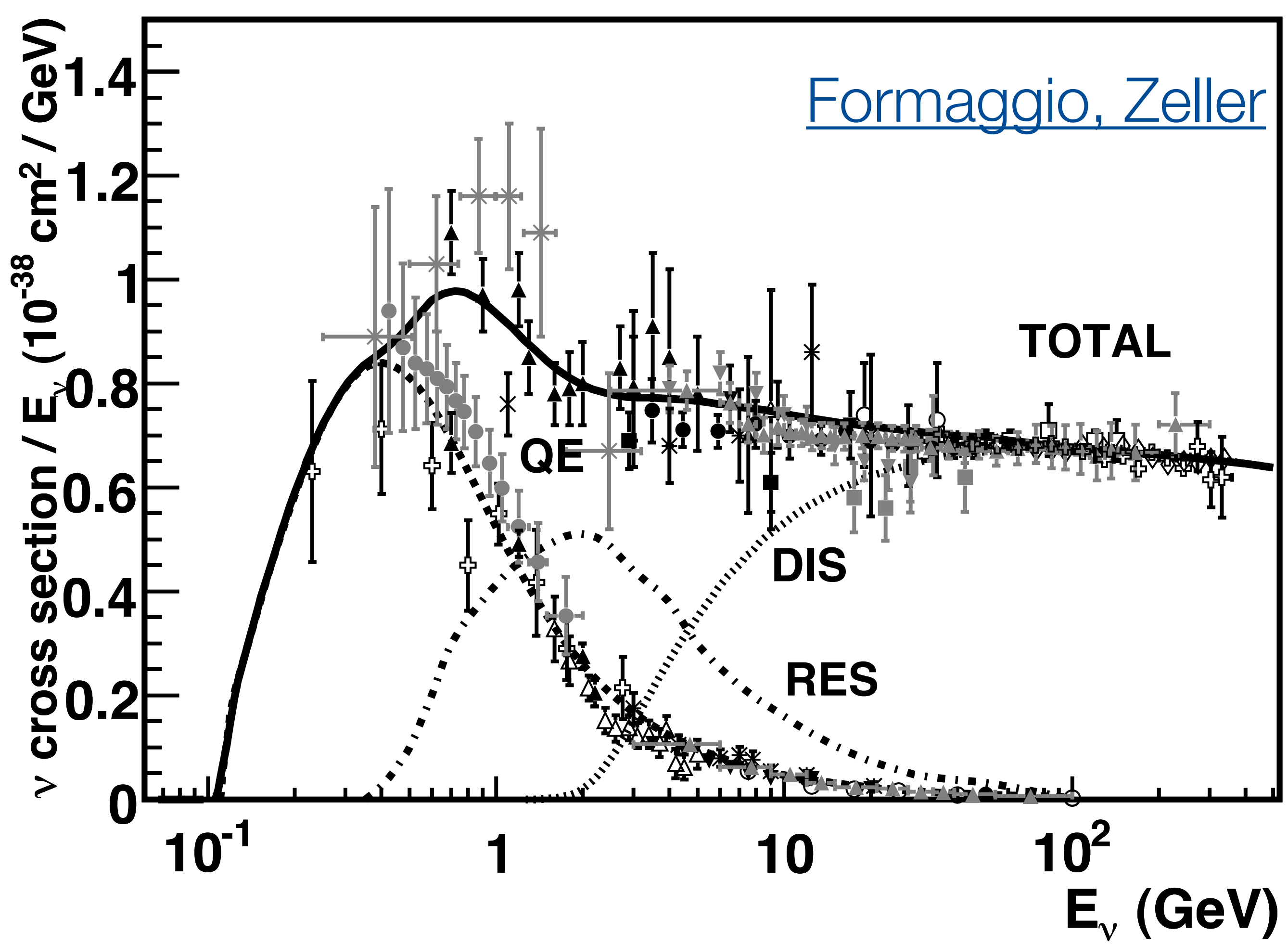
Opportunities for Lattice QCD and Neutrino-Nucleus Scattering

Andreas S. Kronfeld
Theory Division

Neutrinos as a Portal to New Physics and Astrophysics
Kavli Institute for Nuclear Physics
March 1, 2022

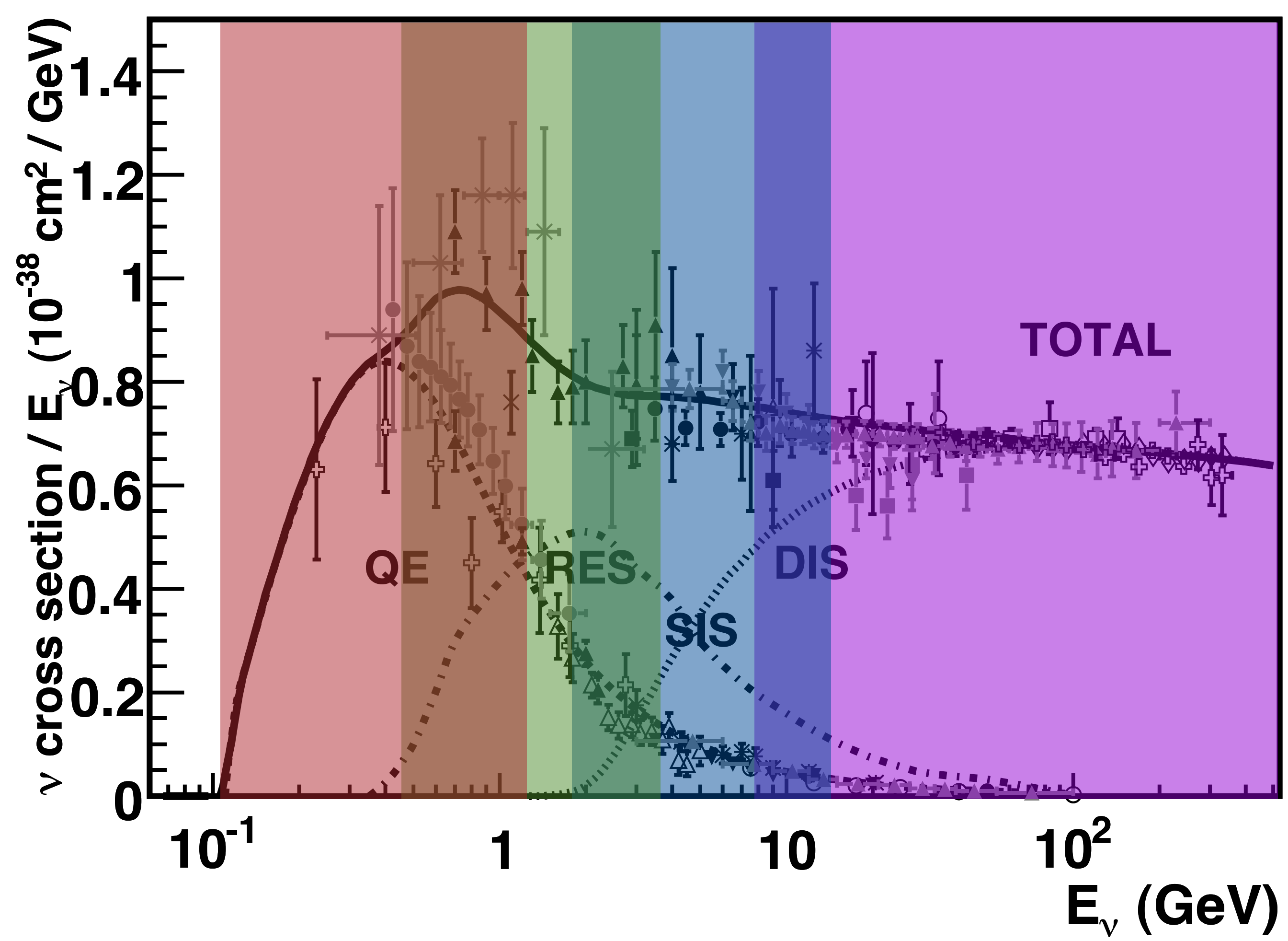
Neutrino-Nucleus Interactions

- Incident neutrino not known and not easily reconstructed because the remnant of the struck nucleus is not detected.
- Event classification at “hadron level” not necessarily clear.
- Paradigm (impulse approx.):
  - nucleon/hadron inputs + chiral effective theory + nuclear many-body theory.
- Not yet possible:
  - direct calculation of ^{12}C or ^{16}O (let alone ^{40}Ar) from QCD.



Opportunities for Lattice QCD

- Various nucleon matrix elements.
- QE: nucleon form factors—electric, magnetic, axial, induced pseudoscalar.
- RES: transition form factors—
  - rough & ready (stable Δ);
  - rigorous.
- SIS: nucleon hadron tensor.
- DIS: moments or Bjorken x dependence of parton distribution functions.



