Emergent phenomena in large interacting communities

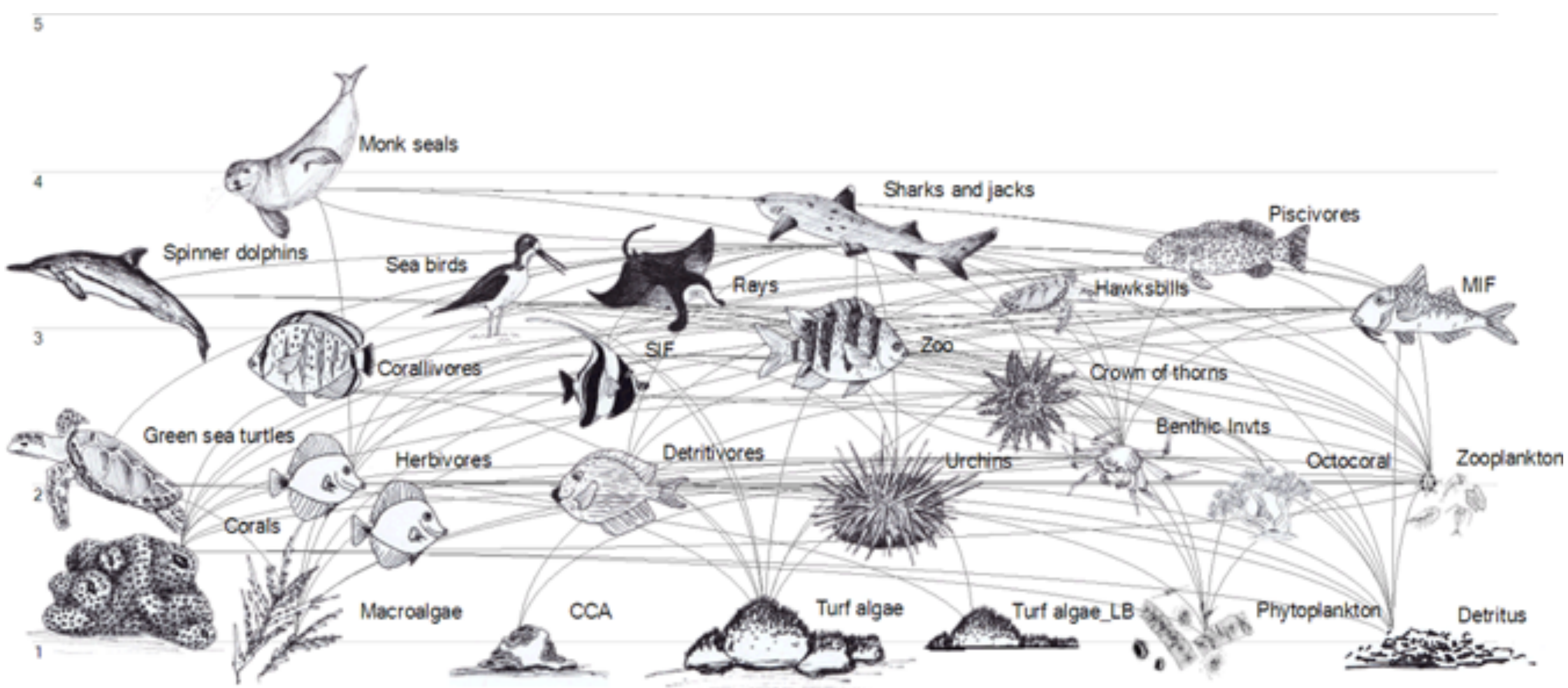
Giulio Biroli
Institute for Theoretical Physics, CEA Saclay, France
Statistical Physics Lab-ENS Paris

Joint works with G. Bunin, C. Cammarota, V. Ros, F. Roy

Ecosystems

- Communities formed by individuals belonging to different species.
- Interactions between individuals intra and inter species.
- Competition for resources--Cooperation.
- Abundances of species vary dynamically due to the births and deaths.

