Neutrino-Nucleon Scattering from Hadronic Tensor with Lattice QCD

- Neutrino-nucleon scattering via Euclidean hadronic tensor
- Inverse problem algorithms
- Elastic scattering and form factors

Neutrino Workshop, KITP Mar. 3, 2022

Hadronic tensor and neutrino-nucleus scattering

- ✦ New long-baseline neutrino experiments are in preparation: T2K, NOvA, PINGU, ORCA, Hyper-Kamiokande, DUNE...
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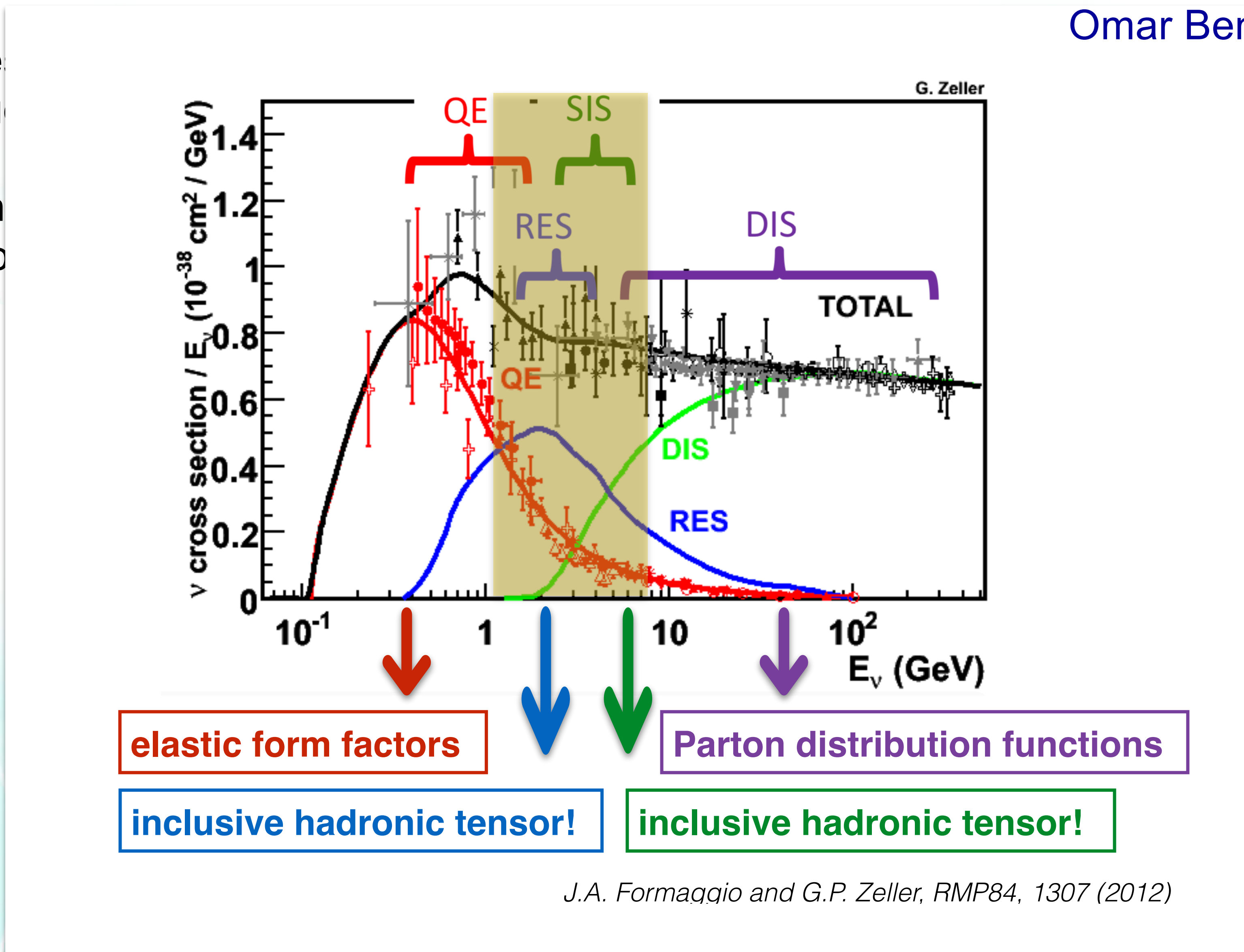


Whitepapers on Neutrino-Nucleus Scattering

- USQCD – A. Kronfeld et al. (Kronfeld:2019nfb)

- Snowmass – M. Wagman et al. (under preparation)

