ν Mass and Structure Formation

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Basics of Structure Formation

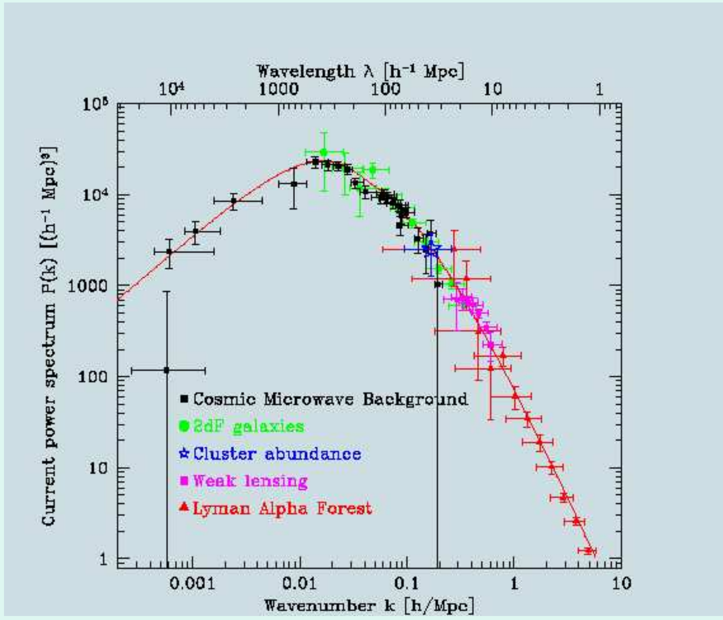
Density Perturbations

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{r})}{\bar{\rho}} - 1 \ ,$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} = 4\pi G\bar{\rho}\delta \ .$$

$$\langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}'} \rangle = (2\pi)^3 \, P(k) \, \delta^{(3)} \left(\mathbf{k} - \mathbf{k}'\right) \ ,$$

$$P(k) = T(k)^2 P_*(k)$$



Tegmark & Zaldarriaga PRD66, 103508 (2002)

$$P_*(k) \propto k^n$$

$$\Delta^2(k)\big|_{z=0} \equiv \frac{k^3}{2\pi^2}P(k) = \delta_H^2 \left(\frac{ck}{H_0}\right)^{3+n} T^2(k),$$

$$\delta_H = 1.95 \times 10^{-5}\Omega_0^{-0.35-0.19\ln\Omega_0-0.17\tilde{n}} e^{-\tilde{n}-0.14\tilde{n}^2} \qquad (\Lambda = 0),$$

$$\delta_H = 1.94 \times 10^{-5}\Omega_0^{-0.785-0.05\ln\Omega_0} e^{-0.95\tilde{n}-0.169\tilde{n}^2} \qquad (\Lambda = 1 - \Omega_0),$$

$$T(k) = \begin{cases} 1; & k<k_{eq} \\ (k_{eq}/k)^2\,; & k>k_{eq} \end{cases}$$

$$k_{\rm eq} \equiv (2\Omega_0 H_0^2 z_{\rm eq})^{1/2} = 7.46 \times 10^{-2}\Omega_0 h^2 \Theta_{2.7}^{-2}\,{\rm Mpc}^{-1},$$

Eisenstein & Hu,
ApJ 496, 605 (1998)

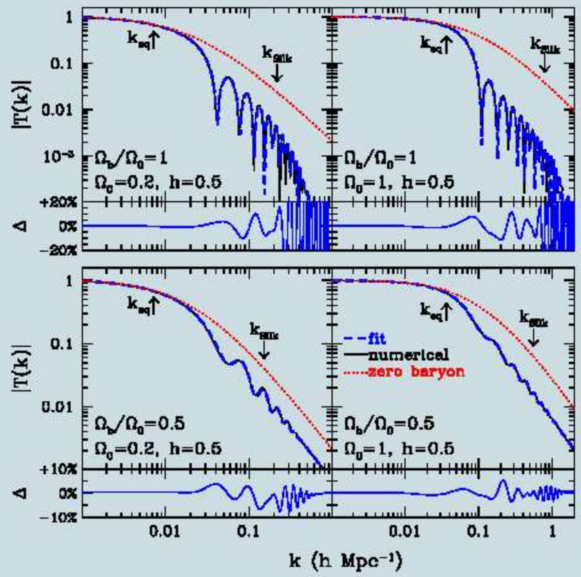


Fig. 3.— Four examples of the fit compared to numerical results. The larger plots show the numerical result (solid) and the fit (dashed). The smaller subplots show the residuals, defined as the difference between the two divided by a non-oscillatory envelope. Note that in the fully baryonic models, the oscillations have alternating sign in the transfer function. Also shown is the zero baryon case (dotted); note the strong suppression on scales below the sound horizon due to the baryons.

Neutrino mass

$$\Omega_\nu h^2 = \frac{m_{\nu,\mathrm{tot}}}{94\ \mathrm{eV}},$$

Neutrino effect on Structure:

hot dark matter suppression

$$k_{\mathrm{nr}} \approx 0.026 \left(\frac{m_\nu}{1\,\mathrm{eV}}\right)^{1/2} \Omega_m^{1/2} h\,\mathrm{Mpc}^{-1}.$$

$$\left(\frac{\Delta P}{P}\right) \approx -8\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{m_\nu}{1\,\mathrm{eV}}\right)\left(\frac{0.1N}{\Omega_m h^2}\right).$$

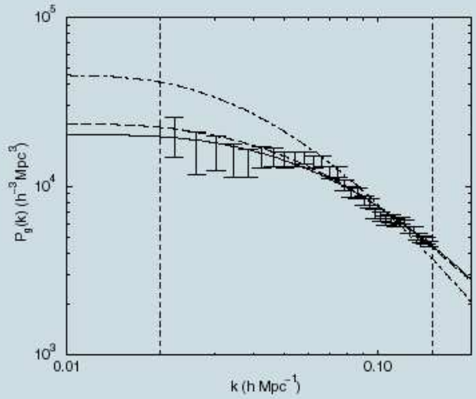


FIG. 1. Power spectra for $\Omega_\nu = 0$ (solid line), $\Omega_\nu = 0.01$ (dashed line), and $\Omega_\nu = 0.05$ (dot-dashed line) with amplitudes fitted to the 2dFGRS power spectrum data (vertical bars) in redshift space. We have fixed $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, and $\Omega_b h^2 = 0.02$. The vertical dashed lines limit the range in k used in the fits.

O. Elgaroy et al (2dFGRS) PRL 89, 061301 (2002)

galaxy survey

Biase

$$\xi_g(r) = b^2 \xi(r) + \tilde{\xi}(r),$$

$$P_g(k) = b^2 P(k) + c,$$

quasi-nonlinear correction (Peacock & Dodds 1994, 1996)

$\Delta^2 \sim 1$, $k \sim 0.2\, h$ Mpc^{-1}

$$\Delta^2_{\rm NL} = f_{\rm NL}[\Delta^2_{\rm L}],$$

$$f_{\rm NL}[x] = x \left(\frac{1 + B\beta x + [Ax]^{\alpha\beta}}{1 + ([Ax]^\alpha g^3(\Omega)/[V x^{1/2}])^\beta} \right),$$

$$g(\Omega) = \frac{\frac{5}{2}\Omega_m}{\Omega_m^{4/7} - \Omega_\Lambda + (1 + \Omega_m/2)(1 + \Omega_\Lambda/70)}.$$

2dFGRS

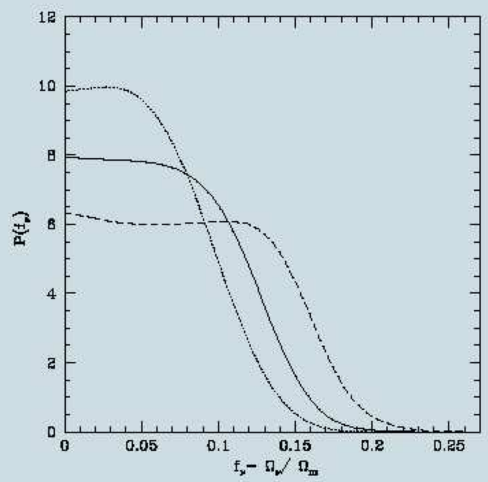


FIG. 3. Probability distributions, normalized so that the area under each curve is equal to 1, for f_ν with marginalization over the other parameters, as explained in the text, for $N_\nu = 3$ massive neutrinos and $n = 0.9$ (dotted line), 1.0 (solid line), and 1.1 (dashed line).

