Status of Neutrino Oscillation Phenomenology

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Interdisciplinary Developments in Neutrino Physics

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March 28, 2022
NEUTRINOS HAVE MASS
[albeit very tiny ones...]

So What?
NEUTRINOS HAVE MASS
[albeit very tiny ones...]

So What?

NEW PHYSICS
Nonzero neutrino masses imply the existence of new fundamental fields \( \Rightarrow \) New Particles

We know nothing about these new particles. They can be bosons or fermions, very light or very heavy, they can be charged or neutral, experimentally accessible or hopelessly out of reach...

There is only a handful of questions the standard model for particle physics cannot explain (these are personal. Feel free to complain).

- What is the physics behind electroweak symmetry breaking? (Higgs \( \checkmark \)).
- What is the dark matter? (not in SM).
- Why is there so much ordinary matter in the Universe? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM).
What is the New Standard Model? [$\nu$SM]

The short answer is – WE DON’T KNOW. Not enough available info!

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the $\nu$SM candidates can do. [are they falsifiable?, are they “simple”? , do they address other outstanding problems in physics?, etc]

We need more experimental input.
Piecing the Neutrino Mass Puzzle

Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique \textit{theoretical} and \textit{experimental} efforts . . .

- understanding the fate of lepton-number. Neutrinoless double-beta decay.
- A comprehensive long baseline neutrino program. (On-going T2K, NO\(\nu\)A, etc. DUNE and HyperK next steps towards the ultimate “superbeam” experiment.)
- Different baselines and detector technologies a must for both over-constraining the system and looking for new phenomena.
- Probes of neutrino properties, including neutrino scattering experiments. And what are the neutrino masses anyway? Kinematical probes.
- Precision measurements of charged-lepton properties \((g - 2, \text{edm})\) and searches for rare processes \((\mu \rightarrow e\text{-conversion the best bet at the moment}).
- Collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- Neutrino properties affect, in a significant way, the history of the universe. These can be “seen” in cosmic surveys of all types.
- Astrophysical Neutrinos – Supernovae and other Galaxy-shattering phenomena.
HOWEVER...  

We have only ever objectively “seen” neutrino masses in long-baseline oscillation experiments. It is one unambiguous way forward!

Does this mean we will reveal the origin of neutrino masses with oscillation experiments? We don’t know, and we won’t know until we try!

The race is not always to the swift, nor the battle to the strong, but that's the way to bet.

— Damon Runyon —
Three Flavor Mixing Hypothesis Fits All* Data Really Well.

<table>
<thead>
<tr>
<th></th>
<th>Normal Ordering (best fit)</th>
<th>Inverted Ordering ($\Delta\chi^2 = 2.6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bfp ±1σ</td>
<td>3σ range</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.304$^{+0.013}_{-0.012}$</td>
<td>0.269 → 0.343</td>
</tr>
<tr>
<td>$\theta_{12} / ^\circ$</td>
<td>33.44$^{+0.77}_{-0.74}$</td>
<td>31.27 → 35.86</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.573$^{+0.018}_{-0.023}$</td>
<td>0.405 → 0.620</td>
</tr>
<tr>
<td>$\theta_{23} / ^\circ$</td>
<td>49.2$^{+1.0}_{-1.3}$</td>
<td>39.5 → 52.0</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.02220$^{+0.00068}_{-0.00062}$</td>
<td>0.02034 → 0.02430</td>
</tr>
<tr>
<td>$\theta_{13} / ^\circ$</td>
<td>8.57$^{+0.13}_{-0.12}$</td>
<td>8.20 → 8.97</td>
</tr>
<tr>
<td>$\delta_{CP} / ^\circ$</td>
<td>194$^{+52}_{-25}$</td>
<td>105 → 405</td>
</tr>
<tr>
<td>$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$</td>
<td>7.42$^{+0.21}_{-0.20}$</td>
<td>6.82 → 8.04</td>
</tr>
<tr>
<td>$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$</td>
<td>+2.515$^{+0.028}_{-0.028}$</td>
<td>+2.431 → +2.599</td>
</tr>
</tbody>
</table>

*Modulo short-baseline anomalies.

http://www.nu-fit.org
Missing Oscillation Parameters: Are We There Yet? \textbf{(NO!)}

- What is the $\nu_e$ component of $\nu_3$? \((\theta_{13} \neq 0!\)\)
- Is CP-invariance violated in neutrino oscillations? \((\delta \neq 0, \pi?)\)
- Is $\nu_3$ mostly $\nu_\mu$ or $\nu_\tau$? \((\theta_{23} > \pi/4, \theta_{23} < \pi/4, \text{ or } \theta_{23} = \pi/4?\)\)
- What is the neutrino mass hierarchy? \((\Delta m_{13}^2 > 0?)\)

\implies\text{ All of the above can “only” be addressed with new neutrino oscillation experiments}

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)
What we ultimately want to achieve:

We need to do this in the lepton sector!
\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

What we have **really measured** (very roughly):

- Two mass-squared differences, at several percent level – many probes;
- \( |U_{e2}|^2 \) – solar data;
- \( |U_{\mu2}|^2 + |U_{\tau2}|^2 \) – solar data;
- \( |U_{e2}|^2 |U_{e1}|^2 \) – KamLAND;
- \( |U_{\mu3}|^2 (1 - |U_{\mu3}|^2) \) – atmospheric data, K2K, MINOS;
- \( |U_{e3}|^2 (1 - |U_{e3}|^2) \) – Double Chooz, Daya Bay, RENO;
- \( |U_{e3}|^2 |U_{\mu3}|^2 \) (upper bound \( \rightarrow \) evidence) – MINOS, T2K.

We still have a ways to go!
A little more quantitative:

<table>
<thead>
<tr>
<th>Unitarity Triangle Closures</th>
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</table>
| \( |U_{\alpha_1}U_{\beta_1}^* + U_{\alpha_2}U_{\beta_2}^* + U_{\alpha_3}U_{\beta_3}^*| \) or \( |U_{\alpha_1}U_{\epsilon_1}^* + U_{\mu_2}U_{\mu_3}^* + U_{\tau_1}U_{\tau_2}^*| \)

<table>
<thead>
<tr>
<th>Normalisations</th>
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<tbody>
<tr>
<td>( \Delta \chi^2 )</td>
</tr>
</tbody>
</table>

\( \alpha, \beta = e, \mu \)
\( \alpha, \beta = e, \tau \)
\( \alpha, \beta = \mu, \tau \)

\( i,j = 1,2 \)
\( i,j = 1,3 \)
\( i,j = 2,3 \)

Golden Opportunity to Understand Matter versus Antimatter?

The SM with massive Majorana neutrinos accommodates five irreducible CP-invariance violating phases.

- One is the phase in the CKM phase. We have measured it, it is large, and we don’t understand its value. At all.

- One is $\theta_{QCD}$ term ($\theta G\tilde{G}$). We don’t know its value but it is only constrained to be very small. We don’t know why (there are some good ideas, however).

- Three are in the neutrino sector. One can be measured via neutrino oscillations. 50% increase on the amount of information.

We don’t know much about CP-invariance violation. Is it really fair to presume that CP-invariance is generically violated in the neutrino sector solely based on the fact that it is violated in the quark sector? Why?

Cautionary tale: “Mixing angles are small.”
What Could We Run Into?

since $m_\nu \neq 0$ and leptons mix ...
What Could We Run Into?

- New neutrino states. In this case, the $3 \times 3$ mixing matrix would not be unitary.

- New short-range neutrino interactions. These lead to, for example, new matter effects. If we don’t take these into account, there is no reason for the three flavor paradigm to “close.”

- New, unexpected neutrino properties. Do they have nonzero magnetic moments? Do they decay? The answer is ‘yes’ to both, but nature might deviate dramatically from $\nu$SM expectations.

- Weird stuff. CPT-violation. Decoherence effects (aka “violations of Quantum Mechanics.”)

- etc.
Physics with Beam $\nu_\tau$’s at the DUNE Far Detector Site


$\nu_\tau$ sample: why?

- **Model independent checks.**
  - Establishing the existence of $\nu_\tau$ in the beam;
  - Is it consistent with the oscillation interpretation $\nu_\mu \rightarrow \nu_\tau$?
  - Measuring the oscillation parameters.
  - Comparison to OPERA, atmospheric samples.

- **Cross-section measurements.**
  - Comparison to OPERA, atmospheric samples.

- **Testing the 3-neutrinos paradigm.**
  - Independent measurement of the oscillation parameters.
  - Is there anything the $\nu_\tau$ sample brings to the table given the $\nu_\mu$, $\nu_e$, and neutral current samples? [model-dependent]
$P(\nu_{\mu} \rightarrow \nu_\tau)$

$E_\nu < E_{\text{threshold}}$

$0.5 < \sin^2(2\theta_{\mu\tau}) < 1$

$E_\nu$ [GeV]
Testing the Three-Massive-Neutrinos Paradigm

\[
\sin^2 2\theta_{\mu e} \equiv 4|U_{\mu 3}|^2|U_{e 3}|^2, \quad \sin^2 2\theta_{\mu \tau} \equiv 4|U_{\mu 3}|^2|U_{\tau 3}|^2, \quad \sin^2 2\theta_{\mu \mu} \equiv 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)
\]

Unitarity Test: \(|U_{e 3}|^2 + |U_{\mu 3}|^2 + |U_{\tau 3}|^2 = 1^{+0.05}_{-0.06} \text{ [one sigma]} \quad (1^{+0.13}_{-0.17} \text{ [three sigma]})\)
DUNE 7 yr. data collection

3.5 yr. Neutrino Mode, 3.5 yr. Antineutrino Mode

$\sin^2 \theta_{12} = 0.310$ (fixed)

$\sin^2 \theta_{13} = 0.02240$ (free)

$\sin^2 \theta_{23} = 0.582$ (free)

$\Delta m^2_{21} = 7.39 \times 10^{-5} \text{ eV}^2$ (fixed)

$\Delta m^2_{31} = +2.525 \times 10^{-3} \text{ eV}^2$ (free, ordering fixed)

$\delta_{CP} = -2.496 \text{ rad} = 217^\circ$ (free)
Case Studies

I will discuss a few case-studies, including the fourth-neutrino hypothesis and non-standard neutral-current neutrino–matter interactions. In general

- I will mostly discuss, for concreteness, the DUNE setup;

- I don’t particularly care about how likely, nice, or contrived the scenarios are. It is useful to consider them as well-defined ways in which the three-flavor paradigm can be violated. They can be used as benchmarks for comparing different efforts, or, perhaps, as proxies for other new phenomena.

