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UNIFICATION OF QUARK AND LEPTON MIXINGS

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- Unification is a old idea.
- "Explain" $\sin^2\theta_W$ and thereby the different strengths of the weak, em & strong ints.
- SUSY required.
- A key ingredient is the unification of quarks & leptons*
- In a quark-lepton unified theory, one may expect that the weak interactions of quarks & leptons parametrized by means of the flavour mixing matrices might be the same.
- But, is it true?

$$U_{CKM} \stackrel{?}{=} U_{PMNS}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{31} & 0 & s_{31} \\ 0 & 1 & 0 \\ -s_{31} & 0 & c_{31} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

(We take all phases to be zero.)

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* However, other possibilities exist $\Big\}$ (trinification & quartification E. Ma's talk)

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Experimental Results (90% CL)

Atmos ν (Super K) ✓ by K2K

$$\boxed{\begin{aligned}\Delta m^2_{23} &= (1.2 - 5) \times 10^{-3}\ eV^2 \\ \sin\theta_{23} &= 0.54 - 0.83\end{aligned}}$$

Solar ν (Cl, Ga, Super K, SNO) ✓ by KamLAND

$$\boxed{\begin{aligned}\Delta m^2_{12} &= (2 - 50) \times 10^{-5}\ eV^2 \\ \sin\theta_{12} &= 0.40 - 0.70\end{aligned}}$$

Reactor ν (CHOOZ, Palo verde)

$$\boxed{\sin\theta_{13} \lesssim 0.16}$$

- Atmos ν won the race for the discovery of m_ν thro' osc.
- Solar ν clinched by SNO (NC) < Solar model confirmed / Solar ν osc confirmed
- LMA picked out by KamLAND (Reactor expt)
- Earlier crucial contribution by CHOOZ reactor expt

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> The signal for unification is the CHOOZ bound : $s_{13} \lesssim 0.16$

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Quark Sector

$$U = \begin{pmatrix} 0.9757 & 0.2205 & -0.0030 \\ -0.2203 & 0.9747 & 0.0378 \\ 0.0053 & -0.0364 & 0.9993 \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Or,

$$U_{us} \simeq \sin\theta_{12} = 0.2205 \simeq \lambda$$
$$U_{cb} \simeq \sin\theta_{23} = 0.0378 \simeq O(\lambda^2)$$
$$U_{ub} \simeq \sin\theta_{13} = 0.003 \simeq O(\lambda^3)$$

Lepton Sector

$$U = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2}\sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0.866 & -0.5 & 0 \\ 0.35 & 0.62 & -0.70 \\ 0.35 & 0.62 & 0.70 \end{pmatrix}$$

where we have put

$\theta_{12} = 30°$ (Solar angle)

$\theta_{23} = 45°$ (Atmos. angle)

$\theta_{31} = 0°$ (CHOOZ angle)

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	Quarks	Leptons
$\sin\theta_{12}$	0.2205	0.5
$\sin\theta_{23}$	0.0378	0.7
$\sin\theta_{31}$	0.003	< 0.16

How can these be unified?

Answer :

- Renormalization at high energies (or, the other way)
- Use Renormalization Group.
- Unification only at high energies
- Radiative magnification of mixing
- Quasifixed point
- Quasidegenerate masses for the 3 neutrinos
- Same CP parity for the 3 neutrinos

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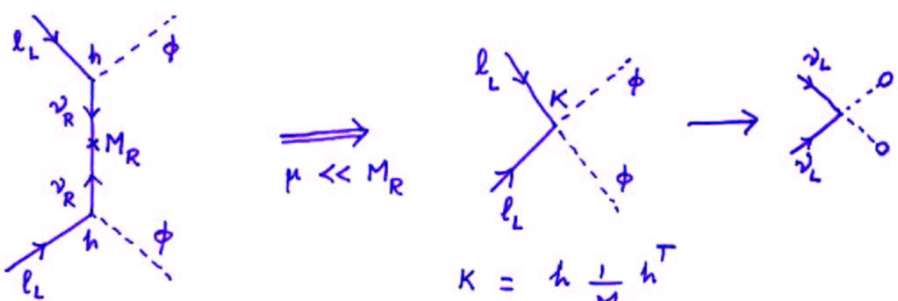
Extension of SM with RH neutrinos and the Seesaw

$$\mathcal{L} = h_{ij}\, \bar{\ell}_{Li}\, \nu_{Rj}\, \phi + \frac{1}{2} M^R_{ij}\, \bar{\nu}^c_{Ri}\, \nu_{Rj} + h.c.$$

$h_{ij}\, \bar{\ell}_{Li}\, \nu_{Rj}\, \phi \rightarrow$ Yukawa couplings $\xrightarrow{\text{SBS}}$ $m_D = h v$; $v = \langle \phi^0 \rangle$

$\frac{1}{2} M^R_{ij}\, \bar{\nu}^c_{Ri}\, \nu_{Rj} \rightarrow$ Majorana mass term



$$K = h \frac{1}{M_R} h^T$$

$$\mathcal{L} \xrightarrow[\mu << M_R]{} \mathcal{L}_{eff} \sim K\, \bar{\ell}_L \phi\, \bar{\ell}_L \phi \quad \text{(5 dim operator)}$$

$$\rightarrow \nu_L^T M_\nu \nu_L$$

where $\boxed{M_\nu = m_D \frac{1}{M_R} m_D^T}$

Seesaw formula

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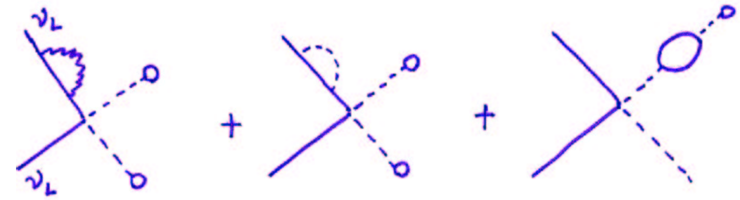
Radiative Corrections and RG evolution



$$t = \ln \mu$$

$$16\pi^2 \frac{dM_\nu}{dt} = \left\{ -\left(\frac{6}{5} g_1^2 + 6 g_2^2\right) + \mathrm{Tr}\left(6\, Y_U Y_U^\dagger\right) \right\} M_\nu + \frac{1}{2} \left\{ \left(Y_E Y_E^\dagger\right) M_\nu + M_\nu \left(Y_E Y_E^\dagger\right)^T \right\}$$

Y_U = 3x3 up-quark Yukawa matrix $\simeq \frac{1}{\sin^2\beta} \begin{pmatrix} 0 & & \\ & 0 & \\ & & h_t^2 \end{pmatrix}$

Y_E = 3x3 Charged lepton Yukawa matrix $\simeq \frac{1}{\cos^2\beta} \begin{pmatrix} 0 & & \\ & 0 & \\ & & h_\tau^2 \end{pmatrix}$

in $\boxed{\text{MSSM}}$

where $\tan\beta = \frac{\langle \phi_u^0 \rangle}{\langle \phi_d^0 \rangle}$

- Chankowski, Królikowski & Pokorski
- Casas, Espinosa, Ibarra and Navarro

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"Diagonalize and Run"

$$\frac{dm_i}{dt} = -2F_\tau m_i U_{\tau i}^2 - m_i F_u \qquad (i = 1, 2, 3)$$

$$\frac{ds_{23}}{dt} = -F_\tau c_{23}^2 \left(-s_{12} U_{\tau 1} D_{31} + c_{12} U_{\tau 2} D_{32}\right)$$

$$\frac{ds_{13}}{dt} = -F_\tau c_{23} c_{13}^2 \left(c_{12} U_{\tau 1} D_{31} + s_{12} U_{\tau 2} D_{32}\right)$$

$$\frac{ds_{12}}{dt} = -F_\tau c_{12} \left(c_{23} s_{13} s_{12} U_{\tau 1} D_{31} - c_{23} s_{13} c_{12} U_{\tau 2} D_{32} + U_{\tau 1} U_{\tau 2} D_{21}\right)$$

where $D_{ij} \equiv \dfrac{m_i + m_j}{m_i - m_j} \quad (i \neq j)$

and

	F_τ	F_u
MSSM	$-\dfrac{h_\tau^2}{16\pi^2 \cos^2\beta}$	$\dfrac{1}{16\pi^2}\left(\dfrac{6}{5} g_1^2 + 6 g_2^2 - \dfrac{6 h_t^2}{\sin^2\beta}\right)$
SM	$\dfrac{3 h_\tau^2}{32\pi^2}$	$\dfrac{1}{16\pi^2}\left(3 g_2^2 - 2\lambda - 6 h_t^2 - 2 h_\tau^2\right)$

$$U = \begin{pmatrix} c_{13} c_{12} & c_{13} s_{12} & s_{13} \\ -c_{23} s_{12} - c_{12} s_{13} s_{23} & c_{12} c_{23} - s_{12} s_{13} s_{23} & c_{13} s_{23} \\ s_{12} s_{23} - c_{12} s_{13} c_{23} & -c_{12} s_{23} - c_{23} s_{13} s_{12} & c_{13} c_{23} \end{pmatrix}$$

$$U^T M_\nu U = \mathrm{diag}\,(m_1, m_2, m_3)$$

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Some simplifications for understanding the evolution:

- In MSSM, F_τ is enhanced by a factor $\sim 10^3$, for $\tan\beta \simeq 50$, as compared to its value in SM. So, the rapid evolution can be attributed to SUSY.
- For quasidegenerate neutrino masses, $D_{ij} \to \infty$. * This contributes to quite rapid evolution.
- At high scale,

$$\left.\begin{array}{l} s_{12} \sim \lambda \sim 0.2 \\ s_{23} \sim O(\lambda^2) \sim 0.035 \\ s_{31} \sim O(\lambda^3) \sim 0.0025 \end{array}\right\} \Rightarrow \begin{array}{l} U_{\tau 1} \sim O(\lambda^3) \\ U_{\tau 2} \sim O(\lambda^2) \end{array}$$

$\Longrightarrow$ Approximate evolution eqs:

$\dfrac{ds_{23}}{dt} \sim \lambda^2 F_\tau D_{32}$ fast; faster than $\dfrac{ds_{12}}{dt}$

$\dfrac{ds_{31}}{dt} \sim \lambda^3 F_\tau (D_{32} + D_{31})$ remains small

$\dfrac{ds_{12}}{dt} \sim \lambda^5 F_\tau D_{21}$ smallness of λ^5 compensated by largeness of D_{21}

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* $D_{ij} \equiv \dfrac{m_i + m_j}{m_i - m_j} \quad (i \neq j)$

$|D_{31}| \simeq |D_{32}| \ll |D_{21}|$

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Procedure

2 Steps:

1. Starting from known values of gauge couplings, masses of quarks and charged leptons and quark mixing angles at low energies, use RG eqs to obtain the corresponding values at high scales: $10^{13} - 10^{16}$ GeV.

2. Assume the neutrino mixing angles at $M_R \sim 10^{13}$ GeV to be the same as the quark mixing angles at high scales.
   - Take the neutrino mass eigenvalues m_i at $M_R \sim 10^{13}$ GeV to be unknown parameters to be determined.
   - Determine these 3 parameters (m_i) in such a way that the solns of the RG eqs at low energies agree with the experimental ranges for the 5 parameters: Δm^2_{12}, Δm^2_{23}, s_{12}, s_{23}, s_{31}

1 is Bottom-up

2 is Top-down

$\bar{m} \equiv \frac{1}{3}(m_1 + m_2 + m_3) \approx m_i$ for any i

Results

- Numerical results in the Tables & Graphs
- Summary: RG evolution yields low energy values for mass squared differences & mixing angles, consistent with experimental data, if the input value of $\bar{m}$ at high scale lies in the range 0.20 to 0.90 eV. The corresponding range of output value of $\bar{m}$ at low scale is 0.15 to 0.65 eV.

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Numerical results on the evolution of neutrino masses and mixing angles

4 Examples are shown below:-

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	At $m_R = 10^{13}$ GeV	At $m_Z = 90$ GeV	
m_1 (eV)	0.2579	0.2066	$\Delta m^2_{12} = 5 \times 10^{-5}$
m_2 (eV)	0.2590	0.2067	$\Delta m^2_{23} = 0.85 \times 10^{-3}$
m_3 (eV)	0.2935	0.2088	
s_{12}	0.20 (input)	0.563	
s_{23}	0.035	0.547	
s_{31}	0.0025	0.080	

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	At $m_R = 10^{13}$ GeV	At $m_Z = 90$ GeV	
m_1 (eV)	0.2983	0.2410	$\Delta m^2_{12} = 4.8 \times 10^{-5}$
m_2 (eV)	0.2997	0.2411	$\Delta m^2_{23} = 1.1 \times 10^{-3}$
m_3 (eV)	0.3383	0.2435	
s_{12}	0.20 (input)	0.568	
s_{23}	0.035	0.680	
s_{31}	0.0025	0.080	

(13)

(3)

	At $m_R = 10^{13}$ GeV	At $m_Z = 90$ GeV
m_1 (eV)	0.4064	0.3244
m_2 (eV)	0.4088	0.3247
m_3 (eV)	0.4621	0.3283
s_{12}	0.20 (input)	0.519
s_{23}	0.035 (input)	0.610
s_{31}	0.0025 (input)	0.080

$\Delta m^2_{12} = 2.0 \times 10^{-4}$

$\Delta m^2_{23} = 2.3 \times 10^{-3}$

(4)

	At $m_R = 10^{13}$ GeV	At $m_Z = 90$ GeV
m_1 (eV)	0.6050	0.4789
m_2 (eV)	0.6087	0.4794
m_3 (eV)	0.6887	0.4847
s_{12}	0.20 (input)	0.731
s_{23}	0.035 (input)	0.644
s_{31}	0.0025 (input)	0.085

$\Delta m^2_{12} = 4.2 \times 10^{-4}$

$\Delta m^2_{23} = 5.1 \times 10^{-3}$

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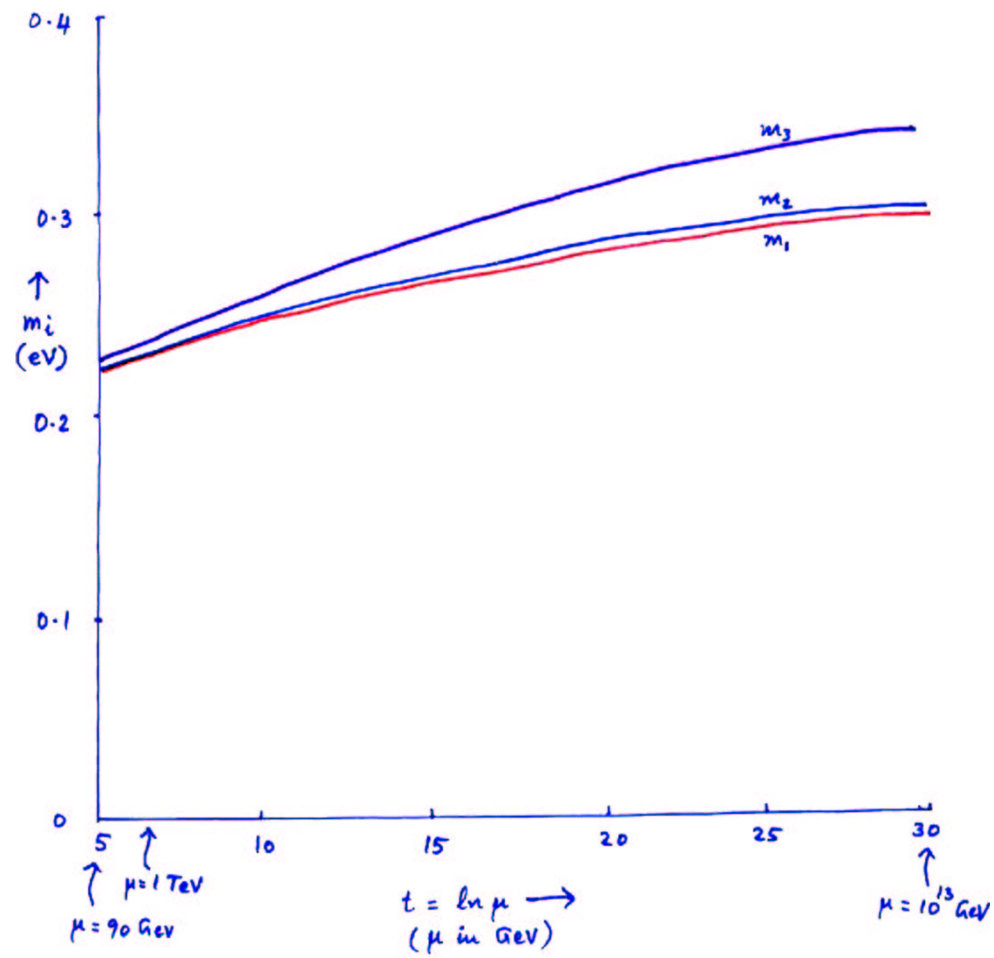

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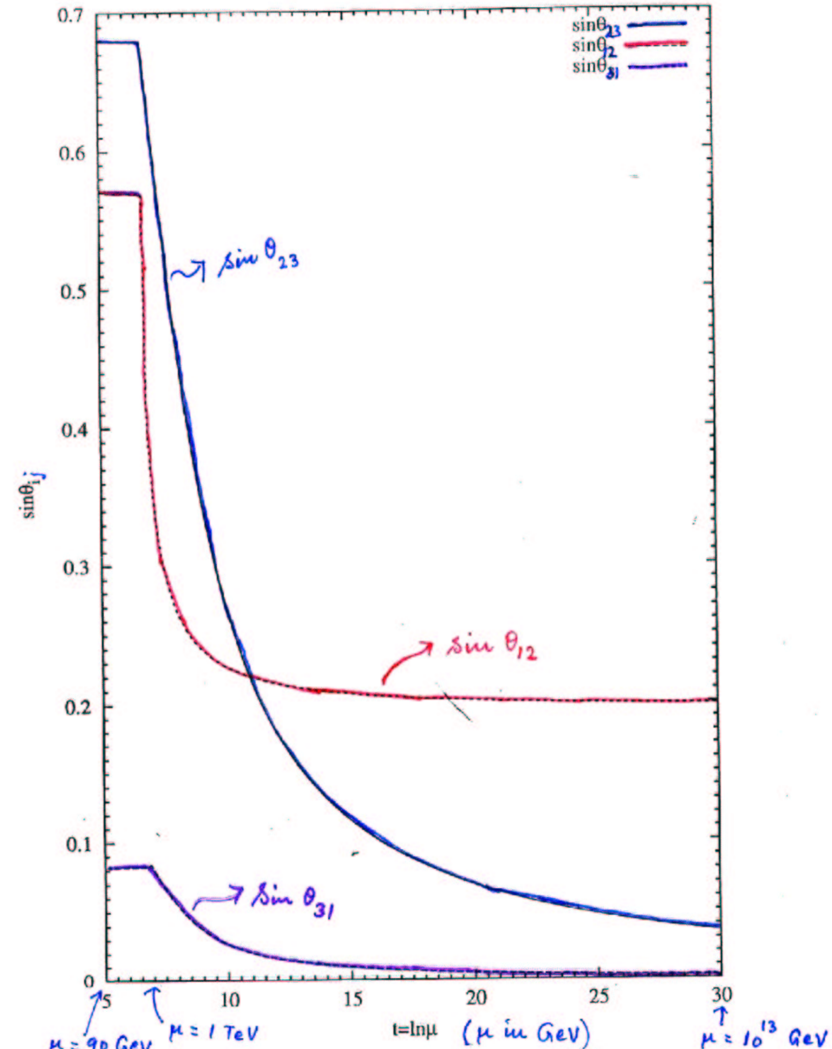

Figure 3: Radiative Magnification with neutrino mixings same as quark mixings at high scale (Case I) in MSSM with $M_{\rm SUSY} = 1$ TeV.

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Unification of quark and lepton mixing

	At low energies		At Unification Scale	Magnification for leptons
	quarks	leptons	quarks/leptons	
$\sin\theta_{12}$	0.2205	0.5	0.20	2.5
$\sin\theta_{23}$	0.0378	0.7	0.035	20
$\sin\theta_{13}$	0.003	0.08	0.0025	32

Quarks : Bottom → up (Low → High) : Slow evolution

Leptons : Top → down (High → Low) : Fast evolution

- s_{23} evolves faster and overtakes s_{12}
- s_{31} magnified even more, but still remains small
- Smallness of $\theta_{\rm CHOOZ}$ is traced to smallness of Wolfenstein's λ^3

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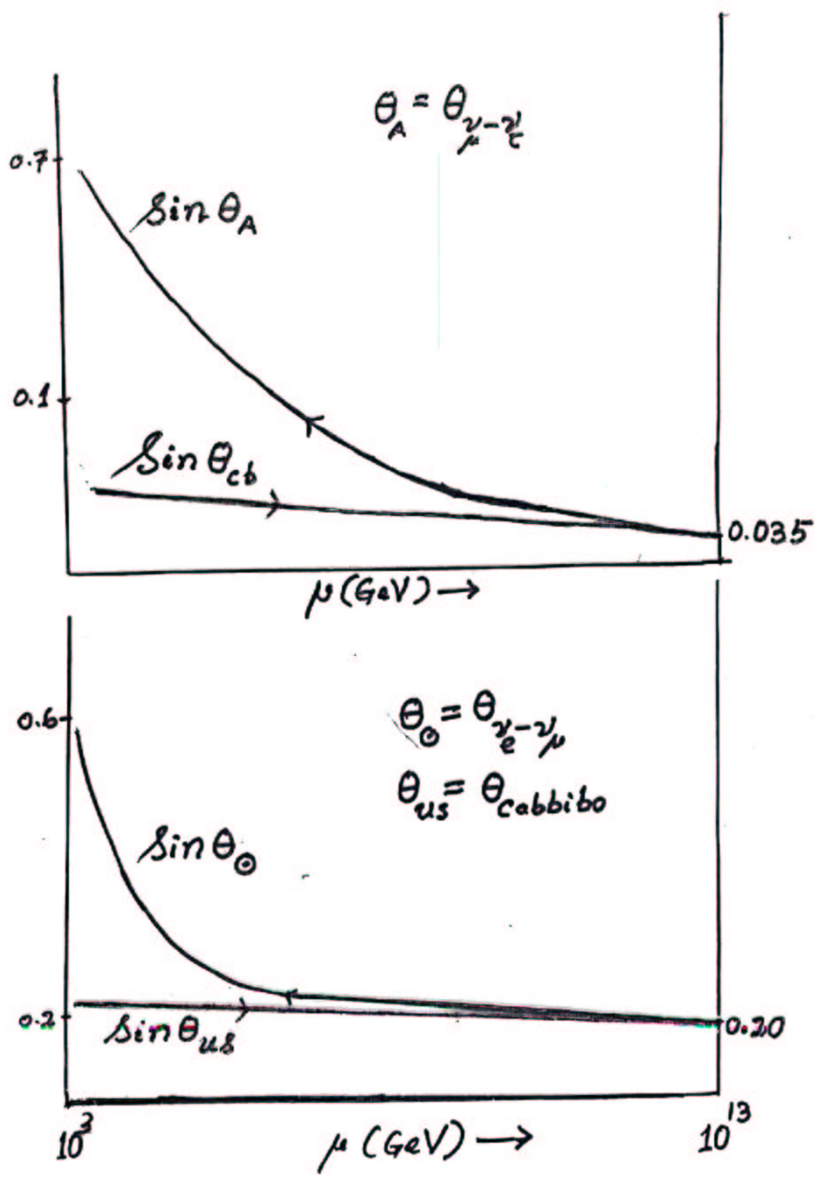

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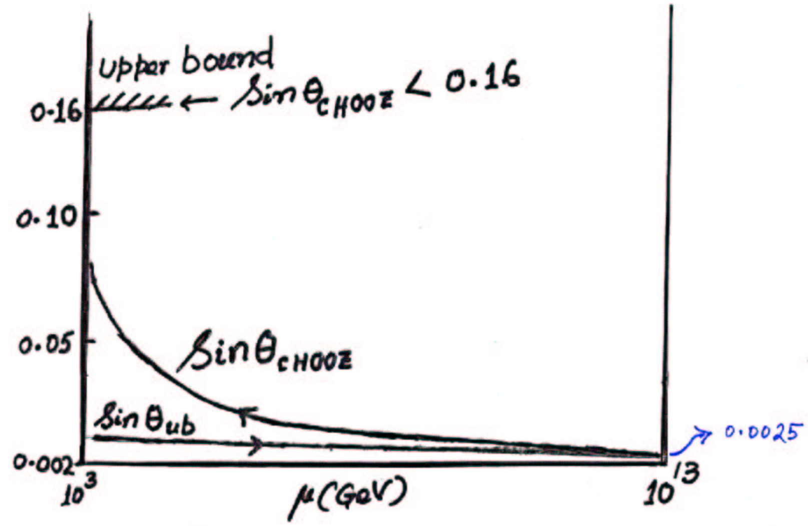

Prediction

(i) $\sin\theta_{CHOOZ}$ is small since in the quark sector $|V_{ub}| = \sin\theta_{ub} = 0.004$ and there is high scale quark-lepton unification

(ii) For future long-baseline expt. we predict $U_{e3} = \sin\theta_{CHOOZ} = 0.08 - 0.10$

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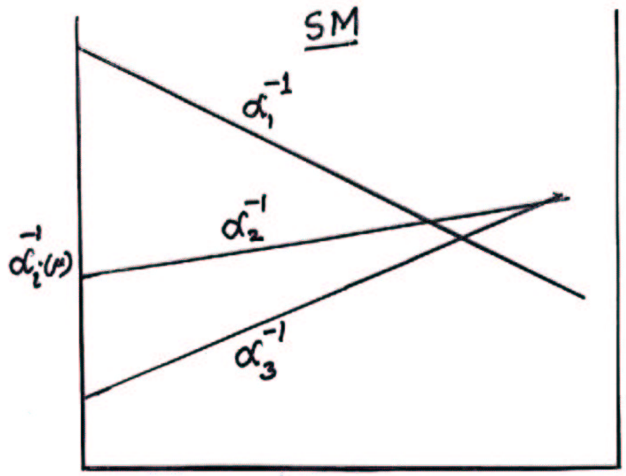
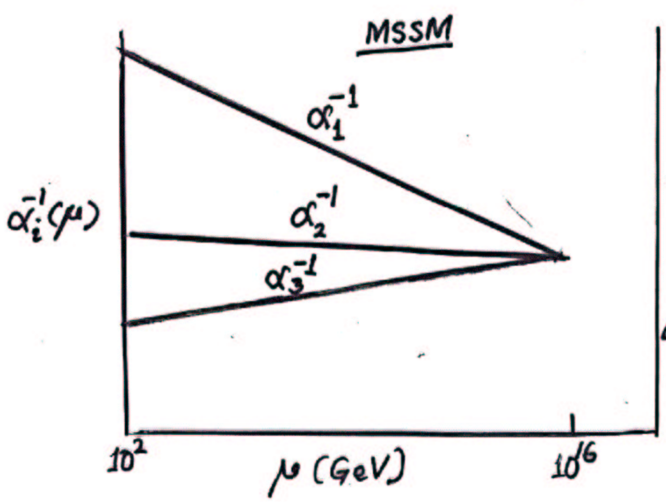
Renormalization and Unity of forces: Gauge forces