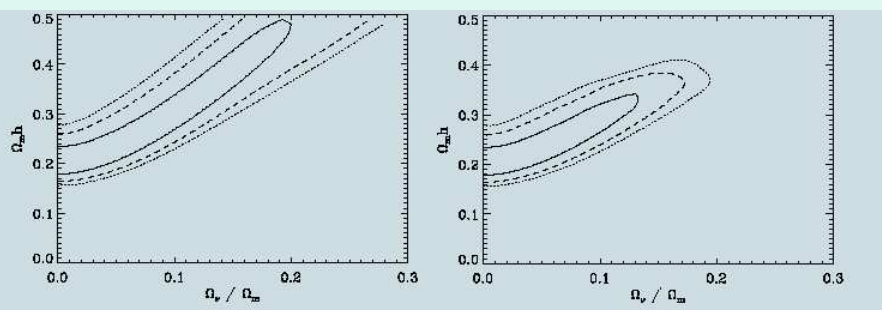


FIG. 2. Confidence contours at 68% (solid line), 95% (dashed line), and 99% (dotted line) in the plane of $f_\nu \equiv \Omega_\nu/\Omega_m$ and $\Gamma \equiv \Omega_m h$, with marginalization over h and $\Omega_b h^2$ using Gaussian priors, and over A using a uniform prior in $0.5 < A < 10$. The left panel shows the case of no prior on Ω_m, and the right panel the case of a uniform "top hat" prior on Ω_m in $0.1 < \Omega_m < 0.5$.

	$\Omega_m = 0.28 \pm 0.14$ (SNIa)		$0.1 < \Omega_m < 0.5$	
n	f_ν	$m_{\nu,tot}$ (eV)	f_ν	$m_{\nu,tot}$ (eV)
0.9	0.12	1.5	0.11	1.5
1.0	0.14	1.8	0.13	1.8
1.1	0.16	2.1	0.16	2.2

TABLE I. Summary of 95% confidence upper bounds on f_ν with our two chosen priors on Ω_m. The conversion of f_ν to $m_{\nu,tot}$ is for $h = 0.7$ and the central values of $\Omega_m = 0.28$ (SNIa case) and $\Omega_m = 0.30$ (uniform prior case).

2dFRGS

CMB

$$\mathcal{C}_\ell = \int_{-\infty}^{\infty} \frac{W_\ell(k)}{T(k)^2} P(k) d\ln k.$$

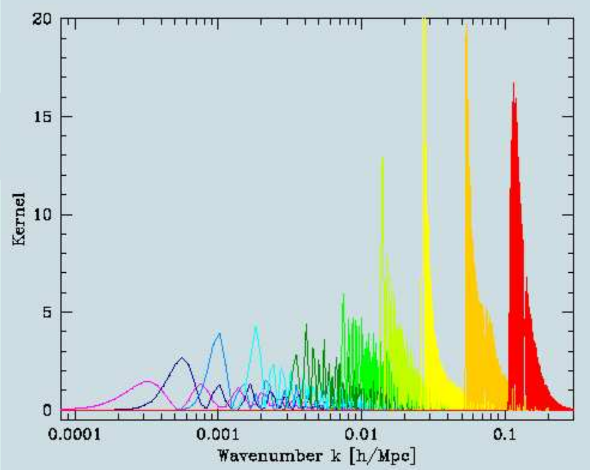


FIG. 4. The curves $kW_\ell(k)$ whose integral give $\mathcal{C}_\ell$ for a scale-invariant spectrum, all rescaled to have unit area. From left to right, the curves are for multipoles ℓ = 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.

