Physics Benchmarks

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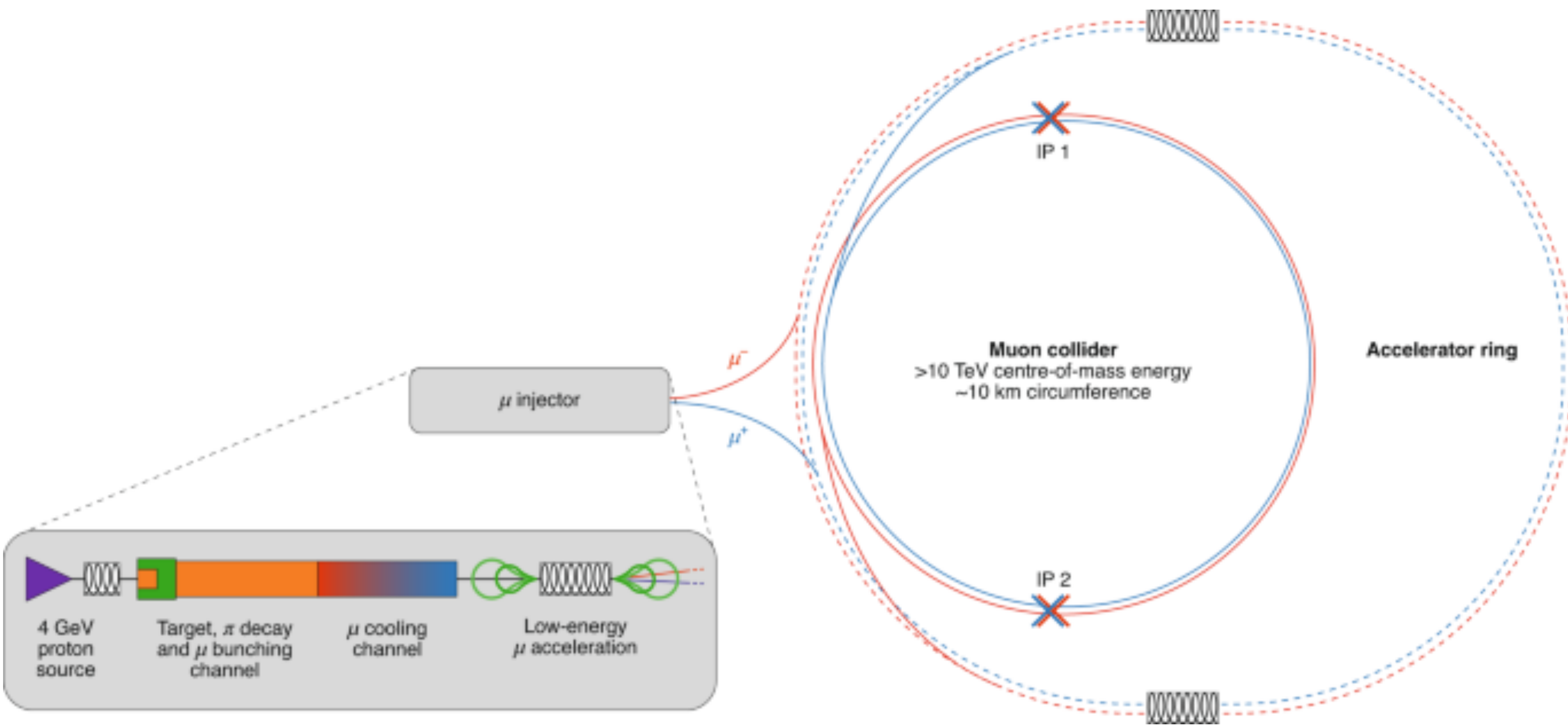
- Argonne/Northwestern
- March 1, 2023



Disclaimer

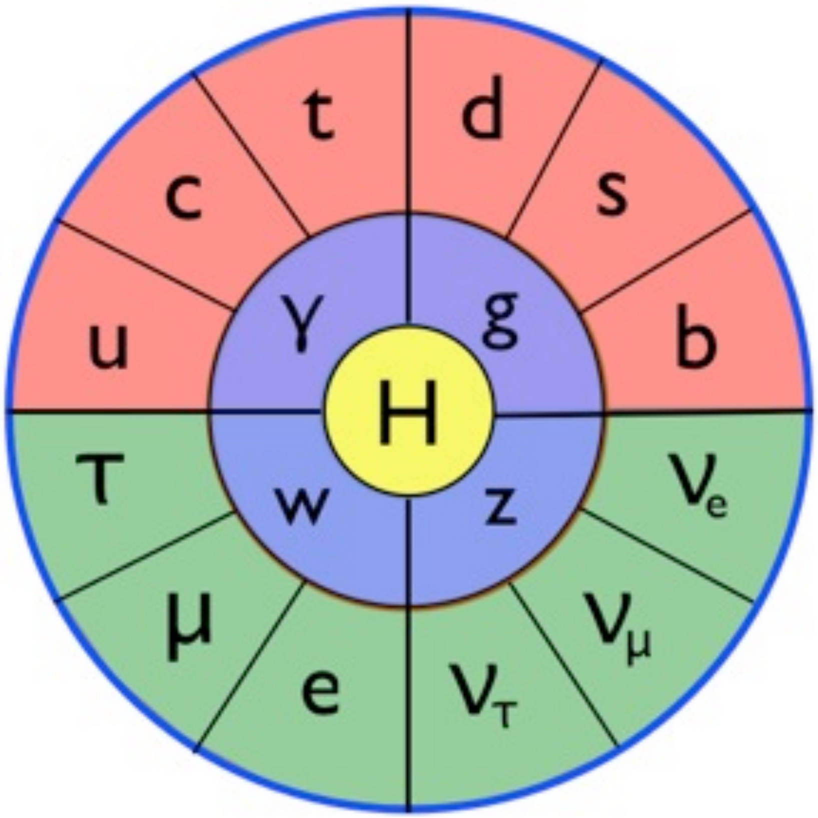
- The space of benchmarks is infinite. 

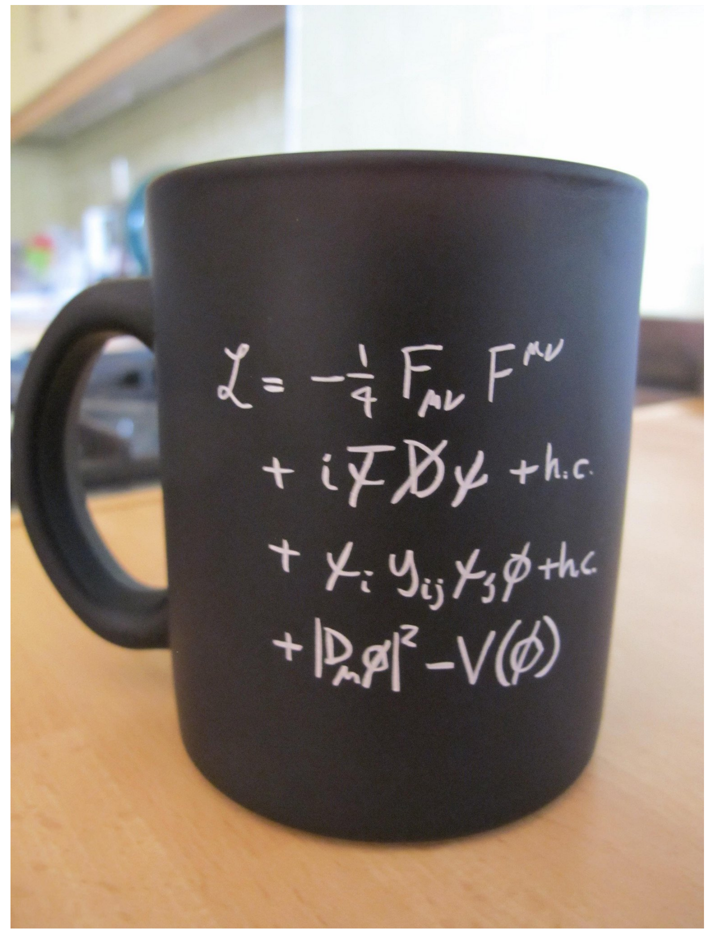
to summarize them all.

- Will attempt to give a broad physics context and provide a roadmap for future explorations.

- **Choice of topics reflects my ignorance –** don’t know everything well enough to comment on everything!

The Standard Model is self-consistent after the discovery of the Higgs:





Where are the physics opportunities?

- Stress-testing SM
  - Predictions of SM which have yet to be observed/tested
  - Over-constrain couplings that have already been established

- Asking the right questions
  - conceptual questions that can’t be answered by the SM
  - empirical questions that can’t be answered by the SM

The SM Higgs boson is very special:

Couplings to massive gauge bosons → $\left(\frac{2m_W^2}{v}\, h\, W_\mu^+ W^{-\mu} + \frac{m_Z^2}{v}\, h\, Z_\mu Z^\mu\right)$

Couplings to massless gauge bosons →

$$+c_g \frac{\alpha_s}{12\pi v} h\, G_{\mu\nu}^a G^{a\,\mu\nu} + c_\gamma \frac{\alpha}{8\pi v} h\, F_{\mu\nu} F^{\mu\nu} + c_{Z\gamma} \frac{\alpha}{8\pi v s_w}\, h\, F_{\mu\nu} Z^{\mu\nu}$$

$$c_g^{(SM)}(125\ \mathrm{GeV}) = 1\ , \qquad c_\gamma^{(SM)}(125\ \mathrm{GeV}) = -6.48\ , \qquad c_{Z\gamma}^{(SM)}(125\ \mathrm{GeV}) = 5.48\ .$$

Couplings to fermions → $\sum_f \frac{m_f}{v} h \bar{f} f$

Self-couplings → $\frac{1}{2} m_h^2 h^2 + \frac{m_h^2}{v} h^3 + \frac{2m_h^2}{v^2} h^4$

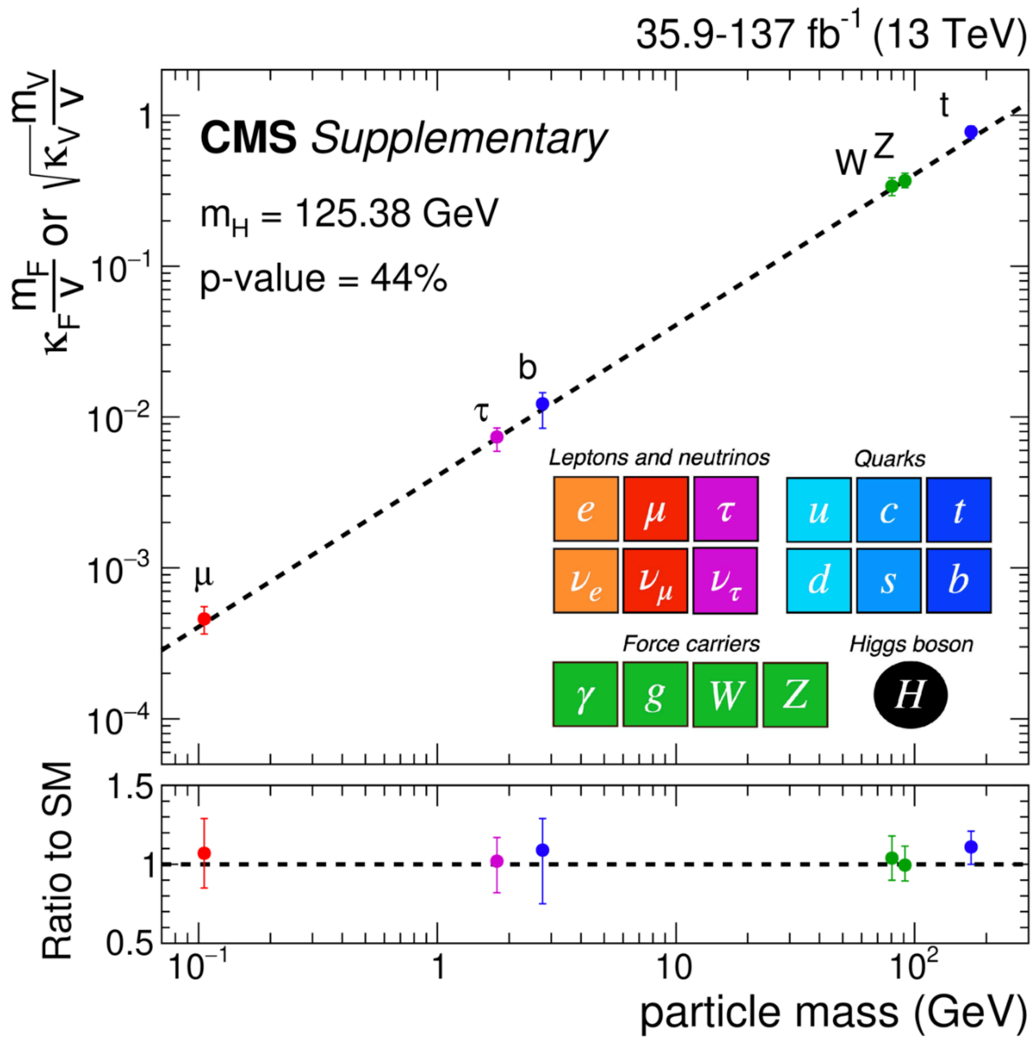
A highly non-trivial prediction:

There is no free parameters (once all masses are measured)!

Stress-testing SM

- **Prioritize couplings which have yet to be established experimentally:**

We need to keep pursuing Yukawa couplings to 1st and 2nd generation fermions.



Muon collider offers some promise in direct measurements from Higgs decays:

Fit Result [%]

	10 TeV Muon Collider	with HL-LHC	with HL-LHC + 250 GeV e^+e^-
κ_W	0.06	0.06	0.06
κ_Z	0.23	0.22	0.10
κ_g	0.15	0.15	0.15
κ_γ	0.64	0.57	0.57
$\kappa_{Z\gamma}$	1.0	1.0	0.97
κ_c	0.89	0.89	0.79
κ_t	6.0	2.8	2.8
κ_b	0.16	0.16	0.15
κ_μ	2.0	1.8	1.8
κ_τ	0.31	0.30	0.27

Table 3: Results of a 10-parameter fit to the Higgs couplings in the κ-framework, based on the attainable precision in each on-shell Higgs production and decay channel listed in Table 2. Additionally, we include the effects of adding data sets projected from the HL-LHC and a 250 GeV e^+e^- Higgs factory. One should keep in mind that a muon collider will also strongly constrain Higgs properties via off-shell measurements, which are not included here.

Muon Smasher's Guide: 2103.14043

To probe the light flavor (u, d, s) Yukawas we have to get creative. There is a proposal using the hadronic event shape at CEPC:

Probing light-quark Yukawa couplings via hadronic event shapes at lepton colliders

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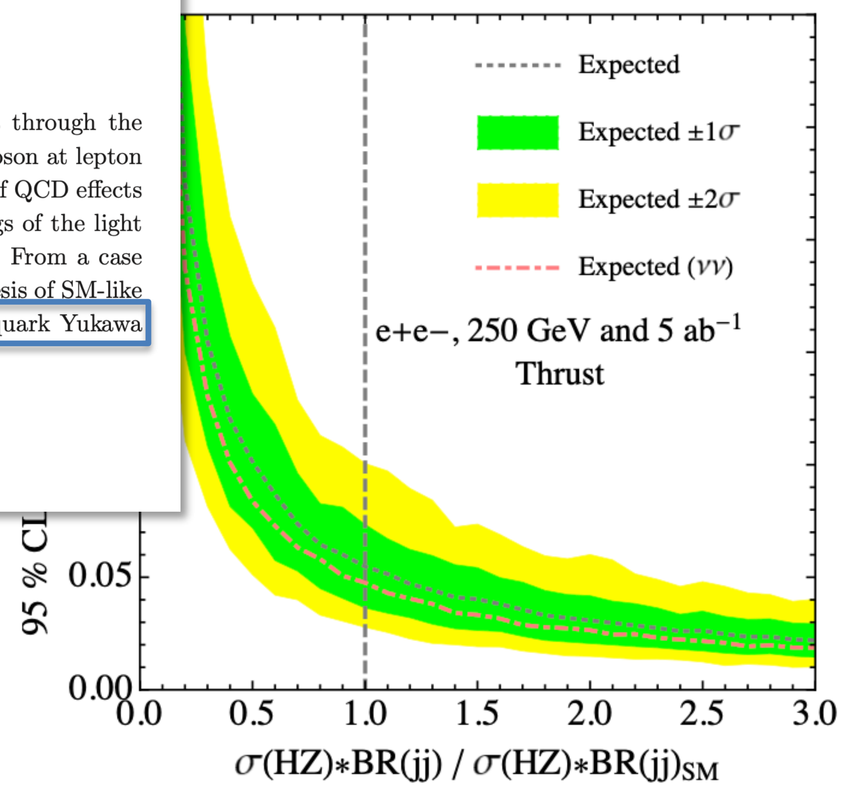
Abstract: We propose a novel idea for probing the Higgs boson couplings through the measurement of hadronic event shape distributions in the decay of the Higgs boson at lepton colliders. The method provides a unique test of the Higgs boson couplings and of QCD effects in the decay of the Higgs boson. It can be used to probe the Yukawa couplings of the light quarks and to further test the mechanism of electroweak symmetry breaking. From a case study for the proposed Circular Electron-Positron Collider, assuming a hypothesis of SM-like theory light-quark couplings with a strength greater than 9% of the bottom-quark Yukawa coupling in the standard model can be excluded.

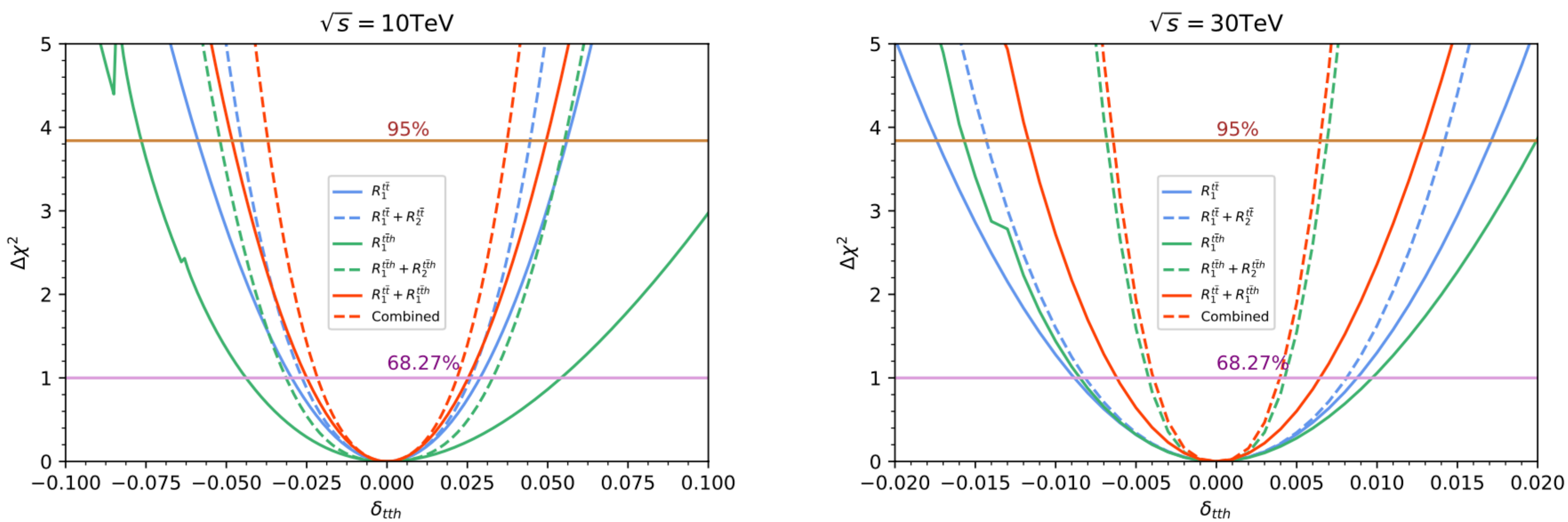
Keywords: Higgs boson, lepton collider, QCD



We need a study for muon collider!