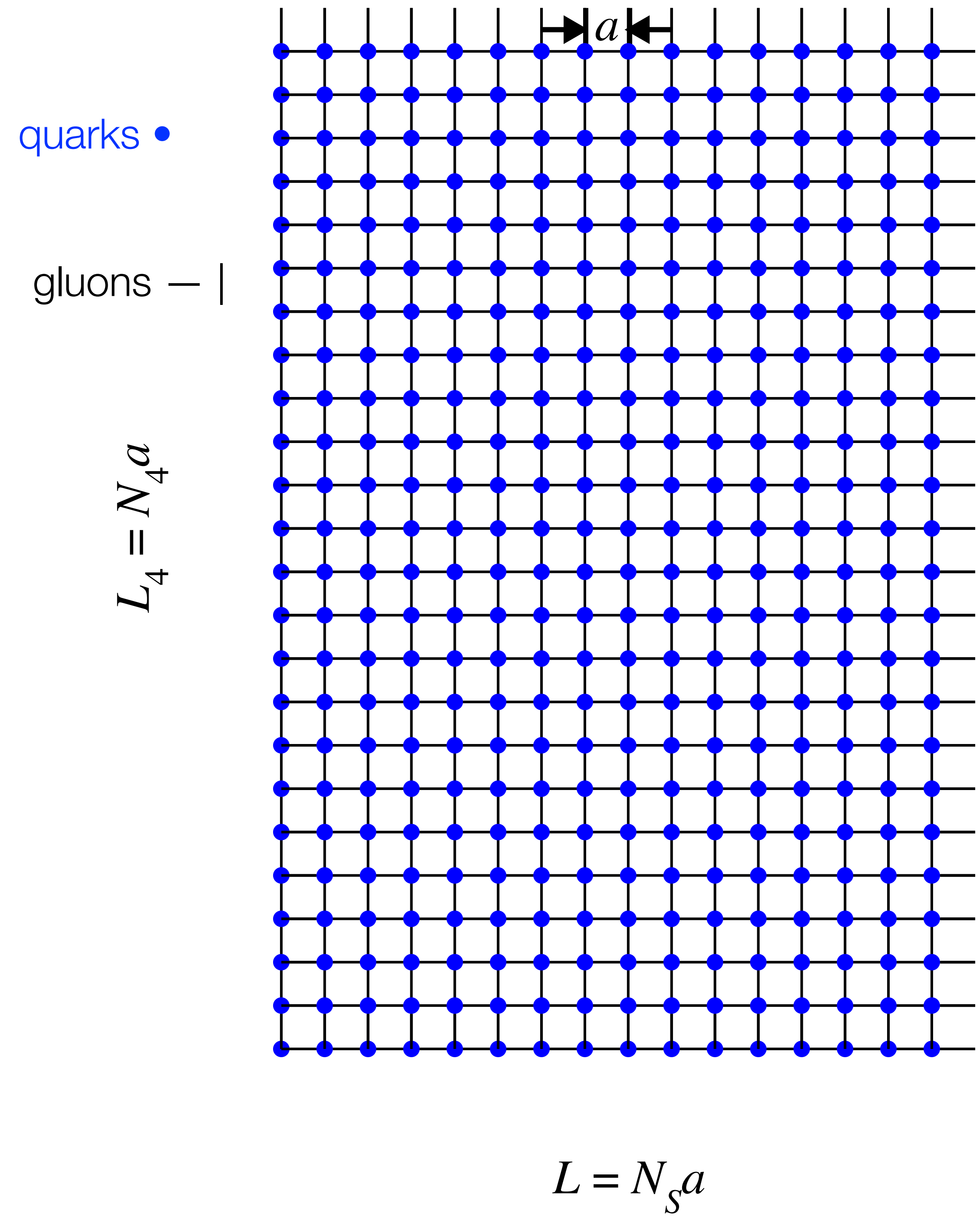
Outline

- Opportunities
- Lattice QCD
- Anatomy of an error budget
- Nucleons are harder

Lattice QCD

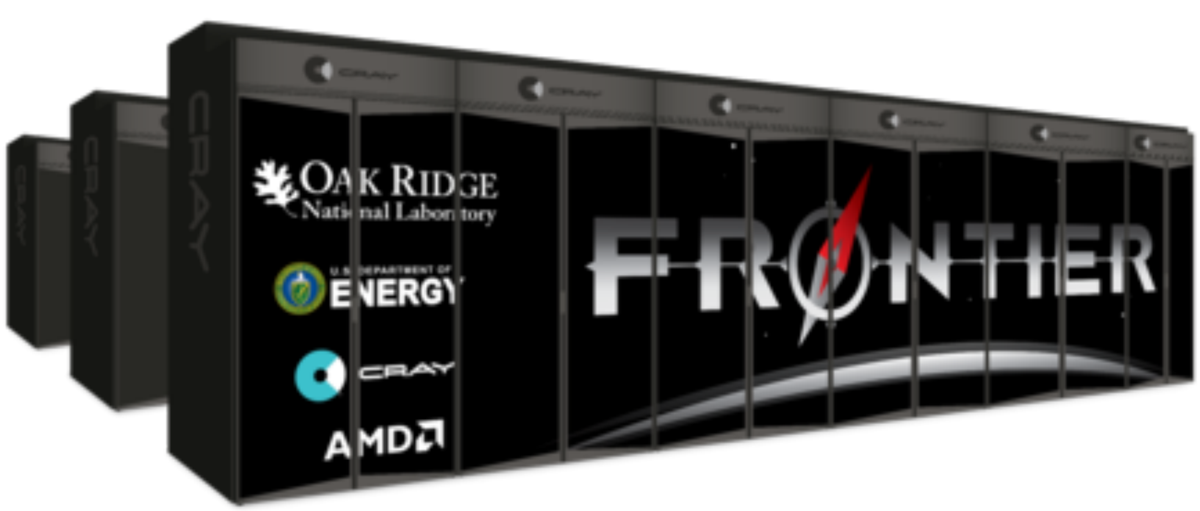
Lattice Gauge Theory

- Start with the QCD Lagrangian.
- Ultraviolet regulator:
  - spacetime lattice.
- Functional integral:
  - SU(3) Haar for gluons;
  - Grassmann for quarks;
  - countable number of d.o.f.
- Mathematical sound framework for QFT.



Lattice Gauge Theory II

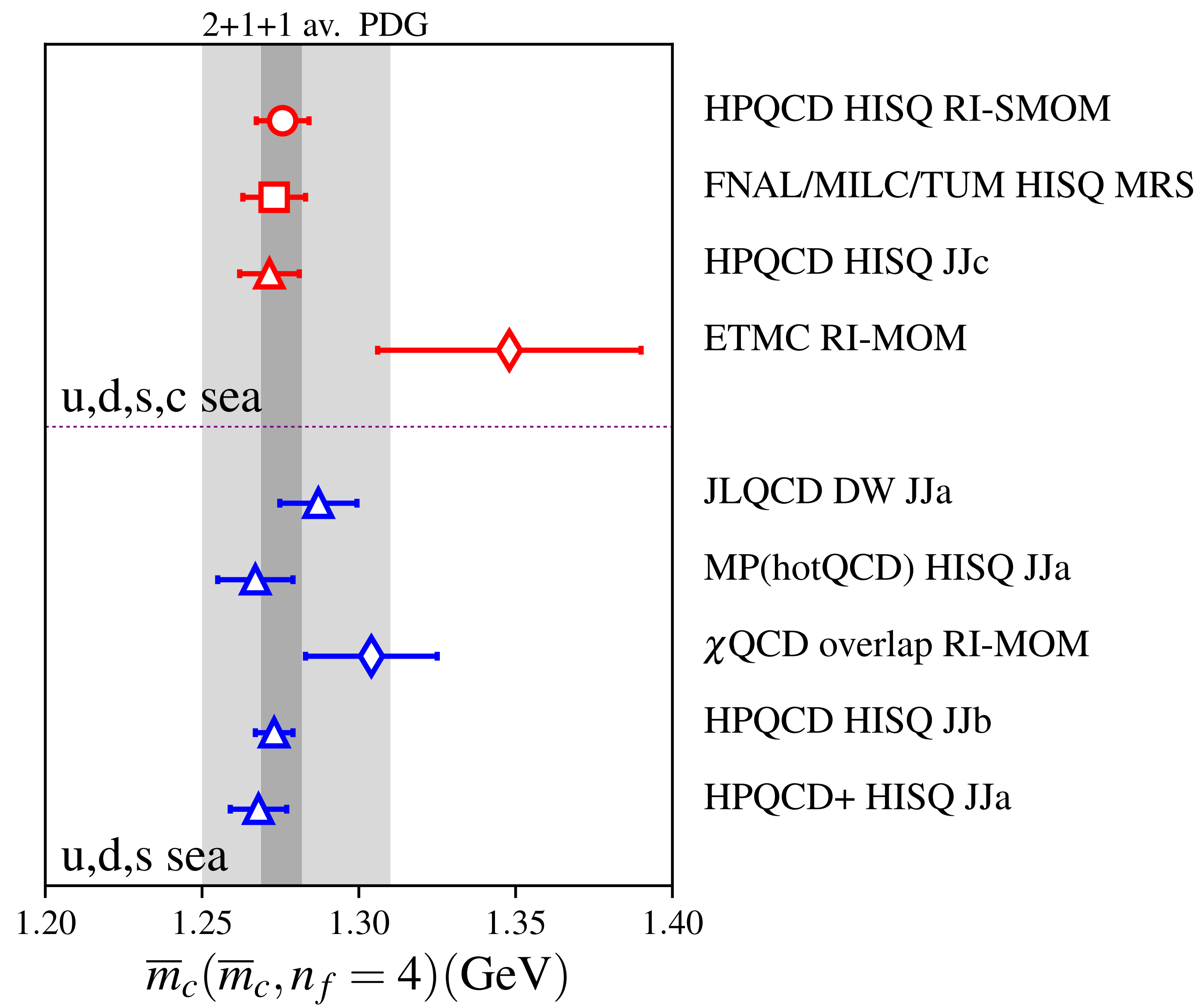
- Imaginary time $x^4 = ix^0$ make integrands damped exponentials.
- Infrared regulator: finite spatial extent & finite temporal extent ($T = 1/L_4$).
- Now the number of integrations is finite (albeit huge in practice ≲ $5.4{\times}10^9$):
  - teach a machine how to compute the integrals—
    Markov chain Monte Carlo



Some Achievements

Quark Masses

- Bottom and charm:
  - sub percent uncertainty;
  - more than one method.
- Strange, down, up:
  - from ratios m_l/m_c;
  - 0.75% on m_s;
  - 1.2% on m_d;
  - 1.9% on m_u.

