Introduction to Lattice QCD and neutrino cross sections

KITP Conference:
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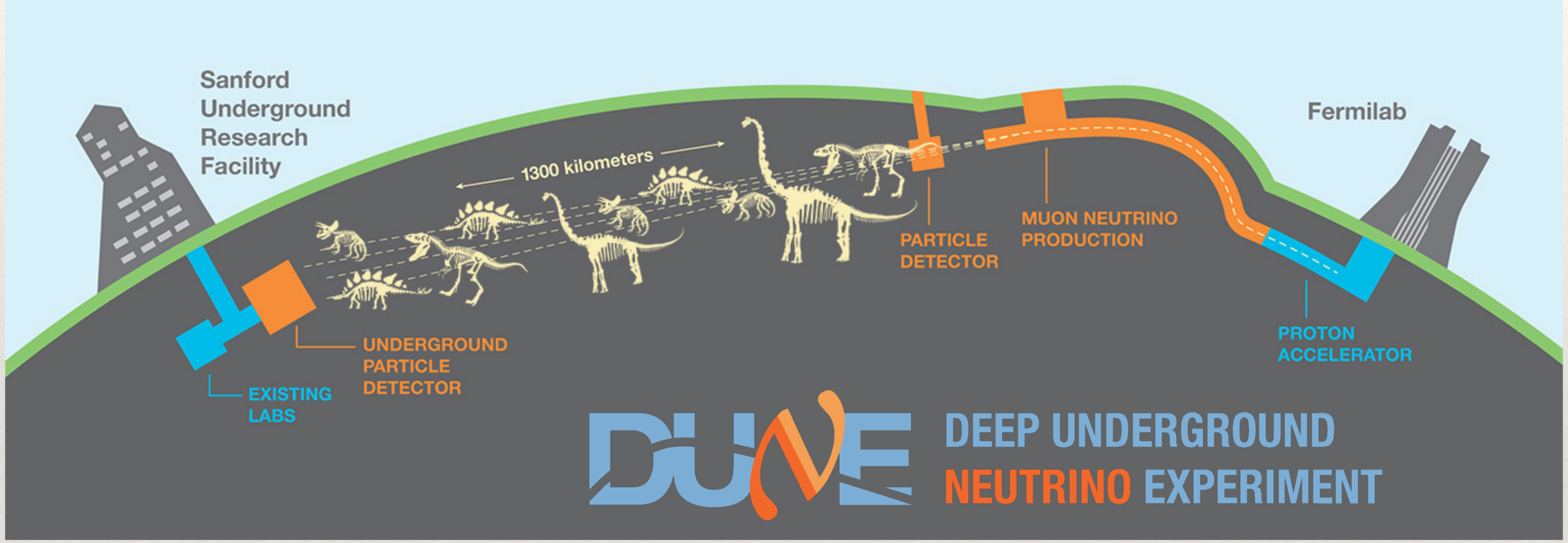
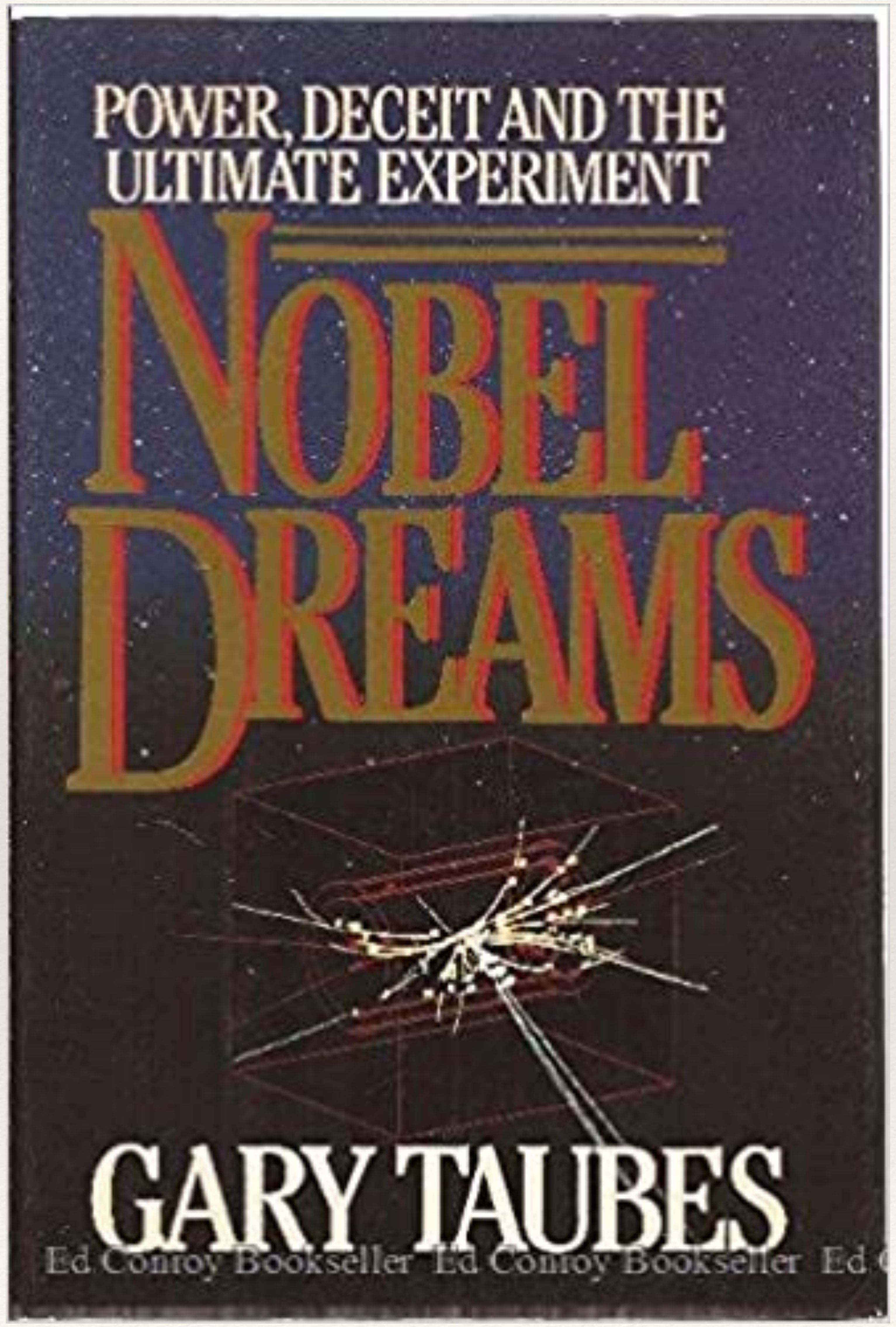
BERKELEY LAB

What is our main goal?

- Theoretical prediction of ν-A cross sections from the Standard Model with full uncertainty quantification





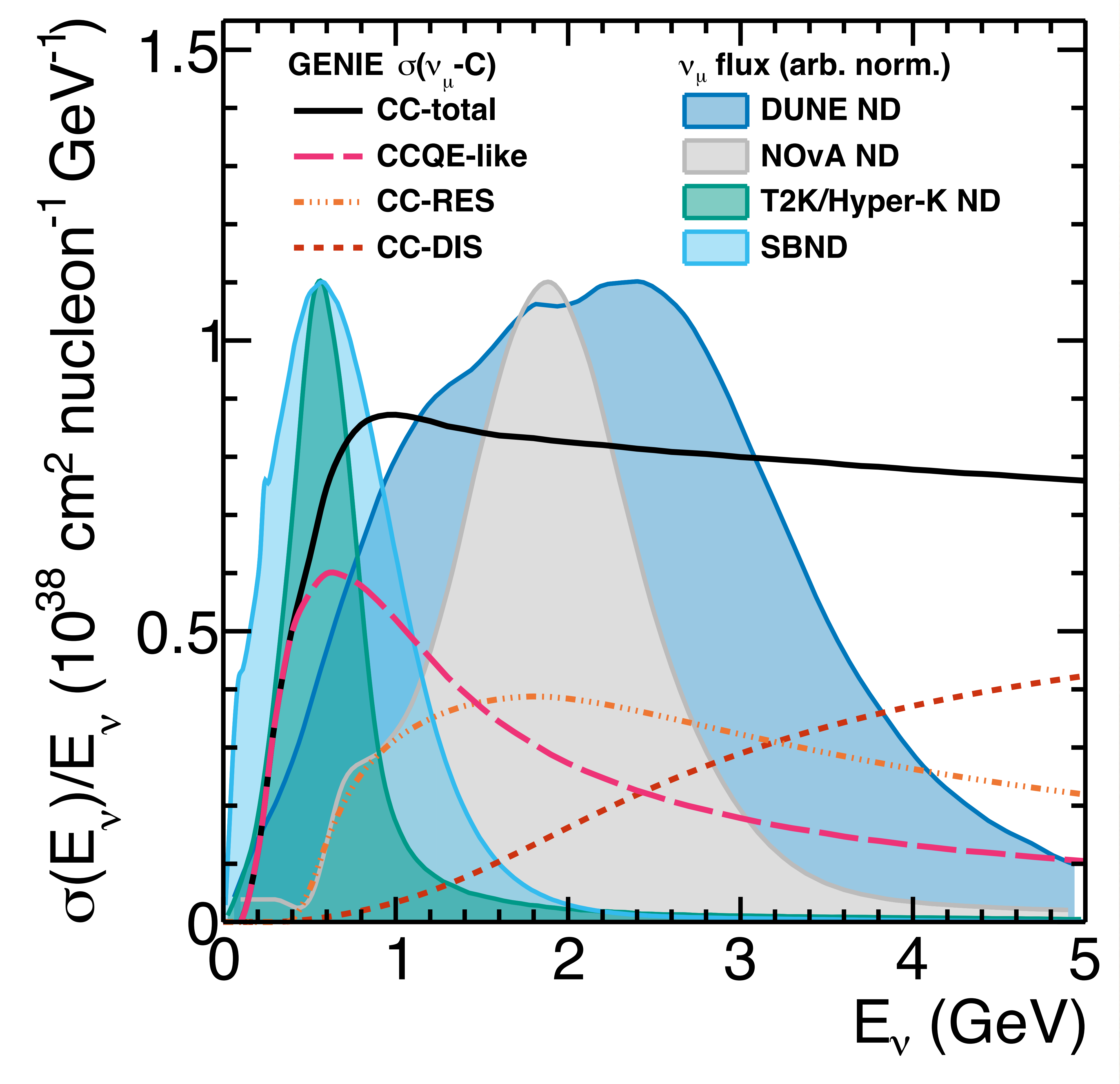
- The analogy is not perfect (→) but, if we aim to find new physics, it is wise to have a prediction of the "background" using the known physics
  [“evidence” of supersymmetry was in fact QCD jet physics]

What is our main goal?

- Theoretical prediction of ν-A cross sections from the Standard Model with full uncertainty quantification

- Very challenging to achieve this goal: Most likely, it is impossible to have a unified theoretical description of ν-A cross sections over the range of ν-energy of interest
  - Lattice QCD: single nucleon, resonance region, …
  - Effective Field Theory (EFT): Low-energy, small-A
  - high energy: DIS and Regge (model)

- The problem demands a description of medium-A
  - over broad range of energy
  - pion production, resonance region
  - final state interactions
  - …



(C. Wilkinson)

What is possible? A “realist” perspective

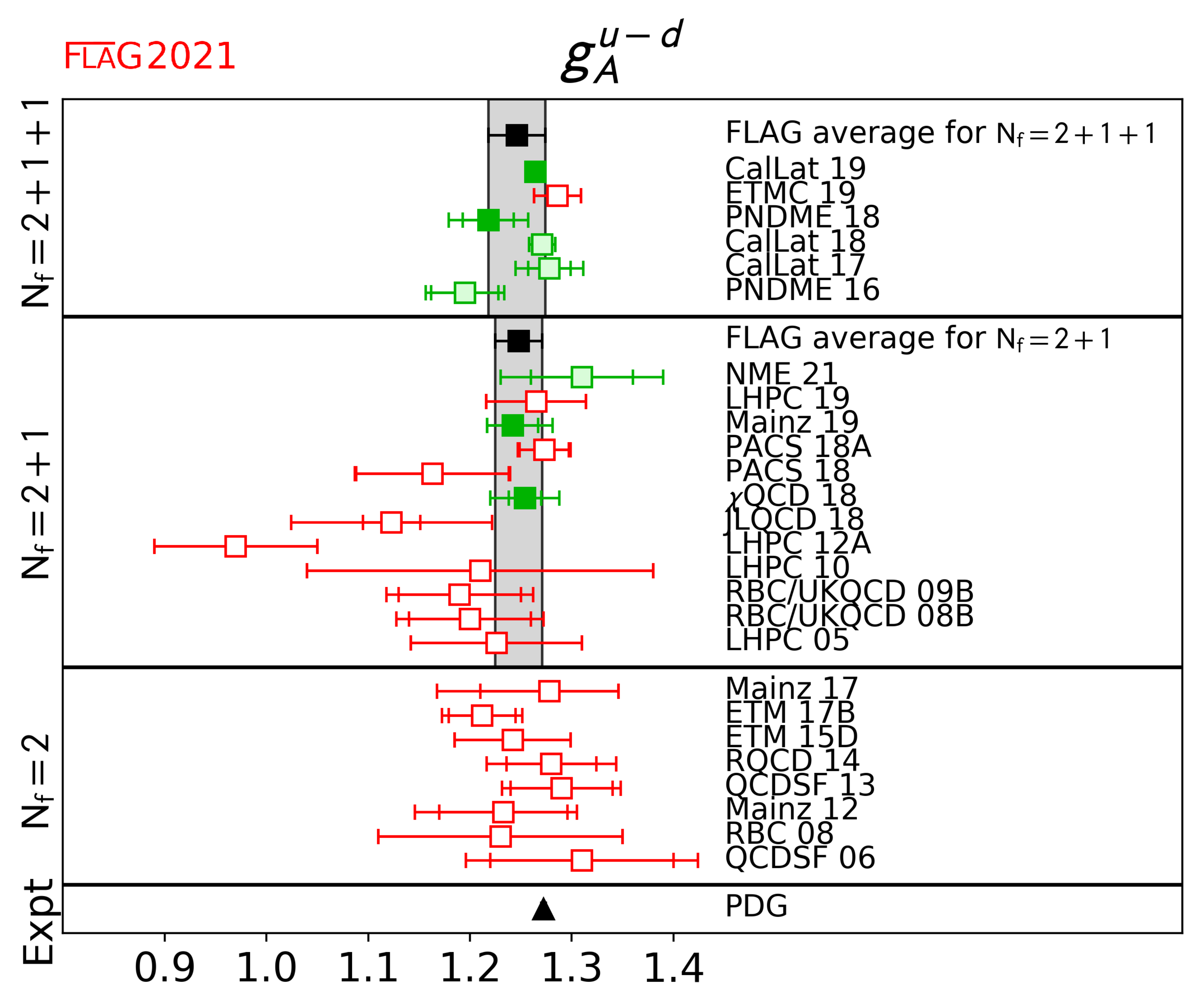
- Lattice QCD (LQCD) can determine single nucleon
  - quasi-elastic
  - resonance region, pion production
  - DIS
  - two-nucleon cross section (corrections)
  - maybe, maybe, light nuclear cross sections

Even, even if we could compute ν-^{12}C, it almost certainly will not be the most economical way to propagate QCD results to nuclear cross sections

- No EFT that can describe ν-A reaction over entire range of E_ν
  - Use EFT, with LQCD input, to describe region of parameter space against which nuclear models can be constrained
    - in this region at least, rigorous, systematically improvable uncertainty rooted in the SM
    - This will allow for calibration of nuclear model uncertainty

Where are we now? (with LQCD)

- Single-nucleon "charges" known at the percent to few-percent level
  - g_A, QED corrections necessary to relate experimental measurement to LQCD results at percent-level
    Cirigliano, de Vries, Hayen, Mereghetti, AWL, 2202.10439
- Control over specific flavor decomposition
  see e.g. Alexandrou et al, ETMC, PRD 104 (2021) 2106.13468
- Nucleon (quasi-)elastic form factors up to Q^2~1-2 GeV^2
  PNDME, NME, RQCD, ETMC, Mainz, LHPC, PACS, CalLat
- Investigations of higher momentum - still "kinda" R&D
  RQCD, LHPC, …
- Very preliminary NN matrix elements
  big caveat - based on assumptions that seem wrong - NN controversy
  no connection to physical pion mass or continuum limit



Outline

- Introduction to lattice QCD
- LQCD systematics for nucleon matrix elements
- Single nucleon quasi-elastic results from LQCD
- Phenomenological Impact
- Future advancements

Introduction to LQCD

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} DU \mathcal{O}(t)\mathcal{O}(0) e^{iS_M[\bar{\psi},\psi,U]}$$

$$= \frac{1}{\mathcal{Z}}\int DU \det\left(\mathrm{i}\not{D} - \mathrm{M}\right) \mathcal{O}(\mathrm{t})\mathcal{O}(0) \mathrm{e}^{\mathrm{i}\mathrm{S}_\mathrm{M}[\bar{\psi},\psi,\mathrm{U}]}$$

Slide adapted from E. Berkowitz

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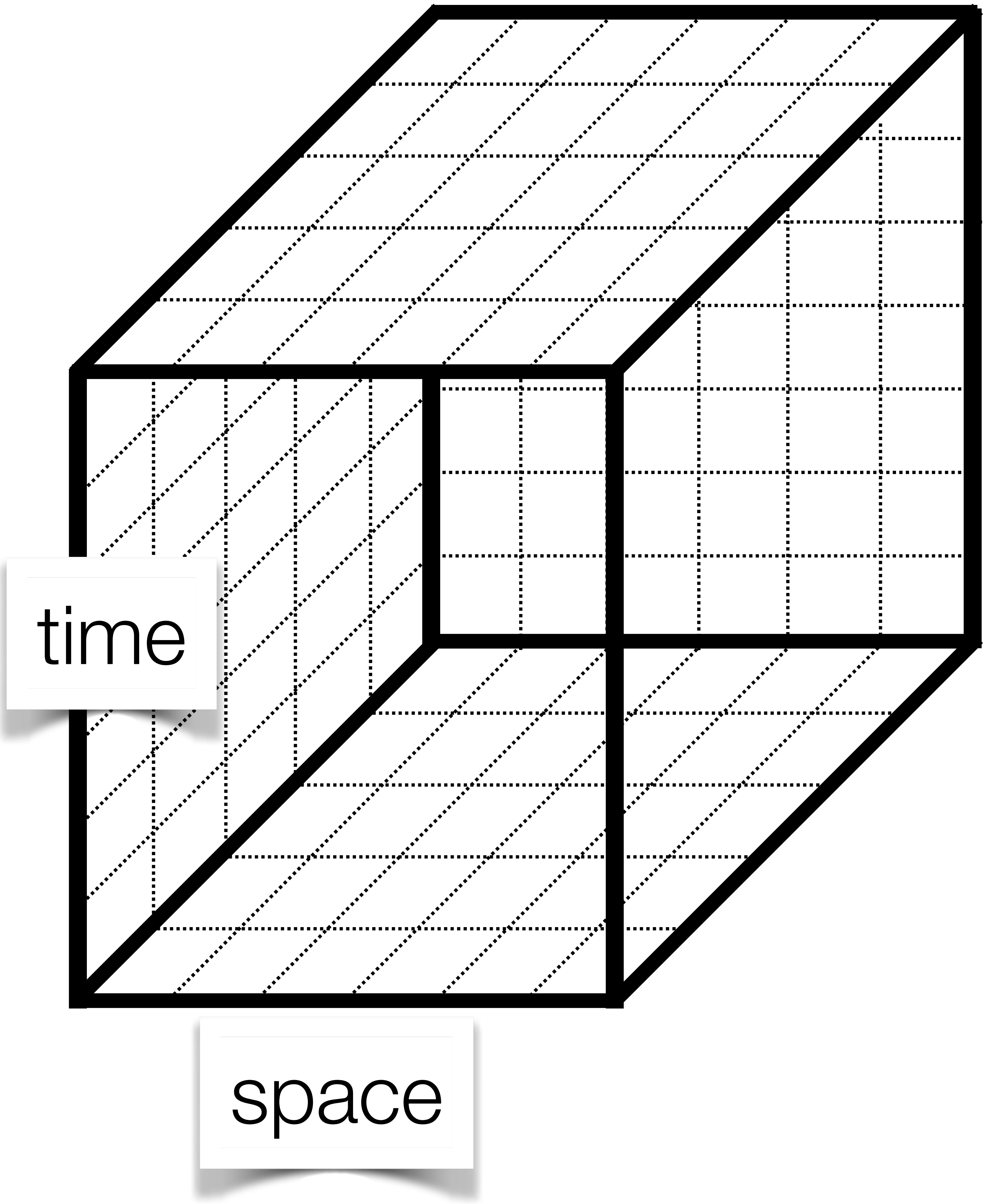
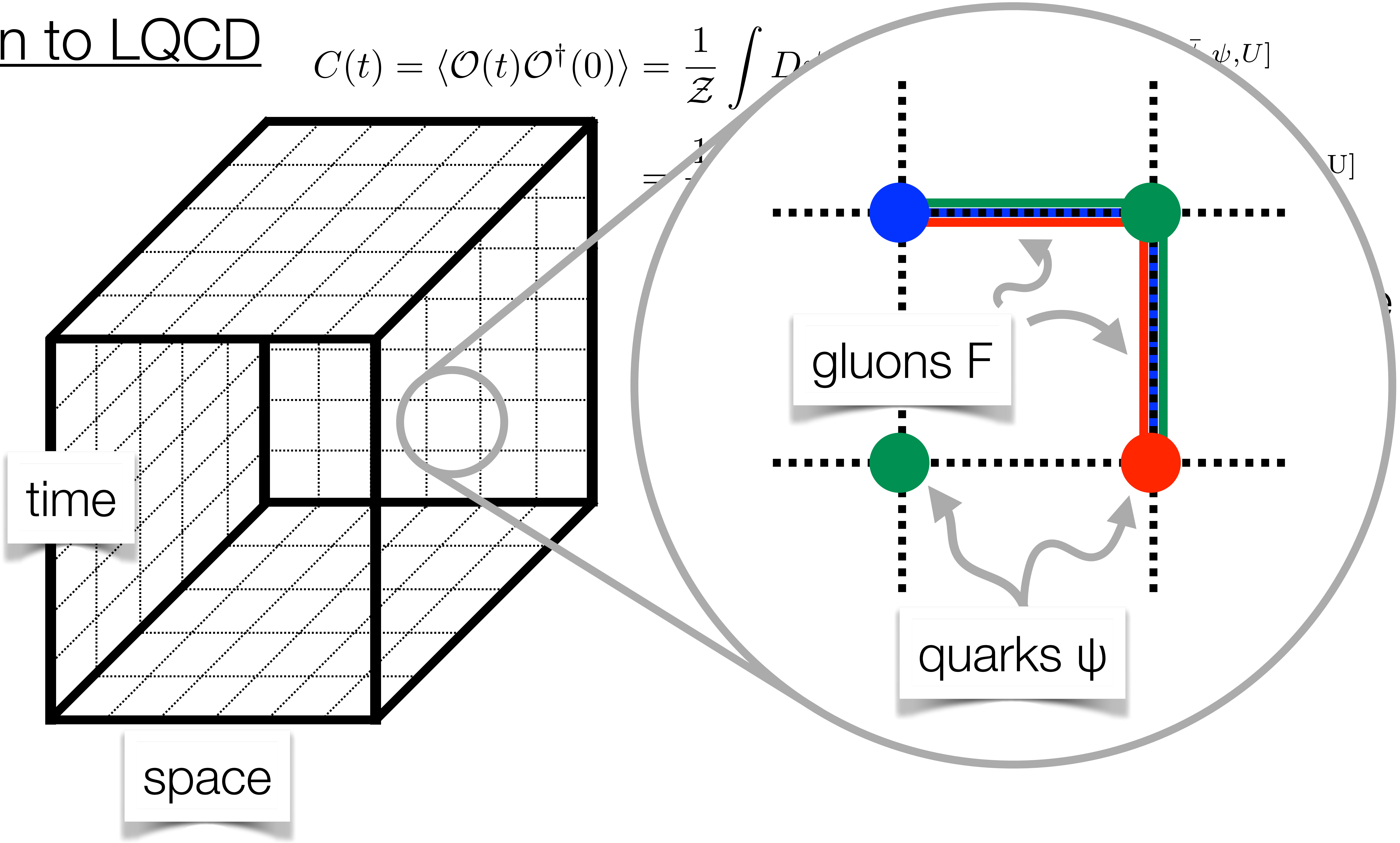


lattice
finite volume

Introduction to LQCD

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \frac{1}{\mathcal{Z}}\int D$$



