

Orientifolds, Mirror Symmetry, and Supersymmetry

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based on

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Work in progress & to appear

4d $\mathcal{N}=1$ Compactification of String theory

- * interesting
- ** relevant for real world physics

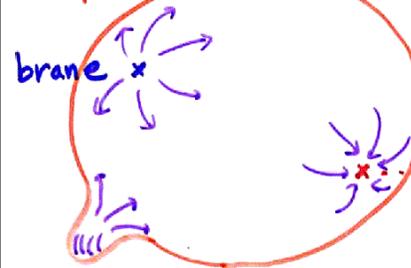
Various ways :- Heterotic string on CY^3 with gauge bundles

- M theory on G_2 holonomy manifolds
- F theory on CY^4

Many of them are dual to:

Type II string theory with space-filling D-branes
(or fluxes)

Compact internal space

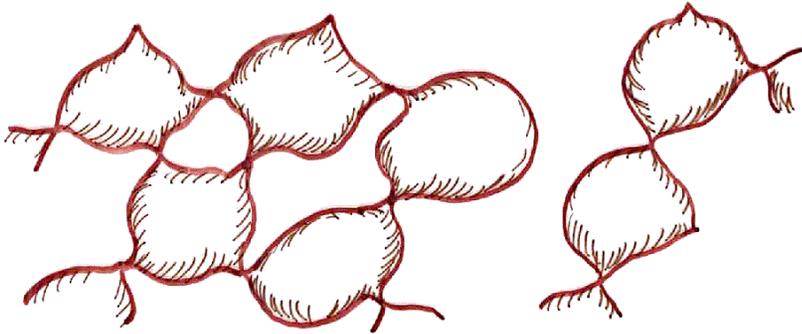


... flux needs to be absorbed



Orientifold is required

4d $\mathcal{N}=2$ compactifications (Type II on CY^3)
Moduli space looks like



What happens to this picture
after Orientifold & addition of
D-branes and fluxes ?

We expect a drastic change

- ① orientifold projection
- ② Open string fields
- ③ Potential

Orientifold is to gauge a parity symmetry

$$P = \tau\Omega : X(t, \sigma) \rightarrow \tau X(\tau, -\sigma)$$



of the worldsheet theory

(2,2) supersymmetry Q_{\pm}, \bar{Q}_{\pm}

- such as
- Non-linear σ -model on Kähler mfd X
 - Landau-Ginzburg model, superpotential W

For 4d $\mathcal{N}=1$ compactification, parity must
preserve a left-right diagonal $\mathcal{N}=2$ (ws) SUSY

Two kinds: (just as D-branes: $Q_{\text{Dyri}} \cdot Q_{\text{Dz}} \cdot Y_{\text{in}}$)

A-parity $Q_{\pm} \xrightarrow{P} \bar{Q}_{\mp}$ (for Type IIA Orientifold)

preserves $Q_A = \bar{Q}_+ + Q_-$, $Q_A^{\dagger} = Q_+ + \bar{Q}_-$

B-parity $Q_{\pm} \xrightarrow{P} Q_{\mp}$ (for Type IIB Orientifold)

preserves $Q_B = \bar{Q}_+ + \bar{Q}_-$, $Q_B^{\dagger} = Q_+ + Q_-$

For NLS-model on a Kähler mfd X $\begin{cases} \text{Kähler form } \omega \\ \text{cplx str } J \end{cases}$
 (with superpotential $W: X \rightarrow \mathbb{C}$ (LG model))

$P = \tau\Omega$ is

an A-parity if $\tau: X \rightarrow X$ **anti-holomorphic isometry**
 $\Rightarrow \tau^*\omega = -\omega$
 ($\tau^*W = \overline{W} + \text{const}$)

a B-parity if $\tau: X \rightarrow X$ **holomorphic isometry**
 $\Rightarrow \tau^*\omega = \omega$
 ($\tau^*W = -W + \text{const}$)

Orientifold plane (O-plane)

$$X^\tau := \{ x \in X \mid \tau(x) = x \}$$

$\tau\Omega$: A-parity $\Rightarrow X^\tau \subset (X, \omega)$ Lagrangian submfd
 ($\text{Im } W = \text{constant on } X^\tau$)

$\tau\Omega$: B-parity $\Rightarrow X^\tau \subset (X, J)$ complex submfd
 ($W = \text{constant on } X^\tau$)

Reduction of Moduli Fields

A-type

Complex structure

$$\tau^{*-1} \overline{\partial}_J \tau^* = \partial_J$$

antiholomorphic condition

(parameter in W
 $\tau^*W = \overline{W}$)
 antihol. condition

Complexified Kähler class

holomorphic condition

$$\tau^*(\omega - iB) = -\omega + iB + \underbrace{\pi i C_1(X)}_{\text{parity anomaly}} \pmod{2\pi i}$$

