Towards a Unified Description of Neutrino-Nucleus Interactions

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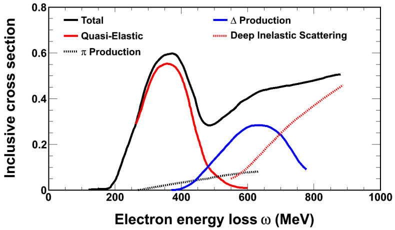
PREAMBLE

- ⋆ Understanding nuclear interactions at fully quantitative level is critical to the interpretation of the events detected by neutrino experiments
- ⋆ Over the past two decades, a great deal of experimental and theoretical work has been specifically devoted to the study of neutrino interactions with nuclei
- ⋆ The ultimate goal of this effort is the development of a unified framework for the description of the variety of reaction mechanisms—leading to single- and multi-nucleon emission, resonance production and deep-inelastic scattering—that contribute to the flux-averaged cross sections.

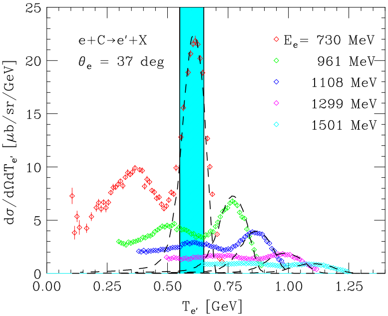
The riddle of flux average

⋆ Consider the electron-carbon x-section at fixed beam energies and scattering angle

- beam energy ∼1 GeV



- beam energies between ∼ 0.7 and ∼1.3 GeV.



⋆ Flux average hampers the determination of the energy transfer to the nuclear target, which largely determines the reaction mechanism

⋆ The flux-averaged cross section at fixed energy of the outgoing lepton picks up contributions from different mechanisms

The inclusive neutrino-nucleus cross sections

- ⋆ Recent measurements of the double-differential neutrino-nucleus cross section include the contributions of all charged-current interactions, regardless of the hadronic final state
- ⋆ Compared to the cross sections corresponding to specific channels, inclusive data allow to perform global tests of theoretical models capable to provide a consistent description of all relevant reaction mechanisms

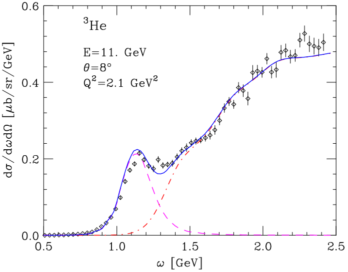
  1. single-nucleon knockout leading to one- and two-nucleon emission
  2. coupling to meson exchange currents
  3. resonance production
  4. non resonant pion production
  5. deep inelastic scattering
  6. final state interactions

- ⋆ The analysis of inclusive data played a critical role in the development of accurate models of electron-nucleus scattering, and greatly helped to pin down the role of different reaction mechanisms in different kinematical regions

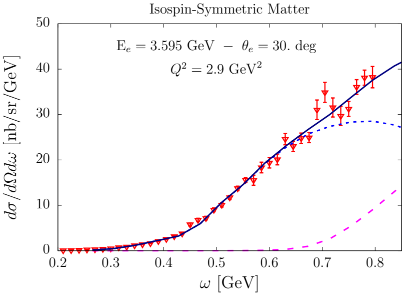
Inclusive electron-nucleus scattering

- ⋆ elastic and inelastic (RES + DIS) processes consistently taken into account (Bodek & Ritchie parametrisation of SLCA data)
- ⋆ no adjustable parameters involved

- SLAC data
  Day *et al*, PRL 43,1143 (1979)

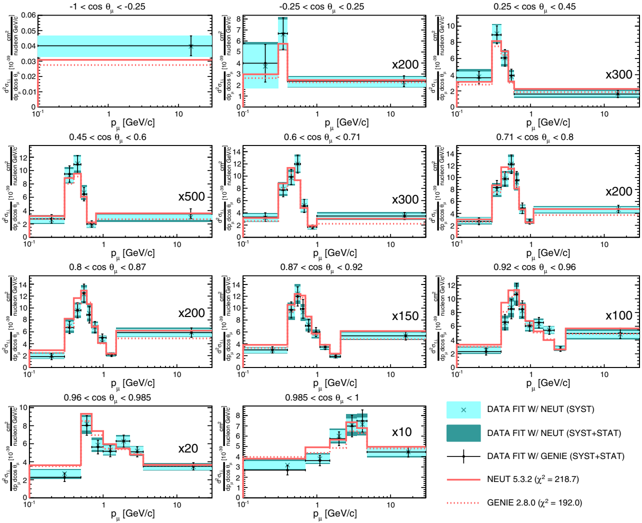


- Extrapolation of SLAC data, taken using targets with $4 \leq A \leq 197$
  Day *et al*, PRC 40, 1011 (1989)



