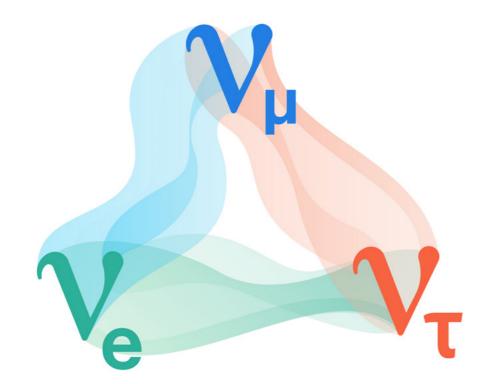
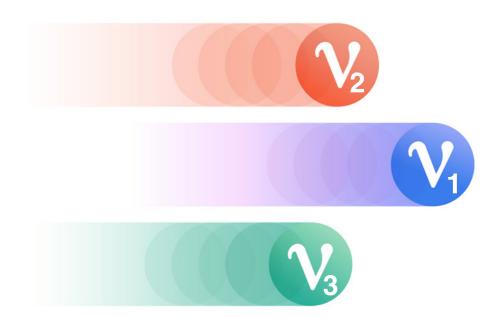
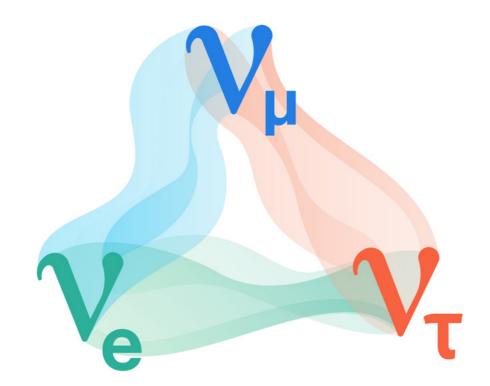
History and present state of neutrino oscillation measurements



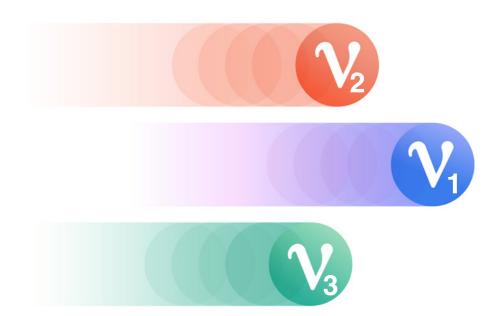
Stephen Parke Theory



History and present state of neutrino oscillation measurements



Stephen Parke Theory



NOBEL 2015

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"





"for the discovery of neutrino flavor transformations, which shows that neutrinos have mass"

39.3 m

See Smirnov arXiv:1609.02386

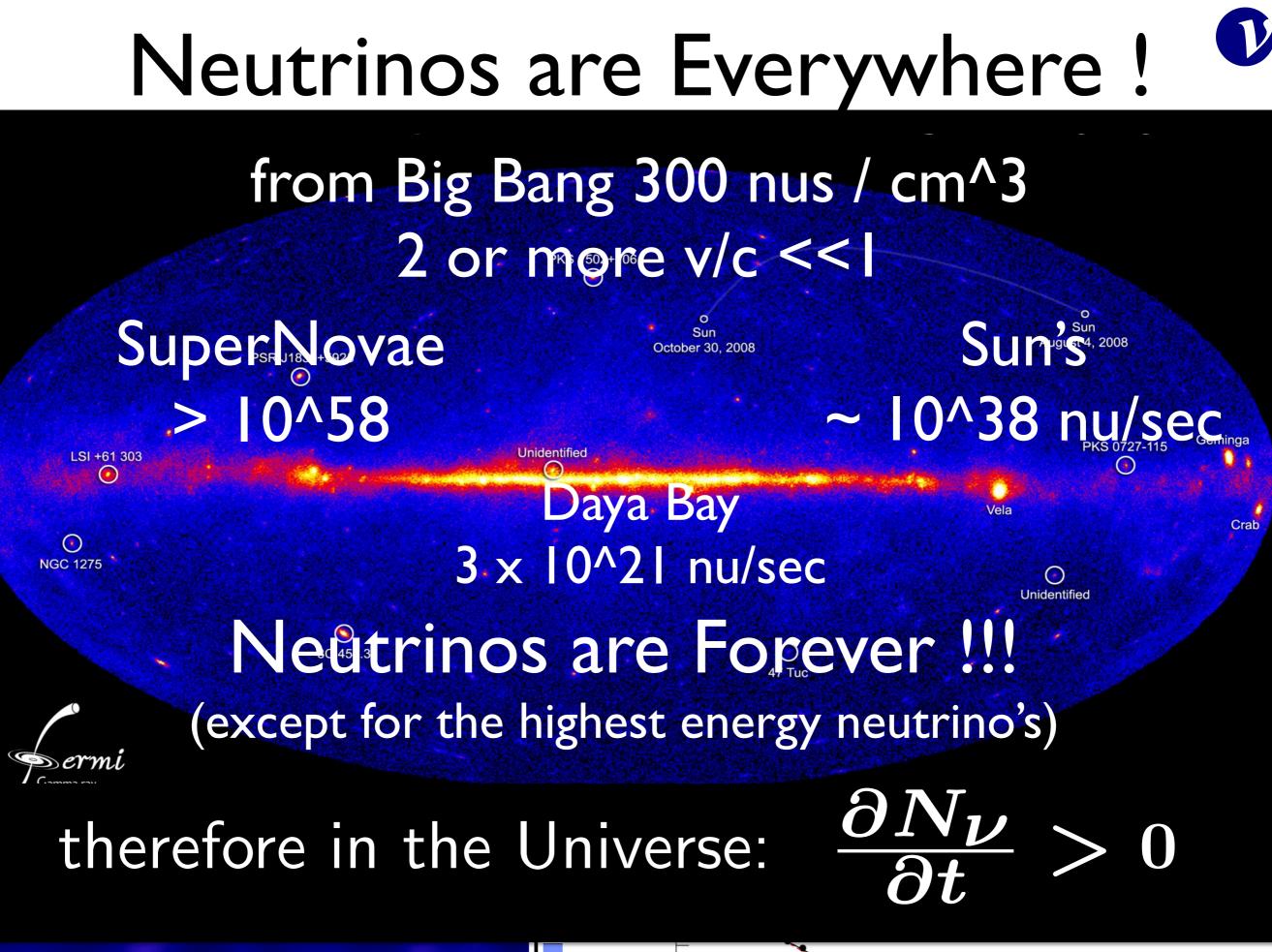
~ vacuum oscillations

Wolfenstein Matter effects dominant flavor transformations

Stephen Parke

Nu22-KITP

3



 $N - 10^{-11}$

— Fit

num = PieChart3D[{157, 353, 490}, ChartStyle → {Cyan}, PlotTheme → "Business", SectorOrigin → {{(-Pi/2+0.15), "Clockwise"}, 0}]

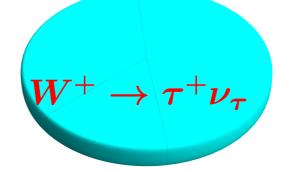
nut = PieChart3D[{157, 353, 490}, 4] massive_neutrinos.nb ChartSt A C, Ref Chilling C, Ref Chilling C, Barton States: Sector C ig C, Clart C, Cla

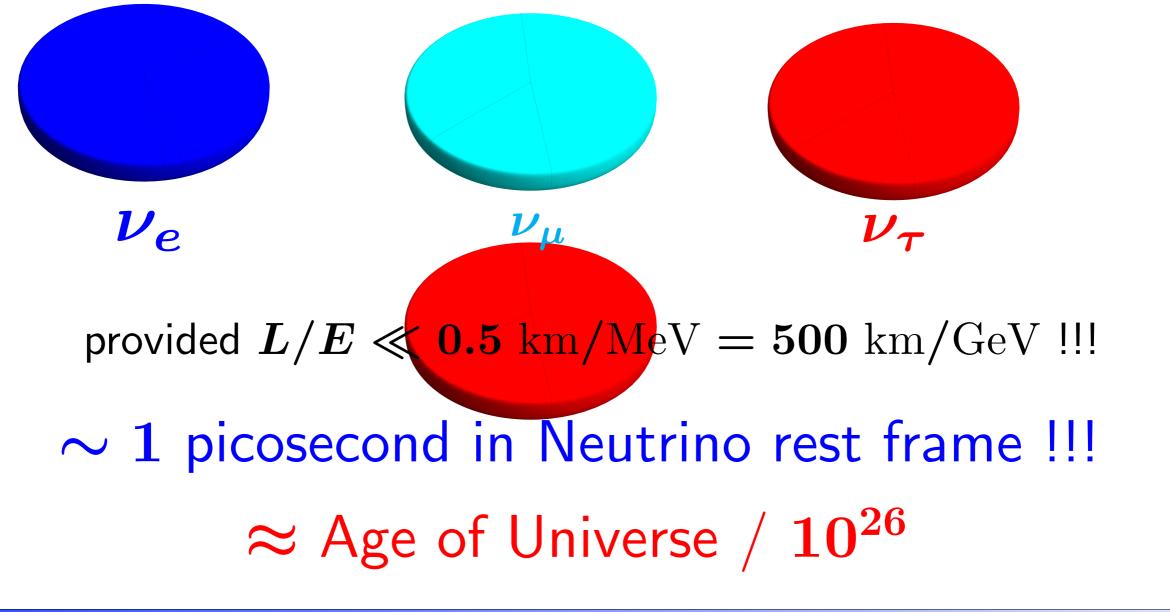
 $ightarrow \mu^+
u_{\mu}$

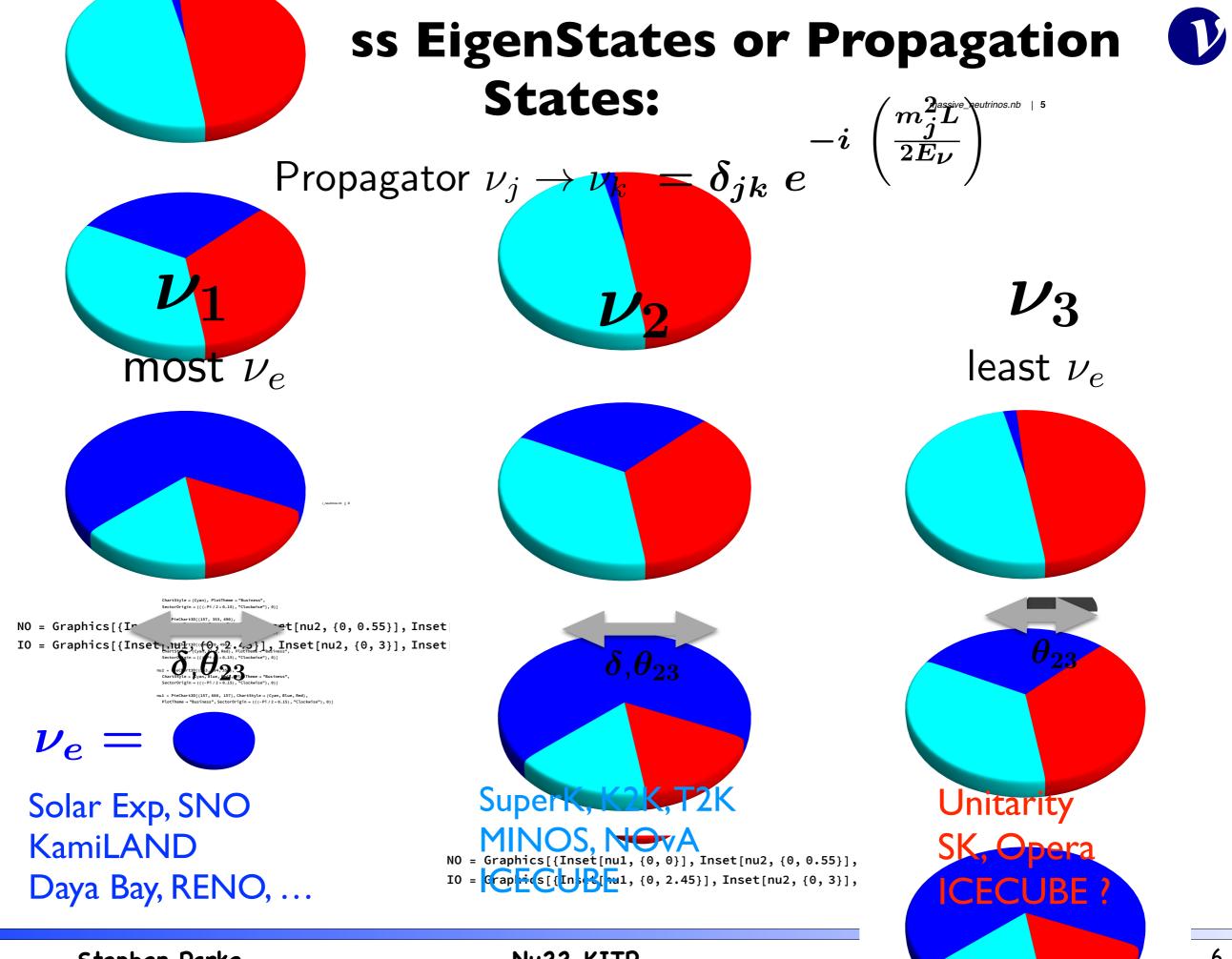
nu3 = PieChart3D[{490, 20, 490}, ChartStyle → {Cyan, Blue, Red}, PlotTheme → "Business", SectorOrigin → {{(-Pi/2+0.15), "Clockwise"}, 0}]

nu2 = PieChart3D[{353, 294, 353}, ChartStyl {Cyan, Blue, Red, PiotTheme → "Business", SectorOrigin → {{(-Pi / 2 + 0.15), "Clockwise"}, 0}]

nu1 = PieChart3D[{157, 686, 157}, ChartStyle \rightarrow {Cyan, Blue, Red}, PlotTheme \rightarrow "Business", SectorOrigin \rightarrow {{(-Pi / 2 + 0.15), "Clockwise"}, 0}]







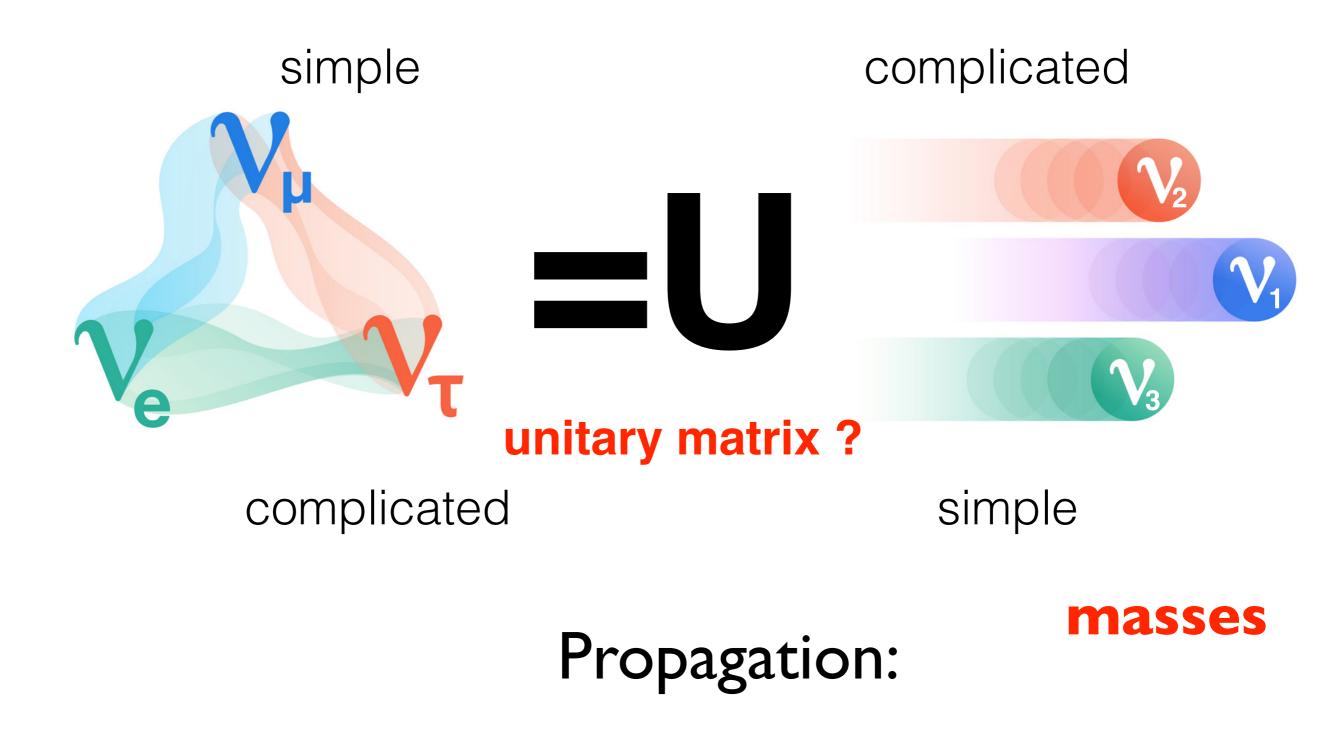
Stephen Parke

Nu22-KITP

6

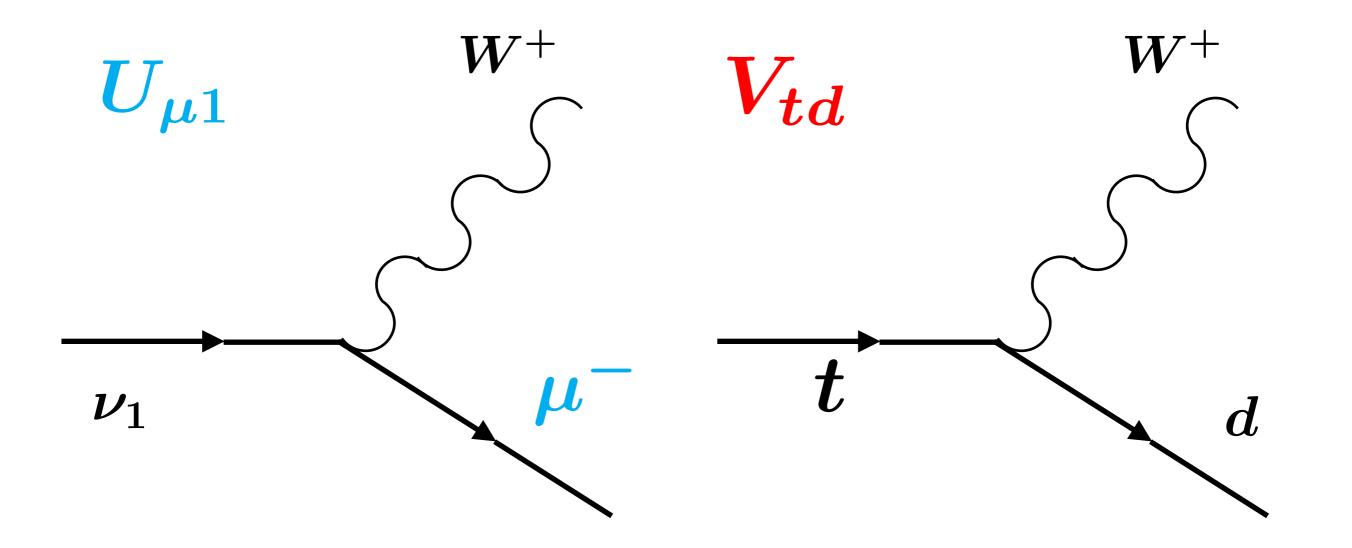


Interactions:



Nu22-KITP





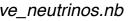
Rates: $|U_{\mu 1}|^2 \& |V_{td}|^2$

NuFIT 5.1 (2021)

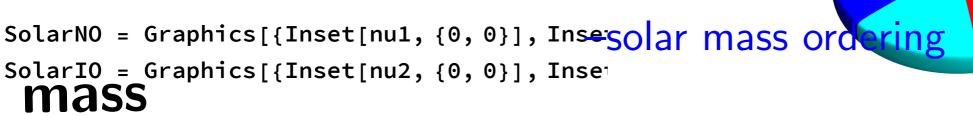
		Normal Ore	lering (best fit)	Inverted Ordering $(\Delta \chi^2 = 2.6)$			
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range		
Ē	$\sin^2 heta_{12}$	$0.304\substack{+0.013\\-0.012}$	$0.269 \rightarrow 0.343$	$0.304\substack{+0.012\\-0.012}$	$0.269 \rightarrow 0.343$		
data	$ heta_{12}/^{\circ}$	$33.44_{-0.74}^{+0.77}$	$31.27 \rightarrow 35.86$	$33.45_{-0.74}^{+0.77}$	$31.27 \rightarrow 35.87$		
heric	$\sin^2 heta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \rightarrow 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \rightarrow 0.623$		
osp	$ heta_{23}/^{\circ}$	$49.2^{+1.0}_{-1.3}$	$39.5 \rightarrow 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \rightarrow 52.1$		
t atn	$\sin^2 heta_{13}$	$0.02220\substack{+0.00068\\-0.00062}$	$0.02034 \rightarrow 0.02430$	$0.02238\substack{+0.00064\\-0.00062}$	$0.02053 \rightarrow 0.02434$		
t SK	$ heta_{13}/^{\circ}$	$8.57^{+0.13}_{-0.12}$	$8.20 \rightarrow 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \rightarrow 8.98$		
without SK atmospheric data	$\delta_{ m CP}/^{\circ}$	194^{+52}_{-25}	$105 \rightarrow 405$	287^{+27}_{-32}	$192 \rightarrow 361$		
W	$\frac{\Delta m_{21}^2}{10^{-5} \ {\rm eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$		
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \rightarrow +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \rightarrow -2.413$		
		Normal Ordering (best fit)		Inverted Ordering $(\Delta \chi^2 = 7.0)$			
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range		
	$\sin^2 heta_{12}$	$0.304\substack{+0.012\\-0.012}$	$0.269 \rightarrow 0.343$	$0.304\substack{+0.013\\-0.012}$	$0.269 \rightarrow 0.343$		
data	$ heta_{12}/^{\circ}$	$33.45_{-0.75}^{+0.77}$	$31.27 \rightarrow 35.87$	$33.45_{-0.75}^{+0.78}$	$31.27 \rightarrow 35.87$		
с С	$\sin^2 heta_{23}$	$0.450\substack{+0.019\\-0.016}$	$0.408 \rightarrow 0.603$	$0.570\substack{+0.016\\-0.022}$	$0.410 \rightarrow 0.613$		
SK atmospheri	$ heta_{23}/^{\circ}$	$42.1^{+1.1}_{-0.9}$	$39.7 \rightarrow 50.9$	$49.0_{-1.3}^{+0.9}$	$39.8 \rightarrow 51.6$		
tmc	$\sin^2 heta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \rightarrow 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \rightarrow 0.02457$		
SK a	$ heta_{13}/^{\circ}$	$8.62_{-0.12}^{+0.12}$	$8.25 \rightarrow 8.98$	$8.61_{-0.12}^{+0.14}$	$8.24 \rightarrow 9.02$		
with	$\delta_{ m CP}/^{\circ}$	230^{+36}_{-25}	$144 \rightarrow 350$	278^{+22}_{-30}	$194 \rightarrow 345$		
·	$\frac{\Delta m_{21}^2}{10^{-5} \ {\rm eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$		
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \rightarrow +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \rightarrow -2.410$		

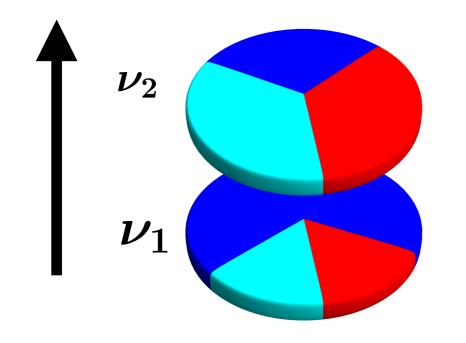
Stephen Parke

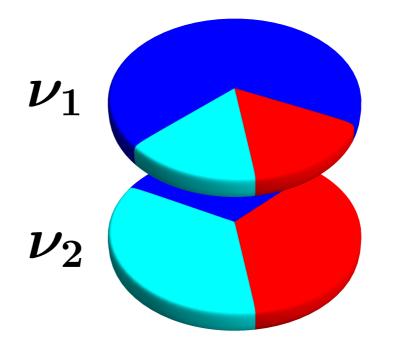
Nu22-KITP

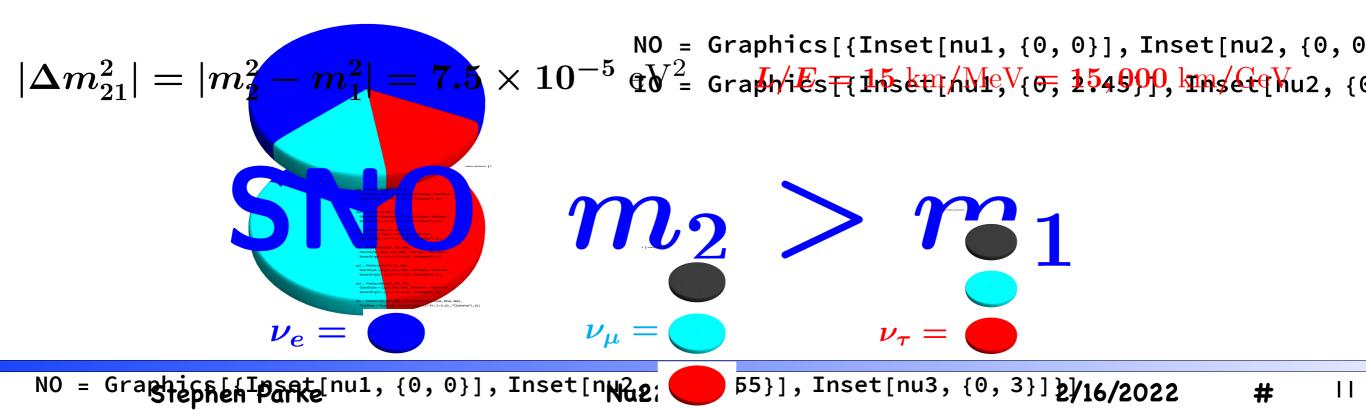


 $u_1, \ \nu_2$ N

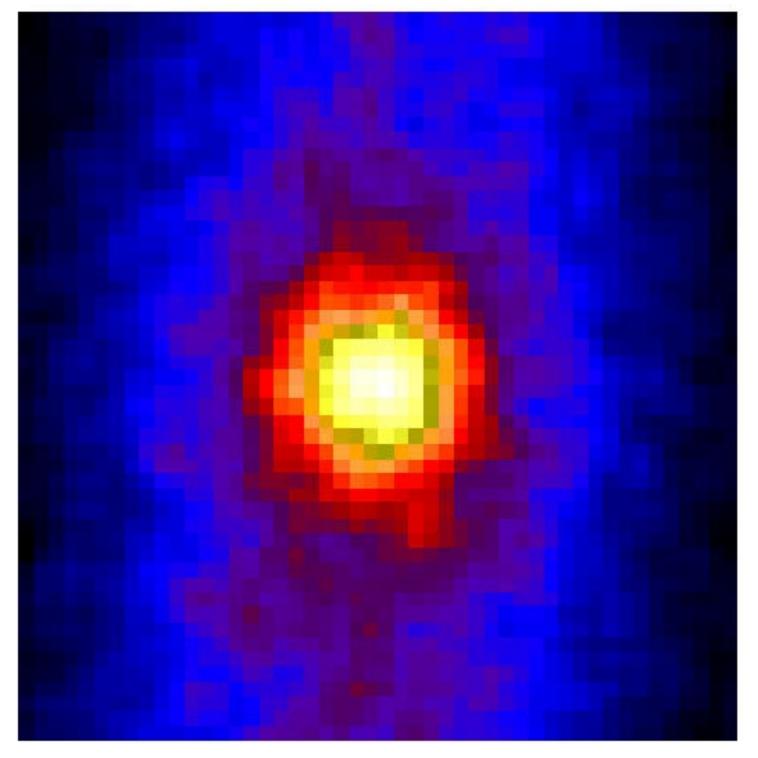








Neutrino Picture of the Sun



 $\nu_e, \nu_\mu, \nu_\tau, \nu_1, \nu_2, \nu_3$ which do

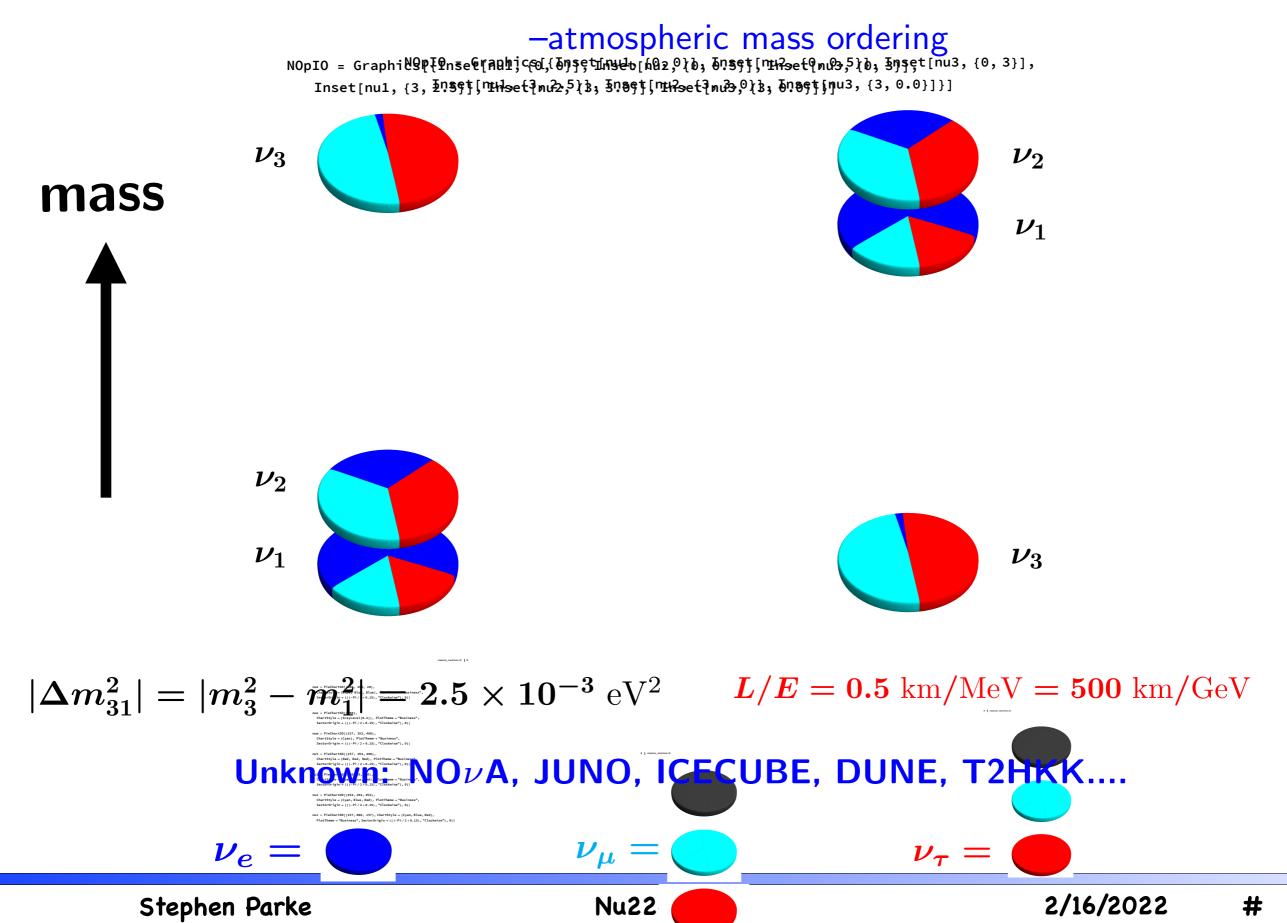
which dominates ?

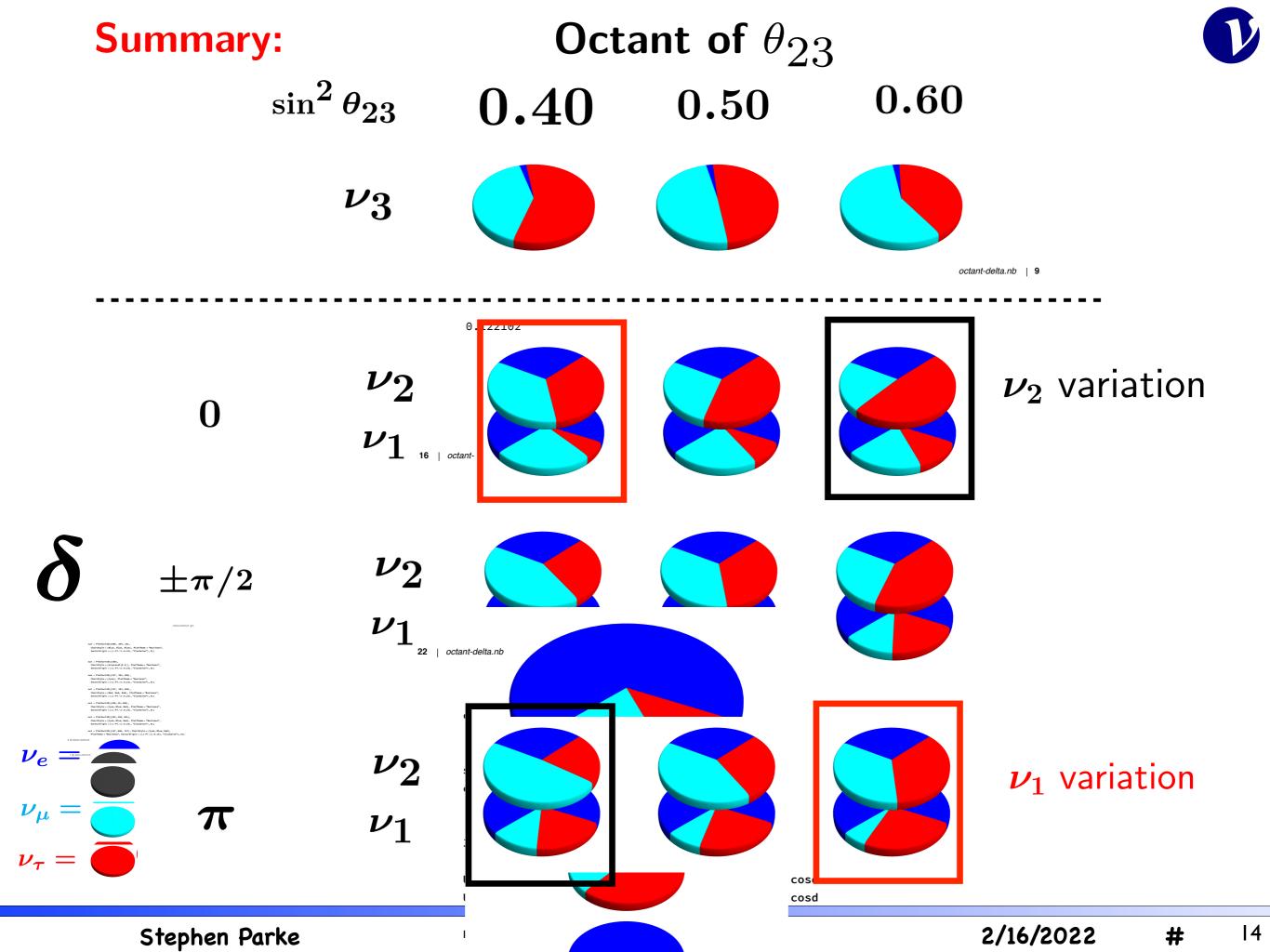
Nu22-KITP



13

^B massive but massive ν_1 massive ν_2 Mass Ordering:





Neutrino Oscillation Amplitudes in vacuum: "the billion \$ process" $P(\nu\mu \rightarrow \nu_e) = |\mathcal{A}\mu e|^2$ $\mathcal{A}_{\mu e} = (2i) [(s_{23}s_{13}c_{13}) [c_{12}^2 e^{-i\Delta_{32}} \sin \Delta_{31} + s_{12}^2 e^{-i\Delta_{31}} \sin \Delta_{32}] + (c_{23}c_{13}s_{12}c_{12}) e^{i\delta} \sin \Delta_{21}]$

maintain the symmetry: $m_1^2 \leftrightarrow m_2^2$ with $heta_{12} o heta_{12} \pm \pi/2$ Denton, Minakata, SP arXiv:1604.08167

$$\Delta P_{CP} = 8 (s_{23}s_{13}c_{13}) (c_{23}c_{13}s_{12}c_{12}) \sin \delta \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32}$$

$$J$$

$$\Delta_{32} \approx \Delta_{31}$$

$$i(\delta \mid \Delta_{21})$$

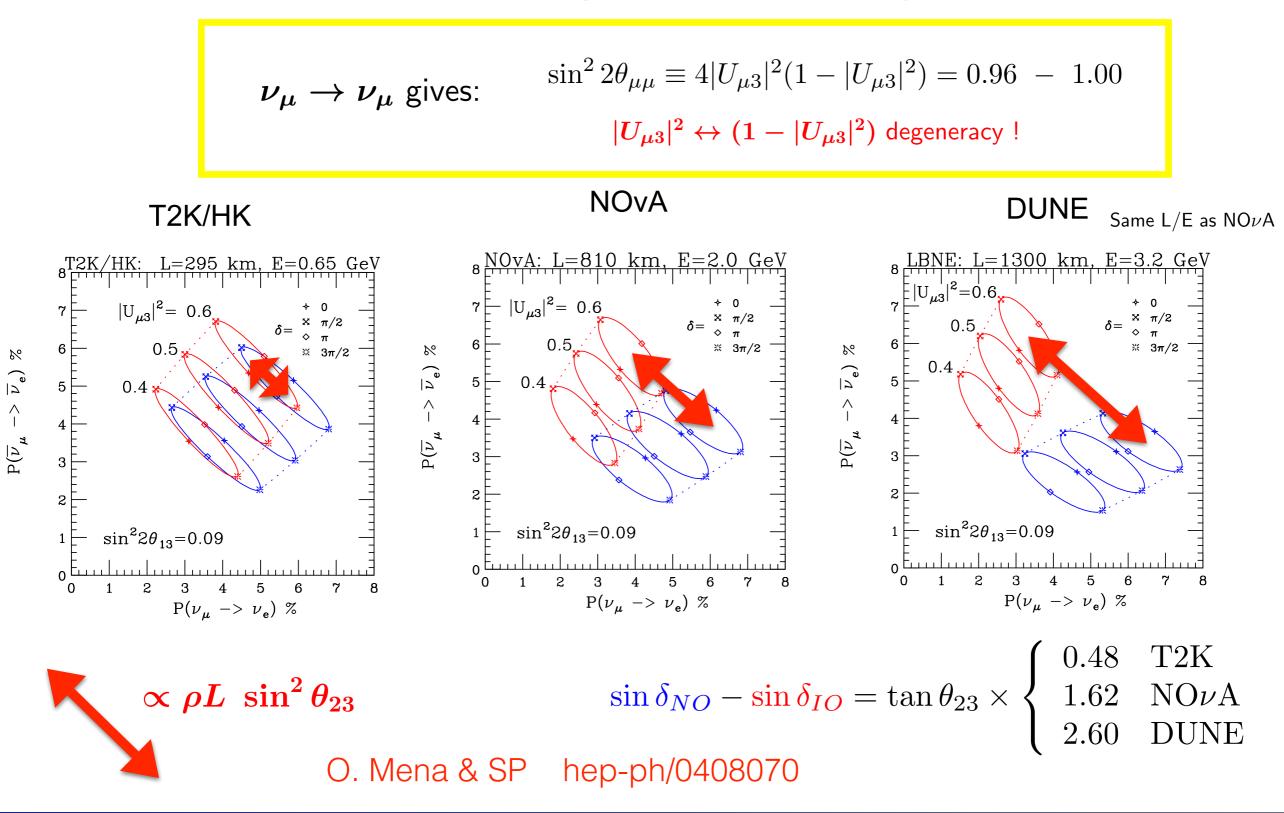
 $\mathcal{A}_{\mu e} ~~pprox~~(2i) \left[\left(s_{23}s_{13}c_{13} \right) \, \sin \Delta_{31} + \, \left(c_{23}c_{13}s_{12}c_{12} \right) \, e^{i(\delta + \Delta_{31})} \, \sin \Delta_{21} \, \right]$

Correlations between



Normal Ordering — Inverted Ordering

 $u_{\mu}
ightarrow
u_{e} \quad ar{
u}_{\mu}
ightarrow ar{
u}_{e}$



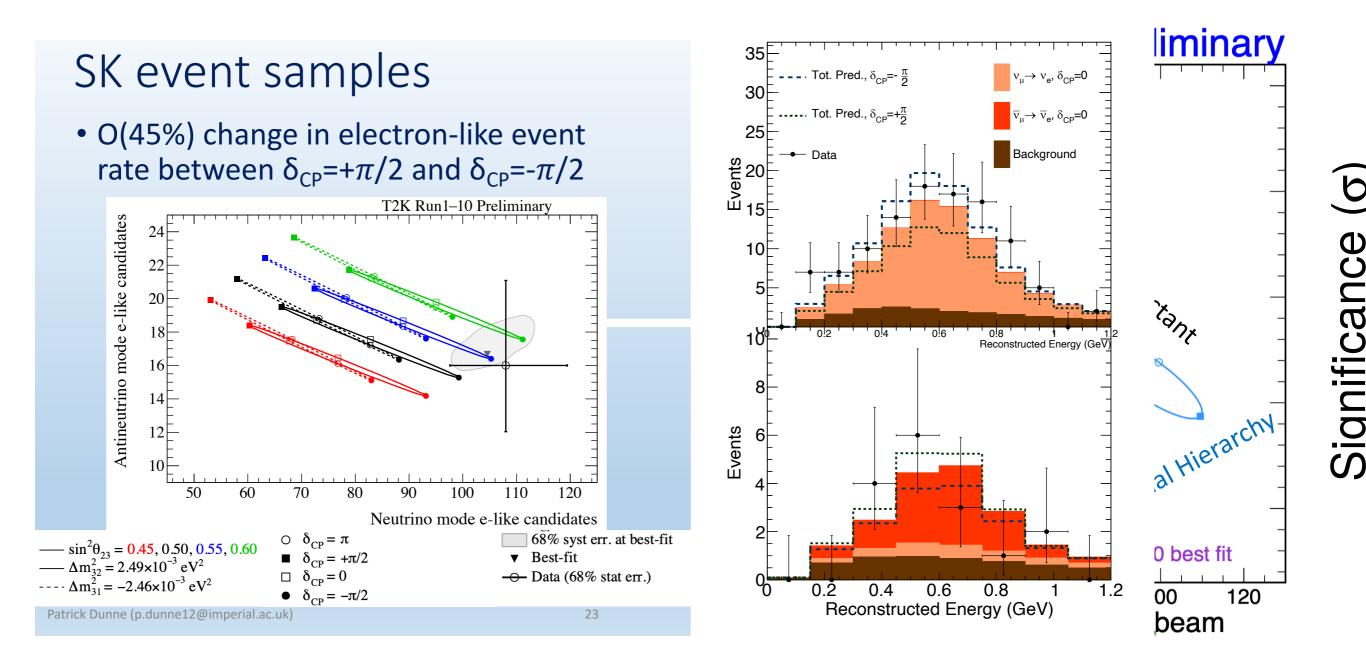
Stephen Parke

Nu22-KITP



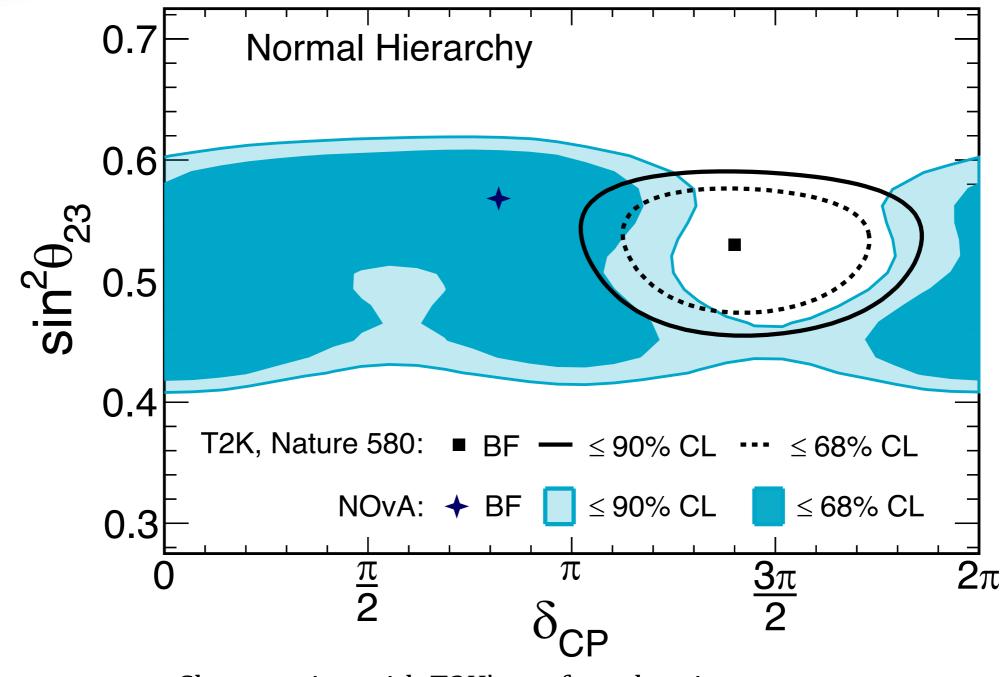
T2K & NOvA

Number of Events proportional to Oscillation Probability



Comparison to T2K

NOvA Preliminary

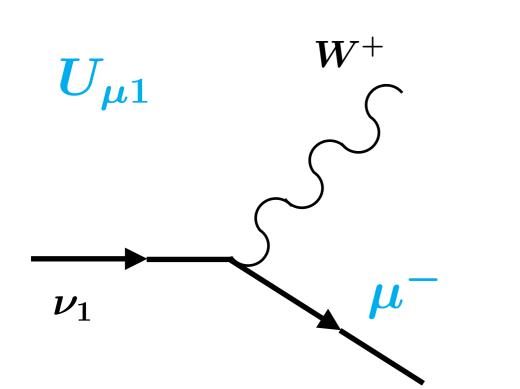


• Clear tension with T2K's preferred region.



Leptons:

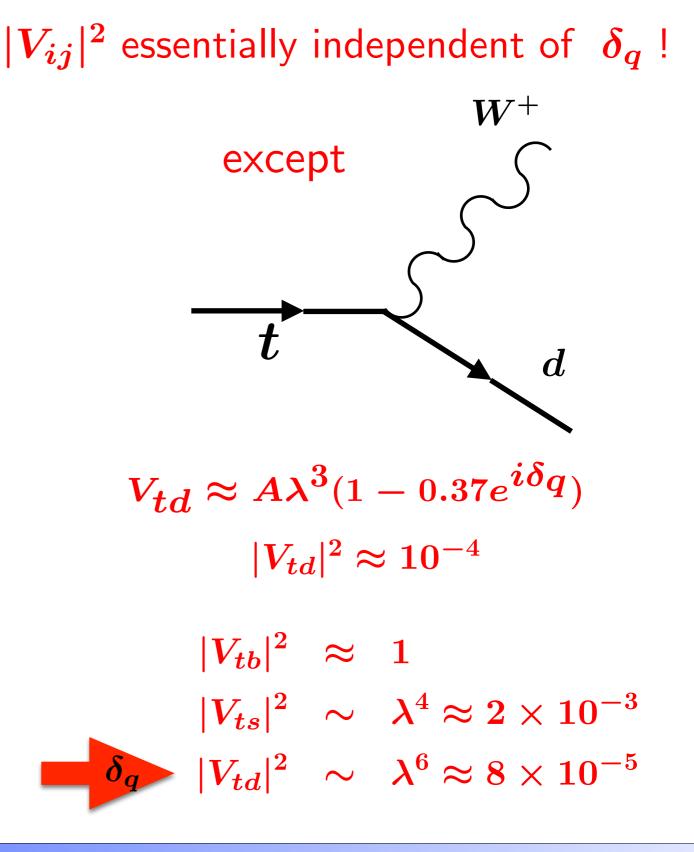




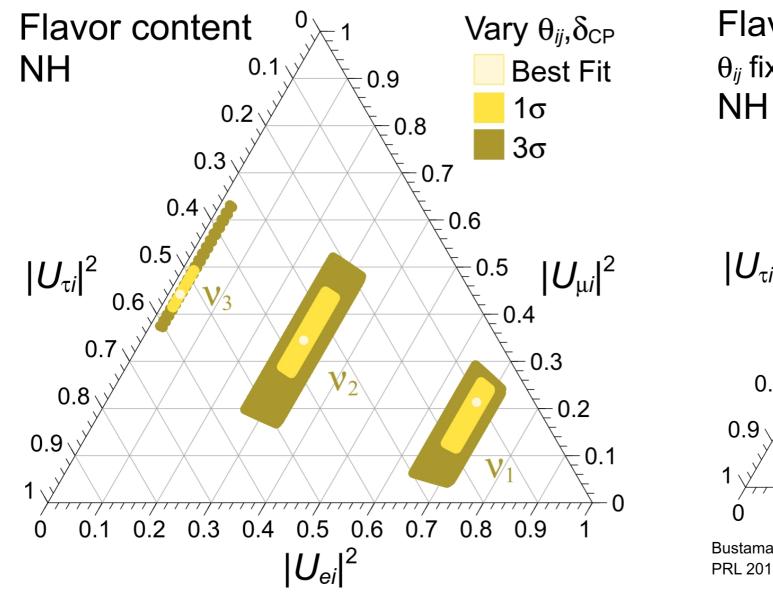
$0.08 < |U_{\mu 1}|^2 < 0.24$ variation in δ only !

factor of 3 diff.

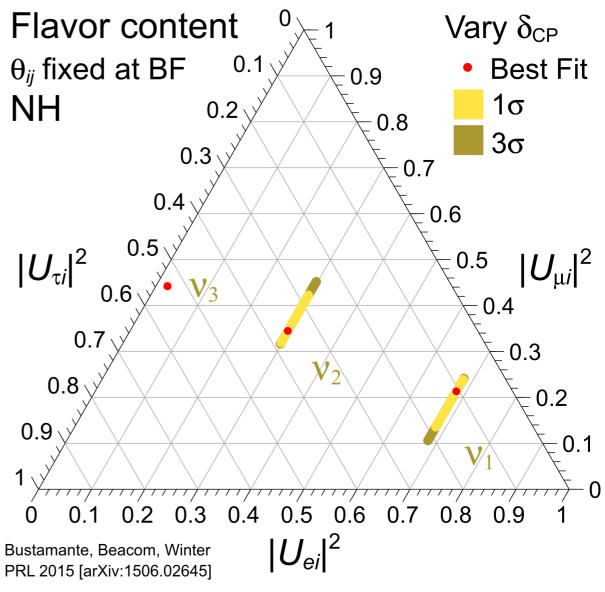
$ U_{\mu3} ^2$	=	0.4 - 0.6
$ U_{\mu2} ^2$	=	0.26 - 0.41
$ U_{\mu 1} ^2$	=	0.08 - 0.24