B-type

Complex structure

$$\tau^{*-1} \overline{\partial}_J \tau^* = \overline{\partial}_J$$

holomorphic condition

(parameter in W
 $\tau^*W = -W$)
 hol. condition

Complexified Kähler class

$$\tau^*(\omega - iB) = \overline{\omega - iB} \pmod{2\pi i}$$

anti-holomorphic condition

Witten indices

P : parity, a, b : branes (preserving same SUSY)

$$I(a, b) = \text{Tr}_{\mathcal{H}_{a, b}} (-1)^F$$



$$I_P = \text{Tr}_{\mathcal{H}_{\text{closed}}} P(-1)^F$$



$$I_P(a, P(a)) = \text{Tr}_{\mathcal{H}_{a, P(a)}} P(-1)^F$$



A-type $\tau: X \rightarrow X$ antiholo, L_a, L_b : Lagrangian subsp of X

$$I(L_a, L_b) = \#(L_a \cap L_b)$$

$$I_{\tau L} = \#(X^\tau \cap X^\tau) \quad \text{intersection #'s}$$

$$I_{\tau L}(L, \tau L) = \#(L \cap X^\tau)$$

B-type $\tau: X \rightarrow X$ holo, E_a, E_b : holo bundles over X

$$I(E_a, E_b) = \int_X \text{ch}(\bar{E}_a \otimes E_b) \text{td}(X) = \sum_{p=0}^n (-1)^p \dim H^{p,p}(X, \bar{E}_a \otimes E_b)$$

$$= \chi(E_a, E_b) \quad \text{Euler characteristic}$$

$$I_{\tau L} = \int_{X^\tau} \frac{\text{ch}(T X^\tau)}{\text{ch}(N X^\tau)} e(N X^\tau) = \sum_{p+q=n} \text{tr}_{H^{p,q}(X)} (\tau^* \otimes *)$$

$$= \text{Sign}(\tau, X) \quad \text{Hirzebruch signature}$$

$$I_{\tau L}(E, \tau^* E) = \int_{X^\tau} \text{ch}(2\bar{E}) \frac{\text{td}(X^\tau)}{\text{ch}(N X^\tau)} = \sum_{p=0}^n (-1)^p \text{tr}_{H^{p,p}(X, \bar{E} \otimes \tau^* \bar{E})} (\tau)$$

$$= L(\tau, E^\vee \otimes \tau^* E^\vee) \quad \text{holomorphic Lefschetz #}$$

RR charges

$|i\rangle$ RR ground states $\leftrightarrow \omega_i \in H^i(X)$

$$\Pi_i^a = \langle B_a | i \rangle$$



$$\Pi_i^P = \langle C_P | i \rangle$$



topologically twisted disc/ $\mathbb{R}P^2$
1-pt function

A-type

independent of Kähler \rightarrow exact computation ^{cat LV}
depends on complex str. $\partial: \Pi_j \sim C_{ij}^k \Pi_k$

$$L \text{ (disk with arrow)} \leftarrow i = \int_L \omega_i$$

$$\tau L \text{ (disk with arrow and X)} \leftarrow i = \int_{X^\tau} \omega_i$$

Period integrals

B-type

indep of cplx str
depends on Kähler $\partial: \Pi_j \sim \tilde{C}_{ij}^k \Pi_k$ ^{corrected}

$$E \text{ (disk with arrow)} \leftarrow i = \int_X e^{B+i\omega} \text{ch}(E) \sqrt{\text{td} X} \omega_i + \dots$$

$$\tau L \text{ (disk with arrow and X)} \leftarrow i = \int_{X^\tau} e^{i\omega} \sqrt{\frac{L(\frac{1}{2}TX^\tau)}{L(\frac{1}{2}NX^\tau)}} \omega_i + \dots$$

Factorization

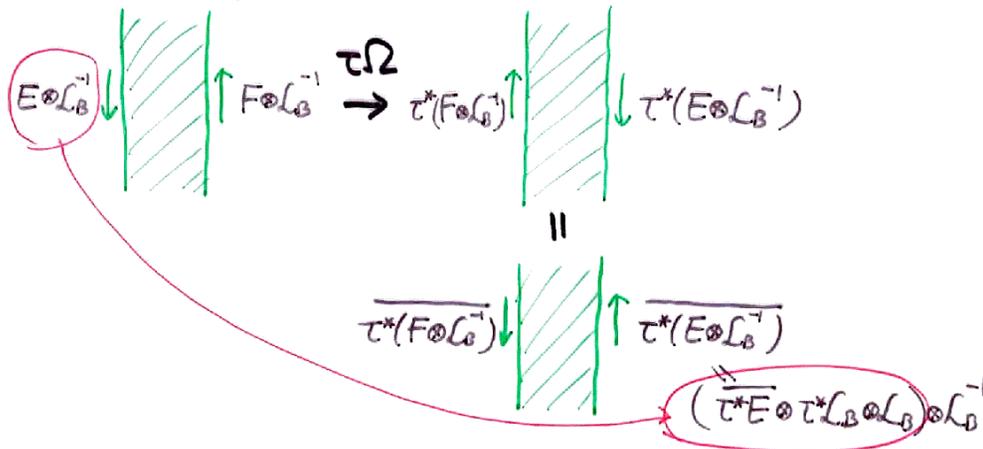


\Leftrightarrow Riemann's bilinear identity

Modification by B-field

A B-field modifies the Chan-Paton factor $E \rightarrow E \otimes \mathcal{L}_B^{-1}$
 "c₁(L_B) = B/2π".

open string ws



$$\tau\Omega: E \rightarrow \overline{\tau^*E} \otimes \mathcal{L}_{\tau^*B+B}$$

one can show $c_1(\mathcal{L}_{\tau^*B+B}) = \frac{\tau^*B+B}{2\pi} \in H^2(X, \mathbb{Z})$

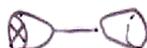
∴ \mathcal{L}_{τ^*B+B} is a good line bundle.