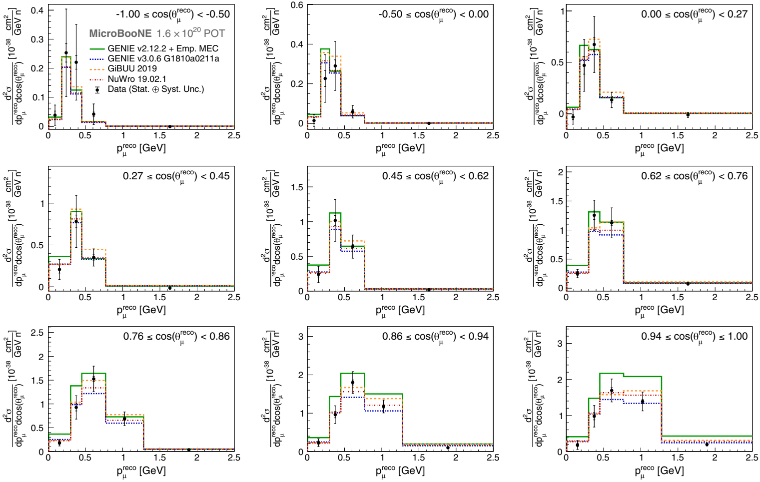
T2K DATA, CARBON TARGET

- Flux-averaged inclusive x-section, PRD 98, 012004 (2018)



μ-BooNE data, argon target

- Flux-averaged inclusive x-section, PRL 123, 131801 (2019)



THE LEPTON-NUCLEUS X-SECTION

- Consider, for example, the cross section of the process $\ell + A \to \ell' + X$ at fixed beam energy

$$d\sigma_A \propto L_{\mu\nu} W_A^{\mu\nu}$$

  - $L_{\mu\nu}$ is fully specified by the lepton kinematical variables
  - The nuclear response tensor

$$W_A^{\mu\nu} = \sum_X \langle 0|J_A^{\mu\,\dagger}|X\rangle\langle X|J_A^{\nu}|0\rangle \delta^{(4)}(P_0 + k - P_X - k')$$

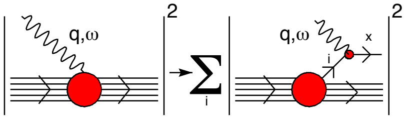
  involves

  1. the target ground state, $|0\rangle$, largely non relativistic
  2. hadronic final state, $|X\rangle$, carrying momentum $q = k - k'$ and possibly involving hadrons other than nucleons $\to$ relativistic treatment needed at high q
  3. the nuclear current operator, explicitly depending on $q \to$ relativistic treatment needed at high q

$$J_A^{\mu} = \sum_i j_i^{\mu} + \sum_{j>i} j_{ij}^{\mu}$$

Impulse approximation and factorisation

⋆ for $\lambda \sim 1/|\mathbf{q}| \ll d_{\rm NN} \sim 1.6$ fm, the average nucleon-nucleon distance in the target nucleus, nuclear scattering reduces to the incoherent sum of scattering processes involving individual nucleons



⋆ Basic assumptions

▷ $J^\mu_A(q) \approx \sum_i j^\mu_i(q)$: single-nucleon coupling

▷ $|X\rangle \to |x(\mathbf{p})\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle$: factorisation of the final state

⋆ Corrections arising from te occurrence of Final State Interactions (FSI) and processes involving two-nucleon Meson-Exchange Currents (MEC) can be consistently included (more on this later)

THE IA CROSS SECTION

- Factorisation allows to rewrite the nuclear transition matrix element as

$$\langle X | J_A^\mu | 0 \rangle \to \sum_i \int d^3k \; M_n(\mathbf{k}) \langle \mathbf{k} + \mathbf{q} | j_i^\mu | \mathbf{k} \rangle$$

  - The nuclear amplitude $M_n = \langle n | a_{\mathbf{k}} | 0 \rangle$ is independent of momentum transfer. It can be accurately calculated within non relativistic many-body theory
  - The matrix element of the current between free-nucleon states can be computed exactly using the fully relativistic expression

- Nuclear x-section

$$d\sigma_A = \int d^3k dE \; d\sigma_N \; P_h(\mathbf{k}, E)$$

- The lepton-nucleon cross section $d\sigma_N$ can be obtained—at least in principle—from proton and deuteron data, theoretical models, or LQCD

- The spectral function $P_h(\mathbf{k}, E)$ describes the probability of removing a nucleon of momentum $\mathbf{k}$ from the nuclear ground state, leaving the residual system with excitation energy E

