PROBING THE SEESAW AND LEPTOGENESIS WITH LOW-ENERGY ν DATA

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Standard parametrization:

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atm. } \nu \text{ osc.}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{reactor } \nu \text{ osc. (short BL)}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar } \nu \text{ osc.}} \cdot K$$

$$= O_{23} \cdot V_{13} \cdot O_{12} \cdot K$$

$$c_{ij} \equiv \cos\theta_{ij}\ ; \quad s_{ij} \equiv \sin\theta_{ij}$$

$$K = \begin{cases} \mathbb{1} & \text{for Dirac } \nu\text{'s} \\ \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2}, 1) & \text{for Majorana } \nu\text{'s.} \end{cases}$$

Majorana phases δ_1 and δ_2 do not contribute to ν oscillations.

$\theta_{12} \sim 0.6$ from $\nu_\odot$ (LMA-MSW-KamLAND)

$\theta_{23} \simeq \frac{\pi}{4}$ from ν_{atm}.

$\theta_{13} \lesssim 0.16$ from CHOOZ

Nothing is known about δ_{CP} !

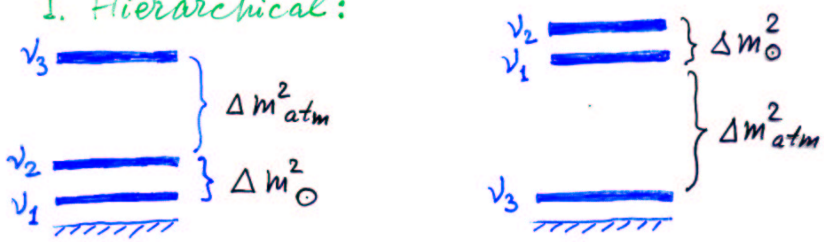
Ignoring the LSND result (not yet independently confirmed):

All data can be explained in terms of oscillations between just 3 known ν species ν_e, ν_μ, ν_τ

$\longleftrightarrow$

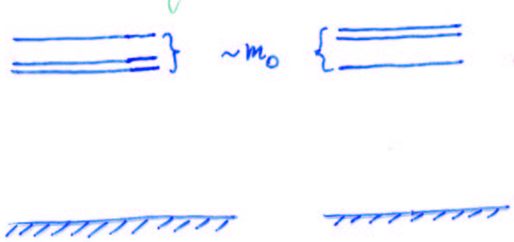
Possible orderings of ν masses:

I. Hierarchical:



Normal hierarchy   Inverted hierarchy

II Quasi-degenerate:



$$\boxed{\Delta m^2_{21} \ll \Delta m^2_{31}}$$

$$\boxed{\nu_{fl} = U\, \nu_{mass}}$$

Summary of ν data (1σ)

- $\Delta m^2_{21} \cong (7.3 \pm 0.8)\cdot 10^{-5}\,eV^2$, $\sin\theta_{12} \equiv s_{12} \cong 0.56 \pm 0.03$ } solar (LMA I)

- $\Delta m^2_{31} \cong (2.6 \pm 0.8)\cdot 10^{-3}\,eV^2$, $\sin^2 2\theta_{23} \gtrsim 0.94$ ($\theta_{23} \cong 45^\circ \pm 7^\circ$) } at-mo-sphe-ric

$|\sin\theta_{13}| = |s_{13}| \lesssim 0.2$ } CHOOZ + solar + atm.

$\longleftrightarrow$

For hierarchical ν masses:

I. Normal hierarchy: ($m_1 \ll m_2 \ll m_3$)

$$m_3 \simeq \sqrt{\Delta m^2_{31}} \equiv \sqrt{\Delta m^2_{atm}} \simeq 0.05\,eV$$

$$m_2 \cong \sqrt{\Delta m^2_{21}} \equiv \sqrt{\Delta m^2_\odot} \simeq 8\cdot 10^{-3}\,eV$$

II. Inverted hierarchy: ($m_3 \ll m_1 \cong m_2$)

$$m_1 \cong m_2 \cong \sqrt{\Delta m^2_{atm}} \simeq 5\cdot 10^{-2}\,eV\,; \quad m_2 - m_1 \simeq \frac{\Delta m^2_\odot}{2\sqrt{\Delta m^2_{atm}}}$$

Overall scale of ν masses

Direct limits: $m_e \lesssim 2.2$ eV (β decay)

Cosmology: $\bar{m} \lesssim 1$ eV

A simple and attractive explanation of the smallness of ν mass:

SEESAW MECHANISM

(Yanagida, 1979; Glashow, 1979; Gell-Mann, Ramond & Slansky, 1979; Mohapatra & Senjanović, 1980)

Simplest version: SM + 3 right-handed (EW singlet) neutrinos N_R

$$-\mathcal{L}_m^{lept} = \bar{\ell}_L m_L \ell_R + \bar{\nu}_L m_D N_R + \frac{1}{2}\overline{N_R^c} M_R N_R + h.c.$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}; \quad n_L = \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$$

$$\boxed{m_L = m_{eff} = -m_D M_R^{-1} m_D^T}$$

N_R are EW singlets $\Rightarrow M_R$ is naturally $\gg v$; $m_D \lesssim v$ ($v = 174$ GeV $\Leftrightarrow \langle H^0 \rangle$)

$$m_{\nu\,light} \sim \frac{v^2}{M_R};$$

For $m_{\nu\,light} \sim (10^{-2} \div 10^{-3})$ eV, $\Rightarrow M_R \sim 10^{15}$ GeV – a very interesting scale (close to GUT)!

Although N_R can be added (and the seesaw mechanism can operate) in the SM, they are most natural in some extensions of the SM (LR, SO(10) GUTs, ...)

A very attractive feature of seesaw:

It has a very simple and attractive built-in mechanism for generating the baryon asymmetry of the Universe!