To specify the parity action of open string, one needs to specify the parity action on \mathcal{L}_{τ^*B+B} over X^τ

$$\left. \begin{array}{l} \tau^2 = 1 \\ \text{rank } \mathcal{L}_{\tau^*B+B} = 1 \end{array} \right\} \Rightarrow \tau = \pm 1 (= \varepsilon_B(i)) \text{ at each component } X_i^\tau$$

$$A: I_{\tau\Omega}^B(L, \tau\Omega) = \#(L \cap \sum_i \varepsilon_B(i) X_i^\tau)$$

$$B: I_{\tau\Omega}^B(E, \tau E) = \int_{X^\tau} \text{ch}(2\overline{E}) \varepsilon_B e^{B/\pi} \frac{\text{td}(X^\tau)}{\text{ch}(\sqrt{N}X^\tau)}$$

By factorization  =  , we find

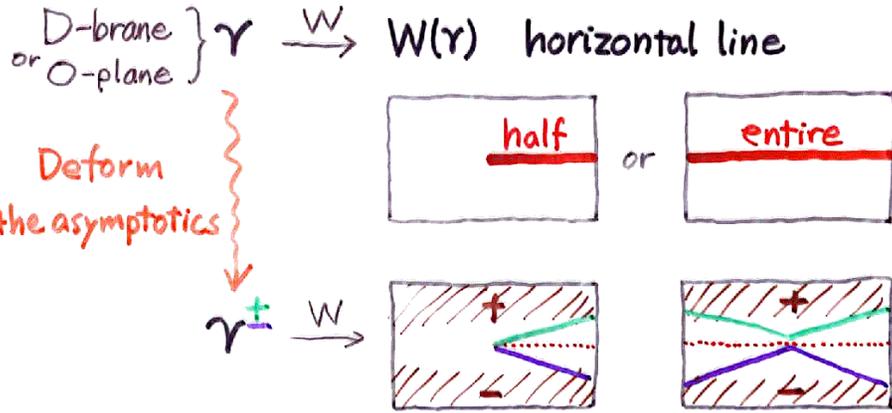
$\mathbb{R}P^2$ diagram

$$\langle \otimes \rangle \leftarrow \phi_i = \int_{X^\tau} \varepsilon_B \omega_i \quad (A)$$

$$= \int_{X^\tau} \varepsilon_B e^{i\omega} \sqrt{\frac{L(\frac{1}{2}TX^\tau)}{L(\frac{1}{2}NX^\tau)}} \omega_i + \dots \quad (B)$$

The sign function $\varepsilon_B: X^\tau \rightarrow \{\pm 1\}$ determines the type of D-planes (SO or Sp)

The case of LG model (A-type) H, Iqbal, Vafa
BH



charge = $\gamma^\pm \in H_n(X^n, B_\pm)$; $B_\pm = \{ \text{Im}W \gtrless 0 \}$

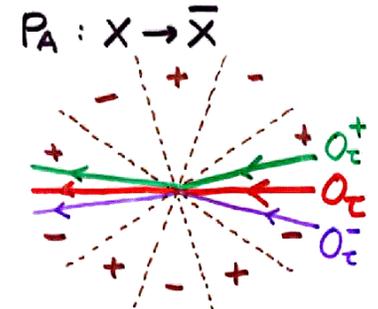
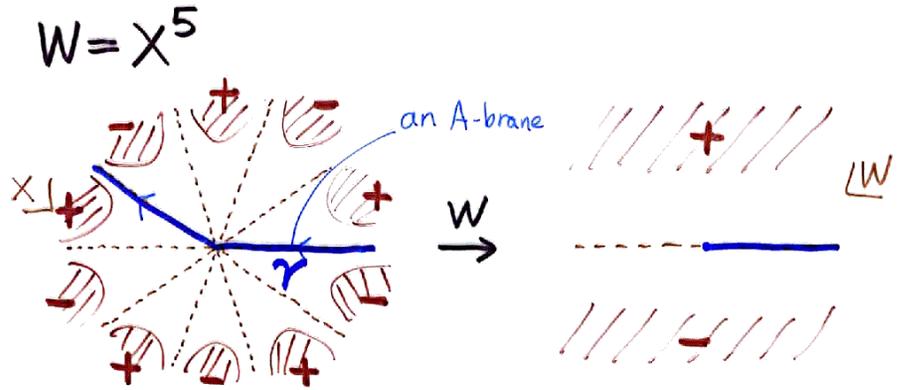
$\gamma \cap i = \int_{\gamma^-} e^{-iW} \phi_i \Omega$ γ : D-brane

$\tau\Omega \cap i = \int_{O_\tau^-} e^{-iW} \phi_i \Omega$ O_τ : O-plane

$\gamma_a \cap \gamma_b = \#(\underline{\gamma_a^-} \cap \underline{\gamma_b^+})$