ANALYTIC STRUCTURE OF THE SPECTRAL FUNCTION

- ⋆ The spectral function, being trivially related to the nucleon Green's function

$$P_h(\mathbf{k},E) = \frac{1}{\pi}\text{Im}\, G_h(\mathbf{k},E) = \sum_n |\langle n|a_{\mathbf{k}}|0\rangle|^2 \delta(E - E_0 + E_n)$$

can be split into pole and smooth contributions

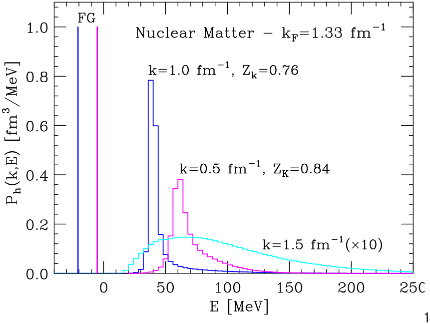
- ⋆ The smooth component originates from effects beyond the mean field approximation underlying the nuclear shell model, notably short range correlations

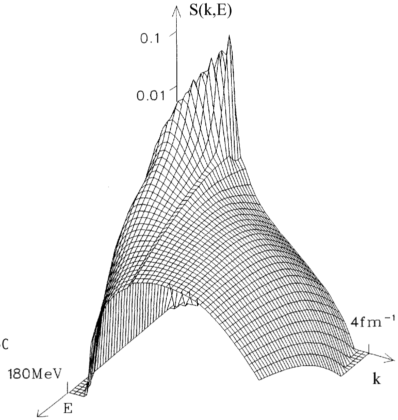
- ⋆ The most significant correlation effects are
  1. multi-nucleon emission in single-nucleon knockout processes
  2. quenching of the strength in the single-nucleon emission sector

They have both been observed by electron-nucleus scattering experiments

ISOSPIN SYMMETRIC NUCLEAR MATTER AT EQUILIBRIUM

- Calculation carried out using a realistic nuclear Hamiltonian model and the formalism of CBF perturbation theory, NPA 505, 267 (1989)

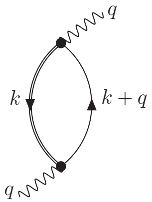




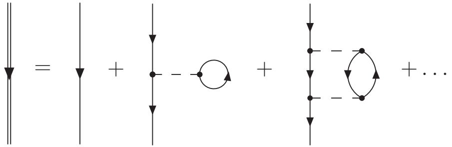
Including interaction effects in the 0π sector

- Single-nucleon emission in $^{12}\mathrm{C}(e,e')$: $|X\rangle = |p, {}^{11}\mathrm{C}\rangle \ , \ |n, {}^{11}\mathrm{B}\rangle$

$W_A^{\mu\nu} =$



$G_h(k, E) =$



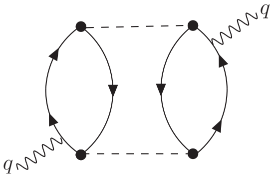
- Warning: the *bare* nucleon-nucleon interaction cannot be used to perform perturbative calculations in the basis of eigenstates of the non-interacting system

- Interactions couple the 1h states of the residual nucleon to 2h1p states, in which one of the spectator nucleons is excited to the continuum. This mechanism leads to the appearance of 2p2h final states.

$$|X\rangle = |pp, {}^{10}\mathrm{B}\rangle \ , \ |np^{10}\mathrm{C}\rangle \ldots$$

- In addition, in the presence of correlations 2p2h states appear through their coupling to the ground state

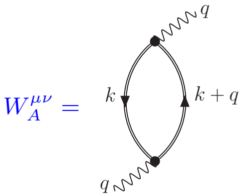
$$W_A^{\mu\nu} =$$



- Note that this process interferes with the other processes leading to the excitation of 2p2h states, that is, coupling to MEC and FSI

Corrections to the IA: FSI

- In principle, the effects of nucleon-nucleon interactions in the final state may be taken into account in a consistent fashion, dressing the particle line with momentum $\mathbf{k} + \mathbf{q}$

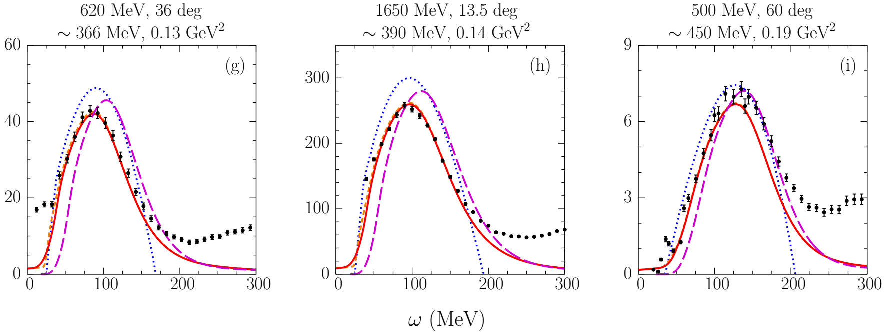


  However, for large $\mathbf{q}$ the Green's function $G_p(\mathbf{k} + \mathbf{q}, E)$, describing the propagation of the outgoing nucleon, cannot be obtained from non relativistic many-body theory. In the absence of a relativistic model of nuclear dynamics, further approximations are needed