One doublet: ϕ

$\langle \phi^0 \rangle = v/\sqrt{2}$



$\alpha_i = \frac{g_i^2}{4\pi}$, $i = 1, 2, 3$

Two doublets and superpartners

ϕ_u, ϕ_d

$\langle \phi_u^0 \rangle = \frac{v_u}{\sqrt{2}} = \frac{v \sin\beta}{\sqrt{2}}$

$\langle \phi_d^0 \rangle = \frac{v_d}{\sqrt{2}} = \frac{v \cos\beta}{\sqrt{2}}$

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z} + (\text{Two-loop}) + (\text{Threshold})$$

(20)

Experimental implications of our results

$\boxed{\text{I}}$ $\bar{m} \equiv \frac{1}{3}(m_1 + m_2 + m_3) \approx 0.15$ eV to 0.65 eV.

This result is relevant for 3 types of expts (a, b, c).
Presently only upper limits are available,
but would be observable in future "improved" expts.

(a) $0\nu\beta\beta$

$|M_{ee}| = |\eta_i m_i U_{ei}^2| = \bar{m}$ (if CP eigenvalues are same for all i and CP violating phases are ignored).

Expt: $|M_{ee}| < (0.33 - 1.35)$ eV (present)
0.1 eV (future)

$|M_{ee}|$ in the range of 0.1 eV or less will be probed in the future (GENIUS, ...)

(b) Tritium β decay: Expt: $\bar{m} < 2.2$ eV (present)
0.35 eV (future)

$\bar{m}$ down to 0.35 eV will be probed in the future (KATRIN)

(c) CMBR anisotropy (COSMOLOGY)

$\sum_i m_i < 0.7$ eV $\Rightarrow$ $\bar{m} < 0.23$ eV (WMAP)

or ($\sum_i m_i < 2.1$ eV $\Rightarrow$ $\bar{m} < 0.7$ eV (WMAP + 2dFGRS)

$\boxed{\text{II}}$ $U_{e3} = \sin\theta_{13} = 0.08 - 0.10$

Accessible to future reactor expts & other LBL expts

(21)

Possible Origin of a quasidegenerate neutrino Spectrum

In L-R sym models, it is more natural to have "Type II" seesaw:

$$\begin{pmatrix} f v_L & M_D \\ M_D^T & f v_R \end{pmatrix}$$

$$M_\nu = f v_L - M_D (f v_R)^{-1} M_D^{-1}$$

Let us assume

$$f \sim 1 f_0$$

$$f v_L \gg M_D (f v_R)^{-1} M_D^T$$

Consequences:

1. Eigenvalues of M_ν are quasidegenerate
2. M_ν is diagonalized by the same matrix that diagonalizes M_D.

- The assumption $f \sim 1 f_0$ can be implemented in a model of S_4 symmetry. (Work in Progress)
- Quasidegenerate masses for ν's are possible in a model of A_4 symmetry too. (E. Ma: Plato's Fire)