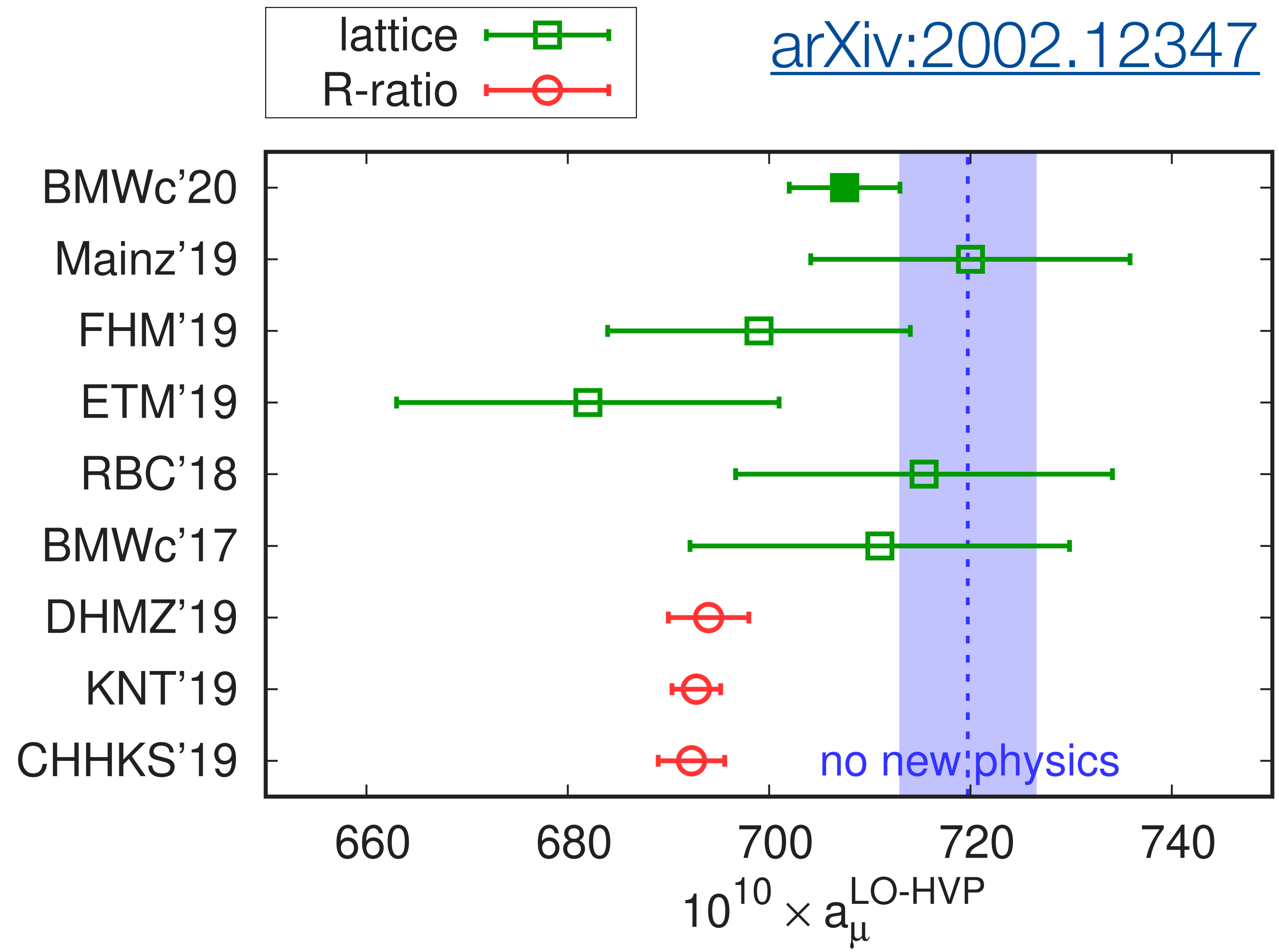


See, e.g., arXiv:1802.04248, arXiv:1805.06225

Muon Anomalous Magnetic Moment

- Hadronic vacuum polarization:
  - several few-percent results (from lattice QCD).
  - one 1% result, with more underway.
  - need 0.5% uncertainty to match needs of Fermilab E989.
- Theory Initiative: [arXiv:2006.04822]

[arXiv:2002.12347]



- Also results using e^+e^- data and dispersion relations.

Anatomy of an Error Budget

Mesons are Easier, so More Developed

- Error budget for meson form factors in the decay $B \to D^* l\nu$:

Table 1: Error budget for all form factors at $w = 1.11 = E_{D^*}/M_{D^*}$.