Traditional ecosystem



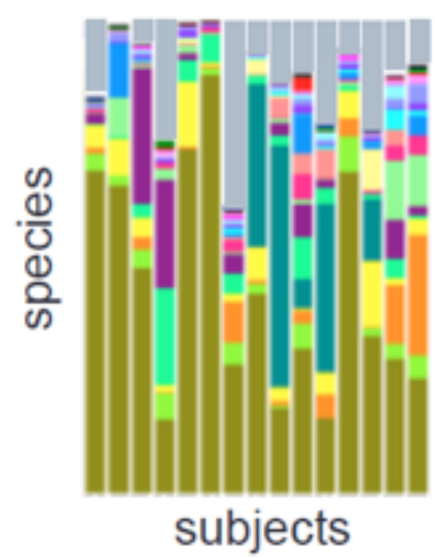
“Modern” ecosystems

“Modern” ecosystems

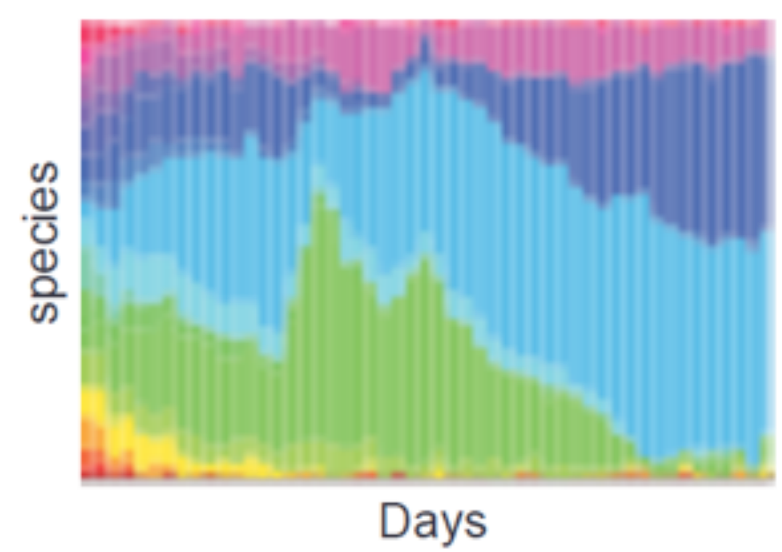
Human microbiome project

hundred-thousand species in a given ecosystem

Abundance profiles



Time dependence



Questions & Motivations

From a few species to many ⟶ STATISTICAL PHYSICS

Emergent properties of complex ecosystems

- How many equilibria for the same ecosystem?
In some ecosystems many, Bashan et al. Nature 2016

- Response to perturbations?
Memory, hysteresis, Dethlefsen, Relman PNAS 2011

- Equilibria or chaotic dynamics?
Chaos in plankton ecosystem, Beninca et al Nature 2008

- What are the factors determining diversity (number of surviving species)?

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j\neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

$$i = 1, \ldots, S$$

$N_i \geq 0$ abundance of species i

S is the number of species

Well-mixed population: no-space dependence

Dynamics due to intra-and inter-species interactions

Properties of the community reached dynamically

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i (K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

$$i = 1, \ldots, S$$

N_i abundance of species i
S is the number of species

$$\frac{dN_i}{dt} = r_i N_i (K_i - N_i)$$

A species alone self-regulates to the abundance K_i

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

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interaction between species

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

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Demographic Noise to model fluctuations in births and deaths

$$\langle \eta_i(t)\eta_j(t')\rangle = 2\omega^2\delta(t-t')$$

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

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Immigration rate

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

$$i = 1, \ldots, S$$

N_i abundance of species i
S is the number of species

Large number of species
(S~50-100 is large)

Lotka-Volterra equations for ecosystems

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

$$i = 1, \ldots, S$$

N_i abundance of species i

S is the number of species

Main assumption: complex -> random

(May in ecology & Wigner in physics)

(Determining interactions network: a key inference problem)

α_{ij} Gaussian RVs i.i.d.

$$\langle \alpha_{ij} \rangle = \frac{\mu}{S} \qquad \langle \alpha_{ij}^2 \rangle_c = \frac{\sigma^2}{S}$$