- I will mostly be interested in three questions:
  - How sensitive are next-generation long-baseline efforts?;
  - How well they can measure the new-physics parameters, including new sources of CP-invariance violation?;
  - Can they tell different new-physics models apart?
Different Oscillation Parameters for Neutrinos and Antineutrinos?


- How much do we know, independently, about neutrino and antineutrino oscillations?
- What happens if the parameters disagree?
DUNE + HK B 99% Cred.

- Neutrino Parameter Measurement
- Antineutrino Parameter Measurement
- Antineutrino Parameter Measurement (no Daya Bay)
- Measurement assuming CPT Conservation

$\Delta \left( \sin^2 \theta_{13} \right)$

$\Delta \left( \Delta m^2_{31} \right) \text{ [eV}^2\text{]}$

$\Delta \left( \delta_{CP} \right)$

A Fourth Neutrino

(Berryman et al, arXiv:1507.03986)

If there are more neutrinos with a well-defined mass, it is easy to extend the paradigm:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_? \\
\vdots
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots \\
U_{?1} & U_{?2} & U_{?3} & U_{?4} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4 \\
\vdots
\end{pmatrix}
\]

- New mass eigenstates easy: \(\nu_4\) with mass \(m_4\), \(\nu_5\) with mass \(m_5\), etc.
- What are these new "flavor" (or weak) eigenstates \(\nu_?\)? Here, the answer is we don’t care. We only assume there are no new accessible interactions associated to these states.
\[ U_{e2} = s_{12}c_{13}c_{14}, \]
\[ U_{e3} = e^{-i\eta_1} s_{13}c_{14}, \]
\[ U_{e4} = e^{-i\eta_2} s_{14}, \]
\[ U_{\mu2} = c_{24}\left(c_{12}c_{23} - e^{i\eta_1} s_{12}s_{13}s_{23}\right) - e^{i(\eta_2-\eta_3)} s_{12}s_{14}s_{24}c_{13}, \]
\[ U_{\mu3} = s_{23}c_{13}c_{24} - e^{i(\eta_2-\eta_3-\eta_1)} s_{13}s_{14}s_{24}, \]
\[ U_{\mu4} = e^{-i\eta_3} s_{24}c_{14}, \]
\[ U_{\tau2} = c_{34}\left(-c_{12}s_{23} - e^{i\eta_1} s_{12}s_{13}c_{23}\right) - e^{i\eta_2} c_{13}c_{24}s_{12}s_{14}s_{34} \]
\[ -e^{i\eta_3}\left(c_{12}c_{23} - e^{i\eta_1} s_{12}s_{13}s_{23}\right)s_{24}s_{34}, \]
\[ U_{\tau3} = c_{13}c_{23}c_{34} - e^{i(\eta_2-\eta_1)} s_{13}s_{14}s_{34}c_{24} - e^{i\eta_3} s_{23}s_{24}s_{34}c_{13}, \]
\[ U_{\tau4} = s_{34}c_{14}c_{24}. \]

When the new mixing angles \( \phi_{14}, \phi_{24}, \) and \( \phi_{34} \) vanish, one encounters oscillations among only three neutrinos, and we can map the remaining parameters \( \{\phi_{12}, \phi_{13}, \phi_{23}, \eta_1\} \rightarrow \{\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}\} \).

Also

\[ \eta_s \equiv \eta_2 - \eta_3, \]

is the only new CP-odd parameter to which oscillations among \( \nu_e \) and \( \nu_\mu \) are sensitive.
Some technicalities for the aficionados

- 34 kiloton liquid argon detector;
- 1.2 MW proton beam on target as the source of the neutrino and antineutrino beams, originating 1300 km upstream at Fermilab;
- 3 years each with the neutrino and antineutrino mode;
- Include standard backgrounds, and assume a 5% normalization uncertainty;
- Whenever quoting bounds or measurements of anything, we marginalize over all parameters not under consideration;
- We include priors on $\Delta m_{12}^2$ and $|U_{e2}|^2$ in order to take into account information from solar experiments and KamLAND. Unless otherwise noted, we assume the mass ordering is normal;
- We do not include information from past experiments. We assume that DUNE will “out measure” all experiments that came before it (except for the solar ones, as mentioned above).
FIG. 1: Expected signal and background yields for six years ($3\nu + 3\overline{\nu}$) of data collection at DUNE, using fluxes projected by Ref. [1], for a 34 kiloton detector, and a 1.2 MW beam. (a) and (b) show appearance channel yields for neutrino and antineutrino beams, respectively, while (c) and (d) show disappearance channel yields. The $3\nu$ signal corresponds to the standard three-neutrino hypothesis, where $\sin^2 \theta_{12} = 0.308$, $\sin^2 \theta_{13} = 0.0235$, $\sin^2 \theta_{23} = 0.437$, $\Delta m^2_{12} = 7.54 \times 10^{-5}$ eV$^2$, $\Delta m^2_{32} = 2.43 \times 10^{-3}$ eV$^2$, $\delta_{CP} = 0$, while the $4\nu$ signal corresponds to $\sin^2 \phi_{12} = 0.315$, $\sin^2 \phi_{13} = 0.024$, $\sin^2 \phi_{23} = 0.456$, $\sin^2 \phi_{24} = 0.023$, $\sin^2 \phi_{34} = 0.030$, $\Delta m^2_{14} = 10^{-2}$ eV$^2$, $\eta_1 = 0$, and $\eta_2 = 0$. Statistical uncertainties are shown as vertical bars in each bin. Backgrounds are defined in the text and are assumed to be identical for the three- and four-neutrino scenarios; any discrepancy is negligible after accounting for a 5% normalization uncertainty.
$|U_{e4}|^2 |U_{\mu 4}|^2$

$\Delta m_{24}^2$ [eV$^2$]

Expected DUNE 95% CL
MINOS 95% CL
IceCube 90% CL

$\sin^2 \phi_{24}$

$\Delta m_{14}^2$ [eV$^2$]

[Berryman et al, arXiv:1507.03986]
3 years $\nu + \bar{\nu}$

$\Delta m_{12}^2 = (7.54 \pm 0.24) \times 10^{-5}$ eV$^2$
$\Delta m_{13}^2 = 2.43 \times 10^{-3}$ eV$^2$
$\Delta m_{34}^2 = 10^{-2}$ eV$^2$

$\sin^2 \phi_{12} = 0.315$
$\sin^2 \phi_{13} = 0.024$
$\sin^2 \phi_{23} = 0.456$
$\sin^2 \phi_{14} = 0.022$
$\sin^2 \phi_{34} = 0.030$

$\eta_1 = \pi/3$
$\eta_s = -\pi/4$
$|U_{e2}|^2 = 0.301 \pm 0.015$

[Berryman et al, arXiv:1507.03986]

FIG. 5: Expected sensitivity contours at 68.3% (blue), 95% (orange), and 99% (red) CL at DUNE with six years of data collection (3y $\nu + 3y \bar{\nu}$), a 34 kiloton detector, and a 1.2 MW beam given the existence of a fourth neutrino with parameters from Case 2 in Table I. Results from solar neutrino experiments are included here as Gaussian priors for the values of $|U_{e2}|^2 = 0.301 \pm 0.015$ and $\Delta m_{12}^2 = 7.54 \pm 0.24 \times 10^{-5}$ eV$^2$ [22].

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin^2 \phi_{14}$</td>
<td>$\sin^2 \phi_{24}$</td>
<td>$\Delta m_{14}^2$ (eV$^2$)</td>
</tr>
<tr>
<td>0.023</td>
<td>0.030</td>
<td>0.93</td>
</tr>
<tr>
<td>0.023</td>
<td>0.030</td>
<td>$1.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.040</td>
<td>0.320</td>
<td>$1.0 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

TABLE I: Input values of the parameters for the three scenarios considered for the four-neutrino hypothesis. Values of $\phi_{12}$, $\phi_{13}$, and $\phi_{23}$ are chosen to be consistent with the best-fit values of $|U_{e2}|^2$, $|U_{e3}|^2$, and $|U_{\mu 3}|^2$, given choices of $\phi_{14}$ and $\phi_{24}$. Here, $\eta_s = \eta_2 - \eta_3$. Note that $\Delta m_{14}^2$ is explicitly assumed to be positive, i.e., $m_1^2 > m_4^2$. 
3 years $\nu + \bar{\nu}$

$\Delta m_{13}^2 = (7.54 \pm 0.24) \times 10^{-5} \text{eV}^2$

$\Delta m_{12}^2 = 2.43 \times 10^{-3} \text{eV}^2$

$\Delta m_{4}^2 = 10^{-5} \text{eV}^2$

$\sin^2 \phi_{12} = 0.321$

$\sin^2 \phi_{13} = 0.024$

$\sin^2 \phi_{23} = 0.639$

$\sin^2 \phi_{14} = 0.040$

$\sin^2 \phi_{24} = 0.326$

$\eta_1 = \pi / 3$

$\eta_2 = -\pi / 4$

$|U_{e2}|^2 = 0.301 \pm 0.015$

FIG. 6: Expected sensitivity contours at 68.3% (blue), 95% (orange), and 99% (red) CL at DUNE with six years of data collection ($3\gamma \nu + 3\gamma \bar{\nu}$), a 34 kiloton detector, and a 1.2 MW beam given the existence of a fourth neutrino with parameters from Case 3 in Table 1. Results from solar neutrino experiments are included here as Gaussian priors for the values of $|U_{e2}|^2 = 0.301 \pm 0.015$ and $\Delta m_{12}^2 = 7.54 \pm 0.24 \times 10^{-5} \text{eV}^2$ [22].
Fourth Neutrino Hypothesis

Non-Standard Neutrino Interactions (NSI)

Effective Lagrangian:

\[ \mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F (\bar{\nu}_\alpha \gamma_\rho \nu_\beta) \sum_{f=e,u,d} (\epsilon_{\alpha\beta}^L f_L^\dagger \gamma^\rho f_L + \epsilon_{\alpha\beta}^R f_R^\dagger \gamma^\rho f_R) + \text{h.c.}, \]

