A Global Analysis of Solar Neutrino and KamLAND Data—— The role of θ_{13}

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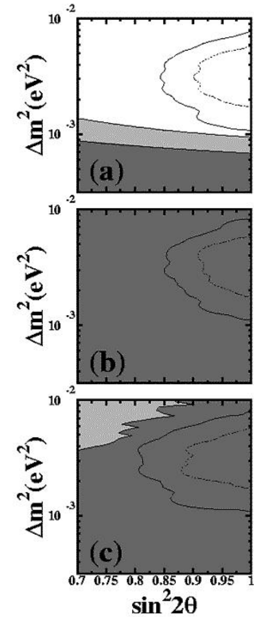
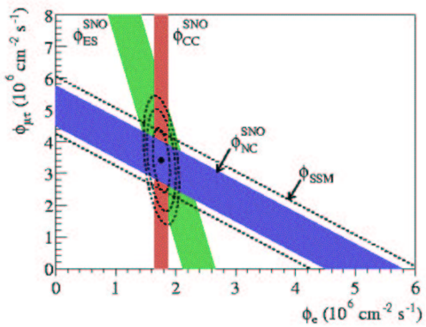
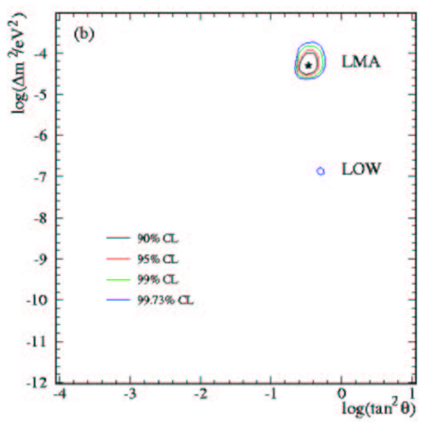
Atmospheric Neutrino Results at Superkamiokande



a) $\nu_\mu \to \nu_\tau$
b) $\nu_\mu \to \nu_s$ $(\delta m^2 > 0)$
c) $\nu_\mu \to \nu_s$ $(\delta m^2 < 0)$
NOTE! : All 2×2 analysis

Sudbury Neutrino Observatory Results (Neutral Current and day-Night)





Atmospheric Neutrinos Fix the θ_{32}.

What about θ_{31}?

The value of θ_{31} is probably the most-pressing open question in neutrino physics:

- CP-violating phase appears together with θ_{13}; Very long baseline neutrino experiments
- Unitarity of neutrino mixing matrix
- Cosmological implications

Three flavor mixing:

$$\Psi_\alpha = \sum_i U_{\alpha i} \Psi_i .$$

$$U_{\alpha i} = T_{23} T_{13} T_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13}^* \\ 0 & 1 & 0 \\ -S_{13} & 0 & C_{13} \end{pmatrix} \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Notation: $C_{13} = \cos\theta_{13}$, etc.

MSW evolution equation:

$$i\frac{\partial}{\partial t}\begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix} = \left[T_{23} T_{13} T_{12} \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} T_{12}^\dagger T_{13}^\dagger T_{23}^\dagger + \begin{pmatrix} V_c + V_n & 0 & 0 \\ 0 & V_n & 0 \\ 0 & 0 & V_n \end{pmatrix} \right] \times \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix}$$

$$V_c(x) = \sqrt{2} G_F N_e(x)$$

$$V_n(x) = -\frac{1}{\sqrt{2}} G_F N_n(x).$$

First Rotation:

$$\tilde{\Psi}_\mu = \cos\theta_{23}\Psi_\mu - \sin\theta_{23}\Psi_\tau ,$$

$$\tilde{\Psi}_\tau = \sin\theta_{23}\Psi_\mu + \cos\theta_{23}\Psi_\tau$$

$$i\frac{\partial}{\partial t}\begin{pmatrix} \Psi_e \\ \tilde{\Psi}_\mu \\ \tilde{\Psi}_\tau \end{pmatrix} = \left[T_{13} T_{12} \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} T_{12}^\dagger T_{13}^\dagger + \begin{pmatrix} V_c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \Psi_e \\ \tilde{\Psi}_\mu \\ \tilde{\Psi}_\tau \end{pmatrix}$$