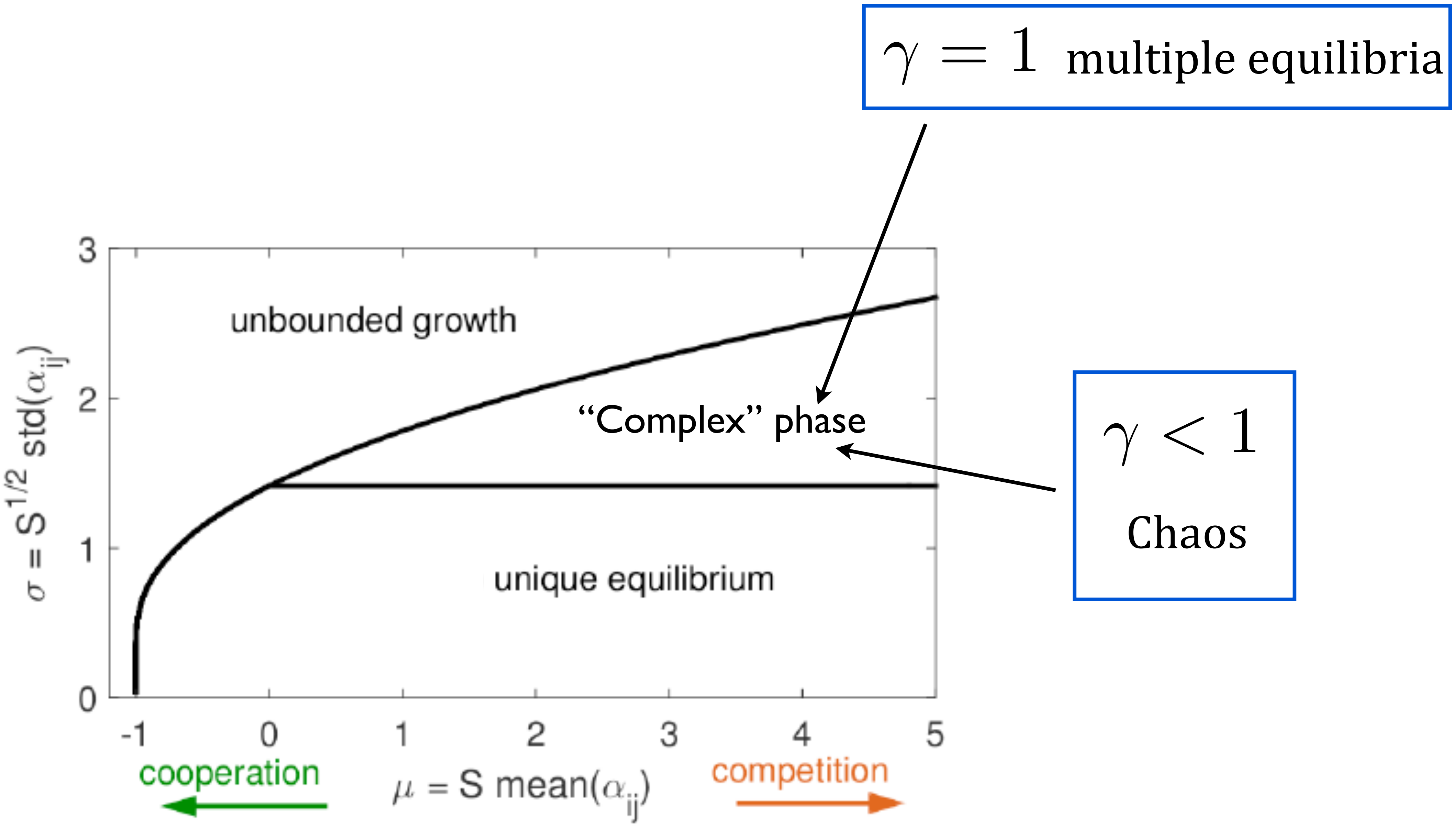
$$\langle \alpha_{ij} \alpha_{ji} \rangle_c = \gamma \langle \alpha_{ij}^2 \rangle_c$$

$\gamma = 1$ symmetric $\qquad -1 \leq \gamma \leq 1$

Small noise and small immigration rate (Ki=1)

Representative model, see
Barbier et al. PNAS to appear

Ecosystems Phase Transitions



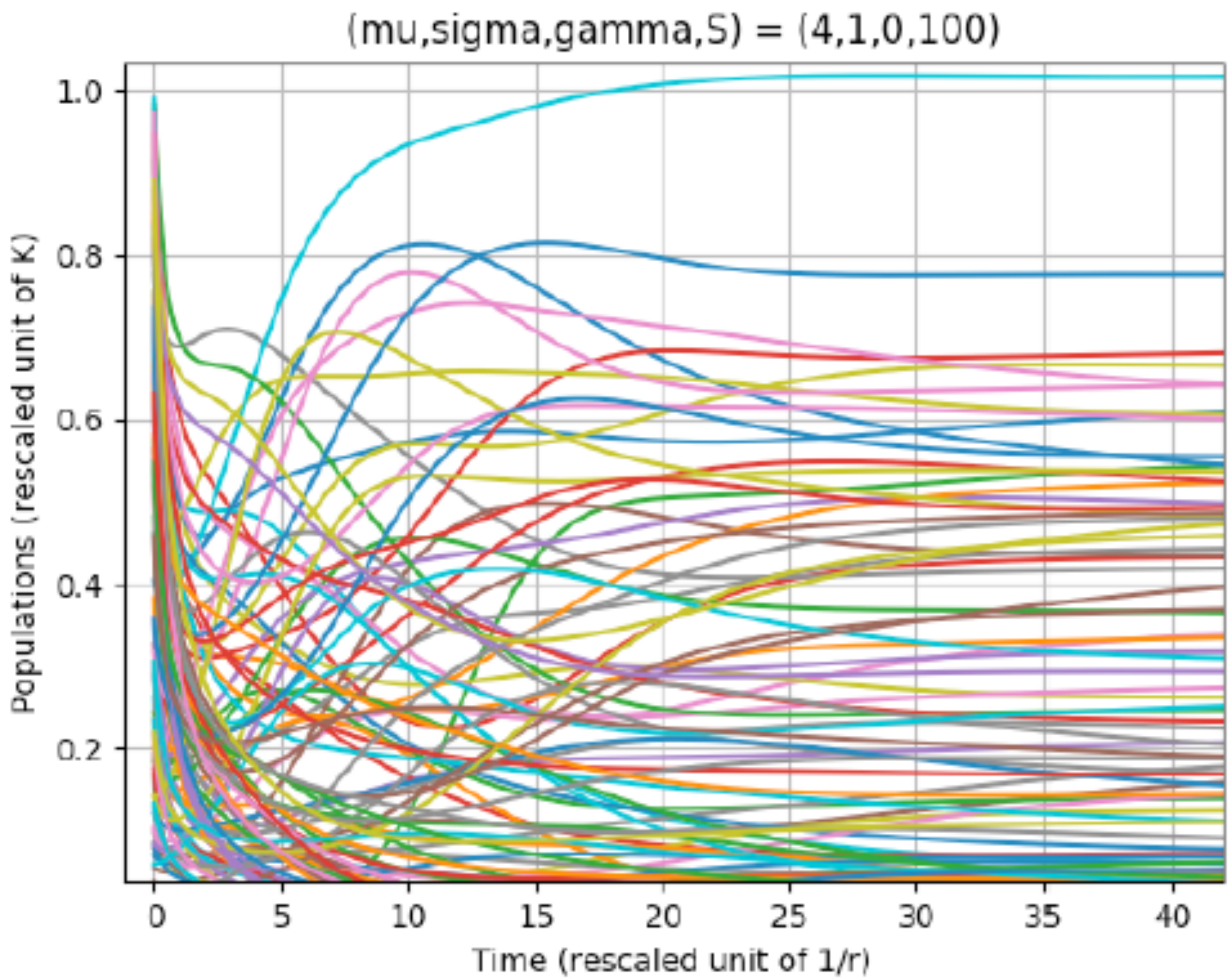
Similar for symmetric and non symmetric interactions (here $\gamma = 0$)