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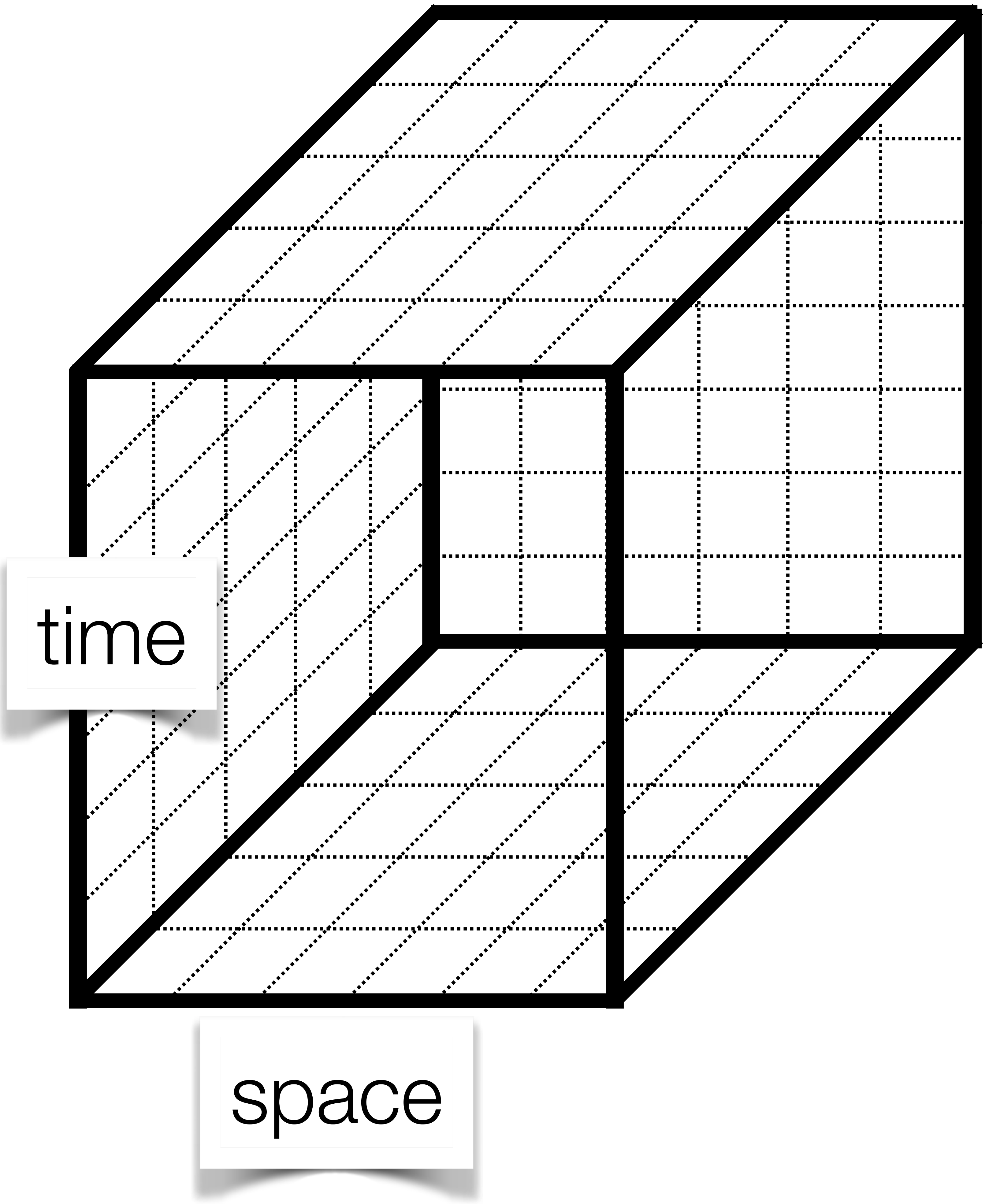

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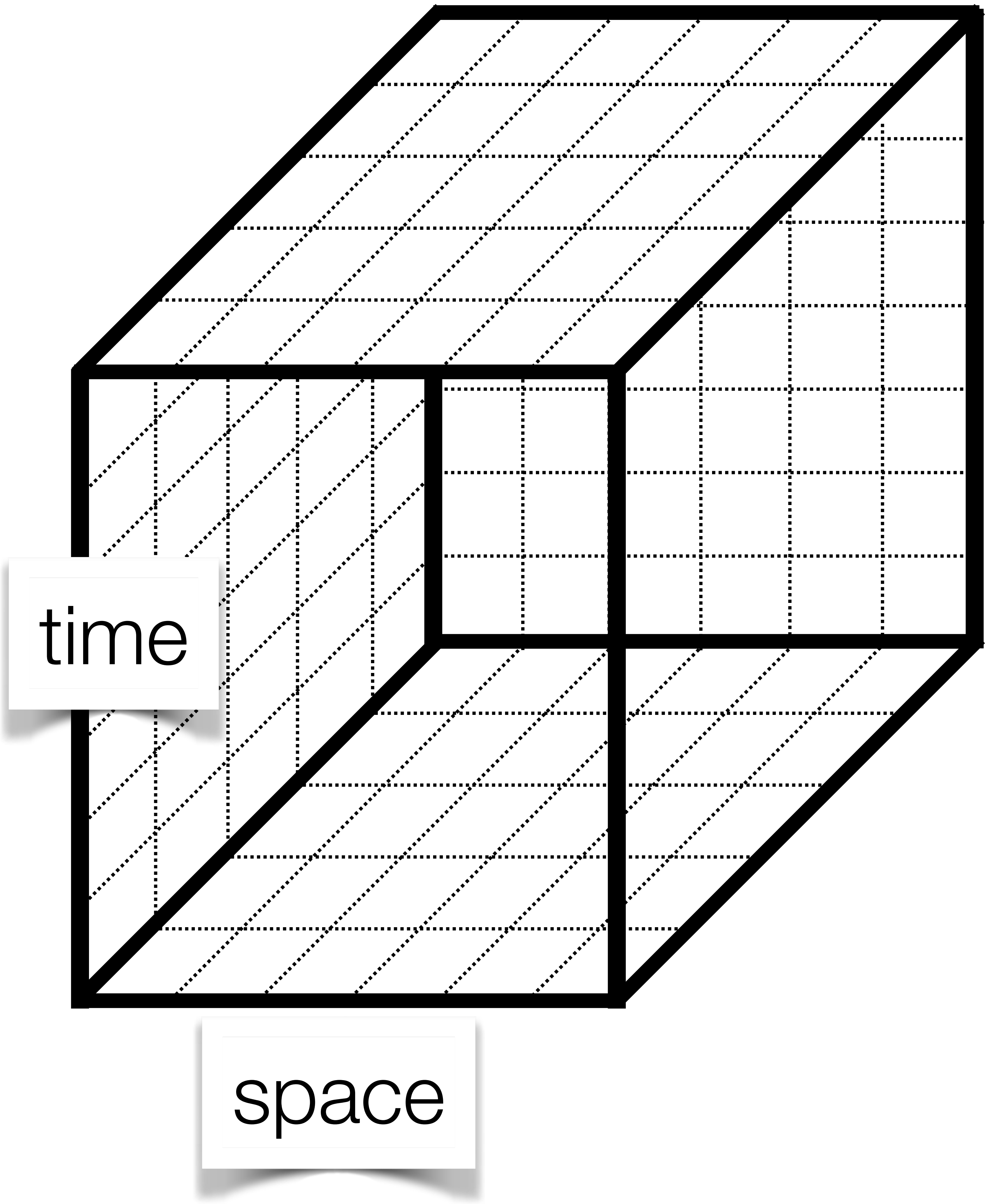

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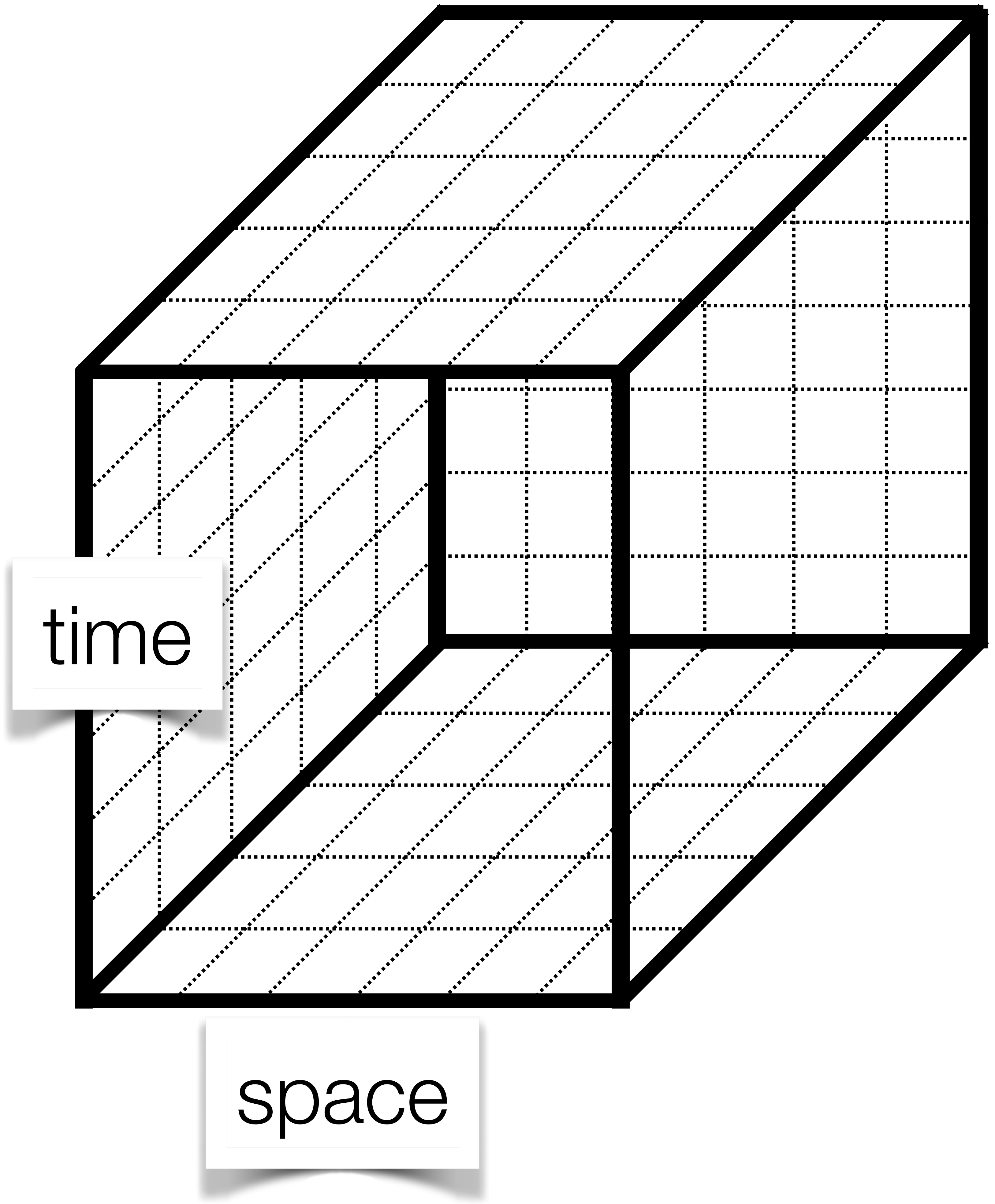

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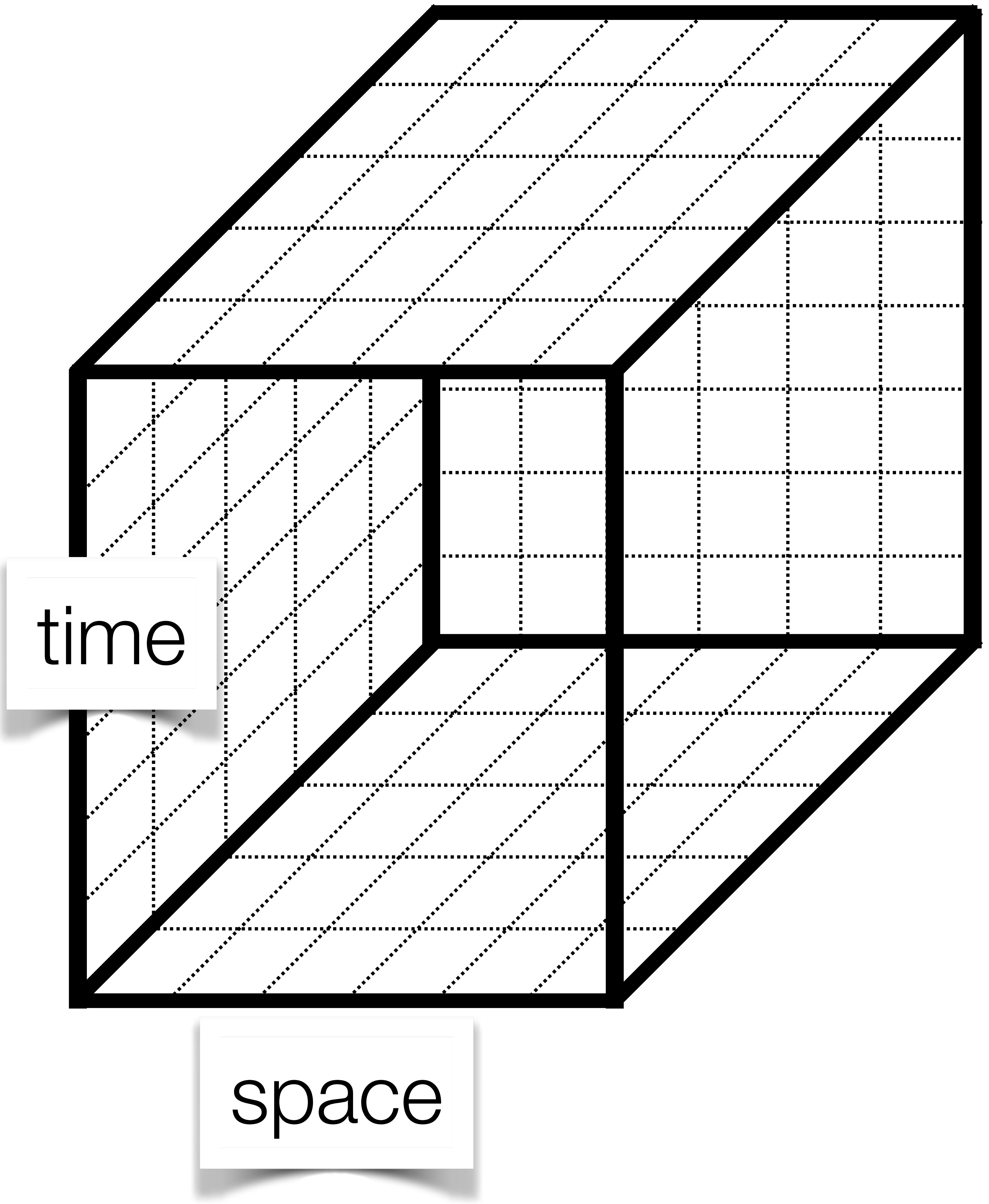

$$\{U_1, U_2, U_3, \ldots, U_N\}$$

Markov Chain Monte Carlo

Slide adapted from E. Berkowitz

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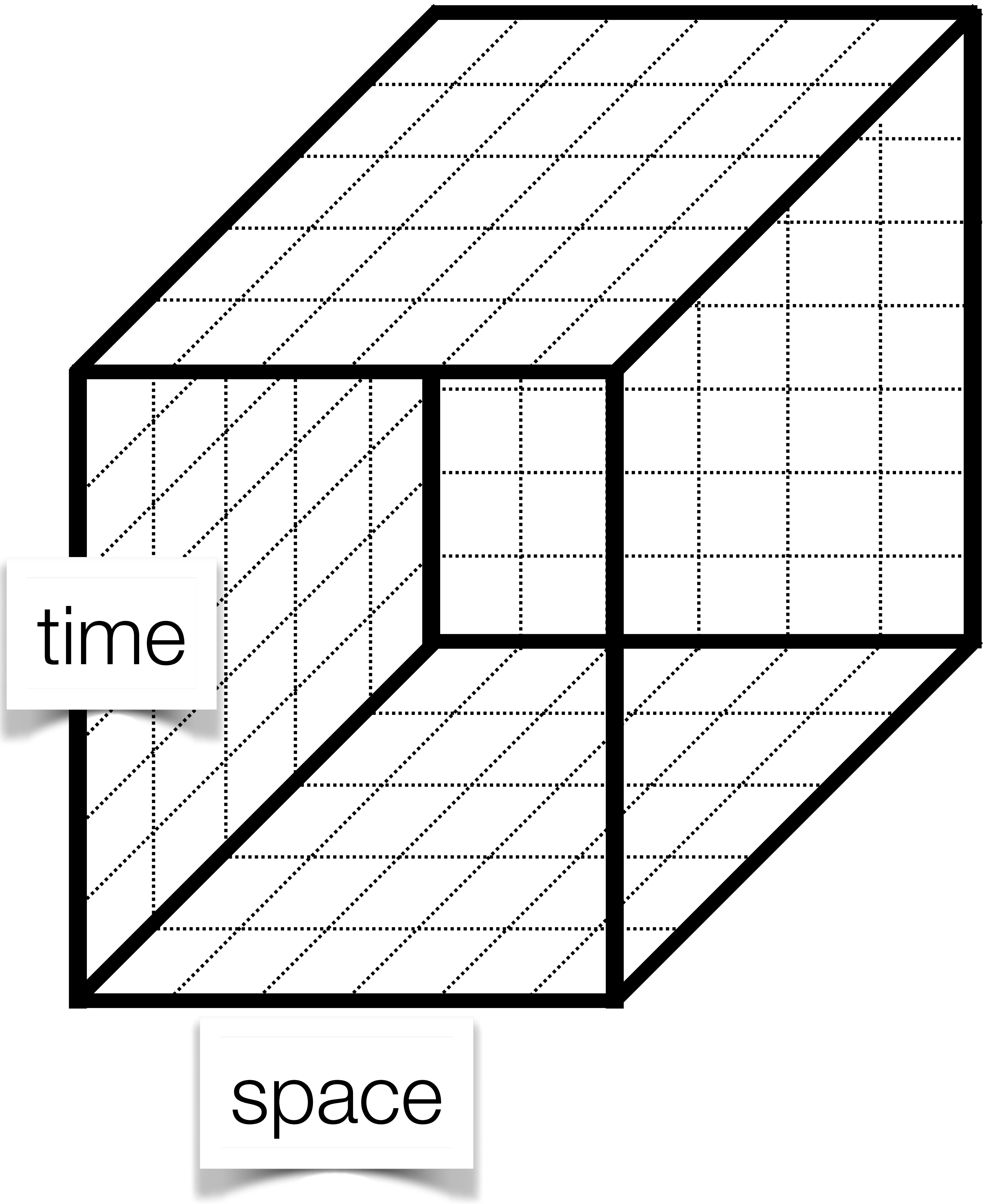

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$$\approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i]$$

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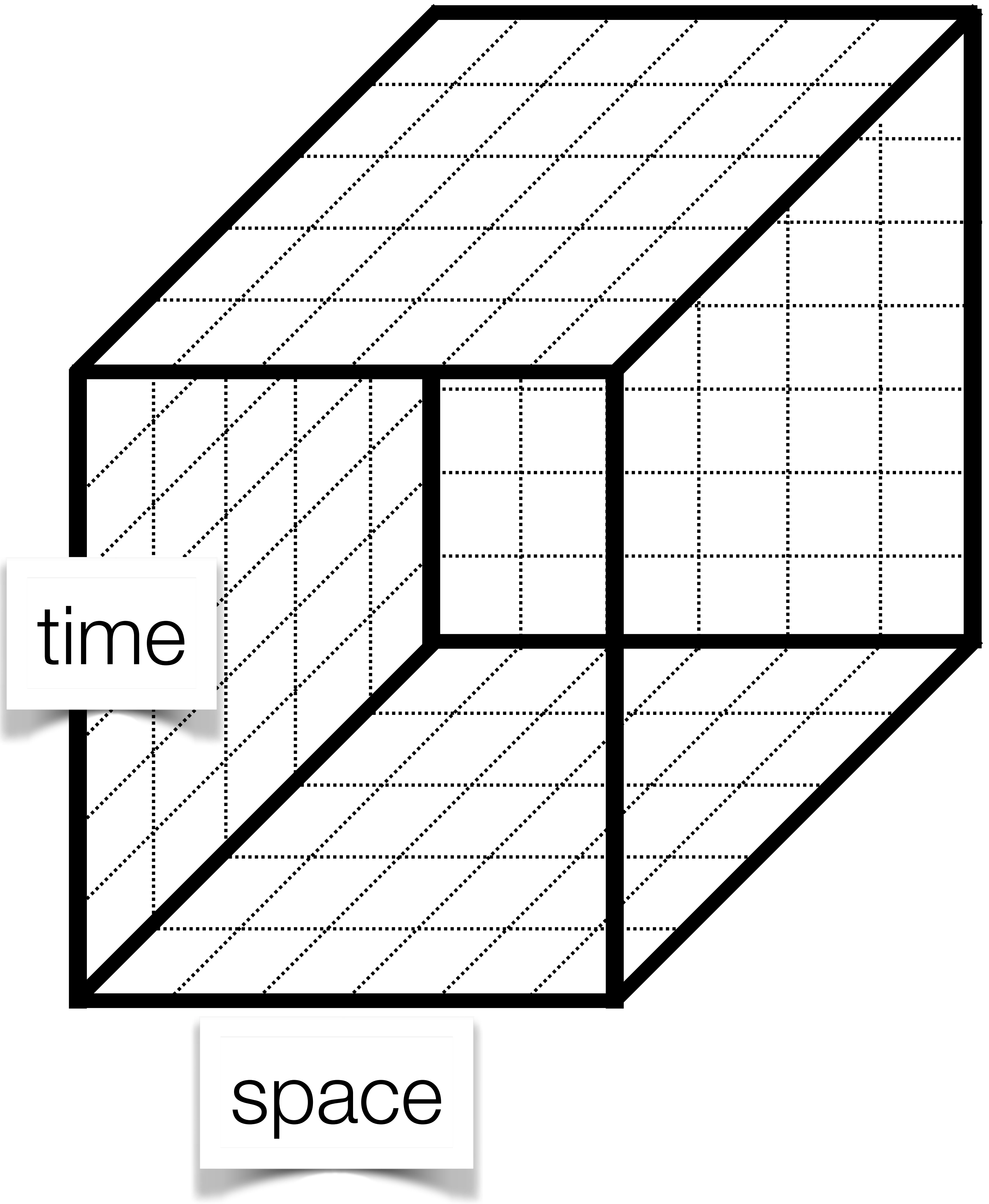

$$\{U_1, U_2, U_3, \ldots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N}\sum_{i=1}^{N} \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$$