Second Rotation:

$$\begin{pmatrix} \varphi_e \\ \varphi_\mu \\ \varphi_\tau \end{pmatrix} = T_{13}^\dagger \begin{pmatrix} \Psi_e \\ \tilde{\Psi}_\mu \\ \tilde{\Psi}_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta_{13}\Psi_e + \sin\theta_{13}\tilde{\Psi}_\tau \\ \tilde{\Psi}_\mu \\ -\sin\theta_{13}\Psi_e + \cos\theta_{13}\tilde{\Psi}_\tau \end{pmatrix}$$

$$i\frac{\partial}{\partial t}\begin{pmatrix} \varphi_e \\ \varphi_\mu \\ \varphi_\tau \end{pmatrix} = \mathcal{H}\begin{pmatrix} \varphi_e \\ \varphi_\mu \\ \varphi_\tau \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} \frac{1}{2}\tilde{V} - \Delta_{21}\cos 2\theta_{12} & \frac{1}{2}\Delta_{21}\sin 2\theta_{12} & -\frac{1}{2}V_c\sin 2\theta_{13} \\ \frac{1}{2}\Delta_{21}\sin 2\theta_{12} & -\frac{1}{2}\tilde{V} + \Delta_{21}\cos 2\theta_{12} & 0 \\ -\frac{1}{2}V_c\sin 2\theta_{13} & 0 & \frac{1}{2}(\Delta_{31}+\Delta_{32}) + V_c - \frac{3}{2}\tilde{V} \end{pmatrix}$$

$$\tilde{V} = V_c\cos^2\theta_{13}$$

$$\Delta_{ij} = \frac{m_i^2 - m_j^2}{2E} = \frac{\delta m_{ij}^2}{2E}$$

Initial conditions:

$$\begin{pmatrix} \varphi_e(t=0) \\ \varphi_\mu(t=0) \\ \varphi_\tau(t=0) \end{pmatrix} = \begin{pmatrix} \cos\theta_{13} \\ 0 \\ -\sin\theta_{13} \end{pmatrix}$$

Atmospheric Neutrino Measurements:

$$\Rightarrow \delta m_{32}^2 = 3 \times 10^{-3}\mathrm{eV}^2$$

Approximate Solar Electron Density:

$$N_e(r) = 245\,\exp(-10.54r/R_\odot)N_A cm^{-3},$$

$$\Rightarrow V_c(r) = 1.87 \times 10^{-5} \times \exp(-10.54r/R_\odot)\ \mathrm{eV}^2/\mathrm{MeV}$$

Consequently

$$\frac{V_c(r)}{\Delta_{31}} < 1$$

everywhere in the Sun (good expansion parameter).

$$i\frac{\partial}{\partial t}\varphi_\tau(t) = a\varphi_e + b\varphi_\tau$$

$$a = -V_c \sin\theta_{13}\cos\theta_{13}$$

$$b = \frac{1}{2}(\Delta_{31}+\Delta_{32}) + V_c - \frac{3}{2}\tilde{V} \simeq \Delta_{31} - \frac{1}{2}V_c(1-3\sin^2\theta_{13}) \equiv \Delta_{31} - \epsilon.$$

$$\Rightarrow$$

$$\varphi_\tau(t) = \sin\theta_{13} e^{-i\int_0^t b(t')dt'}\left[-1 + i\cos\theta_{13}\int_0^t dt' V_c(t')\varphi_e(t')e^{i\int_0^{t'} b(t'')dt''}\right]$$

$$i\int_0^t dt' V_c \varphi_e e^{i\int_0^{t'} b(t'')dt''} = -\frac{V_c(t=0)}{\Delta_{31}}\cos\theta_{13} - \frac{1}{\Delta_{31}}\int_0^t e^{i\Delta_{31}t'}\frac{d}{dt'}\left[V_c(t')\varphi_e(t')e^{i\int_0^{t'}\epsilon(t'')dt''}\right]$$