Hadronic Tensor in Euclidean Path-Integral Formalism

- Lepton Nucleon scattering
  In Minkowski space

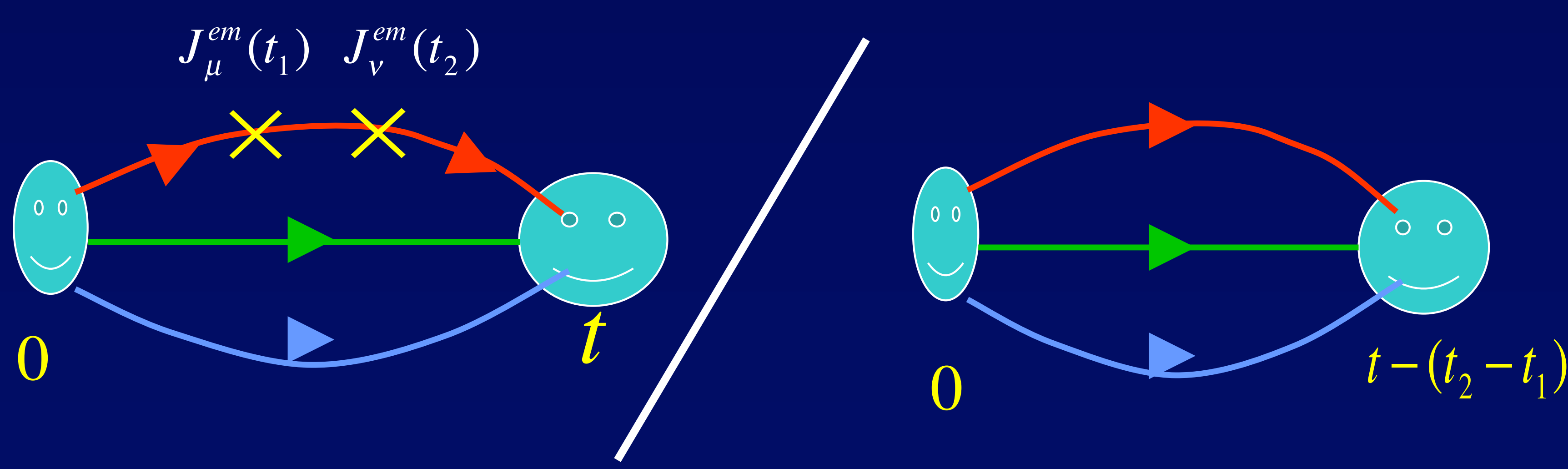
$$\frac{d^2\sigma}{dE' d\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu} = \left\langle N(\vec{p})\right| \int \frac{d^4x}{4\pi}\, e^{iq\cdot x} J_\mu(x) J_\nu(0) \left| N(\vec{p})\right\rangle_{\text{spin avg}}$$

$$= \frac{1}{2}\sum_n \int \prod_{i=1}^{n}\left[\frac{d^3 p_i}{(2\pi)^3 2E_{pi}}\right](2\pi)^3 \delta^4(p_n - p - q) < N(\vec{p}) | J_\mu | n >< n | J_\nu | N(\vec{p}) >_{\text{spin avg}}$$

- Euclidean path-integral

KFL and S.J. Dong, PRL 72, 1790 (1994)
KFL, PRD 62, 074501 (2000)



$W_{\mu\nu}$ in Euclidean Space

$$\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau = t_2 - t_1) = \frac{\frac{E_P}{M_N}\mathrm{Tr} < \Gamma_e \chi_N(\vec{p},t)\sum_{\vec{x}} \frac{1}{4\pi}\; e^{-i\vec{q}\cdot\vec{x}} J_\mu(\vec{x},t_2) J_\nu(0,t_1)\chi_N^\dagger(\vec{p},0) >}{\mathrm{Tr} < \Gamma_e \chi_N(\vec{p},t)\chi_N^\dagger(\vec{p},0) >}$$

$$\xrightarrow[t-t_2>>1/\Delta E_{P,}\; t_1>>1/\Delta E_P]{}$$

$$= \frac{1}{4\pi}\sum_n \left(\frac{2m_N}{2E_n}\right)\; \delta_{\vec{p}_n-\vec{p}-\vec{q}} \;\; < N(\vec{p})\,|\, J_\mu \,|\, n >< n \,|\, J_\nu \;|\, N(\vec{p}) >_{\text{spin avg}} \;\; e^{-(E_n - E_P)\tau}$$

$$= \;\; < N(\vec{p})\,|\sum_{\vec{x}} \frac{e^{-i\vec{q}\cdot\vec{x}}}{4\pi} J_\mu(\vec{x},\tau) J_\nu(0,0)\,|\, N(\vec{p}) >_{\text{spin avg}}$$