For oscillations,

\[ H_{ij} = \frac{1}{2E_\nu} \text{diag} \{0, \Delta m_{12}^2, \Delta m_{13}^2\} + V_{ij}, \]

where

\[ V_{ij} = U_{i\alpha}^\dagger V_{\alpha\beta} U_{\beta j}, \]

\[ V_{\alpha\beta} = A \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}, \]

\[ A = \sqrt{2}G_F n_e. \]  
\[ \epsilon_{\alpha\beta} \]  
are linear combinations of the \( \epsilon_{\alpha\beta}^{fL,R} \). Important: I will discuss propagation effects only and ignore NSI effects in production or detection (\( \epsilon \) versus \( \epsilon^2 \)).
There are new sources of CP-invariance violation! [easier to see T-invariance violation]

![Graph showing CP-invariance violation effects](image)

**FIG. 2:** T-invariance violating effects of NSI at $L = 1300$ km for $\epsilon_{e\mu} = 0.1e^{i\pi/3}$, $\epsilon_{e\tau} = 0.1e^{-i\pi/4}$, $\epsilon_{\mu\tau} = 0.1$ (all other NSI parameters are set to zero). Here, the three-neutrino oscillation parameters are $\sin^2\theta_{12} = 0.308$, $\sin^2\theta_{13} = 0.0234$, $\sin^2\theta_{23} = 0.437$, $\Delta m_{12}^2 = 7.54 \times 10^{-5}$ eV$^2$, $\Delta m_{13}^2 = 2.47 \times 10^{-3}$ eV$^2$, and $\delta = 0$, i.e., no “standard” T-invariance violation. The green curve corresponds to $P_{e\mu}$ while the purple curve corresponds to $P_{\mu e}$. If, instead, all non-zero NSI are real ($\epsilon_{e\mu} = 0.1$, $\epsilon_{e\tau} = 0.1$, $\epsilon_{\mu\tau} = 0.1$), $P_{e\mu} = P_{\mu e}$, the grey curve. The dashed line corresponds to the pure three-neutrino oscillation probabilities assuming no T-invariance violation (all $\epsilon_{\alpha\beta} = 0$, $\delta = 0$).
3 years $\nu + \bar{\nu}$

$\sin^2 \theta_{12} = 0.308$
$\sin^2 \theta_{13} = 0.023$
$\sin^2 \theta_{23} = 0.437$
$\Delta m^2_{21} = (7.54 \pm 0.24) \times 10^{-5} \text{ eV}^2$
$\Delta m^2_{13} = 2.47 \times 10^{-3} \text{ eV}^2$
$\delta = \pi/3$
$|U_{e2}|^2 = 0.301 \pm 0.015$

$\epsilon_{ee} = 0$
$\epsilon_{e\tau} = 0$
$\epsilon_{\mu\mu} = 0$
$\epsilon_{\tau\tau} = 0$
$\epsilon_{\mu\tau} = 0$

$|\epsilon_{\mu\mu}| < 0.04$
$|\epsilon_{\mu\tau}| < 0.18$
$|\epsilon_{\mu\tau}| < 0.01$
$\epsilon_{\tau\tau} = -0.03 \pm 0.08$

FIG. 4: Expected exclusion limits at 68.3% (red), 95% (orange), and 99% (blue) CL at DUNE assuming data consistent with the standard model. The CP violation parameter $\delta$ is equal to $\pi/3$, and the neutrino mass eigenvalues are $\Delta m^2_{21}$ and $\Delta m^2_{13}$. The plots show the allowed regions in the $\epsilon_{ee}$-$\epsilon_{e\tau}$ plane, $|\epsilon_{\mu\mu}|$-$|\epsilon_{\mu\tau}|$, and $|\epsilon_{\mu\tau}|$-$\epsilon_{\tau\tau}$ planes.
\[ \sin^2 \theta_{12} = 0.398 \]
\[ \sin^2 \theta_{13} = 0.023 \]
\[ \sin^2 \theta_{23} = 0.437 \]
\[ \Delta m_{12}^2 = (7.54 \pm 0.24) \times 10^{-3} \text{ eV}^2 \]
\[ \Delta m_{23}^2 = 2.47 \times 10^{-3} \text{ eV}^2 \]
\[ \delta = \pi/3 \]
\[ |U_{e2}|^2 = 0.301 \pm 0.015 \]
\[ c_{ee} = 0.5 \]
\[ c_{ee} = -0.3 \]
\[ c_{ee} = 0 \]
\[ c_{ee} = 0.5 e^{i \pi/3} \]
\[ c_{ee} = 0 \]

\[ \Delta \chi^2 \]
\[ \epsilon_{ee} \]
\[ \epsilon_{\tau \tau} \]
\[ \arg(\epsilon_{ee}) \]
\[ \arg(\epsilon_{\tau \tau}) \]