Related works and phase diagrams
Sompolinsky, Crisanti, Sommers '88 ; Diederich, Opper '89; Fisher, Mehta '14; Kessler, Shnerb '15; Bunin '16

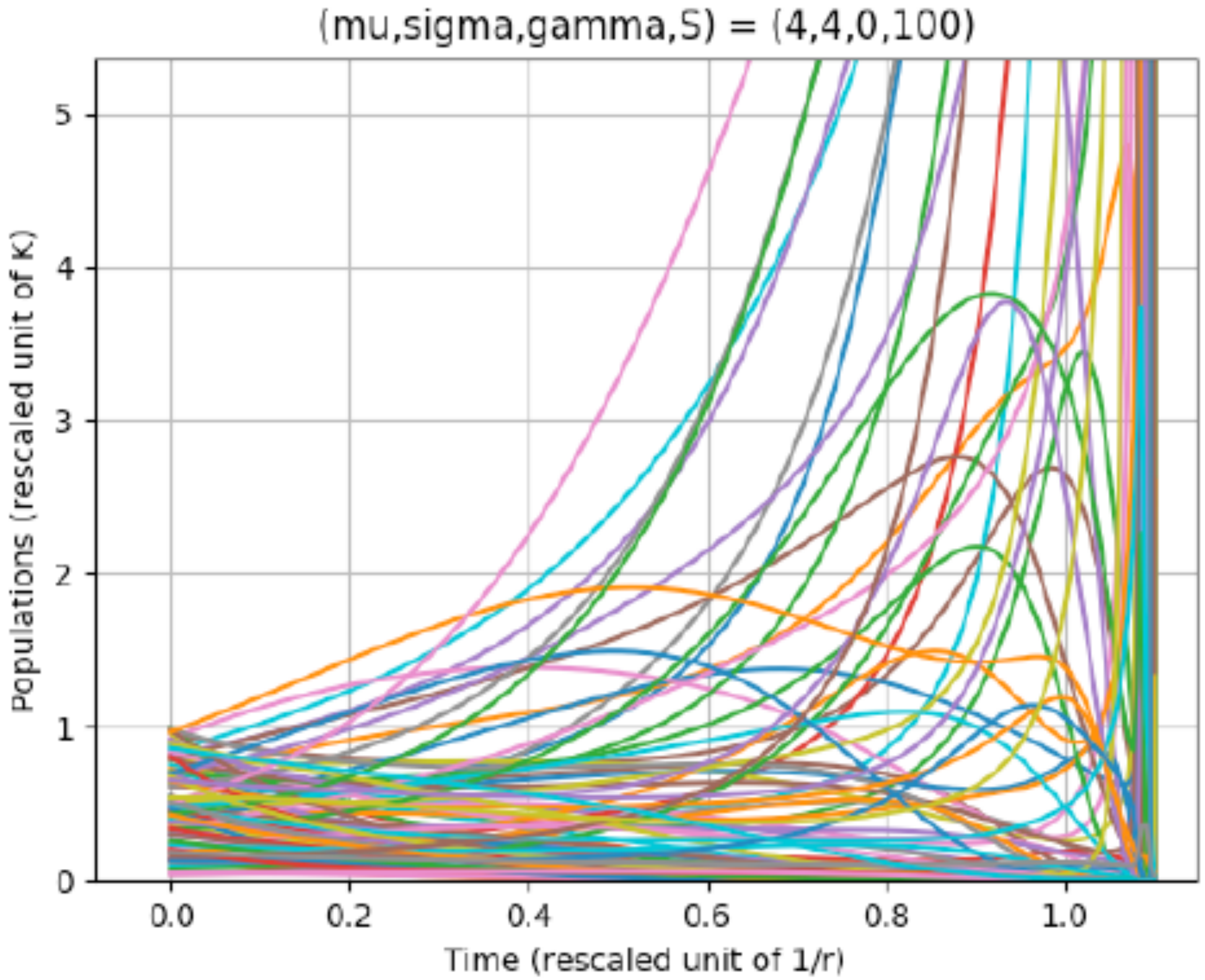
EXACT SOLUTION

G. B., G. Bunin and C. Cammarota arXiv:1710.03606 and works in progress (F. Roy, V. Ros)

Unique Equilibrium Phase



Unbounded Growth Phase



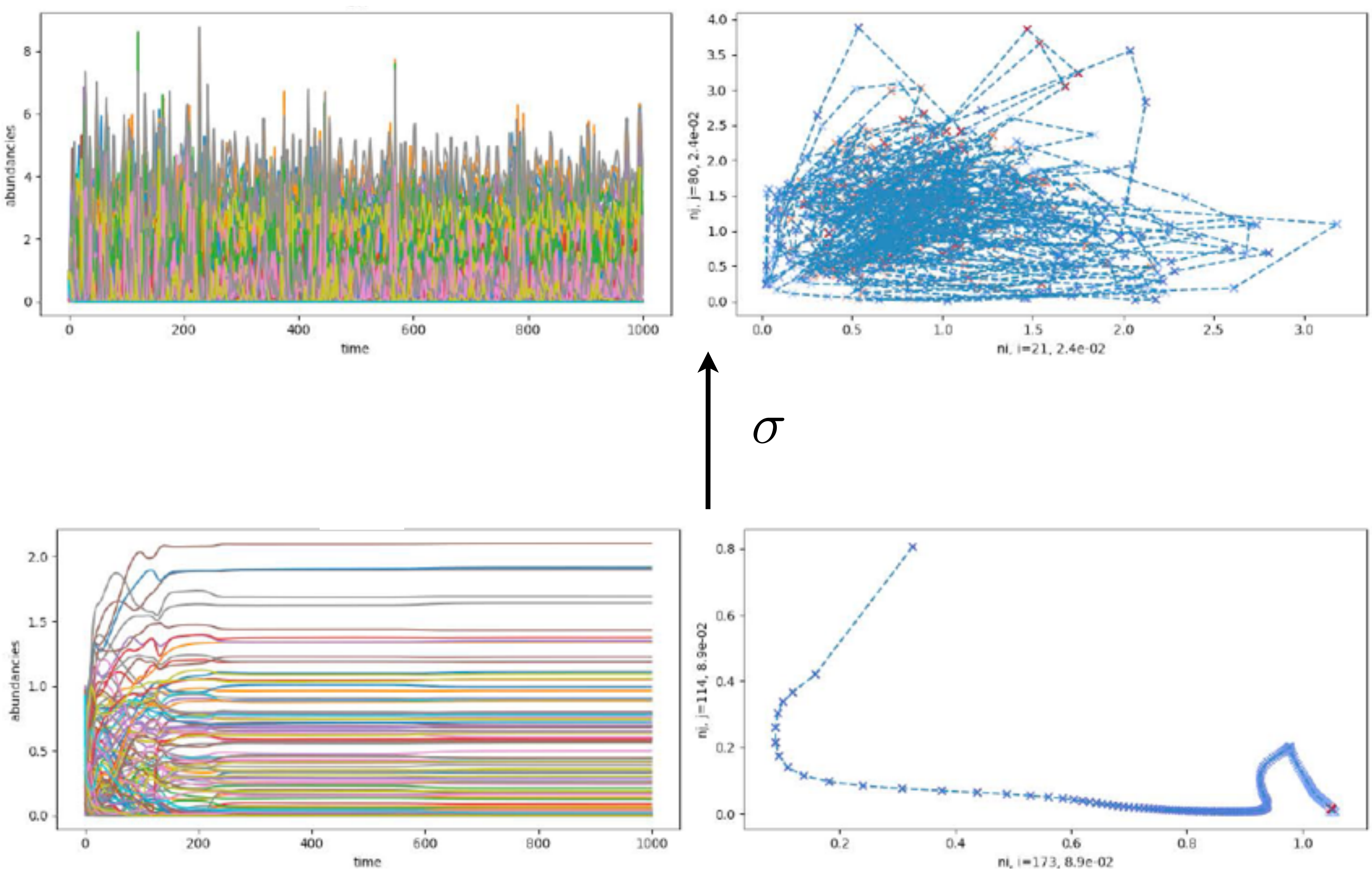
“Complex” Phase

$\gamma = 1$ symmetric interactions: multiple equilibria

$\gamma < 1$ non-symmetric interactions: chaos

Transition to Chaos

$\gamma < 1$



Symmetric interactions

$$\gamma = 1$$

$$\frac{dN_i}{dt} = N_i \left[r_i(K_i - N_i) - \sum_{j,(j \neq i)} \alpha_{ij} N_j \right] + \sqrt{N_i}\eta_i(t) + \lambda_i$$