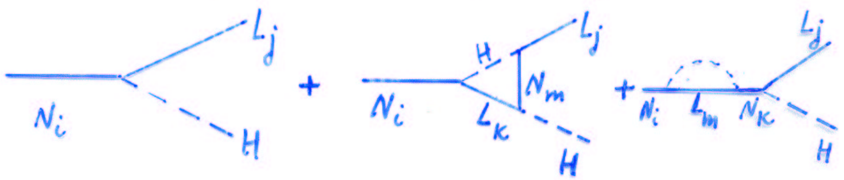
Baryogenesis through leptogenesis

(Fukugita & Yanagida, '86; Luty, '92; Plümacher, '96; Covi et al., '96; Buchmüller & Plümacher, '96; ...)

(1) Out-of-equilibrium L-violating decay of N_{R1} $\Rightarrow$ a net $L \neq 0$ produced

(2) EW sphalerons (which are operative for $T_{EW} \lesssim T \lesssim 10^{12}$ GeV) conserve $(B-L)$ but wash out $(B+L)$, leading to a net $B \neq 0$.

$\cancel{CP}$: due to complexity of Yukawa couplings; comes from interference of tree level and 1-loop diagrams



$\cancel{C}$: by chiral Yukawa couplings of N_R

$\cancel{B}$: by combination of $\cancel{L}$ (due to M_R) and $\cancel{(B+L)}$ by sphalerons.

Out of equilibrium: $\underline{\Gamma(N_1 \Rightarrow HL) < H(M_1)}$

The Hubble parameter:

$$H(T) = 1.66\, g_* \frac{T^2}{M_{Pl}}$$

g_* – # of eff. degrees of freedom:

$$g_* \simeq 100 \quad \left(\frac{434}{4}\right)$$

for SM + 1 RH singlet neutrino.

$$\Gamma_1 < H(M_1) \Rightarrow \underline{\tilde{m}_1 < m_* \simeq 1.1 \cdot 10^{-3}\, \mathrm{eV}}$$

$$\boxed{\tilde{m}_1 = \frac{v^2 (Y_D^\dagger Y_D)_{11}}{M_1}} \qquad m_D = v \cdot Y_D$$

Since the observed $\eta_B = \frac{n_B}{n_\gamma}$ is very small ($\eta_B = [6.1 \pm^{0.3}_{0.2}] \cdot 10^{-10}$), $\tilde{m}_1/m_*$ can be rather large.

Out-of-equilibrium decay condition:

- $$\Gamma_1 = \frac{(Y_D^\dagger Y_D)_{11}}{8\pi} M_1 < H(M_1)$$

Hubble parameter:

$$H(T) = \frac{2\pi^{3/2}}{3\sqrt{5}} g_*^{1/2} \frac{T^2}{M_{Pl}} \cong 1.66\, g_* \frac{T^2}{M_{Pl}}$$

Introduce $\tilde{m}_1 \equiv \frac{(m_D^\dagger m_D)_{11}}{M_1} = 8\pi \frac{v^2}{M_1^2} \Gamma_1$;

the condition $\Gamma_1 < H(T = M_1) \Rightarrow$

$\boxed{\tilde{m}_1 < m_*}$, where

- $$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v^2}{M_{Pl}} \simeq 1.1 \cdot 10^{-3}\ \text{eV}$$

for $g_* = \frac{434}{4}$ (SM + 1 N_R species)

An important parameter: asymmetry produced in decays of N_1 (lightest N_R)

$$\epsilon_1 \equiv \frac{\Gamma_1(N_1 \to L\bar{H}) - \Gamma_1(N_1 \to \bar{L}H)}{\Gamma_1(N_1 \to L\bar{H}) + \Gamma_1(N_1 \to \bar{L}H)}$$

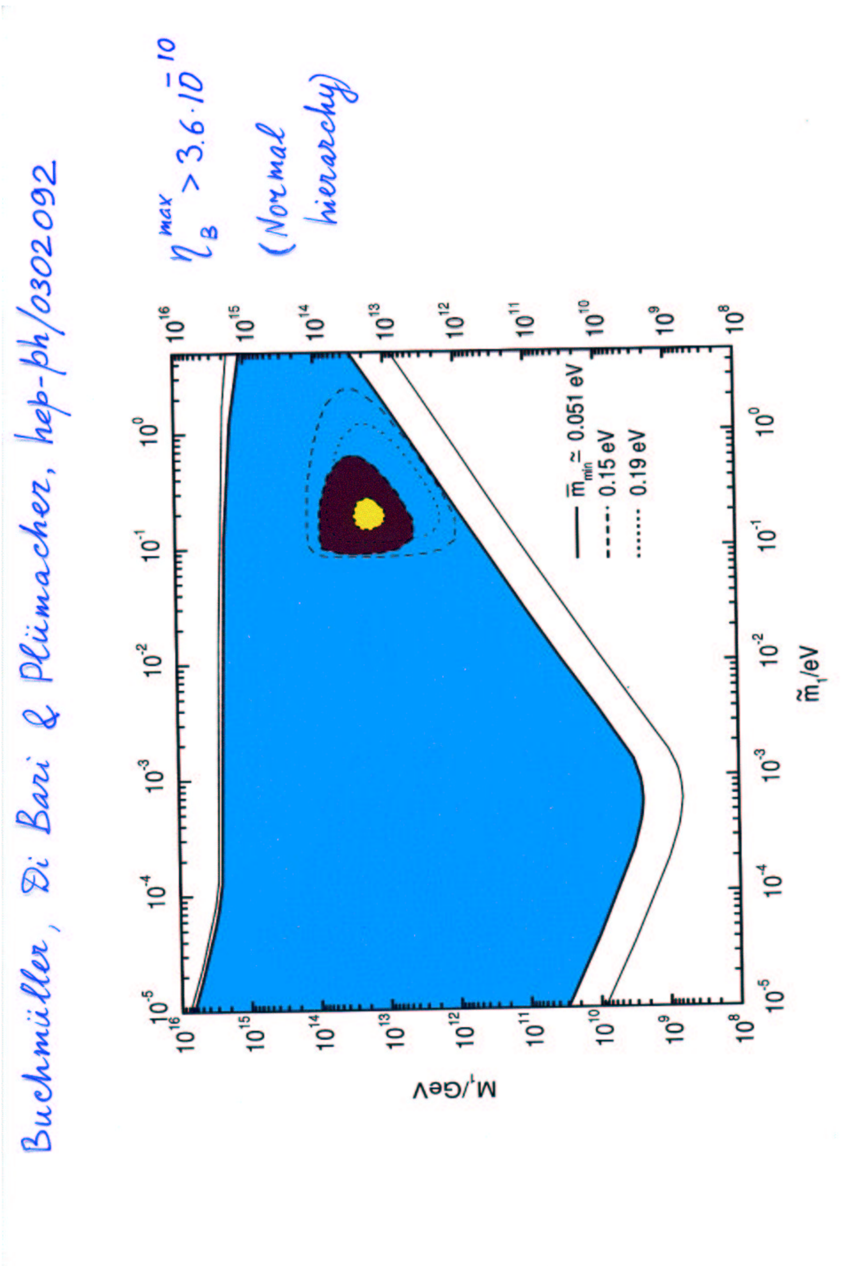
$$\boxed{\epsilon_1 \simeq \frac{1}{8\pi} \frac{1}{(Y_D^\dagger Y_D)_{11}} \sum_{i\neq 1} \mathrm{Im}\left[(Y_D^\dagger Y_D)_{1i}^2\right] \tilde{f}\left(\frac{M_i^2}{M_1^2}\right)}$$