On the other hand, the top Yukawa coupling is of particular interest:

Miranda Chen and Da Liu: 2221.11067



(a) $\mu^+\mu^- \to t\bar{t}\nu\bar{\nu}$ with $\sqrt{s} = 10$ TeV and $L = 10\,\text{ab}^{-1}$.

(b) $\mu^+\mu^- \to t\bar{t}\nu\bar{\nu}$ with $\sqrt{s} = 30$ TeV and $L = 90\,\text{ab}^{-1}$.

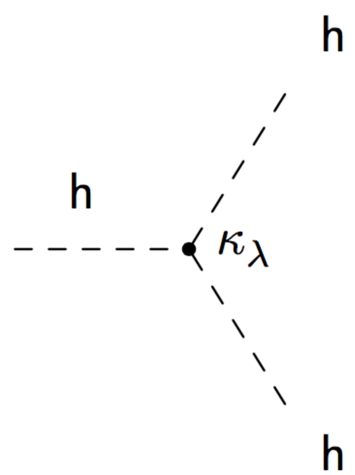
Figure 6: $\Delta\chi^2$ plot as a function of anomalous top Yukawa coupling δ_{tth} for processes $\mu^+\mu^- \to t\bar{t}\nu\bar{\nu}$ and $\mu^+\mu^- \to t\bar{t}\nu\bar{\nu}h$ at 10 TeV (left panel) and 30 TeV (right panel) muon collider. Here $R_1(R_2)$ denotes the interference term and the squared term respectively.

Sensitivity in the ttH channel needs further study!

In addition to Yukawas, there are two important classes of Higgs couplings that have yet to be established <u>experimentally</u>:

- Higgs self-couplings:

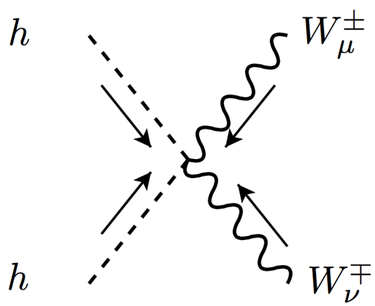
  This can be measured in the double-Higgs production



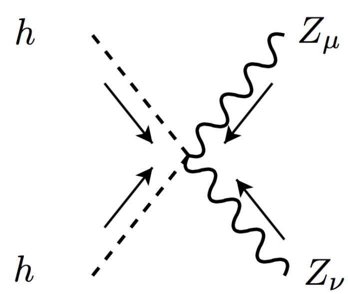
It is difficult to measure at the LHC, but experimental colleagues are making some progress.

- The second class of coupling, however, is still largely missing from the picture -- the HHVV coupling

$$D_\mu H^\dagger D^\mu H \;\supset\; g^2 h^2 V_\mu V^\mu$$



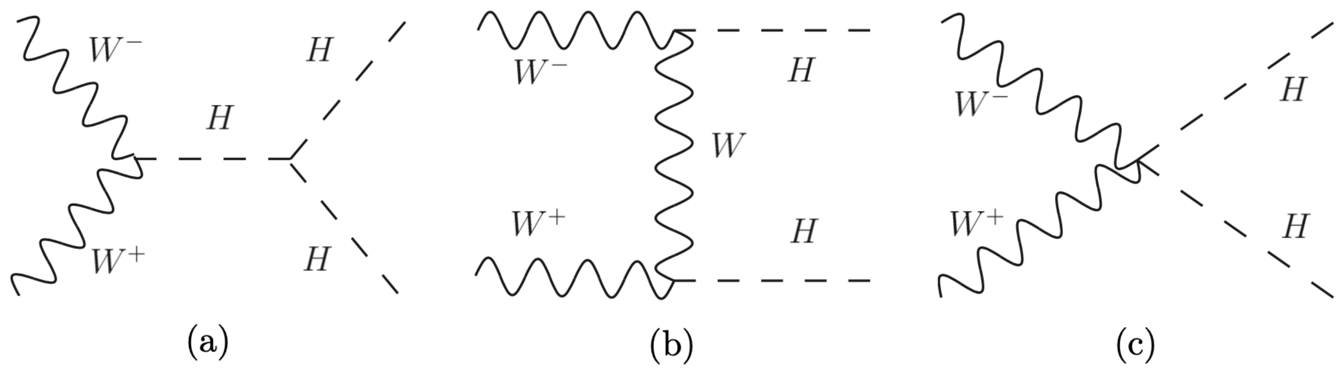
$$\frac{i}{2} g^2 g_{\mu\nu}$$



$$\frac{i}{2} \frac{g^2}{\cos^2\theta_W} g_{\mu\nu}$$

This is a prediction of gauge invariance!

- At a lepton collider, both the trilinear and quartic couplings can be probed in double Higgs production through VBF:



Notice the process is sensitive to **both** HHH and WWHH couplings!

- Using the M_{HH} shape information, it is possible to constrain both couplings at the same time:

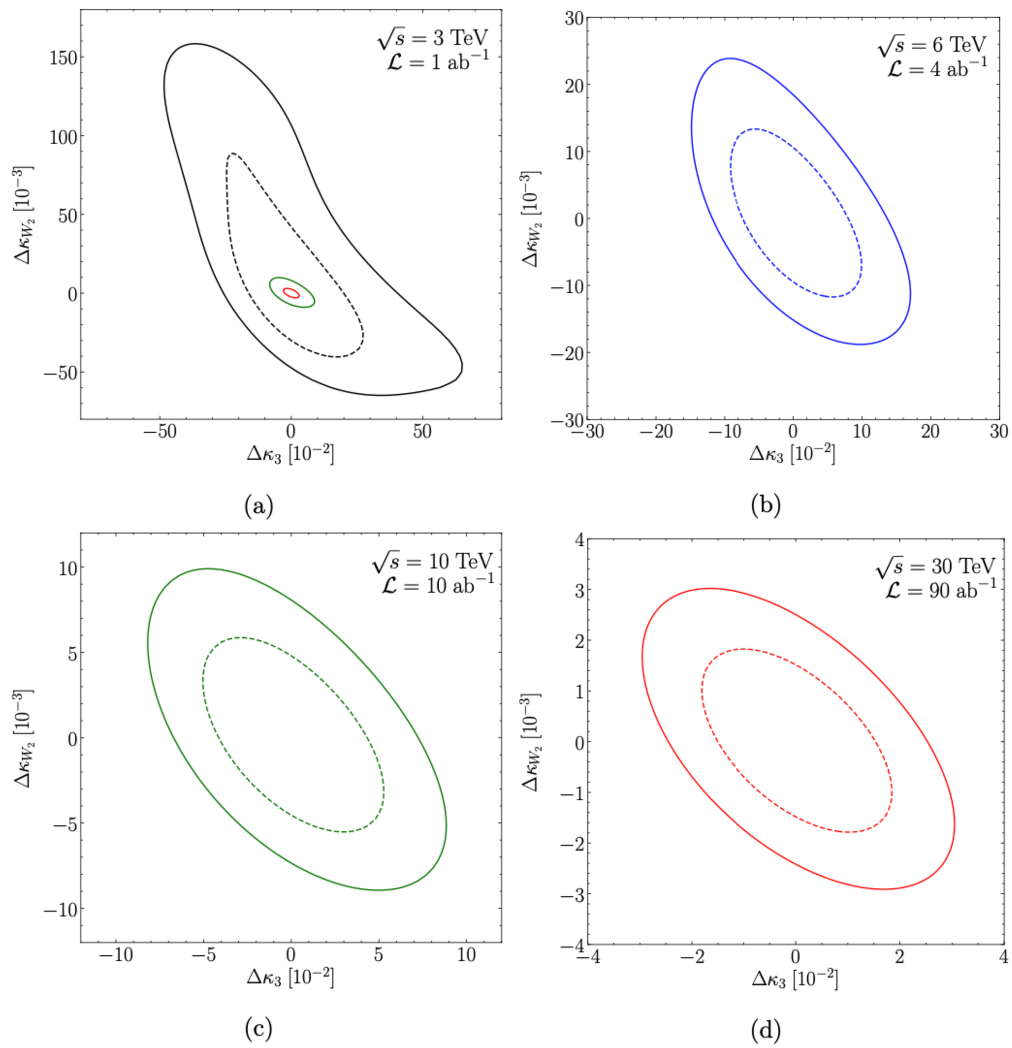


Figure 7: Correlated bounds with 95% C.L. (solid) and 68% C.L. (dashed) in the $\Delta\kappa_{W_2}$-$\Delta\kappa_3$ plane for $\sqrt{s} = 3, 6, 10, 30$ TeV, respectively. In (a), inner ellipses (solid) include the 95% C.L. results for 10 TeV and 30 TeV for comparison.

T. Han, D. Liu, IL, X. Wang: 2008.12204

The ultimate challenge is to disentangle effects of anomalous couplings. For example, one can write down a nonlinear Higgs EFT (a la Chiral Lagrangian):

$$\begin{aligned}\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} &+ \left(2C_0^h\frac{h}{v} + C_0^{2h}\frac{h^2}{v^2}\right)\left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu\right)\\ &+ C_5^h\left(\frac{h}{v}W_\mu^+\mathcal{D}^{\mu\nu}W_\nu^- + \text{h.c.}\right) + C_6^h\frac{h}{v}W_{\mu\nu}^+W^{-\mu\nu}\\ &+ C_5^{2h}\left(\frac{h^2}{v^2}W_\mu^+\mathcal{D}^{\mu\nu}W_\nu^- + \text{h.c.}\right) + C_6^{2h}\frac{h^2}{v^2}W_{\mu\nu}^+W^{-\mu\nu}\\ &+ C_9^{2h}\frac{(\partial_\nu h)^2}{v^2}W_\mu^+W^{-\mu} + C_{10}^{2h}\frac{\partial^\mu h\partial^\nu h}{v^2}W_\mu^+W_\nu^-,\end{aligned}$$

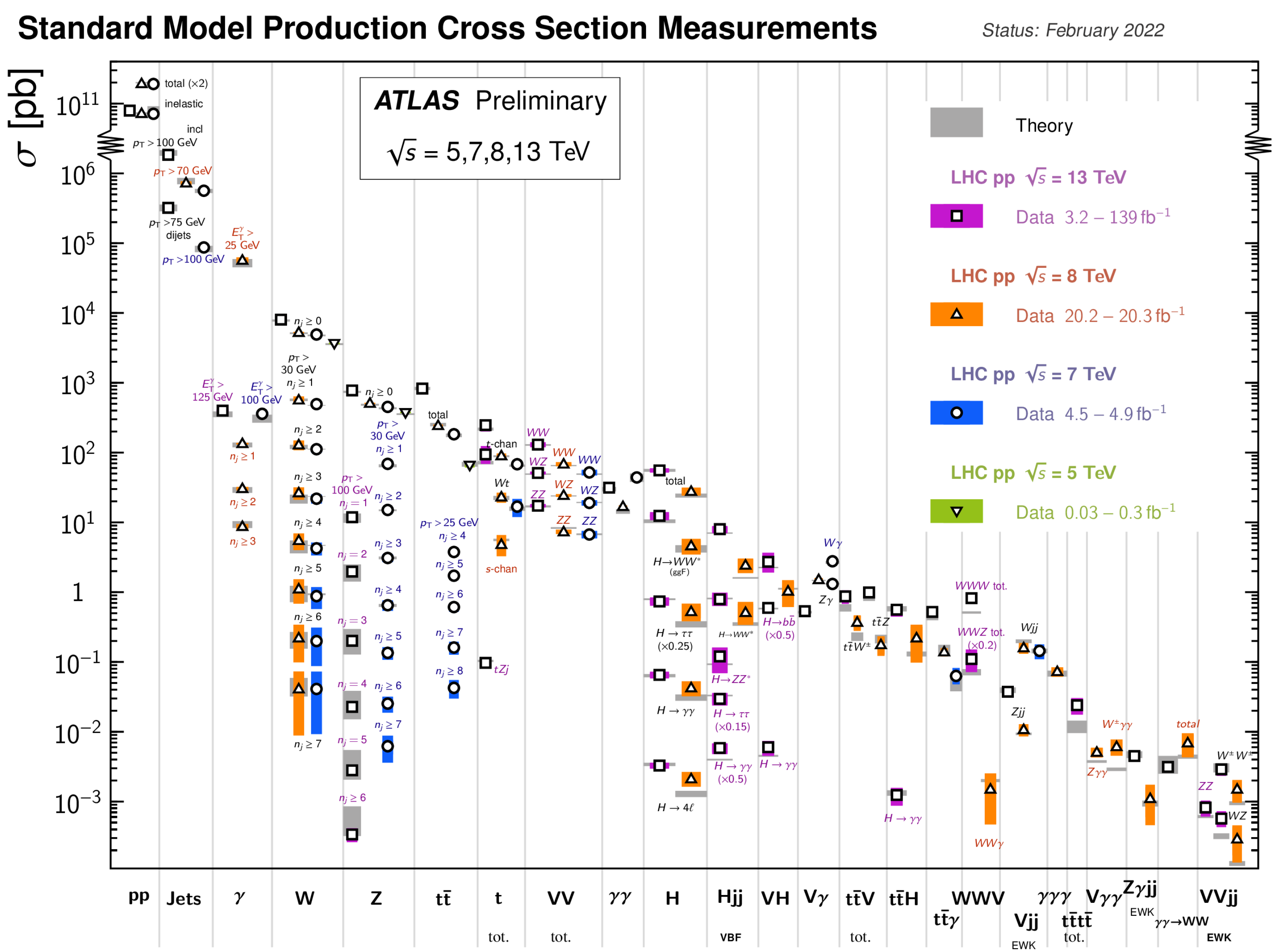
Da Liu, IL, Zhewei Yin: 1805.00489; 1809.09126

Helicity	Diagram	SM	C_5^h	C_6^h	C_5^{2h}	C_6^{2h}	C_9^{2h}	C_{10}^{2h}
$(0,0)$ $(\pm,\pm)$	s-channel	✓	✓	✓	-	-	-	-
	t-channel	✓	✓	✓	-	-	-	-
	u-channel	✓	✓	✓	-	-	-	-
	4-point	✓	-	-	✓	✓	✓	✓
$(\pm,\mp)$	s-channel	-	-	-	-	-	-	-
	t-channel	✓	✓	-	-	-	-	-
	u-channel	✓	✓	-	-	-	-	-
	4-point	-	-	-	-	-	-	✓
$(\pm,0)$ $(0,\pm)$	s-channel	-	-	-	-	-	-	-
	t-channel	✓	✓	✓	-	-	-	-
	u-channel	✓	✓	✓	-	-	-	-
	4-point	-	-	-	-	-	-	✓

T. Han, D. Liu, IL, X. Wang: unpublished

Need a comprehendsive study at the Muon collider!

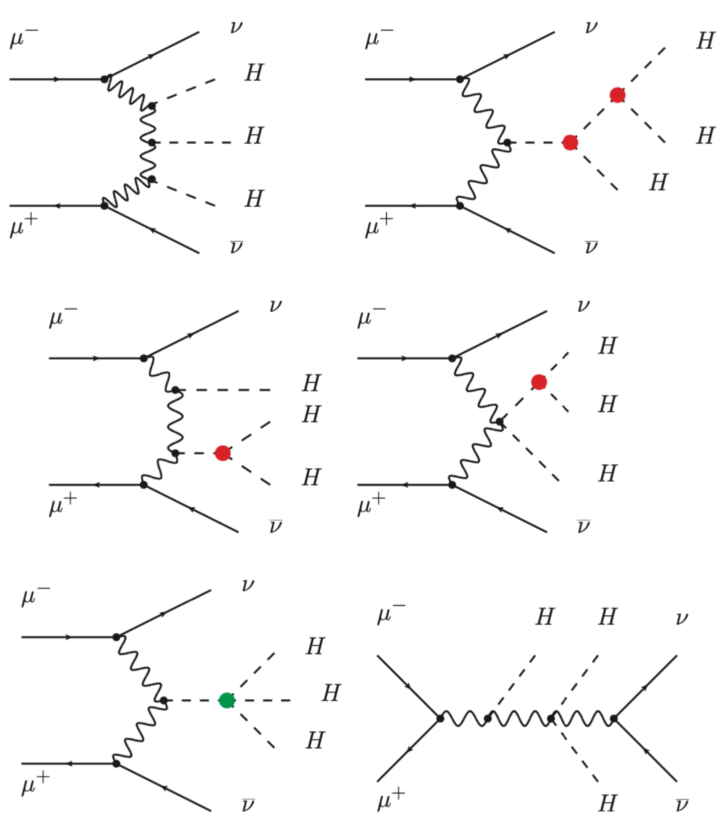
Our experimental colleagues have been systematically stress-testing SM by going to higher multiplicities:

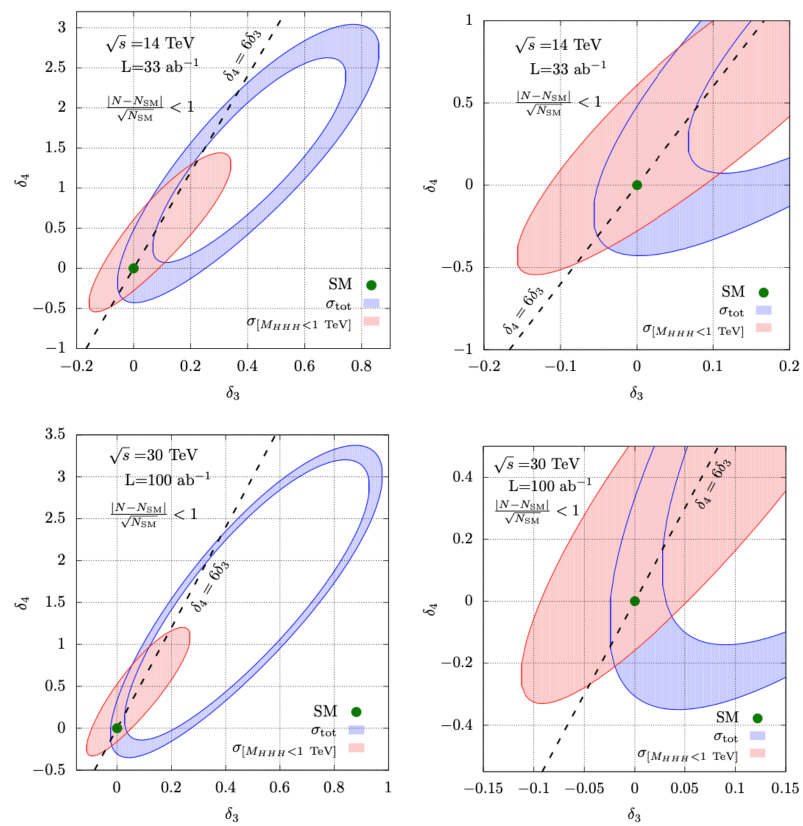


As we go to very high energies, need to do the same for the Higgs!

- HHH and HHHH final states have not been searched for experimentally. **What are the SM predictions??**

This is a new frontier waiting to be explored further. There’s a study on HHH final state at the Muon collider:

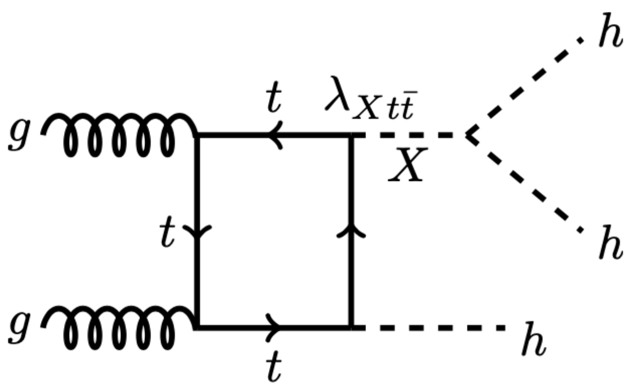


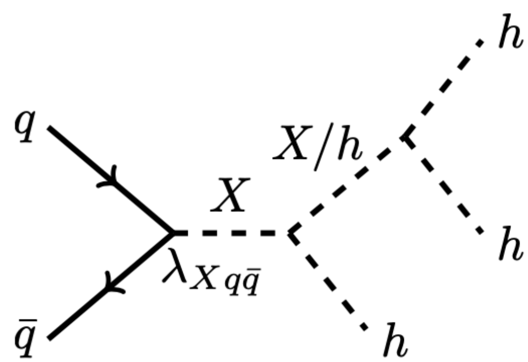


Chiesa et al: 2003.13628

As we go to very high energies, need to do the same for the Higgs!

- 3H and 4H final states can also be produced in simple extensions (2HDM or SM+ singlet) with significant rates at a hadron collider:



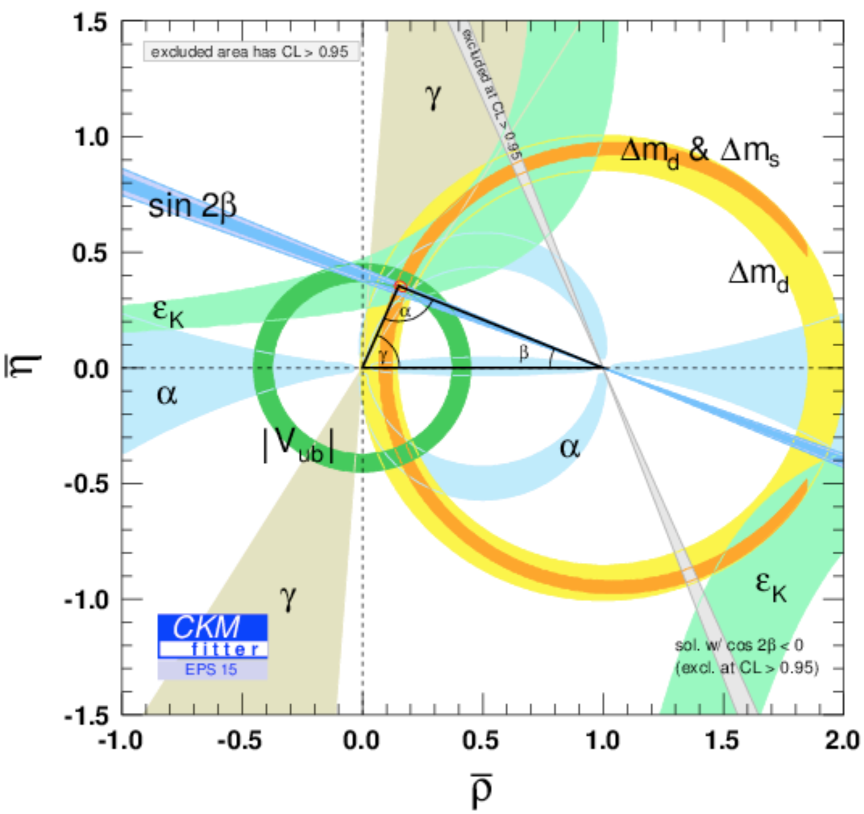


IL, N. Shah, X. Wang: 2012.00773;
Egana-Ugrinovic, Homiller, Meade: 2101.04119
C.-W. Chiang, T.-K. Kuo, IL: 2202.02954

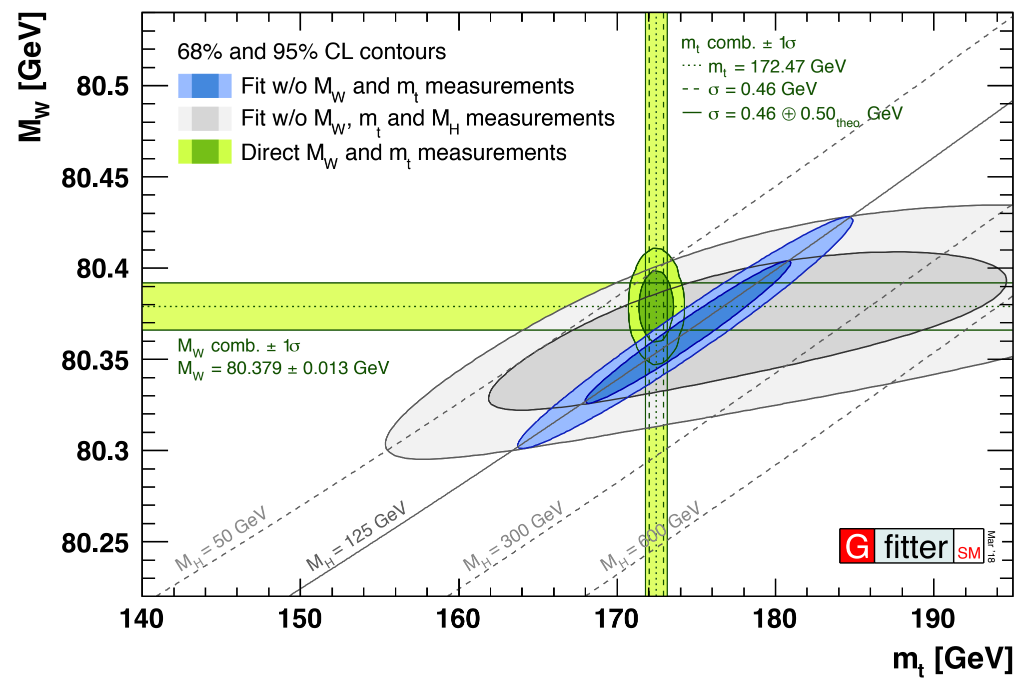
A study for the discovery potential at a high energy lepton collider is currently lacking.

- **For couplings which have been established, we need to over-constrain.**

  Our colleagues in flavor physics and from LEP era are very good at this:



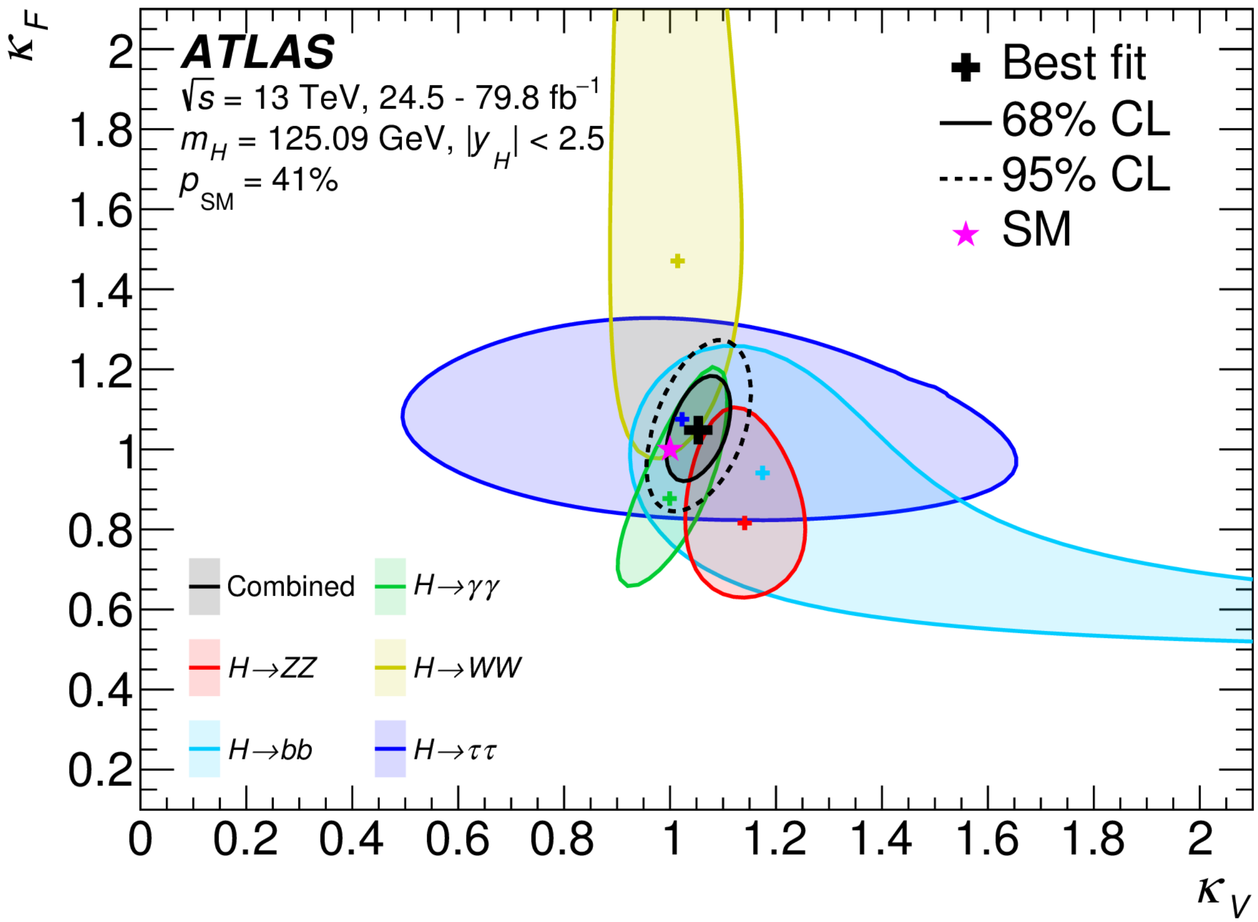
Unitarity triangle



Precision Electroweak Measurements

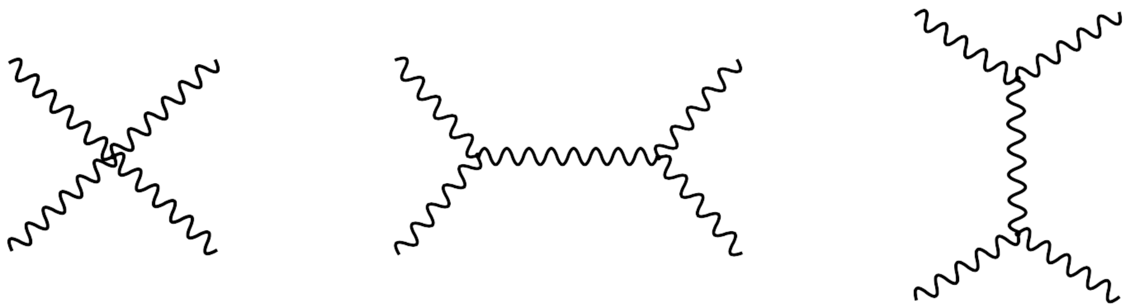
- **For couplings which have been established, we need to over-constrain.**

At the LHC this has been pursued, but we need much better precision!



- Would like to single out one very important prediction of SM Higgs to be measured precisely:

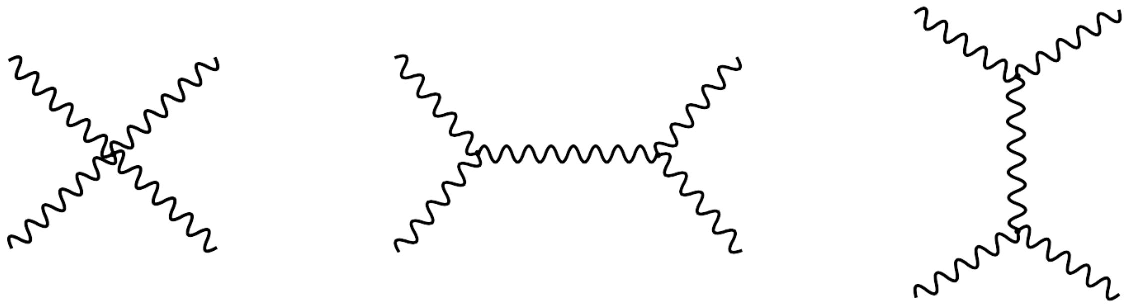
  Without the Higgs, WW scattering amplitude violates unitarity:

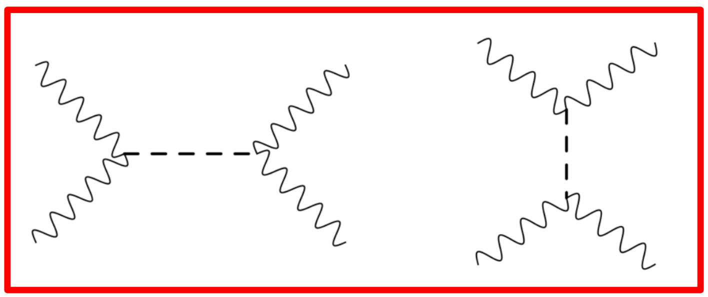


$$\mathcal{A}(W_L^+W_L^- \to W_L^+W_L^-) \simeq \frac{g^2}{4m_W^2}(s+t) \ .$$

- Would like to single out one very important prediction of SM Higgs to be measured precisely:

  Including the Higgs contribution allows the growth to be cancelled completely,





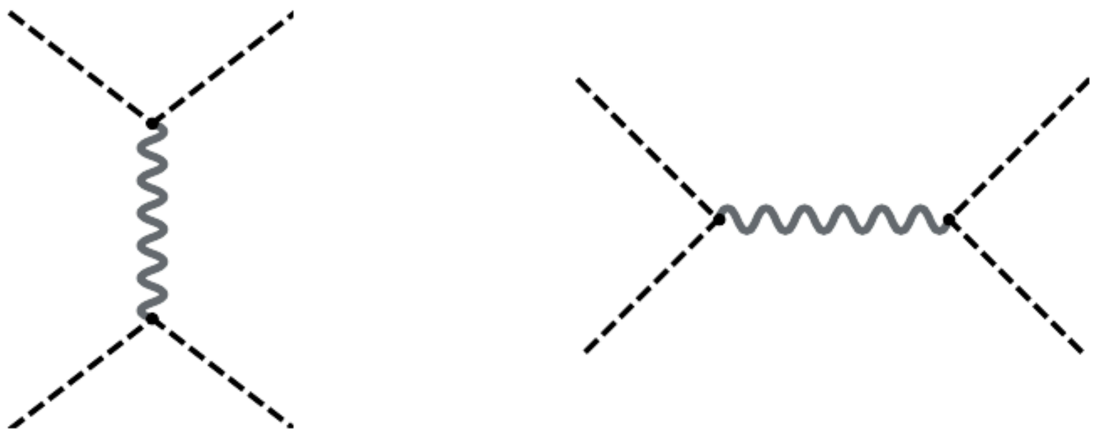
provided the HWW coupling have precisely the form in the SM!

This is an extremely simple and economical solution, except…

Nature has never chosen this simple solution before…

(Recall we have NOT observed a fundamental scalar previously!)

For example, pi-pi scattering in low-energy QCD is unitarized by a series of heavy resonances, including the spin-1 rho meson:



Each resonance only partially unitarizes the pi-pi scattering.

If the 125 GeV Higgs only partially unitarize the VV scattering

→ the HVV coupling will deviate from the SM expectation!!

Unitarization in VV scattering is only tested with O(10%) uncertainty.

→ Clearly not sufficient!

To test this prediction we need

- More precise measurements of HVV couplings.

- Direct measurements of VV scatterings.

How precise is precise enough?

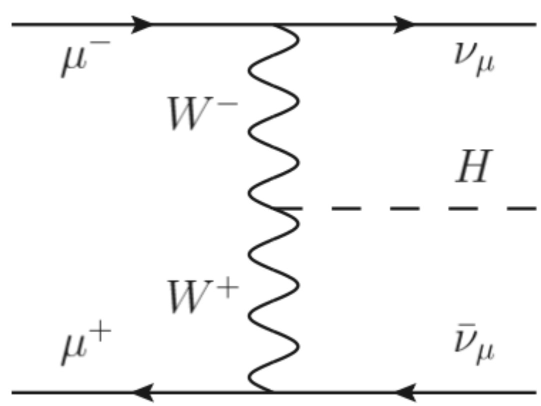
By accident, generic deviations from SM are quadratic in $1/M_{\text{new}}$:

$$\mathcal{O}\left(\frac{v^2}{M_{\text{new}}^2}\right) \sim 5\% \times \left(\frac{1\ \text{TeV}}{\Lambda}\right)^2$$

To establish credible deviations we need percent level precision!

At a high energy muon collider, single Higgs production goes through the VBF topology. Moreover, both WW and ZZ fusion need to be considered:

$$\mu^+\mu^- \to \nu_\mu\bar{\nu}_\mu\ H \qquad (WW\ \text{fusion}),$$
$$\mu^+\mu^- \to \mu^+\mu^-\ H \qquad (ZZ\ \text{fusion}).$$



However, in the ZZ fusion channel, the outgoing muons are very forward and may escape detections:

T. Han, D. Liu, IL, X. Wang: 2008.12204

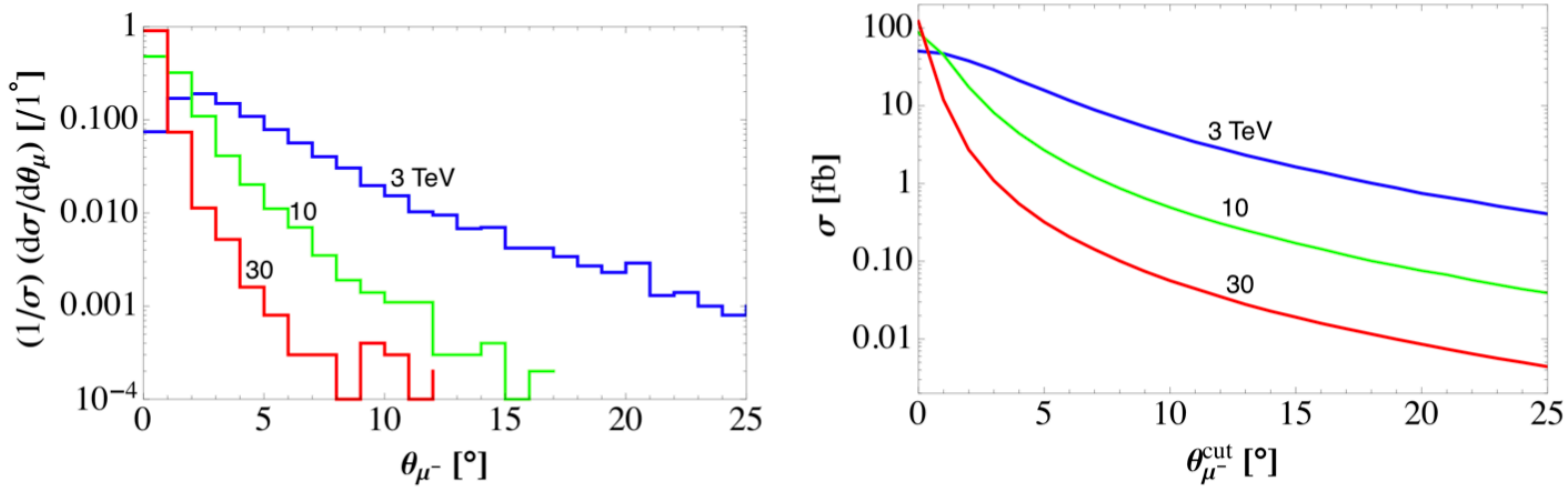


Figure 3: $\mu^+\mu^- \to \mu^+\mu^- H$ via ZZ fusion with $\sqrt{s} = 3, 10$ and 30 TeV for (a) angular distribution θ_{μ^-}, and (b) total cross section versus an angular cut $\theta^{cut}_{\mu^-}$.

This led to the notion of a "inclusive process,"

- Inclusive channel: events from WW fusion and from ZZ fusion without detecting muons;

similar to that at a hadron collider!

A preliminary study using the “kappa” formalism at the muon collider:

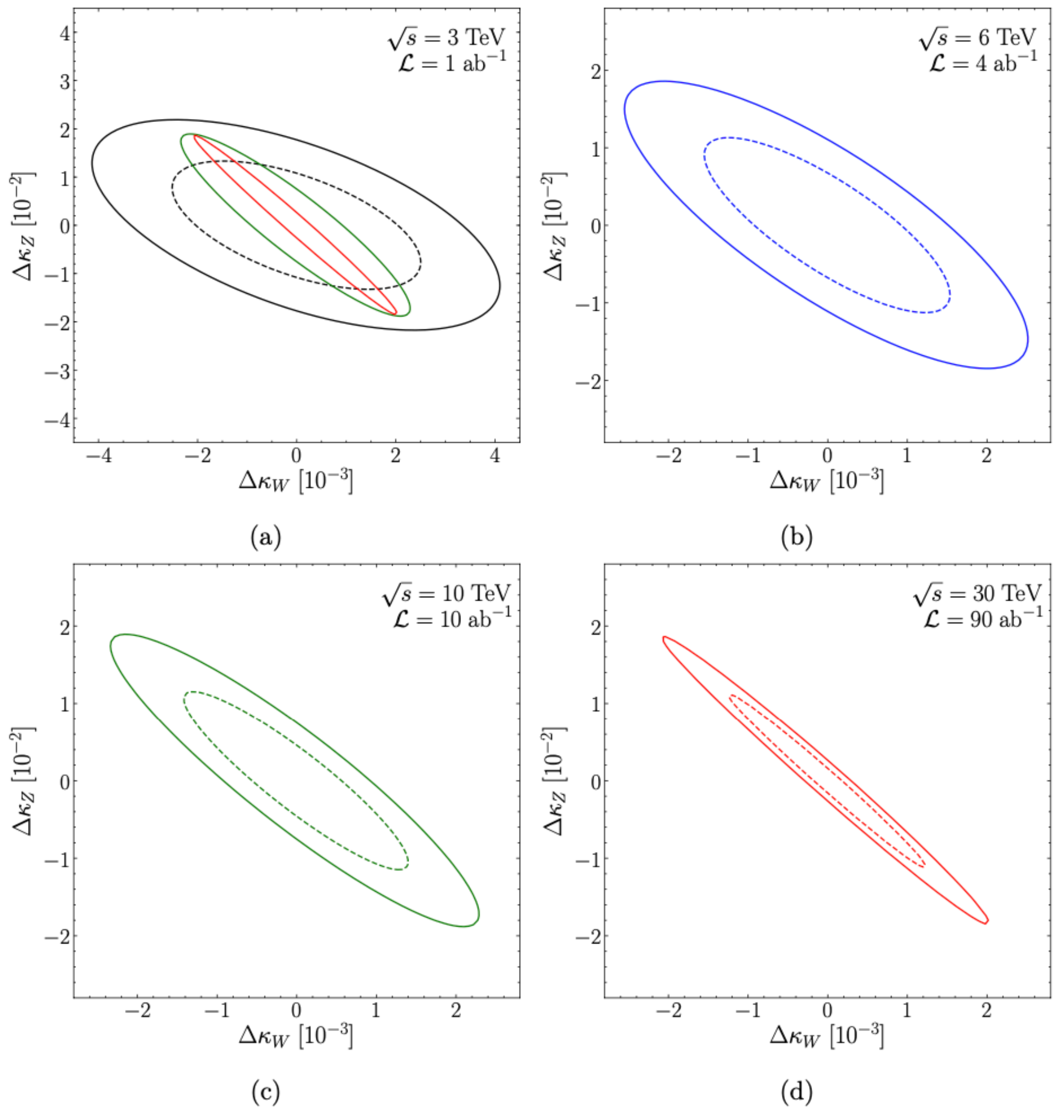
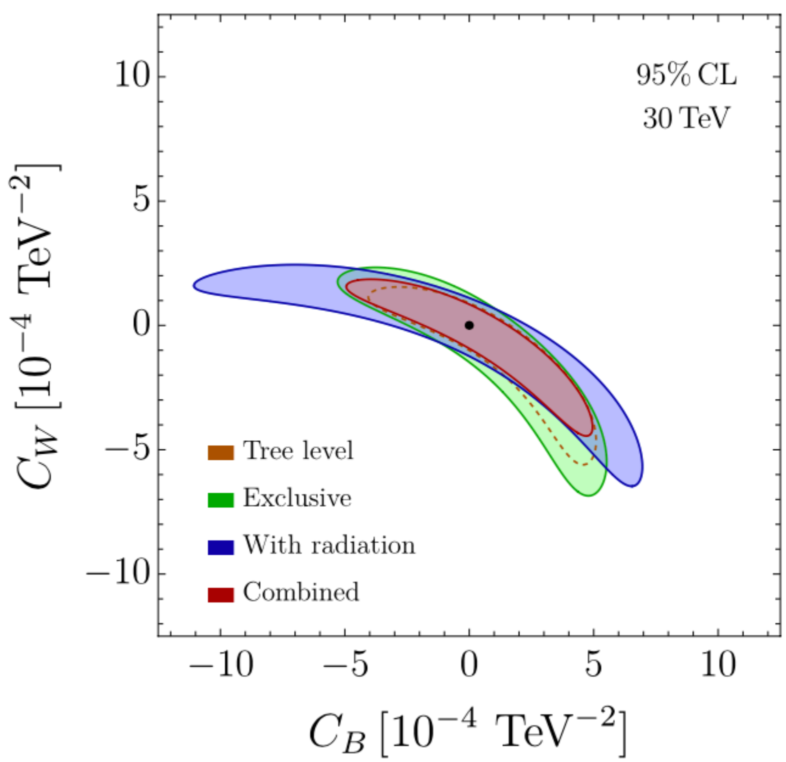
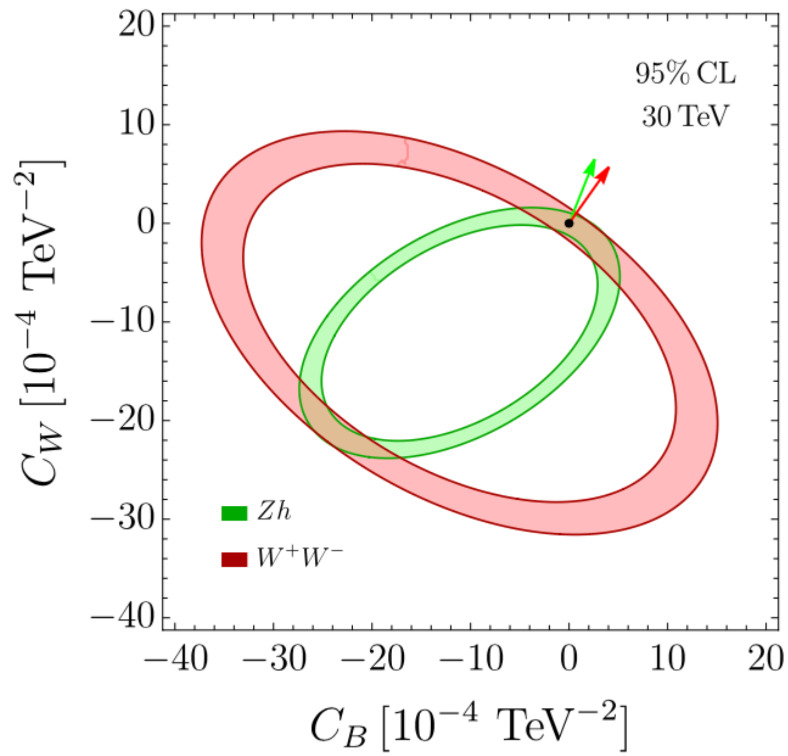


Figure 6: Correlated bounds with 95% C.L. (solid) and 68% C.L. (dashed) in the $\Delta\kappa_W$-$\Delta\kappa_Z$ plane for $\sqrt{s} = 3, 6, 10, 30$ TeV, respectively. In (a), inner ellipses (solid) include the 95% C.L. results for 10 TeV and 30 TeV for comparison.

VV scattering (and diboson final states) have received some attention at the Muon collider:

Di-boson	$O_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D^\nu W^a_{\mu\nu}$ $O_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}_\mu H) \partial^\nu B_{\mu\nu}$



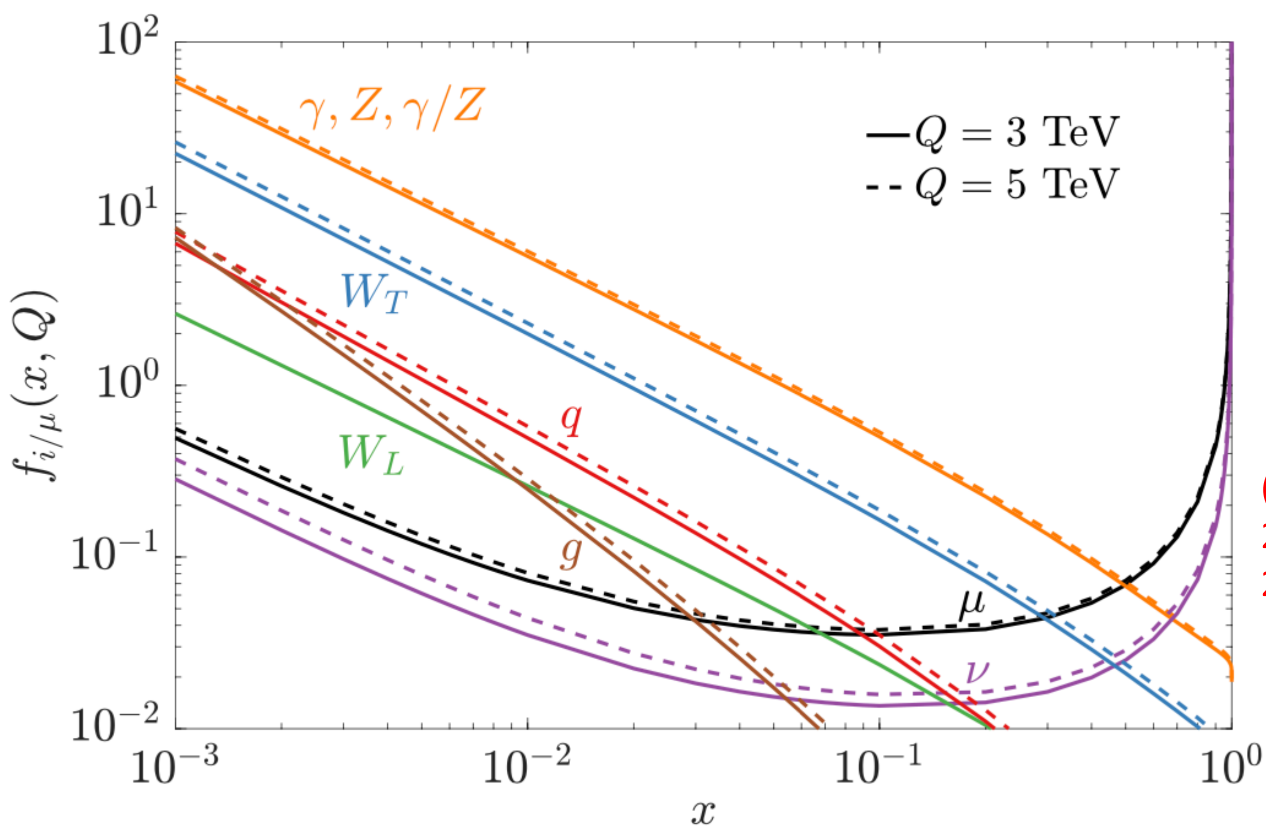


D. Buttazzo, R. Franceschini, A. Wulzer: 2012.11555
A. Wulzer et. al.: 2202.10509

These analyses demonstrate a novel feature of the SM at very high energies:

$$\alpha_{ew} \log \frac{E_{\rm CM}}{m} \sim \mathcal{O}(1)$$

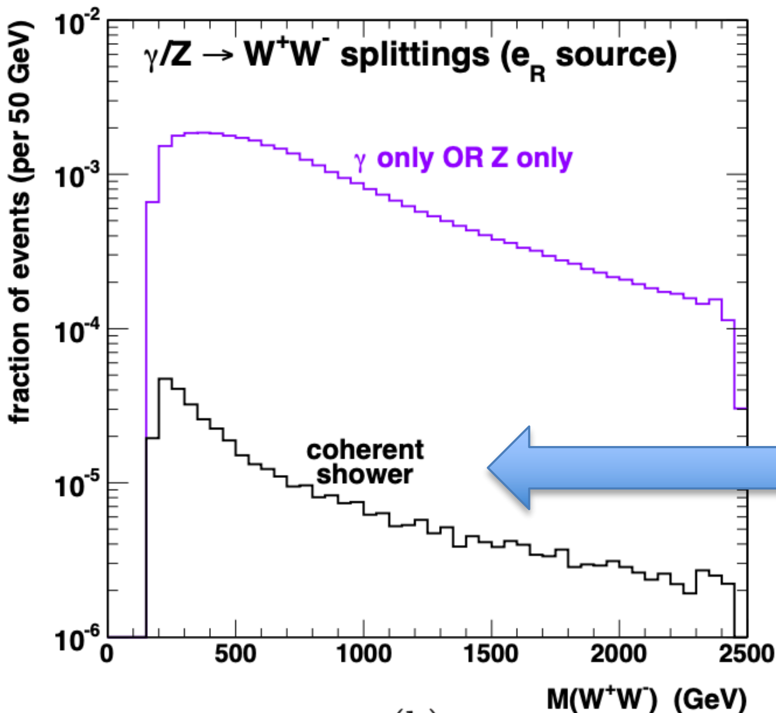
The electroweak Sudakov logs and radiative effects become important and we need to adopt a picture of “electroweak PDF” for the colliding leptons:



T. Han, Y. Ma, K. Xie: 2007.14300; 2103.09844

(See also A. Constantini et. al.: 2005.10289; S. Chen et. al.: 2202.10509.)

More importantly, at energies far above the EW scale, the PDFs evolve according to **unbroken** SU(2)xU(1) gauge theory, meaning it's crucial to take into account B-W^3 mixing and interference effects:



(b)

J. Chen, T. Han, B. Tweedie: 1611.00788

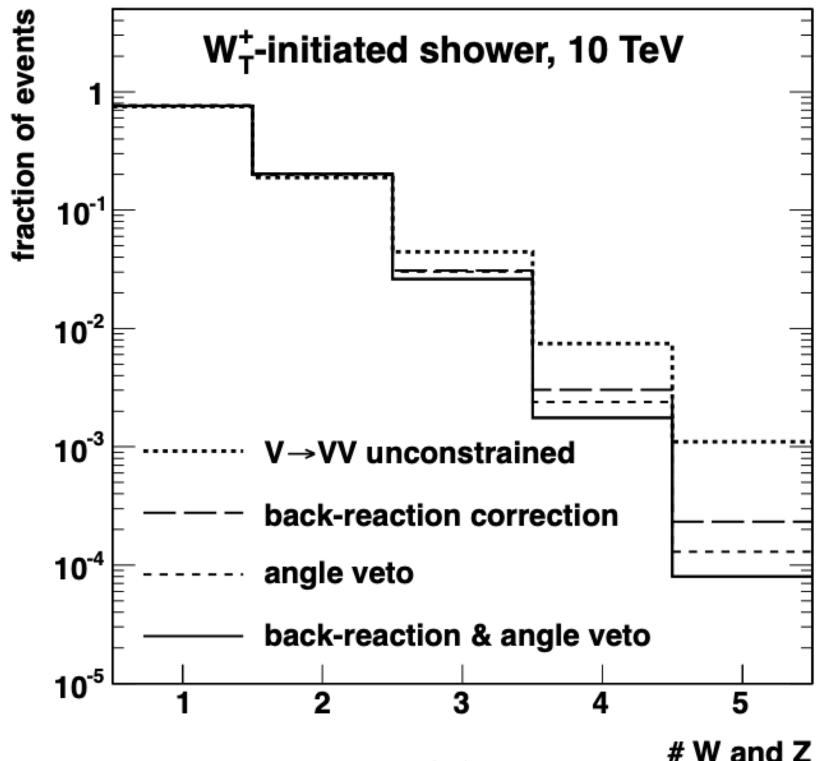
$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -c_W s_W \\ s_W^2 & c_W^2 & c_W s_W \\ 2c_W s_W & -2c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}$$

Retaining B-W^3 interference!

This is an important prediction of SM, which need to be further refined and tested at a high energy lepton collider!

In fact, the need to consider EW "parton showering" gives rise to many novel phenomena as predictions of the SM in the "massless limit."