Tegmark & Zaldarriaga, PRD66, 103508 (2002)

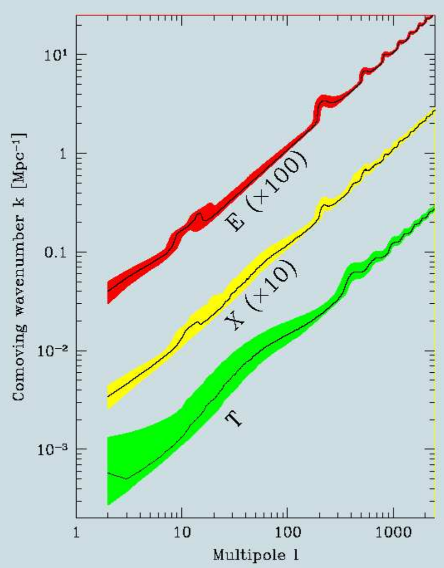


FIG. 5. The correspondence between ℓ-space and k-space for CMB. For each ℓ, the shaded bands indicates the k-range from the 20th to 80th percentile of the distribution $kW_\ell(k)$ (Figure 4), and the black curve shows the median. From top to bottom, the three bands are for the E-polarization, cross-polarization (X) and unpolarized (T) cases, respectively. To avoid clutter, the E and X bands have been multiplied by 10 and 100, respectively.

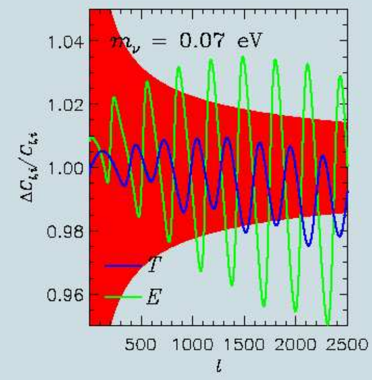


FIG. 2: The change in C_l due to a neutrino mass of 0.07 eV. The shaded band is the cosmic variance error of $\Delta C_l/C_l = 1/\sqrt{2l+1}$.

Hannestad, astro-ph/0205223

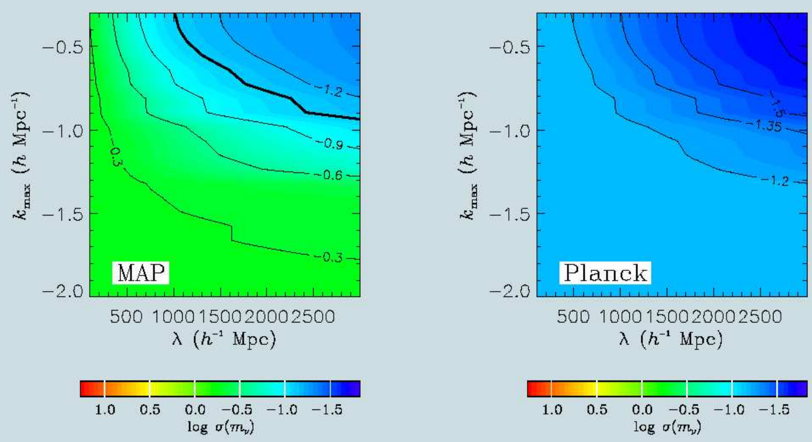


FIG. 4: The estimated 1σ error bar on m_ν using CMB data from MAP combined with a hypothetical LSS survey. The thick solid line corresponds to $m_\nu = 0.07$ eV.

FIG. 5: The estimated 1σ error bar on m_ν using CMB data from Planck combined with a hypothetical LSS survey. The thick solid line corresponds to $m_\nu = 0.07$ eV.

$m_{\nu tot}$ <4.2 eV Wang, Tegmark & Zaldarriaga, PRD65, 123001 (2001)

Hannestad, astro-ph/0205223

TABLE II: Best fit χ^2 and upper limits on $\sum m_{\nu,\mathrm{max}}$ for the three different priors.

prior type	best fit χ^2	$\sum m_{\nu,\mathrm{max}}$ (eV) (95% conf.)
CMB + LSS	24.81	2.96
CMB + LSS + BBN + H_0	25.66	2.65
CMB + LSS + BBN + H_0 + SNIa	25.71	2.47

