The axial form factors from lattice QCD with improved Wilson fermions

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Motivation

In NOvA, T2K, DUNE, Hyper-Kamiokande (HK) muon (anti-)neutrinos are/will be scattered off H_2O, ^{12}C or ^{40}Ar targets.

Clearly, nuclear effects play a role (except for H)
$\rightarrow$ interaction between QCD and nuclear physics communities is needed.

Here we address (quasi-)elastic scattering $\bar{\nu}_\ell p \rightarrow \ell^+ n$, $\nu_\ell n \rightarrow \ell^- p$ via charged current interactions.

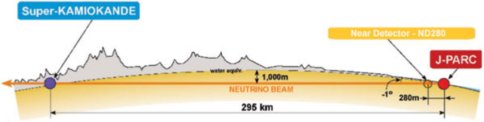
Flavour separation (talk by C Alexandrou) is interesting too, for $\nu N \rightarrow \nu N$, $N \in \{n, p\}$.

Present constraints suggest that there is maximum oscillation $\nu_\mu \rightarrow \nu_e, \nu_\tau$ for $L/E \approx 500\,\mathrm{km/GeV}$.

NOvA: 810 km/2 GeV
T2K & HK: 295 km/0.6 GeV
DUNE: 1300 km/2.5 GeV

Aims: resolving the neutrino mass hierarchy, constraining elements of the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix, in particular the CP violating phase(s), BSM physics.

Experiments



T2K: Tokai to Super-Kamiokande,

$E = 0.6$ GeV, $L/E \approx 500$ **km/GeV.**

Also NOvA, $L/E \approx 400$ km/GeV, DUNE $L/E \approx 520$ km/GeV.

Muon neutrino beam: proton on nucleus $\to$ pions and kaons $\to \mu^+\nu_\mu$ or $\mu^-\bar{\nu}_\mu$.

Near and far detectors.

$$\mathbf{N}^{\mu}_{\text{far}}(\mathbf{E}_\nu) = \mathbf{N}^{\mu}_{\text{near}}(\mathbf{E}_\nu) \times [\text{flux(L)}] \times [\text{detector}] \times [1 - \sum_\beta \mathbf{P}_{\mu\to\beta}(\mathbf{E}_\nu)]$$

E_ν has to be reconstructed from the momentum of the detected charged lepton.

Trivial for $\nu_\mu + n \to \mu^- + p$ if the initial momenta of n and of ν_μ are known.
But...
The neutrino beam is not monochromatic but has a momentum distribution.
The nucleon is bound in a nucleus and has $|\mathbf{p}_{\text{Fermi}}| \sim 200\,\text{MeV}$.
The lepton momentum reconstruction is often incomplete.
Misidentification of inelastic scattering as elastic scattering.
Monte-Carlo simulation needs input regarding the differential cross section.

Form factors (FFs)

$\nu_\ell n \to \ell^- p$ goes via the $V - A$ current. The non-perturbative matrix elements that enter $\mathrm{d}\sigma/\mathrm{d}\Omega$ can be decomposed in terms of four FFs.

Kinematics: $q_\mu = p'_\mu - p_\mu$, $Q^2 = -q_\mu q^\mu \geq 0$, $p'^2 = p^2 = m_N^2 \approx m_n^2 \approx m_p^2$.

$$\langle p(p')|\bar{u}\gamma_\mu d(0)|n(p)\rangle = \bar{u}_p(p')\left[F_1(Q^2)\gamma_\mu + \frac{F_2(Q^2)}{2m_N}\sigma_{\mu\nu}q^\mu\right]u_n(p),$$

$$\langle p(p')|\bar{u}\gamma_\mu\gamma_5 d(0)|n(p)\rangle = \bar{u}_p(p')\left[G_A(Q^2)\gamma_\mu + \frac{G_{\tilde{P}}(Q^2)}{2m_N}q_\mu\right]\gamma_5 u_n(p).$$

Note that $\langle p|\bar{u}\Gamma d|n\rangle = \langle p|(\bar{u}\Gamma u - \bar{d}\Gamma d)|p\rangle$ if $m_u = m_d$, $e_u = e_d$ (isospin limit).

Dirac (vector) FF F_1, Pauli FF F_2, axial FF G_A, induced pseudoscalar FF $G_{\tilde{P}}$.