$$\tilde{f}(x) \equiv \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right) + \frac{1}{1-x}\right]$$

$$\boxed{\eta_B \simeq 10^{-2}\, \epsilon_1 \cdot \varkappa}$$

$\varkappa$ – dilution factor; takes into account washout effects. In general, has to be found by solving a set of coupled Boltzmann equations for N_{N_1}, N_{B-L}.

A useful approx. expression for $K \equiv \frac{\tilde{m}_1}{m_*}$ in the range $K \sim (10 - 10^6)$: $\varkappa \simeq 0.3/[K(\ln K)^{0.6}]$.



- **Goal**: try to study the N_R sector using the low-energy ν data as an input
- Confront the results with baryogenesis through leptogenesis scenario

Strategy:

- Work in the flavour basis (m_ℓ – diagonalized).

$$m_{eff} = - m_D \frac{1}{M_R} m_D^T$$

$$m_{eff} = U \, \mathrm{diag}(m_1, m_2, m_3) U^T$$

- Assume ν mass hierarchy (NH, IH, QD) $\Rightarrow m_1, m_2, m_3$ from Δm^2_{atm}, $\Delta m^2_\odot$ (study case by case).
- Use the standard parametrization for the leptonic mixing matrix U and known values of $\theta_{12}, \theta_{23}, \theta_{13}$ (vary within allowed regions). Keep CP-violating phases free.

A note: we are mostly interested in masses and mixing parameters of N_R $\Rightarrow$ the $\not{CP}$-phases are less important for these purposes.

Dirac-type δ_{CP} in U has small effect on the structure of M_R because it always enters through $s_{13} e^{i\delta_{CP}}$, and θ_{13} is small.

One can eliminate Majorana-type CP-violating phases by allowing $m_{1,2,3}$ to be complex (which we do).

To reconstruct M_R one needs m_D, which is unknown $\Rightarrow$ Some assumptions about the Dirac sector are necessary.

Our assumptions:

- Hierarchical eigenvalues

$$m_D^{diag} = \mathrm{diag}(m_u, m_c, m_t)$$

$m_u \ll m_c \ll m_t$ (but not necessarily equal to corresp. quark masses !)

- Dirac-type LH mixing small (smaller than or of the order of CKM quark mixing)

$$m_D = U_L^\dagger \, m_D^{diag} \, V_R$$

Our assumtion is that U_L (a mismatch between LH rotations diagonalizing m_ℓ and m_D) is close to 1 (or $\sim V_{CKM}$).

U_L - an analog of CKM mixing in the quark sector

$\Rightarrow$ We assume that the large leptonic mixing originates mainly from the N_R sector

$$m_D = U_L^\dagger m_D^{diag} V_R \longrightarrow$$

$$m_{eff} = -\underbrace{U_L^\dagger m_D^{diag} V_R}_{m_D} \frac{1}{M_R} \underbrace{V_R^T m_D^{diag} U_L^*}_{m_D^T}$$

Re-defenition of M_R (N_R basis choice):

$$V_R \frac{1}{M_R} V_R^T \equiv M^{-1}$$

(RH basis where m_D is diagonal) $\Rightarrow$

$$\boxed{m_{eff} = -U_L^\dagger m_D^{diag} \frac{1}{M} m_D^{diag} U_L^*}$$

$$\Downarrow$$

$$\frac{1}{M} = -(m_D^{diag})^{-1} (U_L \; m_{eff} U_L^T)(m_D^{diag})^{-1}$$

$$\boxed{\frac{1}{M} = -(m_D^{diag})^{-1} \cdot m'_{eff} \cdot (m_D^{diag})^{-1}}$$

$$m'_{eff} = U_L \; m_{eff} U_L^T$$

Since $U_L \simeq 1\!\!1$, $m'_{eff} \simeq m_{eff}$ (phenomenologically known ν mass matrix).

Allowing $U_L = (1\!\!1 \div V_{CKM})$ is approx. equivalent to varying $\theta_{12}, \theta_{23}, \theta_{13}$ in their allowed regions. $\Rightarrow$

We take $m'_{eff} = m_{eff}$ and vary all the parameters of m_{eff} in their allowed regions.

U $(\equiv U_{PMNS})$; (m_1, m_2, m_3) $\Rightarrow$

$$m_{eff} = U \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U^T \equiv \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{pmatrix}$$

$(m_{eff})_{ab}$ – simple expressions in terms of $m_1, m_2, m_3, \theta_{ij}, \delta_{cp}$, e.g.