	6 parameters	9 parameters			
	+2dF	no 2dF	+2dF	+2dF	+2dF
	68%-1D	68%-1D	68%-1D	68%-full	95%-full
f_ν	-	< 0.10	< 0.04	< 0.10	< 0.13
w	-	< -0.87	< -0.88	< -0.68	< -0.58
ϵ_1	-	< 0.032	< 0.032	< 0.069	< 0.085
-> m_ν/eV	-	< 0.29	< 0.14	< 0.36	< 0.54 <-
r_{10}	-	< 0.30	< 0.31	< 0.92	< 1.4
$\Omega_b h^2$	0.021 ± 0.001	0.022 ± 0.001	0.022 ± 0.001	$0.018 - 0.025$	$0.017 - 0.026$
$\Omega_{DM} h^2$	0.113 ± 0.008	0.099 ± 0.014	0.106 ± 0.010	$0.082 - 0.130$	$0.072 - 0.142$
h	0.67 ± 0.03	0.67 ± 0.05	0.66 ± 0.03	$0.59 - 0.75$	$0.55 - 0.78$
n_s	0.98 ± 0.04	1.02 ± 0.05	1.03 ± 0.05	$0.91 - 1.13$	$0.87 - 1.19$
Ω_Λ	0.70 ± 0.04	0.72 ± 0.06	0.71 ± 0.04	$0.58 - 0.80$	$0.54 - 0.82$
Ω_m	0.30 ± 0.04	0.28 ± 0.05	0.29 ± 0.04	$0.20 - 0.42$	$0.18 - 0.46$
t_0/Gyr	14.1 ± 0.4	14.3 ± 0.4	14.1 ± 0.4	$13.3 - 15.0$	$13.0 - 15.2$
$\Omega_m h$	0.20 ± 0.02	0.18 ± 0.03	0.19 ± 0.02	$0.15 - 0.25$	$0.13 - 0.26$
σ_8	0.79 ± 0.06	0.54 ± 0.13	0.67 ± 0.08	$0.49 - 0.93$	$0.45 - 0.95$
$\sigma_8 e^{-\tau}$	0.72 ± 0.04	0.50 ± 0.12	0.61 ± 0.07	$0.47 - 0.81$	$0.41 - 0.84$
$\sigma_8 \Omega_m^{0.55}$	0.40 ± 0.05	0.27 ± 0.08	0.34 ± 0.05	$0.22 - 0.51$	$0.19 - 0.53$

TABLE II: Parameter constraints for 6 and 9 parameter flat models with all data with or without 2dF. The top section shows the constraints on the additional parameters that were fixed in the basic 6 parameter model, the bottom half shows the effect these additional parameters have on the results for the basic parameters. 1D limits are from the confidence interval of the fully marginalized 1D distribution, the full limits give the extremal values of the parameters in the full n-dimensional confidence region (see Appendix C for discussion). Bold parameters are base Monte-Carlo parameters, non-bold parameters are derived from the base parameters.

Lewis & Bridle, PRD66, 103511 (2002)

Lyman alpha forest

m_ν <5.5 eV

m_ν <2.4 (Ω_m/0.17-1) eV
[0.2< Ω_m]

but nonlinear-effect?
(Zaldarriaga, Scoccimarro, Hui, astro-ph/0111230)

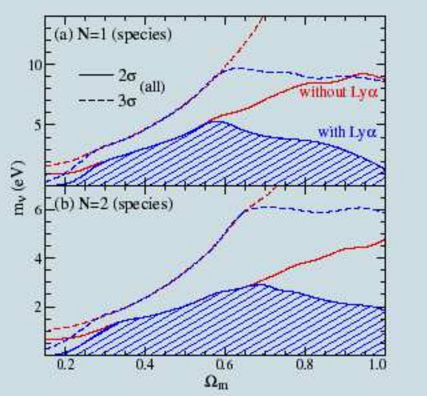


FIG. 2. Constraints on the neutrino mass (a) single massive species (b) two (degenerate) mass species with and without the Lyα constraint.