Inverse Laplace transform – formally correct but not practical

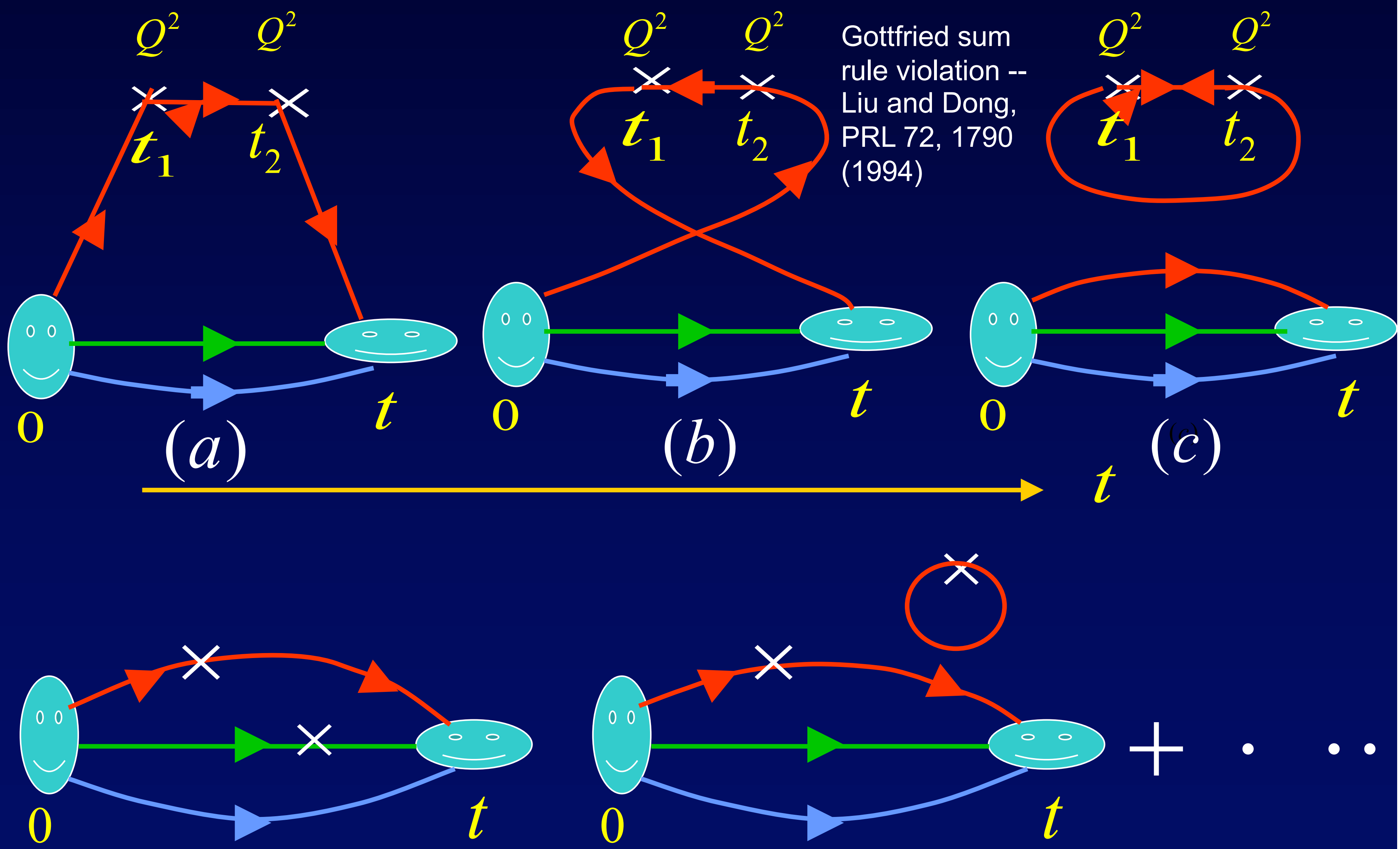
$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau\; e^{\nu\tau}\;\; \tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$$

Hadronic Tensor

$q = q_V + q_{CS}$

Connected sea $\bar{q}_{CS}$

Disconnercted sea $q_{DS} = (\neq ?)\ \bar{q}_{DS}$



Cat's ears diagrams are suppressed by O(1/Q²).

$q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$

Kinematics

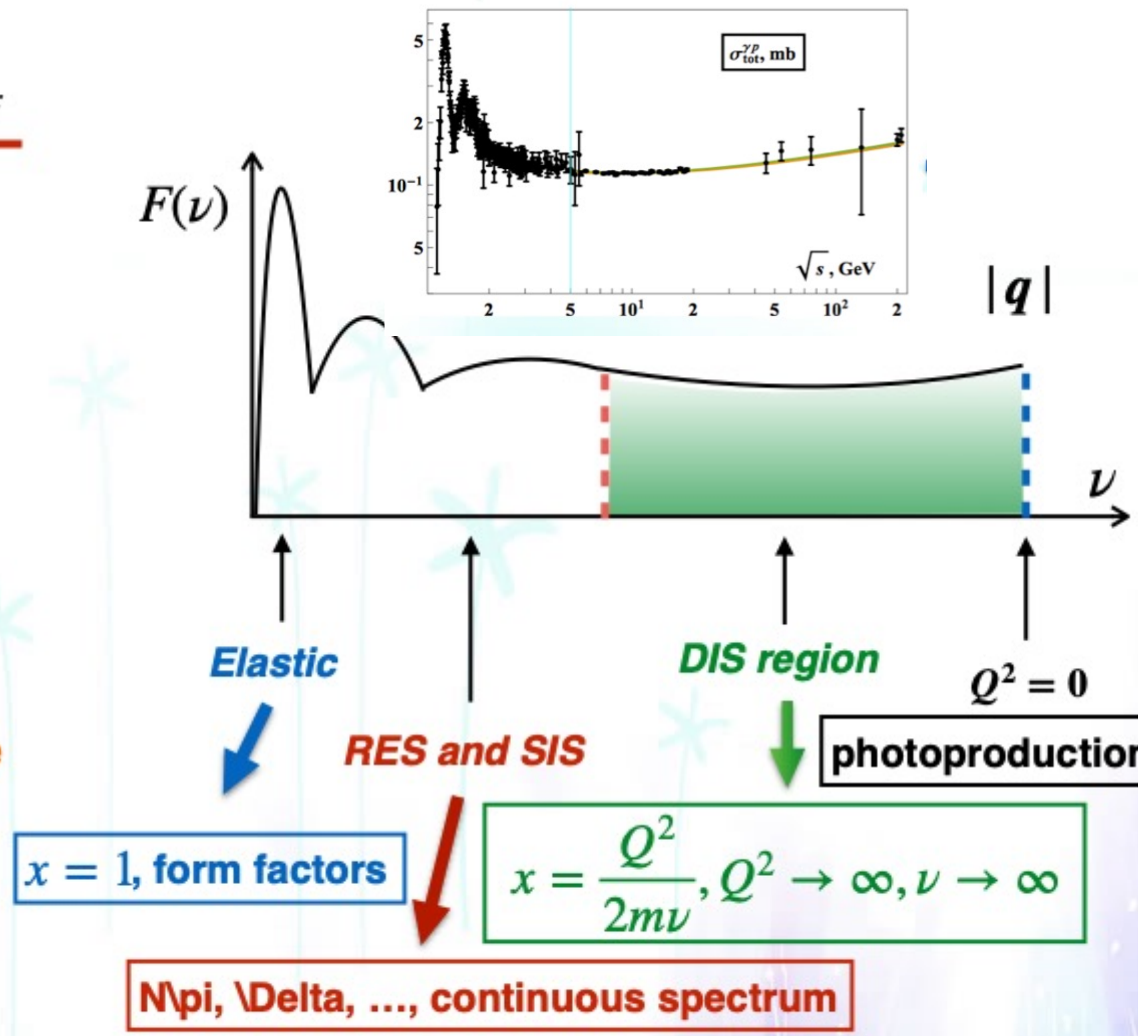
$$\tilde{W}_{\mu\nu}(p,\vec{q},\tau) = \frac{E_p}{m_N}\frac{\mathrm{Tr}[\Gamma_e C_4]}{\mathrm{Tr}[\Gamma_e C_2]} \rightarrow \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s\,|\,J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)\,|\,p,s\rangle$$