$\tau\Omega \cap \tau\Omega = \#(\underline{O_\tau^-} \cap \underline{O_\tau^+})$

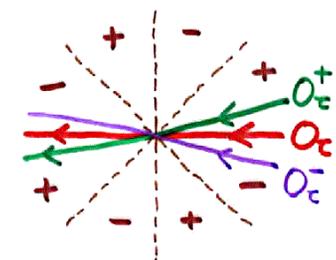
$\gamma \cap \tau\Omega = \#(\underline{\gamma^-} \cap \underline{O_\tau^+})$



$\#(O_\tau^- \cap O_\tau^+) = 0$

$[O_\tau]^+ = [\gamma]^+ \in H_1(\mathbb{C}, B_+)$

$W = X^4, P_A: X \rightarrow \bar{X}$



$\#(O_\tau^- \cap O_\tau^+) = 1$

Linear Sigma Model : $U(1)$ gauge theory

matters $P \quad X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5$
 charges $-1 \quad 1/h_1 \quad 1/h_2 \quad 1/h_3 \quad 1/h_4 \quad 1/h_5 \quad \sum_{i=1}^5 \frac{1}{h_i} = 1$

superpotential $W = P(X_1^{h_1} + X_2^{h_2} + X_3^{h_3} + X_4^{h_4} + X_5^{h_5})$

FI $\gg 0$: NLOM on a Calabi-Yau

$$X_1^{h_1} + \dots + X_5^{h_5} = 0 \quad \text{in a } WCP^4$$

FI $\ll 0$: $\langle P \rangle \neq 0$ Landau Ginzburg Orbifold

- $W = X_1^{h_1} + \dots + X_5^{h_5}$
- $\Gamma =$ generated by $\gamma: X_i \rightarrow e^{\frac{2\pi i}{h_i}} X_i \quad \forall i$
 $\cong \mathbb{Z}_H \quad H = \text{l.c.m.} \{h_1, \dots, h_5\}$

Gepner Model

$$M_{h_1} \times M_{h_2} \times M_{h_3} \times M_{h_4} \times M_{h_5} / \mathbb{Z}_H$$

M_h : IR fixed point of LG $W = X^h$
 $= SU(2)_{k=h-2} / U(1)$ supercoset
 "N=2 minimal model"

A-Parity

$$X_i \rightarrow e^{\frac{2\pi i}{h_i} m_i} X_{\sigma(i)} \quad \bar{P}$$

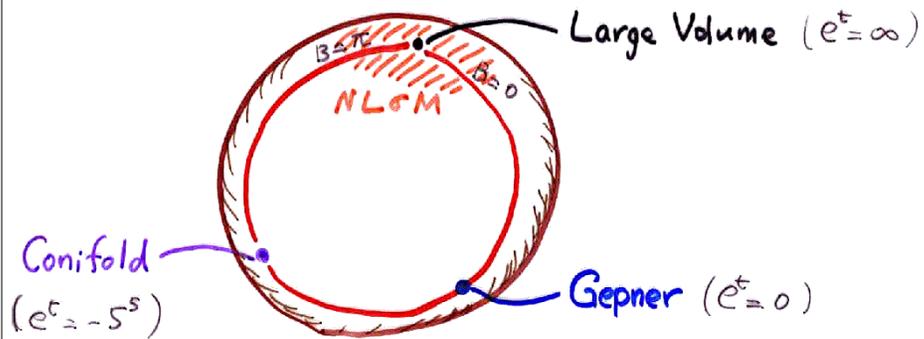
σ : permutation of $\{1, \dots, 5\}$
 $h_{\sigma(i)} = h_i$

B-Parity

$$X_i \rightarrow e^{\frac{2\pi i}{h_i} m_i} X_{\sigma(i)} \quad -P$$

$h_{\sigma(i)} = h_i$
 $m_i + m_{\sigma(i)} = 0 \pmod{h_i}$

Kähler moduli of IIB Orientifolds ④⑤⑥ : $e^t = \text{real}$



* Real but combined with RR holonomy $\begin{cases} A_2 & 09/05 \\ A_4^+ & 07/03 \end{cases}$ to form a complex moduli field

* Spacetime superpotential

* Passes through the conifold point
(other description needed).

* At Gepner point, worldsheet is powerful.

No involutive A/B parities other than ①-⑥

Dressing by $X_i \rightarrow e^{\frac{2\pi i}{5} m_i} X_i$

• makes no difference

or • is impossible

A: $X_i \rightarrow \omega_i \bar{X}_i$

$$\iff X_i' = \pm \omega_i^{\pm \frac{1}{5}} X_i \quad (\pm \omega_i^{\pm \frac{1}{5}})^5 = 1$$

$X_i' \rightarrow \bar{X}_i'$

B: $X_i \rightarrow \omega_i X_i$ not involutive

The situation is different if some h_i even.

1. \exists modification by $X_i \rightarrow e^{\frac{2\pi i}{h_i} m_i} X_i$

2. \exists modification by quantum symmetry at Gepner pt.

To illustrate 1., we consider the example ...

A "Two parameter" Model

$$X_1^8 + X_2^8 + X_3^4 + X_4^4 + X_5^4 = 0 \text{ in } WCP_{11222}^4$$

$$K = 2c$$

$$X_i \rightarrow e^{\frac{2\pi i}{h_i} m_i} \overline{X_i} \quad \text{RRW}$$

$$\text{fixed point : } X_i = e^{\frac{\pi i}{h_i} m_i} x_i \quad x_i \in \mathbb{R}$$

$$(-1)^{m_1} x_1^8 + (-1)^{m_2} x_2^8 + (-1)^{m_3} x_3^4 + (-1)^{m_4} x_4^4 + (-1)^{m_5} x_5^4 = 0$$

(i) $(-1)^{m_i} \equiv 1$: No solution

NO O-plane

(ii) $(-1)^{m_i} = (-1, 1, 1, 1, 1)$

O6 at S^3

(iii) $(-1)^{m_i} = (1, 1, 1, 1, -1)$

O6 at $S^3 \vee S^3$ (meeting at S^1) $\xrightarrow{\text{resolve a single}} S^2 \times S^1$

(iv) $(-1)^{m_i} = (1, 1, 1, -1, -1)$

O6 at T^3

Description at Gepner Point

RCFT technique $\left\{ \begin{array}{l} \text{boundary states Cardy} \\ \text{crosscap states Pradisi-Sagnotti-Stanev} \end{array} \right.$

A-branes

$$|B^A\rangle = \frac{1}{\sqrt{H}} \sum_{\ell=1}^H \gamma^\ell |B^A\rangle^{\text{prod}} \quad \text{product theory (BEFORE GSO)} \\ M_{h_1} \times \dots \times M_{h_5}$$

B-branes

$$|B_\omega^B\rangle = \frac{1}{\sqrt{H}} \sum_{\ell=1}^H \omega^\ell |B^B\rangle_{\gamma^\ell}^{\text{prod}} \quad \omega^H = 1 \\ \text{boundary state in the } \gamma^\ell\text{-twisted circle.}$$

+ transverse space + GSO : Recknagel Schomerus states

Crosscap States (Both A-parity & B-parity)

$$|C_P\rangle = \frac{1}{\sqrt{H}} \sum_{\ell=1}^H |C_{\gamma^\ell P}\rangle^{\text{prod}}$$

Dressing by quantum symmetry $g_\omega = \omega^\ell$ on γ^ℓ -twisted states

$$|C_{g_\omega P}\rangle = \frac{1}{\sqrt{H}} \sum_{\ell=1}^H \omega^{-\ell} |C_{\gamma^\ell P}\rangle^{\text{prod}}$$

A-type : $(g_\omega P)^2 = (\pm 1)^F$ only if $\omega = \pm 1$... possible only if H even (some h_i even)

Full string theory

“Tadpole state” $|T\rangle = \underbrace{|B\rangle}_{\substack{\text{Polchinski-Cai} \\ \text{Callan-Lovelace-Nappi-Yost}}} + \underbrace{|C\rangle}_{\substack{\text{Polchinski-Cai} \\ \text{Callan-Lovelace-Nappi-Yost}}}$

$$|B\rangle_{\text{NSNS}} + |B\rangle_{\text{RR}} \quad |C\rangle_{\text{NSNS}} + |C\rangle_{\text{RR}}$$

$$|B\rangle_{\text{NSNS}} = |B_+\rangle_{\text{NSNS}} + |B_-\rangle_{\text{NSNS}}$$

$$|B\rangle_{\text{RR}} = |B_+\rangle_{\text{RR}} + |B_-\rangle_{\text{RR}}$$

$$|C\rangle_{\text{NSNS}} = |C_{(-1)F_2P}\rangle + |C_{(-1)F_2P}\rangle$$

$$|C\rangle_{\text{RR}} = |C_P\rangle + |C_{(-1)F_2P}\rangle \quad P^2 = 1$$

each term on RHS's $\left| \begin{array}{l} 4d \text{ spacetime} \\ + \text{ ghost} \\ + \text{ superghost} \end{array} \right\rangle \otimes \left| \text{internal} \right\rangle$

Standard Coherent state (universal) What I'm talking about.

Consistency Condition:

$$\langle i | C^{int} \rangle_{\text{RR}} + \frac{1}{4} \langle i | B^{int} \rangle_{\text{RR}} = 0 \quad \forall \text{ (int) RR ground state } |i\rangle_{\text{RR}}$$

$N=1$ supersymmetry:

$$\frac{\langle 0 | C^{int} \rangle_{\text{RR}}}{\langle 0 | C^{int} \rangle_{\text{NSNS}}} = \frac{\langle 0 | B^{int} \rangle_{\text{RR}}}{\langle 0 | B^{int} \rangle_{\text{NSNS}}} \quad |0\rangle_{\text{NSNS}} \xleftrightarrow{\text{spec. flow}} |0\rangle_{\text{RR}}$$

A-type in more detail

All h_i odd $H = \text{l.c.m.}\{h_i\}$ odd, $\{\gamma^{\ell}\}_{\ell=1}^H = \{\gamma^{2\ell}\}_{\ell=1}^H$

$$|C_{\gamma^{2\ell}P}\rangle^{\text{prod}} = \gamma^{\ell} |C_P\rangle^{\text{prod}}$$

$$\therefore |C_P\rangle = \frac{1}{\sqrt{H}} \sum_{\ell=1}^H \gamma^{\ell} |C_P\rangle^{\text{prod}}$$

Some h_i even Haven, $\{\gamma^{\ell}\}_{\ell=1}^H = \{\gamma^{2\ell}\}_{\ell=1}^{H/2} \cup \{\gamma^{2\ell+1}\}_{\ell=1}^{H/2}$

$$|C_{P\pm}\rangle = \frac{1}{\sqrt{H}} \sum_{\ell=1}^{H/2} \left\{ \gamma^{\ell} |C_P\rangle^{\text{prod}} \pm \gamma^{\ell} |C_{\gamma P}\rangle^{\text{prod}} \right\}$$

$+\leftrightarrow-$: dressing by \mathbb{Z}_2 quantum symmetry

- One of \pm continues to Large Volume “Geometric”
- The other is locked at the Gepner point
— Kähler moduli removed “non-geometric”

“Geometric”/“non-geometric” ... distinguished by the **CHARGE**.

e.g. Suppose $P = \tau\Omega$ is “geometric”.

If $[O\text{-plane}]^{\dagger} = [\gamma]^{\dagger} \in H_n(X^n, B_+)$ γ an A-brane

$$\tau: \gamma \rightarrow -e^{i\theta_{\text{LG}}} - i\theta_{\text{CY}} \gamma$$

where $\tau^* \Omega_{\text{LG}} = e^{i\theta_{\text{LG}}} \overline{\Omega_{\text{CY}}}$

Examples

Quintic

$$W = X_1^5 + X_2^5 + X_3^5 + X_4^5 + X_5^5 / \mathbb{Z}_5$$

$$P_A: X_i \rightarrow \bar{X}_i \quad \forall i$$

$$O_{PA} = \frac{1}{\sqrt{5}} (O_1^5 + O_2^5 + O_3^5 + O_4^5 + O_5^5)$$

$$[O_i]^+ = [B_i]^+$$

$$[O_{PA}]^+ = [B]^+$$

$$B = \frac{1}{\sqrt{5}} (B_1^5 + B_2^5 + B_3^5 + B_4^5 + B_5^5) = B_{j_0 n_0 s_0}$$

$$j_0 = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$$

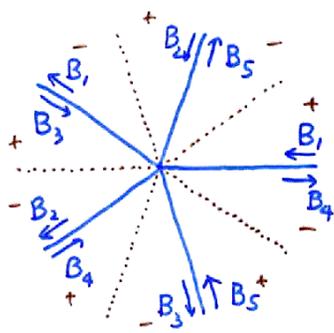
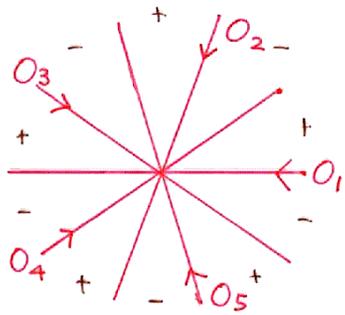
$$n_0 = (2, 2, 2, 2, 2)$$

$$s_0 = (1, 1, 1, 1, 1)$$

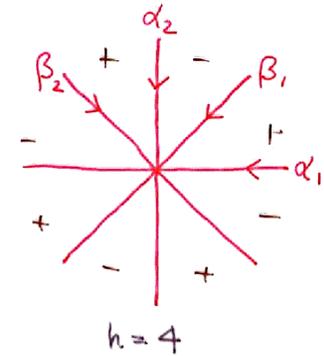
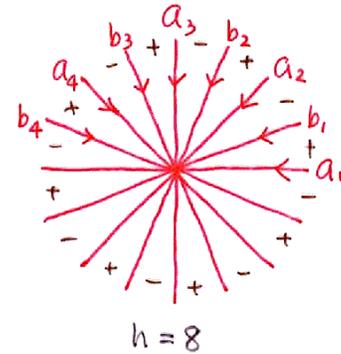
\therefore D-brane on real quintic (RP^5) = $B_{j_0 n_0 s_0}$

$$\begin{aligned} \text{Tension } T_{O_{PA}} &= \langle 0 | C_{(1)}^{F_2 \cdot P_A} \rangle = \frac{5}{\sqrt{5}} \sqrt{\frac{2}{5 \sin(\frac{\pi}{5})}}^5 \\ &= \langle 0 | B_{j_0 n_0 s_0} \rangle_{NSNS} \end{aligned}$$

SO-type O_{PA} & four $B_{j_0 n_0 s_0}$: Consistent & supersymmetric



2 parameter Model $W = X_1^8 + X_2^8 + X_3^4 + X_4^4 + X_5^4 / \mathbb{Z}_8$



$$(i) P_A: X_i \rightarrow \bar{X}_i \quad \forall i$$

$$\begin{aligned} O_{PA\pm} &= \frac{1}{\sqrt{8}} (a_1^2 \alpha_1^3 + a_2^2 \alpha_2^3 + a_3^2 (-\alpha_1)^3 + a_4^2 (-\alpha_2)^3) \\ &\pm \frac{1}{\sqrt{8}} (b_1^2 \beta_1^3 + b_2^2 \beta_2^3 + b_3^2 (-\beta_1)^3 + b_4^2 (-\beta_2)^3) \end{aligned}$$

Homological relations :

$$\begin{aligned} [a_i]^+ &= [b_i]^+ \quad i=1, 2, 3, 4 \\ [\alpha_m]^+ &= [\beta_m]^+ \quad m=1, 2 \end{aligned}$$

$$[O_{PA+}]^+ = \frac{2}{\sqrt{8}} (a_i^2 \alpha_i^3 + \dots)^+ \neq 0$$

$$[O_{PA-}]^+ = 0$$

Large volume : no O-plane

O_{PA-} is the "Geometric" one.

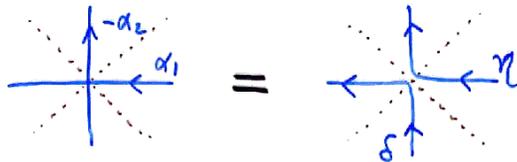
(iii) $\gamma_{(5)PA} : (X_1, \dots, X_5) \rightarrow (\bar{X}_1, \dots, \bar{X}_4, e^{\frac{2\pi i}{5}} \bar{X}_5)$

$$O_{\gamma_{(5)PA}^{\pm}} = \frac{1}{\sqrt{8}} (a_1^2 \alpha_1^2 \beta_1 + a_2^2 \alpha_2^2 \beta_2 + a_3^2 (-\alpha_1)^2 (-\beta_1) + a_4^2 (-\alpha_2)^2 (-\beta_2))$$

$$\equiv \frac{1}{\sqrt{8}} (b_1^2 \beta_1^2 \alpha_2 + b_2^2 \beta_2^2 (-\alpha_1) + b_3^2 (-\beta_1)^2 (-\alpha_2) + b_4^2 (-\beta_2)^2 \alpha_1)$$

"Geometric"

$$[O_{\gamma_{(5)PA}^{\pm}}]^{\dagger} = \frac{1}{\sqrt{8}} [a_1^2 \alpha_1^2 (d_1 - \alpha_2) + a_2^2 \alpha_2^2 (d_2 + \alpha_1) + a_3^2 \alpha_1^2 (-\alpha_1 + \alpha_2) + a_4^2 \alpha_2^2 (-\alpha_2 - \alpha_1)]^{\dagger}$$



$$= \left[\frac{1}{\sqrt{8}} \sum_{l=1}^4 \gamma^l (a_l^2 \alpha_l^2 \eta) + \frac{1}{\sqrt{8}} \sum_{l=1}^4 \gamma^l (a_l^2 \alpha_l^2 \delta) \right]^{\dagger}$$

$$= [B_{j_2 n_2 s_1}]^{\dagger} + [B_{j_3 n_3 s_1}]^{\dagger}$$

$$j_2 = (\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, 0), \quad j_3 = (\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, 1), \quad n_2 = (4, 4, 2, 2, 1)$$

$$\gamma_{(5)PA} \text{ - fixed point set} = S^3 \vee S^3 \leftrightarrow B_{j_1 n_1 s_1} \times B_{j_2 n_2 s_1}$$

$$\text{Tension } T_{O_{\gamma_{(5)PA}^-}} = \langle 0 | C_{(-1)^{F_L} \gamma_{(5)PA}^-} | 0 \rangle = \frac{4}{\sqrt{8}} \sqrt{\frac{2}{8 \sin \frac{\pi}{8}}} \sqrt{\frac{2}{4 \sin \frac{\pi}{4}}} 2 \sin \left(\frac{\pi}{4} \right)$$

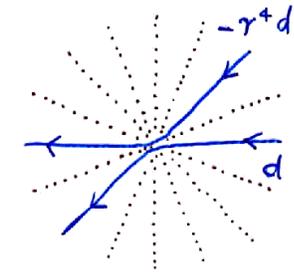
$$= \langle 0 | B_{j_1 n_1 s_1} \rangle_{NSNS} + \langle 0 | B_{j_2 n_2 s_1} \rangle_{NSNS}$$

SO type $O_{\gamma_{(5)PA}^-}$ ← four $B_{j_2 n_2 s_1}$, + four $B_{j_3 n_3 s_1}$: consistent & supersymmetric

"Non-geometric"

$$[O_{\gamma_{(5)PA}^+}]^{\dagger} = \frac{1}{\sqrt{8}} \left[(a_1 + a_2) a_1 \alpha_1^3 + (a_2 + a_3) a_2 \alpha_2^3 - (a_3 + a_4) a_3 \alpha_1^3 - (a_4 - a_1) a_4 \alpha_2^3 \right]^{\dagger}$$

$$a_1 + a_2 =$$



$$= \frac{1}{\sqrt{8}} \left[\sum_{l=1}^8 \gamma^l (d a_l \alpha_l^3) \right]^{\dagger} = [B_{j'_1 n'_1 s_1}]^{\dagger}$$

$$j'_1 = (2, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$$

$$n'_1 = (5, 4, 2, 2, 2)$$

$$\text{Tension } T_{O_{\gamma_{(5)PA}^+}} = \langle 0 | C_{(-1)^{F_L} \gamma_{(5)PA}^+} | 0 \rangle = \frac{4}{\sqrt{8}} \sqrt{\frac{2}{8 \sin \frac{\pi}{8}}} \sqrt{\frac{2}{4 \sin \frac{\pi}{4}}} 2 \cos \left(\frac{\pi}{8} \right)$$

$$= \langle 0 | B_{j'_1 n'_1 s_1} \rangle_{NSNS}$$

SO-type $O_{\gamma_{(5)PA}^+}$ & four $B_{j'_1 n'_1 s_1}$: consistent & supersymmetric