↓

Langevin equation

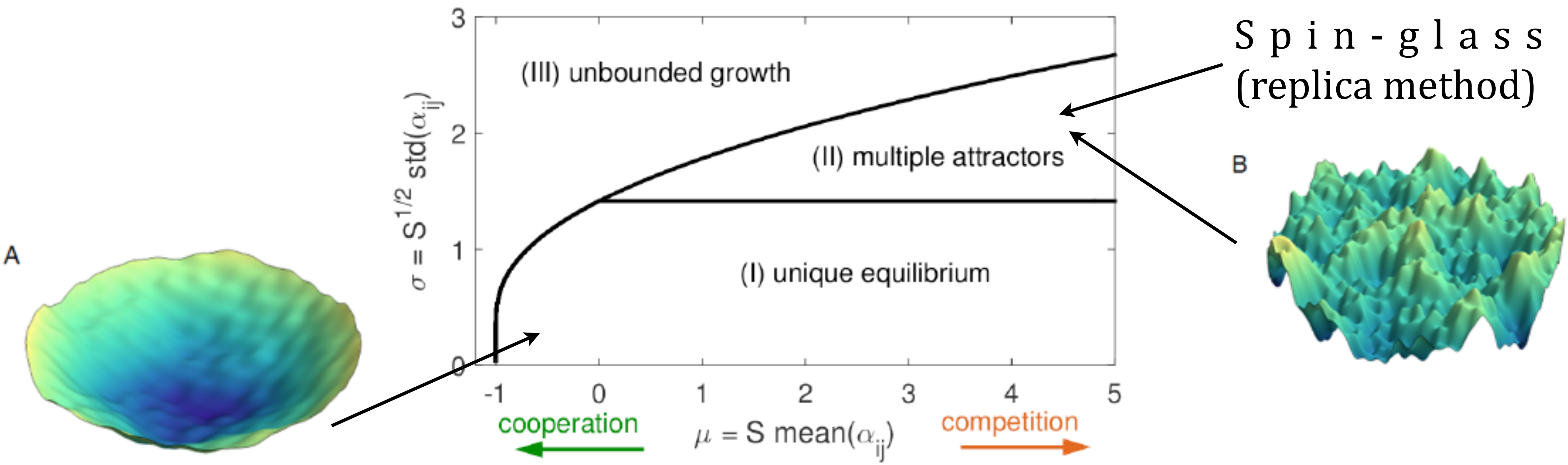
$$\frac{dN_i}{dt} = -N_i \partial_{N_i} E(\{N_i\}) + \sqrt{N_i}\eta_i(t)$$

$$E = \sum_i V_i(N_i) + \frac{1}{2}\sum_{i \neq j} \alpha_{ij} N_i N_j + \sum_i (\omega^2 - \lambda_i) \log N_i$$

Stochastic dynamics of a disordered system
(~spin-glass)

small or zero noise ⟶ Low temperature physics
$T = \omega^2$

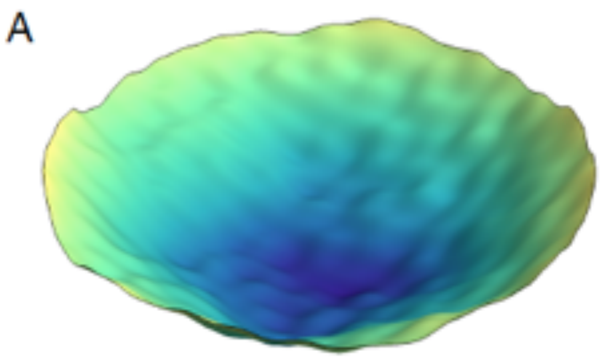
The phase diagram & the energy landscape

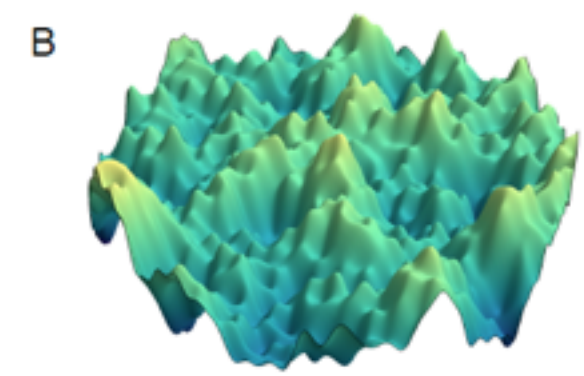


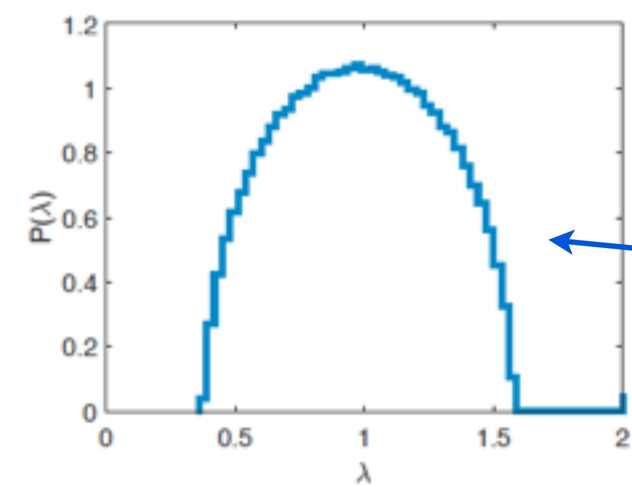
The Two Phases

One equilibrium

Multiple Equilibria
(Critical Spin-Glass Phase)