Slide adapted from E. Berkowitz

Introduction to LQCD

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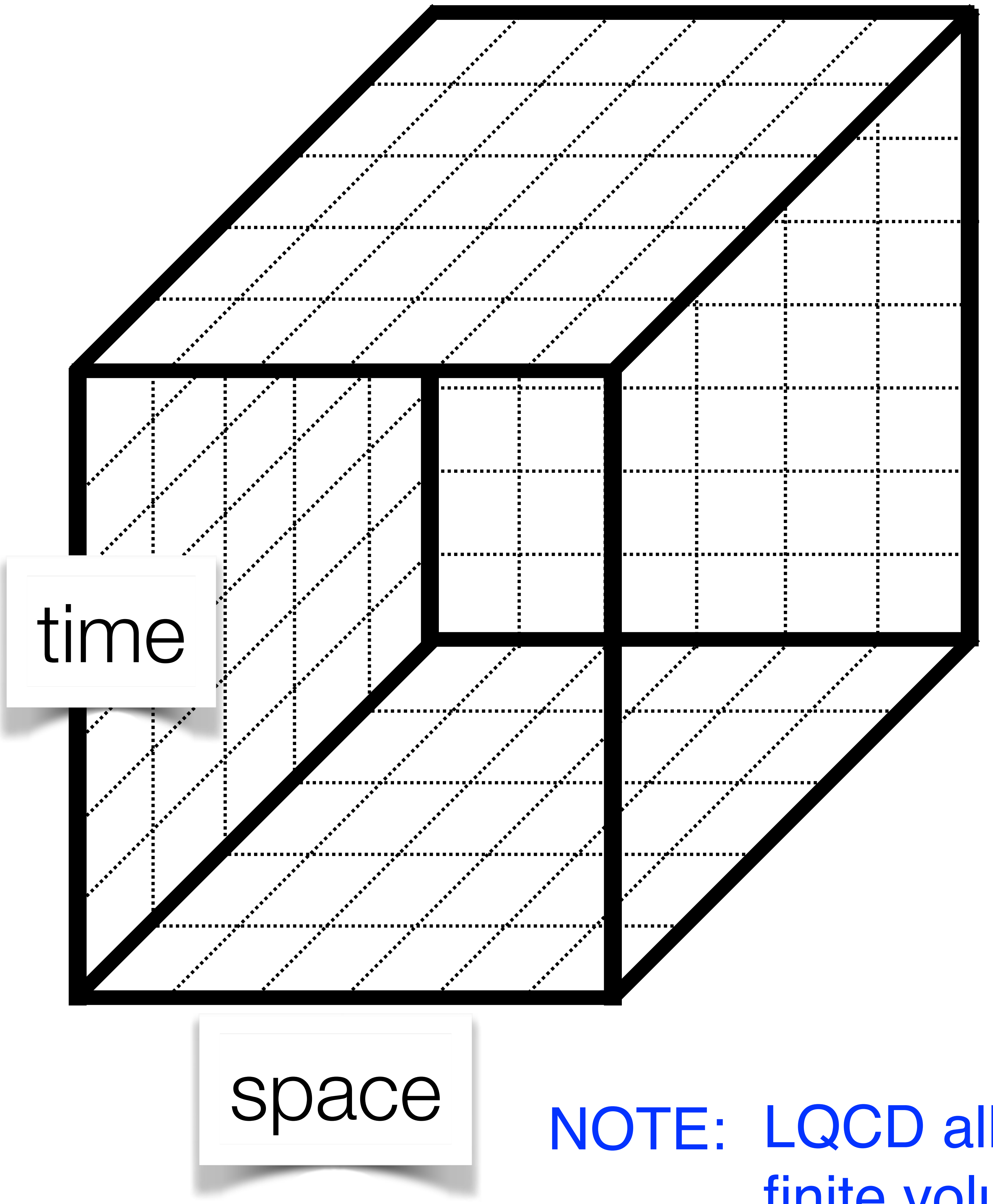

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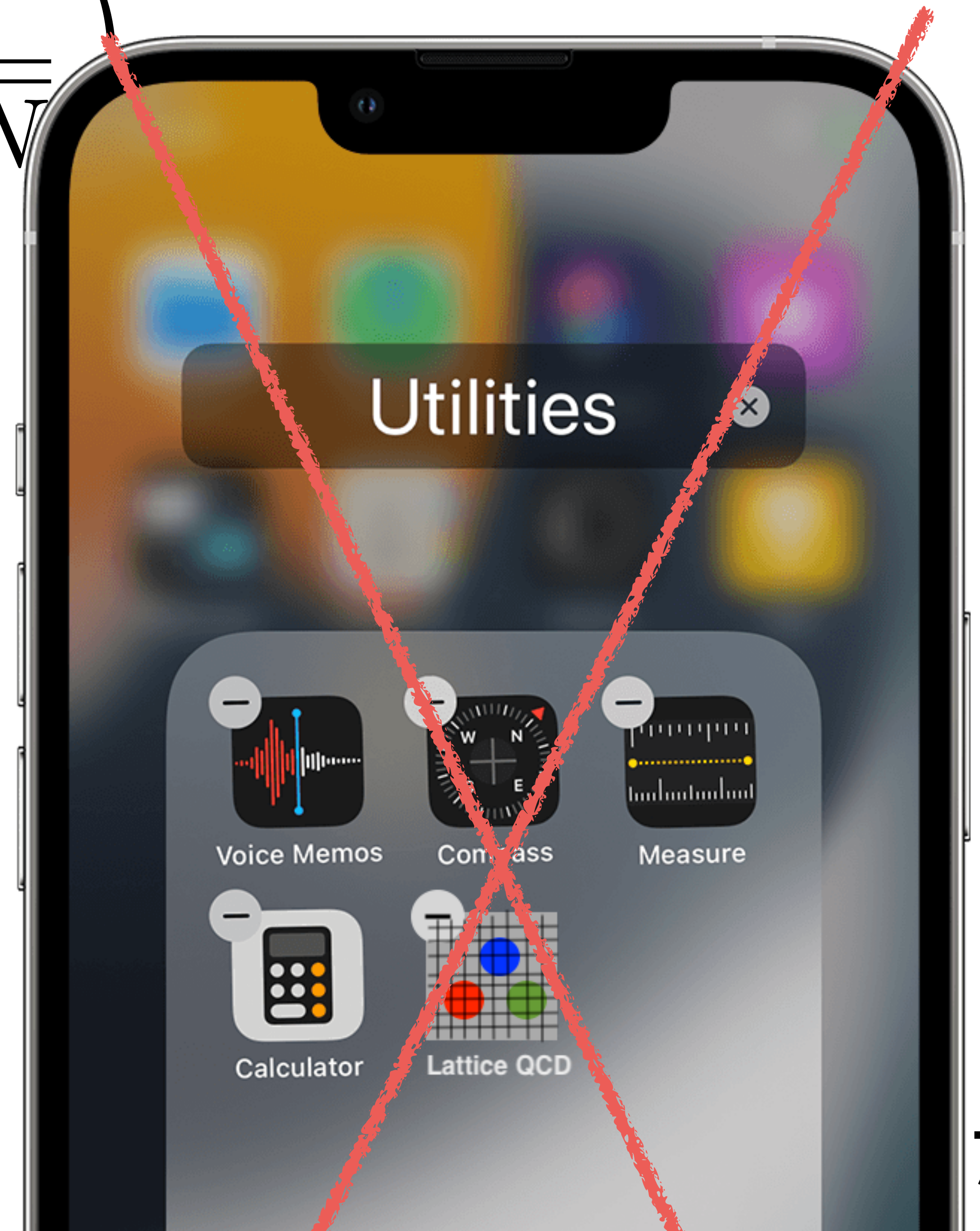
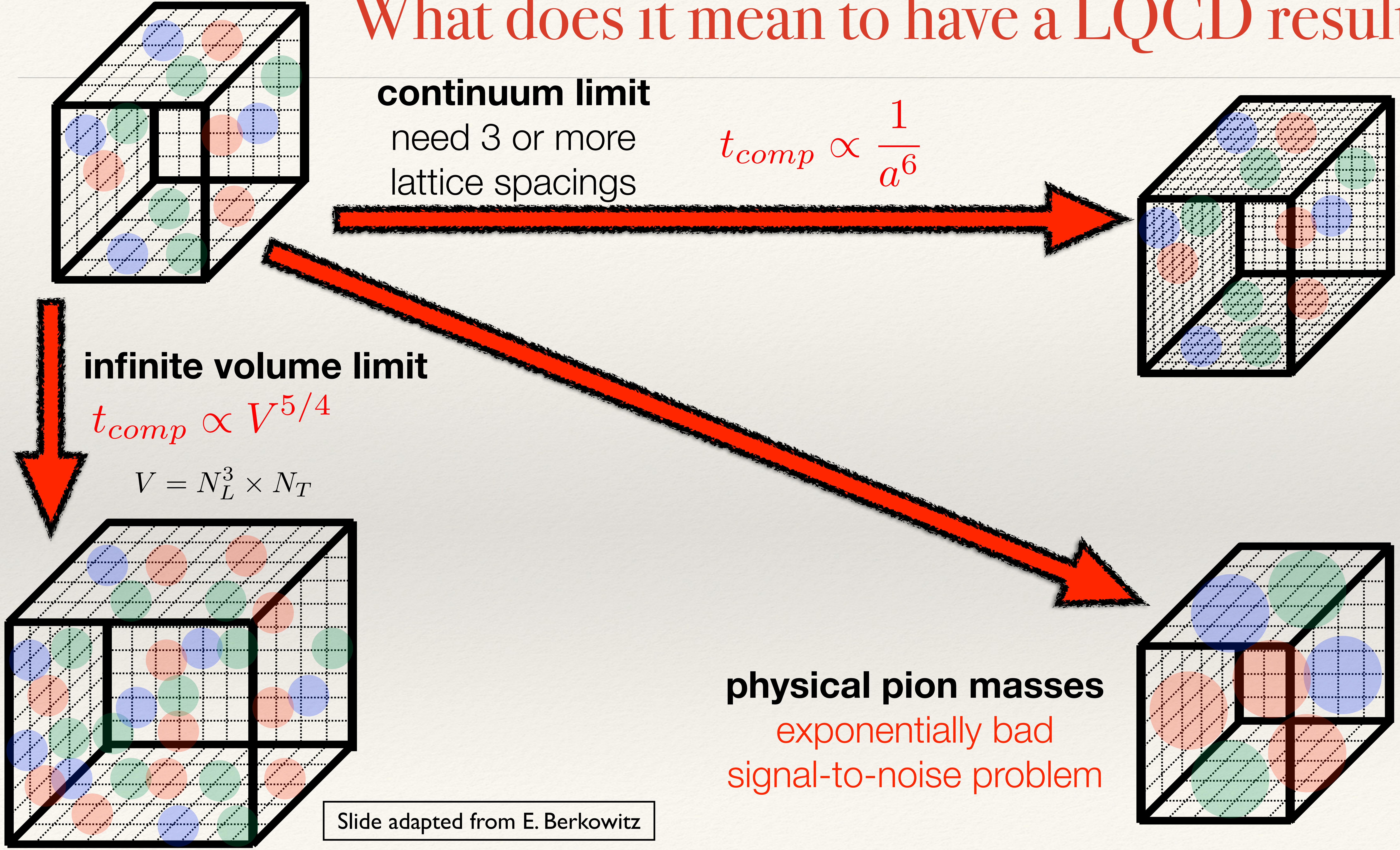


NOTE: LQCD allows us to compute Euclidean space, finite volume, correlation functions

Non-trivial numerical analysis (and sometimes formalism) to extract spectrum, matrix elements, form factors, …

Slide adapted from E. Berkowitz

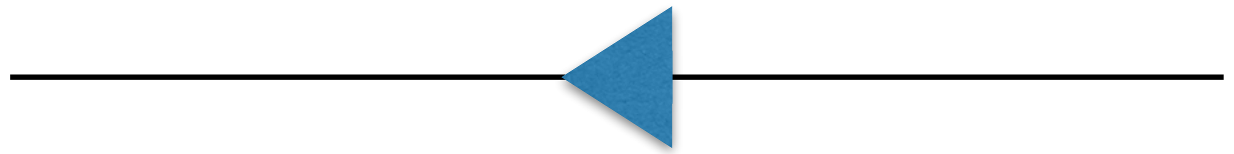
What does it mean to have a LQCD result?



LQCD challenges for NP

Most difficult challenge: an exponentially bad signal-to-noise problem

Lepage, TASI 1989

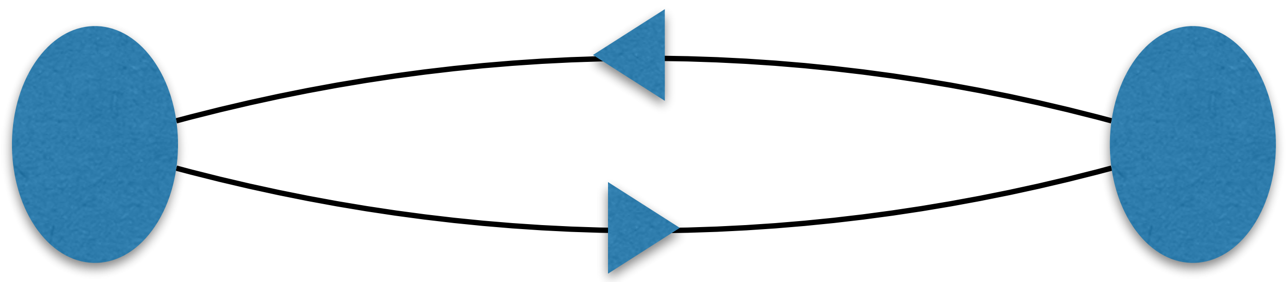


$$\sim e^{-\frac{1}{2}m_\pi t} + e^{-\frac{1}{3}m_N t} + \cdots$$

Each **quark propagator** carries information about pions and nucleons (conversations with David Kaplan)

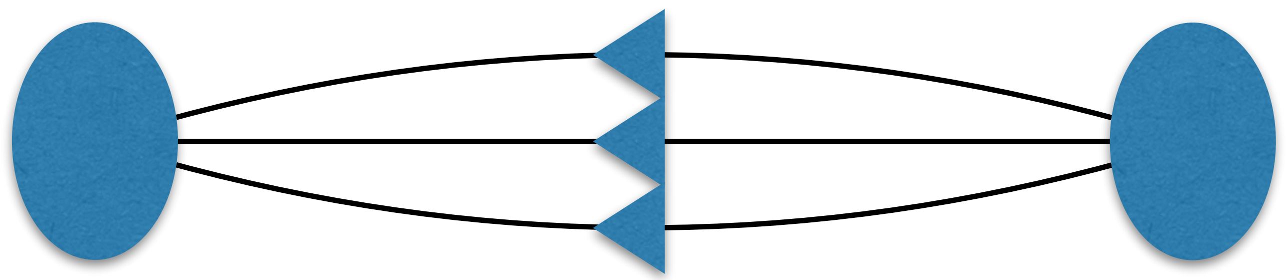
$$\lambda_\pi(t) \gg \lambda_N(t)$$

$$\lambda_i(t) \sim e^{-E_i t}$$



$$\bar{d}\gamma_5 u : C(t) = A_\pi e^{-m_\pi t} + \cdots$$

Large pion eigenvalues must cancel to expose small nucleon eigenvalues $e^{-m_N t} \ll e^{-m_\pi t}$



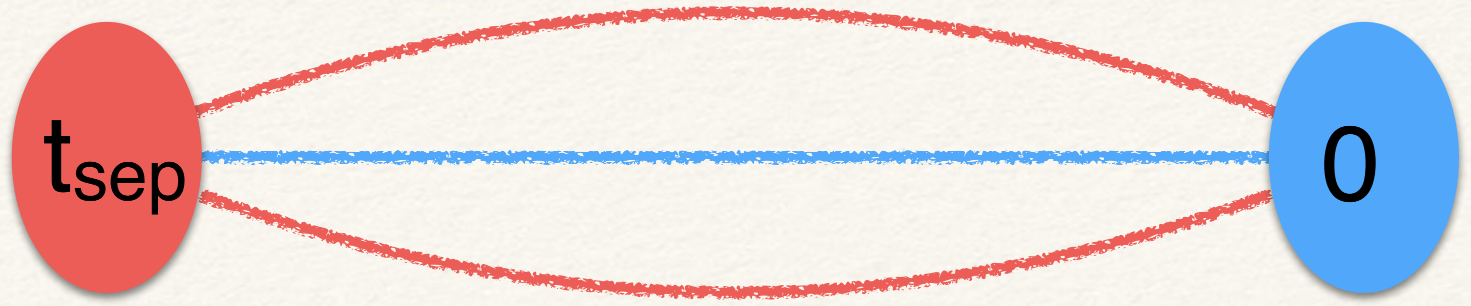
$$(u^T C\gamma_5 d)u : C(t) = A_N e^{-m_N t} + \cdots$$

$$\frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N} \exp\left[-A\left(m_N - \frac{3}{2}m_\pi\right)t\right]$$

→ exponential noise power-law statistics

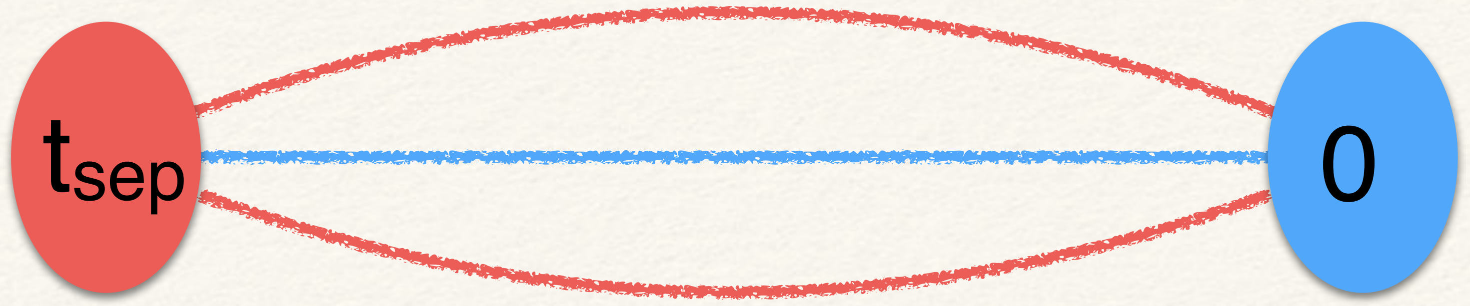
LQCD: 2 point functions

$$C(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} \langle \Omega | O(t,\mathbf{x}) O^\dagger(0,\mathbf{0}) | \Omega \rangle$$



$$\begin{aligned}
C(t) &= \sum_{\mathbf{x}} \langle \Omega | O(t,\mathbf{x}) O^\dagger(0,\mathbf{0}) | \Omega \rangle && \text{focus on 0-momentum} \\
&= \sum_{\mathbf{x}} \langle \Omega | e^{\hat{H}t} O(0,\mathbf{x}) e^{-\hat{H}t} O^\dagger(0,\mathbf{0}) | \Omega \rangle && \text{time-evolve operator} \\
&= \sum_{n} \sum_{\mathbf{x}} \langle \Omega | e^{\hat{H}t} O(0,\mathbf{x}) e^{-\hat{H}t} | n \rangle \langle n | O^\dagger(0,\mathbf{0}) | \Omega \rangle && \text{multiply by 1,} \quad 1 = \sum_n |n\rangle\langle n| \\
&= \sum_{n} e^{-E_n t} \sum_{\mathbf{x}} \langle \Omega | O(0,\mathbf{x}) | n \rangle \langle n | O^\dagger(0,\mathbf{0}) | \Omega \rangle && \text{define vacuum to have 0-energy} \\
&= \sum_{n} e^{-E_n(\mathbf{p}=0)t} \langle \Omega | O(0) | n, \mathbf{p}=0 \rangle \langle n, \mathbf{p}=0 | O^\dagger(0) | \Omega \rangle \\
&= \sum_{n} e^{-E_n t} z_n z_n^\dagger && \text{sum of exponentials}
\end{aligned}$$

LQCD: 2 point functions



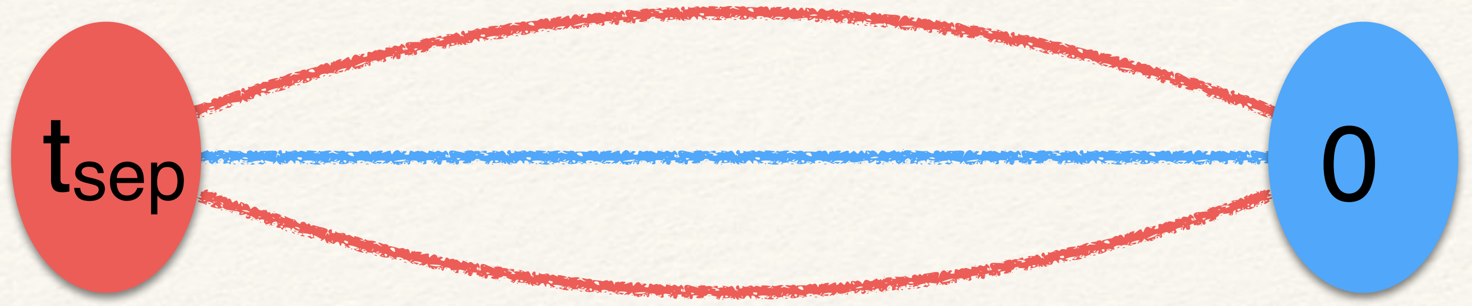
$$C(t,\mathbf{p}=0) = \sum_{\mathbf{x}} \langle \Omega | N(t,\mathbf{x}) N^\dagger(0,\mathbf{0}) | \Omega \rangle$$
$$= A_0 e^{-E_0 t}\left(1 + \sum_{n>0} r_n e^{-\Delta_{n0} t}\right)$$
$$\Delta_{n0} = E_n - E_0$$

$$m_{\text{eff}}(t) = \ln\left(\frac{C(t)}{C(t+1)}\right) \underbrace{\longrightarrow}_{\text{large } t} E_0 + \sum_{n>0} r_n \left(e^{-\Delta_{n0} t} - e^{-\Delta_{n0} t + 1}\right)$$