$$\tilde{W}_{\mu\nu}\left(p,\vec{q},\tau\right) = \int d\nu W_{\mu\nu}\left(p,\vec{q},\nu\right)e^{-\nu\tau}$$

$$Q^2 = -q^2 = |\vec{q}|^2 - \nu^2$$

$$x = \frac{Q^2}{2m\nu}$$

$$W^2 = m^2 + 2m\nu - Q^2$$



1. Both Q^2 and ν need to be large (difficulty?)

2. Will have a range of ν (feature?)

x

Inverse problems are ubiquitous

- Extracting spectral functions from lattice data: $c_2(t) = \int d\omega e^{-\omega t} \boldsymbol{\rho(\omega)}$

Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

- Global fittings of PDFs: $F_i = \sum_a C_i^a \otimes \boldsymbol{f_a}$

- Lattice calculation of Quasi-PDFs: $\tilde{q}(x, P_3) = \frac{2P_3}{4\pi} \sum_{z=-z_{\max}}^{z_{\max}} e^{-ixP_3 z} h_\Gamma(P_3, z)$

J. Karpie et. al., JHEP04, 057 (2019)

$$\tilde{q}(x, \mu^2, P^z) = \int_0^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right)$$

X. Xiong et. al., PRD90:014051 (2014)

- Lattice cross sections:

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^1 \frac{dx}{x} f_a(x, \mu^2) \times K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + O(\xi^2 \Lambda_{\mathrm{QCD}}^2)$$

Y.-Q. Ma and J.-W. Qiu, PRL120, 022003 (2018)

- Lattice calculation of Pseudo-PDFs: $\mathfrak{M}_R(\nu, \mu^2) \equiv \int_0^1 dx \, \cos(\nu x) \, q_v(x, \mu^2)$

K. Orginos et al., PRD96, 094503 (2017)

Visualize 2-point Functions

- ✦ **Backus-Gilbert (BG)** *G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)*
- ✦ **Maximum Entropy (ME)** *E Rietsch et. al., JOURNAL OF GEOPHYSICS, 42:489 (1977)* *M. Asakawa et. al., Prog. Part. Nucl. Phys. 46, 459 (2001)*
- ✦ **Bayesian Reconstruction (BR)** *Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)*

$$P[\omega\,|\,D,\alpha,m] \propto e^{Q'(\omega)} \qquad Q' = \alpha S - L - \gamma(L - N_\tau)^2$$

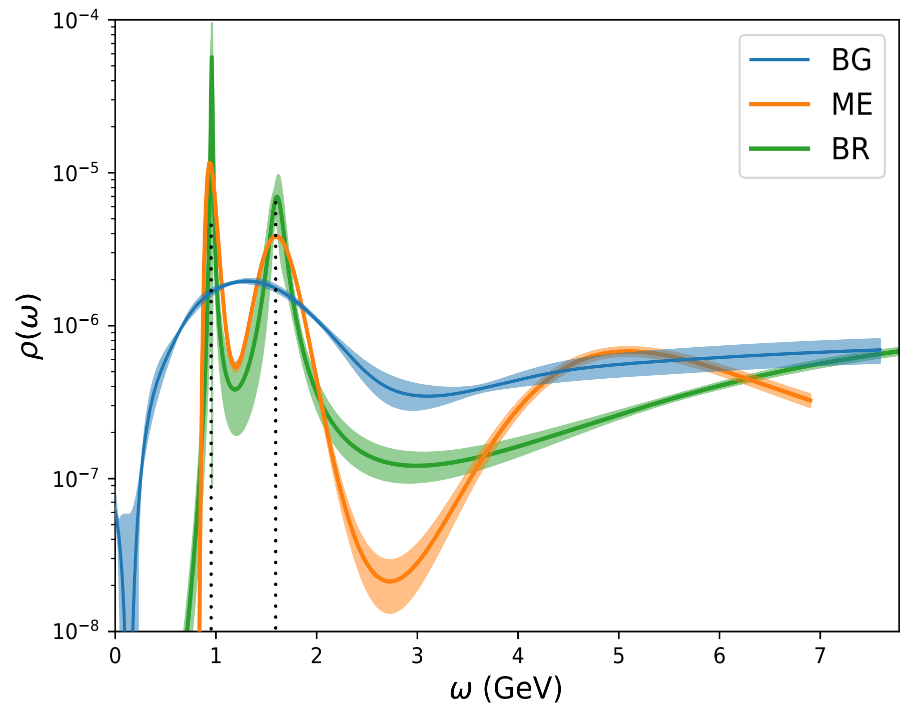
$$S = \sum_\nu \left[1 - \frac{\omega(\nu)}{m(\nu)} + \log\left(\frac{\omega(\nu)}{m(\nu)}\right)\right]\Delta\nu$$

$$P[\omega\,|\,D,m] = \frac{P[D\,|\,\omega,I]}{P[D\,|\,m]}\int d\alpha P[\alpha\,|\,D,m]$$