- $m_{ee} \cong c_{12}^2 m_1 + s_{12}^2 m_2 + s_{13}^2 e^{-2i\delta_{cp}} m_3$
- $m_{e\mu} \cong \frac{1}{\sqrt{2}}\left[(m_3 - m_{ee}) s_{13} e^{-i\delta_{cp}} + (m_2 - m_1) s_{12} c_{12}\right]$
- $m_{e\tau} \cong \frac{1}{\sqrt{2}}\left[(m_3 - m_{ee}) s_{13} e^{-i\delta_{cp}} - (m_2 - m_1) s_{12} c_{12}\right]$

. . . .

$(|s_{13}| \ll 1;\ \theta_{23} \simeq 45°)$.

- $\left(m_D^{diag}\right)^{-1} = \begin{pmatrix} m_u^{-1} & 0 & 0 \\ 0 & m_c^{-1} & 0 \\ 0 & 0 & m_t^{-1} \end{pmatrix}$; $\Rightarrow$

- $$\frac{1}{M} = -\begin{pmatrix} \frac{m_{ee}}{m_u^2} & \frac{m_{e\mu}}{m_u m_c} & \frac{m_{e\tau}}{m_u m_t} \\ \cdots & \frac{m_{\mu\mu}}{m_c^2} & \frac{m_{\mu\tau}}{m_c m_t} \\ \cdots & \cdots & \frac{m_{\tau\tau}}{m_t^2} \end{pmatrix}$$

An important point:

LMA solution of the solar ν problem $\Rightarrow$ all entries of $(m_{eff})_{ab}$ are roughly of the same order (within ~1 order of magnitude of each other). ← Except in a few special cases

Neutrino mass textures (in the limit $\Delta m^2_\odot \Rightarrow 0$)

NH: $$m_{eff} \sim \begin{pmatrix} \otimes & \otimes & \otimes \\ \otimes & 1 & 1 \\ \otimes & 1 & 1 \end{pmatrix}$$

IH: $$\begin{pmatrix} \pm 2 & \otimes & \otimes \\ \otimes & 1 & -1 \\ \otimes & -1 & 1 \end{pmatrix}; \quad \begin{pmatrix} \otimes & 1 & 1 \\ 1 & \otimes & \otimes \\ 1 & \otimes & \otimes \end{pmatrix}$$

Zeros have to be filled in with small elements. Since for LMA $\Delta m^2_\odot \simeq \Delta m^2_{atm}/30$ ($m_2 \sim m_3/6$ for NH) these elements are actually not too small!

(Even more so for QD case).

Eigenvalues of m_D: $m_u \ll m_c \ll m_t$.

Take $\frac{m_u}{m_c} \simeq \frac{m_c}{m_t} = \lambda$ $(\lambda \sim 5 \cdot 10^{-3})$.

Since the elements of m_{eff} are of the same order, M^{-1} has the structure

$$M^{-1} \sim -\frac{m_{ee}}{m_u^2} \left(\begin{array}{cc|c} 1 & \lambda & \lambda^2 \\ \lambda & \lambda^2 & \lambda^3 \\ \hline \lambda^2 & \lambda^3 & \lambda^4 \end{array} \right)$$

Generic case

Dominant (1-2) block. N_R masses and mixing easily found.

$$M_1^{-1} \cong (M^{-1})_{11} = -\frac{m_{ee}}{m_u^2}$$

$$M_2^{-1} \cong M^{-1}_{22} - \frac{(M^{-1}_{12})^2}{(M^{-1}_{11})}$$

– from the determinant of (1-2) block (seesaw like)!

$$M_3^{-1} = -\frac{m_1 m_2 m_3}{m_u^2 m_c^2 m_t^2} \cdot M_2 M_3$$

from the determinant of the seesaw formula.

$$m_{eff} = -m_D \frac{1}{M} m_D^T$$

Taking det of both sides:

- $$m_1 m_2 m_3 = -\frac{m_u^2 m_c^2 m_t^2}{M_1 M_2 M_3}$$

$\underbrace{\text{RH}}$ mixing also readily found

Important for leptogenesis!

U_R: $\quad M^{-1} = U_R \left(M_{diag}^{-1}\right) U_R^T \; ; \quad \Rightarrow$

$$M = U_R^* \; M_{diag} \; U_R^\dagger \, .$$

Simplify notation: $\boxed{M^{-1} \equiv \tilde{M}}$

- $U_{11}^R \simeq U_{22}^R \simeq U_{33}^R \simeq 1$
- $U_{21}^R \simeq -U_{12}^{R\,*} \simeq \dfrac{\tilde{M}_{12}}{\tilde{M}_{11}}$
- $U_{31}^R \simeq \dfrac{\tilde{M}_{13}}{\tilde{M}_{11}}$
- $U_{13}^{R\,*} \simeq \dfrac{\tilde{M}_{12}\tilde{M}_{23} - \tilde{M}_{13}\tilde{M}_{22}}{\tilde{M}_{11}\tilde{M}_{22} - \tilde{M}_{12}^2}$
- $U_{32}^R \simeq -U_{23}^{R\,*} \simeq \dfrac{\tilde{M}_{11}\tilde{M}_{23} - \tilde{M}_{12}\tilde{M}_{13}}{\tilde{M}_{11}\tilde{M}_{22} - \tilde{M}_{12}^2}$

Calculation of $\underbrace{(m_D^\dagger m_D)_{ij}}_{M_R-diag}$ in the basis where m_D is diagonal:

- $(m_D^\dagger m_D)_{ij} = v^2 (Y_D^\dagger Y_D)_{ij} \Rightarrow \left[U_R^\dagger \left(m_D^{diag}\right)^2 U_R\right]_{ij}$

In terms of low-energy observables (m_{eff})

$$U^R_{11} \cong U^R_{22} \cong U^R_{33} \cong 1$$

$$U^R_{21} \cong -U^{R*}_{12} \cong \frac{m_{e\mu}}{m_{ee}} \cdot \frac{m_u}{m_c} \qquad O(\lambda)$$

$$U^R_{31} \cong \frac{m_{e\tau}}{m_{ee}} \cdot \frac{m_u}{m_t} \qquad O(\lambda^2)$$

$$U^{R*}_{13} \cong \frac{m_{e\mu}\, m_{\mu\tau} - m_{\mu\mu}\, m_{e\tau}}{m_{ee}\, m_{\mu\mu} - m_{e\mu}^2} \cdot \frac{m_u}{m_t} \qquad O(\lambda^2)$$

$$U^R_{32} \cong -U^{R*}_{23} \cong \frac{m_{ee}\, m_{\mu\tau} - m_{e\mu} m_{e\tau}}{m_{ee}\, m_{\mu\mu} - m_{e\mu}^2} \cdot \frac{m_c}{m_t} \qquad O(\lambda)$$

$\Rightarrow$ RH mixing is small!
So is LH mixing by our assumption ($U_L \cong 1$) $\Longrightarrow$ Large leptonic mixing in U appears to emerge from nothing!
Seesaw enhancement of leptonic mixing (Smirnov 1993; Tanimoto 1995; Altarelli, Feruglio & Masina, 2000).