NOTE: if the creation operator is conjugate to the annihilation operator

$$r_n \geq 0$$

LQCD: 2 point functions



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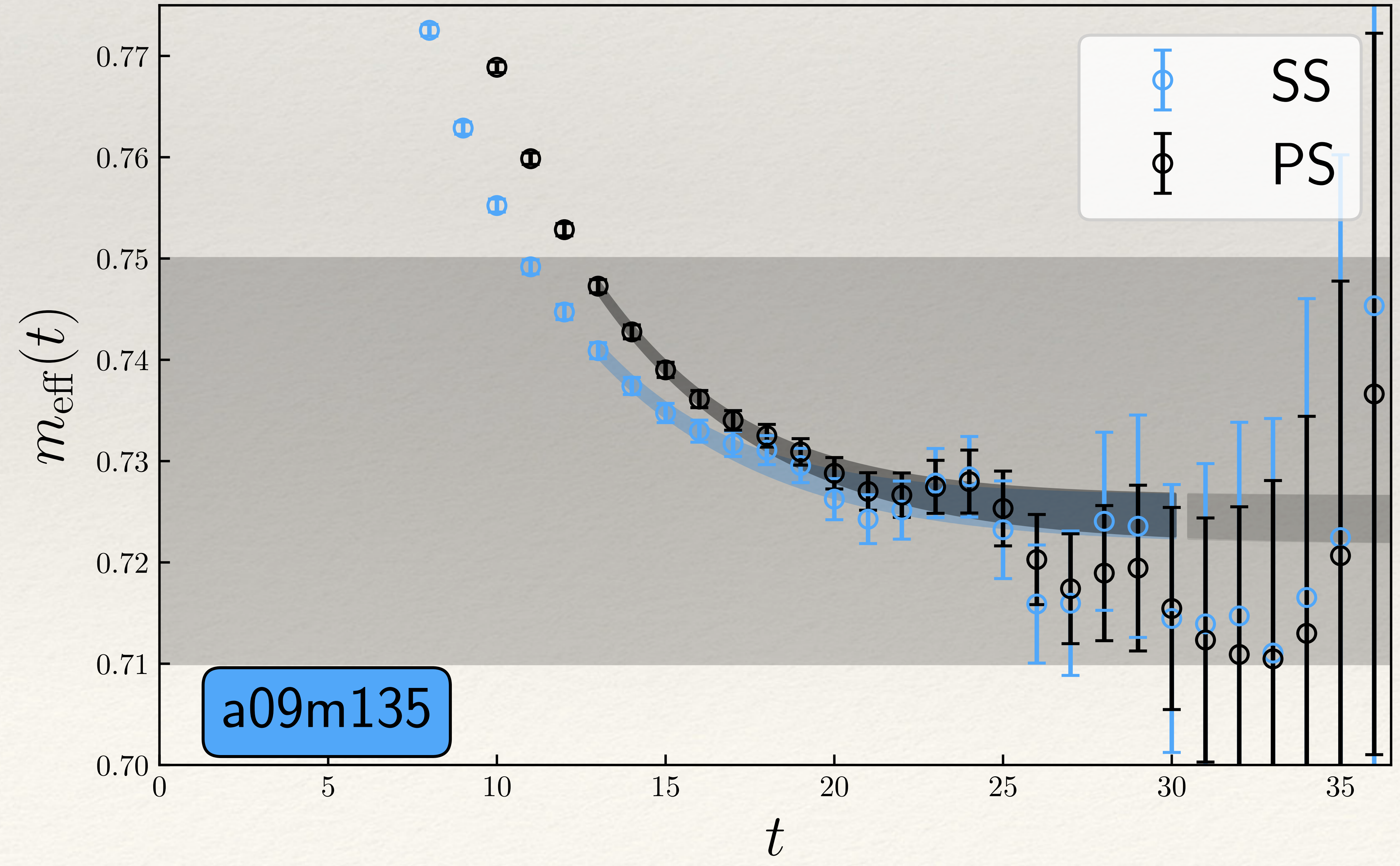
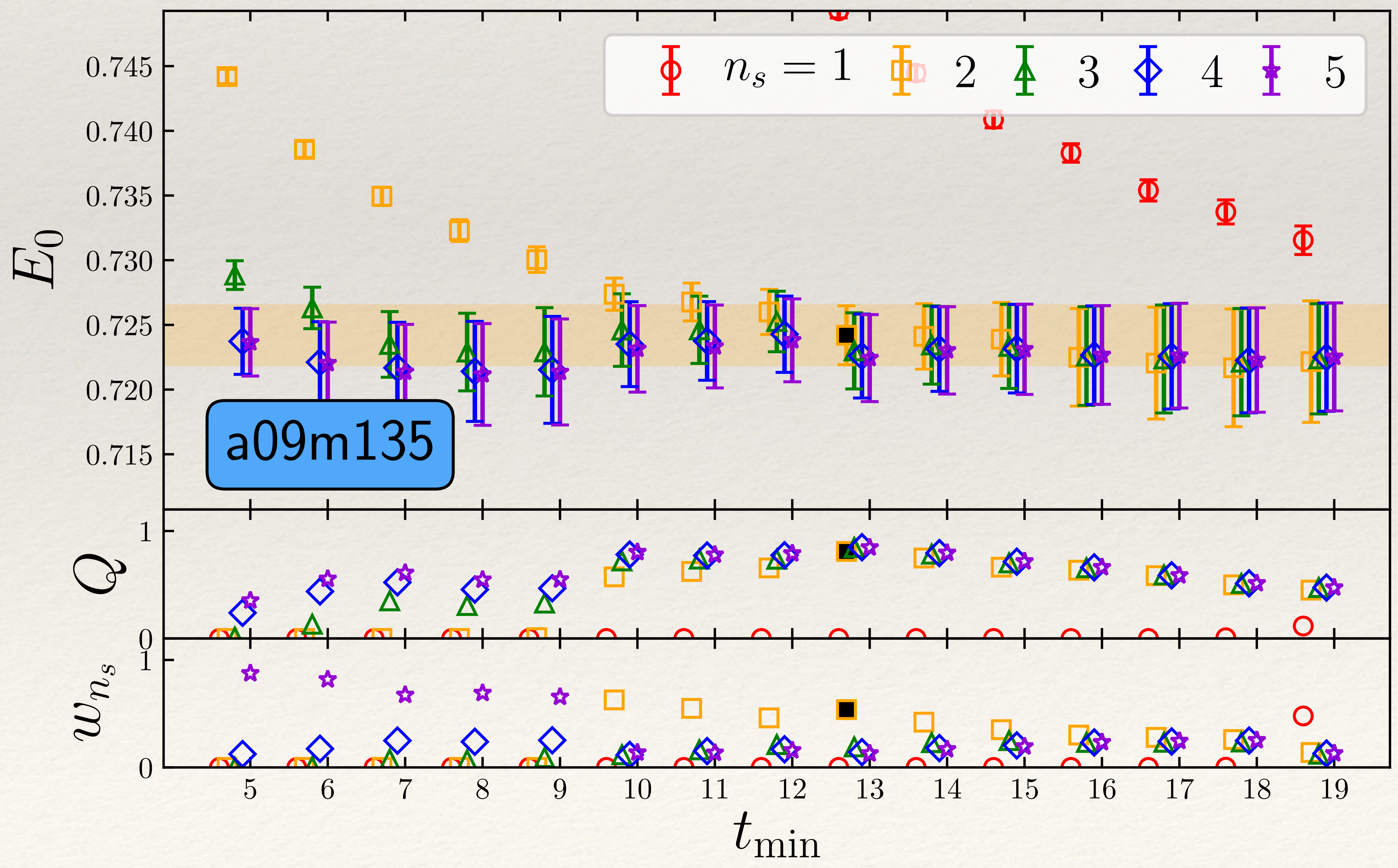
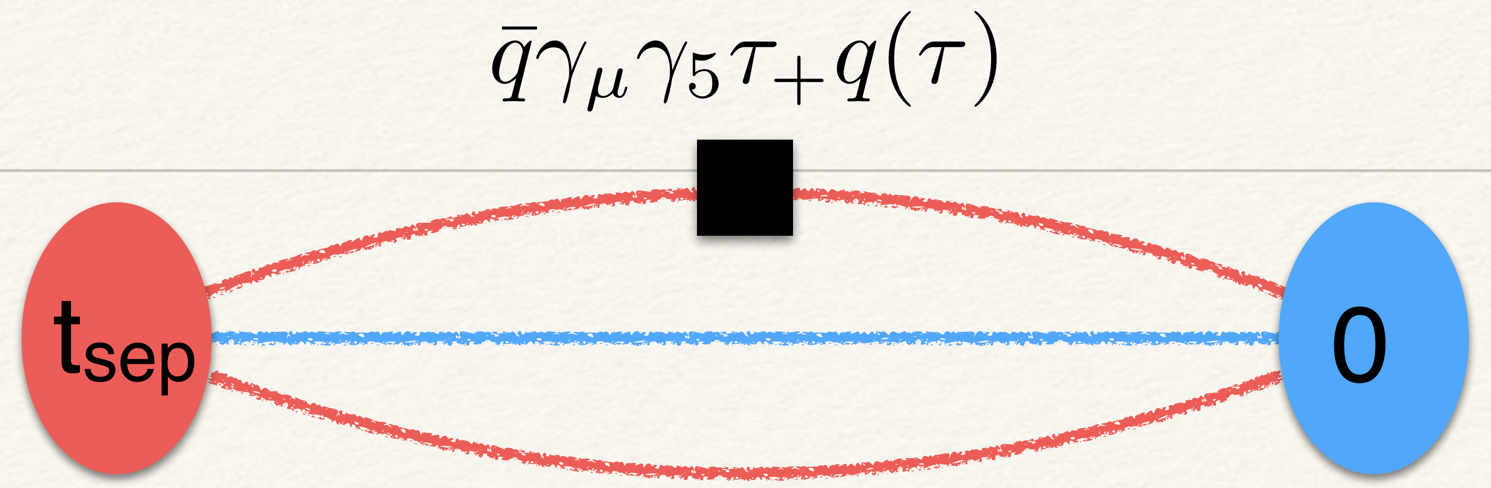
but… signal-to-noise - can not simply "wait till long time" to get ground state (g.s.)





LQCD: 3 point functions

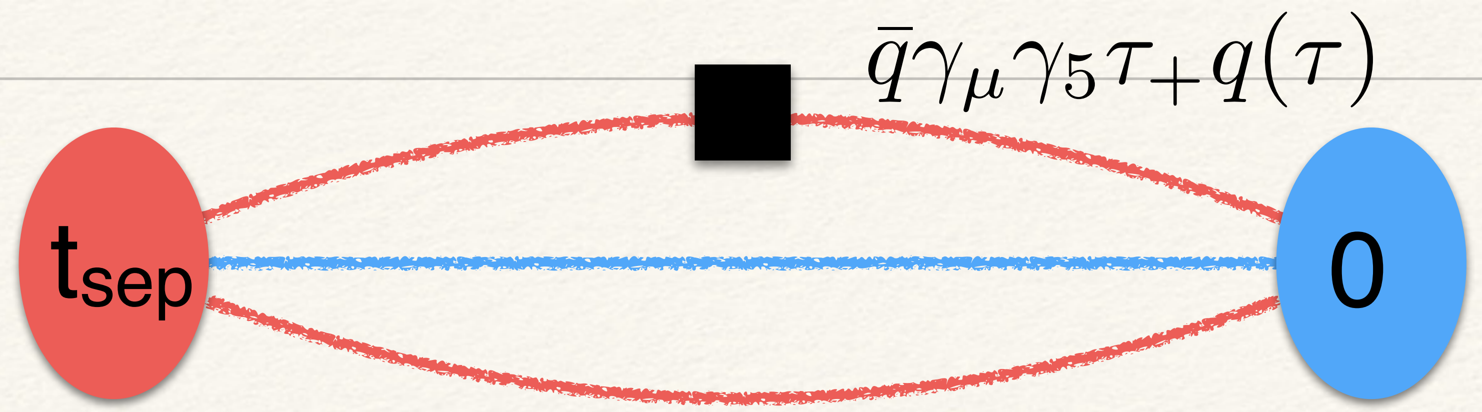


- The most common method (sub-optimal) to compute nucleon matrix elements
  - For a few values of t, compute the 3-point function for all τ

$$C_\Gamma(t,\tau,\mathbf{p},\mathbf{q}) = \sum_{\mathbf{y},\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{x}} \langle\Omega|N(t,\mathbf{y})j_\Gamma(\tau,\mathbf{x})N^\dagger(0,\mathbf{0})|\Omega\rangle$$

  - Each choice of t is a new, expensive computation
  - Ideally, $t \sim 2\, t_{2pt\text{-}gs}$, but, S/N prevents that
  - The g.s. matrix-element/form-factor must be determined through an extrapolation in t and τ after numerical analysis

LQCD: 3 point functions



- Consider zero-momentum (p=0) and zero momentum transfer (q=0)

use "multiply by 1" trick

$$1 = \sum_n |n\rangle\langle n|$$

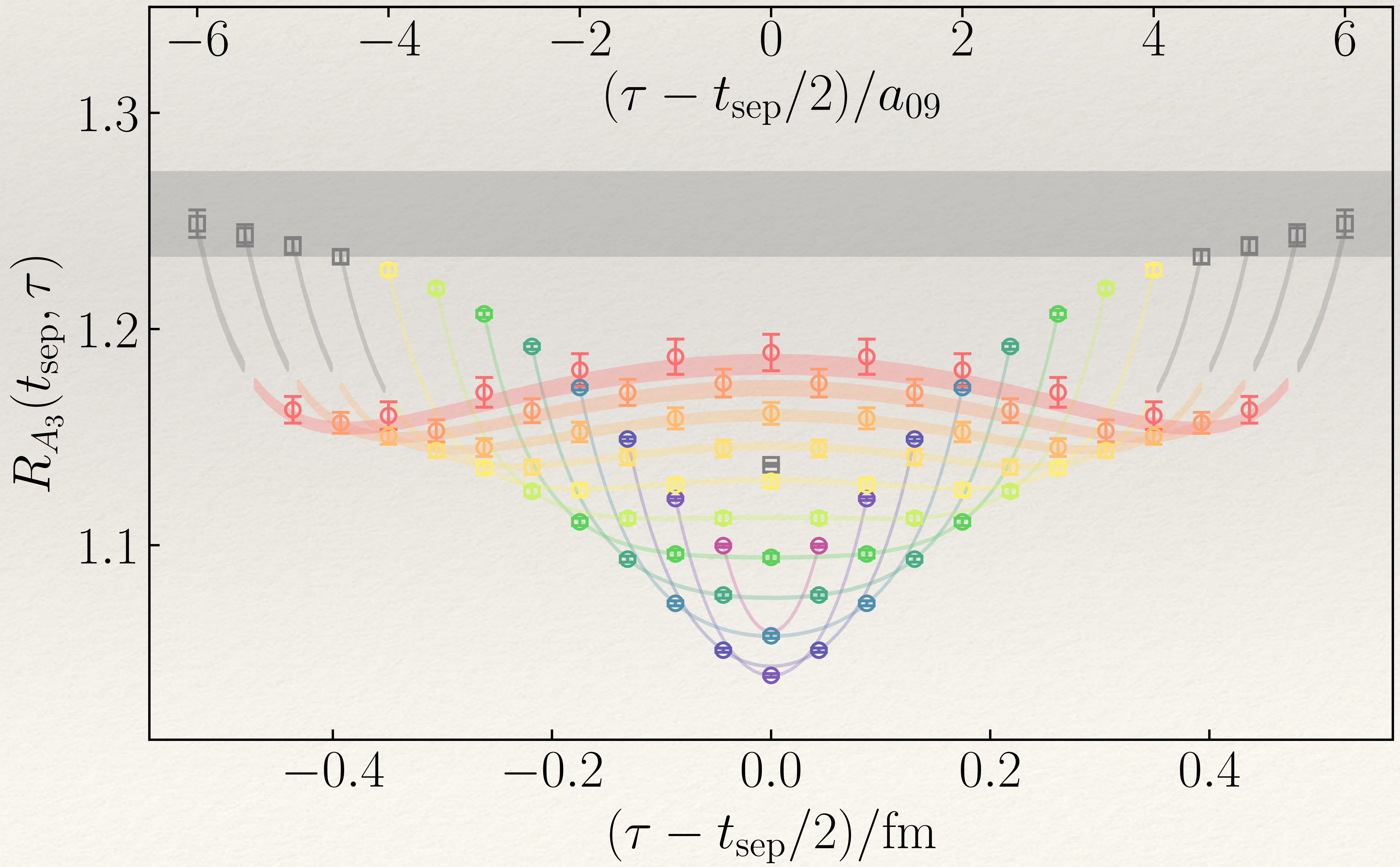
$$C_\Gamma(t,\tau) = \sum_{\mathbf{y},\mathbf{x}} \langle \Omega | N(t,\mathbf{y}) j_\Gamma(\tau,\mathbf{x}) N^\dagger(0,\mathbf{0}) | \Omega \rangle$$

$$= |z_0|^2 g_{00}^\Gamma e^{-E_0 t} + \sum_{n>0} |z_n|^2 g_{nn}^\Gamma e^{-E_n t} + 2 \sum_{n<m} z_n z_m^\dagger g_{nm}^\Gamma e^{-(E_n + \frac{\Delta_{mn}}{2} t)} \cosh\left[\Delta_{mn}\left(\tau - \frac{t}{2}\right)\right]$$

"scattering" (sc) excited states

"transition" (tr) excited states

- scattering excited states only depend on t
- transition excited states depend on t and τ
- NOTE: for intermediate t, there is a conspiracy of excited states that give the appearance of no excited state contamination