Hyper parameter alpha is integrated over

Maximum search is in the entire parameter space ($O(10^3)$)

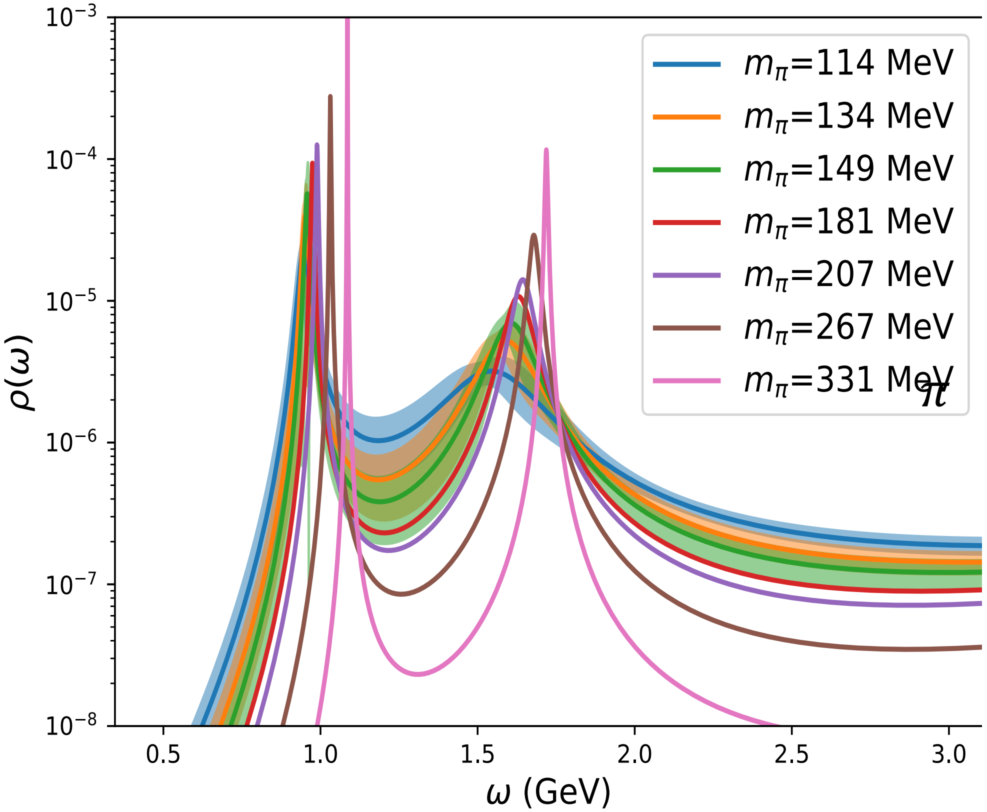
High precision architecture is needed (e.g., 512-bit floating point number).