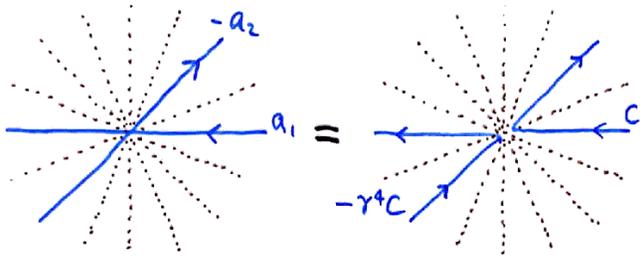
Moduli field of total size removed.

$$(ii) \gamma_{(1)PA} : (X_1, X_2, X_3, X_4, X_5) \rightarrow (e^{\frac{2\pi i}{8}} \bar{X}_1, \bar{X}_2, \bar{X}_3, \bar{X}_4, \bar{X}_5)$$

$$O_{\gamma_{(1)PA}+} = \frac{1}{\sqrt{8}} (b_1 a_1 \alpha_1^3 + b_2 a_2 \alpha_2^3 + b_3 a_3 (-\alpha_1)^3 + b_4 a_4 (-\alpha_2)^3) \\ \pm \frac{1}{\sqrt{8}} (a_2 b_1 \beta_1^3 + a_3 b_2 \beta_2^3 + a_4 b_3 (-\beta_1)^3 + (-a_1) b_4 (-\beta_2)^3)$$

"Geometric"

$$[O_{\gamma_{(1)PA}-}]^{\dagger} = \frac{1}{\sqrt{8}} [(a_1 - a_2) a_1 \alpha_1^3 + (a_2 - a_3) a_2 \alpha_2^3 + (-a_3 + a_4) a_3 \alpha_1^3 + (-a_4 - a_1) a_4 \alpha_2^3]^{\dagger}$$



$$= \frac{1}{\sqrt{8}} \left[\sum_{i=1}^8 \gamma^2 (c a_i \alpha_i^3) \right]^{\dagger} = [B_{j_i, n_i, s_i}]^{\dagger}$$

$$j_i = (0, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \quad n_i = (1, 4, 2, 2, 2) \quad s_i = (1, 1, 1, 1, 1)$$

The fixed pt set of $\gamma_{(1)PA} \cong S^3 \leftrightarrow B_{j_i, n_i, s_i}$

$$\text{Tension } T_{O_{\gamma_{(1)PA}-}} = \langle 0 | C_{(-1)^{F_L} \gamma_{(1)PA}-} \rangle = \frac{4}{\sqrt{8}} \sqrt{\frac{2}{8 \sin \frac{\pi}{8}}}^2 \sqrt{\frac{2}{4 \sin \frac{\pi}{8}}}^3 \cdot 2 \sin \left(\frac{\pi}{8} \right) \\ = \frac{1}{\sqrt{2}} = \langle 0 | B_{j_i, n_i, s_i} \rangle_{NSNS}$$

SO-type $O_{\gamma_{(1)PA}-}$ & four B_{j_i, n_i, s_i} : consistent & supersymmetric

Math Problems

Both related to spacetime superpotential and other holomorphic terms in string theory effective action

① Gromov-Witten invariants for unoriented strings

$$\begin{array}{ccc} \Sigma & \xrightarrow{f} & X \\ \Omega \downarrow & \curvearrowright & \downarrow \tau \\ \Sigma & \xrightarrow{f} & X \end{array}$$

X toric CY : $\mathbb{R}P^2$ amplitudes are computed by

- Mirror Symmetry Acharya-Aganagic-H-Vafa
- Localization Diaconescu-Florea-Misra

τ free \Rightarrow No "framing ambiguity"
(No dependence of torus weights)

② Categorical description of D-branes in Type II Orientifolds

e.g. Type IIB : $\mathcal{D}^*(X)$

... refinement of $K(X)$ Minasian-Moore
D-brane charge Witten

D-brane charge in Type II Orientifolds :

$$KR^{-(9-p)}(X) \quad \text{for } X = \mathbb{R}^{p+1} \times \mathbb{R}^{9-p} \text{ (or } \mathbb{T}^{9-p}) \quad \text{Hori:}$$

+ \bar{O}_p with $O\bar{p}$ Bergman-Gimon-Horava

- What is it for general (X, τ) ?
- How to refine it ?

Hint $[X, \mathbb{F}(H_{\mathbb{R}} \oplus C \oplus C_H)]^{\mathbb{Z}_2} \cong KR^p(X)$

$\underset{G_{\tau}}{\downarrow} \quad \underset{J_{c.c.}}{\downarrow}$ Atiyah-Singer '69