N_R masses (small s_{13} limit, NH):

- $M_1 \simeq \dfrac{m_u^2}{s_{12}^2\, m_2} \sim 10^7$ GeV
- $M_2 \simeq \dfrac{2\, m_c^2}{m_3} \sim 10^{11}$ GeV
- $M_3 \simeq \dfrac{m_t^2\, s_{12}^2}{2\, m_1} \gtrsim 10^{15}$ GeV

Very strong mass hierarchy.
Seesaw formulas for $m_{\nu\,light}$:

- $m_1 \cong \dfrac{m_t^2}{2 M_3}\, s_{12}^2$
- $m_2 \cong \dfrac{m_u^2}{M_1}\, \dfrac{1}{s_{12}^2}$
- $m_3 \cong \dfrac{2\, m_c^2}{M_2}$

$m_1 < m_2 \ll m_3$

$\Leftarrow$ Very different from naive expectations

$m_1 \sim \dfrac{m_u^2}{M_1}$

$m_2 \sim \dfrac{m_c^2}{M_2}$

$m_3 \sim \dfrac{m_t^2}{M_3}$

An "effective mass" parameter

$$\tilde{m}_1 = \frac{(Y_D^\dagger Y_D)_{11} v^2}{M_1} \simeq \frac{m_1^2 c_{12}^2 + m_2^2 s_{12}^2}{m_1 c_{12}^2 + m_2 s_{12}^2}$$

For $m_1 \ll m_2$, $\tilde{m}_1 \simeq m_2 = \sqrt{\Delta m^2_\odot}$.

$\longleftrightarrow$

Inverted hierarchy ($m_3 \ll m_1 \approx m_2$):

$$M_1 \simeq \frac{m_u^2}{m_1} \sim 10^5 \text{ GeV}$$

$$M_2 \simeq \frac{2 m_c^2}{m_1} \sim 10^{10} \text{ GeV}$$

$$M_3 \simeq \frac{m_t^2}{2 m_3} \gtrsim 10^{15} \text{ GeV}$$

$\longleftrightarrow$

For quasi-degenerate light ν's –

$$M_1 \sim \underbrace{\frac{m_u^2}{m_0}}_{\sim 10^5 \text{ GeV}}; \quad M_2 \sim \underbrace{\frac{m_c^2}{m_0}}_{\sim 10^{10} \text{ GeV}}; \quad M_3 \sim \underbrace{\frac{m_t^2}{m_0}}_{\sim 10^{15} \text{ GeV}}$$

for $m_1 \simeq m_2 \simeq m_3 \equiv m_0$, $m_0 \simeq 0.1$ eV.

In all cases (NH, IH, QD) in the generic case M_1 is below the absolute lower limit ($\sim 5 \cdot 10^9$) of allowed values consistent with baryogenesis through leptogenesis!

Can this be cured?

For hierarchical RH neutrinos ($M_1 \ll M_2 \ll M_3$):

$$\boxed{\epsilon_1 \simeq \frac{3}{16\pi} \frac{1}{(Y_D^\dagger Y_D)_{11}} \sum_{i=2,3} \mathrm{Im}\left[(Y_D^\dagger Y_D)^2_{1i} \right] \frac{M_1}{M_i}}$$

For successful baryogenesis via leptogenesis one needs $\epsilon_1 \gtrsim 10^{-6}$.

Consider NH.

- $(Y_D^\dagger Y_D)_{11} \simeq 3 \frac{m_u^2}{v^2}$

- $(Y_D^\dagger Y_D)_{12} \simeq \left\{ \frac{m^*_{e\mu} + m^*_{e\tau}}{m_{ee}} \right\} \frac{m_u m_c}{v^2}$

- $(Y_D^\dagger Y_D)_{13} \cong \left(\frac{m_{e\tau}}{m_{ee}}\right)^* \frac{m_u m_t}{v^2}$

$\Downarrow$ $\tilde{m}_1 \simeq m_2 \cong \sqrt{\Delta m^2_\odot}$

$$\boxed{\begin{aligned}\epsilon_1 &\cong \frac{1}{32\pi}\frac{1}{s_{12}^4}\left(\frac{m_3}{m_2}\right)^3 \cdot s_{13}^2 \cdot \mathrm{Im}(phase) \cdot \frac{m_u^2}{v^2} \\ &\sim 7\cdot 10^{-11} \cdot \mathrm{Im}(phase) \cdot \left(\frac{m_u}{3\,MeV}\right)^2\end{aligned}}$$

m_u – the smallest eigenvalue of m_D.

To obtain $\epsilon_1 \gtrsim 10^{-6}$ one would need

- $\boxed{m_u \gtrsim 350\ MeV}$

Difficult to reconcile with GUTs

Another option:

A suppressed (compared to the typical magnitudes) value of m_{ee}.

E.g., $\leftarrow$ Buchmüller & Yanagida, 1998

$$m_{eff} \sim \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} \quad \text{or} \sim \begin{pmatrix} \varepsilon^3 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}$$

$\boxed{\varepsilon \sim 0.1}$

To remain in the generic case one would still need

$$\tilde{M}_{11} = \frac{m_{ee}}{m_u^2} \gg \tilde{M}_{12} = \frac{m_{e\mu}}{m_u m_c} \Rightarrow \boxed{\frac{m_{ee}}{m_{e\mu}} \gg \frac{m_u}{m_c}}$$

even though $\frac{m_{ee}}{m_{e\mu}} \ll 1$.