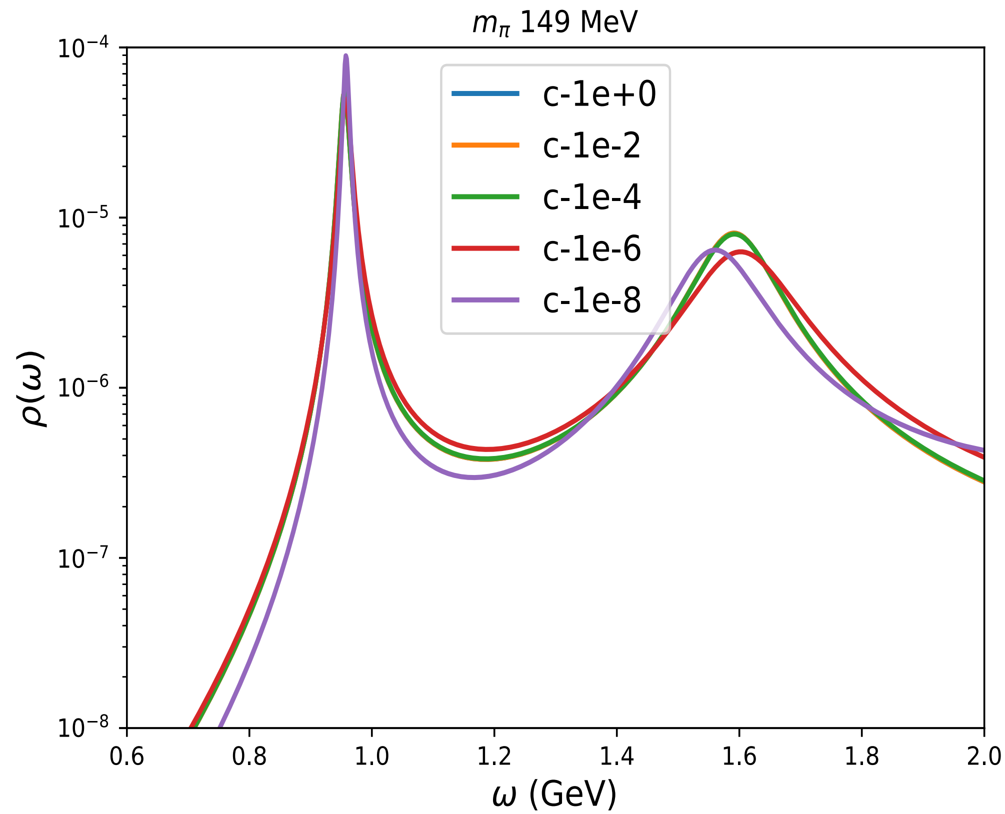


RBC/UKQCD 48I, physical pion mass, a~0.11 fm

Overlap fermion on 2+1 flavor DWF, $48^3 \times 96$, m_π = 139 MeV, L = 5.5 fm

Visualize 2-point Functions



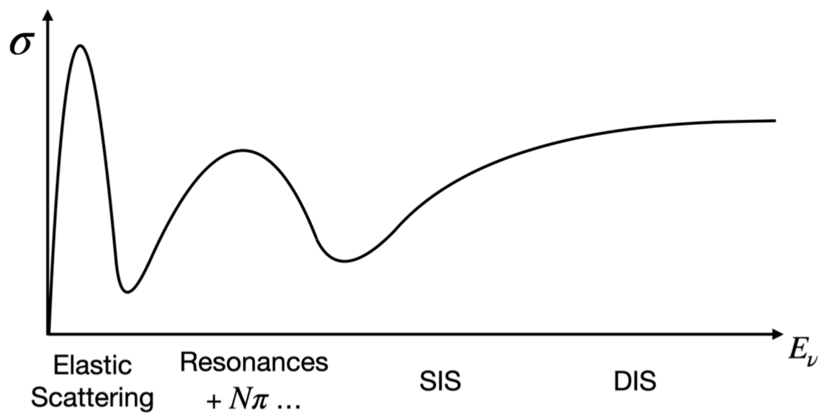


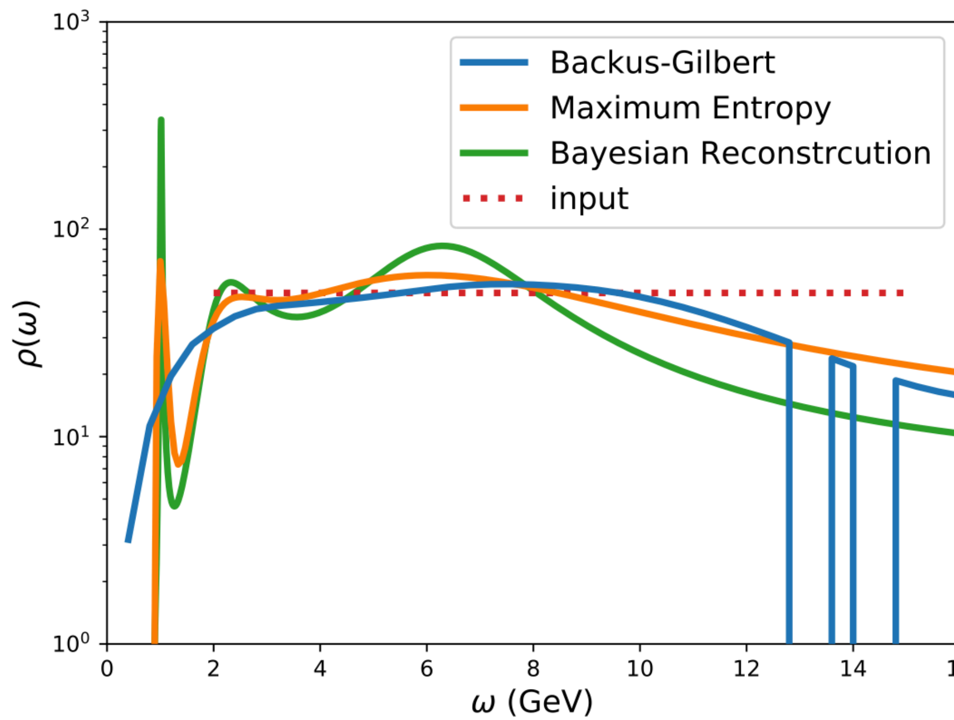
RBC/UKQCD 48I, physical pion mass, a~0.11 fm

Smeared grid-source, using t=[1,15], c=1e-6

Overlap fermion on 2+1 flavor DWF,
48^3 x 96, m_π = 139 MeV, L = 5.5 fm

Mock Data Test





Mock data contain two **isolated states of mass 1.0 and 1.5 GeV,** and **a constant dense spectrum (simulating the continuous spectrum) from 2 GeV to 15 GeV**. The lattice spacing is set to be 0.02 fm and the number of time slices is 100. The signal-to-noise ratio is set to be 100.