One example is multiple collimated EW bosons initiated from transverse gauge bosons, giving rise to "weak jets":



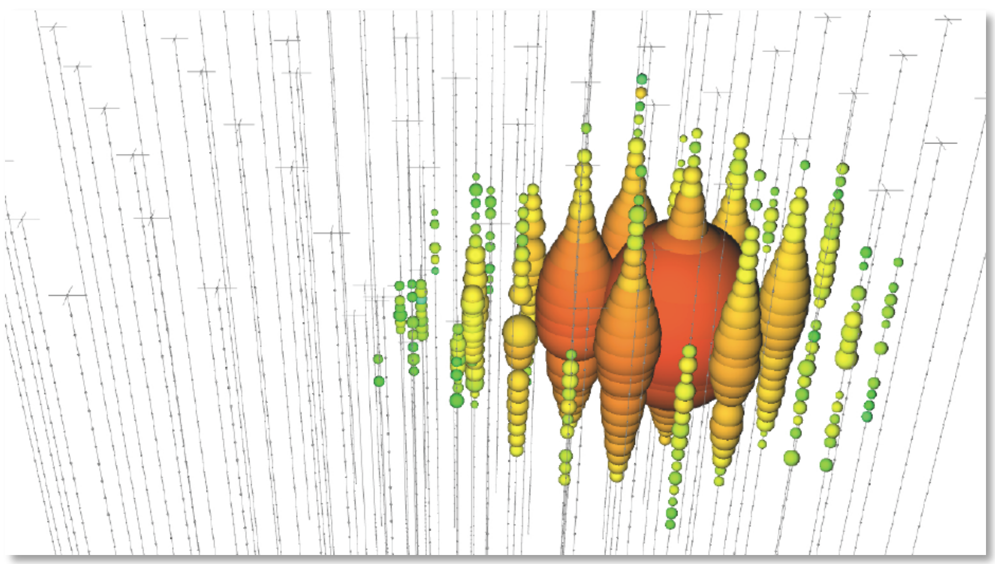
(b)

J. Chen, T. Han, B. Tweedie: 1611.00788

One of the most interesting questions (benchmarks) is the EW parton showering of a high energy neutrino:

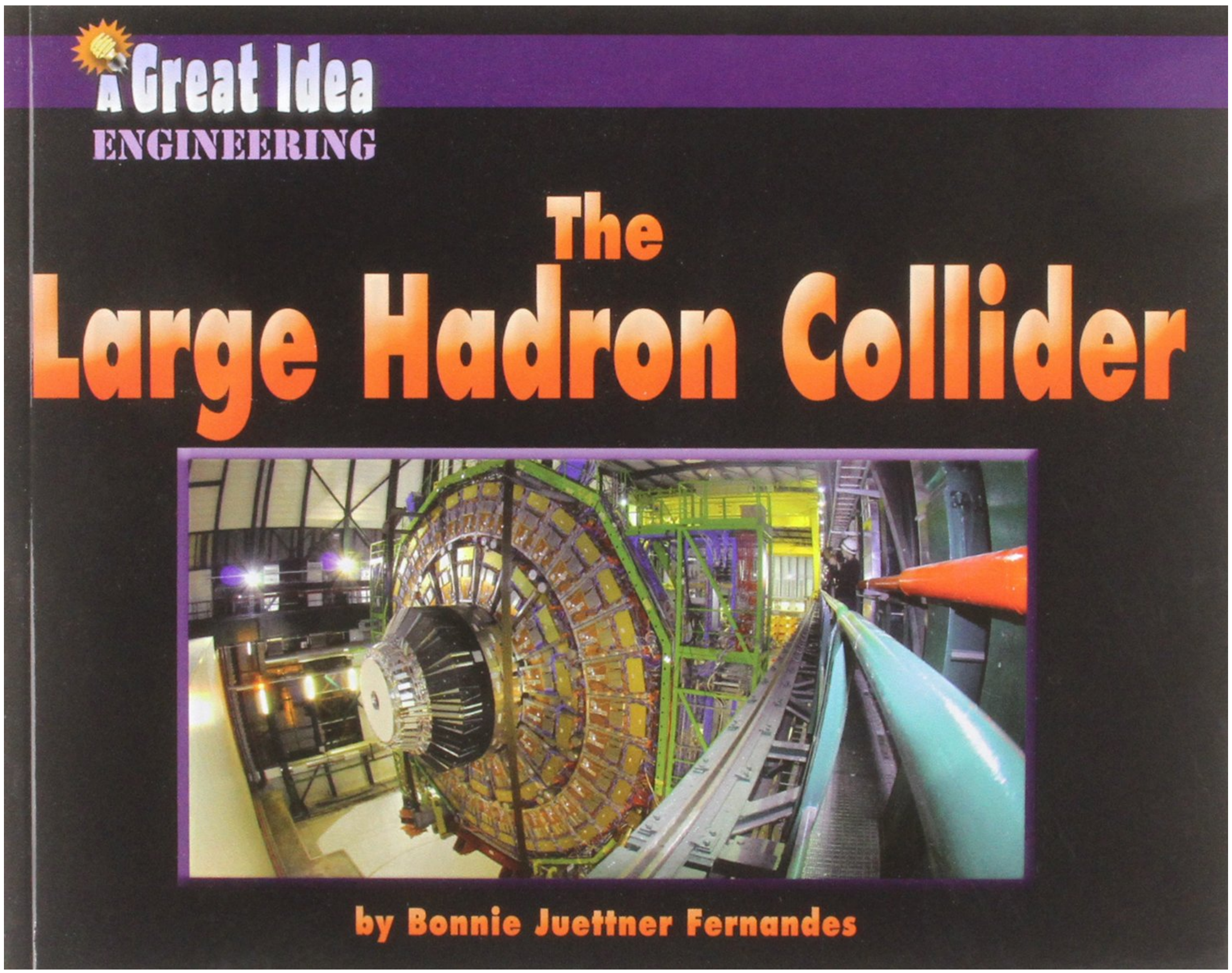
- Can a very energetic neutrino be "seen" via the final state radiation?
  (A ν-jet ?)

- What about through its interactions with detector materials ??
  (A ν-calorimeter?)



Asking the right questions

I was reminded of one such question a few years ago, when I was reading my kids a nice children's book on the LHC:



For elementary school age.
ISBN: 9781603575805

My then 7-year-old asked the following question that we still have no answer to today:

What is the Higgs made of?

(When I couldn't answer his question, this is what his face looked like: )

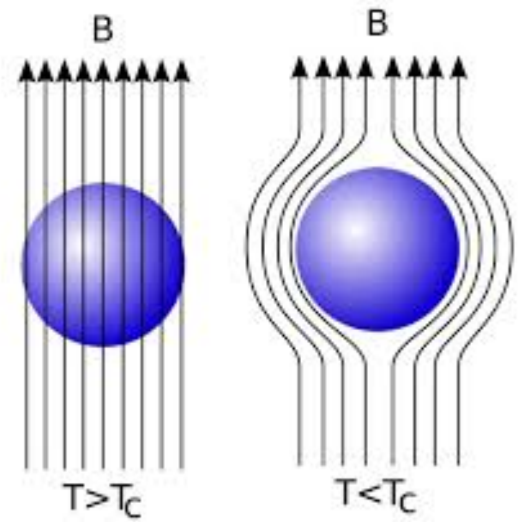
A physics Ph.D could rephrase the question in a slightly more sophisticated fashion:

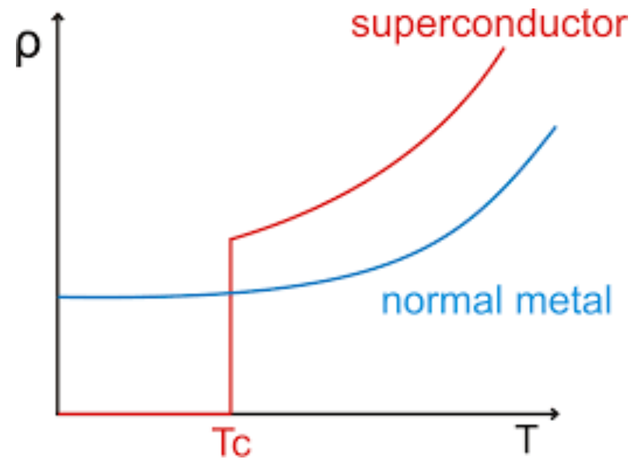
What is the microscopic theory that gives rise to the Higgs boson and its potential?

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4$$

Our colleagues in condensed matter physics are very used to asking, and studying, this kind of questions.

One of the most beautiful examples is the superconductivity discovered in 1911:





Ginzburg-Landau theory from 1950 offered a **macroscopic** (ie effective) theory for conventional superconductivity,

$$V(\Psi) = \alpha(T)|\Psi|^2 + \beta(T)|\Psi|^4 \qquad \alpha(T) \approx a^2(T - T_c) \qquad \text{and} \qquad \beta(T) \approx b^2$$

What is the **microscopic** origin of the Ginzburg-Landau potential for superconductivity?

In 1957 Bardeen, Cooper and Schrieffer provided the **microscopic** (fundamental) theory that allows one to

1) interpret $|\Psi|^2$ as the number density of Cooper pairs

2) calculate coefficients of $|\Psi|^2$ and $|\Psi|^4$ in the potential.

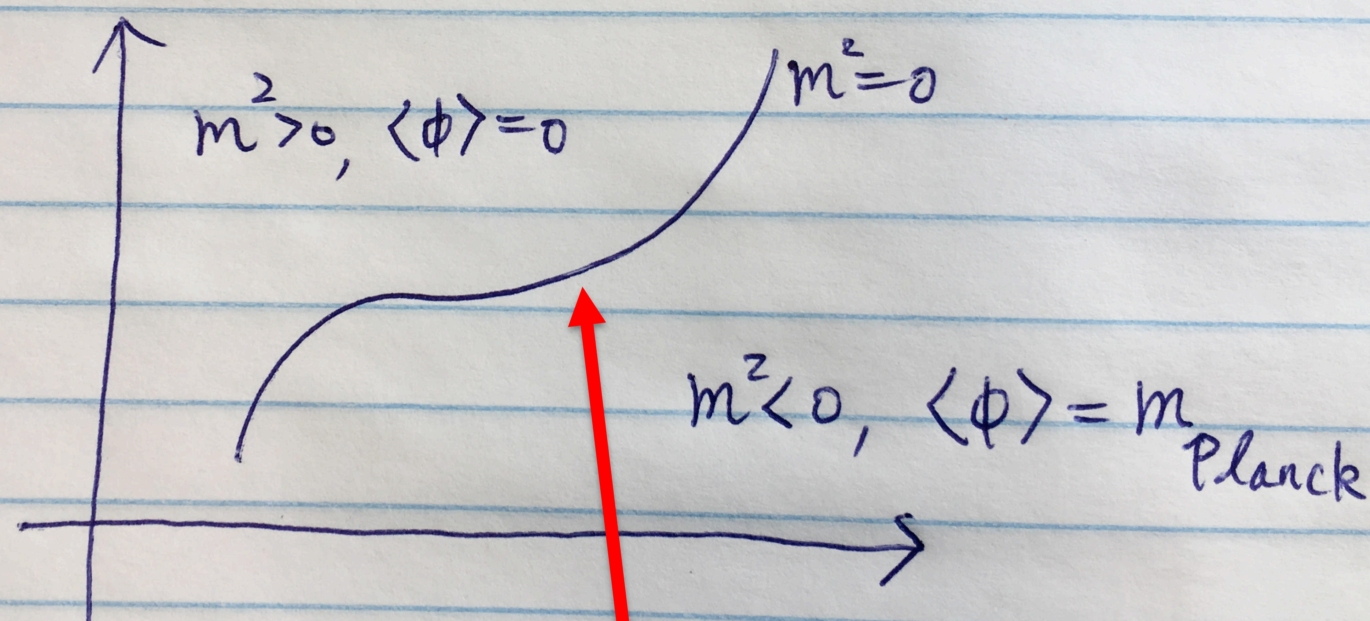
We do not have the corresponding **microscopic** theory for the Higgs boson.

In fact, we have NOT even measured the Ginzburg-Landau potential of the Higgs!

The question can be reformulated in terms of **Quantum Criticality**:

$$V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4$$

Quantum Phase Diagram of EWSB



Mh=125 GeV. We are sitting extremely close to the criticality. **WHY??**

One appealing possibility – the critical line is selected dynamically.

This is the analogy of BCS theory for electroweak symmetry breaking. It goes by the name of "technicolor," which is strongly disfavored experimentally.

Two popular "explanations:"

1. Postulate new global symmetries above the weak scale, and the Higgs boson arises as a (pseudo) Nambu-Goldstone boson.

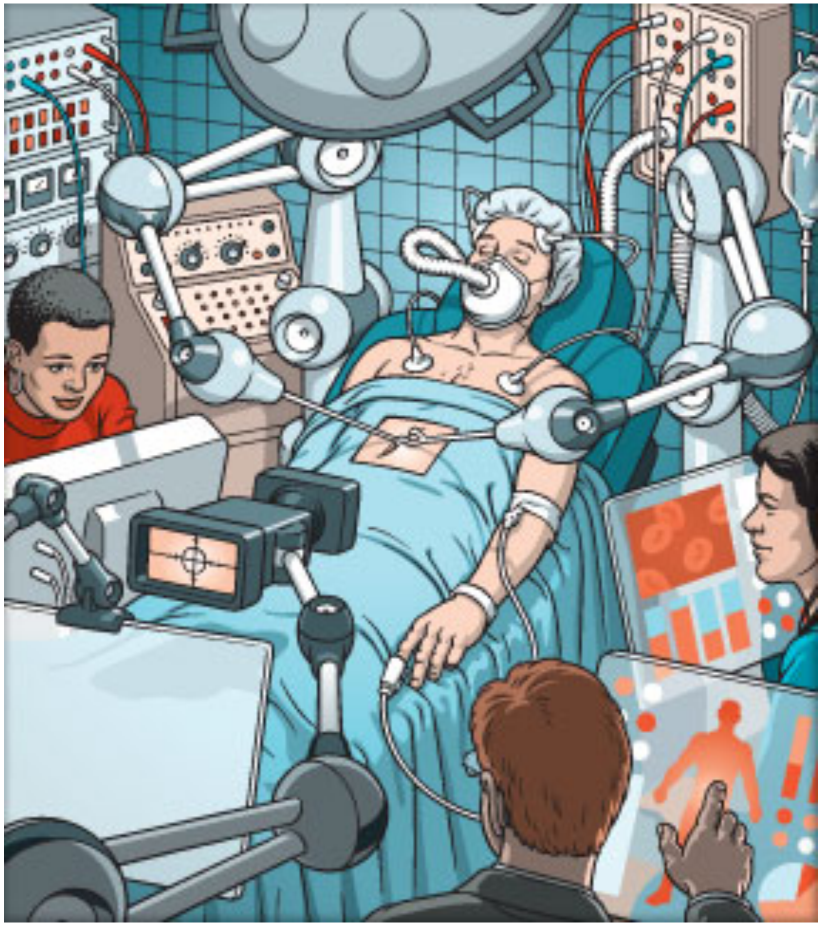
➔ This class goes by the name of "composite Higgs models."

2. The critical line is a locus of enhanced symmetry.

➔ This is the (broken) supersymmetry.

Supersymmetry v.s. Composite Higgs:

Neither of them is doing great --



Although that may be a difference of opinion…

We have not seen any signs of SUSY or CHM.

This only **deepens** the mystery, of why we are sitting close to the critical line of EWSB!

However, we do know that electroweak symmetry breaking more exotic than the BCS theory of superconductivity.

“The Universe is not a piece of crappy metal!”

by a prominent HEP theorist.

From this perspective, the Higgs boson is the most exotic state of quantum criticality.

Some people argued that there could be nothing because the SM by itself is UV-complete.

But this is a reasoning that has failed **many times** through out the course of the history:

- QED (photons+electrons) is a UV-complete theory. But physics didn’t stop there.
- QCD (gluons+quarks) is also a UV-complete theory. Again physics didn’t stop there.
- SM with one generation of fermion is UV-complete. “WHO ORDERED THAT?”

It is a somewhat embarrassing realization that, after 40 years, our understanding of the electroweak symmetry breaking is still at the level of Ginzburg-Landau level!

In order to understand the *microscopic* nature of the Higgs, we can measure:

- Deviations in H(125) coupling <u>structure</u>.
- Rare and new decay channels of H(125).
- Partners of the SM top quark that couple significantly to H(125).
- Additional Higgs bosons.

An important benchmark:

Simultaneous measurements on HVV and HHVV coupling structures allows to detect the presence of possible new symmetry in the Higgs sector.

If the Higgs is a composite particle like the pions (pNGB), there will be a nonlinear symmetry relating multi-Higgs self-interactions.

Such a nonlinear symmetry appears prominently in nuclear physics, relating the self-interactions of pions.

Let me elaborate –

Suppose the SM is just an effective description:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^n} \mathcal{O}_i^{(n-4)}$$

At the weak scale, the HVV and HHVV couplings deviate from their SM expectations, both in <u>coupling strength</u> and the <u>tensor structure</u>,

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu h \partial^\mu h + b_{nh} \left(\frac{h}{v}\right)^n \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right)$$

There are also operators carrying "four-derivative":

$$\frac{h}{v} V_{1\,\mu} \mathcal{D}^{\mu\nu} V_{2\,\nu} \,, \qquad \frac{h}{v} V_{1\,\mu\nu} V_2^{\mu\nu} \,, \qquad \mathcal{D}^{\mu\nu} = \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^2$$

$$\frac{h^2}{v^2} V_{1\,\mu} \mathcal{D}^{\mu\nu} V_{2\,\nu} \,, \qquad \frac{h^2}{v^2} V_{1\,\mu\nu} V_2^{\mu\nu} \,, \qquad \frac{\partial_\mu h \partial_\nu h}{v^2} V_1^\mu V_2^\nu$$

In a given BSM model, coefficients of these corrections can be calculated.

Generically, these coefficients are independent parameters depending on various masses and couplings in the UV model.

However, in composite Higgs models these anomalous HVV and HHVV couplings are controlled by only a small number of parameters

→ because there is a nonlinear symmetry relating the coefficients.

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu h \partial^\mu h + b_{nh}\left(\frac{h}{v}\right)^n \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right)$$

$$\begin{aligned} b_h &= 1 - 2\xi & b_{2h} &= 2\sqrt{1-\xi}\ , \\ b_{3h} &= -\frac{4}{3}\xi\sqrt{1-\xi}\ , & b_{4h} &= \frac{1}{3}\xi(2\xi-1)\ , \\ b_{5h} &= \frac{4}{15}\xi^2\sqrt{1-\xi}\ , & b_{6h} &= \frac{2}{45}\xi^2(1-2\xi)\ , \\ &\ldots & &\ldots \end{aligned}$$

This is in complete parallel to pions in low-energy QCD:

$$\mathscr{L} = -\frac{1}{2}\frac{\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}}{(1+\vec{\pi}^2/F^2)^2} \, . \qquad (19.5.18)$$

Weinberg QFT, Vol II

At the two-derivative level, everything is controlled by "F".

For composite Higgs models, the two-derivative Lagrangian can be written in a compact way:

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{g^2 f^2}{4}\sin^2(\theta + h/f)\left(W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu\right)$$

$$\sin^2\theta = \xi = \frac{v^2}{f^2}$$

In the unitary gauge, the "symmetry" that enforces this particular form is highly disguised and non-trivial.