$$\Rightarrow$$

$$\varphi_\tau(t) = \sin\theta_{13}\, e^{-i\int_0^t b(t')dt'} \times \left\{-1 + \cos^2\theta_{13}\left[-\xi\left(1 + \frac{\delta m^2_{21}}{\delta m^2_{31}} + \cdots\right) + \xi^2\left(1 - 2\sin^2\theta_{13} + \cdots\right)\right]\right\}$$

$$\xi \equiv \frac{V_c(t=0)}{\Delta_{31}}.$$

$$i\frac{\partial}{\partial t}\begin{pmatrix}\varphi_e\\ \varphi_\mu\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\tilde{V} - \Delta_{21}\cos 2\theta_{12} & \frac{1}{2}\Delta_{21}\sin 2\theta_{12}\\ \frac{1}{2}\Delta_{21}\sin 2\theta_{12} & -\frac{1}{2}\tilde{V} + \Delta_{21}\cos 2\theta_{12}\end{pmatrix}\begin{pmatrix}\varphi_e\\ \varphi_\mu\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}\varphi_e(t)\\ \varphi_\mu(t)\end{pmatrix} = \begin{pmatrix}\Phi_e(t) & -\Phi^*_\mu(t)\\ \Phi_\mu(t) & \Phi^*_e(t)\end{pmatrix}\begin{pmatrix}\cos\theta_{13}\\ 0\end{pmatrix} \equiv \hat{U}\begin{pmatrix}\cos\theta_{13}\\ 0\end{pmatrix}$$

where where $\Phi_e(t)$ and $\Phi_\mu(t)$ are solutions of the evolution equation with the initial conditions $\Phi_e(t=0)=1$ and $\Phi_\mu(t=0)=0$.

We need the electron neutrino amplitude:

$$\Psi_e = \cos\theta_{13}\,\varphi_e - \sin\theta_{13}\,\varphi_\tau.$$

$$\Psi_e = \cos^2\theta_{13}\,\Phi_e - \sin^2\theta_{13} e^{-i\int_0^t b(t')dt'} D$$

$$D \equiv \varphi_\tau e^{i\int_0^t b(t')dt'} / \sin\theta_{13}$$

$$\Rightarrow$$

$$P_{3\times3}(\nu_e \to \nu_e) = \cos^4\theta_{13}\, P_{2\times2}(\nu_e \to \nu_e \text{ with } N_e\cos^2\theta_{13}) + \sin^4\theta_{13}\left[1 + 2\xi\cos^2\theta_{13}\left(1 + \frac{\delta m^2_{21}}{\delta m^2_{31}}\cos 2\theta_{12}\right) + \xi^2\cos^4\theta_{13}\left(2\frac{\delta m^2_{21}}{\delta m^2_{31}}\cos 2\theta_{12} - 1\right)\right] + \mathcal{O}(\xi^3)$$

$$\xi \equiv \frac{V_c(t=0)}{\Delta_{31}}.$$

Best fit:

$$\tan^2\theta_{12} \sim 0.46$$

$$\cos^4\theta_{13} \sim 1$$

$$\delta m^2_{12} \sim 7.1 \times 10^{-5} \quad \mathrm{eV}^2$$

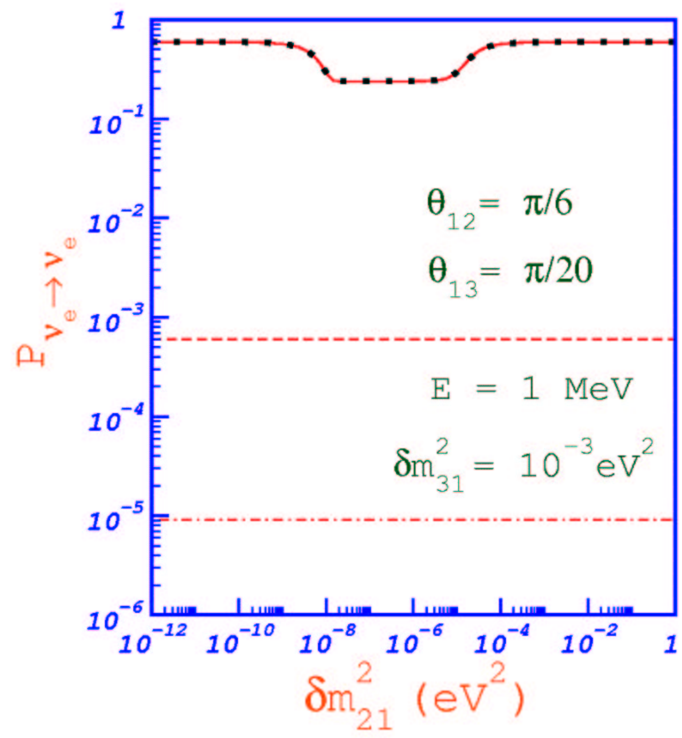


FIG. 1:

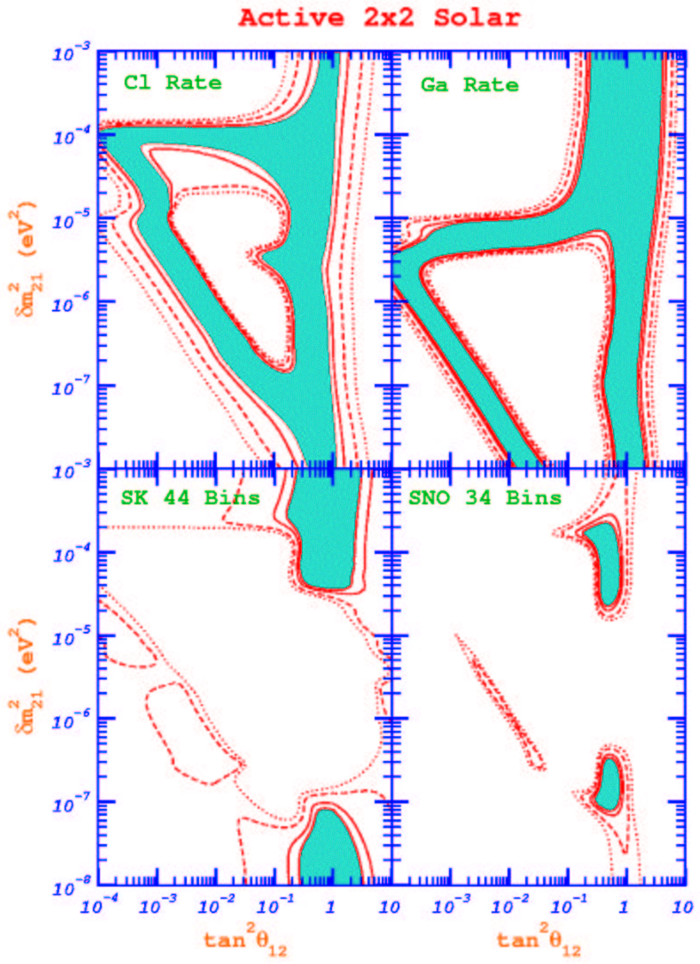

Active 2x2 Solar
Cl Rate
Ga Rate
SK 44 Bins
SNO 34 Bins
δm^2_{21} (eV^2)
$\tan^2\theta_{12}$

FIG. 2:

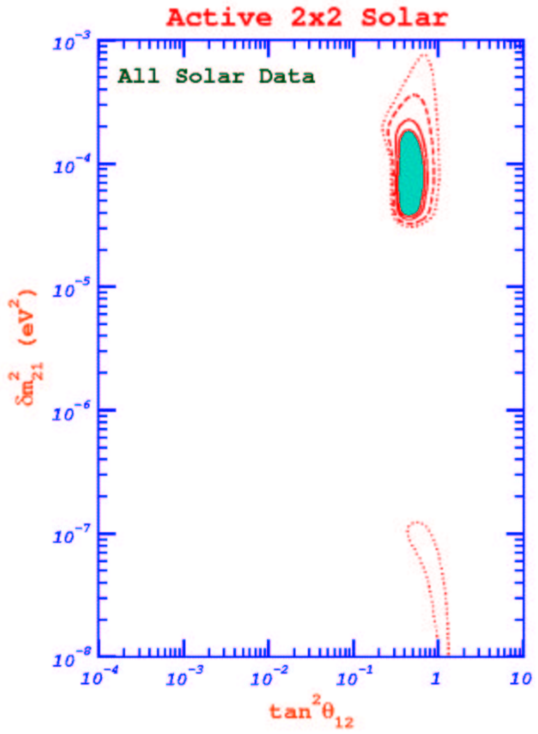

Active 2x2 Solar
All Solar Data
δm^2_{21} (eV^2)
$\tan^2\theta_{12}$

FIG. 3:

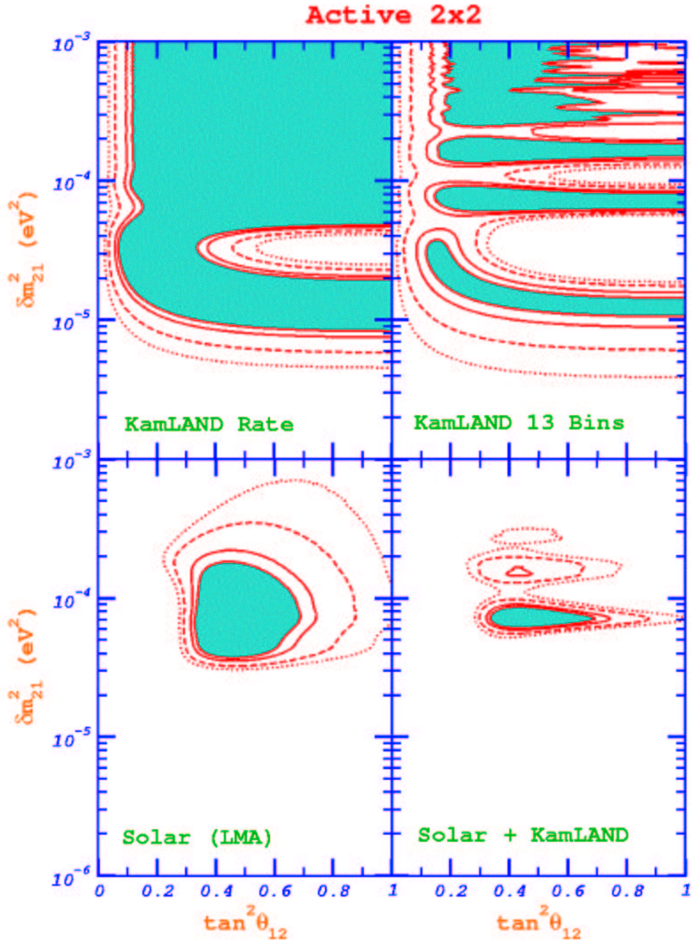

Active 2x2
KamLAND Rate
KamLAND 13 Bins
Solar (LMA)
Solar + KamLAND
δm^2_{21} (eV^2)
$\tan^2\theta_{12}$

FIG. 4:

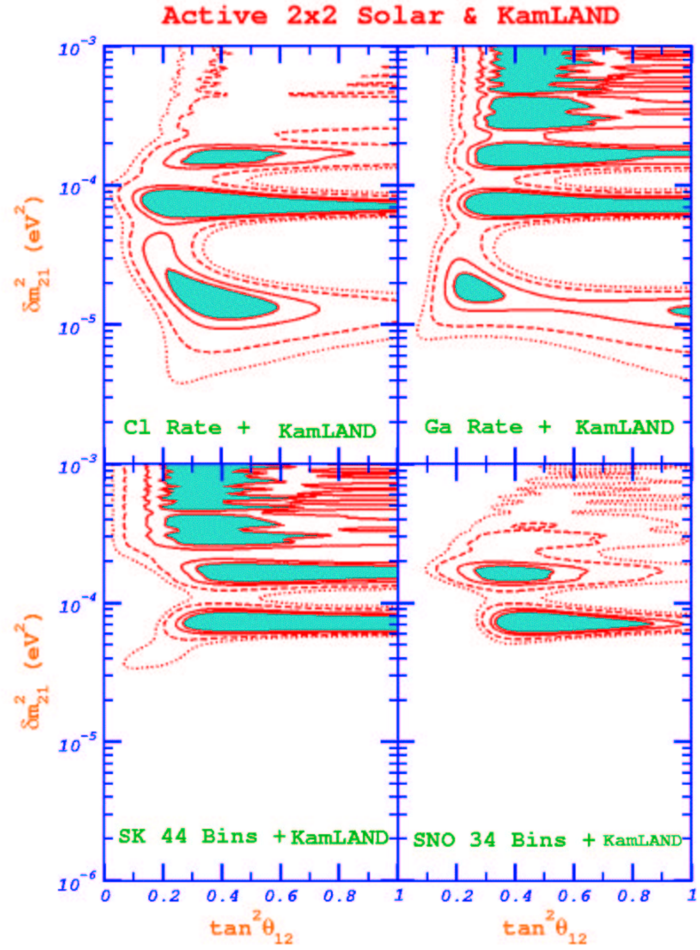

Active 2x2 Solar & KamLAND
Cl Rate + KamLAND
Ga Rate + KamLAND
SK 44 Bins +KamLAND
SNO 34 Bins +KamLAND
δm^2_{21} (eV^2)
$\tan^2\theta_{12}$

FIG. 5:

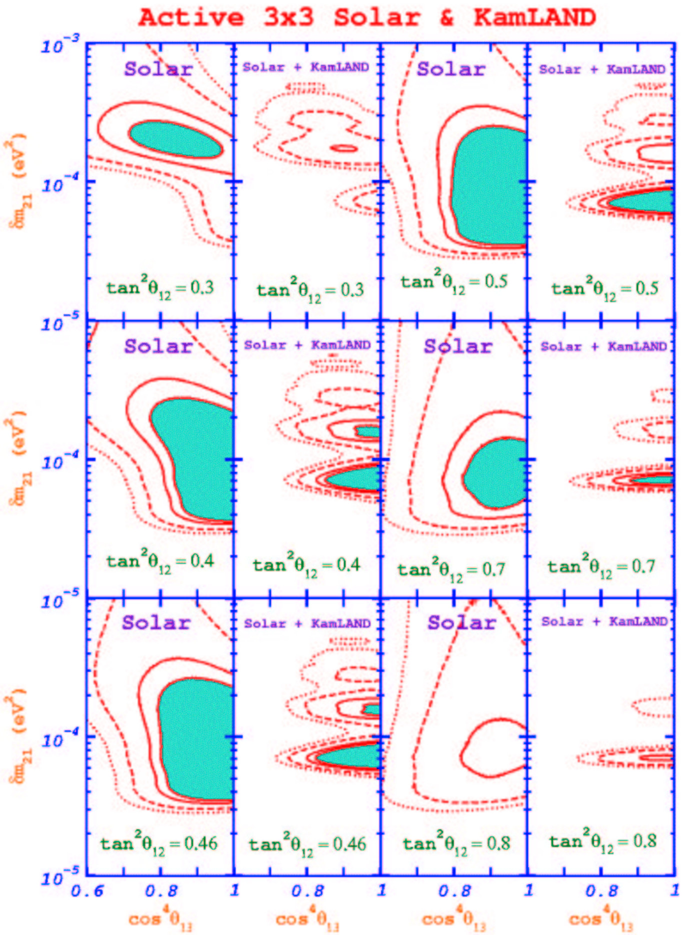

Active 3x3 Solar & KamLAND
Solar
Solar + KamLAND
$\tan^2\theta_{12} = 0.3$
$\tan^2\theta_{12} = 0.5$
$\tan^2\theta_{12} = 0.4$
$\tan^2\theta_{12} = 0.7$
$\tan^2\theta_{12} = 0.46$
$\tan^2\theta_{12} = 0.8$
δm^2_{21} (eV^2)
10^{-3}
10^{-4}
10^{-5}
0.6
0.8
1
$\cos^4\theta_{13}$

FIG. 6: