredients:

- Set of admissible configurations $\mathcal{S}$,
- partial order $\prec$
- absorbing states α, ω,
- transition maps U, D.

- Trapping fields: for each configuration σ, define a pair $F_-(\sigma) < F_+(\sigma)$
- Stability: Configuration σ is stable for forces F

$$F_-(\sigma) < F < F_+(\sigma),$$

- Abs. states: $F_-(\alpha) = -\infty$, $F_+(\omega) = +\infty$.
- Monotonicity: For all configurations σ,

$$F_+(U\sigma) > F_+(\sigma), \quad F_-(D\sigma) < F_-(\sigma).$$

- Evolution: given σ apply force: move along orbit until you reach first stable config.
- Trapping fields F for loop (μ, ν):

$$F_-(\mu) < F < F_+(\nu).$$

Intra-loop forcing and marginality

$F_-(\boldsymbol{\sigma}), F_+(\boldsymbol{\sigma})$

$\boldsymbol{\sigma}$

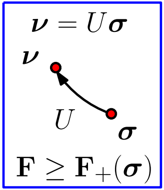
$\boldsymbol{\sigma}$ stable for:

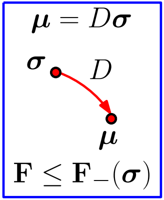
$F_-(\boldsymbol{\sigma}) < F < F_+(\boldsymbol{\sigma})$

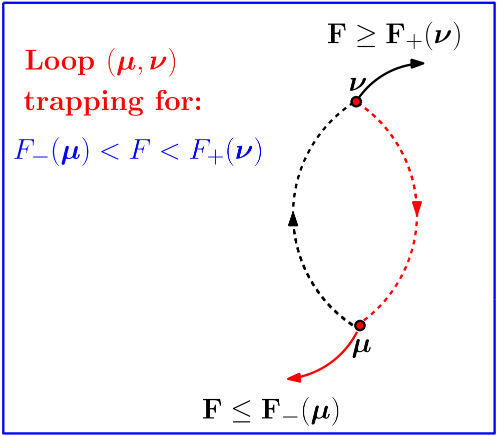
Montonicity:

$F_+(U\boldsymbol{\sigma}) > F_+(\boldsymbol{\sigma})$

$F_-(\boldsymbol{\sigma}) > F_-(D\boldsymbol{\sigma})$







Intra-loop forcing and marginality

$F_-(\boldsymbol{\sigma}), F_+(\boldsymbol{\sigma})$

$\boldsymbol{\sigma}$

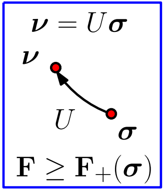
$\boldsymbol{\sigma}$ stable for:

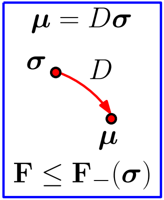
$F_-(\boldsymbol{\sigma}) < F < F_+(\boldsymbol{\sigma})$

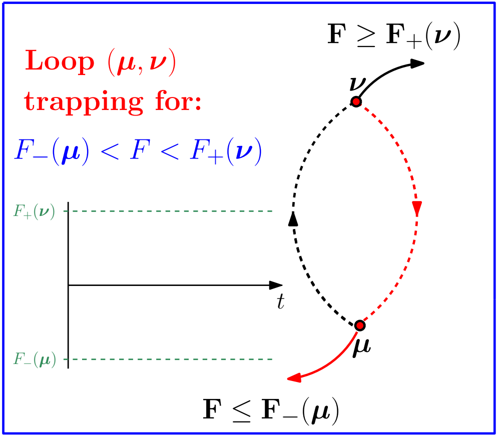
Montonicity:

$F_+(U\boldsymbol{\sigma}) > F_+(\boldsymbol{\sigma})$

$F_-(\boldsymbol{\sigma}) > F_-(D\boldsymbol{\sigma})$







Intra-loop forcing and marginality

$F_-(\boldsymbol{\sigma}), F_+(\boldsymbol{\sigma})$

$\boldsymbol{\sigma}$

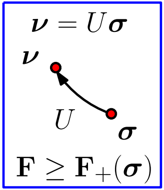
$\boldsymbol{\sigma}$ stable for:

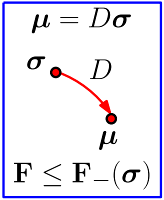
$F_-(\boldsymbol{\sigma}) < F < F_+(\boldsymbol{\sigma})$

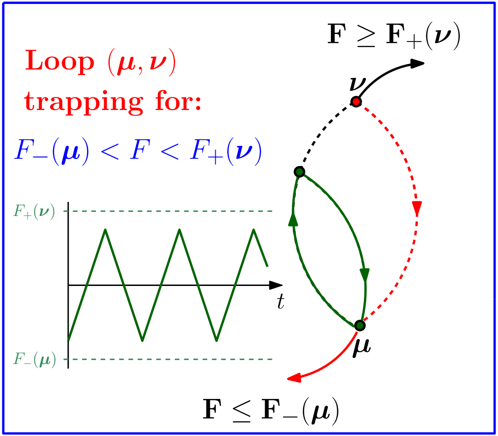
Montonicity:

$F_+(U\boldsymbol{\sigma}) > F_+(\boldsymbol{\sigma})$

$F_-(\boldsymbol{\sigma}) > F_-(D\boldsymbol{\sigma})$



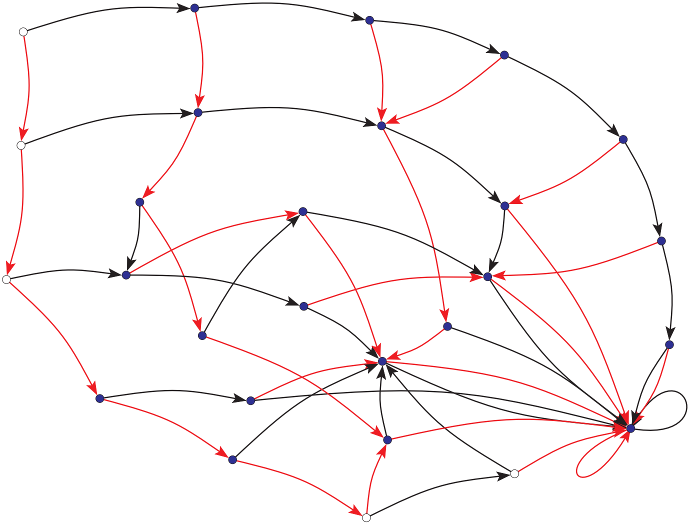




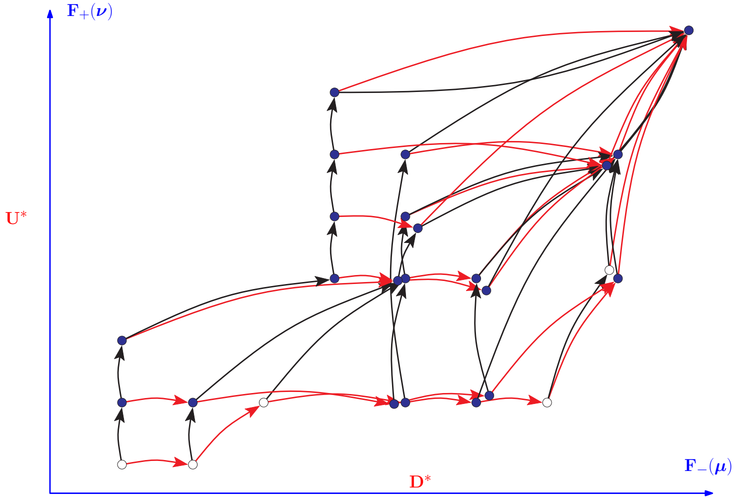
Inter-loop graphs and forcing

What if the amplitude of my forcing is larger than what the loop can trap?

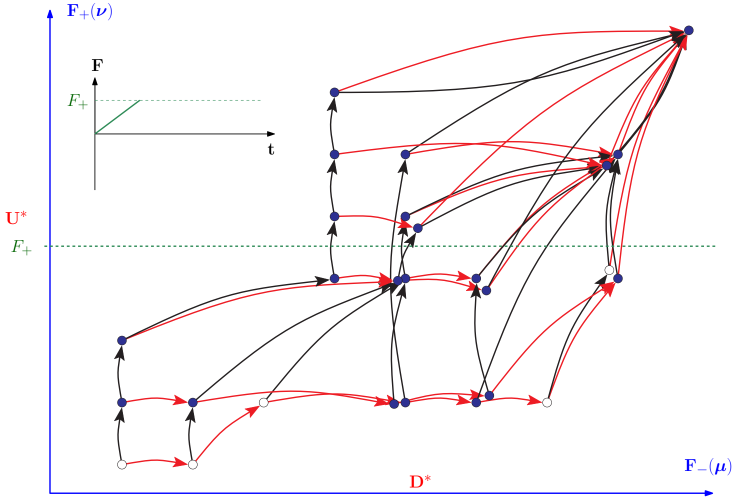
Inter-loop graph



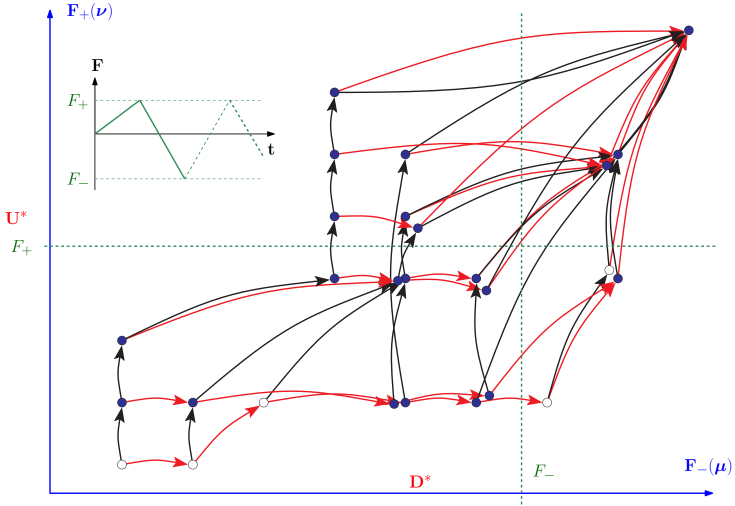
Inter-loop structure: Loop trapping graph



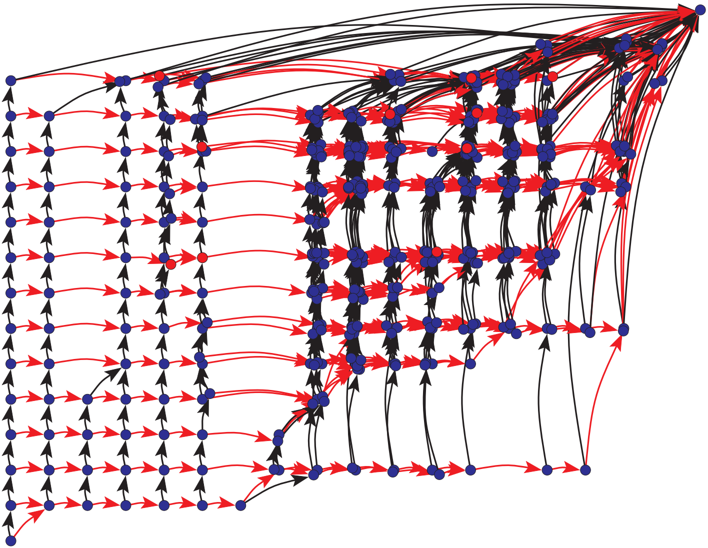
Inter-loop structure: Loop trapping graph



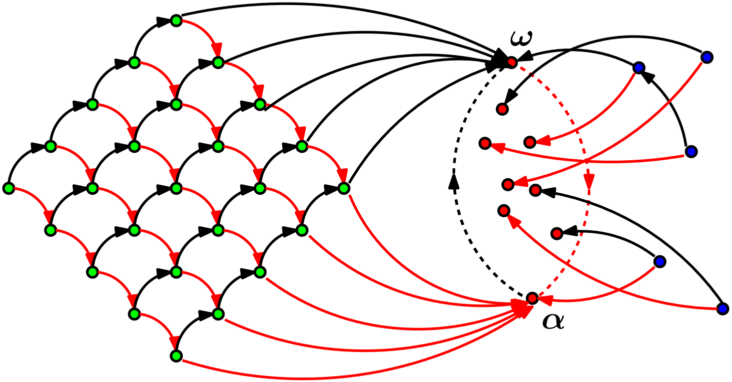
Inter-loop structure: Loop trapping graph



Inter-loop structure: Loop trapping graph for bigger system



Inter-loop structure: with RPM only



Summary & Conclusion

- The depinning via triggering of avalanches is a paradigm for dynamic criticality
- For such out-of-equilibrium models, emergence of criticality and universal properties still not well understood
- Presented a toy model that is analytically tractable
- Key insights gained:
  - Started with a microscopic model.
  - By analysing avalanche dynamics, found that dynamics is governed by evolution of ARs,
  - Evolution of ARs is a coagulation process $\Rightarrow$ evolution of avalanche size $\mathcal{S}$ and correlation length ξ.
  - On the macroscopic scale, few ingredients of the microscopic model survived.
  - An example for how universal features might emerge from different microscopic evolution rules.
  - Coagulation linked to inter-loop structure.

DAMAs Conclusion & Outlook

- DAMA arose out of abstracting systems like RFIM and depinning toy model. Can use this to count states for the toy model: For α, ω-loop: $n_{\mathrm{states}} \sim \ell^{\gamma}$, with $\gamma = (\sqrt{17} - 1)/2$.
- Disorder enters through the maps U and D: they are random objects.
- Weak version of RPM sufficient to prescribe intra-loop structure, tree, one periodic transients inside loops.
- In DAMA transients longer than one perior are due to inter-loop structure!
- DAMA has built in notion of marginality.
- RPM without no-passing: spin ice, spin glasses with mixed interactions.

References

- **The Toy Model:**

  - D.C. Kaspar and M. Mungan "Subthreshold behavior and avalanches in an exactly solvable Charge Density Wave system," *EPL*, **103** (2013) 46002.
  - D.C. Kaspar and M. Mungan "Exact results for a toy model exhibiting dynamic criticality," *Ann. H. Poinc*, **16** (2015), 2837-2879,
  - M. Ișeri, D.C. Kaspar and M. Mungan "Depinning as a Coagulation Process," *EPL*, **115** (2016) 46003,
  - M. Mungan and M.M. Terzi, "The structure of state transition graphs in hysteresis models with return point memory. I. General Theory," submitted to `arXiv`.