Croft, Hu & Dave, PRL 83, 1092 (1999)

Cluster abundance

$$n(M,z) = \sqrt{\frac{2}{\pi}}\frac{\rho\delta_c}{M}\frac{1}{\sigma^2(R,z)}\left|\frac{d\sigma(R,z)}{dM}\right|e^{-\frac{\delta_c^2}{2\sigma^2(R,z)}}$$

$$\sigma^2(R,z) = \frac{1}{2\pi^2}\int_0^\infty P(k,z)|W(kR)|^2 k^2 dk,$$

$$W(x) = \frac{3}{x^3}(\sin x - x cos x)$$

$$P(k,z) = Ak^n T^2(k,z)\left[\frac{D(z)}{D(0)}\right]^2$$

$$D(z) = \frac{g(\Omega_m(z))}{1+z}$$

$$T_g = \frac{7.75}{\beta}\left(\frac{6.8}{5X+3}\right)M_{15}^{\frac{2}{3}}\left(\frac{\Omega_m}{\Omega_m(z)}\right)^{\frac{1}{3}}\left(\frac{\Delta_{cr}}{178}\right)^{\frac{1}{3}}(1+z)\ \mathrm{Kev}$$

$0 < f_\nu < 0.2$ (Arhipova et al, astro-ph/0110426)

$m_\nu < 3x0.6$ eV (Fukugita et al, PRL 84, 1082(2000)

strong lensing

substructure -> lensing flux ratio

$$f_{\rm sat}(> M_{\rm sat}) = A\left[\mathrm{erfc}\left(\sqrt{\frac{a\nu}{2}}\right) + \frac{\Gamma(\frac{1}{2}-p, a\nu/2)}{2^p\sqrt{\pi}}\right] \quad (1)$$

$$\nu = \frac{\left[\delta_c\left(D(z_{\rm sat})^{-1} - D(z_{\rm gal})^{-1}\right)\right]^2}{\sigma^2(M_{\rm sat}) - \sigma^2(M_{\rm gal})}. \quad (2)$$

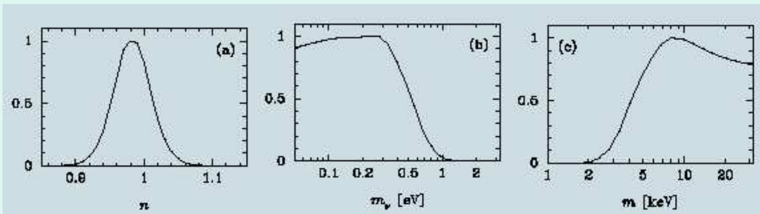


FIG. 2: Likelihood of (a) tilt of primordial power spectrum, (b) neutrino mass, (c) dark matter particle mass.

Dalal & Kochanek, astro-ph/0202290

Weak Lensing

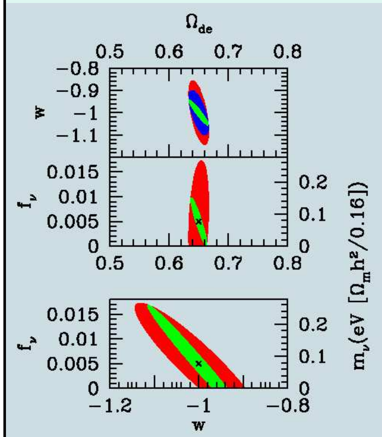


FIG. 3: Projected errors (all 1-σ) on the dark energy density, equation of state, and neutrino mass from 4000 square degree weak lensing survey. In each case, the innermost region is the constraint arising from fixing the parameter not shown (e.g., neutrino mass in the upper panel), while the outermost constraint comes from marginalizing over the third parameter. The middle region in the top panel emerges if the laboratory constraint on the neutrino mass is 0.1 eV.

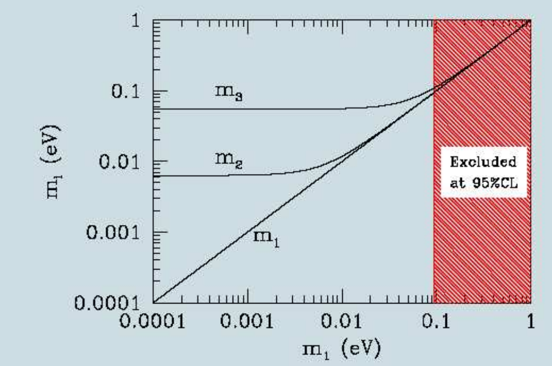


FIG. 4: Masses of the three neutrino species as a function of the lightest neutrino mass in a hierarchical mass scheme [19]. At large m_1, all three must be nearly degenerate to account for the solar and atmospheric oscillations. The projected cosmological upper limit excludes the shaded region at 95% CL.

Abazajian & Dodelson, astro-ph/0212216