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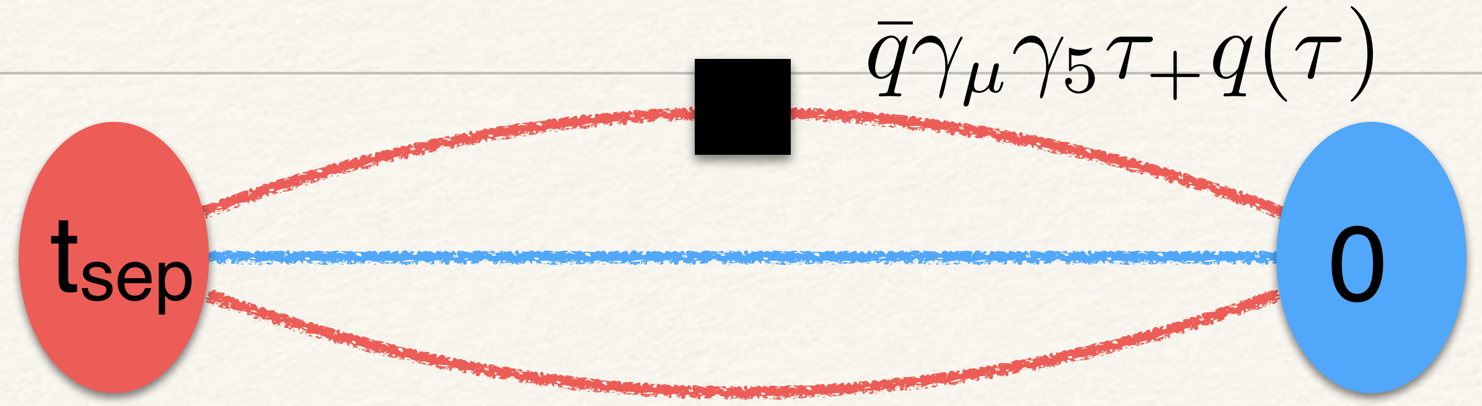
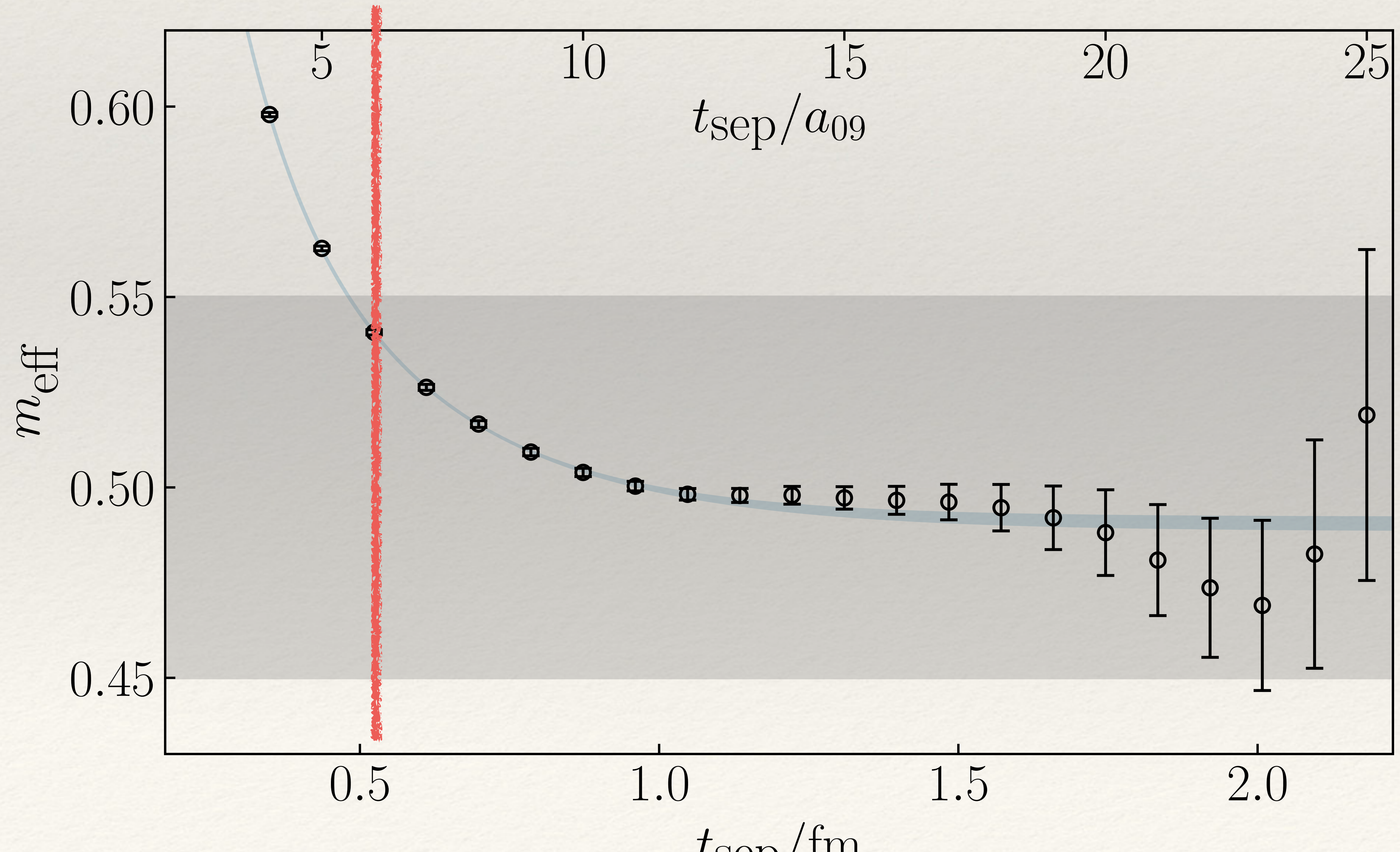
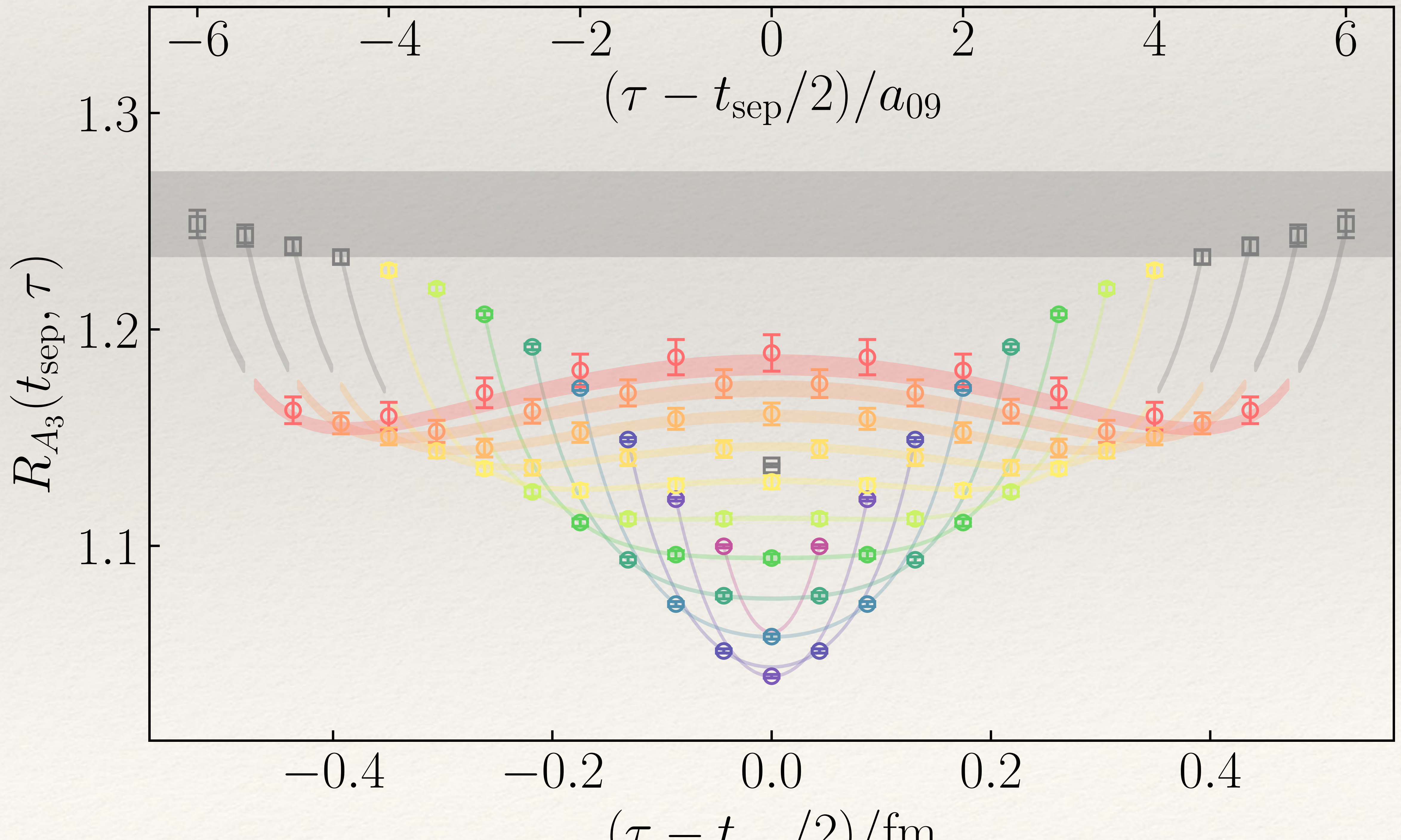


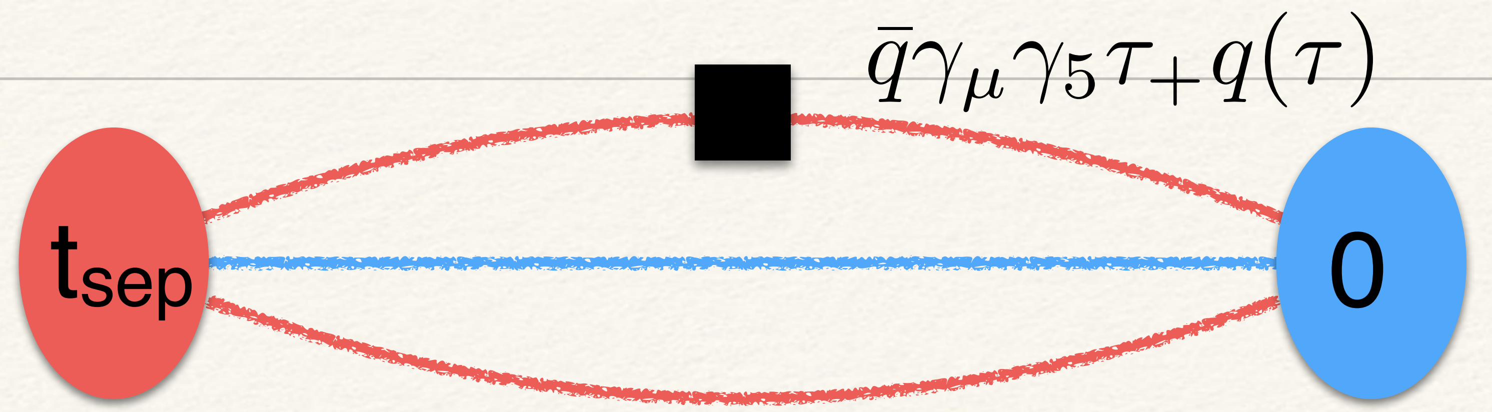
- Consider zero-momentum (p=0) and zero momentum transfer (q=0)

- NOTE: 2pt at t=6 (similar to 3pt with t=12, τ=6) has significant excited state contamination

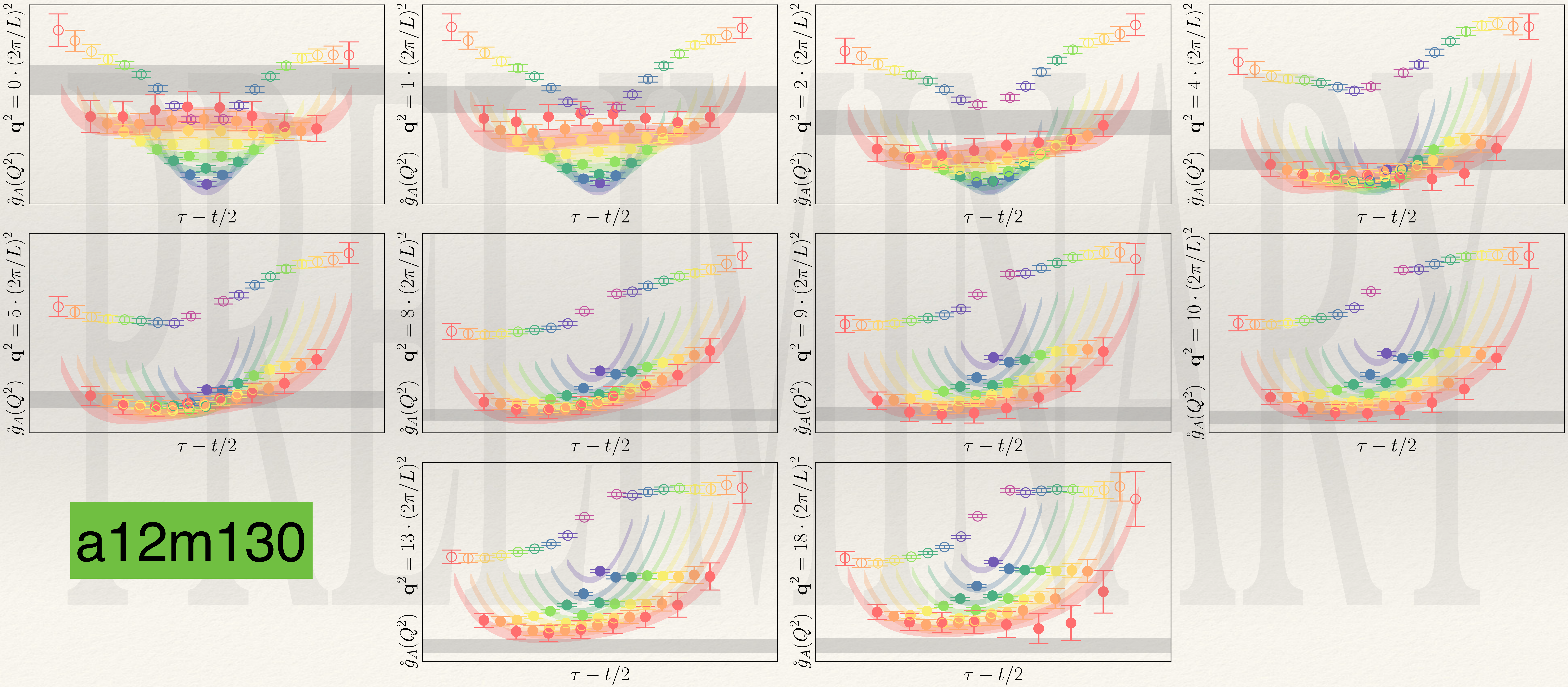




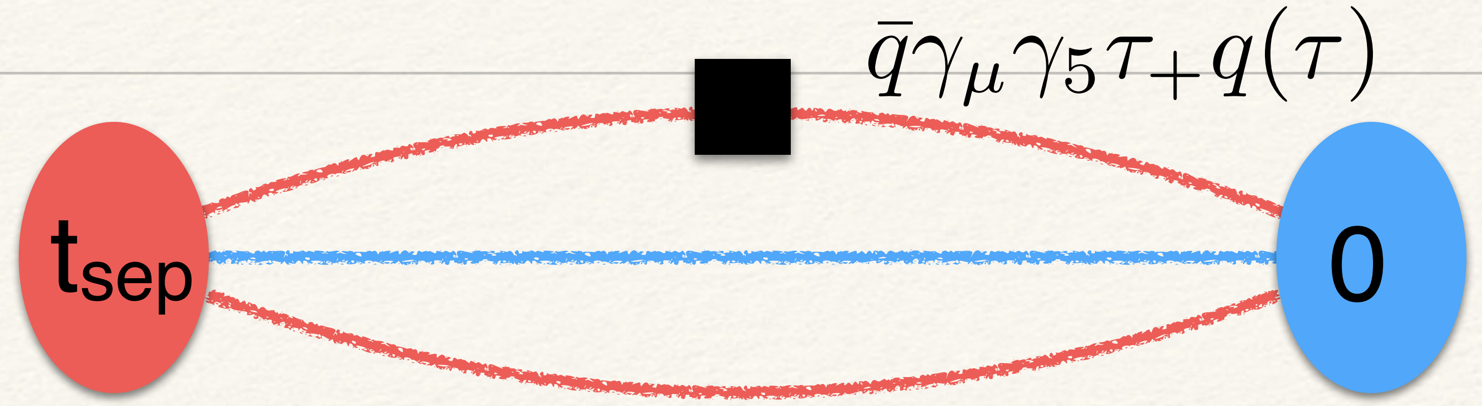
LQCD: 3 point functions



- zero-momentum sink (p=0), non-zero momentum transfer (q≠0)
  source has momentum -q by momentum conservation

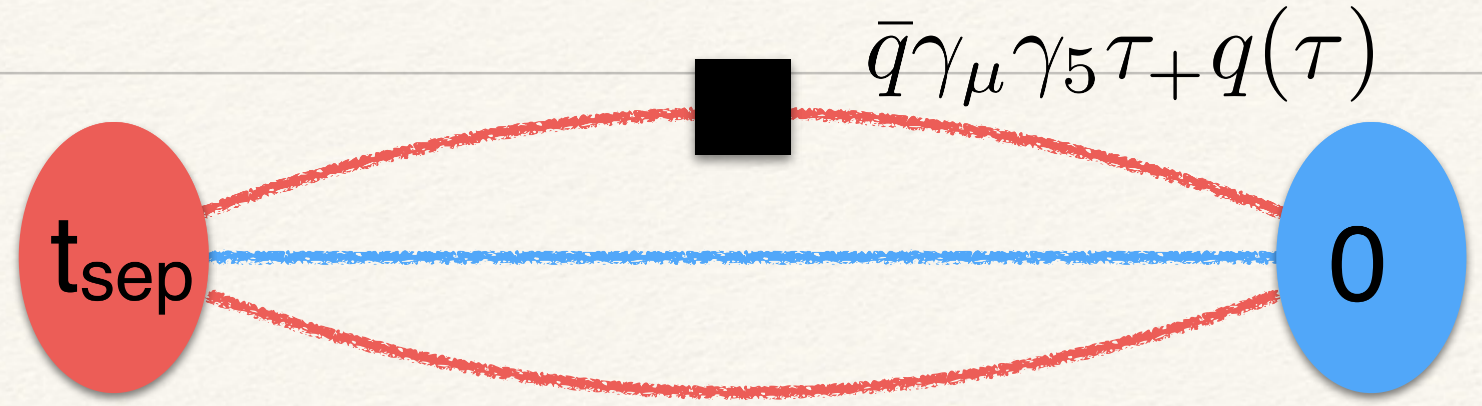


LQCD: 3 point functions



- Lessons learned:
  - Excited state (e.s.) contamination was the main cause of the g_A deficiency
  - It is not practical to pull source/sink far enough apart to eliminate e.s. contamination
  - Need at least 3 values of t (t_{sep}) to control extrapolation
    - most groups advocate high statistics, O($5e^5$), at large t_{sep} values
    - some groups advocate medium statistics, O($2e^4$), with O(10) t_{sep} values
  - Numerical analysis is critical:
    large, highly correlated data sets with under-sampled covariance
    insufficient to get spectrum from 2pt and use that in 3pt analysis - need global analysis
  - Excited states are also important for form-factors - see Sara Collins talk next
  - The operators we use are sub-optimal to eliminate e.s. contamination
    Some analysis tricks can suppress e.s. for forward matrix elements

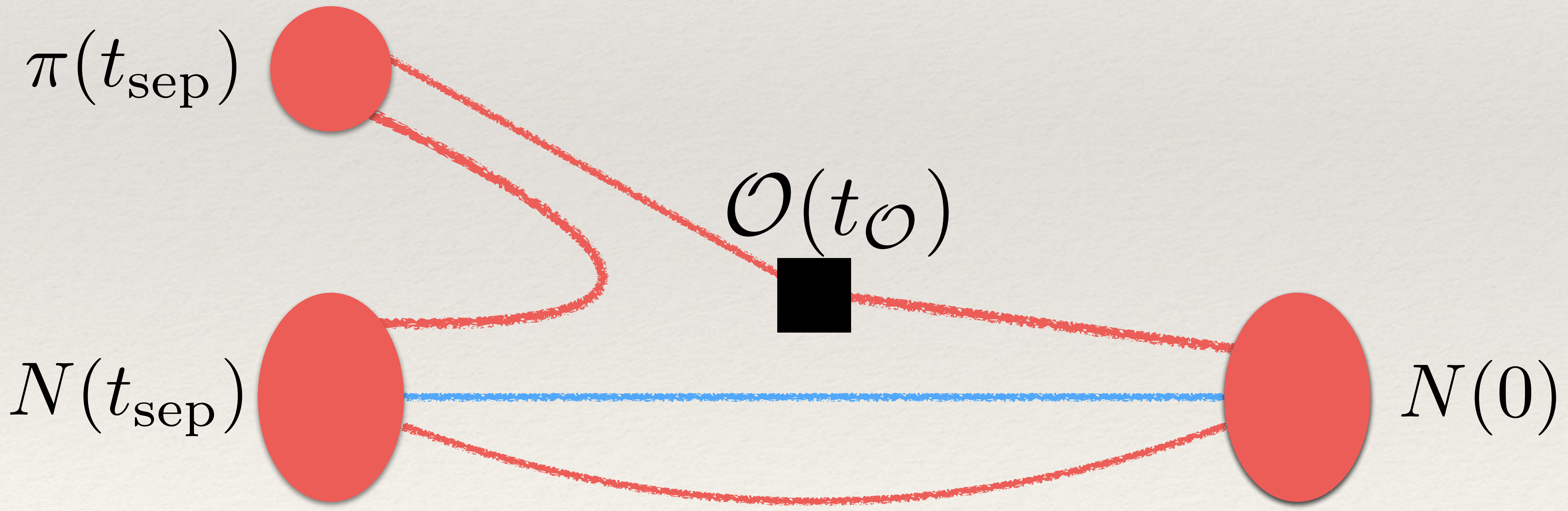
LQCD: 3 point functions

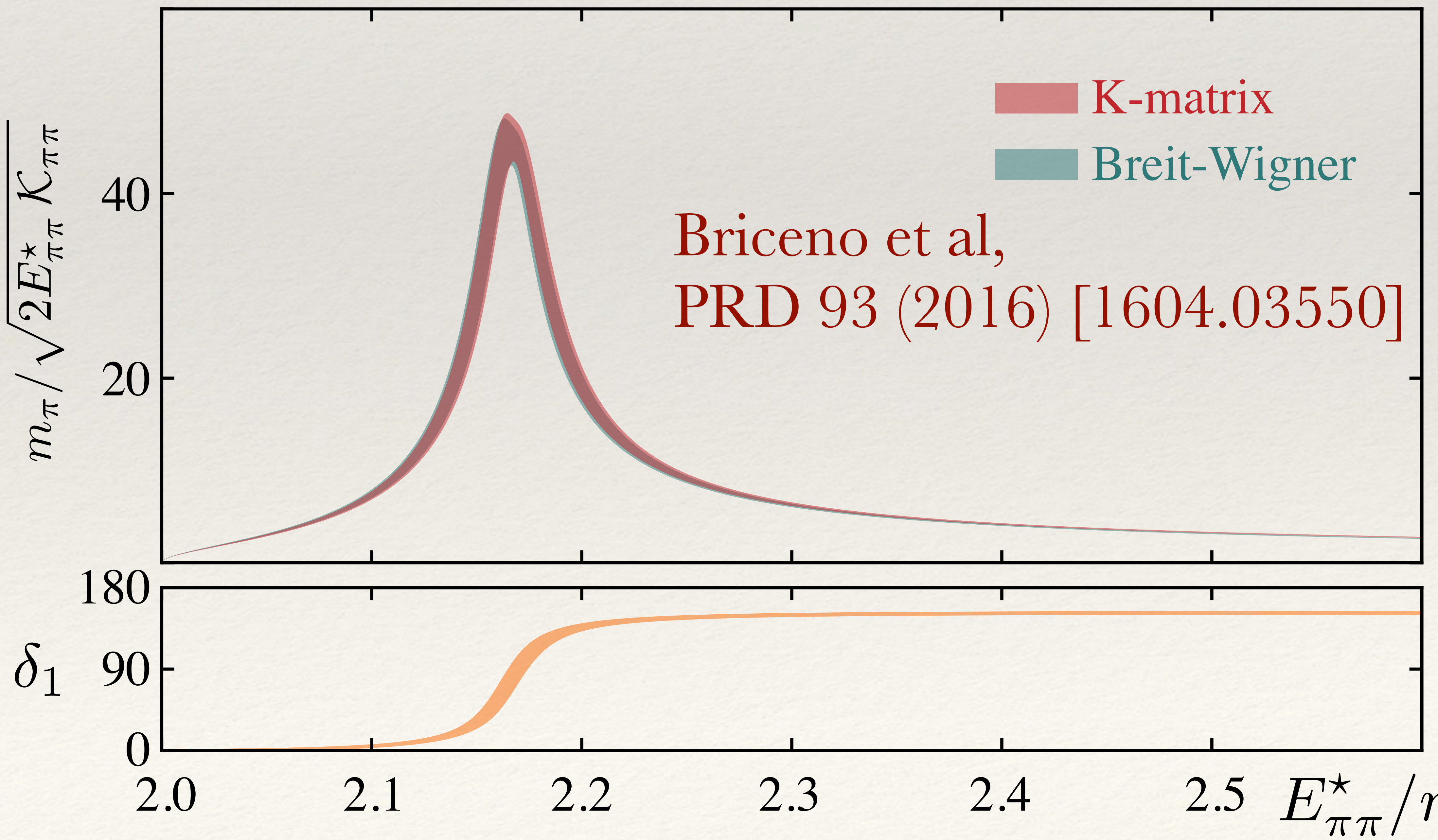


- Lessons learned:
  - We really need to incorporate variational basis in our computations
    - hopefully, will enable removal of dominant e.s.
      - identify "optimal" nucleon

$$|N_{\rm opt}\rangle = \alpha_N|N\rangle + \alpha_{N\pi}|N\pi\rangle + \cdots$$

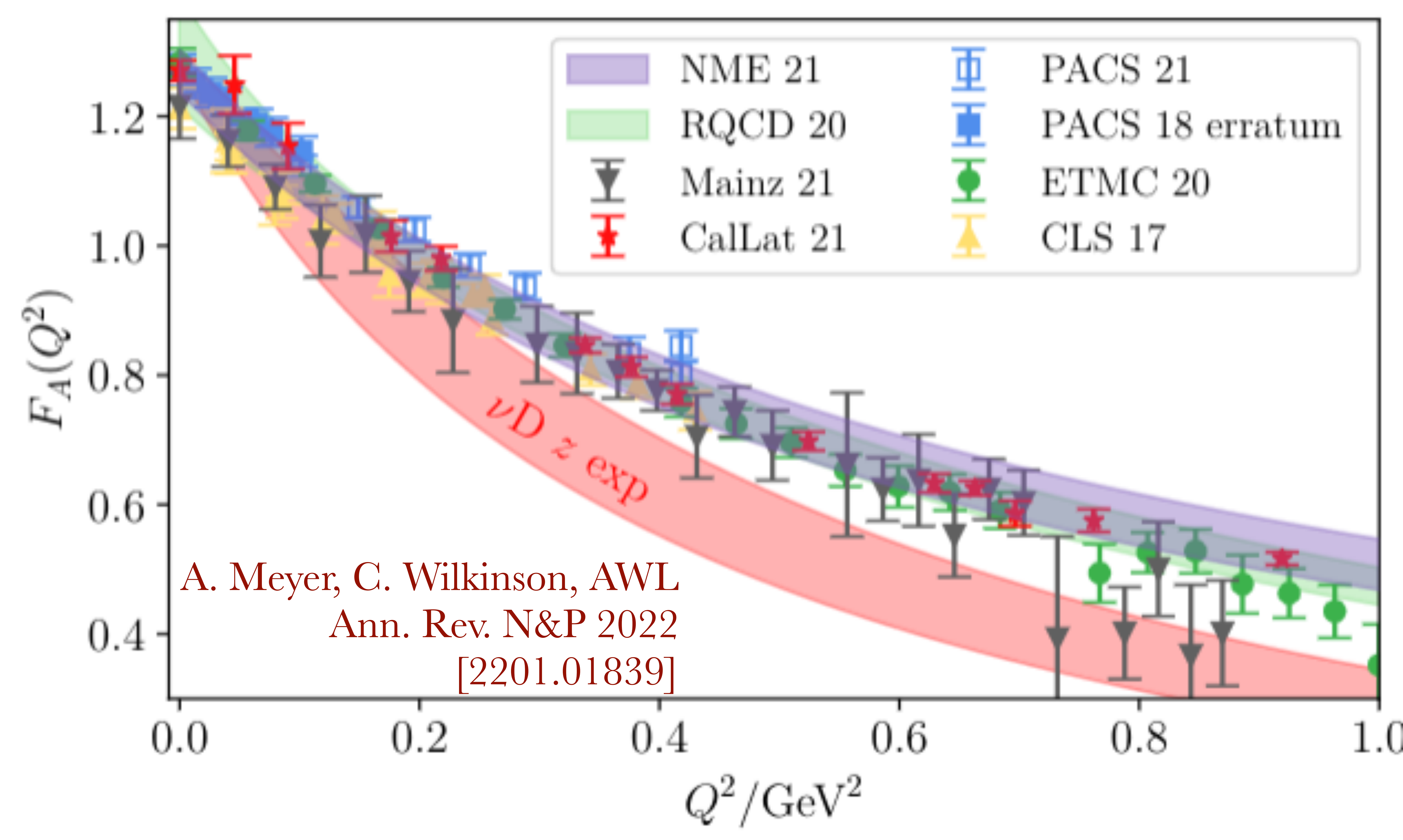
    - allows for Breit-Frame (p_f = -p_i) (which allows for better control of e.s. systematic uncertainty)
    - enables proper study of resonance region and pion-production