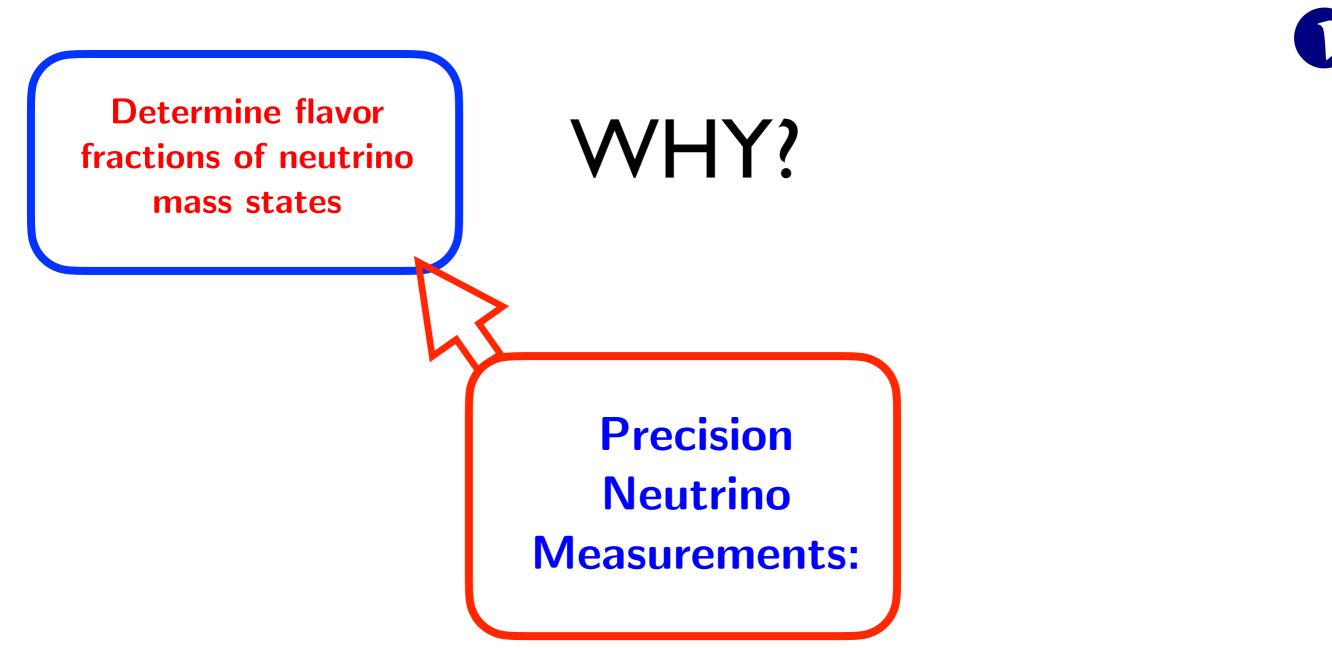
D



 $\delta \& heta_{23}$ uncertainty



no θ_{23} uncertainty

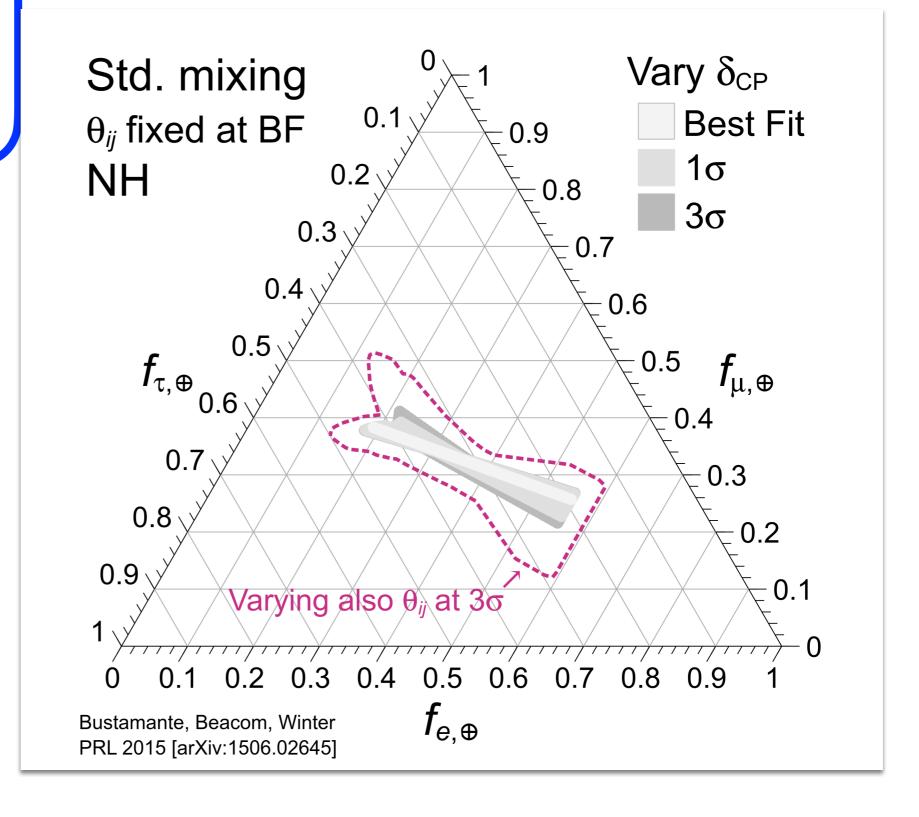


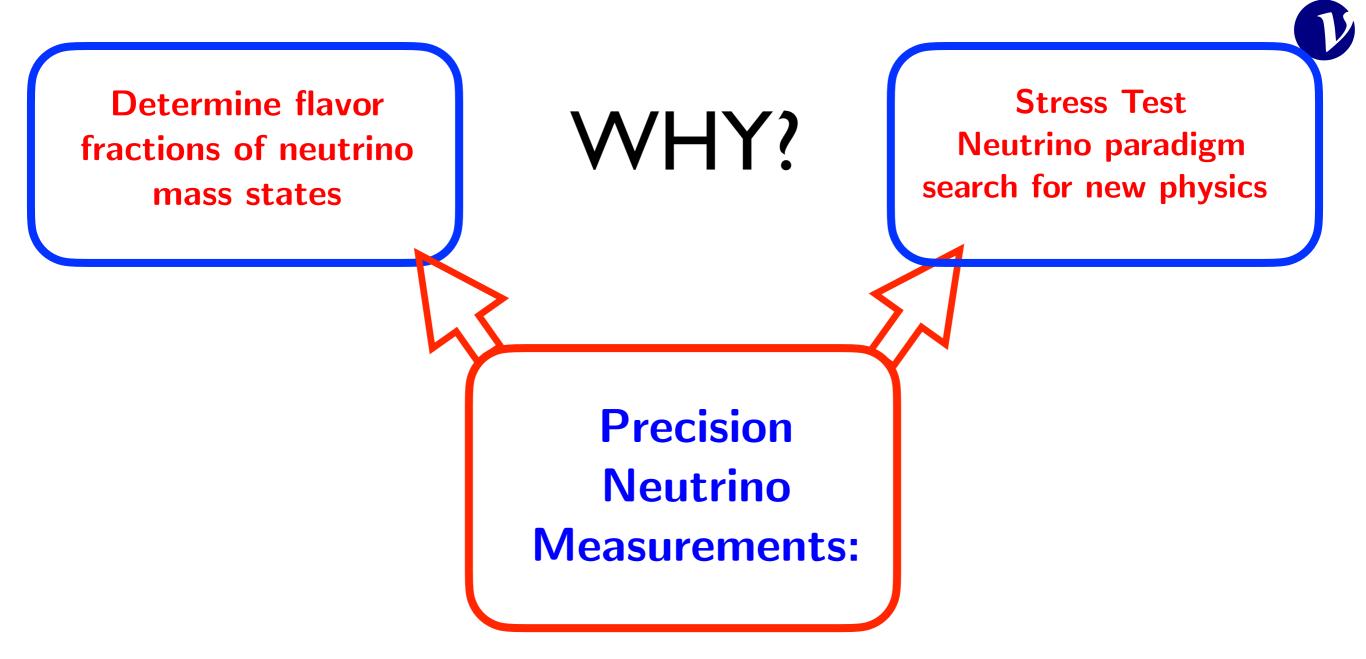
To discover neutrino BSM, one needs precision predictions for nuSM

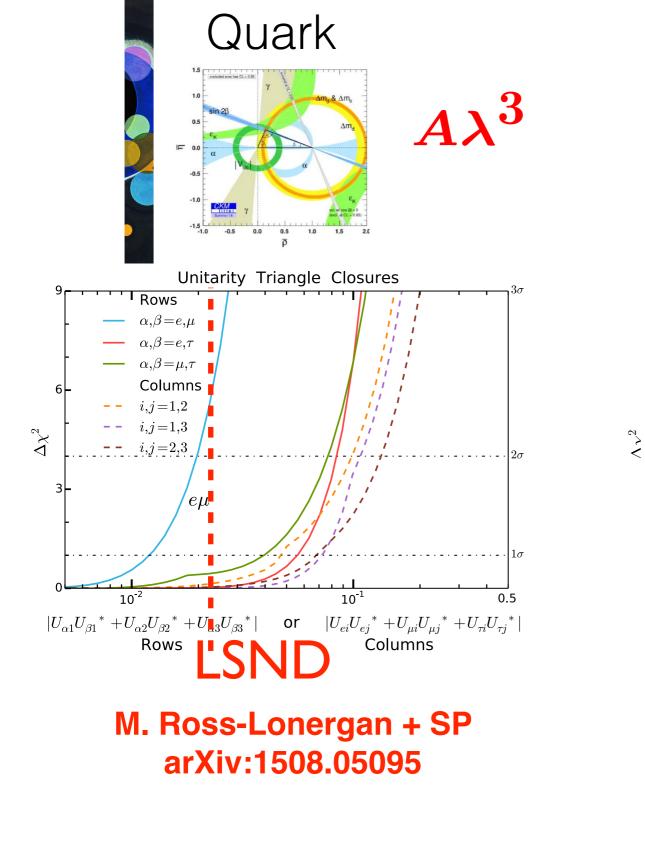


Determine flavor fractions of neutrino mass states

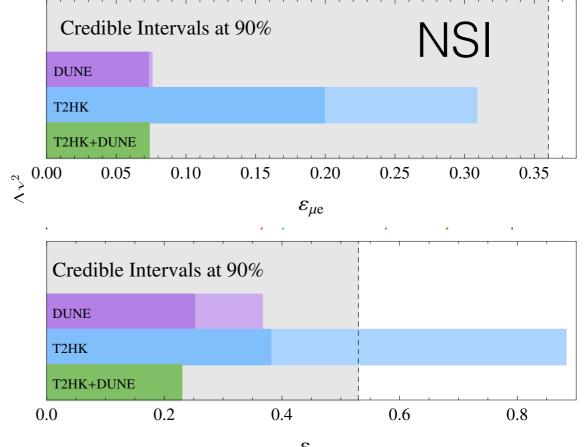
Precision Predictions for flavor ratios at ICECUBE.