Source	h_V (%)	h_{A_1} (%)	h_{A_2} (%)	h_{A_3} (%)
Chiral-continuum fit error	4.2	2.0	17.4	6.9
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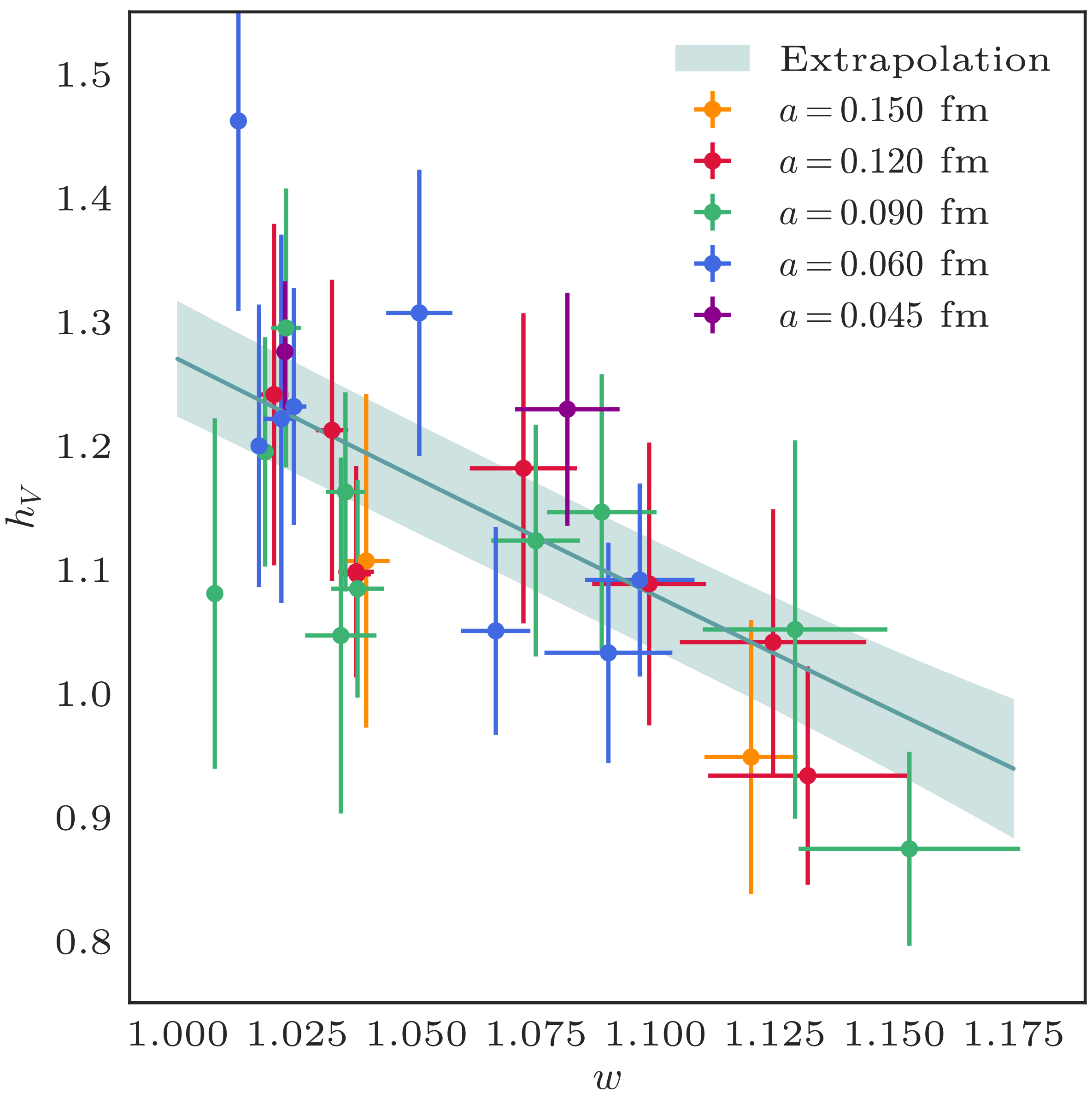
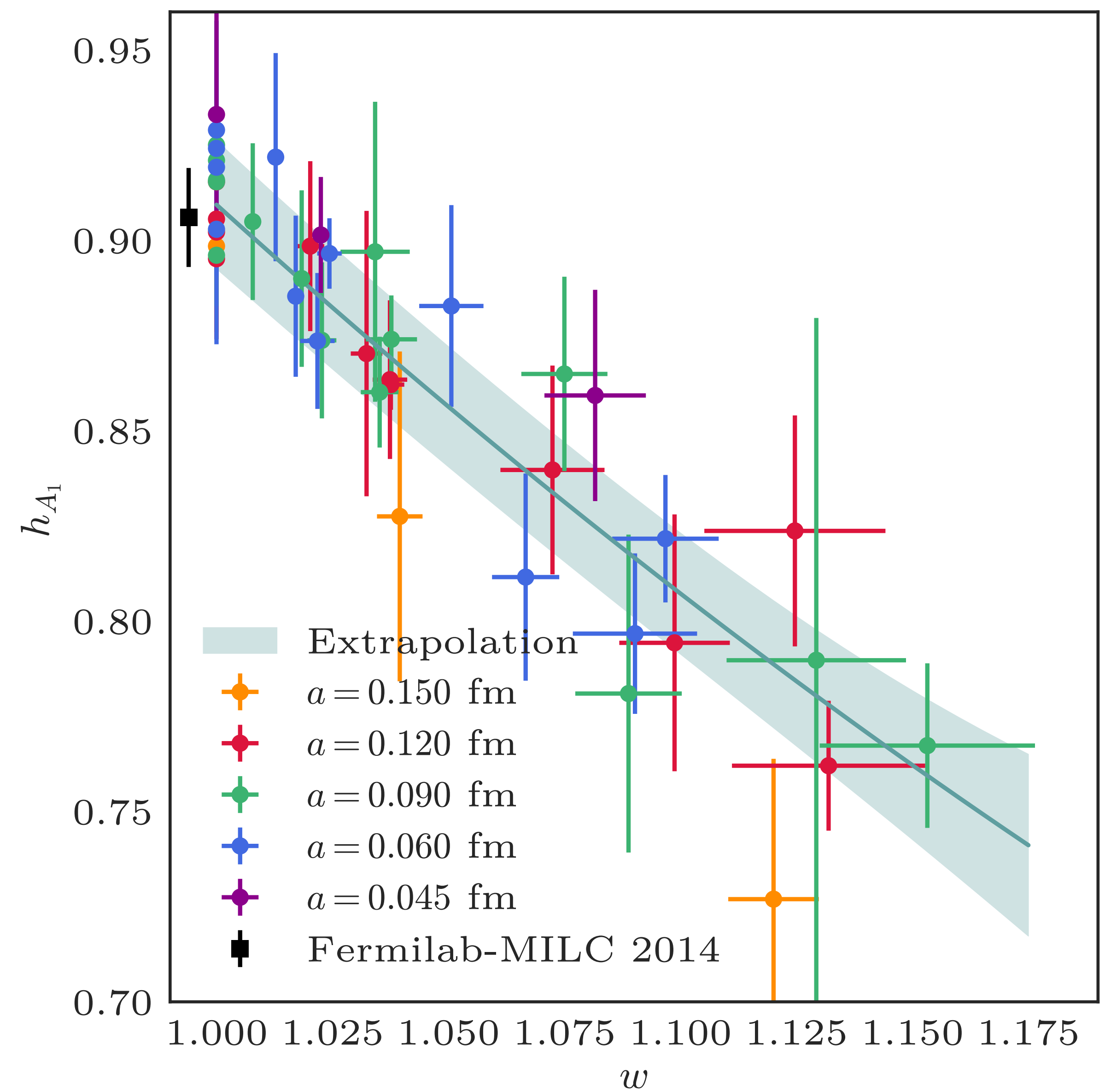
LQ mistuning	0.0	0.0	0.1	0.0
Scale setting	0.0	0.0	0.3	0.1
Isospin effects	0.1	0.2	1.2	0.5
Finite volume	—	—	—	—
Total error	4.2	2.0	17.4	6.9

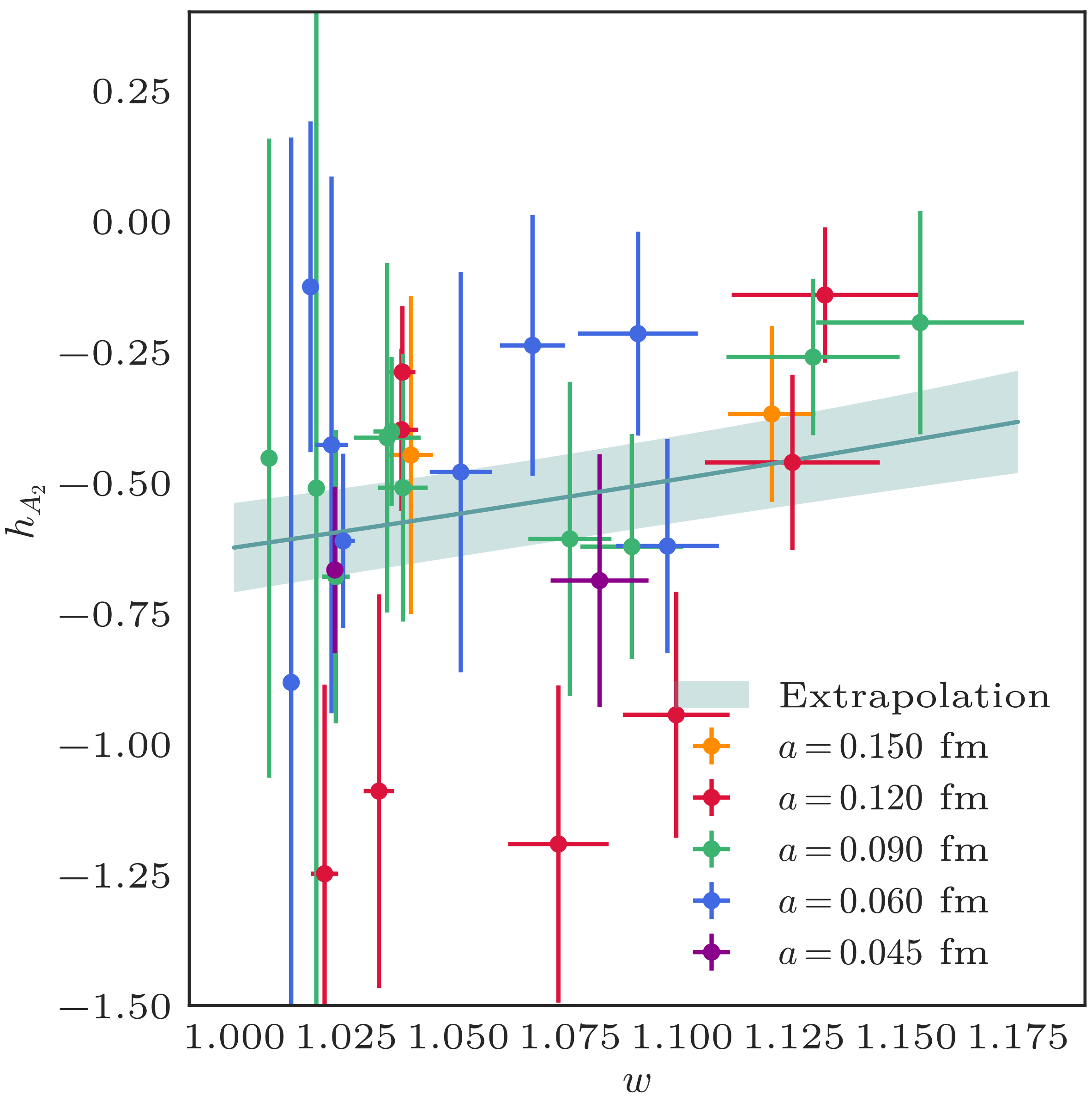
$$q^2 = M_B^2 + M_{D^*}^2 - 2wM_BM_{D^*}$$