- Extensive studies of FSI in electron-nucleus scattering have been performed within the approach based on the eikonal approximation and a generalisation of the optical potential model

Interactions effects (no MEC yet)

- nuclear mean field → cross section shifted
- nucleon-nucleon correlations → coupling between 1p1h and 2p2h final states. Peak quenched, appearance of tails at both low and high energy transfer, ω.
- FSI → cross section shifted and broadened

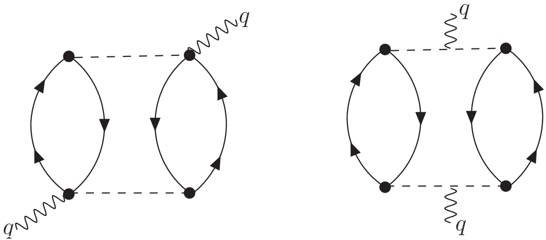


A. Ankowski *et al*, PRD **91** 033005, (2015)

Corrections to the IA: MEC

- Two-nucleon currents naturally couple the nuclear ground state to 2p2h final states, e.g. through the processes

$$W_A^{\mu\nu} =$$



as well as through similar processes involving the excitation of a Δ-resonance

THE EXTENDED FACTORISATION *ansatz*

- ⋆ Highly accurate and consistent calculations of processes involving MEC can be carried out in the non relativistic regime
- ⋆ Fully relativistic MEC studied mainly within the Fermi gas model
- ⋆ Using relativistic MEC and a realistic description of the nuclear ground state requires the extension of the IA scheme to treat two-nucleon emission amplitudes
  - ▸ Rewrite the hadronic final state $|n\rangle$ in the factorized form

$$|n\rangle \to |\mathbf{p},\mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)},\mathbf{p},\mathbf{p}'\rangle$$

$$\langle X|j^{\mu}_{ij}|0\rangle \to \int d^3k\, d^3k'\, M_n(\mathbf{k},\mathbf{k}')\,\langle \mathbf{p}\mathbf{p}'|j^{\mu}_{ij}|\mathbf{k}\mathbf{k}'\rangle\,\delta(\mathbf{k}+\mathbf{k}'+\mathbf{q}-\mathbf{p}-\mathbf{p}')$$

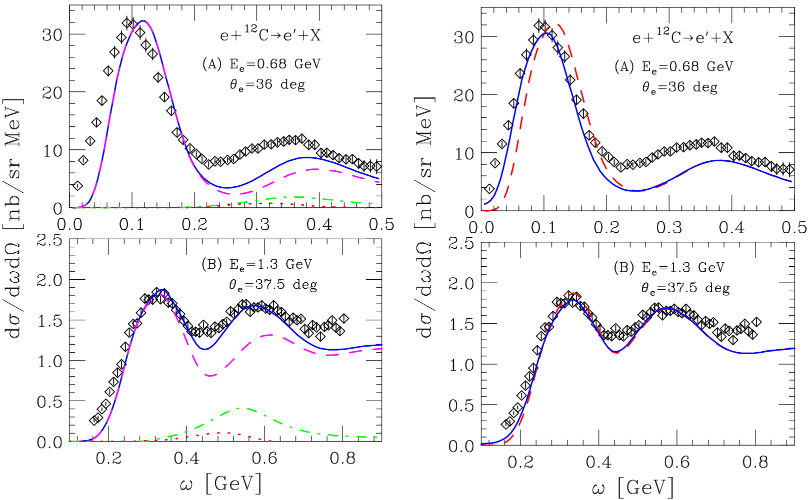
The amplitudes

$$M_n(\mathbf{k},\mathbf{k}') = \langle n_{(A-2)},\mathbf{k},\mathbf{k}'|0\rangle$$

are independent of q , and can be obtained from non relativistic many-body theory. They contribute to the two-nucleon Green's function

Pinning down FSI & MEC

- N. Rocco, OB, and A. Lovato, PRL 116, 192501 (2016)

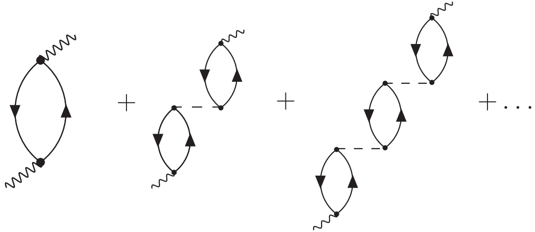


Non factorizable interactions

- At low momentum transfer, mechanisms involving many nucleons may become important. Within the ring approximation—also referred to as Tamm-Dancoff approximation—the nuclear final state is written in the form

$$|X\rangle = \sum_i C_i |p_i h_i\rangle$$

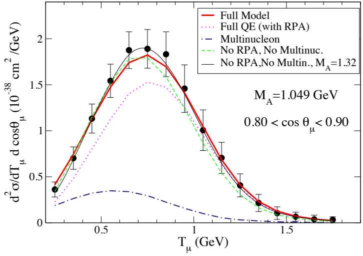
$$W_A^{\mu\nu} =$$

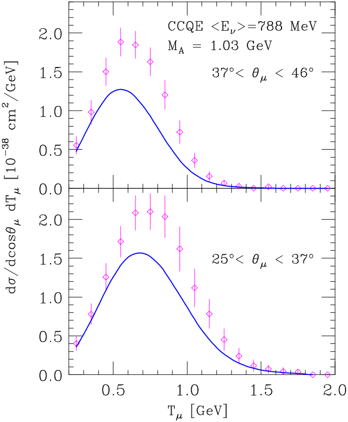


Note: the Random-Phase-Approximation (RPA) is a generalisation of the above scheme

Neutrino-nucleus cross sections: why worry

- x-section models based on different dynamical assumptions and including different reaction mechanisms yield similar results



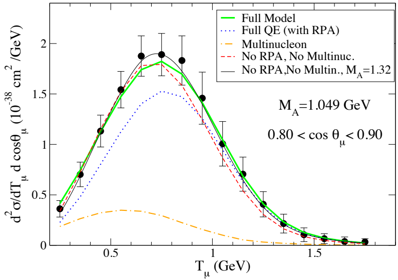


- From the numerical point of view, RPA effects turn out to be similar to the quenching of the single-nucleon knock-out x-section arising from nucleon-nucleon correlations

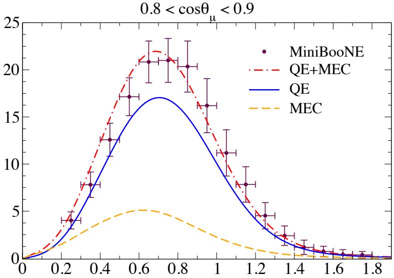
Valencia Model *vs* Superscaling

- ⋆ Comparison to the flux-integrated cross section measured by the MiniBooNE Collaboration.

- ▶ Nieves *et al*



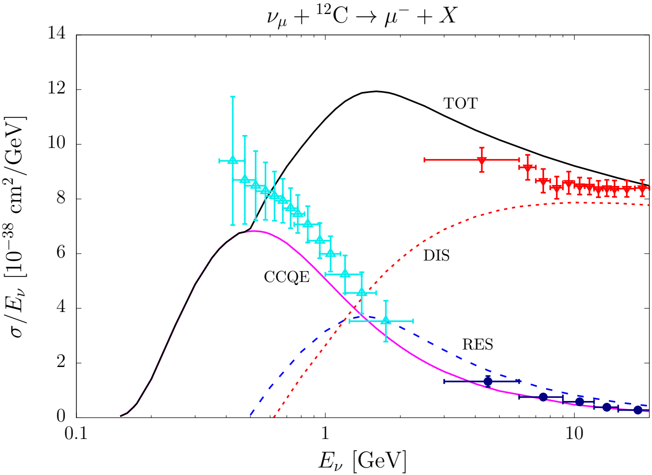
- ▶ Megias *et al*



- ⋆ The result of Nieves et al show a significant contribution arising from the excitation of nuclear collective modes (RPA), which is not included in the approach of Megias et al
- ⋆ The inclusive flux-averaged neutrino-nucleus cross sections provide information useful to resolve the degeneracy between theoretical models

EXTENSION TO INELASTIC PROCESSES

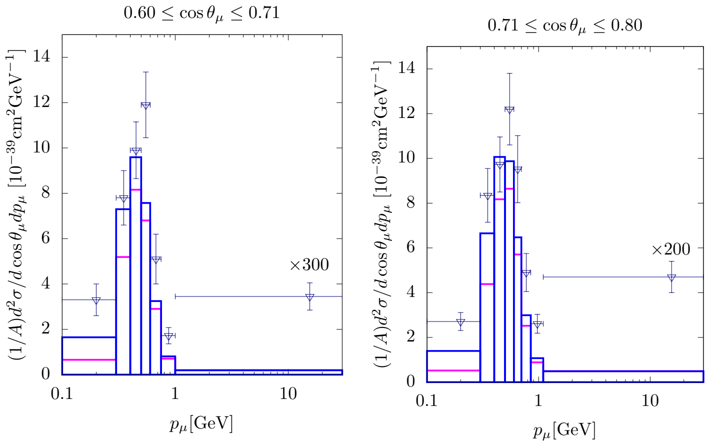
- Total ν-Carbon Cross Section, E. Vagnoni *et al*, PRL 118, 142502 (2017)