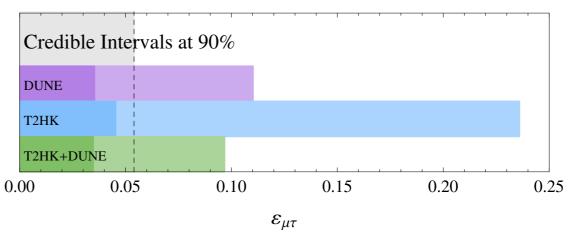




Stress Test Neutrino paradigm search for new physics

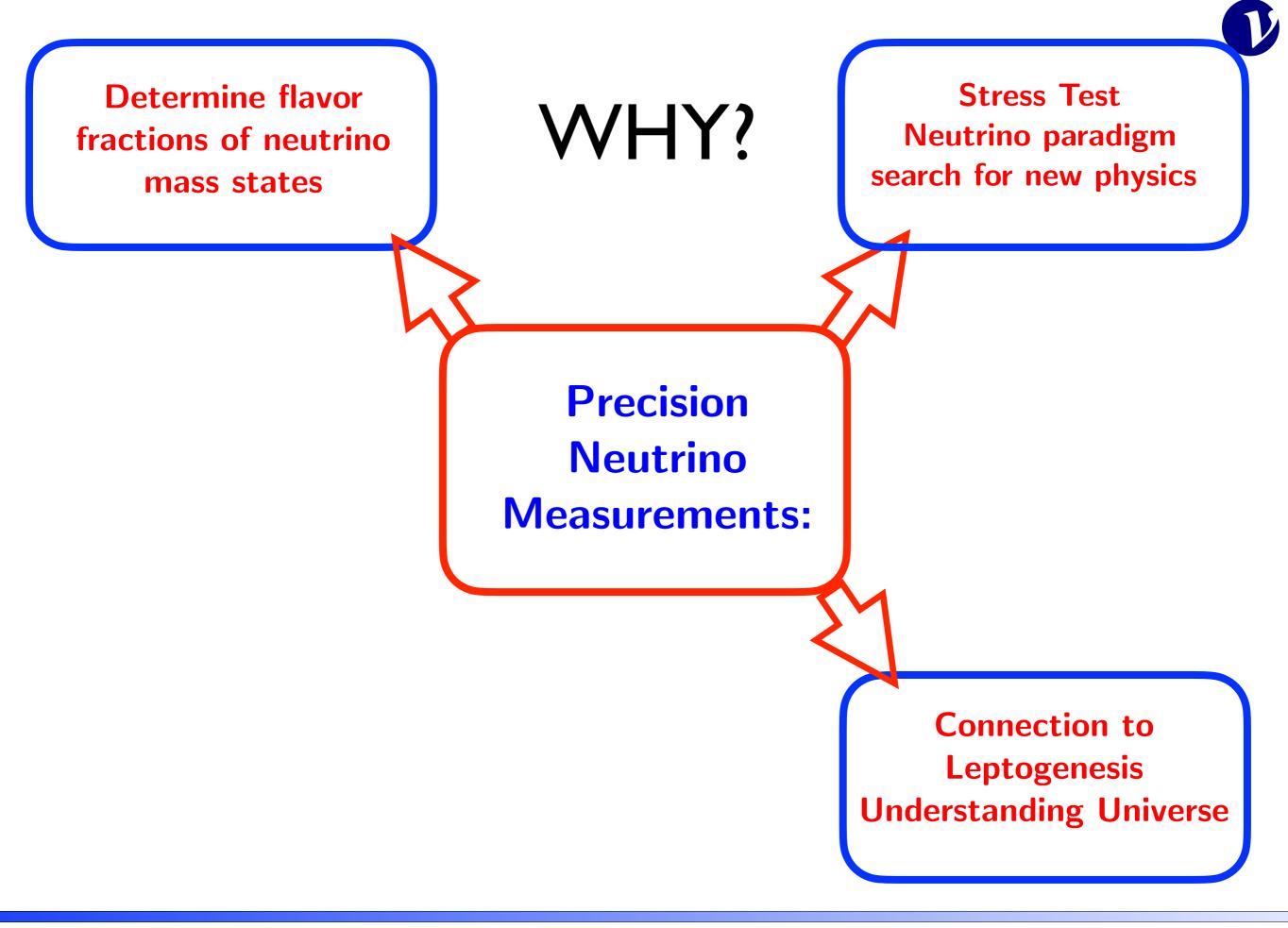




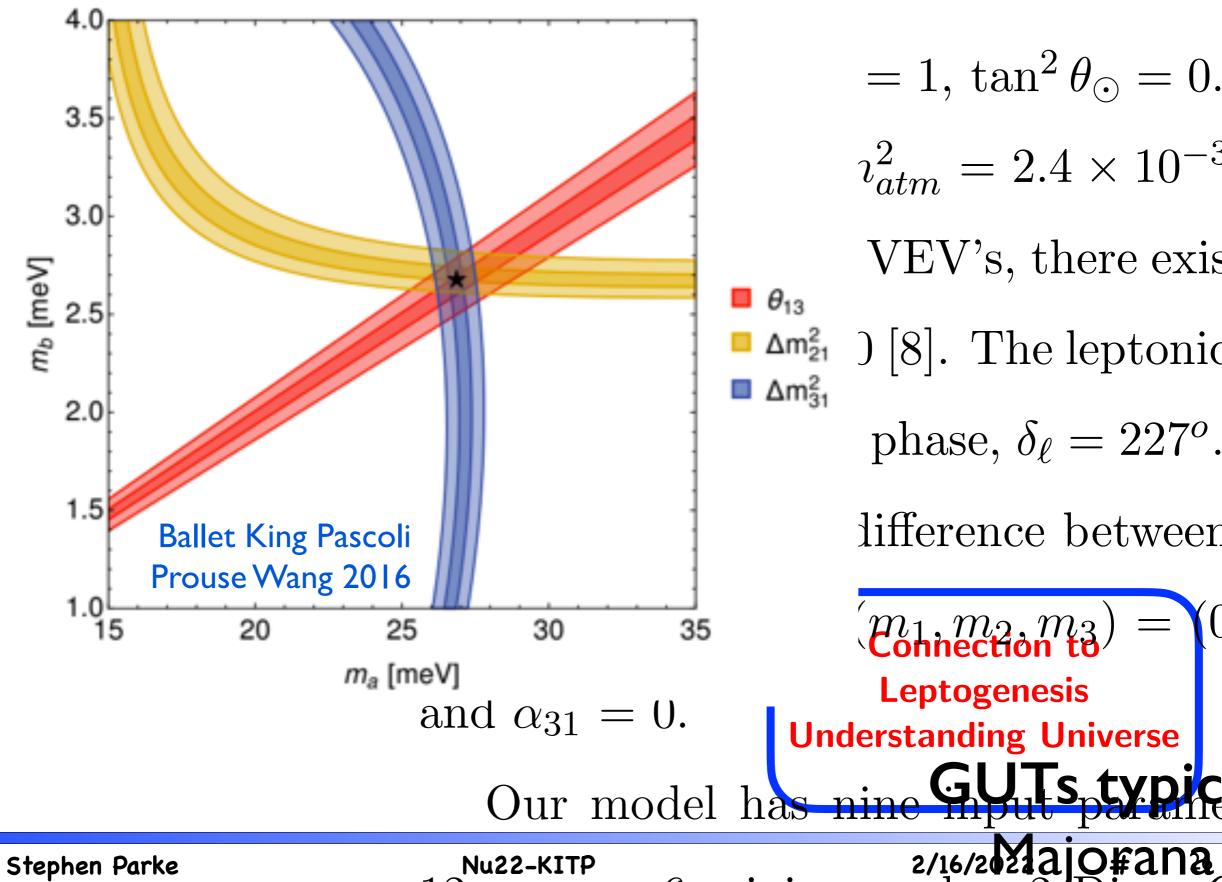


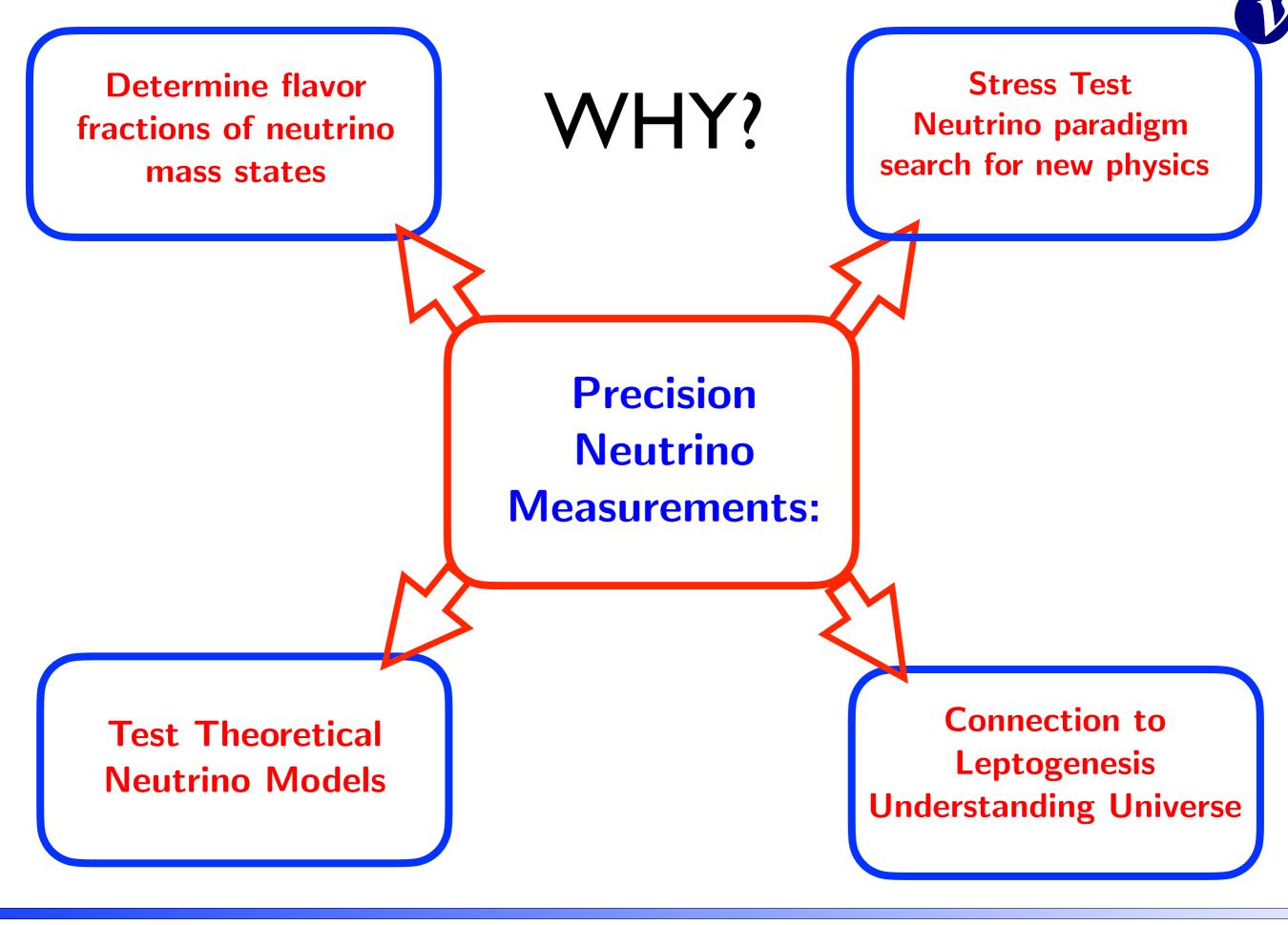
P.Coloma

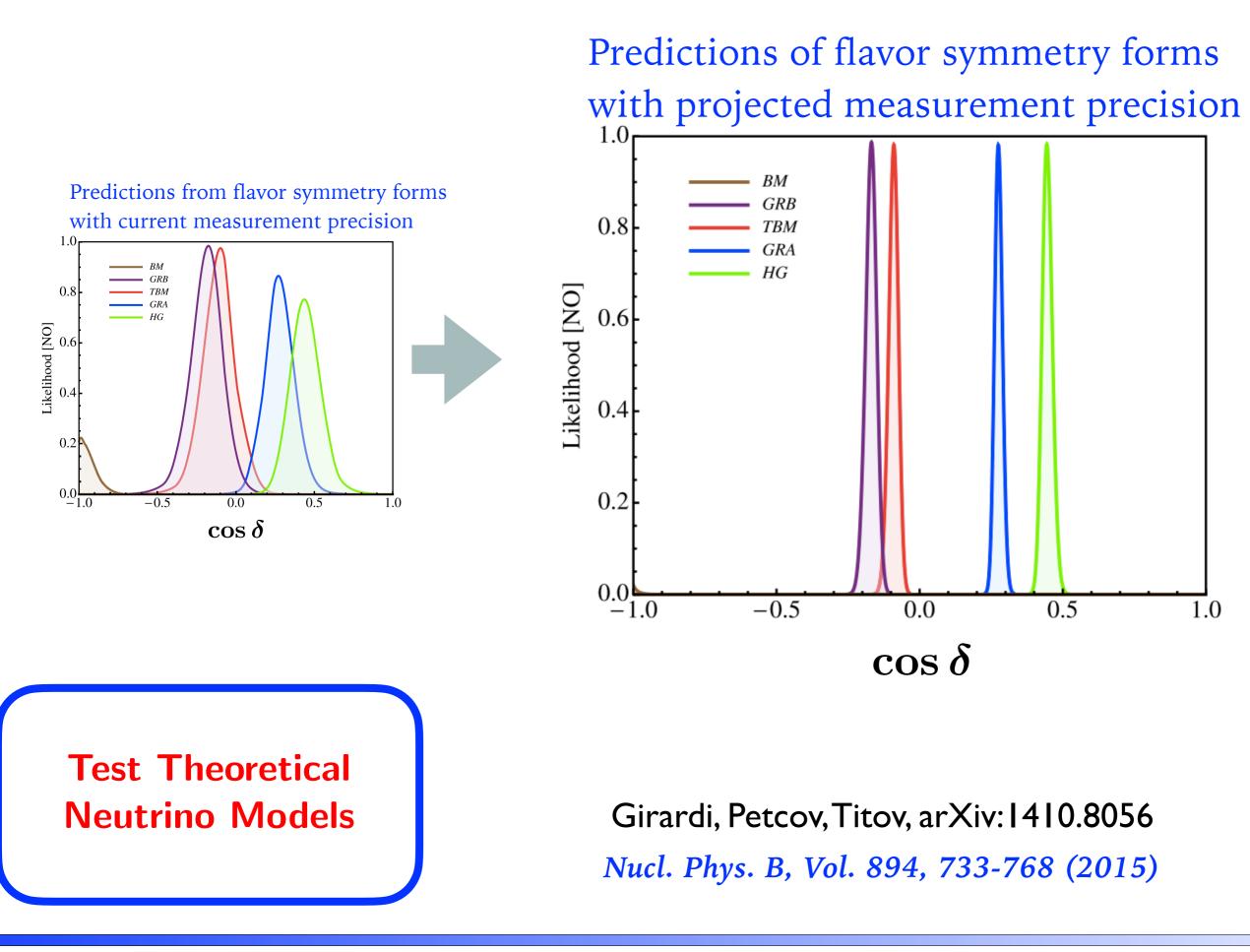
arXiv:1511.06357

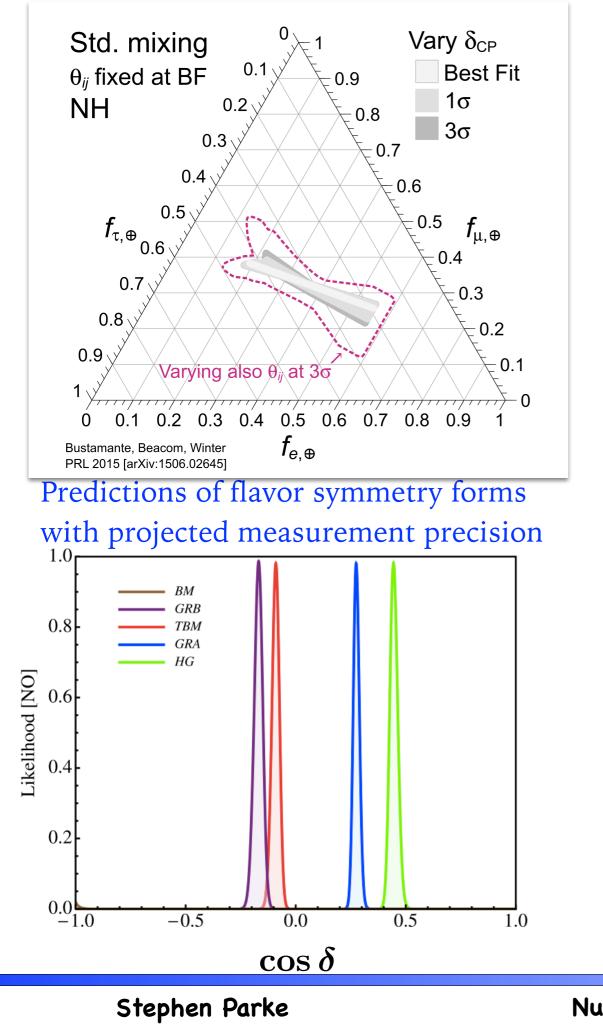


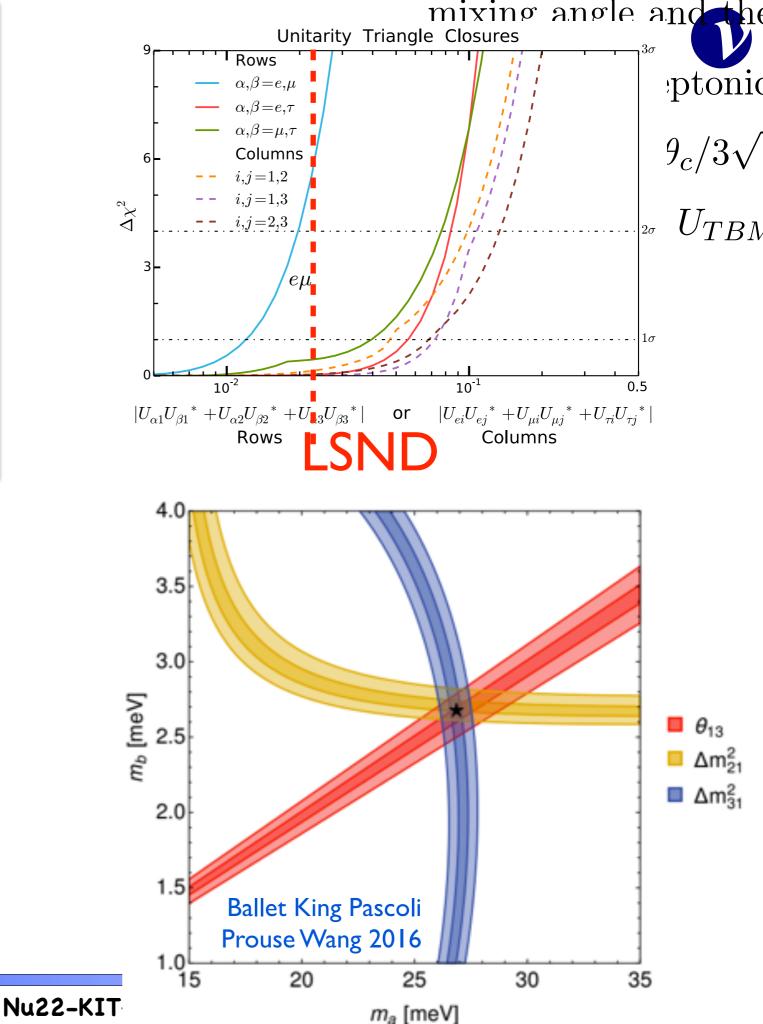














$\Delta m_{31}^2 \& \Delta m_{32}^2$ V $\Delta m_{ee}^2 \& \Delta m_{\mu\mu}^2$

Stephen Parke

2/16/2022 # 30



Effective Δm_{eff}^2 for Δm_{31}^2 & Δm_{32}^2 at L/E ~ 500 km/GeV = 0.5 km/MeV:

Channel Dependent:

(...) is ν_e average of 1 & 2

$$\Delta m_{ee}^2 \equiv m_3^2 - (c_{12}^2 m_1^2 + s_{12}^2 m_2^2)$$
$$= c_{12}^2 \Delta m_{31}^2 + s_{12}^2 \Delta m_{32}^2$$

 $1 - P(\nu_e \to \nu_e) \approx 4 |U_{e3}|^2 (1 - |U_{e3}|^2) \sin^2 \Delta_{ee}$

Daya Bay and RENO.

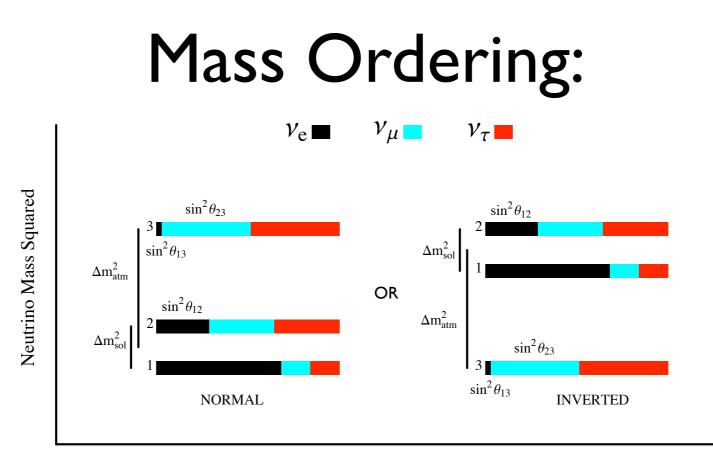
(...) is u_{μ} average of 1 & 2

$$\Delta m_{\mu\mu}^2 \equiv m_3^2 - (s_{12}^2 m_1^2 + c_{12}^2 m_2^2)$$
$$= s_{12}^2 \Delta m_{31}^2 + c_{12}^2 \Delta m_{32}^2$$

 $1 - P(
u_{\mu} o
u_{\mu}) pprox 4 |U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) \sin^2 \Delta_{\mu \mu}$

T2K, NOvA,

Nunokawa, SP, Zukanovich hep/0503283



Fractional Flavor Content

Normal Ordering: $|\Delta m^2_{ee}(NO)| > |\Delta m^2_{\mu\mu}(NO)|$

Inverted Ordering: $|\Delta m^2_{ee}(IO)| < |\Delta m^2_{\mu\mu}(IO)|$

Difference is $\cos 2\theta_{12}\Delta m^2_{21} \approx 1.2\%$



How does this come about ?

Hamiltonian in flavor basis $= \frac{1}{2E} U_{23}U_{13}U_{12} M^2 U_{12}^{\dagger}U_{13}^{\dagger}U_{23}^{\dagger}$

$$U_{12} \operatorname{diag}(m_1^2, m_2^2, m_3^2) U_{12}^{\dagger} = \begin{pmatrix} c_{12}^2 m_1^2 + s_{12}^2 m_2^2 & s_{12} c_{12} \Delta m_{21}^2 \\ s_{12} c_{12} \Delta m_{21}^2 & s_{12}^2 m_1^2 + c_{12}^2 m_2^2 \\ & m_3^2 \end{pmatrix}$$

For ν_e disappearance U_{13} is most important:

$$m_3^2 - (c_{12}^2 m_1^2 + s_{12}^2 m_2^2) \equiv \Delta m_{ee}^2$$

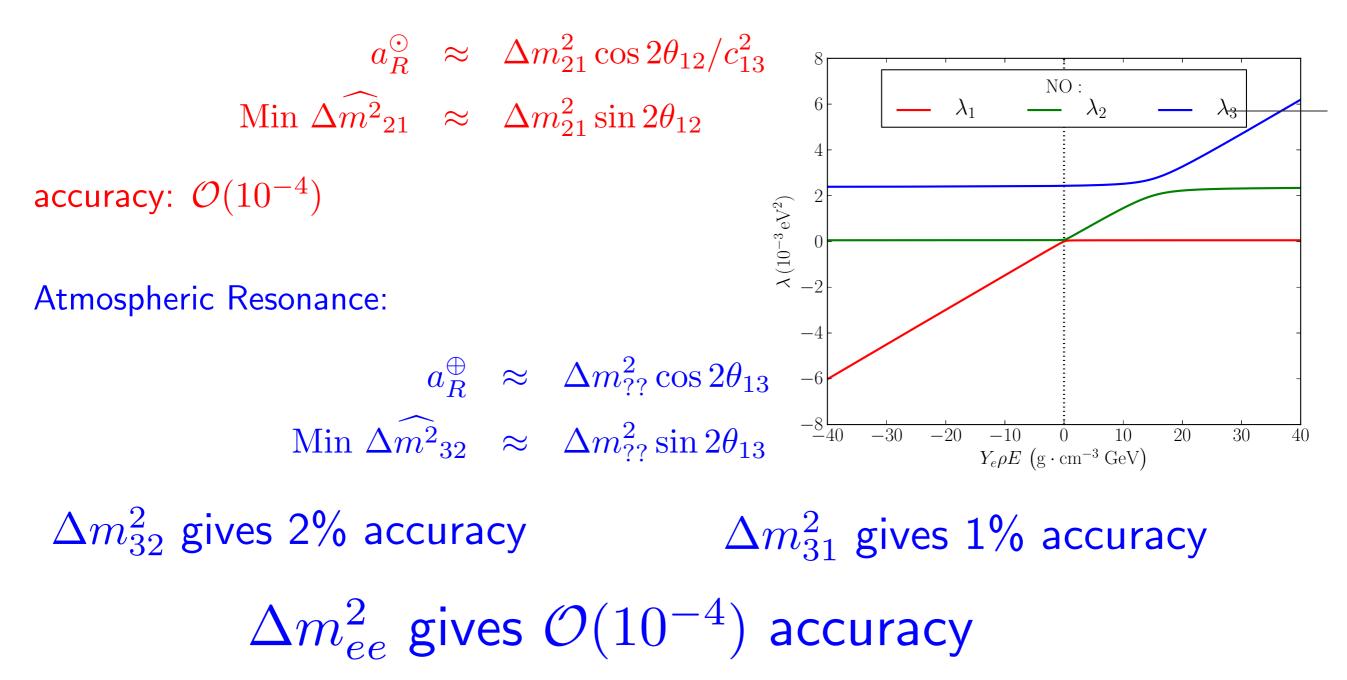
For ν_{μ} disappearance U_{23} is most important:

$$m_3^2 - (s_{12}^2 m_1^2 + c_{12}^2 m_2^2) \equiv \Delta m_{\mu\mu}^2$$

Even in Matter Δm_{ee}^2 is useful:

Defn $a \equiv 2\sqrt{2}G_F N_e E_{\nu}$, the Wolfenstein Matter Potential

Solar Resonance:



SP 2012.07186 (hep-ph)





At oscillation maximum in vacuum:

 $P(\bar{\nu}_{\mu} \to \bar{\nu}_{e}) - P(\nu_{\mu} \to \nu_{e}) = \pi J_{0} \left(\frac{\Delta m_{21}^{2}}{\Delta m_{ee}^{2}}\right)$

where J_0 is Jarlskog Invariant (1986):

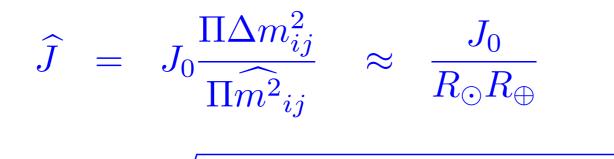


 $J = \sin 2\theta_{12} \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{23} \sin \delta \approx 0.3 \sin \delta$

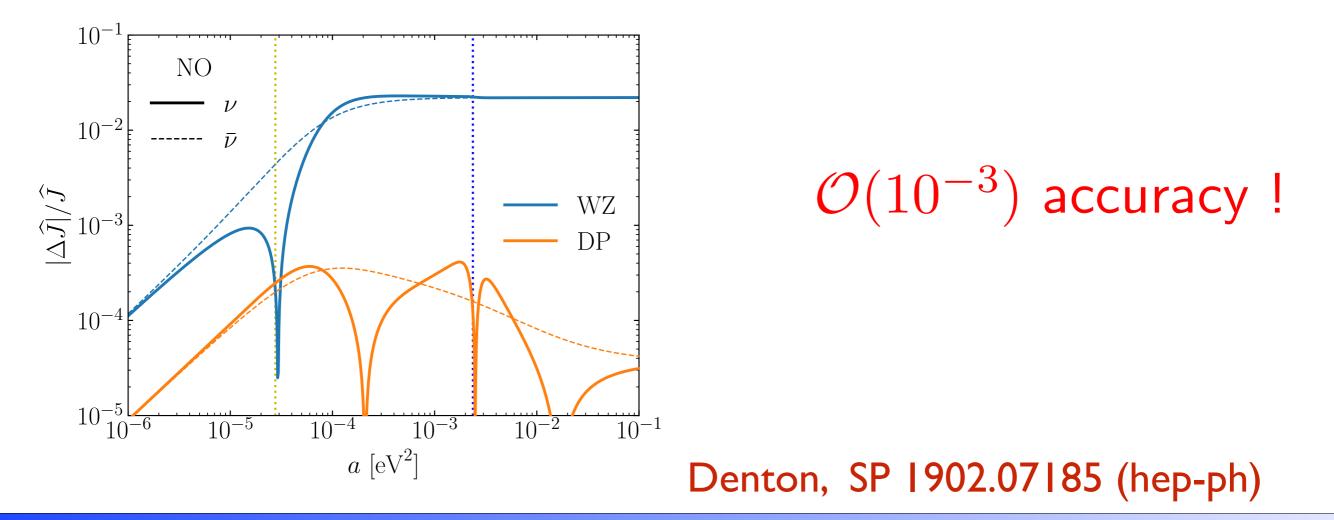
Nu22-KITP



Jarlskog Invariant in Matter



$$R_{\odot} = \sqrt{(\cos 2\theta_{12} - ac_{12}^2/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}}$$
$$R_{\oplus} = \sqrt{(\cos 2\theta_{13} - a/\Delta m_{ee}^2)^2 + \sin^2 2\theta_{13}}$$





JUNO by Nu2024 or Nu2026

Best measurements of Δm^2_{21} , $\sin^2 \theta_{12}$ and Δm^2_{ee} : accuracy $\Rightarrow \sim 0.5\%$

(JUNO value of $\sin^2 \theta_{13}$ will not be more accurate than Daya Bay)

$$1 - P(\bar{\nu}_e \to \bar{\nu}_e) = 4c_{13}^4 s_{12}^2 c_{12}^2 \sin^2 \Delta_{21} + 2s_{13}^2 c_{13}^2 \left(1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}} \cos\left[2|\Delta_{ee}| \pm \Phi(\Delta_{21})\right]\right)$$

Amplitude modulation

Phase advance(NO)/retardation(IO)