Take, e.g., $\frac{m_{ee}}{m_{e\mu}} \simeq \frac{1}{30}$. To obtain $\epsilon_1 \gtrsim 10^{-6}$ one would need

$$\boxed{m_u \gtrsim 50\ MeV}$$

Special cases

I. Subdeterminant $\tilde{M}_{(1-2)} \cong 0$

($\Leftrightarrow$ subdet. $m_{eff\,(1-2)} \cong 0$,

$m_{ee} m_{\mu\mu} - m_{e\mu}^2 \cong 0$)

$\Rightarrow$ $\tilde{M}_2$ ($= M_2^{-1}$) cannot be found directly from det $[\tilde{M}_{(1-2)}]$.

All special cases (but I) are only possible for NH!

II. $m_{ee} \cong 0 \Rightarrow \tilde{M}_{11} \cong 0$.

More precisely:

$$\underbrace{\frac{m_{ee}}{m_u^2}}_{\tilde{M}_{11}} << \underbrace{\frac{m_{e\mu}}{m_u m_c}}_{\tilde{M}_{12}} \Longleftrightarrow m_{ee} \ll \frac{m_u}{m_c} \cdot m_{e\mu}$$

$\tilde{M}_1$ ($= M_1^{-1}$) is no longer given by $\tilde{M}_{11}$.

III. $m_{ee} \cong 0$ and $m_{e\mu} \cong 0$ (Comb. I + II).

$\Rightarrow m_{ee} \cong 0$ and $\det(m_{eff})_{12} \equiv m_{ee} m_{\mu\mu} - m_{e\mu}^2 \cong 0$

Special case I: det $\tilde{M}_{(1-2)} \cong 0$.

In the limit $s_{13} \Rightarrow 0$ this requires

$$m_1 c_{12}^2 + m_2 s_{12}^2 \cong - \frac{m_1 m_2}{m_3}$$

$\boxed{M_1 \cong \frac{m_u^2}{m_{ee}}}$ $\Leftarrow$ The same expression as in the generic case.

$M_{2,3}$ – difficult to find from $\tilde{M} = M^{-1}$, but easy to find from M.

Alternatively, $M_{2,3}$ can be found from determ. condition

$$M \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \sim\lambda^2 & \lambda \\ \lambda^2 & \lambda & \sim 0 \end{pmatrix}$$

$M_{2,3} \cong \pm M_{23} \Rightarrow \boxed{M_{2,3} \cong \pm \frac{m_c m_t}{\sqrt{\Delta m^2_{atm}}}}$

Numerically:

$\underbrace{M_1 \sim (10^7 \div 10^8)\ \mathrm{GeV}}$; $M_{2,3} \simeq \pm 4 \cdot 10^{12}$ GeV

Still too small for a successful baryogenesis through leptogenesis

RH mixing parameters (special case I):

- $U^R_{11} \simeq 1$; $U^R_{22} \simeq U^R_{33} \simeq \frac{1}{\sqrt{2}}$

  $U^R_{23} \simeq \frac{1}{\sqrt{2}}$; $U^R_{32} = -\frac{1}{\sqrt{2}}$

- $U^R_{21} \simeq \frac{\tilde{M}_{12}}{\tilde{M}_{11}} \simeq \frac{m_{e\mu}}{m_{ee}} \cdot \frac{m_u}{m_c} \quad \sim O(\lambda)$

- $U^R_{31} \simeq \frac{\tilde{M}_{13}}{\tilde{M}_{11}} = \frac{m_{e\tau}}{m_{ee}} \cdot \frac{m_u}{m_t} \quad \sim O(\lambda^2)$

- $U^{R*}_{12} \simeq U^{R*}_{13} \simeq -\frac{1}{\sqrt{2}} \frac{m_{\mu\mu}}{m_{e\mu}} \cdot \frac{m_u}{m_c} \quad \sim O(\lambda)$

$$\longleftrightarrow$$

$$(Y_D^\dagger Y_D)_{11} \simeq \frac{m_u^2}{v^2}\left[1 + \frac{|m_{e\mu}|^2 + |m_{e\tau}|^2}{|m_{ee}|^2}\right];$$

$$(Y_D^\dagger Y_D)_{12} \simeq \frac{1}{\sqrt{2}}\left(\frac{m_{e\tau}}{m_{ee}}\right)^* \frac{m_u m_t}{v^2} \simeq (Y_D^\dagger Y_D)_{13};$$

$$\epsilon_1 \simeq 8.8 \cdot \left(\frac{m_u}{v}\right)^2 \cdot \mathrm{Im}(\text{phase}) \sim 2.6 \cdot 10^{-9} \left(\frac{m_u}{3\,\text{MeV}}\right)^2.$$

For $\epsilon_1 \sim 10^{-6}$ one needs $\boxed{m_u \gtrsim 60\ \text{MeV}}$

Special case II: $m_{ee} \simeq 0$ $(\tilde{M}_{11} \simeq 0)$

$$\tilde{M} \sim \left(\begin{array}{cc|c} 0 & \lambda & \lambda^2 \\ \lambda & \lambda^2 & \lambda^3 \\ \hline \lambda^2 & \lambda^3 & \lambda^4 \end{array}\right); \quad \tilde{M}_{1,2} \simeq \pm \tilde{M}_{12} \ (\sim\lambda).$$

M_3 - from det. condition

$m_{ee} \simeq m_1 c_{12}^2 + m_2 s_{12}^2$ ($s_{13} \Rightarrow 0$ limit).

$\underline{|m_1| < |m_2|}$ (NH); $\theta_{12} < 45° \Rightarrow \underline{c_{12}^2 > s_{12}^2}$

Cancellation possible if $\boxed{m_1 \simeq -\tan^2\theta_{12} \cdot m_2}$

N.B.: Parameter m_{ee} is responsible for $2\beta 0\nu$ decay (remember $U_L \simeq \mathbb{1}$).