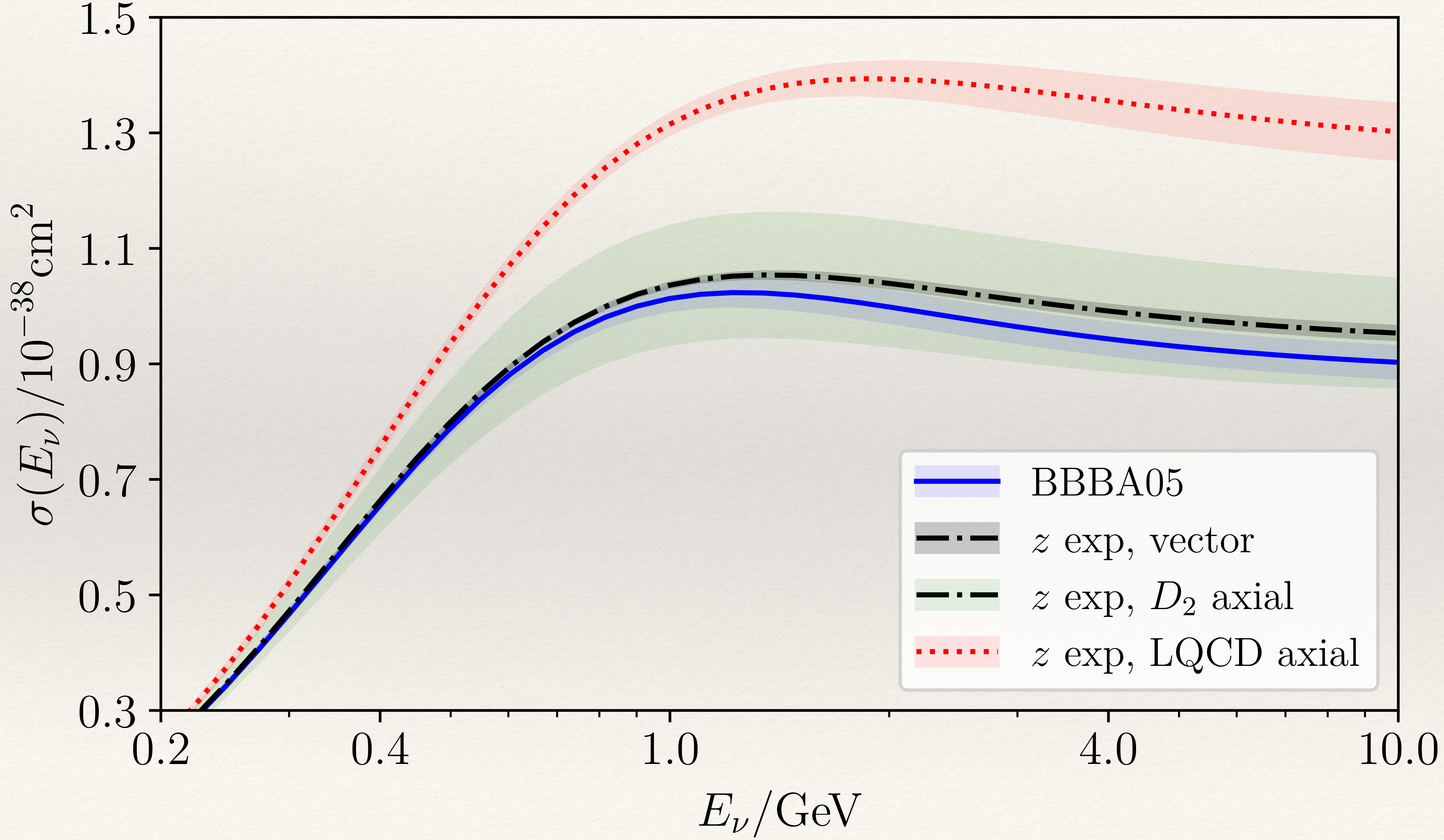
LQCD results for $F_A(Q^2)$

- ~1-2 years for "fully" quantified LQCD results for FA
- 2 groups have done all extrapolations (NME, RQCD)
- Others have not - but use different "actions" - consistency indicative that continuum is small systematic
- See. Konstantine Ottnad's talk today for more detailed examination of FF calculations
- Consitency: worth comparing with phono-extraction



Phenomenological Implications - why we have to care

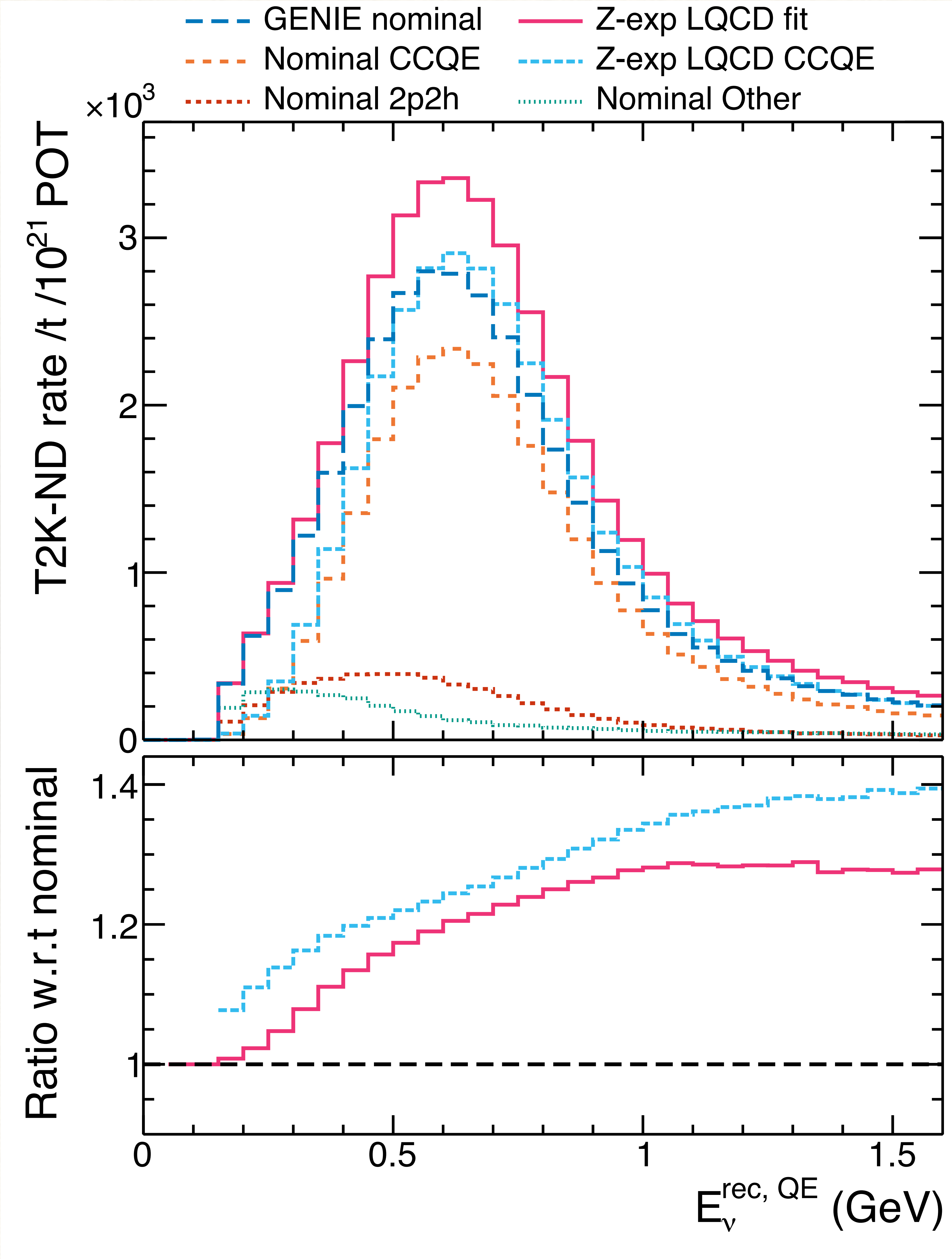
- Compare single-nucleon cross section - 30% larger than deuterium extraction



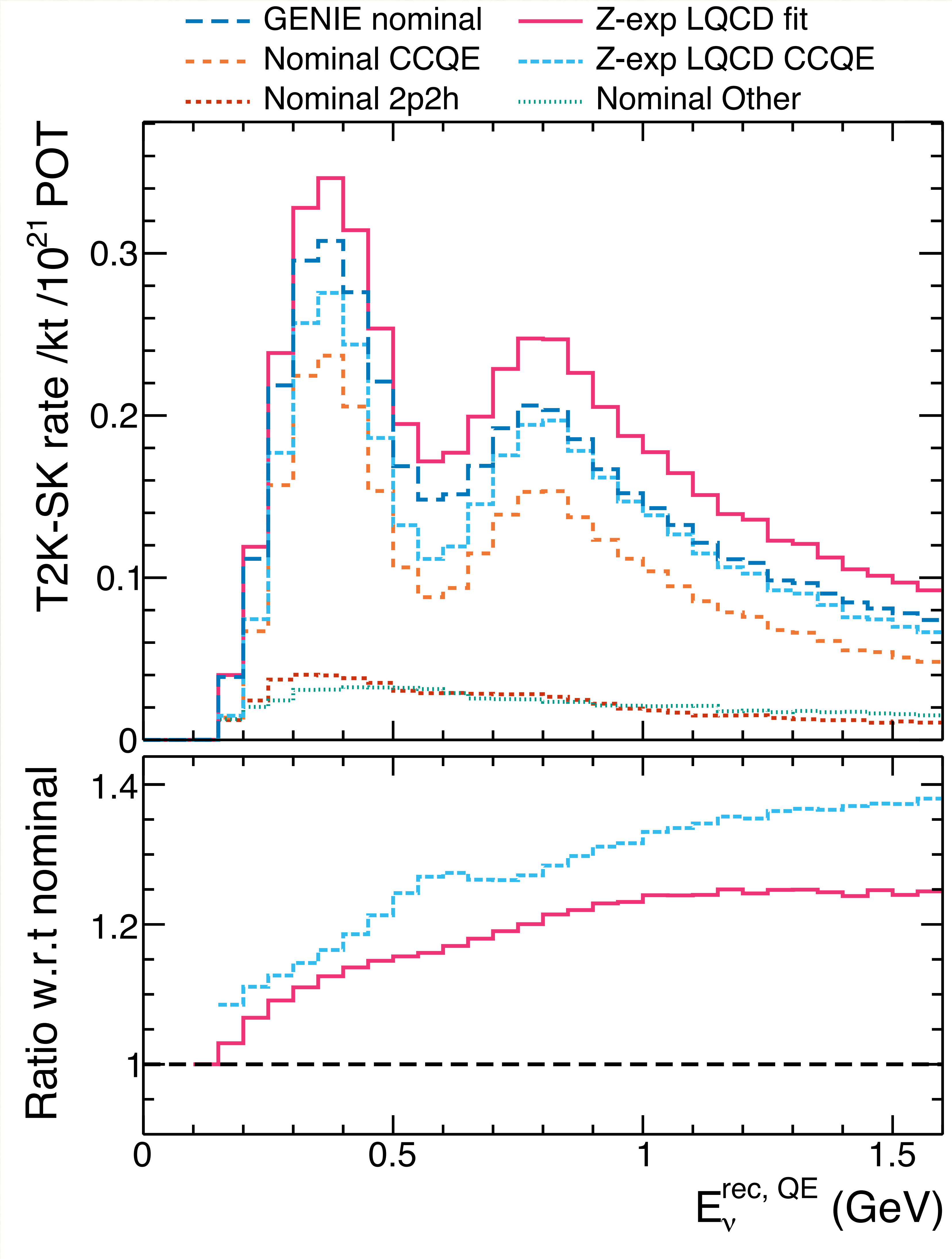
A. Meyer, C. Wilkinson, AWL
Ann. Rev. N&P 2022
[2201.01839]

Phenomenological Implications - why we have to care

- Freeze everything but single-nucleon to see impact of changing this one “moving piece”



T2K



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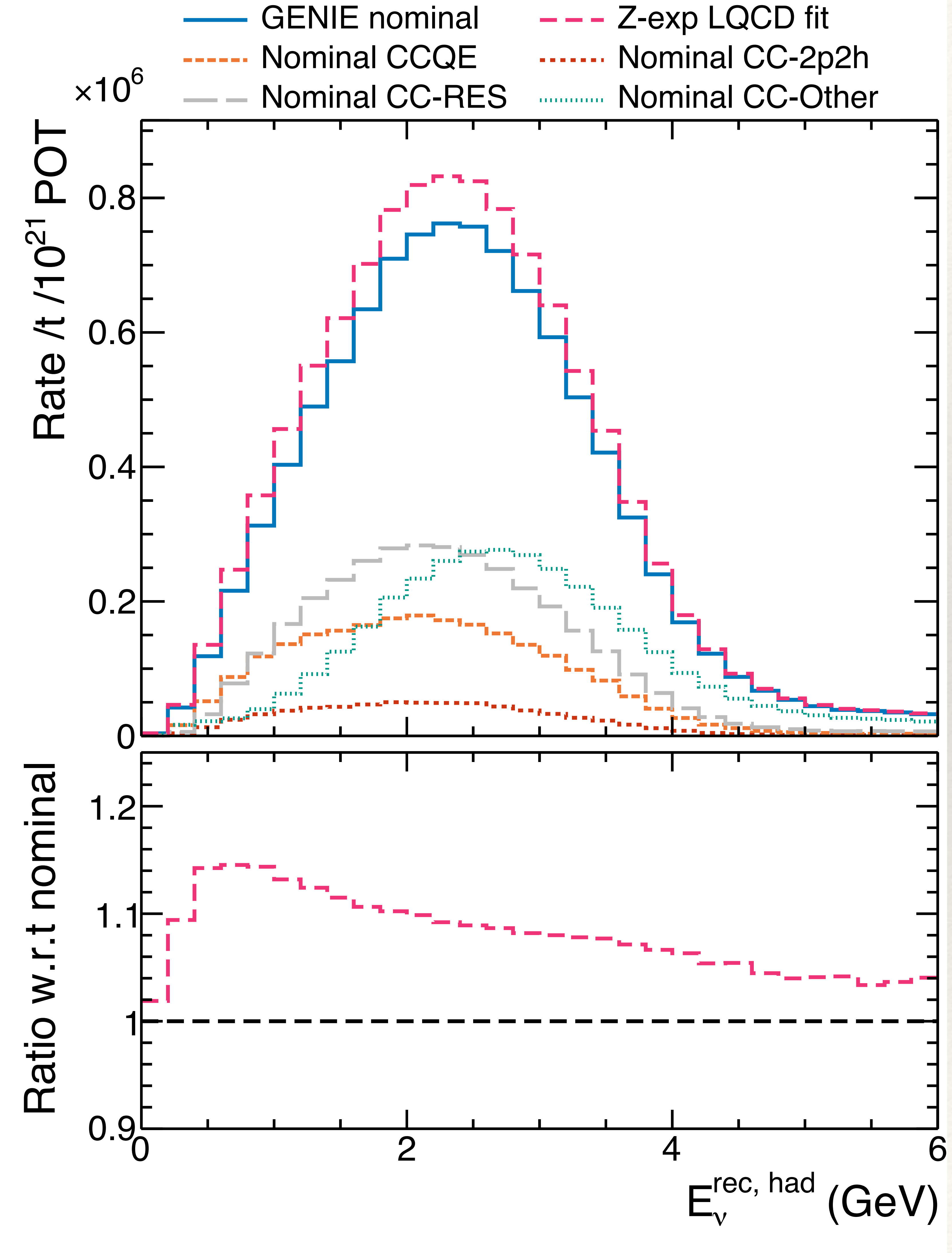
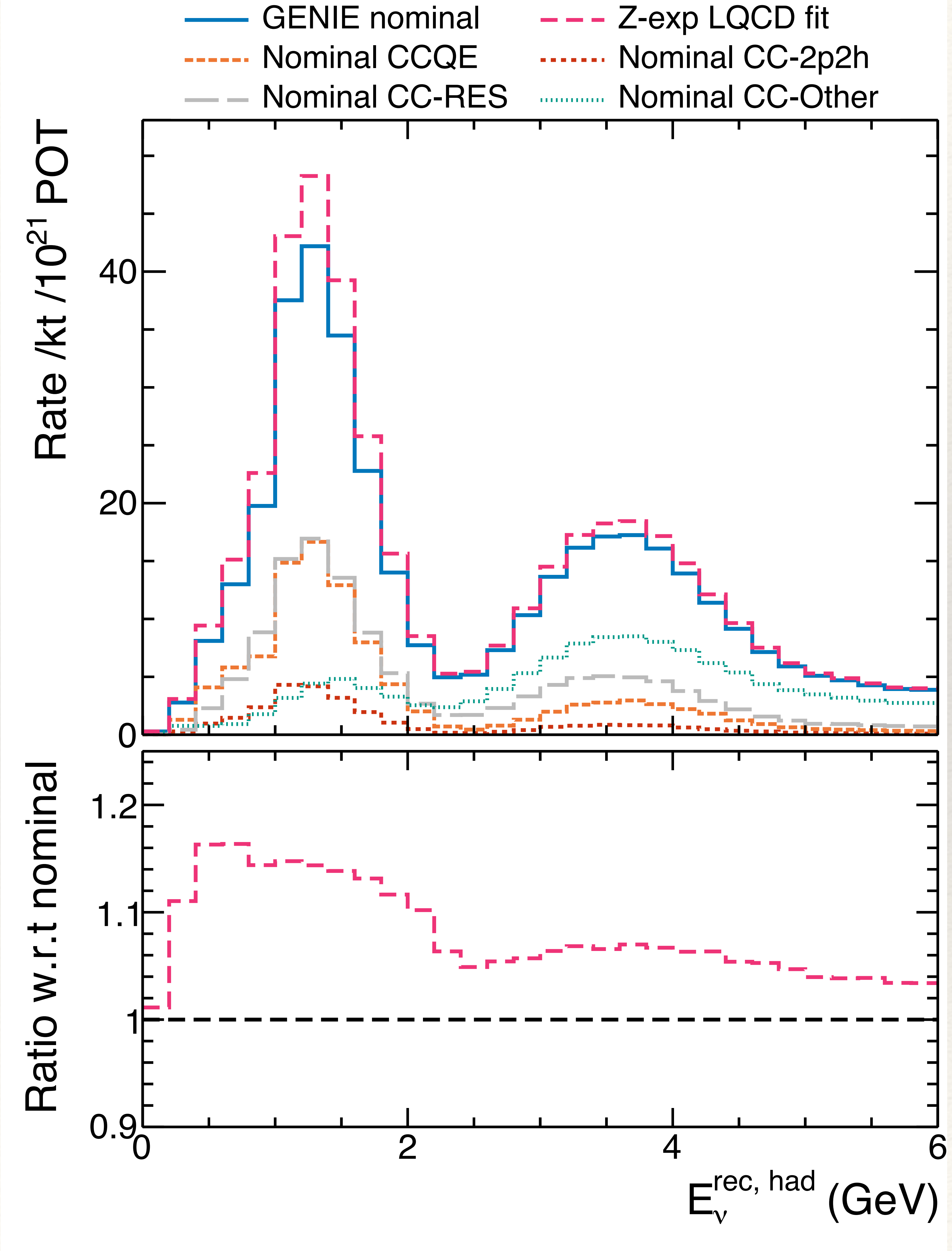