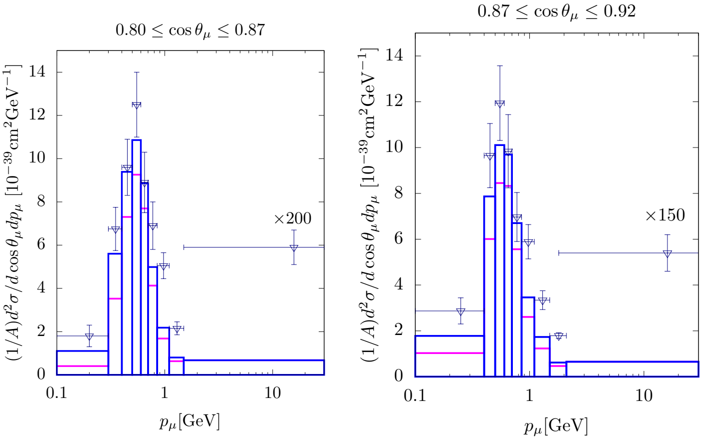


- σ_{CCQE}: NOMAD, PLB 660, 19 (2008), MiniBooNE, PRD 81, 092005 (2010)
- σ_{TOT}: NOMAD, EPJC 63, 555 (2009)

Preliminary: flux-averaged inclusive cross setion

- ⋆ Comparison with the inclusive flux-averaged ν_μ-Carbon CC cross section measured by the T2K collabortion, PRD 98, 012004 (2018). Inelastic structure functions from T. Sato, No MEC, no FSI.




$0.80 \leq \cos\theta_\mu \leq 0.87$
$(1/A)d^2\sigma/d\cos\theta_\mu dp_\mu$ $[10^{-39}\text{cm}^2\text{GeV}^{-1}]$
$\times 200$
p_μ[GeV]
$0.87 \leq \cos\theta_\mu \leq 0.92$
$(1/A)d^2\sigma/d\cos\theta_\mu dp_\mu$ $[10^{-39}\text{cm}^2\text{GeV}^{-1}]$
$\times 150$
p_μ[GeV]

Summary & Outlook

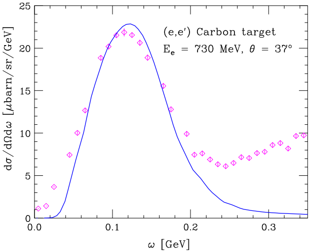
- ⋆ Despite the complexity of flux average, a consistent description of the neutrino-nucleus cross in both elastic and inelastic channels appears to be possible within the approach based on factorisation.
- ⋆ Most theoretical models employed of neutrino-nucleus interactions involve some level of factorisation. However, to fully exploit its potential, this scheme must be implemented using spectral functions providing an accurate description of the initial state. Valuable new information will be provided by electron scattering experiments, notably the measurement of the ${}^{40}_{18}\mathrm{Ar}(e, e'p)$ cross section in Jlab Hall A
- ⋆ A better understanding of the interaction vertices, involving vector and axial form factors, and structure functions in the resonance production and DIS regions, is needed
- ⋆ The present development of the treatment of FSI, while being adequate for inclusive processes in the 0π sector, need to be improved and generalised to treat both exclusive and inelastic processes
- ⋆ Long-range correlations and the breakdown of factorisation at low momentum transfer need to be carefully investigated

Thank you!

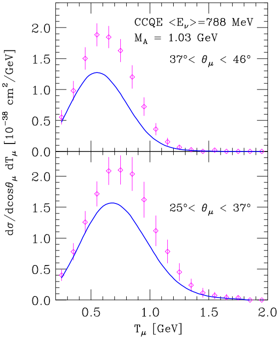
Backup slides

COMPARING e- AND ν_μ-CARBON 0π CROSS SECTIONS

▷ Electron scattering



▷ MiniBooNe CCQE cross section

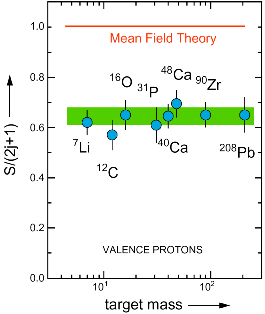


- Theoretical calculations carried out using the same formalism, PRL 105, 132301 (2010)
- Owing to flux average, reaction mechanisms other than the dominant single-nucleon knock out contribute to the neutrino cross section

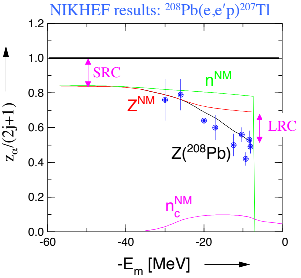
QUENCHING OF THE 1P1H STRENGTH

⋆ Nucleon-nucleon correlations move strength from the 1p1h sector to the 2p2h sector