<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_{ee}$</th>
<th>$\epsilon_{e\mu}$</th>
<th>$\epsilon_{e\tau}$</th>
<th>$\epsilon_{\mu\mu}$</th>
<th>$\epsilon_{\mu\tau}$</th>
<th>$\epsilon_{\tau\tau}$</th>
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<tr>
<td>Case 1</td>
<td>0</td>
<td>0.15$e^{i \pi/3}$</td>
<td>0.3$e^{-i \pi/4}$</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>Case 2</td>
<td>-1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.5</td>
<td>0</td>
<td>0.5$e^{i \pi/3}$</td>
<td>0</td>
<td>0</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

**TABLE I:** Input values of the new physics parameters for the three NSI scenarios under consideration. The star symbol is a reminder that, as discussed in the text, we can choose $\epsilon_{\mu\mu} \equiv 0$ and reinterpret the other diagonal NSI parameters.
Telling Different Scenarios Apart:

FIG. 8: Sensitivity contours at 68.3% (blue), 95% (orange), and 99% (red) for a four-neutrino fit to data consistent with Case 2 from Table I. All unseen parameters are marginalized over, and Gaussian priors are included on the values of $\Delta m_{12}^2$ and $|U_{e2}|^2$. See text for details.


<table>
<thead>
<tr>
<th>Fit</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3$\nu$ with Solar Priors</td>
<td>217/114 $\sim$ 5.4$\sigma$</td>
<td>186/114 $\sim$ 4.2$\sigma$</td>
<td>118/114 $\sim$ 4.3$\sigma$</td>
</tr>
<tr>
<td>3$\nu$ without Priors</td>
<td>172/114 $\sim$ 3.4$\sigma$</td>
<td>134/114 $\sim$ 1.6$\sigma$</td>
<td>154/114 $\sim$ 2.7$\sigma$</td>
</tr>
<tr>
<td>4$\nu$ with Solar Priors</td>
<td>193/110 $\sim$ 4.8$\sigma$</td>
<td>142/110 $\sim$ 2.3$\sigma$</td>
<td>153/110 $\sim$ 2.8$\sigma$</td>
</tr>
</tbody>
</table>

TABLE II: Results of various three- or four-neutrino fits to data generated to be consistent with the cases listed in Table I. Numbers quoted are for $\chi^2_{\text{min}}$/dof and the equivalent discrepancy using a $\chi^2$ distribution.
How Do We Learn More – Different Experiments!

- Different $L$ and $E$, same $L/E$ (e.g. HyperK or ESSnuSB versus DUNE);
- Different matter potentials (e.g. atmosphere versus accelerator);
- Different oscillation modes (appearance versus disappearance, e’s, μ’s and τ’s).

![Graph showing oscillation probabilities](image)


**FIG. 9:** Oscillation probabilities for three-neutrino (dashed) and NSI (solid) hypotheses as a function of $L/E_\nu$, the baseline length divided by neutrino energy, for the DUNE (purple) and HyperK (green) experiments. Here, $\delta = 0$ and the three-neutrino parameters used are consistent with Ref. [47].
Precision Meas. of Oscillation Parameters. Why and How Much?

A word from flavor models:

Figure 2: $P_{\cos \delta}$ as a function of $\cos \delta$ for various mixing patterns. Here we have assumed that $P_z(z)$ is a Gaussian centered at the experimental best-fit value of $z$, with width of $1\sigma$.

[Everett et al., arXiv:1912.10139]
More General Comments.

If there is an underlying structure behind the values of the lepton masses and mixing angles...

- it may lead to relations among the parameters: **sum rules**.
  \[ f(\theta_{12}, \theta_{13}, \theta_{23}, \delta, m_1, m_2, m_3) = 0. \]

- it may lead to relations between PMNS and CKM parameters.
  \[ f(\text{PMNS}) = g(\text{CKM}). \]

- etc.

These provide guidance for precision.

- Sum rules need all oscillation parameters to be known with similar precision: \( \theta_{23}, \delta \) are the obvious outliers.

- On the CKM side, \( \theta_{12} = 13.04^\circ \pm 0.05^\circ, \ \theta_{13} = 0.201^\circ \pm 0.011^\circ, \ \theta_{23} = 2.38^\circ \pm 0.06^\circ, \ \delta = 68.8^\circ \pm 4.5^\circ. \) (several percent to sub percent).
Anarchy vs. Order — more precision required!

Order: $\sin^2 \theta_{13} = C \cos^2 2\theta_{23}$, $C \in [0.8, 1.2]$  

[AdG, Murayama, 1204.1249]
In Conclusions

1. We still know very little about the new physics uncovered by neutrino oscillations.

2. Neutrino masses are very small – we don’t know why, but we think it means something important.

3. Neutrino mixing is “weird” – we don’t know why, but we think it means something important.
4. **We need more experimental input** These will come from a rich, diverse experimental program which relies heavily on the existence of underground facilities capable of hosting large detectors (double-beta decay, precision neutrino oscillations, supernova neutrinos, proton decay, etc).

5. **Precision measurements of neutrino oscillations are sensitive to several new phenomena, including new neutrino properties, the existence of new states, or the existence of new interactions.** There is a lot of work to be done when it comes to understanding which new phenomena can be probed in long-baseline oscillation experiments (and how well) and what are the other questions one can ask – related and unrelated to neutrinos – of these unique particle physics experiments.