J. Liang *et. al., Phys.Rev.D 101 (2020) 11, 114503*

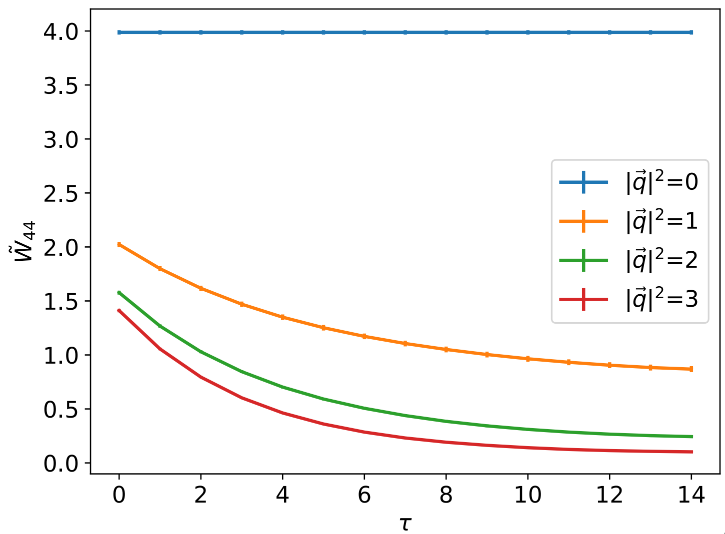
J. Liang and KFL
(Liang:2020sqi)

Form Factor Case for Elastic Scattering

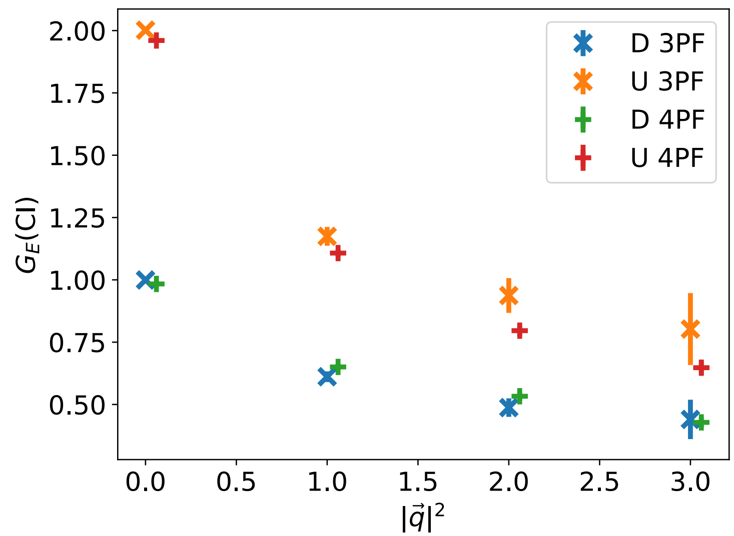
$$\widetilde{W}_{44}(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s\,|\,J_4(\vec{x}_2,t_2)J_4(\vec{x}_1,t_1)\,|\,p,s\rangle = \sum_n A_n e^{-\nu_n\tau}$$

$$A_0 = \langle p,s\,|\,J_4(\vec{q})\,|\,n=0\rangle\langle n=0\,|\,J_4(-\vec{q})\,|\,p,s\rangle = G_E^2(Q^2)$$

RBC/UKQCD 32IF lattice (a ~ 0.063 fm, pion mass ~ 370 MeV), clover on domain wall



u quark



***J. Liang** and K.-F. Liu PoS LATTICE2019 (2020) 046*

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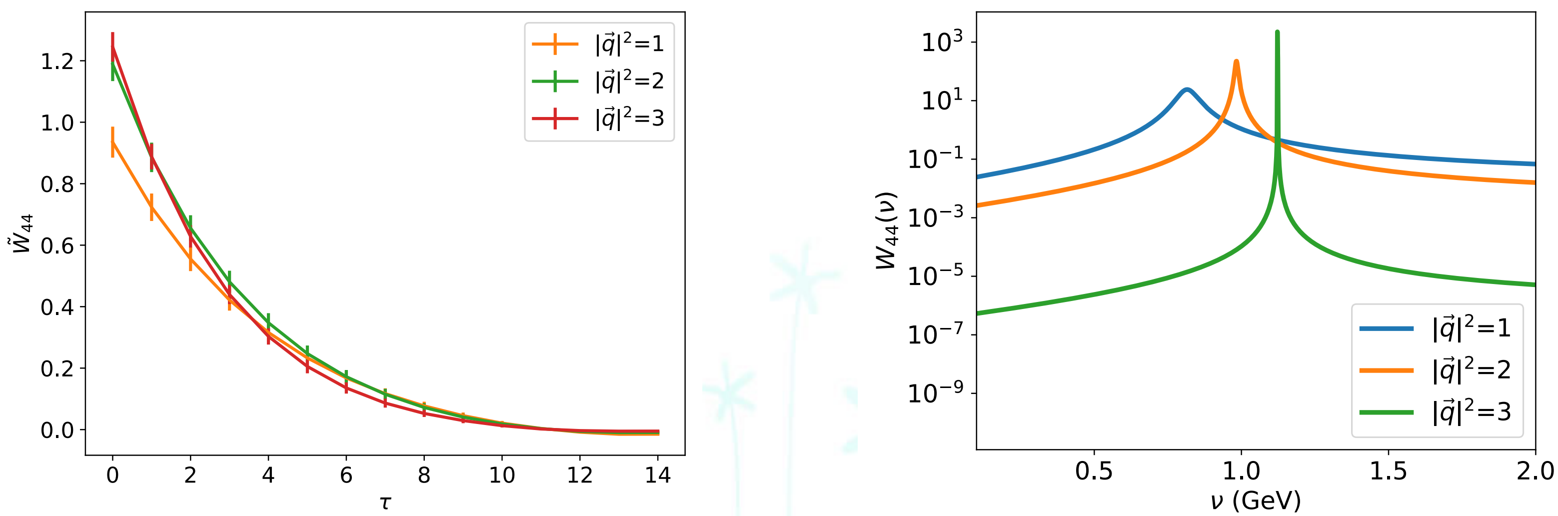
Beyond the Elastic Contribution



u quark with elastic contribution subtracted

Non-zero inelastic contribution

Need more data with finer lattice spacings and better understanding of the reconstruction