DUNE



A. Meyer, C. Wilkinson, AWL
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Future directions

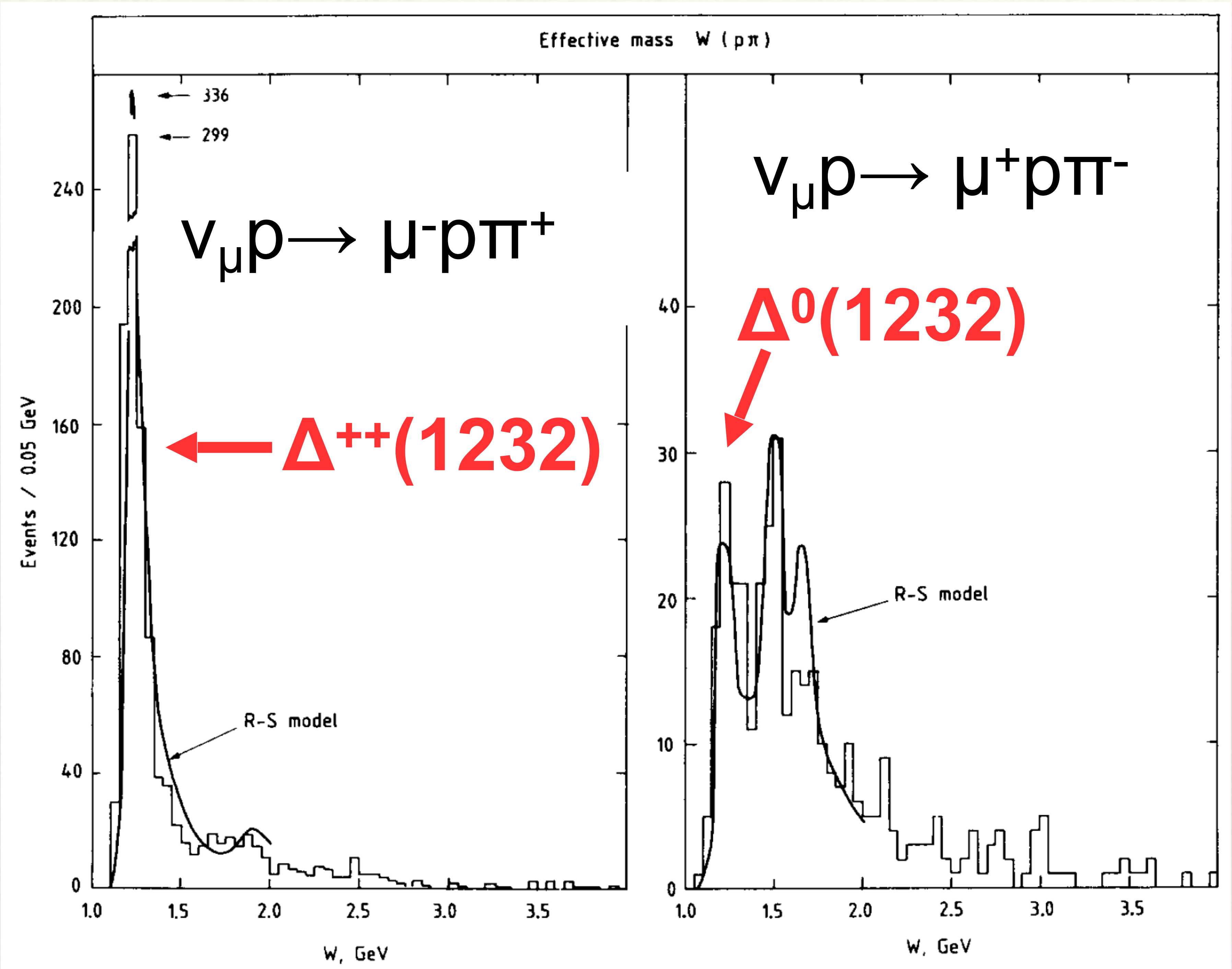
- 1-2 year, fully quantified LQCD results for F_A up to $Q^2 \sim 2\text{GeV}^2$
  - Several groups with full continuum, physical pion mass extrapolations
  - need to verify excited states are under control

- We need to implement variational basis that uses multi-hadron creation/annihilation operators (see RQCD preliminary work in this direction - Barca, Bali, Collins, LATTICE2021 - 2110.11908)
  - This will also critically allow for resonance-region and pion production

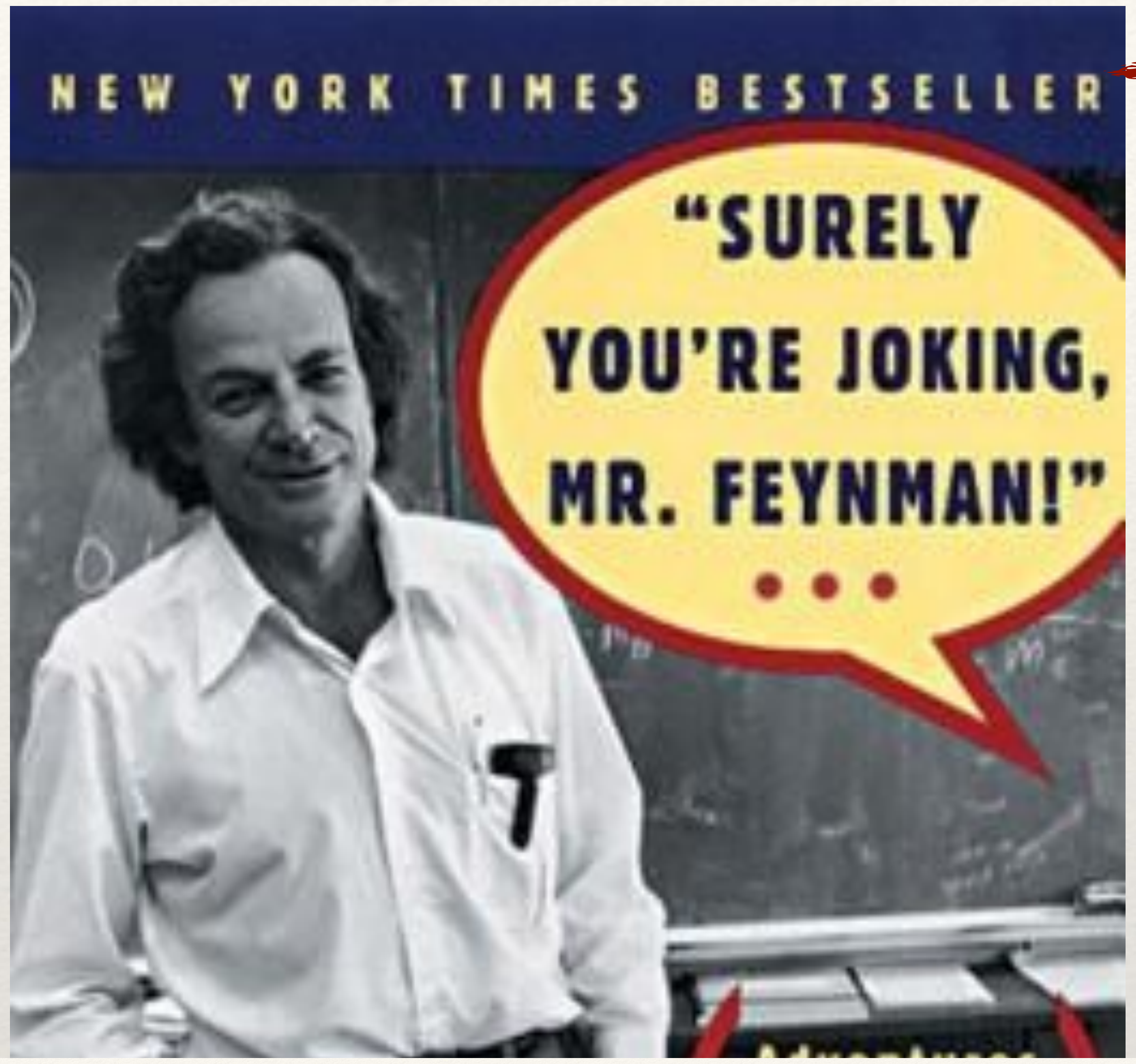
Future directions

2. State of the Field

Nucl. Phys. B264 221 (1986)



$$W^2 = (\Sigma E)^2 - |\Sigma \mathbf{p}|^2$$



Indeed not!

Our pion production model uses a description of resonance production that is "naive and obviously wrong in its simplicity" [F.K.R. PRD3 (1971)]

I trust some bright motivated physicists will fix this soon

- Current models are unsatisfactory:
  - Simplistic description of neutrino-nucleon interaction
  - Unsophisticated description of the nucleus
- Heavy reliance on old data (experiments shut down)
- ~10% uncertainties on effective parameters at best

Future directions

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  - indications that delta-less SU(2) Baryon XPT does not converge
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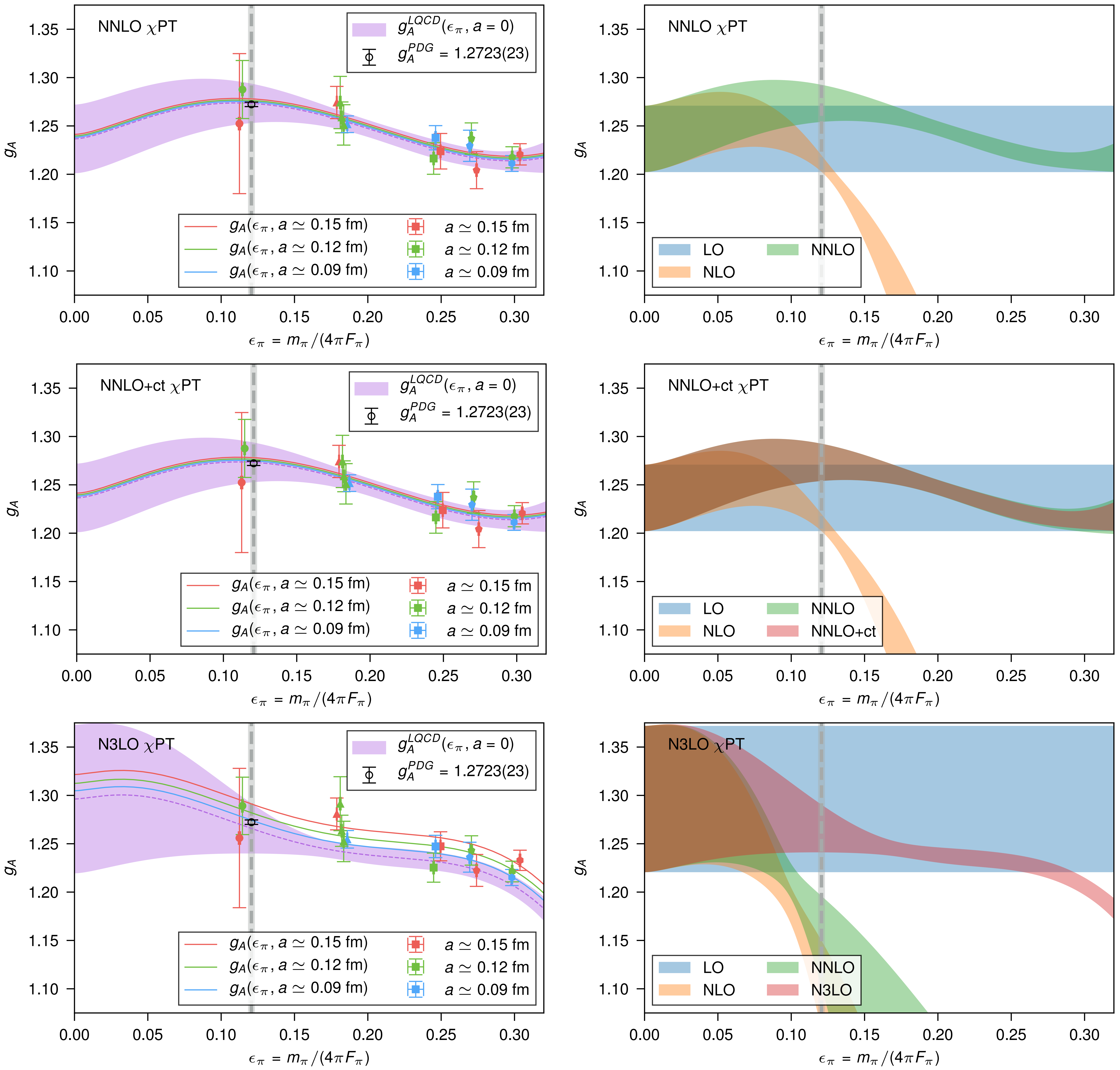
    - if single nucleon does not converge, we have to perform calculation at the physical pion mass
    - VERY DIFFICULT for NN

convergence of the chiral expansion…

Chang et al. CalLat
Nature 558 (2018)
1805.12130



$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4\epsilon_\pi^4$$

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Bernard and Meissner (CD06)
Phys.Lett.B639 [hep-lat/0605010]
F —> F_π

convergence of the chiral expansion…

Chang et al. CalLat
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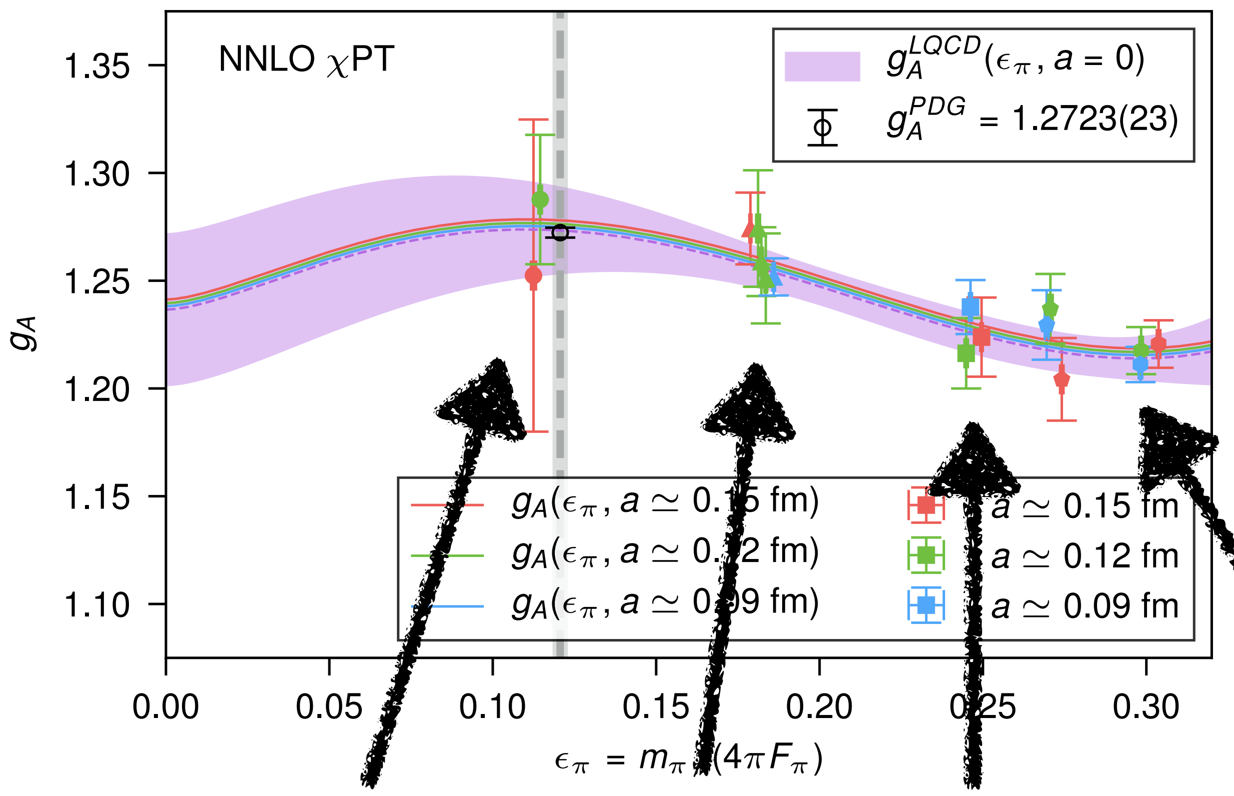

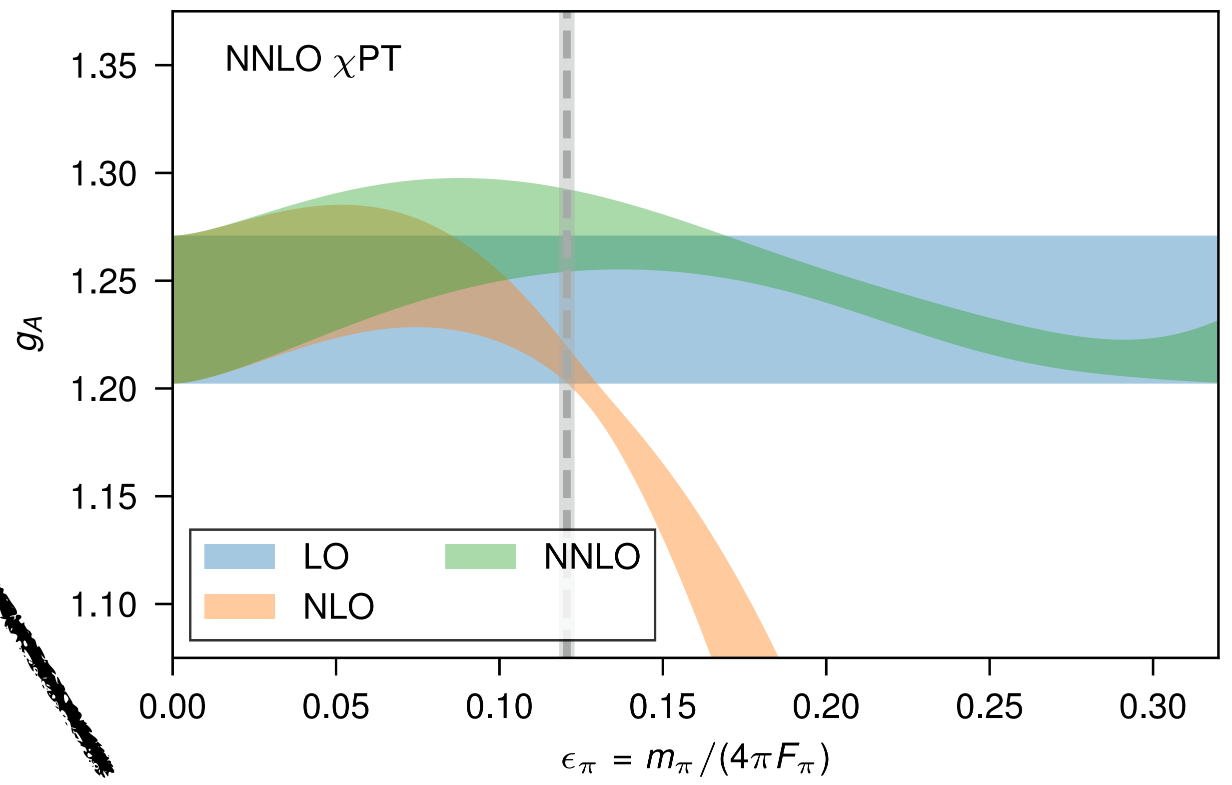

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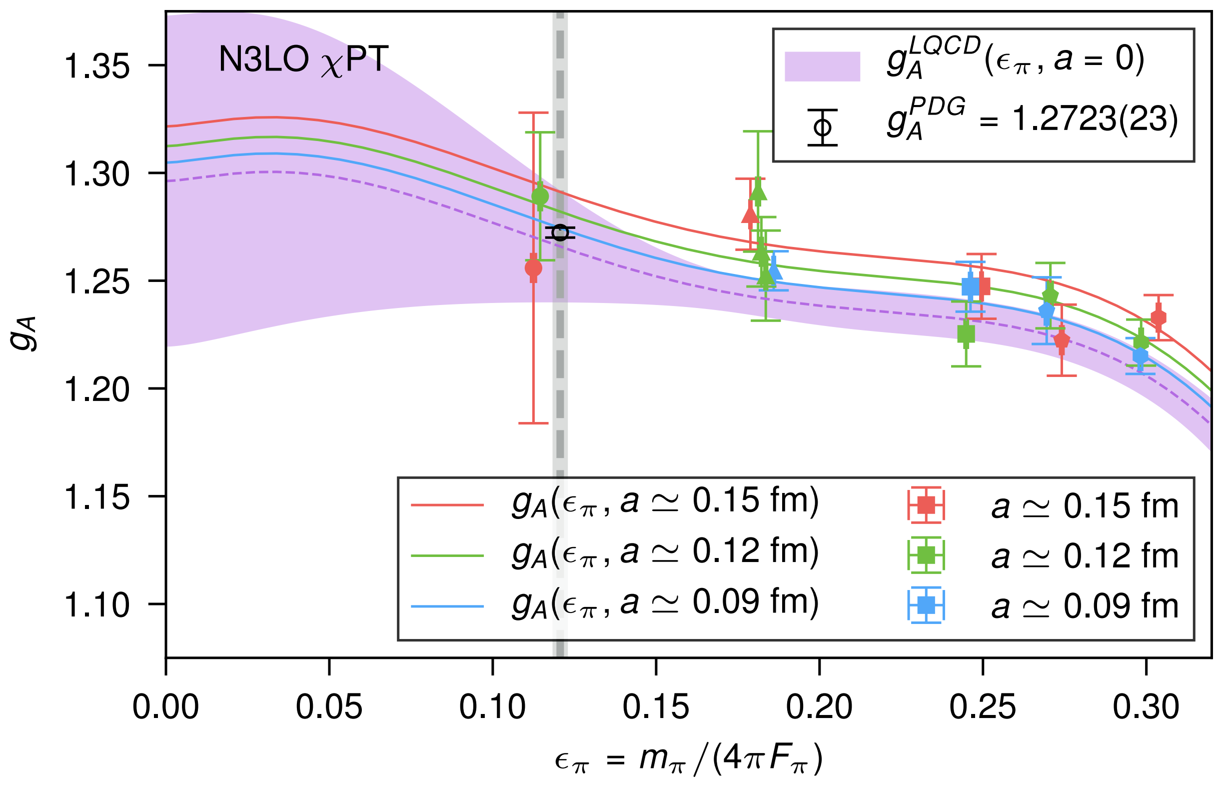
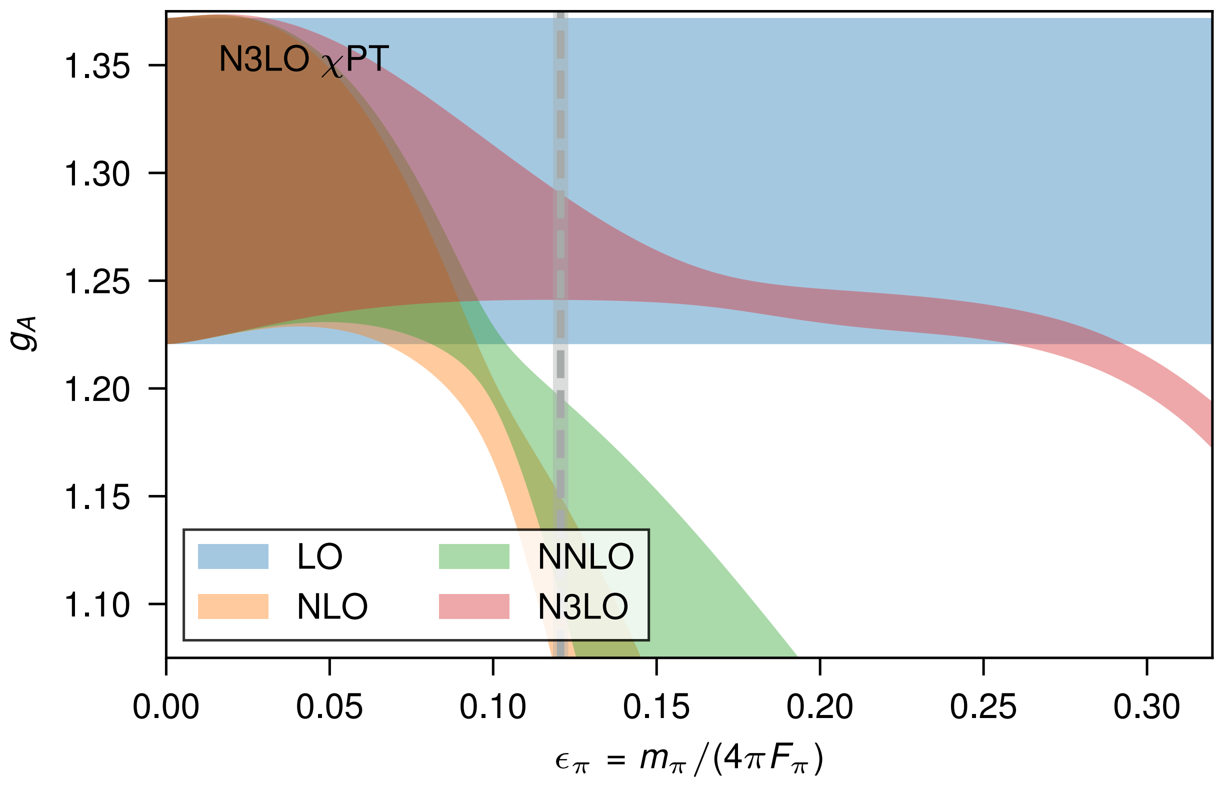
$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$m_\pi \sim 130$ MeV 220 310 400

can we trust extrapolation of quantities
with chiraly-enhanced behavior?
if the single nucleon is not converging, would you
trust chiral extrapolations of two or more nucleons?