F_1 and F_2 are relatively well-known (using isospin symmetry) from lepton-proton and lepton-neutron/deuteron scattering (but not their slope at $Q^2 = 0$!).

$g_A = G_A(0)$ is well determined from β-decay. $G_A(Q^2)$ information from neutrino scattering and (indirectly) through exclusive pion electroproduction $e^- p \to \pi^- \nu p$.

Using $G_A \approx g_A$, F_1 and F_2 as input, $G_{\tilde{P}}(0.88\, m_\mu^2)$ can be determined from muon capture $\mu^- p \to \nu_\mu n$ in muonic hydrogen.

PCAC and PPD relations

The impact of $G_{\tilde{P}}(Q^2)$ on the cross section is suppressed by a factor $m_\ell^2/m_N^2 \approx 0.01$ for $\ell = \mu$. Therefore, it is only relevant for small Q^2, where this form factor is large (e.g., at the muon capture point).

We define the pseudoscalar FF:

$$\langle p(p')|\bar{u}i\gamma_5 d(0)|n(p)\rangle = \bar{u}_p(p') G_P(Q^2) i\gamma_5 u_n(p).$$

Abbreviations to be used: $A_\mu = \bar{u}\gamma_\mu\gamma_5 d$, $P = \bar{u}i\gamma_5 d$.

Consequence of the PCAC relation, i.e. the axial Ward-Takahashi identity ($\partial_\mu A_\mu = 2m_{ud}P$ and $\bar{u}_N\gamma_\mu\gamma_5 u_N = 2m_N\bar{u}_N i\gamma_5 u_N$):

$$2m_N G_A(Q^2) = 2m_{ud} G_P(Q^2) - \frac{Q^2}{2m_N} G_{\tilde{P}}(Q^2).$$

With complete non-perturbative order-a improvement, this relation will receive $\mathcal{O}(a^2\Lambda^2, a^2Q^2, a^2 m_{ud}\Lambda, \ldots)$ lattice spacing corrections.

Current algebra gives the Pion pole dominance (PPD) relation

$$G_{\tilde{P}}(Q^2) \approx \frac{4m_N^2}{M_\pi^2 + Q^2} G_A(Q^2).$$

Unless $M_\pi^2 = 0$, also in the continuum this relation is only approximate.

Lattice calculation

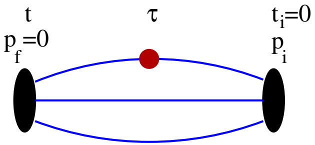
Ideally, one would compute the FFs directly from QCD via lattice simulation, without additional assumptions.

Apart from attaining a meaningful statistical error, this requires taking the following limits:

- Continuum limit: $a^2 \to 0$.
- Infinite volume limit: $L = N_s a \to \infty$. Due to the mass gap, these effects are exponential $\sim \exp(-LM_\pi)$, however, large N_s become necessary at small M_π and at small a. ChPT $\to$ FVE are most relevant at small Q^2 (which is why we do not divide by $G_A(0)$).
- Physical point: Results must be extra-/interpolated to physical quark masses or, equivalently, physical pion and kaon masses.
- Extrapolation to infinite Euclidean time separations, where the ground state dominates.

Spectral decomposition

In our case Wick contractions give only **connected diagrams** (isospin limit).



$$C_{2pt}(\mathbf{p},t) = (Z_0^{\mathbf{p}})^2\, e^{-E_N(\mathbf{p})t}\left[1+\sum_{k>0}\left(\frac{Z_k^{\mathbf{p}}}{Z_0^{\mathbf{p}}}\right)^2 e^{-\Delta E_k(\mathbf{p})t}\right],$$

$$C_{3pt}(\mathbf{p}'=\mathbf{p},\tau,t) = (Z_0^{\mathbf{p}})^2\,\langle 0|A|0\rangle e^{-E_N(\mathbf{p})t}\left[1+\sum_{k>0}\frac{Z_k^{\mathbf{p}}}{Z_0^{\mathbf{p}}}\frac{\langle k|A|N\rangle}{\langle 0|A|0\rangle}\left(e^{-\Delta E_k(\mathbf{p})(t-\tau)}+e^{-\Delta E_k(\mathbf{p})\tau}\right)\right.$$
$$\left.+\left(\frac{Z_k^{\mathbf{p}}}{Z_0^{\mathbf{p}}}\right)^2\frac{\langle k|A|k\rangle}{\langle 0|A|0\rangle}e^{-\Delta E_k(\mathbf{p})t}\right].$$

For simplicity, above $q=0$.
$\langle 0|A|0\rangle$ is the (purely real or imaginary) Euclidean matrix element of interest for the nucleon ground state $|0\rangle = |N\rangle$, $Z_k \propto \langle\Omega|\mathcal{N}|k\rangle = Z_k^*$ are the so-called overlap factors and $\Delta E_k = E_k - E_N$ with $E_0 = E_N$.

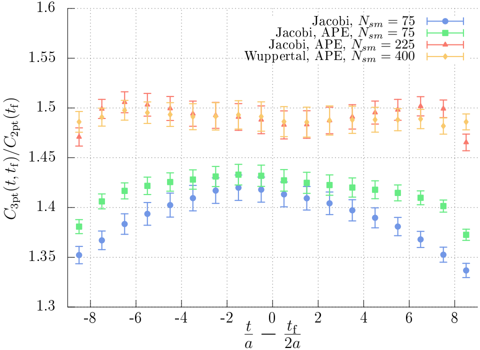
2pt-function: excited state suppression with $\delta_k^2 = Z_k^2/Z_0^2$.

3pt-function: suppression only with $\delta_k\langle k|A|0\rangle/\langle 0|A|0\rangle$. What if a $\langle k|A|0\rangle$ is large?

Excited state pollution

The signal decreases exponentially, noise/signal increases exponentially with the source-sink separation time. So one cannot achieve arbitrary large separations between source, current and sink.

“Smearing” enables the construction of an “interpolator” $\overline{\mathcal{N}}$ that creates a combination of energy eigenstates, $\overline{\mathcal{N}}|\Omega\rangle = c_0|N\rangle + c_1|N'\rangle + \cdots$, with $|c_0| \gg |c_k| \quad \forall \quad k > 0$. Then all δ_k are small.



Example of a ratio of 3pt-over 2pt-functions for g_A: “good” smearing vs. “not so good” smearing (not renormalized).

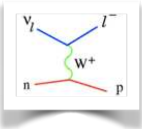
Problem with the axial current: It couples well to pions since $\langle\Omega|A_\mu|\pi^+(q)\rangle = i\sqrt{2}F_\pi q_\mu$. Matrix elements “$\langle N\pi|A_\mu|N\rangle$” may be enhanced!

Neutrino-nucleus scattering

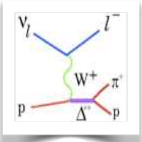
This also happens in experiment. Our interpolators do not couple to Δ, i.e. $N\pi$ with spin $3/2$, but they couple to P-wave $N\pi$ with spin $1/2$.

Current project (L Barca): Compute axial $N \to N\pi$ transition form factors.

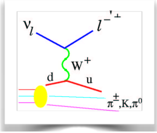
Quasi-elastic scattering (QE)

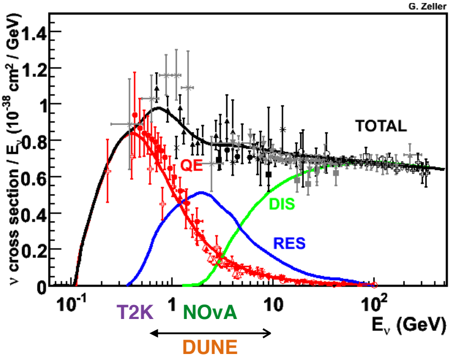


Resonance production (RES)



Deep Inelastic scattering (DIS)





J.A. Formaggio, G. Zeller, Reviews of Modern Physics, 84 (2012)

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Coordinated Lattice Simulations (CLS)

CLS members/groups at

- HU Berlin
- CERN
- TC Dublin
- Mainz
- UA Madrid
- Milano Bicocca
- Münster
- Odense/CP3 Origins
- Regensburg
- Roma I + II
- Wuppertal
- DESY/Zeuthen

Coordinated generation of gauge ensembles using openQCD
`https://luscher.web.cern.ch/luscher/openQCD/`
[M Lüscher, S Schaefer, 1206.2809].

$N_f = 2 + 1$ flavours of non-perturbatively order-a improved Wilson fermions on tree level Symanzik improved glue.

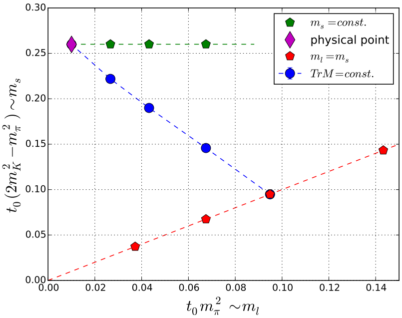
Complete non-perturbative (NP) $\mathcal{O}(a)$ improvement of action and operators.

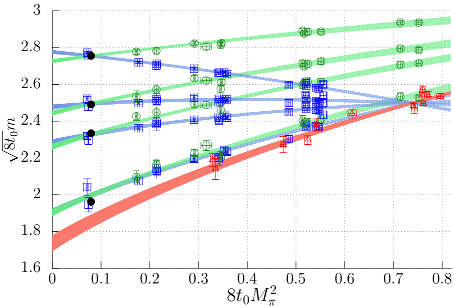
NP renormalization.

Keep it simple and local: no smeared action etc.

Simulation strategy

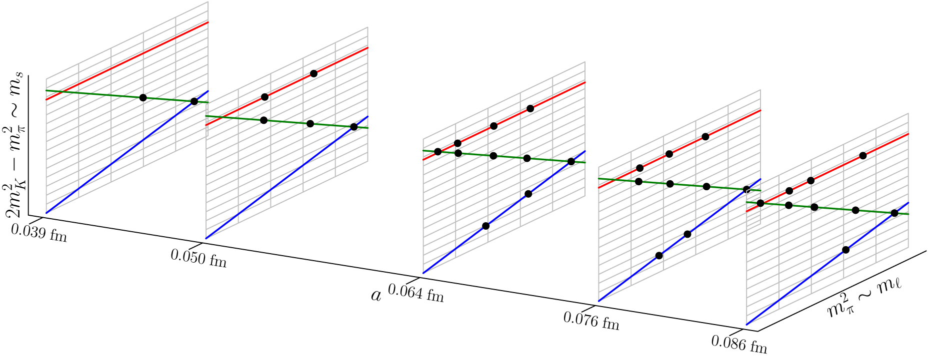
Simulate along $m_s + 2m_\ell = \mathrm{const}$ [QCDSF+UKQCD: W Bietenholz et al, 1003.1114], **and** $\widehat{m}_s \approx \mathrm{const}$ [G Bali et al, 1606.09039; 1702.01035], enabling Gell-Mann–Okubo/SU(3) and SU(2) ChPT extrapolations.





Right: octet baryon masses: Ξ, Σ, Λ, N and $\mathcal{O}(p^3)$ SU(3) EOMS BChPT.

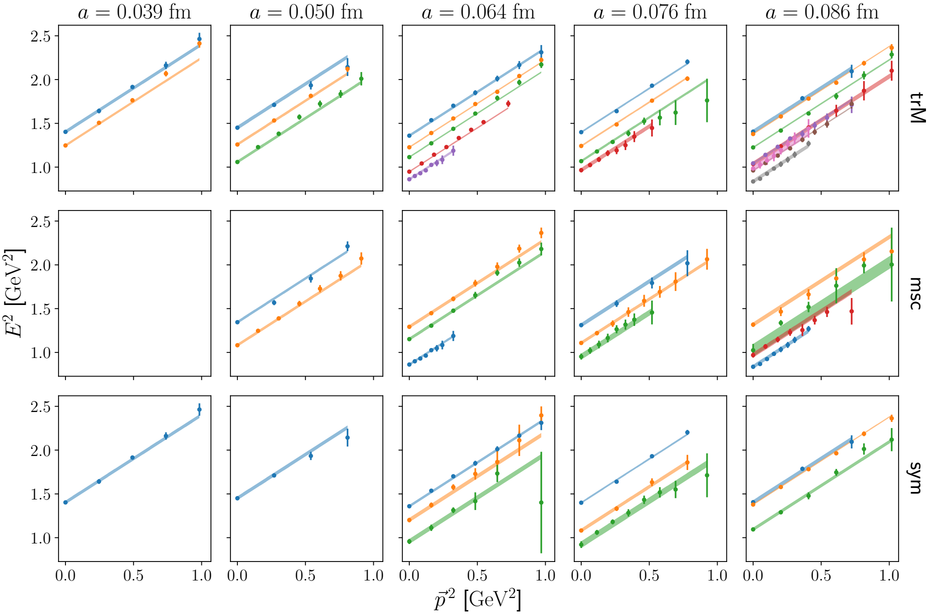
Analysed ensembles



The number of lattice points varies from $24^3 \cdot 128$ to $96^3 \cdot 192$.

For all the $a < 0.06\,\mathrm{fm}$ ensembles and many of the other ensembles, we use open boundary conditions in time.

Checking dispersion relation for the nucleon



Physical point extrapolation

The Q^2 values differ, depending on the volume, the lattice spacing and the quark masses: to carry out the continuum limit, an interpolation is required.

Within global fits we use the dipole ansatz as well as z-expansions to parameterize $G_A(Q^2)$, $(Q^2 + M_\pi^2)G_{\tilde{P}}(Q^2)$ and $(Q^2 + M_\pi^2)G_P(Q^2)$ in the continuum limit.

We also carry out fits, imposing the PCAC relation in the continuum limit.

Each of the 2–4 fit parameters (for each of the form factors) have

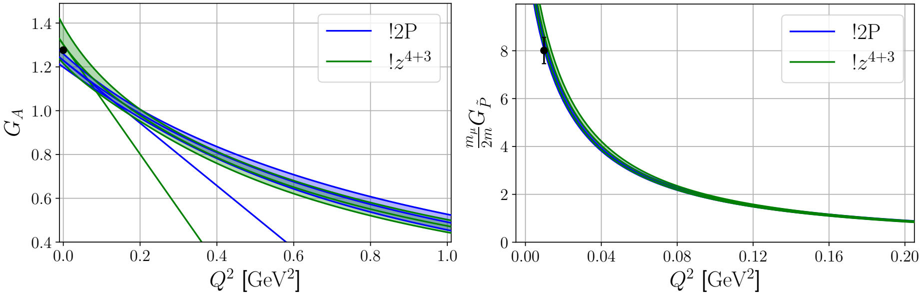
- mass effects, quadratic in the pseudoscalar masses,
- finite volume effects $\propto M_P^2 e^{-M_P L}/\sqrt{M_P L}$,
- lattice spacing effects $\propto a^2/t_0$, $\propto a^2(2M_K^2 + M_\pi^2)$ and $\propto a^2(M_K^2 - M_\pi^2)$.

The ansätze for the mass and volume dependence are inspired by ChPT but phenomenological since ChPT does not apply to $Q^2 \gg M_\pi^2$. Systematics explored by different excited state fits, cuts on the quark masses and the lattice spacing.

Details can be found in RQCD: G Bali, L Barca, S Collins, P Wein, S Weishäupl, T Wurm et al, 1911.13150.

Since the continuum fit parameters are correlated and the fit function is somewhat involved for the z-expansion, data files can be found in the arXiv submission. Please use them! All systematics are included.

Results: physical point, continuum limit



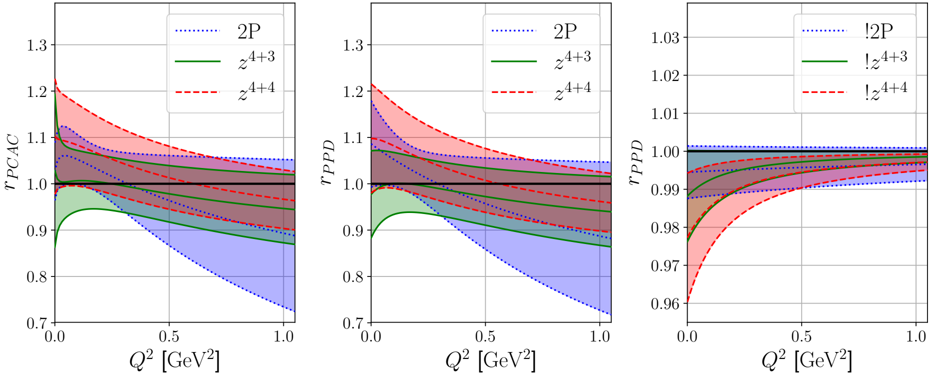
Black points are experimental values for $g_A = G_A(0)$ and $g_P = m_\mu/(2m_N) G_{\tilde{P}}(0.88 m_\mu^2)$ at the muon capture point.

Straight lines are the slopes at $Q^2 = 0$: the axial radius is very parametrization dependent and smaller for dipole fits.

The axial radius is not at all important for neutrino scattering! Even the electric charge radius of the proton is not too well known from charged lepton scattering!

$\exists$ als work by Mainz on CLS ensembles: D Djukanovic et al, 2112.00127

Results: PCAC and PPD relations in the continuum limit



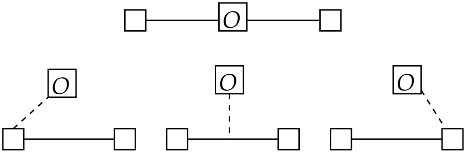
Right: PCAC is imposed.

$$r_{\mathrm{PCAC}} = \frac{2m_{ud}\, G_P(Q^2) + \frac{Q^2}{2m_N}\, G_{\tilde{P}}(Q^2)}{2m_N\, G_A(Q^2)} = 1, \qquad r_{\mathrm{PPD}} = \frac{M_\pi^2 + Q^2}{4m_N^2}\, \frac{G_{\tilde{P}}(Q^2)}{G_A(Q^2)} \approx 1.$$

Violations of the pion pole dominance relation are very small.

Discussion of $N \to N\pi$ pollution

Chiral perturbation theory (ChPT) Tree-level diagrams:



Top diagram:

$$
\begin{array}{ll}
\sim G_A & \text{for } \mathcal{O} = A_\mu \\
= 0 & \text{for } \mathcal{O} = P
\end{array}
$$

Bottom centre diagram:

$$
\begin{array}{ll}
\sim G_{\tilde{P}} + \text{excited states} & \text{for } \mathcal{O} = A_4 \\
\sim G_P + \text{excited states} & \text{for } \mathcal{O} = P
\end{array}
$$

Other diagrams: only contribute to the excited states.

In ChPT these contributions are enhanced by a factor m_N/M_π in A_4 with respect to the ground state but also present in A_j.

ChPT predicts the $N\pi$ energy level (all momentum transferred to π at tree-level) and the coupling.

$N\pi$ excited state contributions

[Bär,1906.03652,1812.09191]: $N\pi$ contributions to a combination R_4 of C_{3pt}^{O} and C_{2pt} for $O = A_4$ in leading one-loop order of SU(2) covariant ChPT.

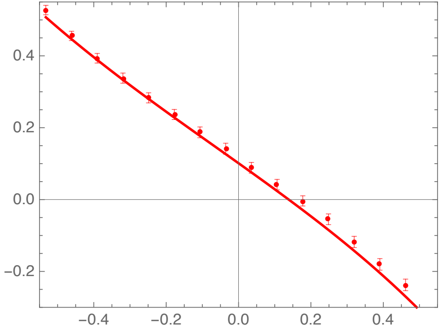
[Bär,1907.03284]

R_4 vs. $\tau/a - t_f/(2a)$

$t_f \to \infty$: $R_4 \to \mathrm{const.}$.

Data: [RQCD,1810.05569]:

$M_\pi \sim 150$ MeV, $a = 0.07$ fm, $t_f = 1.06$ fm, $\mathbf{p}' = \mathbf{0}$, $|\mathbf{q}| = 2\pi/(64a)$



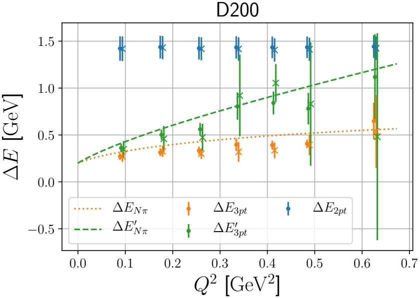
$$C_{3pt}^{A_4}(\mathbf{p}'=\mathbf{0}, \mathbf{p}=-\mathbf{q}, t_f, \tau) = C_{3pt,N}^{A_4}(\mathbf{q}, t_f, \tau) + C_{3pt,N\pi}^{A_4}(\mathbf{q}, t_f, \tau) = \mathcal{O}\left(\frac{M_\pi}{m_N}\right) + \mathcal{O}(1)$$

In this channel $N(\mathbf{0})\pi(-\mathbf{q}) \to N(\mathbf{0})$ is enhanced, relative to $N(-\mathbf{q}) \to N(\mathbf{0})$.