- Spectroscopic factors of valence states (Lapikas, 1993)



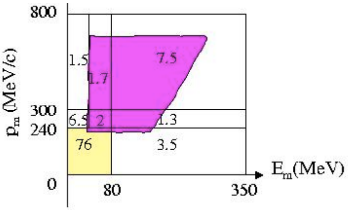
- Spectroscopic factors of the shell model states of ^{208}Pb (OB et al, 1991)

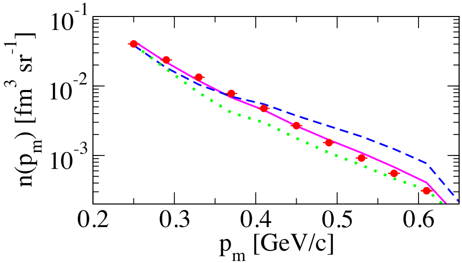


⋆ Short range correlations account for more than $\sim$ 70% of the observed quenching

Measured Correlation Strength

- The correlation strength in the 2p2h sector has been investigated by the JLAB E97-006 Collaboration using a carbon target



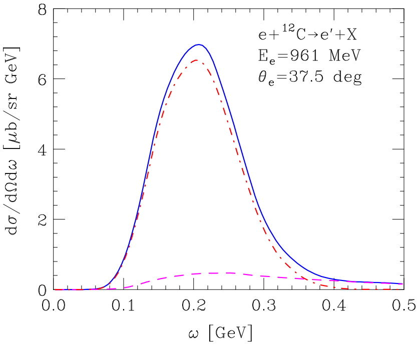


- Measured correlation strength

Experiment	0.61 ±0.06
Greens function theory [3]	0.46
CBF theory [2]	0.64
SCGF theory [4]	0.61

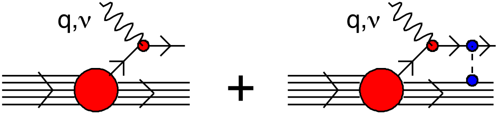
Correlation Effects on the QE Cross Section

- Correlations move strength from the 1p1h sector—in which the residual system bound state—to the 2p2h sector—in which one spectator nucleon excited to the continuum—leading to a quenching of the peak and to the appearance of a tail extending to large energy loss



Final State Interactions (FSI)

- The measured $(e, e'p)$ x-sections provide overwhelming evidence of the occurrence of significant FSI effects in the QE sector



$$d\sigma_A = \int d^3k dE \, d\sigma_N \, P_h(\mathbf{k}, E) P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$$

- the particle-state spectral function $P_p(|\mathbf{k}+\mathbf{q}|, \omega - E)$ describes the propagation of the struck particle in the final state
- the IA is recovered replacing

$$P_p(|\mathbf{k}+\mathbf{q}|, \omega - E) \to \delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$

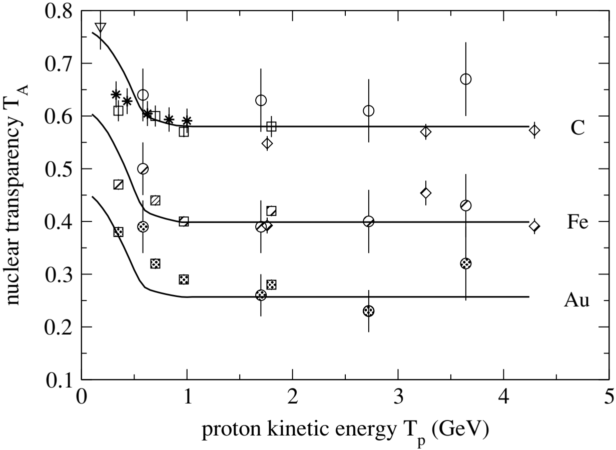
- effects of FSI on the inclusive cross section
  - ⋆ shift in energy transfer due to the mean field of the spectator nucleons
  - ⋆ redistributions of the strength due to rescattering of the knocked out nucleon
- high energy (eikonal) approximation
  - ⋆ the struck nucleon moves along a straight trajectory with constant velocity
  - ⋆ the fast struck nucleon "sees" the spectator system as a collection of fixed scattering centers

$$\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}) \to \sqrt{T_{|\mathbf{k}+\mathbf{q}|}}\,\delta(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2})$$
$$+(1 - \sqrt{T_{|\mathbf{k}+\mathbf{q}|}})\,f(\omega - E - \sqrt{|\mathbf{k}+\mathbf{q}|^2 + m^2}))$$

- the nuclear transparency T is measured by $(e, e'p)$ experiments, and the folding function f can be computed within nuclear many-body theory using as input nucleon-nucleon scattering data
- complex pattern of significant medium effects

Gauging FSI: Nuclear Transparency from $(e, e'p)$

- Nuclear transparency, measured by the ratio $\sigma_{\text{exp}}/\sigma_{\text{IA}}$. PRC 72, 054602 (2005)