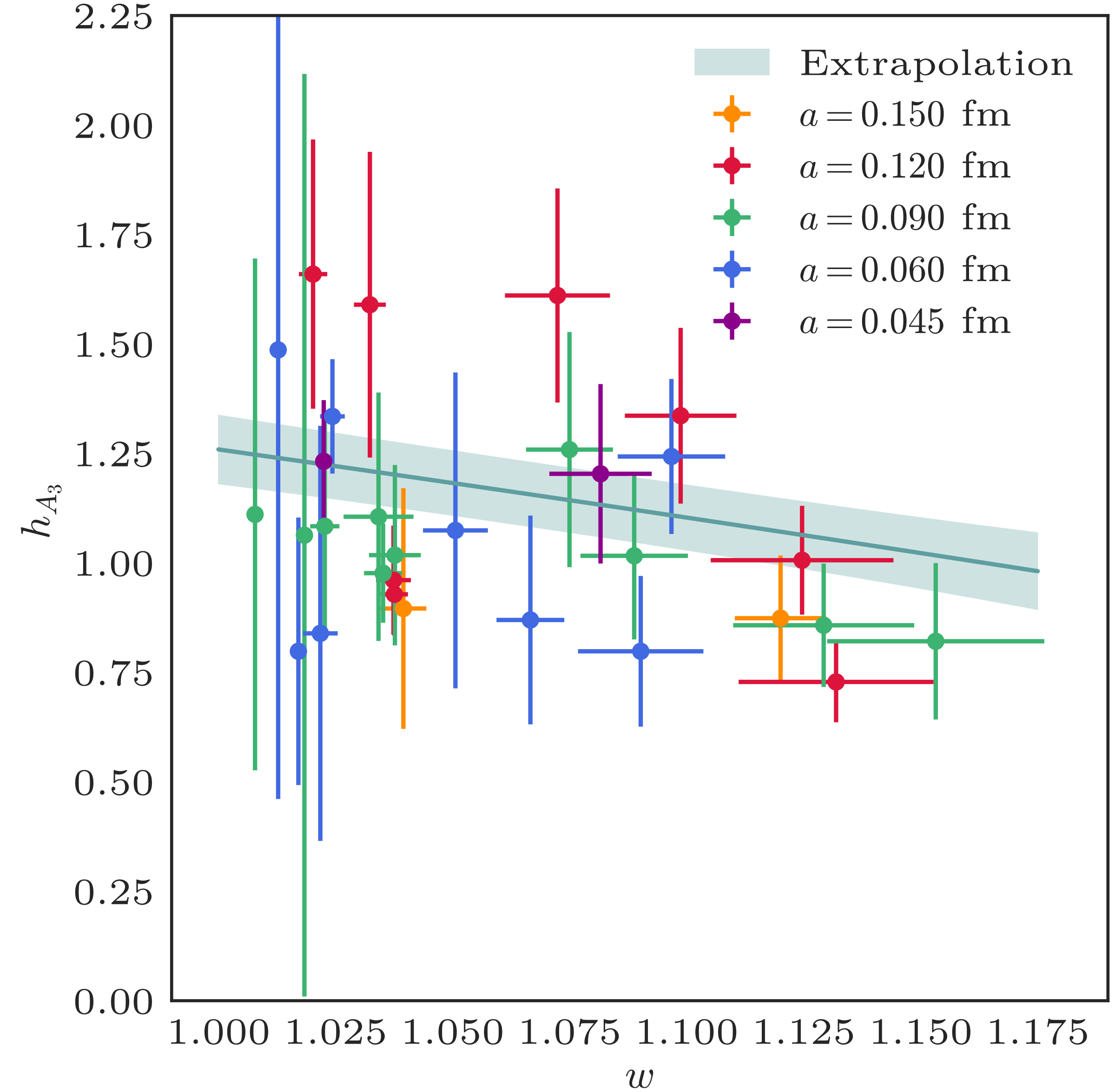
$$(M_B - M_{D^*})^2 > q^2 > m_\ell^2 \Rightarrow 1 \le w < 1.6$$





- Vector current: 1 form factor.
- Most precise ⇐ heavy-quark sym.





- Suppressed ⇐ heavy-quark sym.
- Third of 3 axial form factors.

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- Fermilab Lattice & MILC, arXiv:2105.14019.

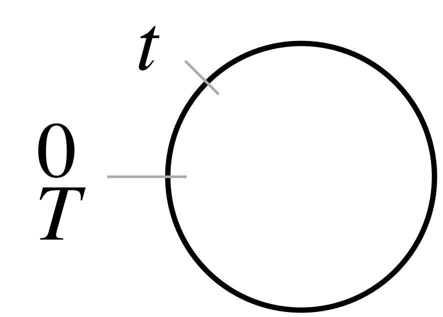
Statistics

- Correlation functions from path integral:

$$\langle O_i(t) O_j^\dagger(0)\rangle = \frac{1}{Z}\int \mathscr{D}U \,\mathrm{Det}(\not{D}+m)e^{-S_g} O_i(t) O_j^\dagger(0)$$

$$= \frac{1}{C}\sum_{c=0}^{C-1} O_i^{(c)}(t) O_j^{(c)\dagger}(0)$$

Monte Carlo with importance sampling.



- Euclidean time evolution:

$$\langle O_i(t) O_j^\dagger(0)\rangle = \frac{\sum_{rs}\langle r|O_i|s\rangle\langle s|O_j^\dagger|r\rangle e^{-tE_s-(T-t)E_r}}{\sum_r \langle r|r\rangle e^{-TE_r}}$$

$$\stackrel{T\to\infty}{=} \sum_s \langle 0|O_i|s\rangle\langle s|O_j^\dagger|0\rangle e^{-t(E_s-E_0)}$$

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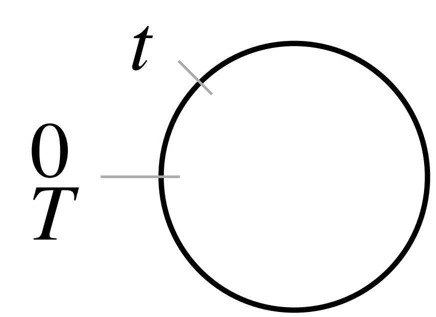

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$$1 \sum^{C-1} O^{(c)}(t)\, O^{(c)\dagger}(0)$$

statistical estimates of
energies (*e.g.*, masses) for given lattice spacing a
and choices of the quark masses

$$\langle O_i(t) O_j^\dagger(0)\rangle = \frac{\sum_{rs}\langle r|O_i|s\rangle\langle s|O_j^\dagger|r\rangle e^{-tE_s-(T-t)E_r}}{\sum_r \langle r|r\rangle e^{-TE_r}}$$

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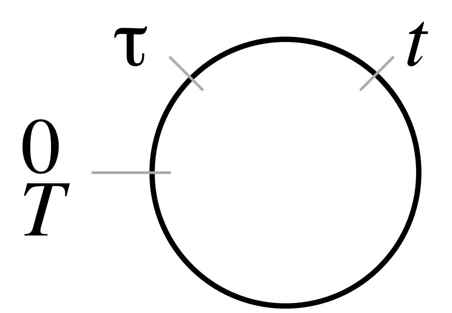
Three-Point Functions

- Matrix elements like form factors from three-point funtions—

$$\langle O_i(t) Z_A A^\mu(\tau) O_j^\dagger(0)\rangle \stackrel{T\to\infty}{=} \sum_{rs} \langle 0|O_i|r\rangle\langle r|Z_A A^\mu|s\rangle\langle s|O_j^\dagger|0\rangle e^{-\tau E_s-(t-\tau)E_r}$$

$$Z_A A^\mu \doteq \mathscr{A}^\mu$$

with similar Euclidean time evolution:



- Choose quantum numbers of O_j and O_i to get $N \to N$ resp. $N \to \Delta$.

- Matching factor Z_A to give continuum normalization to lattice current.

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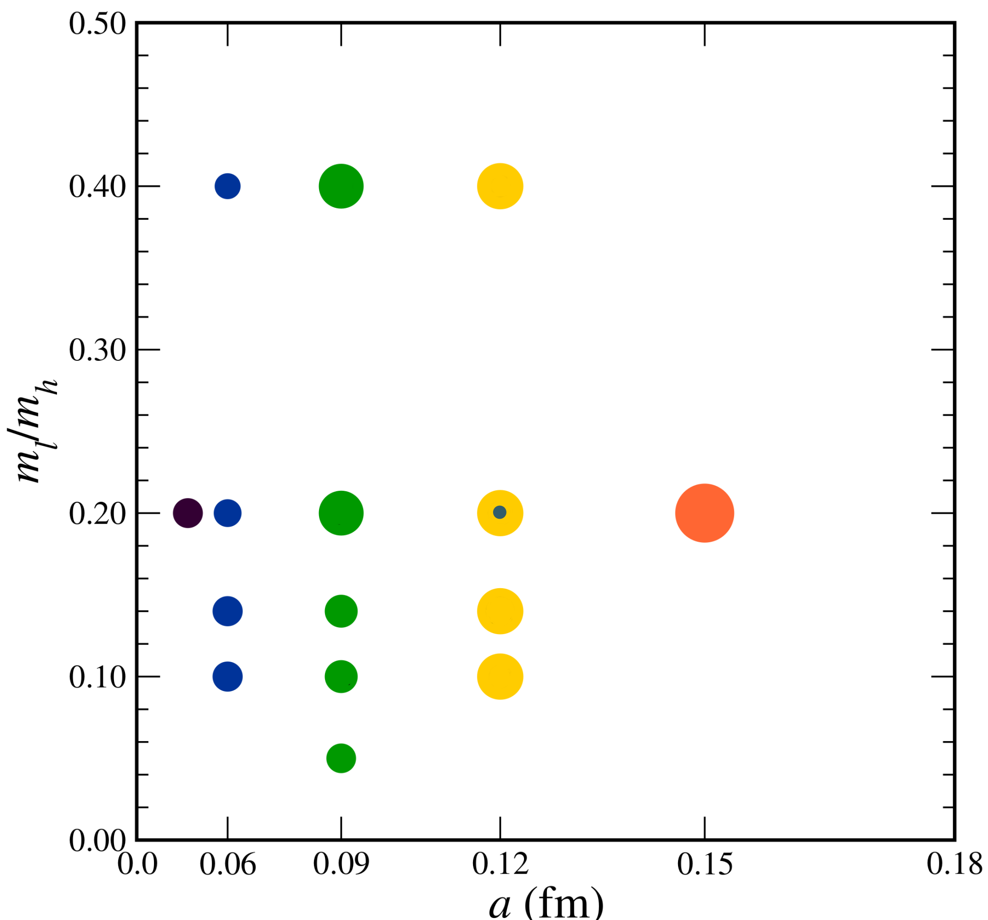
Effective Field Theory