- $M_{1,2} \simeq \pm\left(2\sqrt{2} \cdot \frac{\sqrt{\cos 2\theta_{12}}}{\sin 2\theta_{12}}\right) \frac{m_u m_c}{\sqrt{\Delta m^2_\odot}} \quad \sim 8 \cdot 10^8$ GeV

- $M_3 \simeq \frac{m_t^2}{2\sqrt{\Delta m^2_{atm}}} \quad \sim 3 \cdot 10^{14}$ GeV $\quad (m_u \simeq 3$ MeV)

$\Delta M_{1,2}/M_1 \sim \lambda^2/\lambda \sim \lambda$ (splitting $\simeq \tilde{M}_{22}$).

Can be made much smaller if instead of $m_{ee} = 0$ one requires m_{ee} small $(|\tilde{M}_{11}| \simeq |\tilde{M}_{22}|)$

RH mixing parameters (special case II):

- $U^R_{11} \simeq U^R_{22} \simeq U^R_{12} \simeq \frac{1}{\sqrt{2}}$
- $U^R_{21} \simeq -\frac{1}{\sqrt{2}}$
- $U^R_{31} \simeq -U^R_{32}\, \frac{m_{e\tau}}{m_{e\mu}} \cdot \frac{m_e}{m_t} \cdot \frac{1}{\sqrt{2}}$ $\qquad O(\lambda)$
- $U^{R*}_{23} \simeq -\frac{m_{e\tau}}{m_{e\mu}} \cdot \frac{m_e}{m_t}$ $\qquad O(\lambda)$
- $U^{R*}_{13} \simeq -\left(\frac{m_{e\mu} m_{\mu\tau} - m_{e\tau} m_{\mu\mu}}{m_{e\mu}^2}\right)\frac{m_u}{m_t}$ $\qquad O(\lambda^2)$

$$\longleftrightarrow$$

- $(m_D^\dagger m_D)_{ij} = v^2 (Y_D^\dagger Y_D)_{ij} = (m_D^{diag})_k U^{R*}_{ki} U^R_{kj}$
- $(Y_D^\dagger Y_D)_{11} \simeq \frac{m_c^2}{v^2}$;
- $(Y_D^\dagger Y_D)_{12} \simeq -\frac{1}{2}\frac{m_c^2}{v^2}\left[1 + \left|\frac{m_{e\tau}}{m_{e\mu}}\right|^2\right]$
- $(Y_D^\dagger Y_D)_{13} \simeq -\frac{1}{\sqrt{2}}\left(\frac{m_{e\tau}}{m_{e\mu}}\right)^* \cdot \frac{m_c m_t}{v^2}$

For quasi-degenerate N_1 and N_2:

$$\epsilon_1 + \epsilon_2 \simeq 2\epsilon_1 \simeq \frac{1}{4\pi}\frac{1}{(Y_D^\dagger Y_D)_{11}}\left\{ \mathrm{Im}\left[(Y_D^\dagger Y_D)_{12}^2\right] \cdot \left(\frac{M_1}{\Delta}\right) - \frac{3}{2}\mathrm{Im}\left[(Y_D^\dagger Y_D)_{13}^2\right] \cdot \frac{M_1}{M_3} \right\}$$

Enhancement factor $\left(\frac{M_1}{\Delta}\right)$:

$$\Delta = \max\{|M_1 - M_2|, \Gamma_1\}$$

$$\Gamma_1 \simeq \Gamma_2 \simeq \frac{1}{8\pi}(Y_D^\dagger Y_D)_{11} \cdot M_1 \simeq \frac{1}{8\pi}\frac{m_c^2}{v^2} M_1 ;$$

$\boxed{\frac{\Gamma_1}{M_1} \sim 10^{-6}}$ – a strong enhancement is in principle possible

Effective mass parameter:

$$\tilde{m}_1 \simeq \tilde{m}_2 \simeq \frac{m_c^2}{M_1} \simeq 1\,eV.$$

- $\epsilon_1 + \epsilon_2 \simeq 4 \cdot 10^{-6}\left(\frac{M_1}{\Delta}\right) \cdot \mathrm{Im}(phase)$

Enhancement $\left(\frac{M_1}{\Delta}\right) \sim 10^4$ necessary because of large $\tilde{m}_{1,2}$ $\Rightarrow$ in principle OK.

Special case III: both m_{ee} and $m_{e\mu} \cong 0$.

$$m_{eff} \cong \begin{pmatrix} 0 & 0 & m_{e\tau} \\ 0 & m_{\mu\mu} & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{pmatrix} \Rightarrow \tilde{M} \cong \begin{pmatrix} 0 & 0 & \tilde{M}_{13} \\ 0 & \tilde{M}_{22} & \tilde{M}_{23} \\ \tilde{M}_{13} & \tilde{M}_{23} & M_{33} \end{pmatrix}$$

$$\det(m_{eff})_{(1-2)} = m_{ee} m_{\mu\mu} - m_{e\mu}^2 = 0$$

$$\Rightarrow \det(\tilde{M})_{(1-2)} = 0.$$

We had:

(I) $\det(\tilde{M})_{(1-2)} = 0$, $m_{ee} \neq 0 \Rightarrow M_2 \simeq M_3$

II $\det(\tilde{M})_{(1-2)} \neq 0$, $m_{ee} = 0 \Rightarrow M_1 \cong M_2$.

Case III is equiv. (I) + (II) $\Rightarrow M_1 \simeq M_2 \simeq M_3$.

$$M_1 \cong \tilde{M}_{22}^{-1} = \frac{m_c^2}{m_{\mu\mu}}; \quad M_{2,3} \cong \pm \frac{m_u m_t}{m_{e\tau}}$$

We have: $m_u m_t \simeq m_c^2$; $m_{\mu\mu} \approx m_{e\tau}$

$\Downarrow$

- $M_1 \simeq M_2 \simeq M_3$

Having $m_{ee} \cong 0$ and $m_{e\mu} \cong 0$ simultaneously requires $s_{13} \cong -(m_2/m_3)(s_{12}/c_{12}) \simeq 0.1$

Special case III: $m_{ee} \cong 0 \cong m_{e\mu}$

- $$M_1 \cong \frac{2 m_c^2}{m_3 + \frac{\cos 2\theta_{12}}{\cos^2\theta_{12}} m_2} \sim 5.7 \cdot 10^{10} \text{ GeV}$$

- $$M_{2,3} \cong \pm \frac{m_u m_t}{\sqrt{2}\, m_2} \cdot \cot\theta_{12} \sim 6.4 \cdot 10^{10} \text{ GeV}$$

Stronger degeneracy between M_1 and $M_{2,3}$ can be achieved by a slight change of m_c and/or $m_u m_t$.