Two-Nucleon Spectral Function

⋆ Calculations have been carried out for uniform isospin-symmetric nuclear matter

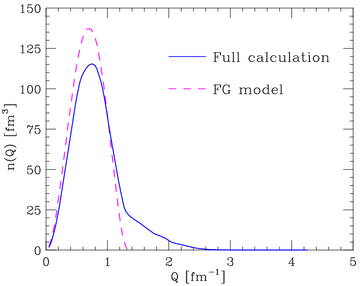
$$P(\mathbf{k}_1, \mathbf{k}_2, E) = \sum_n |M_n(k_1, k_2)|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k}_1, \mathbf{k}_2) = \int dE\, P(\mathbf{k}_1, \mathbf{k}_2, E)$$

⋆ Relative momentum distribution

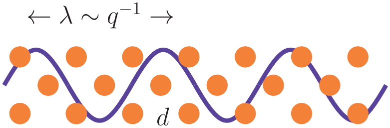
$$n(\mathbf{Q}) = 4\pi |\mathbf{Q}|^2 \int d^3q\, n\left(\frac{\mathbf{Q}}{2} + \mathbf{q}, \frac{\mathbf{Q}}{2} - \mathbf{q}\right)$$

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2 \ \ , \ \ \mathbf{Q} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}$$



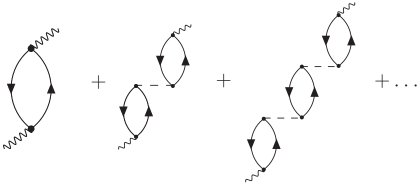
Long-Range Correlations

- At low momentum transfer the space resolusion of the neutrino becomes much larger than the average NN separation distance (~ 1.5 fm), and the interaction involves many nucleons



- Write the nuclear final state as a superposition of 1p1h states (RPA scheme)

$$|n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i\rangle$$



TAMM-DANCOFF (RING) APPROXIMATION

- ⋆ Propagation of the particle-hole pair produced at the interaction vertex gives rise to a collective excitation. Replace

$$|ph\rangle \to |n\rangle = \sum_{i=1}^{N} C_i \, |p_i h_i)$$

- ⋆ The energy of the state $|n\rangle$ and the coefficients C_i are obtained diagonalizing the hamiltonian matrix

$$H_{ij} = (E_0 + e_{p_i} - e_{h_i})\delta_{ij} + (h_i p_i|V_{\rm eff}|h_j p_j)$$

$$e_k = \frac{k^2}{2m} + \sum_{\mathbf{k}'} \langle \mathbf{k}\mathbf{k}'|V_{\rm eff}|\mathbf{k}\mathbf{k}'\rangle_a$$

- ⋆ The appearance of an eigenvalue, ω_n, lying outside the particle-hole continuum signals the excitation of a collective mode

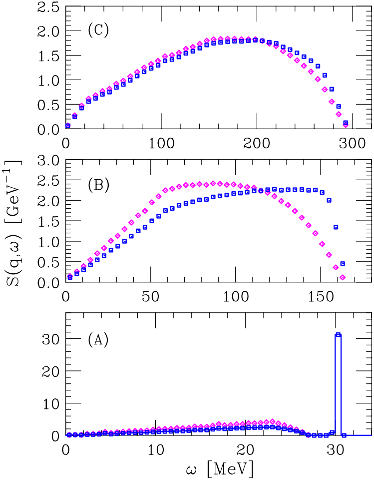
Beyond Factorisation: Long-Range Correlations

- $|\mathbf{q}|$-evolution of the density-response of isospin-symmetric nuclear matter, PLB 680, 305 (2009)

$|\mathbf{q}| \approx 480$ MeV

$|\mathbf{q}| \approx 300$ MeV

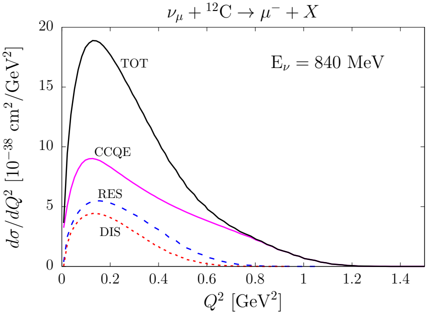
$|\mathbf{q}| \approx 60$ MeV



Factorisation in the inelastic channels

⋆ Q^2-distribution at fixed neutrino energy

- E. Vagnoni, OB, and D. Meloni, PRL 118, 142502 (2017)
- Nucleon structure functions:
  - CCQE: BBBA vector form factors + dipole fit of the axial form factor
  - RES: model of Lalakulich, Paschos, and Sakuda
  - DIS: parton distributions of Glück, Reya, and Vogt



- Note: $E_\nu = 840$ MeV is the average energy of the T2K flux