One way to "detect" the presence of such nonlinear symmetry is to measure HVV and HHVV couplings to see if they are controlled by the same parameter:

$$\mathcal{L}_{\rm NL} = \sum_i \frac{m_W^2}{m_\rho^2}\left(C_i^h \mathcal{I}_i^h + C_i^{2h}\mathcal{I}_i^{2h} + C_i^{3V}\mathcal{I}_i^{3V}\right)$$

$\mathcal{I}_i^h$	$\mathcal{I}_i^{2h}$
(1) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$	(1) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$
(2) $\frac{h}{v} Z_{\mu\nu} Z^{\mu\nu}$	(2) $\frac{h^2}{v^2} Z_{\mu\nu} Z^{\mu\nu}$
(3) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$	(3) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$
(4) $\frac{h}{v} Z_{\mu\nu} A^{\mu\nu}$	(4) $\frac{h^2}{v^2} Z_{\mu\nu} A^{\mu\nu}$

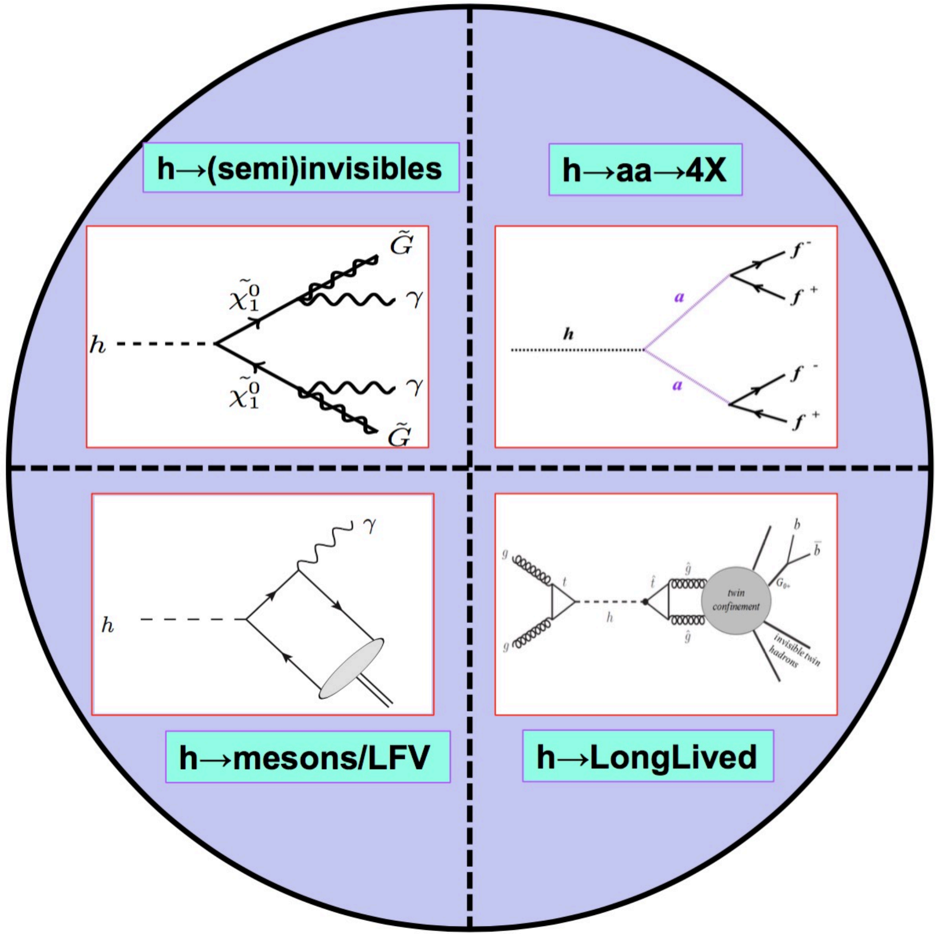
$$\frac{C_3^{2h}}{C_3^h} = \frac{C_4^{2h}}{C_4^h} = \frac{1}{2}\cos\theta = \frac{1}{2}\sqrt{1-\xi}$$

- This is an example of several "universal relations."
- There are also universal relations in aTGC.

Z. Yin, D. Liu and IL: 1805.00489; 1809.09126

→ **Opens up a new experimental frontier**

- Rare and new decay channels of H(125), a.k.a. “Exotic Higgs decays”, are getting more attention lately.



There are about 10 million Higgs bosons produced at a high energy muon collider. We need a careful study to understand the reach of these searches.

There are several broad categories:

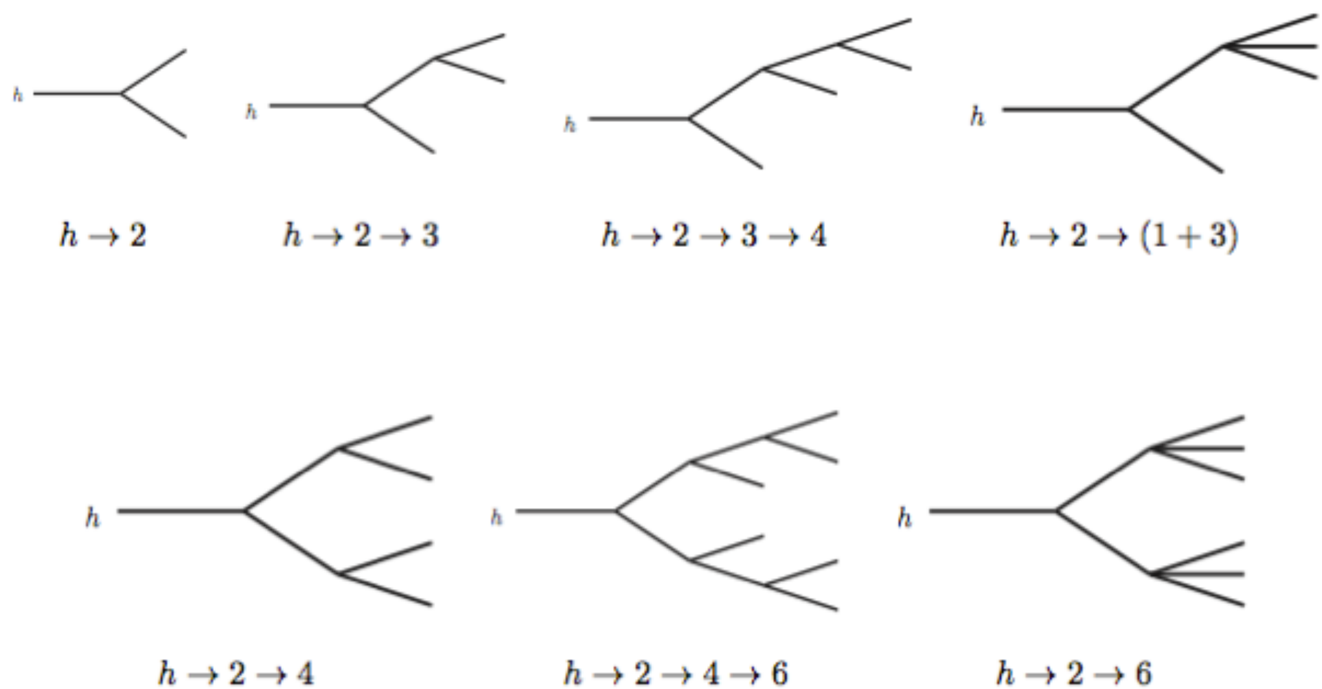
- Rare mesonic exclusive and flavor-violating decays:
  - Providing a unique window into the H(125) couplings to light quark flavors.
  - Testing the “flavor symmetry” of the SM lagrangian.

- New particles in the decay of H(125):
  - New intermediate particles into SM final states.
  - New “invisible particles” in the decays of H(125).
  - New long-lived particles in the decay.

Mass of the Higgs is only 125 GeV, searches often face experimental challenges in triggering, detector response, MC simulations of signal samples, and etc.

→ Nice playground for theorists and experimentalists alike!

For example, theorists have proposed a comprehensive list of exotic Higgs decay signatures:



The exotic Higgs decay topologies we consider. Intermediate lines represent an *on-shell, neutral* particle, which is either a Z-boson or a BSM particle.

See 1312.4992

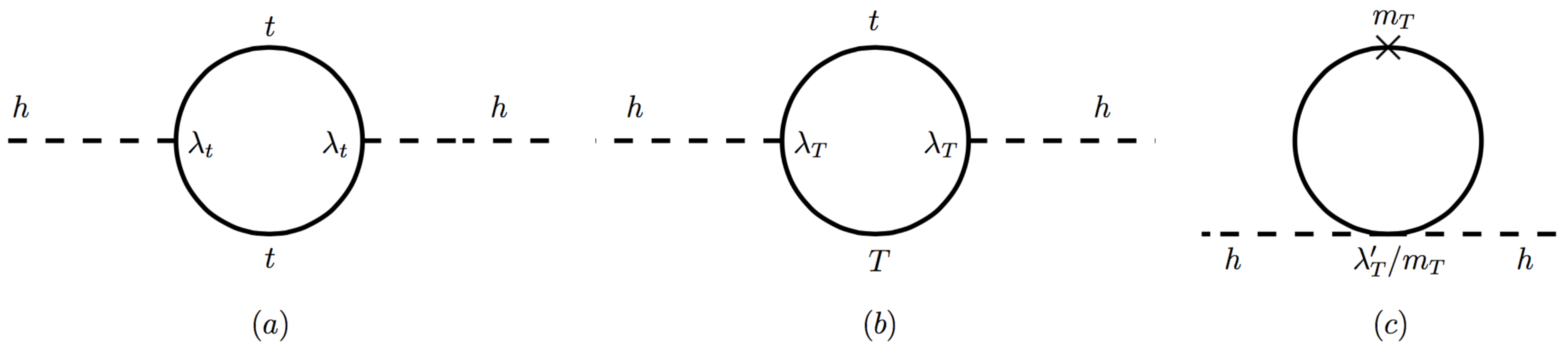
- Top partners can be either spin-0 in supersymmetry (the top squark) or spin-1/2 in composite Higgs models (the vector-like quark).

Their existence provides a "microscopic origin" for the special "minus sign" in the Higgs potential:

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4$$

This sign could be generated by top partners at the loop-level through the celebrated Coleman-Weinberg mechanism.

In addition, the top partners are also responsible for cancelling the top quadratic divergences in the Higgs mass-squared:



The Naturalness relation: $\lambda'_T = \lambda_t^2 + \lambda_T^2$

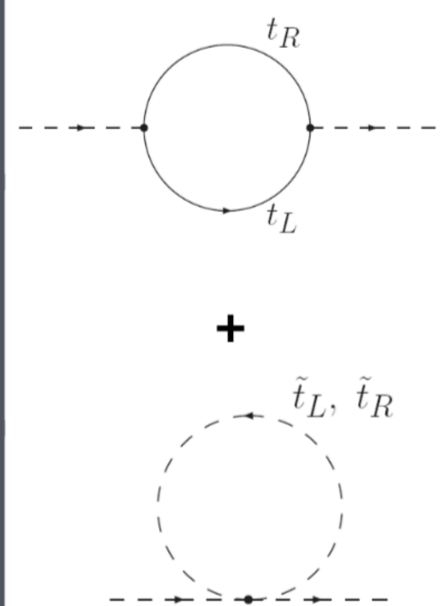
They must have a significant coupling to the Higgs, but they are not necessarily colored!

The uncolored top partners (neutral naturalness) present special challenge for its discovery.

	Scalar Top Partner	Fermionic Top Partner
Colored Top Partner	Supersymmetry	Compositeness/Warped Extra Dimensions
EW-charged Top Partner	Folded SUSY [Chacko, Harnik, Goh, Burdman]	Quirky Little Higgs [Cai, Cheng, Terning]
Neutral Top Partner	Hyperbolic Higgs [Cohen, Craig, Giudice, McCullough] Tripled Top [Cheng, Li, Salvioni, Verhaaren]	Twin Higgs [Chacko, Harnik, Goh] [Barbieri, Gregoire, Hall]

Table from B. Batell

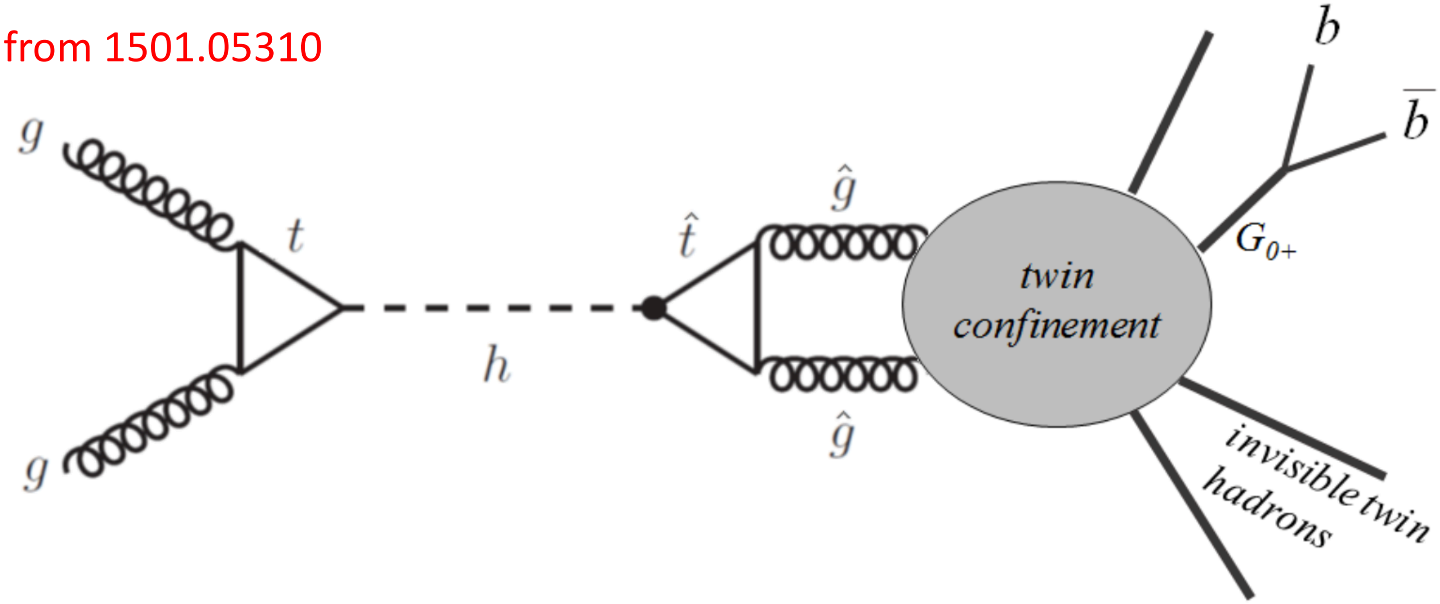
[Table inspiration from Curtin, Verhaaren]



Slides by A. Martin

However, one might be able to infer neutral naturalness from exotic Higgs decays:

Figure from 1501.05310



This is the most salient feature common to popular models explaining the naturalness problem:

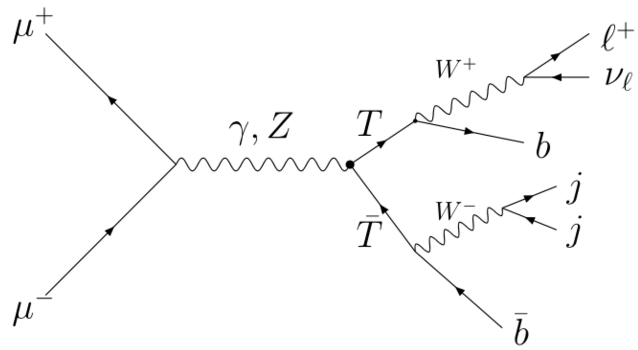
The existence of the symmetry-partner of the top!

Their presence often modifies the top Yukawa coupling.

Three routes to measuring naturalness:

- Direct searches of the colored top partner.
- Indirect searches of the uncolored top partner through exotic decays of the 125 GeV Higgs.
- Precise measurements of the top Yukawa coupling. (See earlier slide.)

A preliminary study on vector-like top partners at the muon collider:



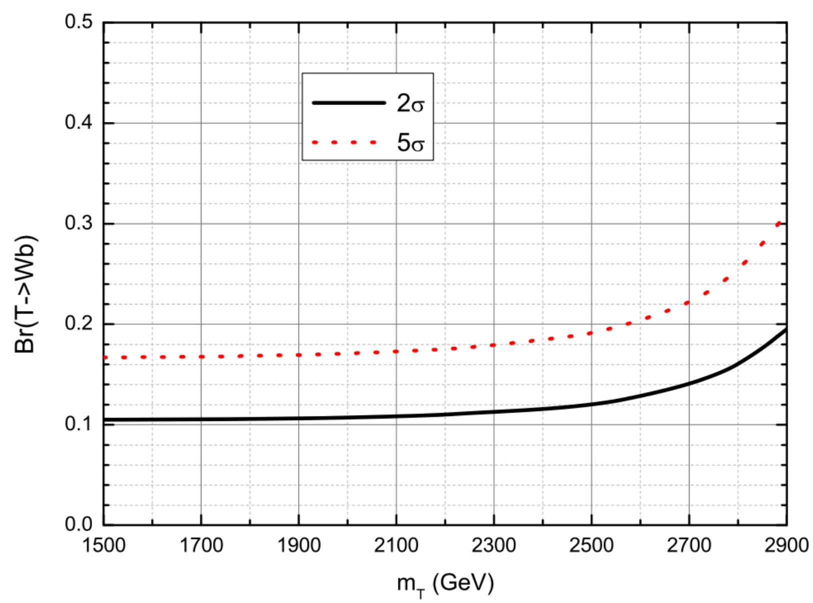
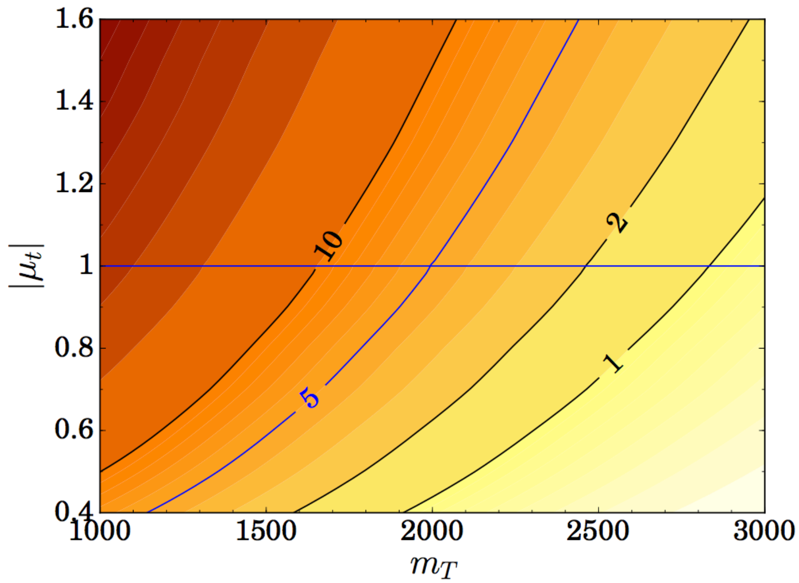


Fig. 5. 2σ exclusion limit and 5σ discovery prospects contour plots for the signal in $(Br(T \to Wb) - m_T)$ planes at muon collider with $\sqrt{s} = 6$ TeV and $\mathcal{L} = 3600$ fb^{-1}.