N_1 has a large component along m_c direction

$$U^R_{21} \cong 1, \quad U^R_{11} \sim U^R_{31} \sim 0$$

$N_{2,3}$ have large components ($\sim \frac{1}{\sqrt{2}}$) along m_u and m_t directions:

$$U^R_{12} \simeq \frac{1}{\sqrt{2}}, \; U^R_{32} \cong -\frac{1}{\sqrt{2}}; \; U^R_{13} \cong U^R_{33} \cong \frac{1}{\sqrt{2}}$$

$\Downarrow$

$$\tilde{m}_1 \cong \frac{m_c^2}{M_1} \simeq \frac{m_3}{2} \simeq 2.5 \cdot 10^{-2} \text{ eV}; \quad \tilde{m}_{2,3} \simeq \frac{m_t^2}{2 M_2} \sim 250 \text{ eV}$$

Huge $\tilde{m}_{2,3} \Rightarrow$ a very strong washout effect.

For $\tilde{m}_1 \simeq 2.5\cdot 10^{-2}$ eV, $M_1 \simeq 6\cdot 10^{10}$ GeV is only ~ a factor of $(2\div 3)$ below the value required for a successful baryogenesis through leptogenesis ($M_1 \sim 1.5\cdot 10^{11}$ GeV). In principle, mass degeneracy could help.

But: strong washout effects due to $\not{L}$ processes involving close in mass N_2 and N_3 may erase the asymmetry produced by the decays of N_1.

Requires further study.

We were assuming small LH mixing, $U_L \simeq \mathbb{1}$ (or $\sim V_{CKM}$), so that

$$m'_{eff} = U_L\, m_{eff}\, U_L^T \simeq m_{eff}.$$

What happens if strong LH rotations are allowed?

Typically, the elements of m_{eff} are reshuffled, making them even more homogeneous $\Rightarrow$ enforces the generic case.

Situation would change if there are (consistent with low-energy phenomenology) LH rotations which would break the $\tilde{M}$(1-2) dominance.

Any pattern that reproduces correctly $\Delta m^2_\odot$ and Δm^2_{atm} would do $\Rightarrow$ the LH rotation can always be chosen so as to reproduce the correct U_{PMNS}!

An example

$$\tilde{M} \simeq \begin{pmatrix} \sim A & \sim A & 0 \\ \sim A & \sim A & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$$

Eigenvalues: $\sim 0,\ 2A,\ \varepsilon$

$$m'_{eff} \simeq \begin{pmatrix} \sim A \cdot m_u^2 & \sim A\, m_u m_c & 0 \\ \sim A\, m_u m_c & \sim A\, m_c^2 & 0 \\ 0 & 0 & \varepsilon \cdot m_t^2 \end{pmatrix}$$

Eigenvalues: $\underbrace{\sim 0}_{m_1},\ \underbrace{A \cdot (m_u^2 + m_c^2)}_{m_2},\ \underbrace{\varepsilon \cdot m_t^2}_{m_3}$

$\Rightarrow A \simeq 6 \cdot 10^{-12}\ \text{GeV}^{-1}\ ;\quad \varepsilon \simeq 1.6 \cdot 10^{-15}\ \text{GeV}^{-1}$!

$M_1 \simeq (2A)^{-1} \simeq 8.3 \cdot 10^{10}$ GeV

$M_2 \simeq \varepsilon^{-1} \simeq 6.1 \cdot 10^{14}$ GeV

$M_3 \to \infty$

$\theta_{12}^R \simeq 45^\circ\ ;\quad \theta_{23}^R = \theta_{13}^R = 0.$

$\tilde{m}_1 \simeq \frac{m_c^2}{2M_1} \simeq m_2 \simeq 8 \cdot 10^{-2}$ eV – OK

- m'_{eff} almost diagonal $\Rightarrow U \simeq U_L \Leftrightarrow$ strong lept. mixing from D. sector

Conclusions

- 1. Within the seesaw mechanism, a systematic study of the RH sector was performed under assumptions of hierarchical m_D eigenvalues and small LH rotation. Input: low E ν data.
- 2. Generically, one obtains a very strong hierarchy of N_R masses (and small RH mixing), with $M_1 \sim (10^5 \div 10^7)$ GeV, $M_2 \sim (10^{10} \div 10^{11})$ GeV, $M_3 \gtrsim 10^{15}$ GeV. $\Leftarrow$ M_1 too small for a successful leptogenesis.
- 3. In addition to the generic case, there are 3 special cases:
  - (I) $M_1 \simeq (10^7 \div 10^8)$ GeV; $M_2 \simeq M_3 \simeq 4 \cdot 10^{12}$ GeV

    $\Uparrow$ Still too low for a successful leptog. $\quad\Uparrow$ quasi-degenerate

- (II) $\underbrace{M_1 \simeq M_2 \cong 8 \cdot 10^8 \text{ GeV}}_{\text{Quasi-degenerate}}$; $M_3 \simeq 3 \cdot 10^{14}$ GeV

  Successful leptogenesis possible

- (III) All 3 RH neutrinos of about the same mass:

$$M_1 \sim M_2 \sim M_3 \sim 6 \cdot 10^{10} \text{ GeV}$$

  Strong washout effects due to N_2 and N_3 - requires a special study.

- 4. The case when large LH mixing is allowed is less restrictive.

  At least one case leading to a successful leptogenesis has been identified. Others may exist.