6. **There is plenty of room for surprises**, as neutrinos are potentially very deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are “quantum interference devices” – potentially very sensitive to whatever else may be out there (e.g., $\Lambda \simeq 10^{14}$ GeV).
Backup Slides . . .
Solar Neutrinos

We are not done yet!

- see “vaccum-matter” transition
- probe for new physics: NSI, pseudo-Dirac, ...
- probe of the solar interior! “solar abundance problem” (see e.g. 1104.1639)

‘CNO neutrinos may provide information on planet formation!'

FIG. 1: Recent SNO solar neutrino data [18] on $P(\nu_e \rightarrow \nu_e)$ (blue line with 1 σ band). The LMA MSW solution (dashed black curve with gray 1 σ band) appears divergent around a few MeV, whereas for NSI with $\epsilon_{\nu_e \tau} = 0.4$ (thick magenta), the electron neutrino probability appears to fit the data better. The data points come from the recent Borexino paper [19].
<table>
<thead>
<tr>
<th></th>
<th>OSC</th>
<th></th>
<th>+COHERENT</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LMA</td>
<td>LMA + LMA-D</td>
<td>LMA</td>
<td>LMA + LMA-D</td>
</tr>
<tr>
<td>$\varepsilon_{ee}^u - \varepsilon_{\mu\mu}^u$</td>
<td>$[-0.020, +0.456]$</td>
<td>$[-1.192, -0.802]$</td>
<td>$[-0.008, +0.618]$</td>
<td>$[-0.008, +0.618]$</td>
</tr>
<tr>
<td>$\varepsilon_{\tau\tau}^u - \varepsilon_{\mu\mu}^u$</td>
<td>$[-0.005, +0.130]$</td>
<td>$[-0.152, +0.130]$</td>
<td>$[-0.111, +0.402]$</td>
<td>$[-0.111, +0.402]$</td>
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<tr>
<td>$\varepsilon_{e\mu}^u$</td>
<td>$[-0.060, +0.049]$</td>
<td>$[-0.060, +0.067]$</td>
<td>$[-0.060, +0.049]$</td>
<td>$[-0.060, +0.049]$</td>
</tr>
<tr>
<td>$\varepsilon_{e\tau}^u$</td>
<td>$[-0.292, +0.119]$</td>
<td>$[-0.292, +0.336]$</td>
<td>$[-0.248, +0.116]$</td>
<td>$[-0.248, +0.116]$</td>
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<tr>
<td>$\varepsilon_{\mu\tau}^u$</td>
<td>$[-0.013, +0.010]$</td>
<td>$[-0.013, +0.014]$</td>
<td>$[-0.012, +0.009]$</td>
<td>$[-0.012, +0.009]$</td>
</tr>
<tr>
<td>$\varepsilon_{ee}^d - \varepsilon_{\mu\mu}^d$</td>
<td>$[-0.027, +0.474]$</td>
<td>$[-1.232, -1.111]$</td>
<td>$[-0.012, +0.565]$</td>
<td>$[-0.012, +0.565]$</td>
</tr>
<tr>
<td>$\varepsilon_{\tau\tau}^d - \varepsilon_{\mu\mu}^d$</td>
<td>$[-0.005, +0.095]$</td>
<td>$[-0.013, +0.095]$</td>
<td>$[-0.103, +0.361]$</td>
<td>$[-0.103, +0.361]$</td>
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<tr>
<td>$\varepsilon_{e\mu}^d$</td>
<td>$[-0.061, +0.049]$</td>
<td>$[-0.061, +0.073]$</td>
<td>$[-0.058, +0.049]$</td>
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<tr>
<td>$\varepsilon_{e\tau}^d$</td>
<td>$[-0.247, +0.119]$</td>
<td>$[-0.247, +0.119]$</td>
<td>$[-0.206, +0.110]$</td>
<td>$[-0.206, +0.110]$</td>
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<tr>
<td>$\varepsilon_{\mu\tau}^d$</td>
<td>$[-0.012, +0.009]$</td>
<td>$[-0.012, +0.009]$</td>
<td>$[-0.011, +0.009]$</td>
<td>$[-0.011, +0.009]$</td>
</tr>
<tr>
<td>$\varepsilon_{ee}^p - \varepsilon_{\mu\mu}^p$</td>
<td>$[-0.041, +1.312]$</td>
<td>$[-3.328, -1.958]$</td>
<td>$[-0.010, +2.039]$</td>
<td>$[-0.010, +2.039]$</td>
</tr>
<tr>
<td>$\varepsilon_{\tau\tau}^p - \varepsilon_{\mu\mu}^p$</td>
<td>$[-0.015, +0.426]$</td>
<td>$[-0.424, +0.426]$</td>
<td>$[-0.364, +1.387]$</td>
<td>$[-0.364, +1.387]$</td>
</tr>
<tr>
<td>$\varepsilon_{e\mu}^p$</td>
<td>$[-0.178, +0.147]$</td>
<td>$[-0.178, +0.178]$</td>
<td>$[-0.179, +0.146]$</td>
<td>$[-0.179, +0.146]$</td>
</tr>
<tr>
<td>$\varepsilon_{e\tau}^p$</td>
<td>$[-0.954, +0.356]$</td>
<td>$[-0.954, +0.949]$</td>
<td>$[-0.860, +0.350]$</td>
<td>$[-0.860, +0.350]$</td>
</tr>
<tr>
<td>$\varepsilon_{\mu\tau}^p$</td>
<td>$[-0.035, +0.027]$</td>
<td>$[-0.035, +0.035]$</td>
<td>$[-0.035, +0.028]$</td>
<td>$[-0.035, +0.028]$</td>
</tr>
</tbody>
</table>