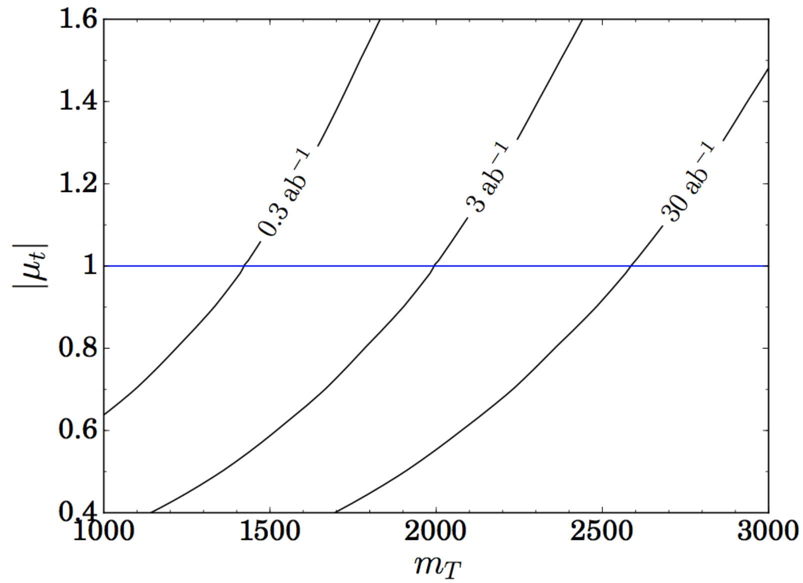
Lv, Cui, Li, Liu: NPB 985 (2022) 116016

In this aspect, the ambition should not stop at discovering a top partner. We need to also test the "naturalness relation" in order to detect the presence of a new symmetry in the top sector. This has been studied at a 100 TeV pp collider:

C.-R. Chen, T. Liu, J. Hajer, IL and H. Zhang: 1705.07743



(a) Luminosity of $3\,\mathrm{ab}^{-1}$.



(b) Significance of 5σ.

Figure 3: Discovery reach defined as $Z(b|b+s)$ for pair production of top partners in association with one Higgs boson at 100 TeV. We present the reaches for a fixed luminosity of $3\,\mathrm{ab}^{-1}$ in Figure (a) and for a fixed significance of 5σ with luminosities of 0.3, 3, $30\,\mathrm{ab}^{-1}$ in Figure (b).

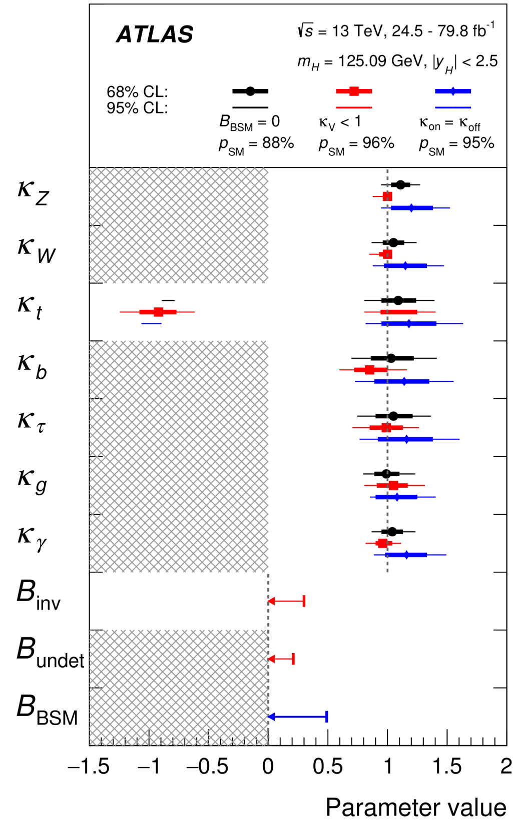
Need to pursue a similar program for muon collider!

Where are the additional Higgs bosons?

Recall the generic expectation on the possible deviation in the signal strength of h125:

$$\mathcal{O}\left(\frac{v^2}{M_{\rm new}^2}\right) \sim 5\% \times \left(\frac{1\ \mathrm{TeV}}{\Lambda}\right)^2$$

We have measured a SM-like 125 GeV Higgs.



What does H(125) tell us about the additional Higgs bosons??

- New Higgs bosons are heavy >~ 500 GeV by decoupling.
- Alignment without decoupling → a somewhat light Higgs is still possible.

In fact, it was pointed out more than 10 years ago that, there could be a SM-like Higgs without "heavy" non-SM scalars:

Gunion and Haber, hep-ph/0207010

V. A SM-LIKE HIGGS BOSON WITHOUT DECOUPLING

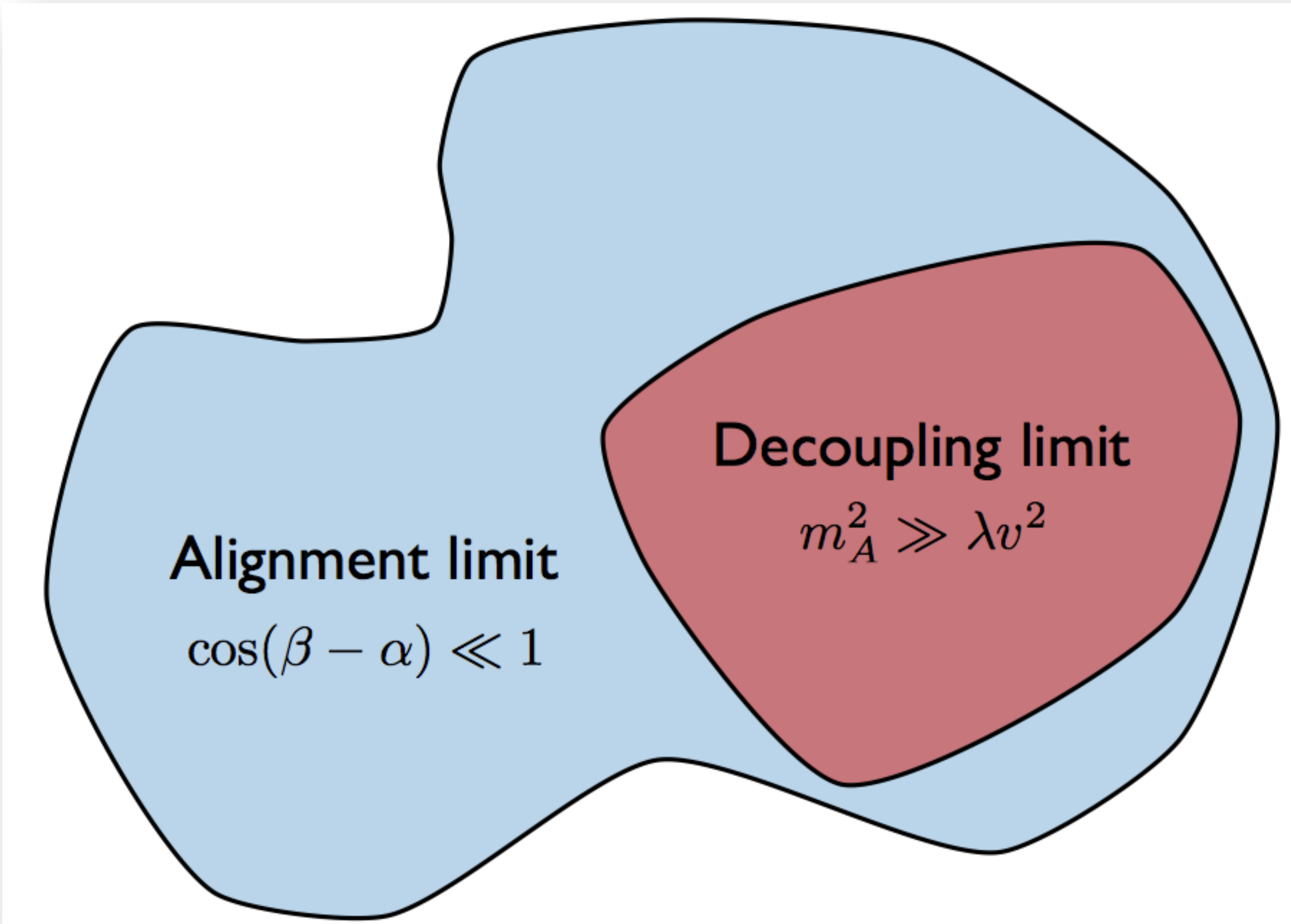
We have demonstrated above that the decoupling limit (where $m_A^2 \gg |\lambda_i|v^2$) implies that $|c_{\beta-\alpha}| \ll 1$. However, the $|c_{\beta-\alpha}| \ll 1$ limit is more general than the decoupling limit. From eq. (36), one learns that $|c_{\beta-\alpha}| \ll 1$ implies that either (i) $m_A^2 \gg \lambda_A v^2$, and/or (ii) $|\widehat{\lambda}| \ll 1$ subject to the condition specified by eq. (33). Case (i) is the decoupling limit described in

"Alignment without decoupling" was (re)discovered by two groups:

- MSSM augmented by a triplet scalar in 1303.0800 by Delgado, Nardini and Quiros.
- Studies on the parameter space of general THDMs by Craig, Galloway and Thomas in 1305.2424.

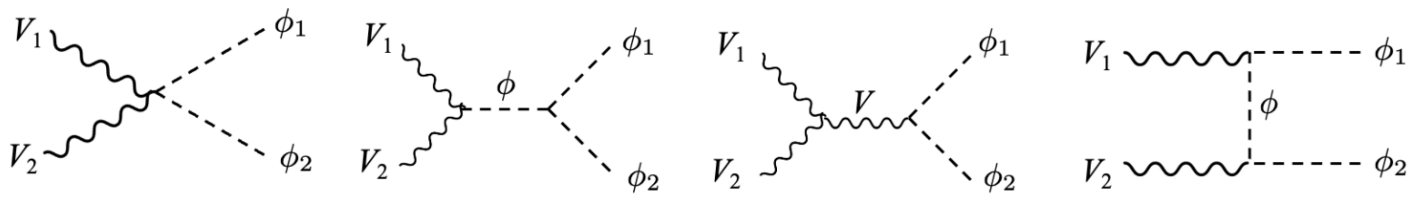
See also Carena, IL, Shah, Wagner: 1310.2248; Carena, Haber, IL, Shah and Wagner: 1410.4969

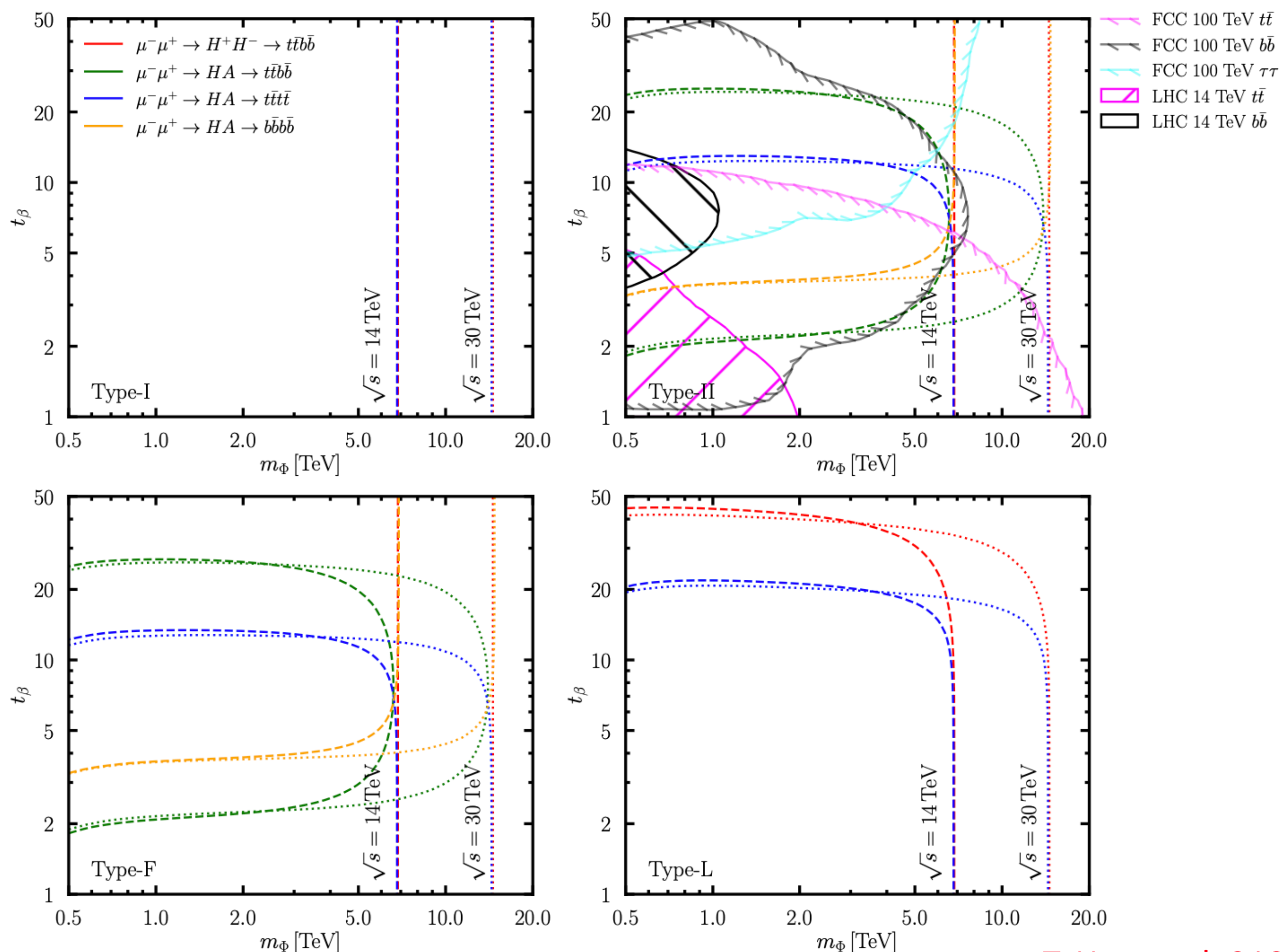
A SM-like Higgs does NOT imply new degrees of freedom are heavy:



Art work by N. Craig

At a high energy muon collider, the production goes through VBF topology:

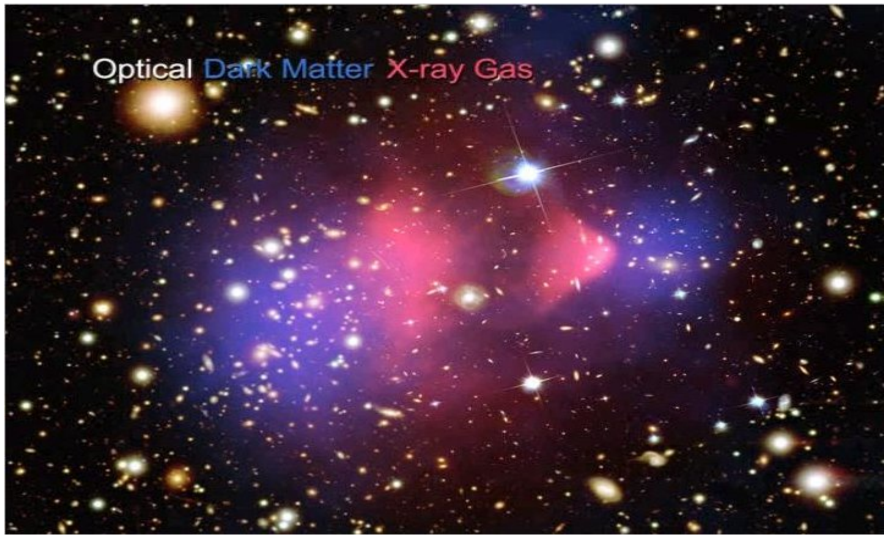


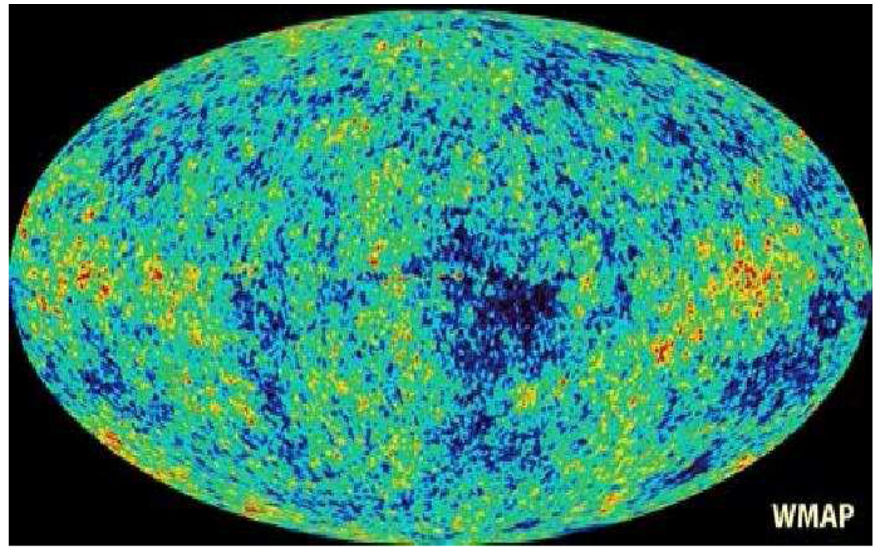


T. Han et al: 2102.08386

Some excellent empirical questions SM cannot answer:

- Dark matter/Dark sector:





We (most people) are convinced about the existence of dark matter. What is it??

In principle, a high energy collider could produce dark matter particles with mass around E_{CM} /2.