Maybe correct lattice data by subtracting the expectation? Problem: systematics.

Instead: simultaneous fit to different channels

D200 CLS ensemble: $M_\pi \sim 200$ MeV, $a = 0.064$ fm.

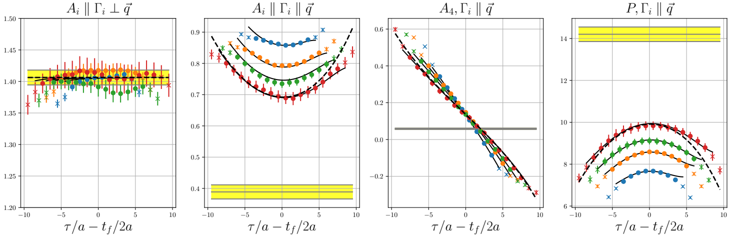


$$\Delta E_{N\pi} \approx E_\pi(-\mathbf{q}) + E_N(\mathbf{0}) - E_N(-\mathbf{q})$$

$$\Delta E'_{N\pi} \approx E_\pi(\mathbf{q}) + E_N(-\mathbf{q}) - E_N(\mathbf{0})$$

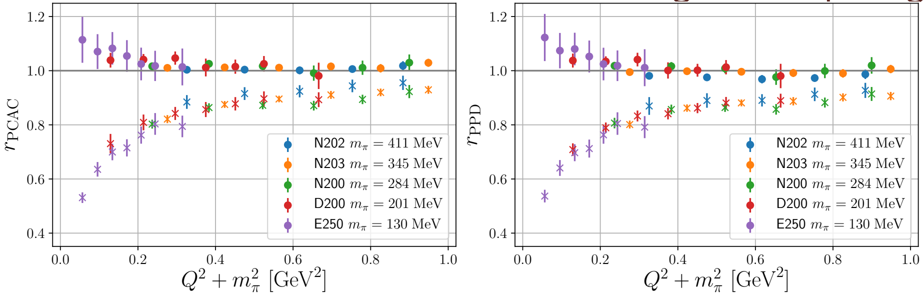
since $\mathbf{p}' = \mathbf{0}$ and $\mathbf{p} = -\mathbf{q}$.

Include $N\pi$ levels of tree-level ChPT leads to reasonable $\chi^2/\mathrm{d.o.f}$ ($|\mathbf{q}| = 2\pi/(64a)$.)



The yellow result respects the known PCAC relation (up to lattice artefacts).

PCAC and PPD relations at a non-vanishing lattice spacing



Crosses: taking the excited state gap from the two-point function.

Circles: simultaneous fit to all 3pt-functions, including A_4.

The simultaneous fit was carried out with a free second excited state gap on either side and

- fixing $\Delta E_{N\pi}$ and $\Delta E'_{N\pi}$ from the non-interacting levels,
- fitting these energies from the 3pt-function data.

The difference is included in the systematic error of the final result.
Note that $G_A(Q^2)$, extracted from A_i alone via a naive fit, is only marginally different. However, the effect on $G_{\tilde{P}}$ and G_P is huge at small Q^2 and M_π^2.

Summary

- We have determined the axial form factor in the range of momentum transfers that is relevant for long baseline neutrino experiments.
- Using this could improve our understanding of nuclear effects, and help to entangle different processes, ultimately leading to higher precision.
- A problem in the determination of the induced pseudoscalar and pseudoscalar form factors is the enhancement of $N\pi$ states. This cannot be eliminated by "smearing": also the most ideal nucleon interpolator will couple to these! Once this was understood, also these form factors could be determined reliably.
- The PCAC relation between the form factors is satisfied in the continuum limit. It is still slightly violated at $a > 0$. Violations of pion pole dominance are found to be just a few per cent at physical quark masses.
- It would be nice to further confirm this picture by explicitly computing axial $N \to N\pi$ matrix elements, also using five-quark interpolators. This (and the determination of transition form factors) is in progress (L Barca).