Table 1. 2σ allowed ranges for the NSI couplings $\varepsilon_{\alpha\beta}^u$, $\varepsilon_{\alpha\beta}^d$ and $\varepsilon_{\alpha\beta}^p$ as obtained from the global analysis of oscillation data (left column) and also including COHERENT constraints. The results are obtained after marginalizing over oscillation and the other matter potential parameters either within the LMA only and within both LMA and LMA-D subspaces respectively (this second case is denoted as LMA + LMA-D).
Figure 6. Two-dimensional projections of the allowed regions onto different vacuum parameters after marginalizing over the matter potential parameters (including $\eta$) and the undisplayed oscillation parameters. The solid colored regions correspond to the global analysis of all oscillation data, and show the 1$\sigma$, 90%, 2$\sigma$, 99% and 3$\sigma$ CL allowed regions; the best-fit point is marked with a star. The black void regions correspond to the analysis with the standard matter potential (i.e., without NSI) and its best-fit point is marked with an empty dot. For comparison, in the left panel we show in red the 90% and 3$\sigma$ allowed regions including only solar and KamLAND results, while in the right panels we show in green the 90% and 3$\sigma$ allowed regions excluding solar and KamLAND data, and in yellow the corresponding ones excluding also IceCube and reactor data.

I. Esteban et al, 1805.04530 [hep-ph]
The Physics Behind NSI – Comments and Concerns

There are two main questions associated to NSI’s. They are somewhat entwined.

1. What is the new physics that leads to neutrino NSI? or are there models for new physics that lead to large NSIs? Are these models well motivated? Are they related to some of the big questions in particle physics?

2. Are NSIs constrained by observables that have nothing to do with neutrino physics? Are large NSI effects allowed at all?
Effective Lagrangian:

\[ \mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F \epsilon^{\alpha\beta}(\bar{\nu}_\alpha\gamma_\rho\nu_\beta)(\bar{f}\gamma^\rho f). \]

This is not $SU(2)_L$ invariant. Let us fix that:

\[ \mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F \epsilon^{\alpha\beta}(\bar{L}_\alpha\gamma_\rho L_\beta)(\bar{f}\gamma^\rho f). \]

where $L = (\nu, \ell^-)^T$ is the lepton doublet. This is a big problem. Charged-Lepton flavor violating constraints are really strong (think $\mu \to e^+e^-e^+$, $\mu \to e$-conversion, $\tau \to \mu+$hadrons, etc), and so are most of the flavor diagonal charged-lepton effects.

There are a couple of ways to circumvent this...
1. Dimension-Eight Effective Operator

\[ \mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F \epsilon^{\alpha\beta} (\bar{\nu}_\alpha \gamma_\rho \nu_\beta) (\bar{f} \gamma^\rho f). \]

This is not \(SU(2)_L\) invariant. Let us fix that in a different way

\[ \mathcal{L}^{\text{NSI}} = -2\sqrt{2}G_F \frac{\epsilon^{\alpha\beta}}{v^2} ((HL)^\dagger_\alpha \gamma_\rho (HL)_\beta) (\bar{f} \gamma^\rho f). \]

where \(HL \propto H^+\ell^- - H^0\nu\). After electroweak symmetry breaking \(H^0 \rightarrow \nu + h^0\) and we only get new neutrino interactions.

Sadly, it is not that simple. At the one-loop level, the dimension-8 operator will contribute to the dimension-6 operator in the last page, as discussed in detail in [Gavela et al, arXiv:0809.3451 [hep-ph]]. One can, however, fine-tune away the charged-lepton effects.
2. Light Mediator

(Overview by Y. Farzan and M. Tórtola, arXiv:1710.09360 [hep-ph])

\[ \mathcal{L}^{\text{NSI}} = -2\sqrt{2} G_F \epsilon^{\alpha\beta} (\bar{\nu}_\alpha \gamma_\rho \nu_\beta) \left( \bar{f} \gamma^\rho f \right). \]

This may turn out to be a good effective theory for neutrino propagation but a bad effective theory for most charged-lepton processes. I.e.

\[ \mathcal{L}^{\text{NSI}} = -2\sqrt{2} G_F \epsilon^{\alpha\beta} (\bar{L}_\alpha \gamma_\rho L_\beta) \left( \bar{f} \gamma^\rho f \right). \]

might be inappropriate for describing charged-lepton processes if the particle we are integrating out is light (as in lighter than the muon). Charged-lepton processes are “watered down.” Very roughly

\[ \epsilon \rightarrow \epsilon \left( \frac{m_{Z'}}{m_\ell} \right)^2 \]

where \( m_{Z'} \) is the mass of the particle mediating the new interaction, and \( m_\ell \) is the mass associated to the charged-lepton process of interest.