Fast ν–ν flavor processes in the early universe, 1 MeV$<T<$ 50 MeV

(with some general comments on the FF formalism)

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Conventional view on early U:

"It's very near flavor equilibrium---and ν - $\bar{\nu}$ equilibrium

……. So nothing much can happen"

But a lot happens--

1. Each individual ν undergoes full flavor oscillations at a rate approx. = G_F N/Vol., N =no. of all ν's in box. ("FF" rate)

2. This can have two important cosmological consequences

A. At around T=5 MeV the electron-photon temp. is getting a little higher than the ν temp, but there is some heating of ν_ε 's through $e^+ + e^- \leftrightarrow \nu_e + \bar{\nu}_e$ that puts more energy on the ν side. Source of the .04 in "3.04 effective species of ν "

(Dolgov Phys. Rep. review of 2002).

.04 will be greatly enhanced due to FF oscillations!

B. With added heavyish (1KeV ?) sterile coupled weakly through a mass-like term…….

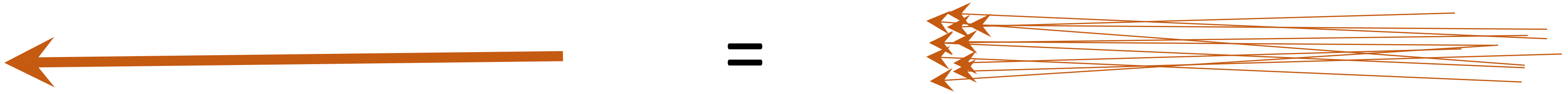
It makes an efficient sterile production mechanism.

But first - general methods in the FF business

Beam A

maybe 10^{10} angles

Beam B



In any of our beams, the angles are all different, one from another.

$$|\Psi_B\rangle = e^{i\sum_j \phi_j} \prod_{j=1}^{N} |\mathrm{flav}_j^B\rangle$$

$$|\Psi_A\rangle = e^{i\sum_j \phi_j} \prod_{j=1}^{N} |\mathrm{flav}_j^A\rangle$$

None of those lines changes directions, ever ! --------

(“ever” = “on the 1 cm. scale for fast processes”)

Luckily, we can show how

$$|\Psi_A\rangle = e^{i\sum_j \phi_j} \prod_{j=1}^{N} |\mathrm{flav}_j^A\rangle$$

becomes

$$|\Psi_A\rangle = e^{i\sum_j \phi_j} |\mathrm{flav}^{\mathrm{A}}\rangle$$

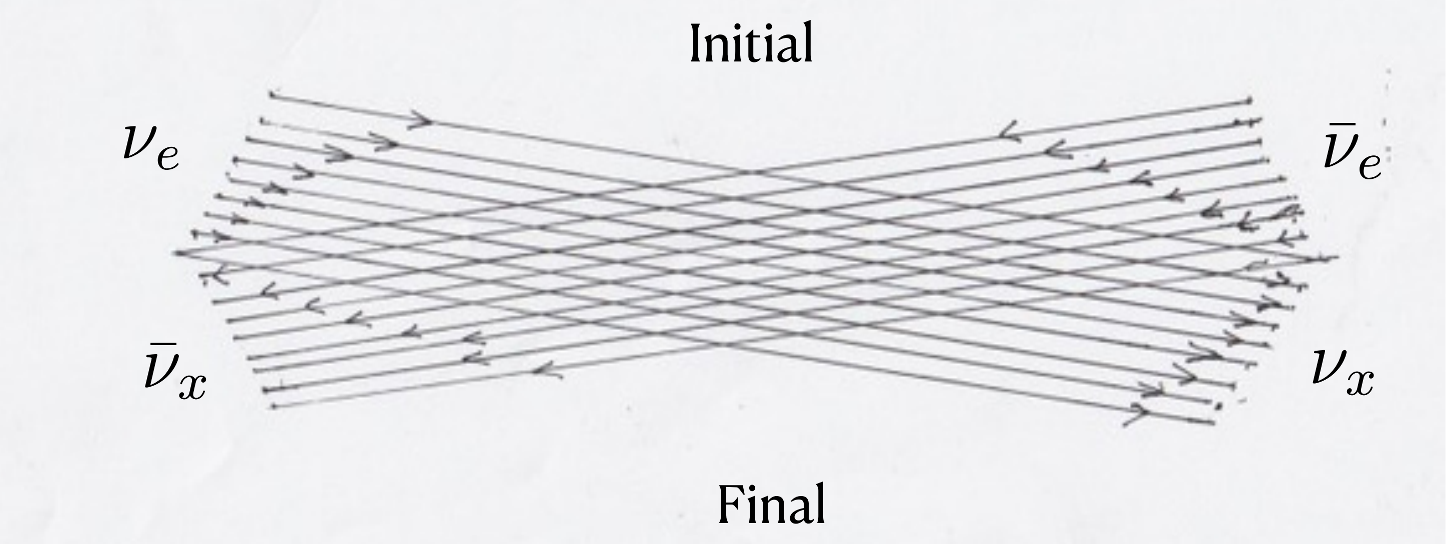
when a collective coordinate is introduced.

In the process the original coupling constant within our 1 cm. size box, G_F /Vol. becomes G_F n, and flavor is to be manipulated with Pauli matrices, beam by beam---

--Raffelt-Sigl equations regained.

Fast Example 1 (two "flows")

time



Chakraborty, Hansen, Izaguirre, Raffelt, arXiv:1602.00698

Used the equations of Raffelt and Sigl 1990's

Each of those beams in the diagram above is one of those conglomerates of 10^{10} ν's

But now consider an E.U. that is initially in thermal and flavor equilibrium.

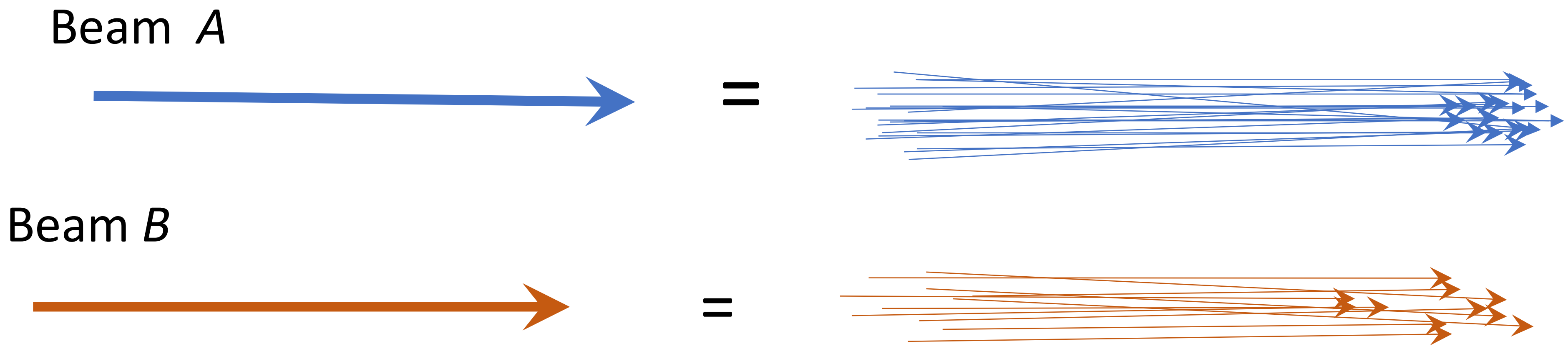
One playbook says:

“Nothing happens on the fast (1 cm. or less) time-scale because--- Along any line in space we have an equal number of each flavor and lepton number going in each direction. That is: of some +1 and -1 eigenvalues of the flavor-diagonal operators (“σ_3 ”) that fix the initial flavor.

Therefore in any particular direction we can take a single beam with initial value $\sigma_3=0$.”

(Also, your formalism might be doing that behind your back}

But .....