- We have data with
  - 5 different lattice spacings, $0.045\ \text{fm} < a < 0.15\ \text{fm}$;
  - {1,4,5,4,1} different light quark masses m_l (in the sea), $m_q = m_l$;
  - varying m_b, m_c on $(0.12\ \text{fm}, 0.2m_s)$.
- Effective field theories for combining data:
  - Symanzik effective field theory, a^2;
  - chiral perturbation theory, m_l;
  - heavy-quark effective theory, m_b, m_c.



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variations of χPT (NNLO vs. NLO), changing data sets

variations in discretization effects $\alpha_s m_b a$, $(m_s a)^2$, …

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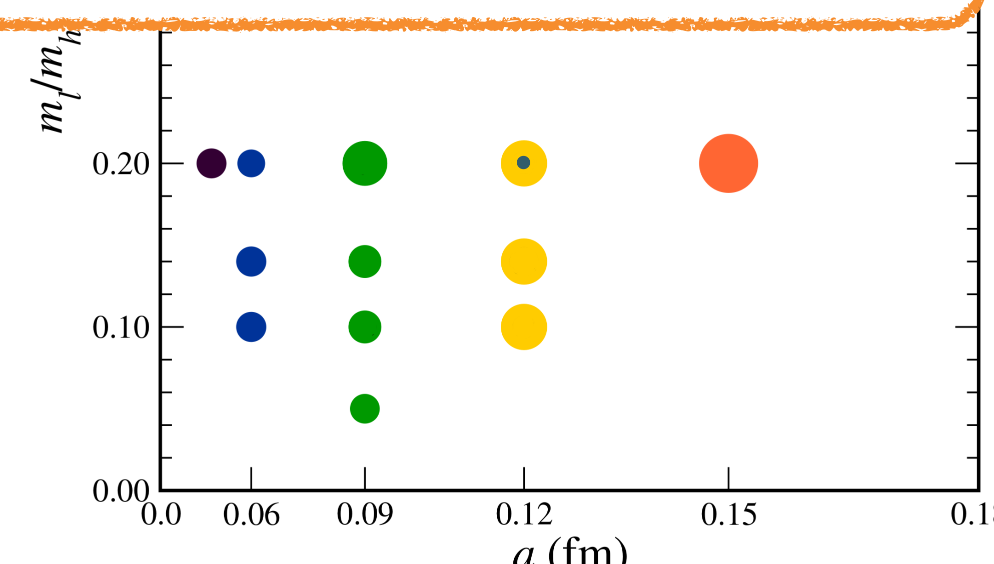

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- Propagate errors in the study of m_b, m_c dependence.

- Actually, all sources of uncertainty incorporated into the EFT fit: reflects growth in statistical uncertainties from interpolation and extrapolation.

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- Then undertake a subtraction in quadrature exercise.

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- Repeat with extrapolated $m_l \to (m_u+m_d)/2$ shifted by $\pm 1\sigma$.

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- Convert from lattice units to “r1” units and then to MeV.

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- Repeat extrapolating m_l to m_d or m_u.

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- Finite-volume χPT results no different than infinite volume.

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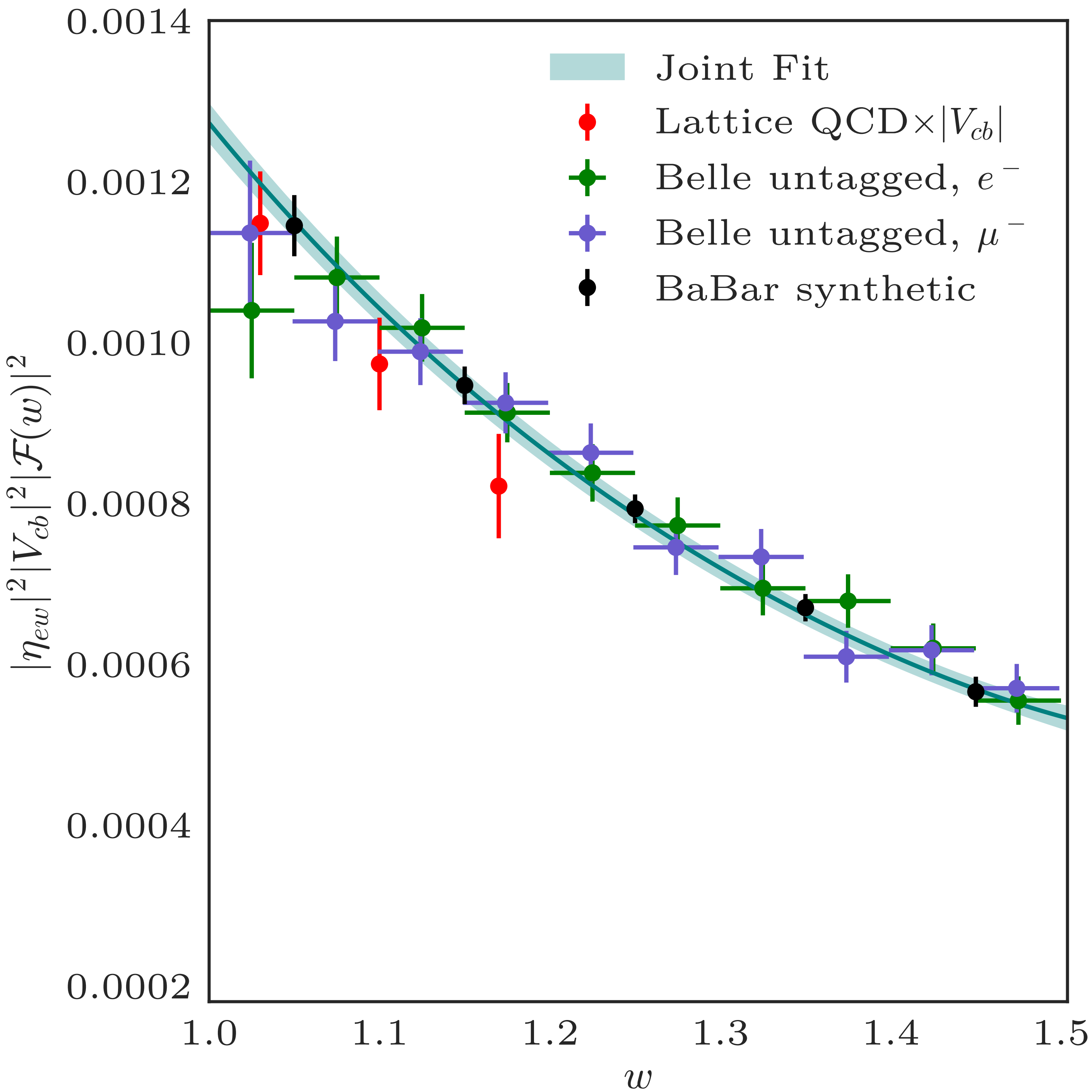
- Quadrature sum.

Blinding

- B-physics experimenters very often "blind" their results:
  - insert unknown but removable offsets into raw data;
  - remove blinding when systematic error analysis is complete.
- Here, multiply all matching factors by a common factor $\beta \approx 1$:
  - run analysis with $Z_A \to \beta Z_A$; then restore Z_A and re-run.
- In my opinion, highly advisable for both normalization (g_A) and slope (r_A) of nucleon form factors:
  - try $Z_A \to \beta_A Z_A$ and $r_1/a \to \beta_r r_1/a$.

z Expansion

- Extend to full kinematic range with z expansion (the one you know about).
- Fit lattice QCD.
- Fit experiment:
  - quadruply differential cross section $d^4\sigma/dw\,dc_D\,dc_l\,d\chi$.
- Fit both with floating relative normalization: $|V_{cb}|$.



Nucleons are Harder

Signal-to-Noise Ratio

- General: if signal is $\langle O_i(t) O_j^\dagger(0) \rangle$ then noise is $\langle O_i O_i^\dagger(t)\, O_j^\dagger O_j(0) \rangle$, and $O_j^\dagger O_j$ always creates n pions (O_j is n-quark operator).
- Consider a pion: $\dfrac{e^{-M_\pi t}}{e^{-M_\pi t}} \sim 1$
- Consider the vector channel (w/ rho resonance) $\sim \dfrac{e^{-M_\rho t}}{e^{-M_\pi t}} = e^{-(M_\rho - M_\pi)t}$
- Consider nucleon: $\sim \dfrac{e^{-M_N t}}{e^{-\frac{3}{2} M_\pi t}} = e^{-(M_N - \frac{3}{2} M_\pi)t}$
- Nucleus: $\sim e^{-(M_A - \frac{3}{2} A M_\pi)t}$
- A sign problem: pion correlation function is a sum (over spin) of squares; all other particles have terms of various signs/phases.

Way Out

- Extract masses (and matrix elements) and pre-asymptotic times (see blackboard).
- Then excited states contribute—
  - also worse for nucleons than mesons, because, e.g., $N\pi$ has the same quantum numbers as N.
- Matrix correlation functions + variational calculation:
  - find linear combos of operators orthogonal to first few excited states;
  - "generalized eigenvalue problem", "multi-state fitting", ….

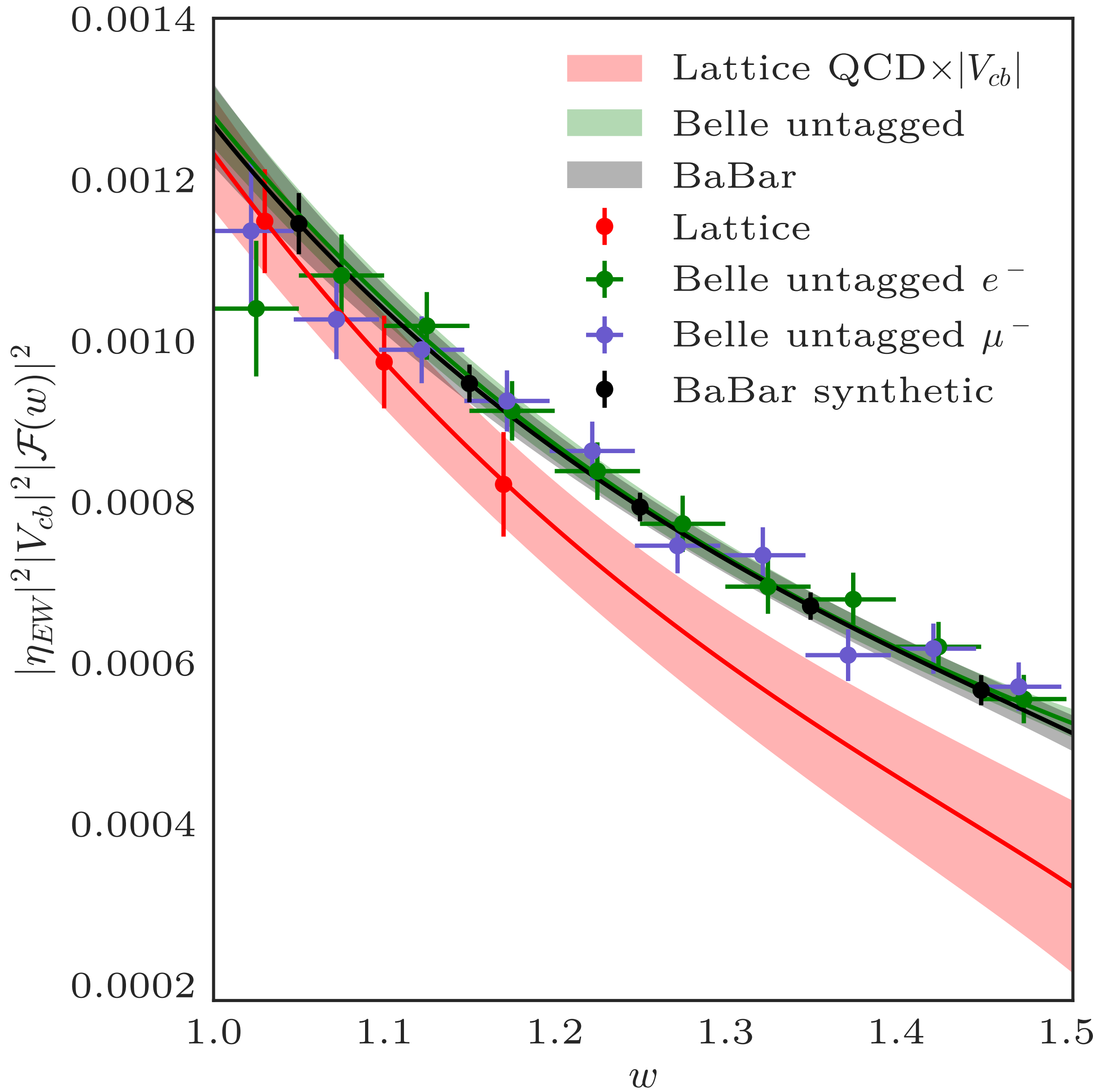
Outlook

What, When?

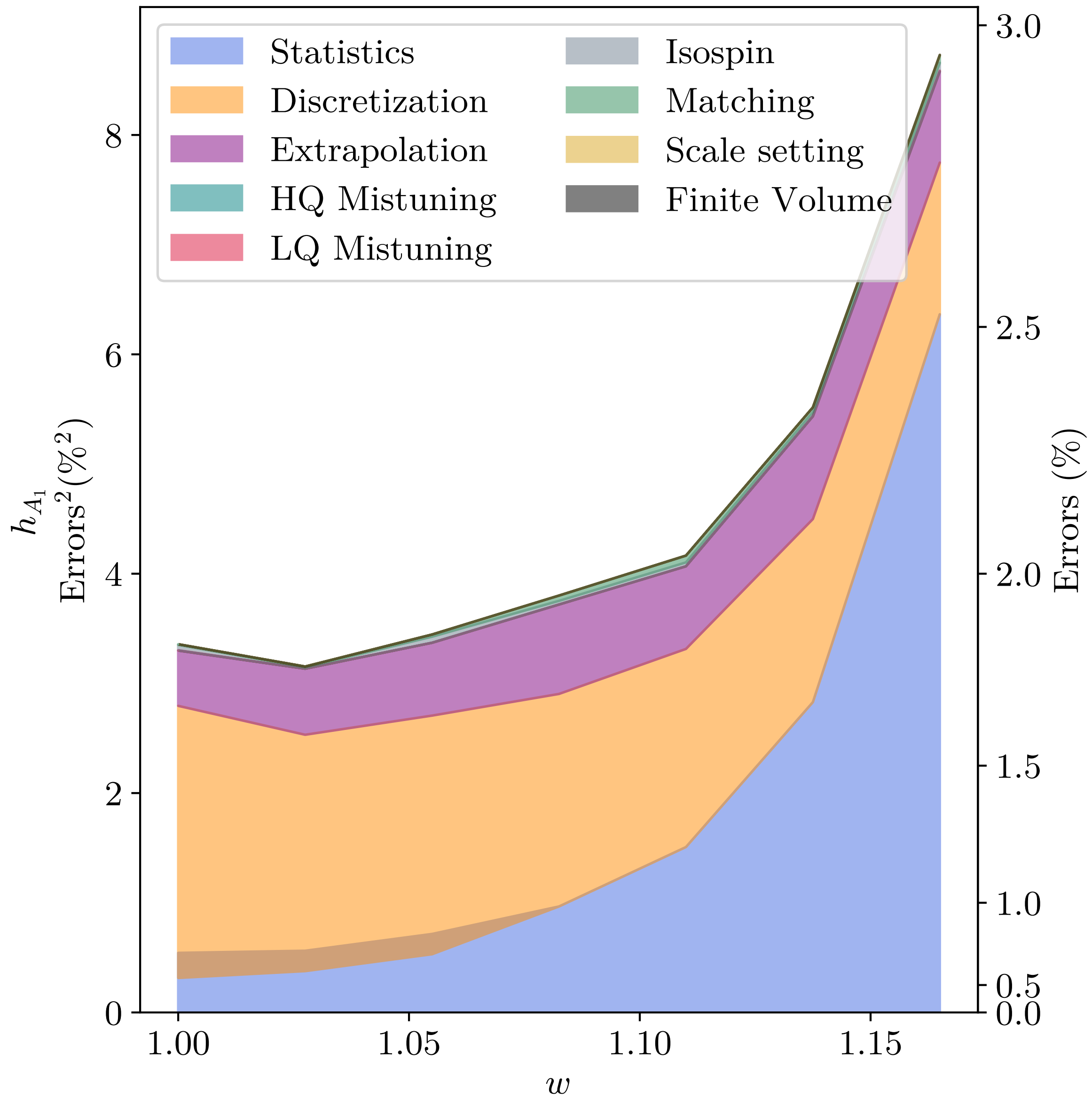
- Axial form factor with all systematics controlled in ~now (NME).
- PDFs with all systematics controlled (*e.g.*, physical pions) ~few years.
- Resonant properties & hadron tensor: “first” calculations ~few years; full error budgets perhaps ~decade.
- From a QCD perspective, nuclei are hadrons too:
  - perhaps ^{12}C and ^{16}O are possible on DUNE/HyperK timescale 🤔
- Tasks for a quantum computer: semi-exclusive final states; ^{40}Ar.
- Ongoing: turn “lattice QCD + nuclear MBT” from an idea into a formalism.