Compare different inverse methods

Phenomenological input

$$W^2 = M_N^2 + 2M_N\nu - Q^2$$

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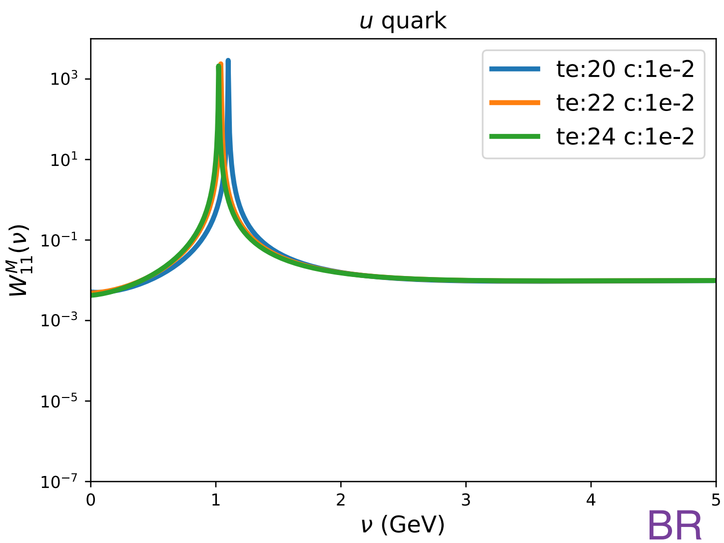
J. Liang et al.
(Liang:2019frk)

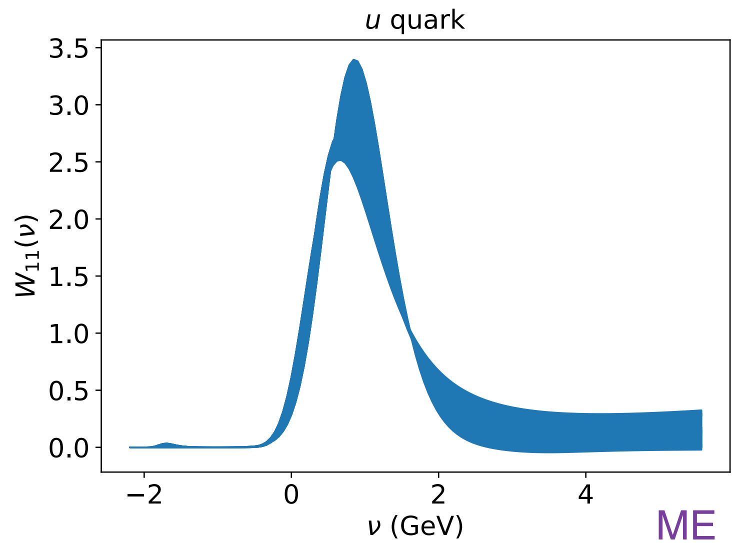
Large Momentum Transfer Case

$\mu = \nu = 1$ and $p_1 = q_1 = 0 \quad W_{11}(\nu) = F_1(x, Q^2)$

$24^3 \times 128,\ a_s \sim 0.12$ fm, $\xi \sim 3.5,\ m_\pi \sim 380$ MeV, $\frac{2\pi}{L} \sim 0.42$ GeV

$\boldsymbol{p}$	$\boldsymbol{q}$	E_p	$E_{n=0}$	$\lvert \boldsymbol{q} \rvert$	ν	Q^2	x
(0,3,3)	(0,-6,-6)	2.15	2.15	3.57	[2.96, 3.28]	[4, 2]	[0.16, 0.07]





J. Liang et. al., Phys.Rev.D 101 (2020) 11, 114503

Fine lattice spacings!

Better solving the inverse problem!

Q^2 = 11.7 GeV^2

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$$W^2 = (p+q)^2 = M_N^2 - Q^2 + 2E_p\nu - 2\vec{p}\cdot\vec{q}$$

W = 2.5 GeV

$$|\vec{p}+\vec{q}| = 1.8\ \text{GeV}$$

Other approaches

- OPE with heavy quark -- W. Detmold and D. Lin (Detmold:2005gg)
- OPE with Compton Amplitude – A.J. Chambers et al. (Chambers:2017dov)
- Modified Barkus-Gilbert -- M. Hansen et al. (Hansen:2018ght)
- Chebyshev approximation – H. Fukaya et al. (Fukaya:2020wpp)

Summary and Challenges

- How high an excitation energy can be reached on today's lattice?

$$W^2 = (p+q)^2 = M_N^2 - Q^2 + 2E_p\nu - 2\vec{p}\cdot\vec{q}$$

$$\mathrm{Take}\ |\tilde{\mathrm{p}}| = 1\mathrm{GeV}, |\tilde{\mathrm{q}}| = 3\mathrm{GeV}, \nu = 2.5\mathrm{GeV} \longrightarrow \mathrm{Q}^2 = 2.75\mathrm{GeV}^2, \mathrm{W} = 3.35\mathrm{GeV}$$

- DIS is perhaps reachable on lattices with a = 0.06 fm or less, but will need multi-hadron interpolation field (?)

- Spectral decompositions in lattice 2-, 3-, and 4-point functions of baryons have seen limited success. Need better data and robust inverse algorithm.

- Spectral decomposition of 2- and 3-pt functions can be improved with a larger basis of interpolation fields in a variational approach. How about 4-pt functions?