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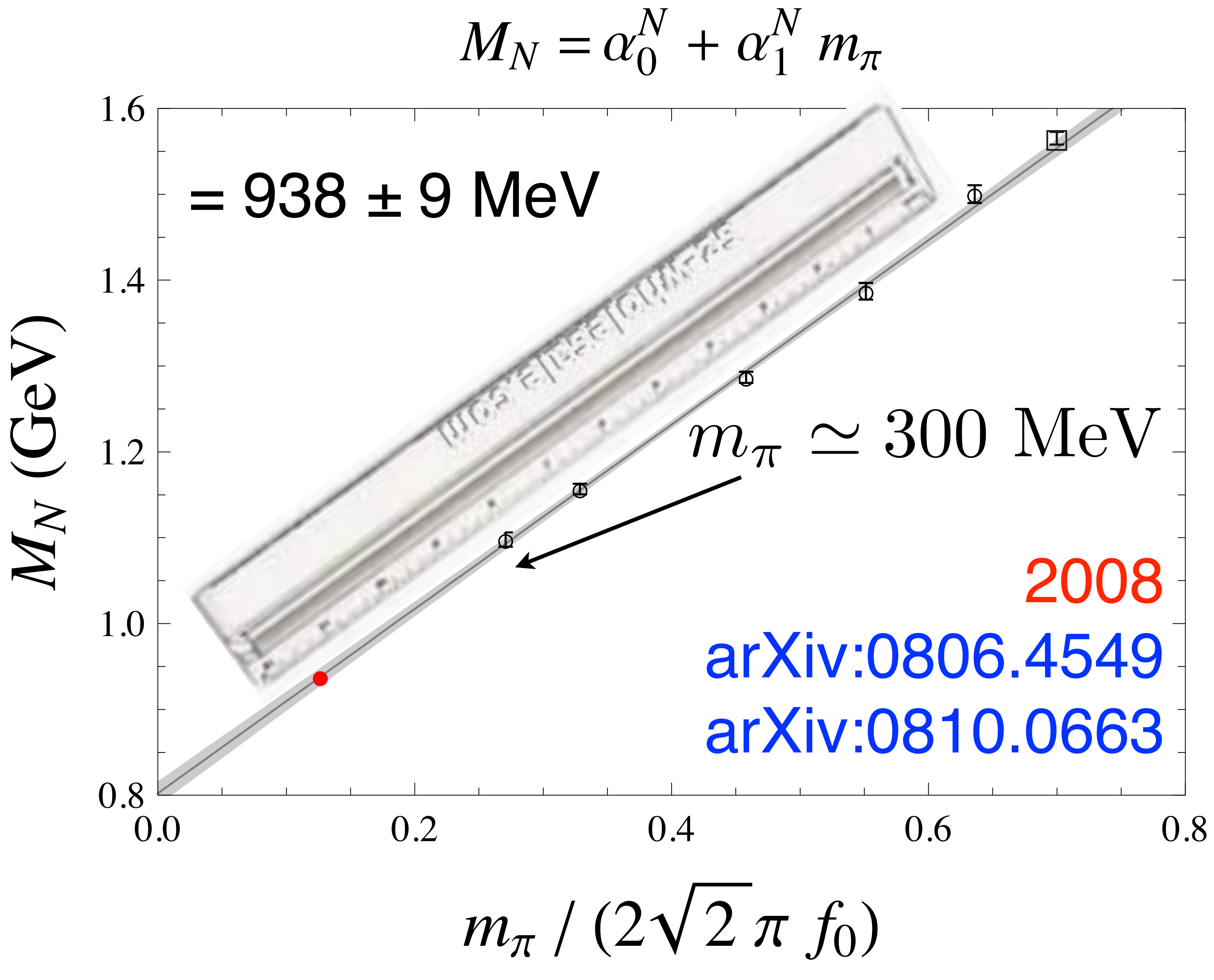
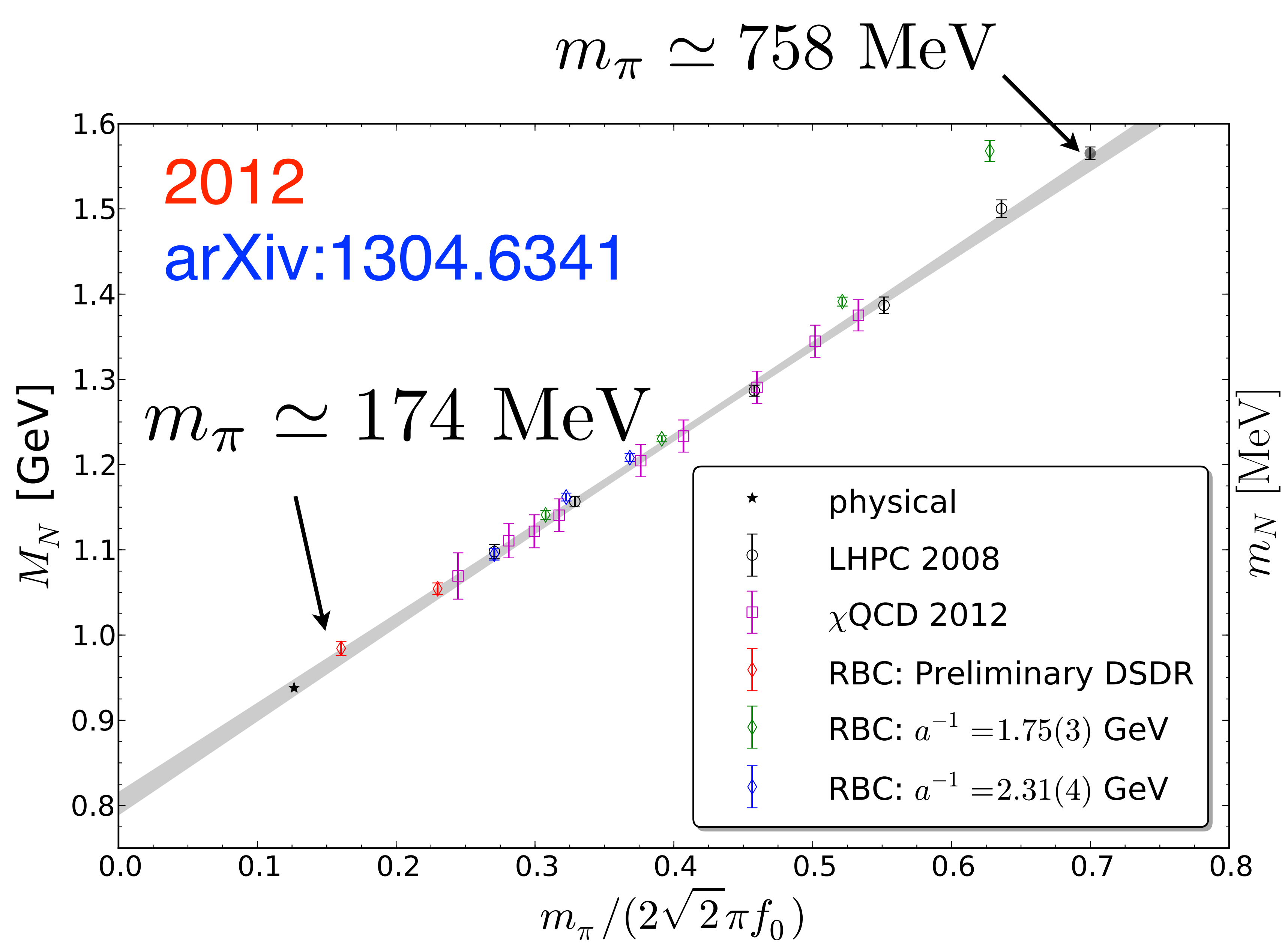
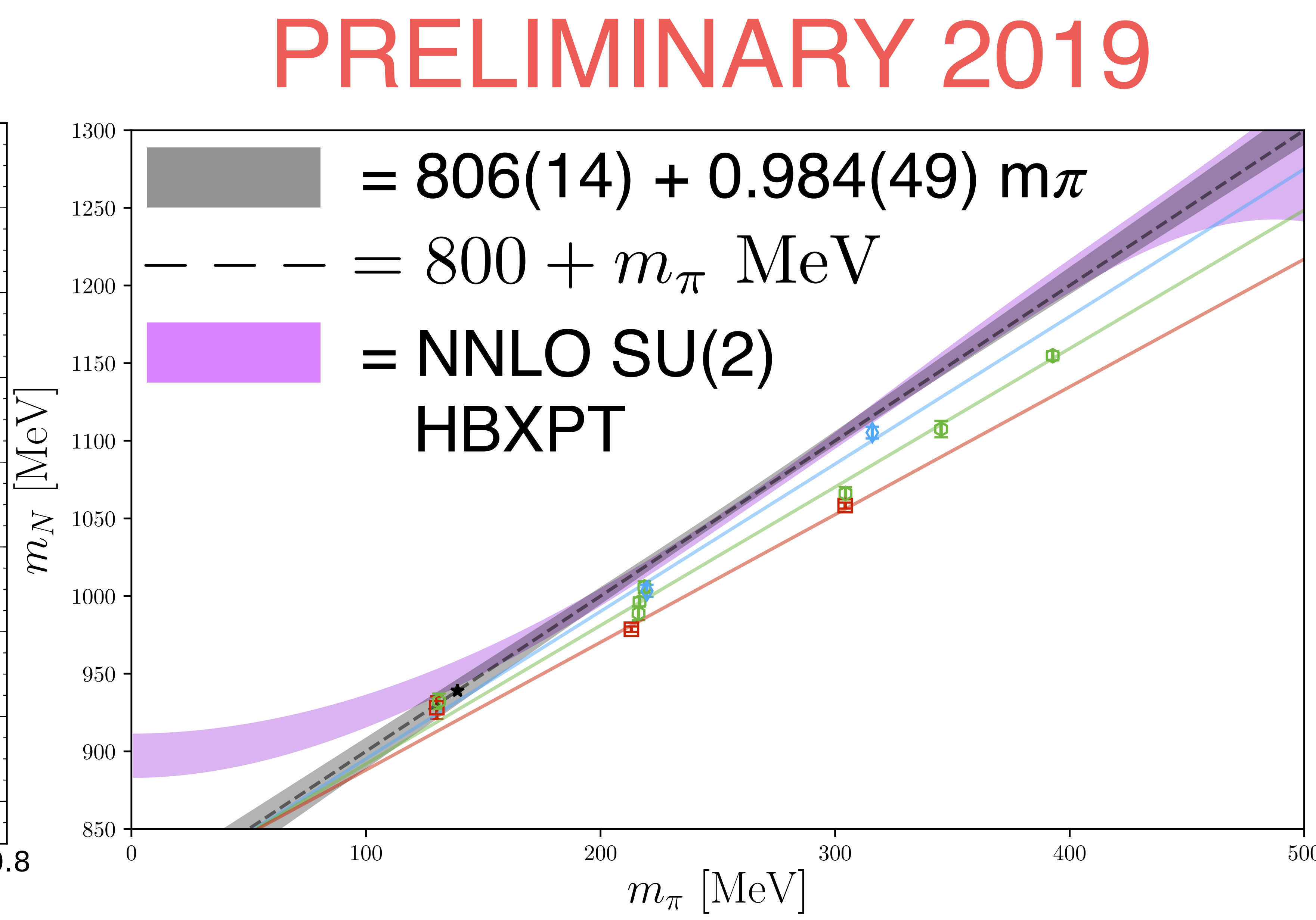
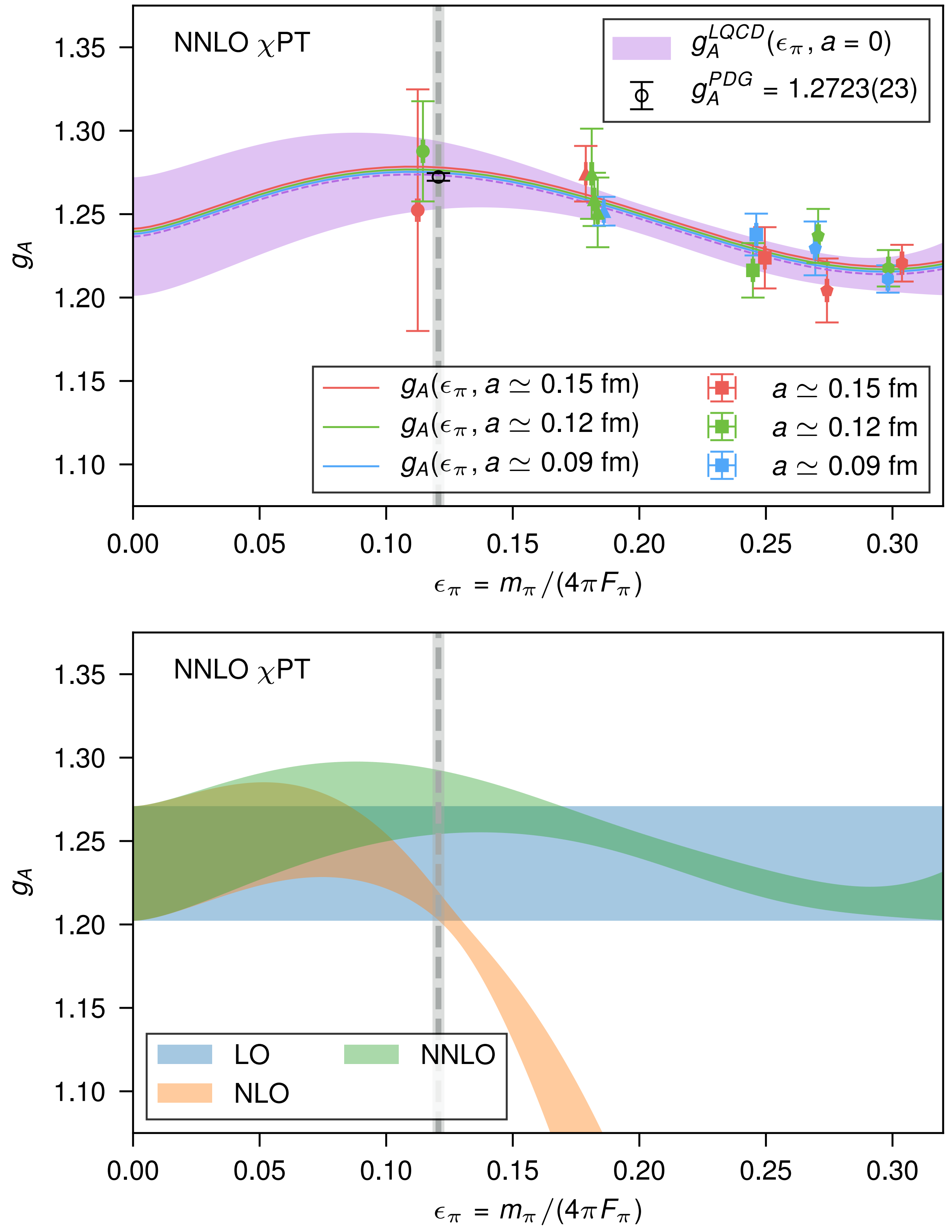






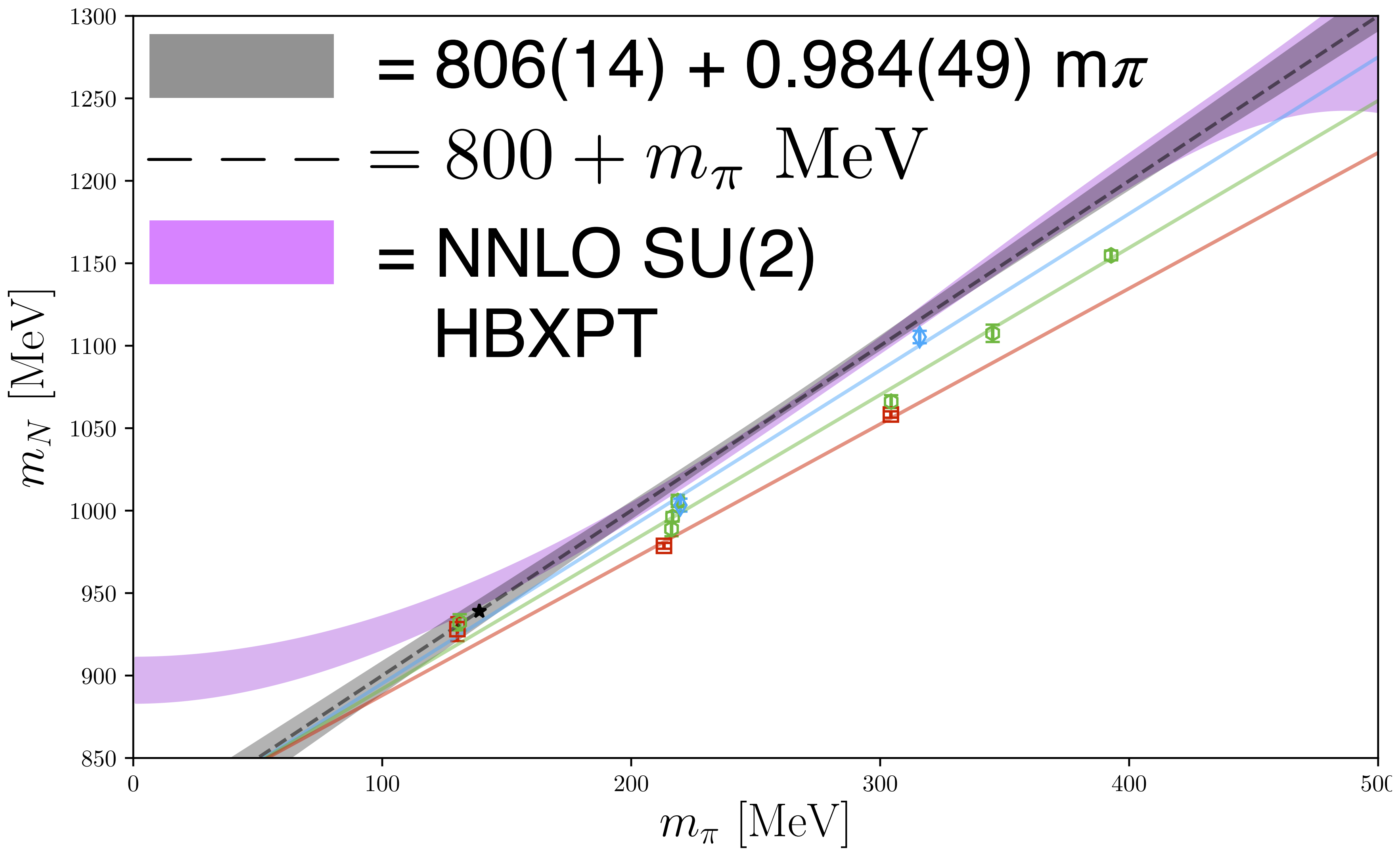
- The point is not - will this ruler approximation hold to arbitrary precision?
  - Already, it holds to sufficient precision that it demonstrates a strong cancellation between different orders in the expansion
  - Is it sufficiently accurate that one can extract the pion-nucleon sigma term? $\sigma_{\pi N} = 67 \pm 4$ MeV
  - BMW find a much smaller sigma term, 38(3)(3) PRL 116 (2016) [arXiv:1510.08013]
  - Almost all other lattice groups find similarly small values - in significant tension with the phenomenological determinations [Hoferichter, Ruiz de Elvira, Kubis, Meissner PhysRept 625 (2016)]
  - Gupta et al. PRL 127 (2021) [2105.12095] find large value - if e.s. are forced to be N(p)π(-p)

convergence of the chiral expansion…



Drischler, Haxton, McElvain, Mereghetti,
Nicholson, Vranas, AWL
Prog.Part.Nucl.Phys. 121 (2021)

PRELIMINARY 2019 [1910.07961]



- Chiral corrections to g_A from $SU(2)$ HBχPT(Δ̸) at the physical pion mass

N^nLO	LO	NLO	N^2LO	N^3LO
N^2LO	1.237(34)	-0.026(30)	0.062(14)	–
N^3LO	1.296(76)	-0.19(12)	0.045(63)	0.117(66)

- $SU(2)$ HBχPT(Δ̸) is a failed expansion…

- Worth noting - if you use $SU(2)$ HBχPT(Δ) and force the delta-axial couplings, the value of the pion-nucleon sigma term is also large
- large Nc gives de-coherent nucleon and delta loop corrections to g_A, but coherent to M_N
- $SU(2)$ HBχPT(Δ) has a chance of being a converging expansion - but it won't be pretty

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- Build up nuclear corrections - NN matrix elements
  - Better resolve the NN-controversy first! It seems the old NPLQCD results are wrong - and their are also indications of large discretization effects (Mainz)

Conclusions

- Lattice QCD can provide valuable input and "break the degeneracy" of nuclear model effects in modeling ν-A cross sections
- Lattice QCD is not a "black box" that returns results - but requires very sophisticated (at least by theory standards) data analysis methods to obtain robust matrix elements and form factors with fully quantified uncertainties
- Lattice QCD is around the corner from providing impactful results to ν-A cross section modeling - 1-2 years for robust quasi-elastic F_A predictions
- The near horizon includes understanding the resonance region and pion-production and two-nucleon charges and form-factors
- These results also need to be coupled to EFTs of nuclear physics and many-body nuclear models and integrated into the event generators

Thank You