Questions?

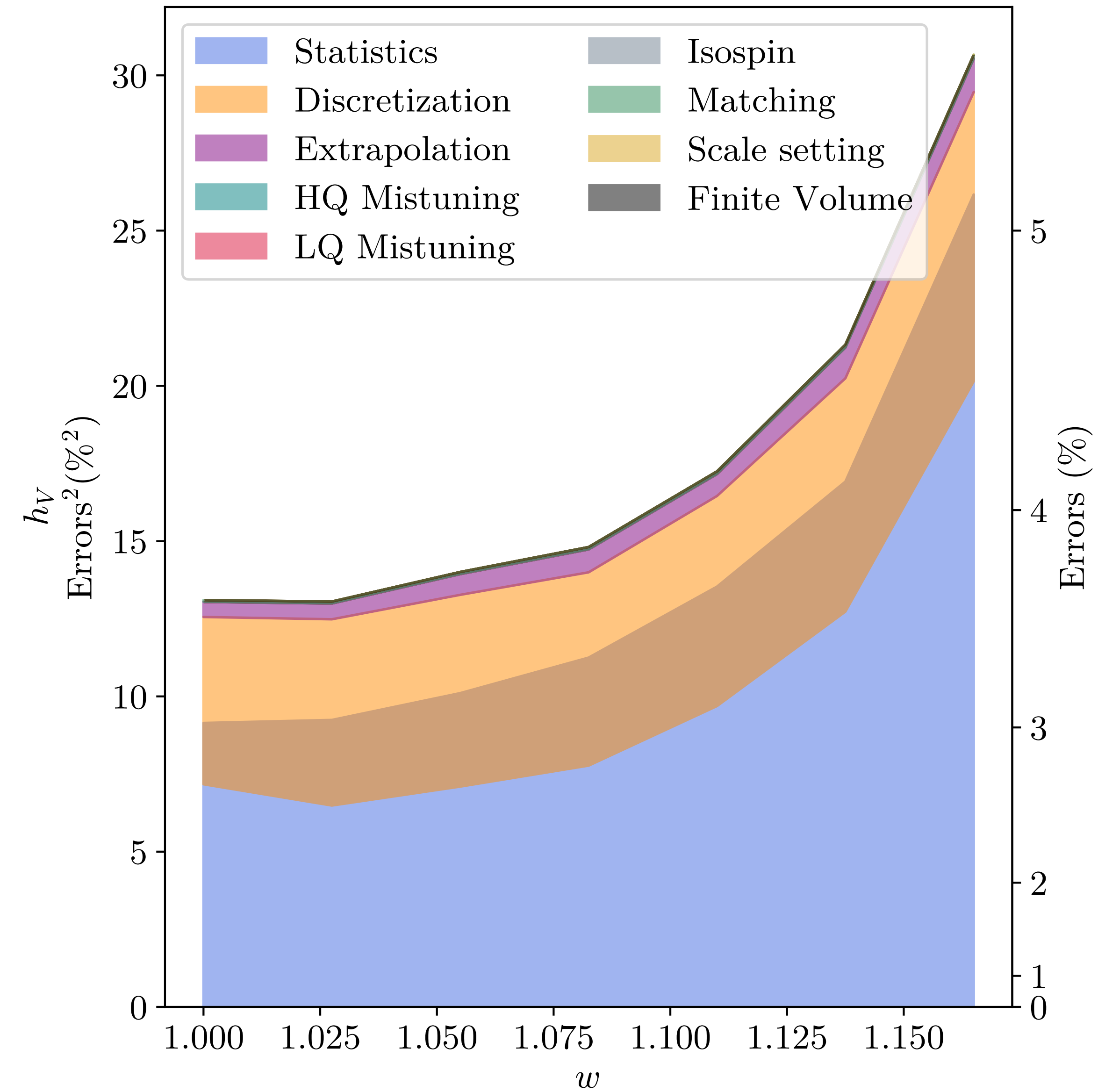
Backup



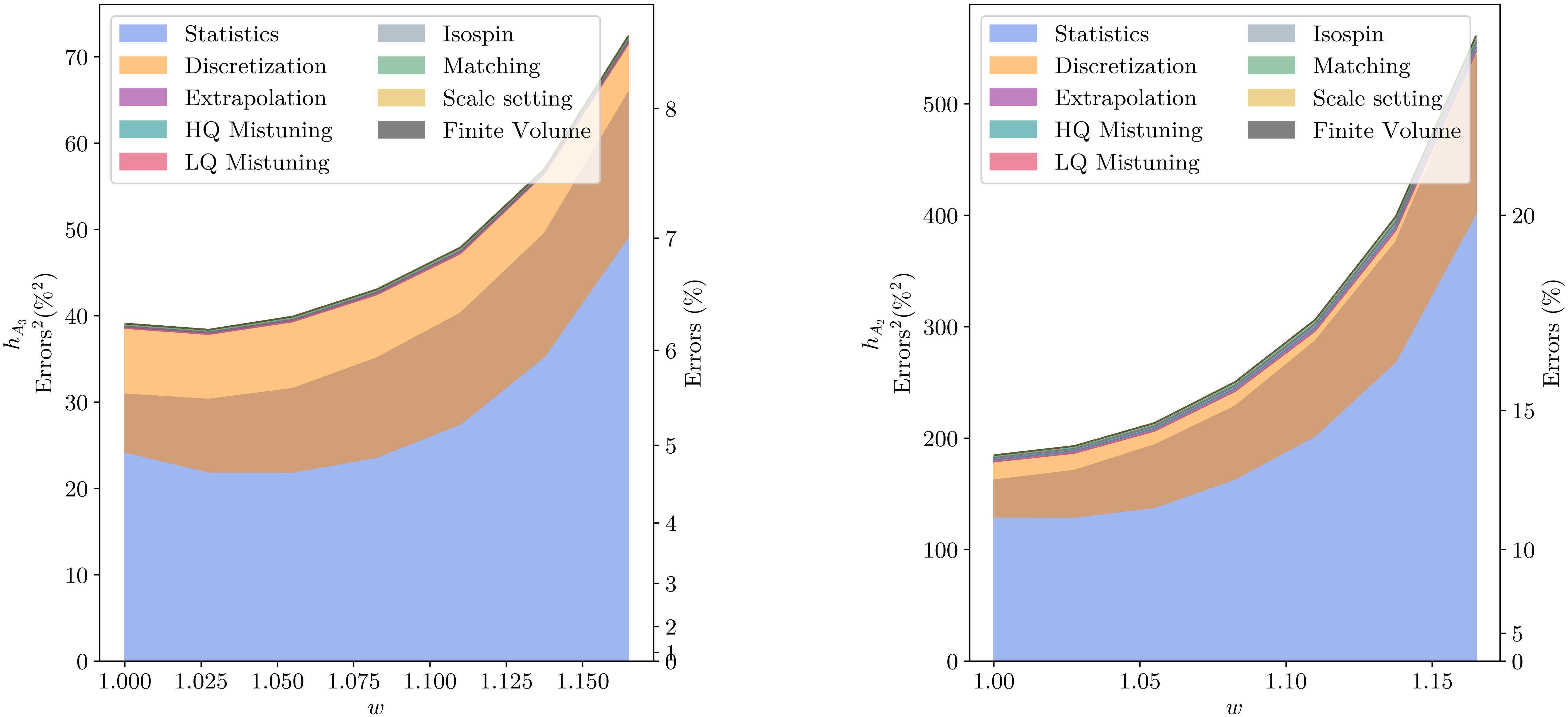
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
$|\eta_{EW}|^2|V_{cb}|^2|\mathcal{F}(w)|^2$
1.0
1.1
1.2
1.3
1.4
1.5
w
Lattice QCD×$|V_{cb}|$
Belle untagged
BaBar
Lattice
Belle untagged e^-
Belle untagged μ^-
BaBar synthetic



Statistics
Discretization
Extrapolation
HQ Mistuning
LQ Mistuning
Isospin
Matching
Scale setting
Finite Volume
h_{A_1}
Errors^2(%^{2})
Errors (%)
w



Statistics
Discretization
Extrapolation
HQ Mistuning
LQ Mistuning
Isospin
Matching
Scale setting
Finite Volume
h_V
Errors^2(%^{2})
Errors (%)
w


Statistics
Discretization
Extrapolation
HQ Mistuning
LQ Mistuning
Isospin
Matching
Scale setting
Finite Volume
h_{A_3}
Errors^2 (%^{2})
Errors (%)
w
h_{A_2}
Errors^2 (%^{2})
Errors (%)
w