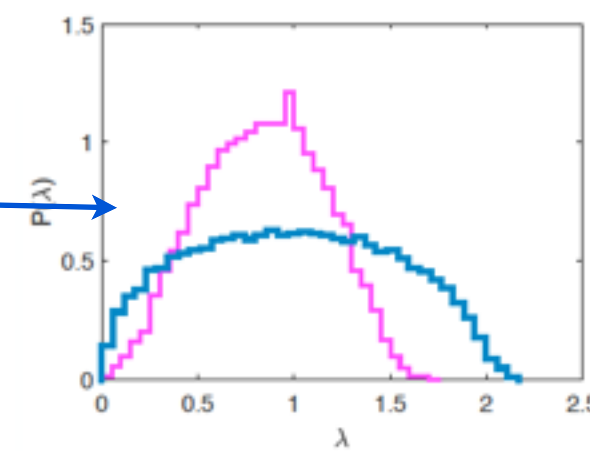






Inverse eigenvalue distribution of the stability matrix $\frac{\partial N_i^*}{\partial K_j}$

Eigenvalue distribution of the Hessian

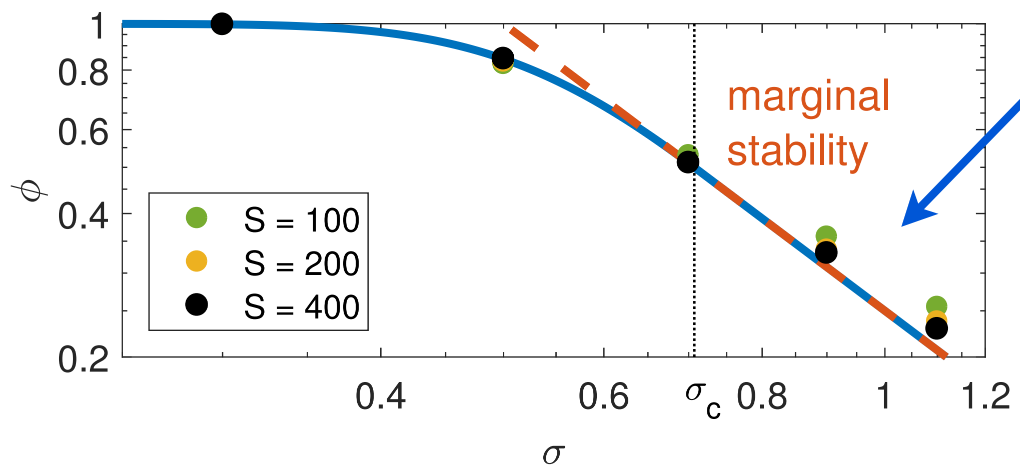


One single stable equilibrium

All equilibria marginally stable

Critical Multiple Equilibria Phase

Marginal stability fixes dynamically the diversity



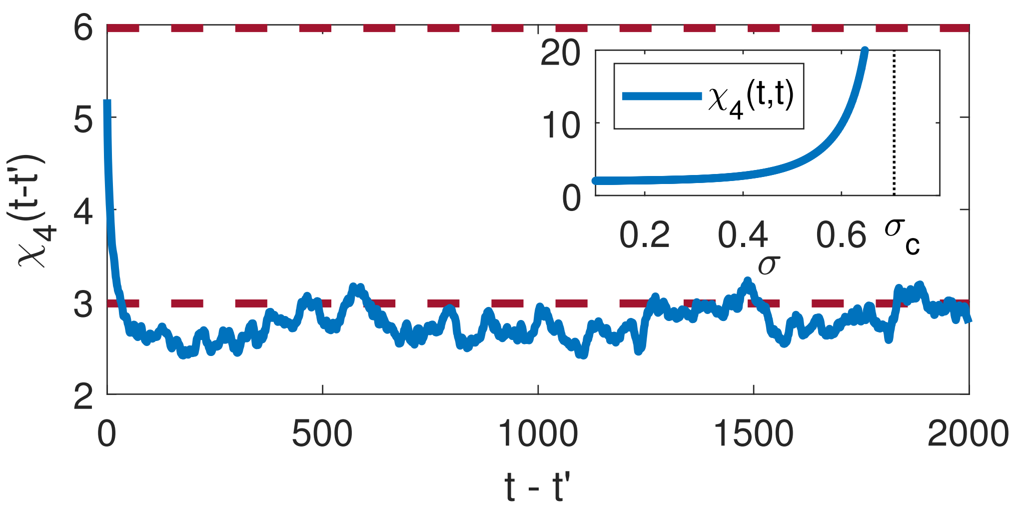
May's bound

Analogous phenomenon in jamming of hard spheres: isostaticity of packings

- Extreme susceptibility to perturbations (memory only in the one equilibrium phase)