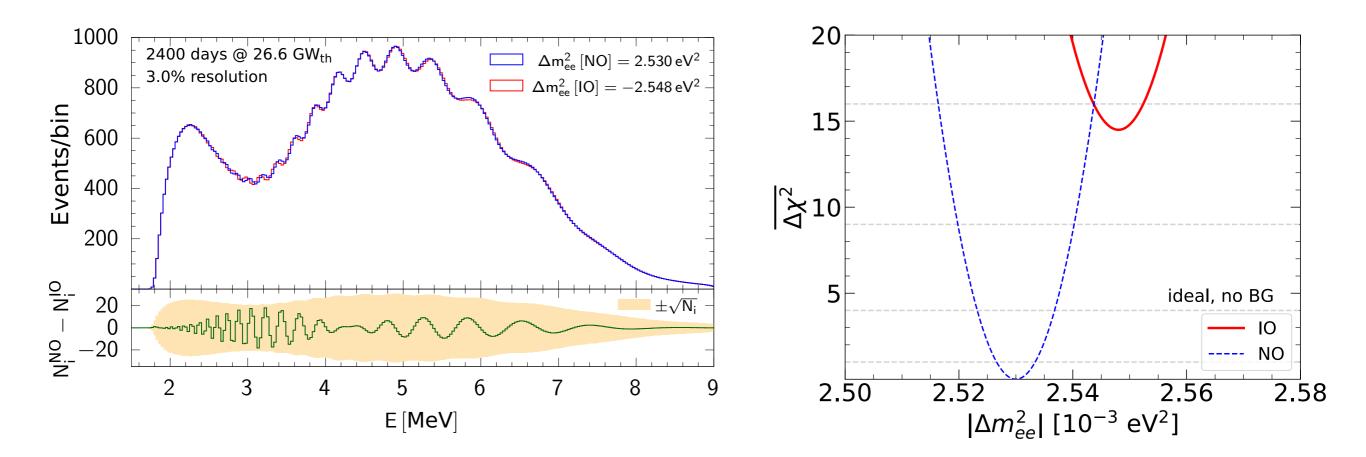
 $\Phi(\Delta_{21}) = \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \cos 2\theta_{12} \Delta_{21} = \mathcal{O}(\Delta_{21}^3)$

$$\Phi(\Delta_{21} = \pi/2) = \pi \sin^2 \theta_{12}$$

Minakata, Nunokawa, SP, Zukanovich hep/0701151



JUNO Events Spectra

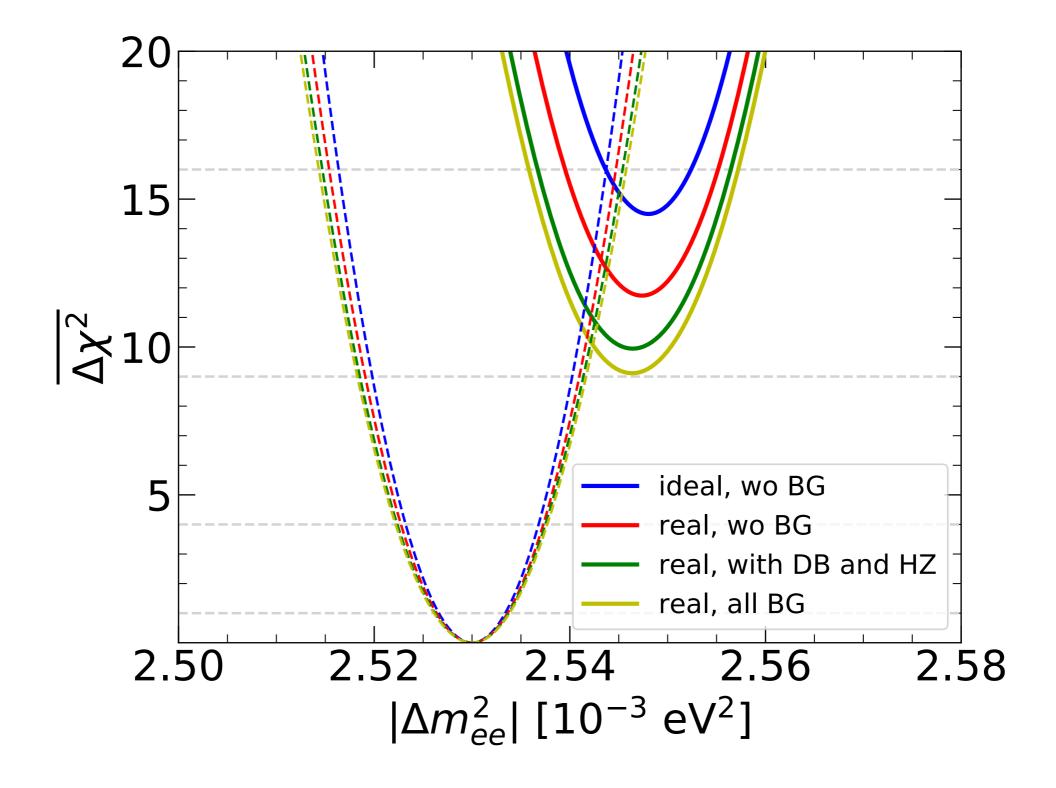


8 years, 26.6 GW_th 52.5 km, baseline 3% resolution

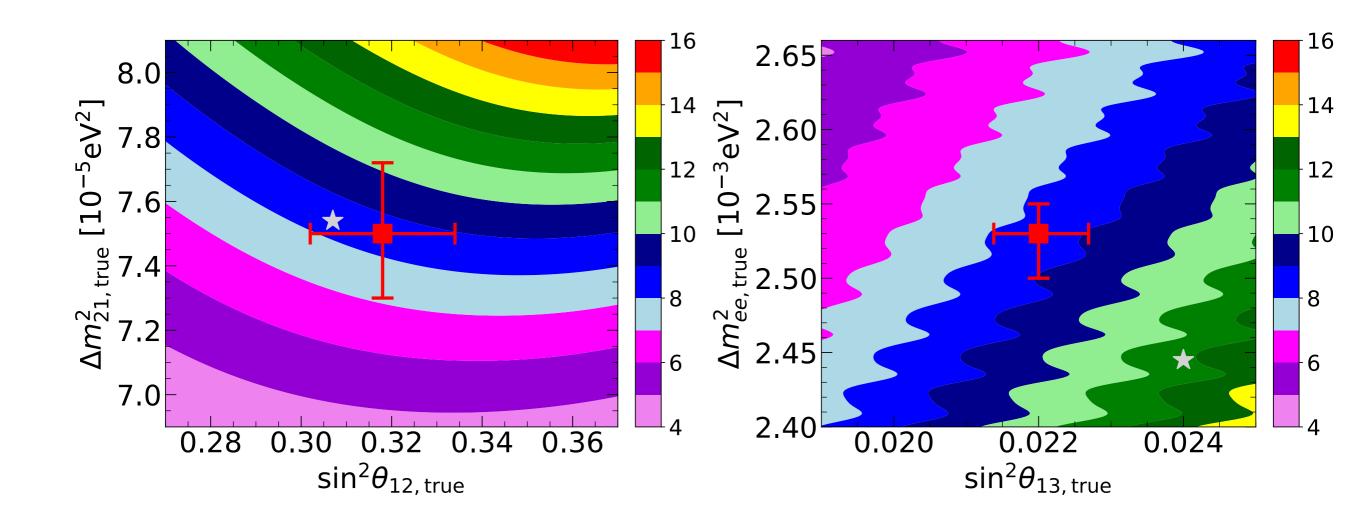
No backgrounds No Systematics

Forero, SP, Ternes, Zukanovich 2107.12410

Real Baseline Distribution + Backgrounds

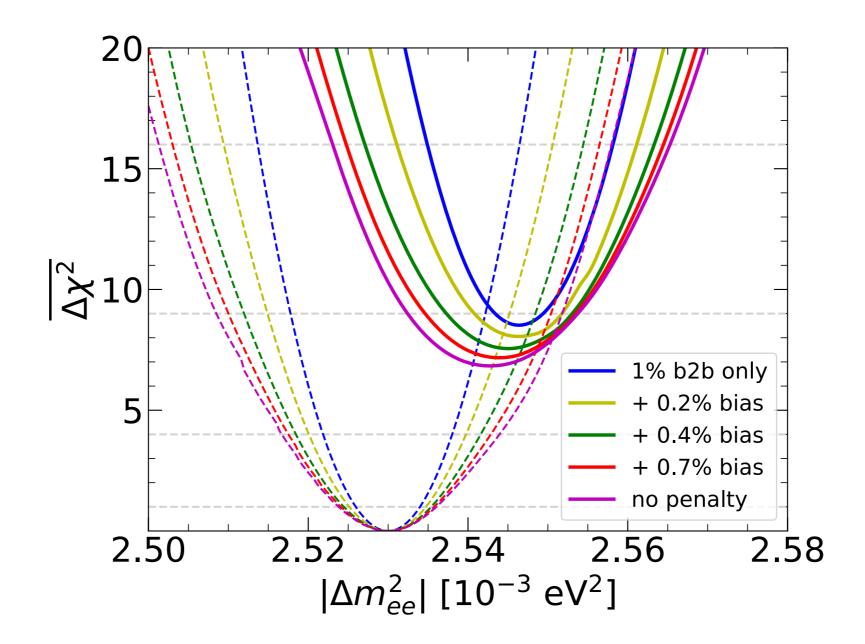


Parameter Sensitivity:

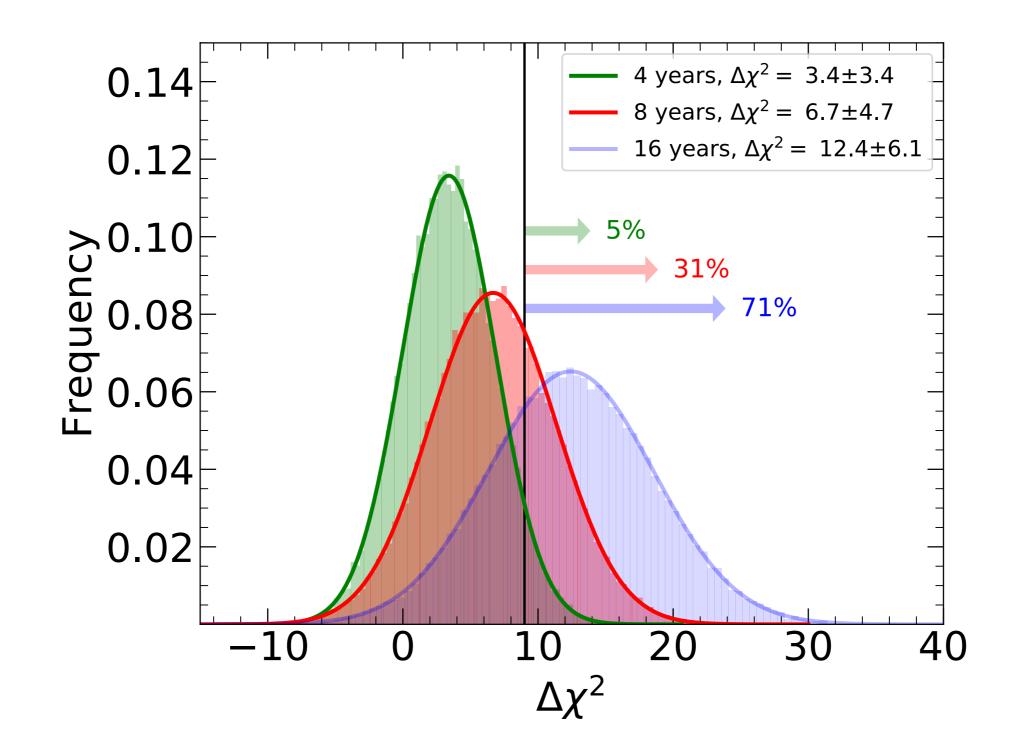


V

Non-linear Energy Response



JUNO probability of determining Mass Ordering

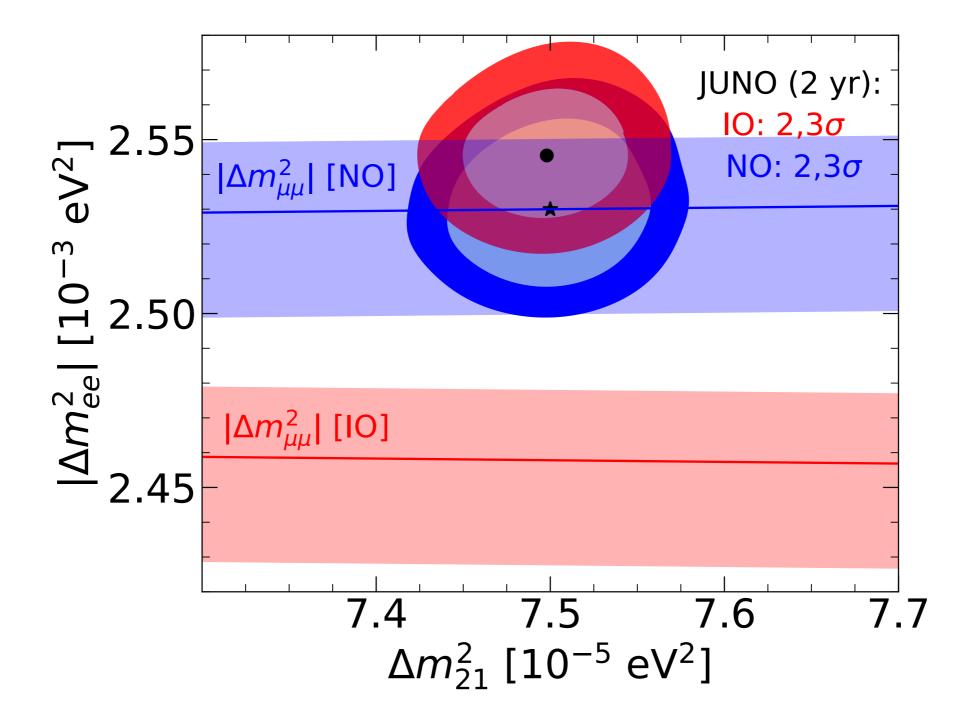




GLOBAL DATA:







Summary:



- from Nu1998 to now, tremendous exp. progress on Neutrino SM: more at Nu2022. May 31-June 4, 2022
- LSND Sterile Nu's neither confirmed or ruled out at acceptable CL: – ultra short baseline reactor exp. —microBooNE
- Great Theoretical progress on understand many aspects of Quantum Neutrino Physics: – Oscillations, Decoherence, Osc. Probabilities in Matter, Leptogenesis,
- Still searching for convincing model of Neutrino masses and mixings: with testable and confirmed predictions !