For the simplest WIMP scenarios, the thermal target is well above 1 TeV:

Thermal targets

Model (color, n, Y)		Therm. target
(1,2,1/2)	Dirac	1.1 TeV
(1,3,0)	Majorana	2.8 TeV
$(1,3,\epsilon)$	Dirac	2.0 TeV
(1,5,0)	Majorana	14 TeV
$(1,5,\epsilon)$	Dirac	6.6 TeV
(1,7,0)	Majorana	48.8 TeV
$(1,7,\epsilon)$	Dirac	16 TeV

Correct relic abundance
$\Rightarrow$ Thermal targets

Reach up to thermal target
$\approx$
complete coverage for WIMP candidate

Mitridate, Redi, Smirnov, Strumia, 1702.01141

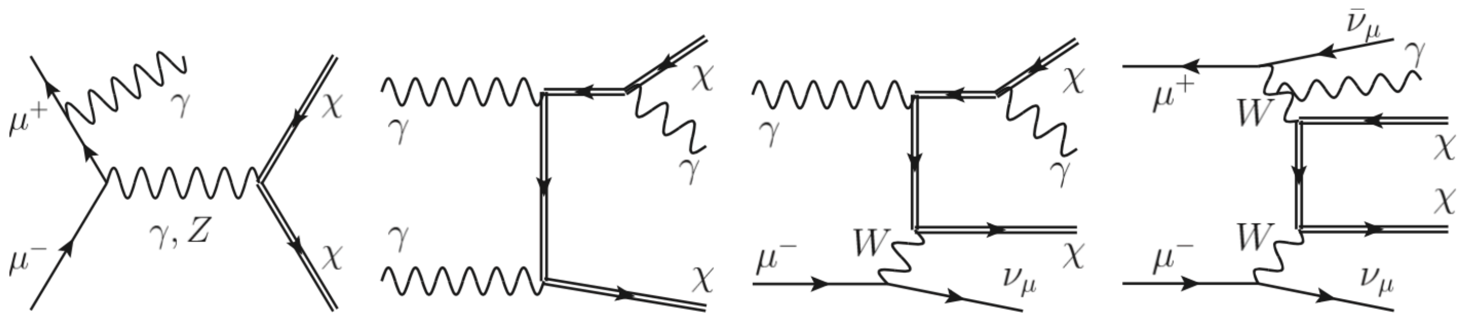
S. Bottaro, D. Buttazzo, M. Costa, R. Franceschini, P. Panci, D. Redigolo, L. Vittorio, 2107.09688

Slide from L.-T. Wang

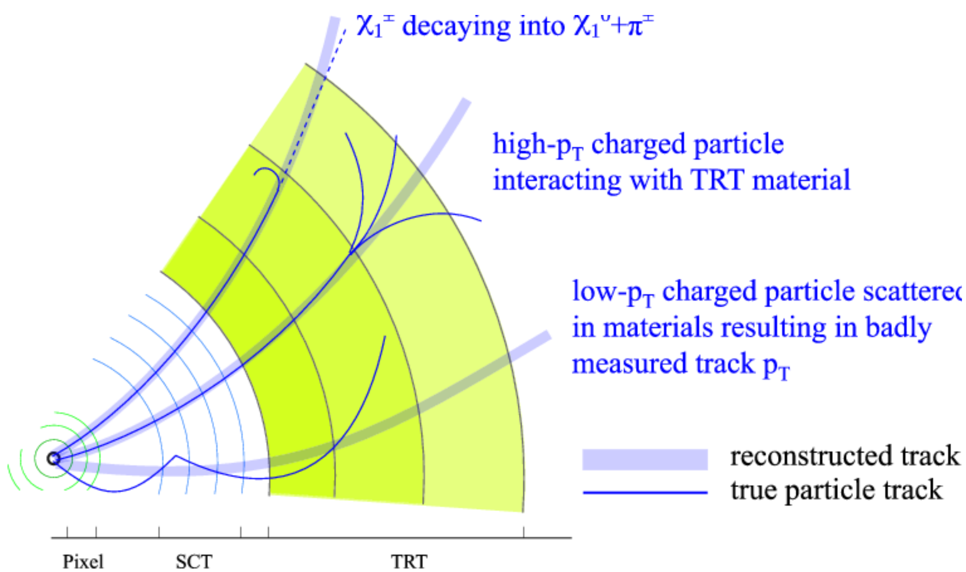
There are two classes of signatures at a Muon collider:

- Mono-X signatures

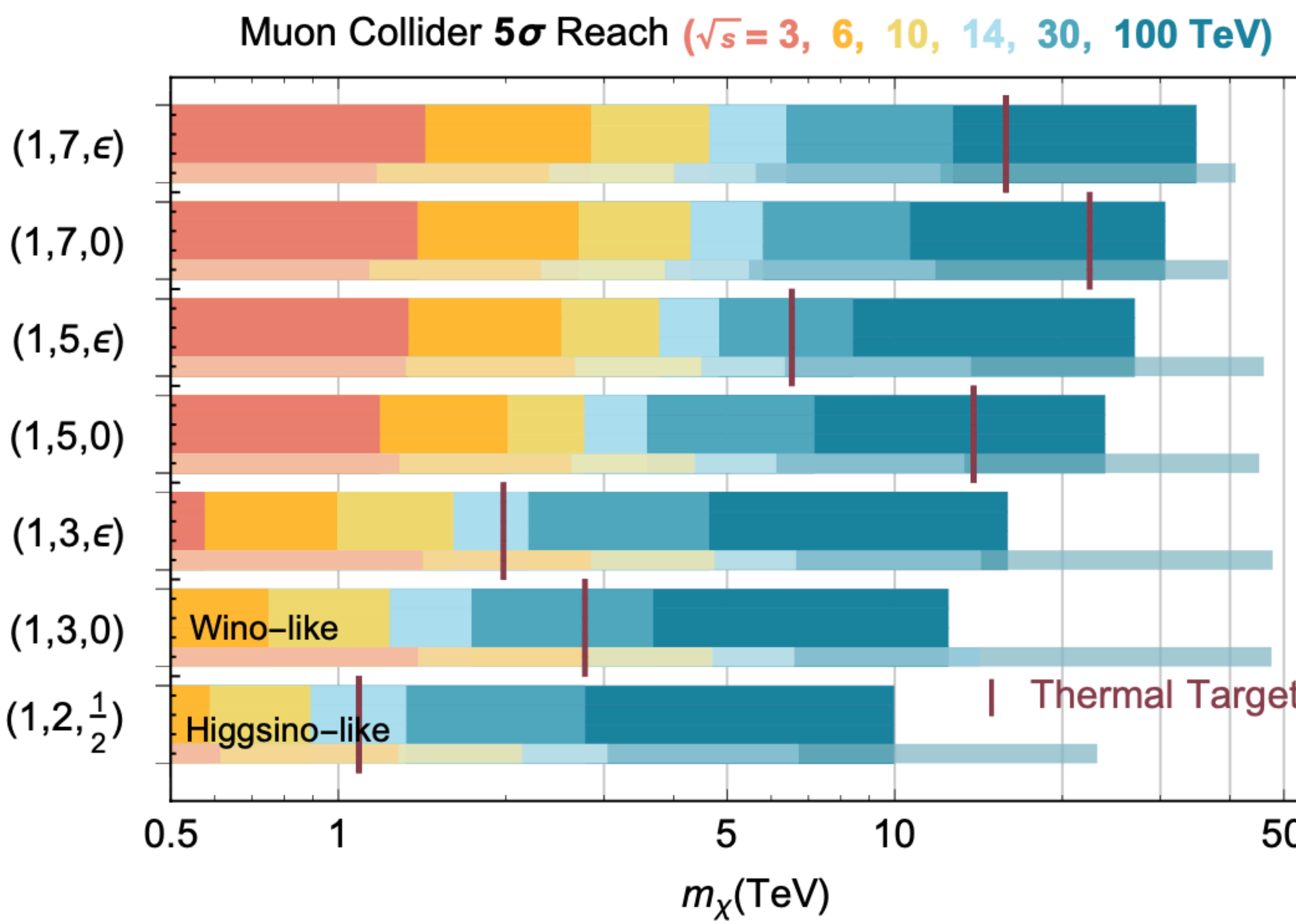
T. Han, Z. Liu, L.-T. Wang, X. Wang: 2009.11287



- Disappearing tracks:

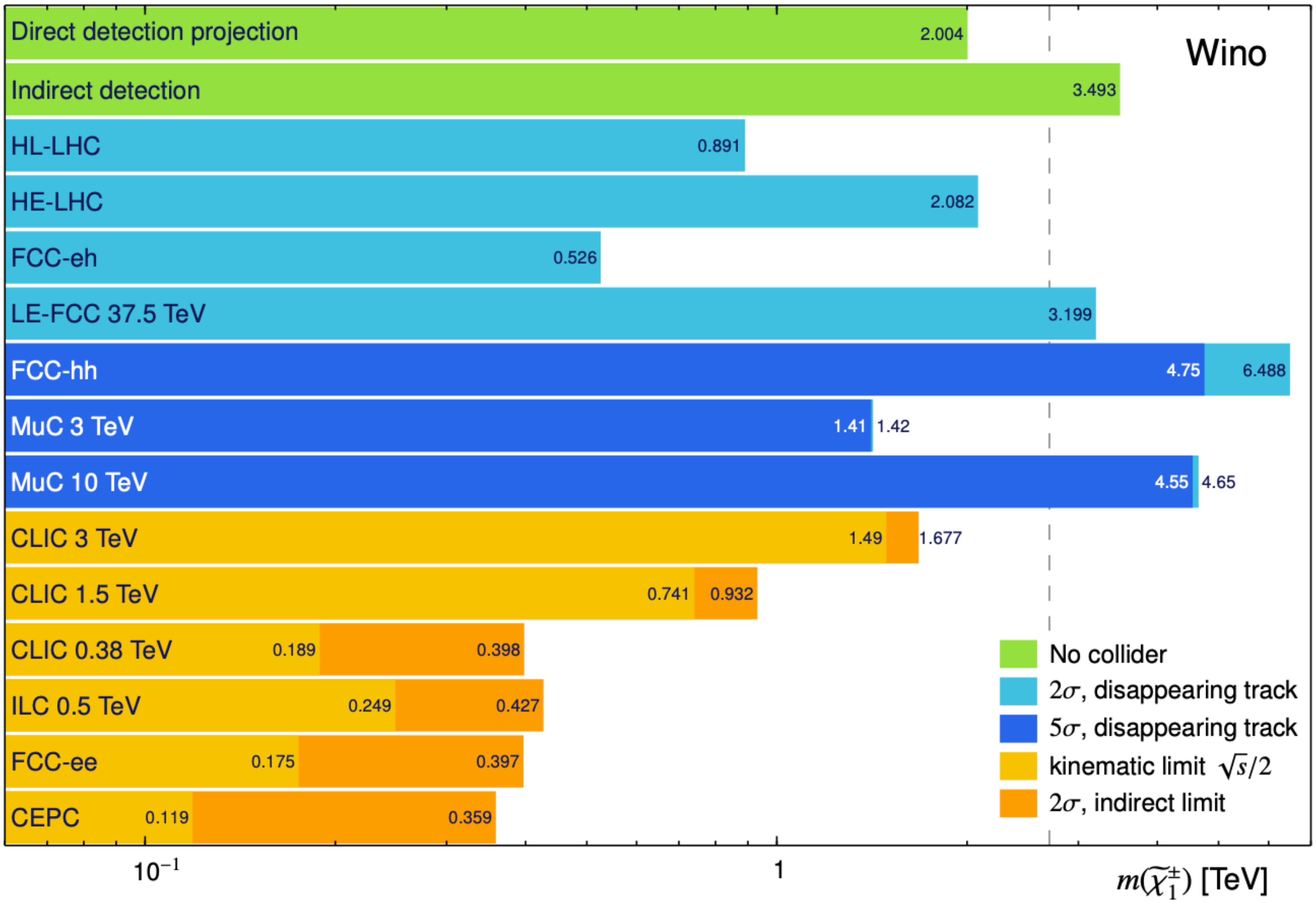


Preliminary study of searching for "minimal" WIMPs:



T. Han, Z. Liu, L.-T. Wang, X. Wang: 2009.11287

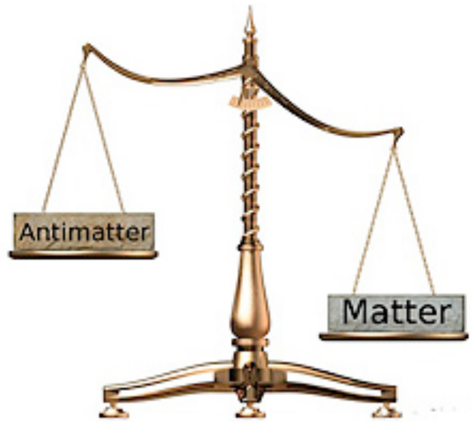
Dedicated study using disappearing tracks on Wino/Bino:



R. Capdevilla, F. Meloni, R. Simoniello, J. Zurita: 2102.11292

Some excellent empirical questions SM cannot answer:

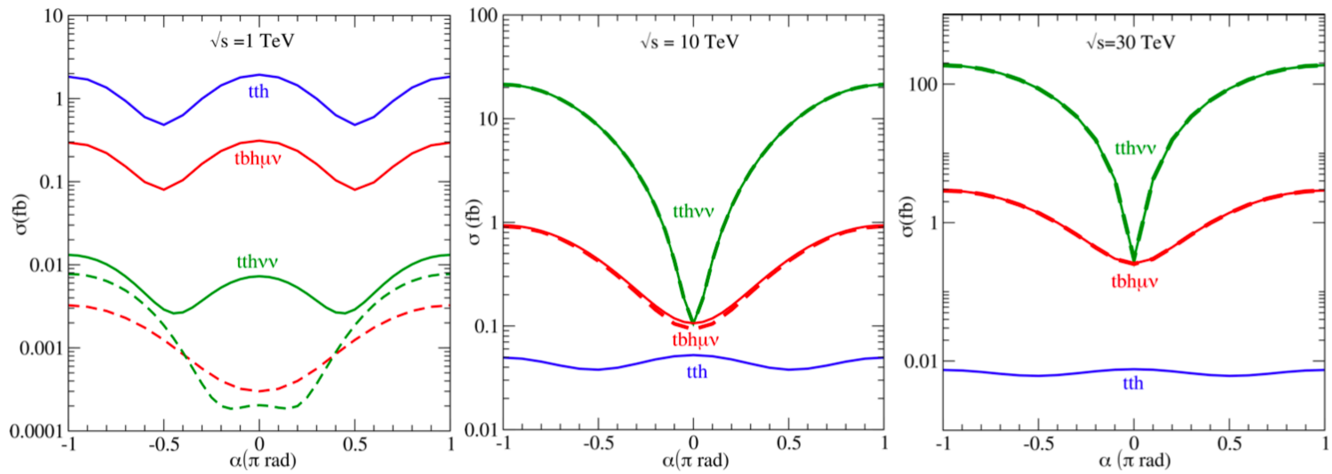
- CP-violation and baryon asymmetry.



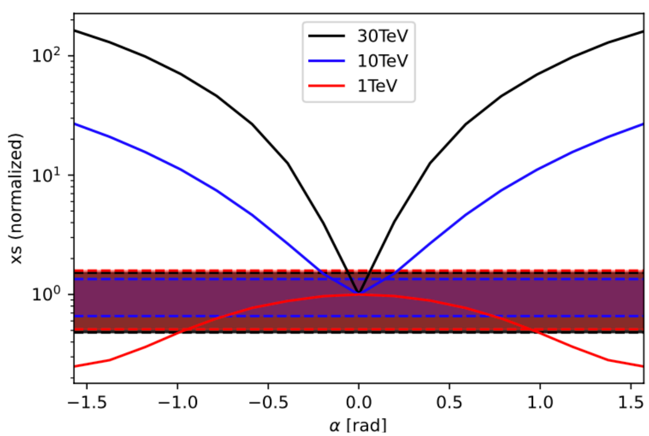
New sources of CP-violation in the Higgs couplings? One example is the top Yukawa coupling:

$$\mathcal{L} \supset -\frac{m_t}{v}\kappa_t \bar{t}\left(\cos\alpha + i\gamma_5 \sin\alpha\right) t\, h$$

The production x-sections depend on the CP-phase alpha:



ions for $\mu^+\mu^- \to t\bar{t}h$ (blue), $t\bar{t}h\nu\bar{\nu}$ (green), and $tbh\mu\nu$ (red) at 1 TeV (left), 10 TeV
V muon colliders (right) with CP violating phase α from $-\pi$ to π. For $tbh\mu\nu$ a cut of
een applied. Dashed lines are for the VBF-like contributions.



Barman et al: 2203.08127

FIG. 8: 2σ exclusion on α at (red) 1 TeV, (blue) 10 TeV and (black) 30 TeV muon colliders with luminosities of 100 fb^{-1}, 10 ab^{-1}, and 10 ab^{-1}, respectively. The solid lines show the combined signal cross-section before cuts normalized to the SM prediction. The horizontal lines represent the projected bounds on the cross-section normalized to the SM production cross-section for each energy. These bounds are statistical only.

A related question is the pattern of fermion masses and mixings -- the flavor symmetry:

$\mu^+\mu^- \to \mu\,\tau$	$\mu^+\mu^- \to \mu\,\tau\,\nu_\mu\,\nu_\tau$	$\mu^+\mu^- \to \tau^+\tau^-$

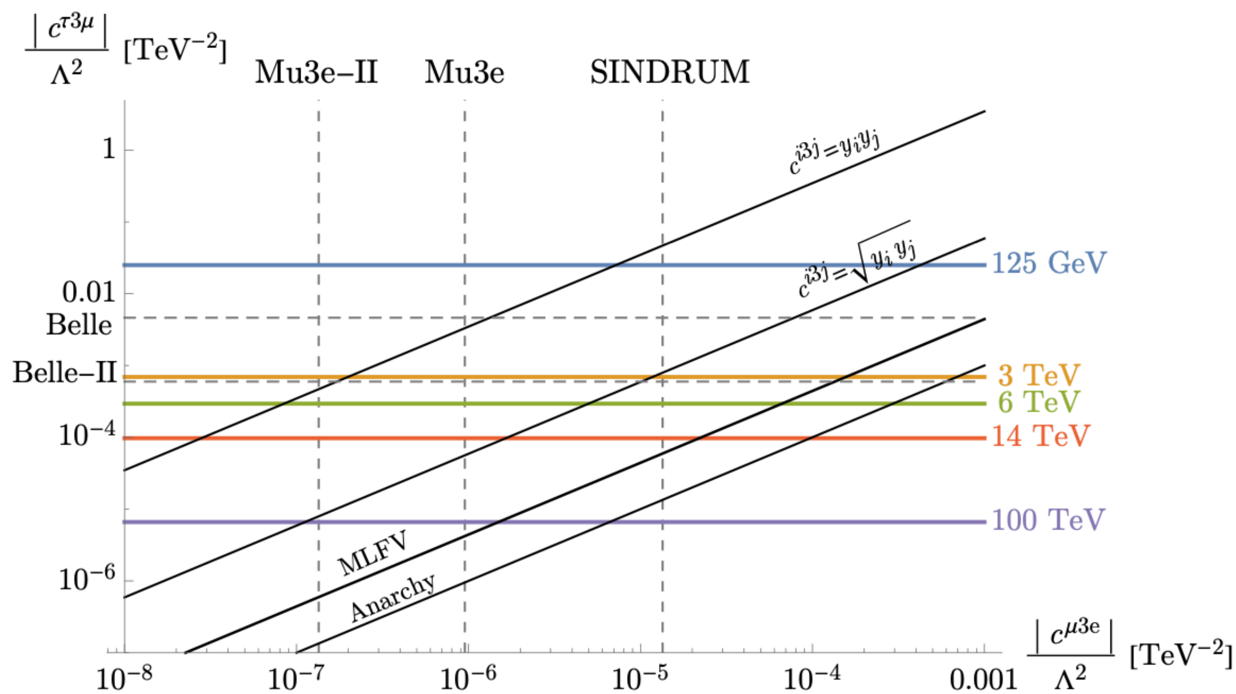


Figure 22: Summary of muon collider and precision constraints on flavor-violating 3-body decays. The colored horizontal lines show the sensitivity to the $\tau 3\mu$ operator at various energies, all assuming 1 ab^{-1} of data. The dashed horizontal (vertical) lines show the current or expected sensitivity from $\tau \to 3\mu$ ($\mu \to 3e$) decays for comparison. The diagonal black lines show the expected relationship between different Wilson coefficients with various ansatz for the scaling of the flavor-violating operators (e.g., "Anarchy" assumes that all Wilson coefficients are $\mathcal{O}(1)$).

Muon Smasher's Guide: 2103.14043

What is the physics case for Muon collider?

Standard Model is our no-lose theorem!!