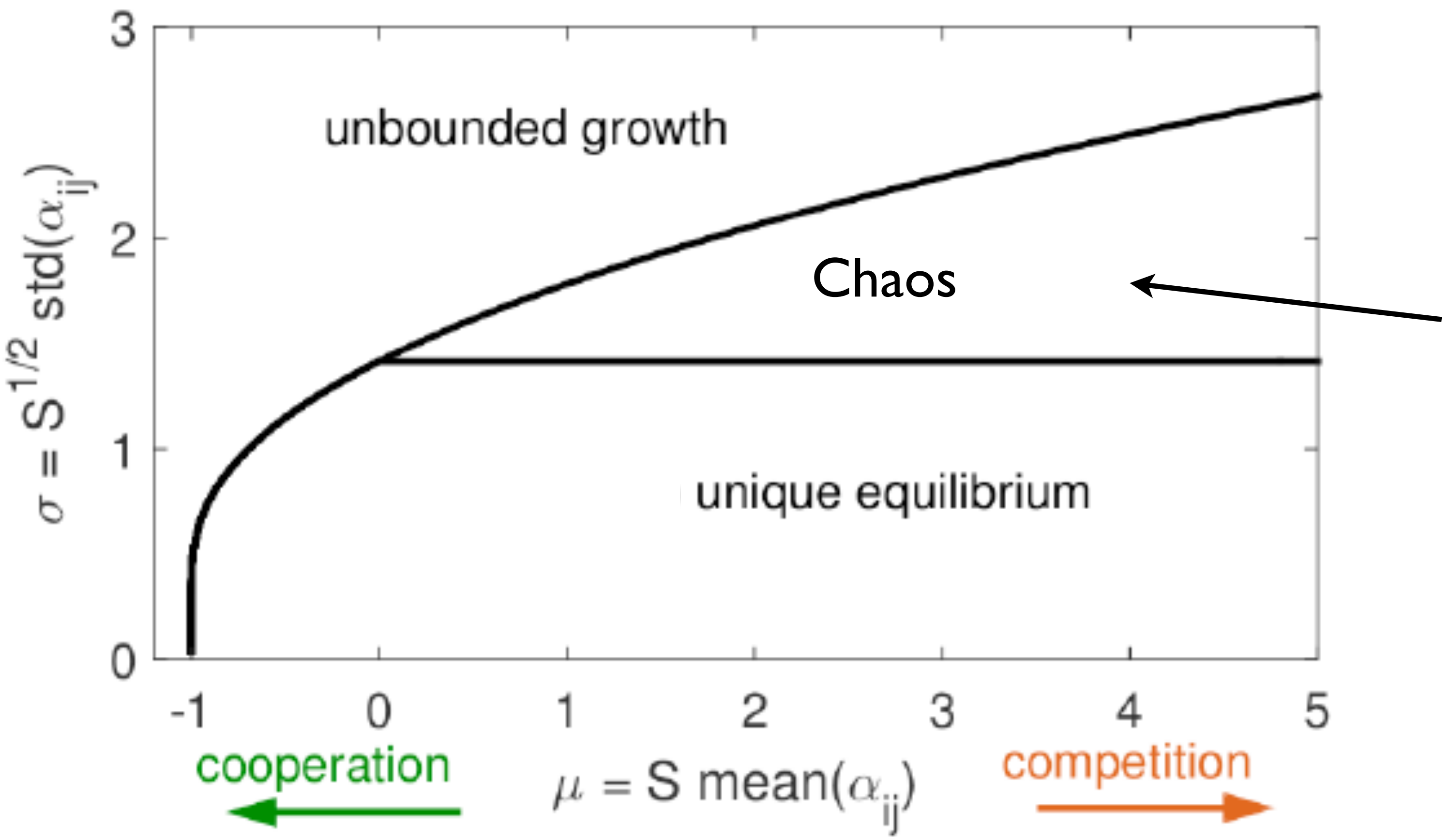
- Large fluctuations-correlations

$$\chi_4(t,t') = \frac{1}{S}\sum_{ij}\left[\langle \delta N_i(t)\delta N_i(t')\delta N_j(t)\delta N_j(t')\rangle - \right.$$
$$\left. -\langle \delta N_i(t)\delta N_i(t')\rangle\langle \delta N_j(t)\delta N_j(t')\rangle\right]$$



Dynamics and Transition to Chaos

$\gamma < 1$



All equilibria are unstable
(Kac-Rice Method)

Chaotic dynamics
(dynamical mean-field theory)

Ongoing: charaterize chaotic dynamics, properties of the transition to chaos

Related works: Sompolinsky, Crisanti, Sommers '88 ; Kessler, Shnerb '15

Emergent phenomena in interacting communities

- Different phases of ecosystems from the exact solution of the Lotka-Volterra model of ecosystems

- An entire region with multiple equilibria poised at the edge of stability:
  - marginal phase, extreme susceptibility to perturbations, large correlations, ...
  - diversity is dynamically fixed by the requirement of being marginal stable: May's bound is saturated

- Chaotic phase where all equilibria are unstable

- Generality beyond the particular model we studied: emergent properties as for phases of matter.

- Many perspectives: Chaotic dynamics, slow dynamics, avalanches, other functional responses, retardation effects, space dependence, ...