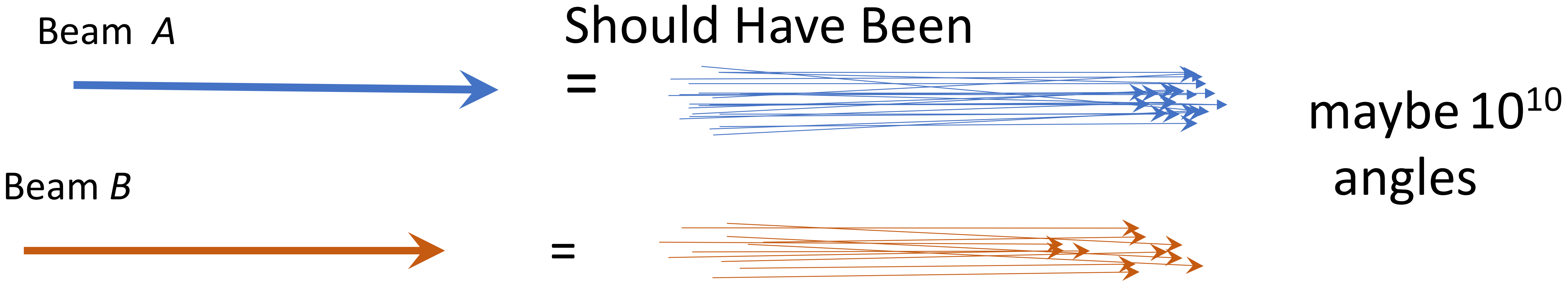
maybe 10^{10} angles

$$|\Psi_A\rangle + |\Psi_B\rangle = e^{i\sum_j \phi_j}|\mathrm{flav}^{\mathrm{A}}\rangle + e^{i\sum_j \phi'_j}|\mathrm{flav}^{\mathrm{B}}\rangle \qquad ?$$

On the RHS the red and blue direction sets are completely disjoint.

In an initial condition the above addition is clear nonsense .

The whole point of beams to replace individual particles was that the product of the unknowable phase factors remains as a multiplicative factor in the entire wave function.



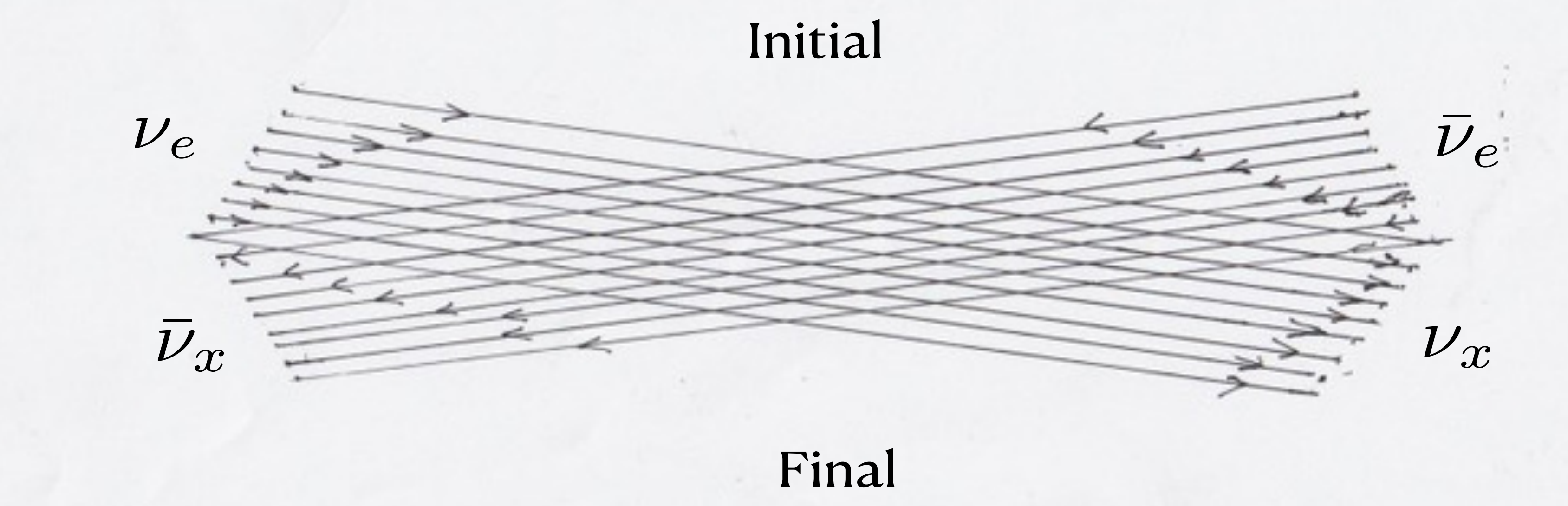
$$|\Psi_A\rangle \times |\Psi_B\rangle \Rightarrow e^{i\sum_j \phi_j}|\mathrm{flav}^{\mathrm{A}}\rangle \times e^{i\sum_j \phi'_j}|\mathrm{flav}^{\mathrm{B}}\rangle \times \text{ two more}$$

The product of unknowable phases remains:

a multiplicative factor in the entire wave function.

In the end we find that each beam pictured in the initial flows, and at each angle, must be replaced by a set.....

time



of four separate beams, ν_e , ν_x , $\bar{\nu}_e$, $\bar{\nu}_x$

So , just to simulate the evolution of these two flows, at an angle, one to another,

I need 16 beams interacting mutually with each other, with their separate initial pure flavors. Equations of evolution are standard .

Outcome: Look at the whole system's expectation value of the number operator for any one of the four species --you find that it is constant in time—of course.

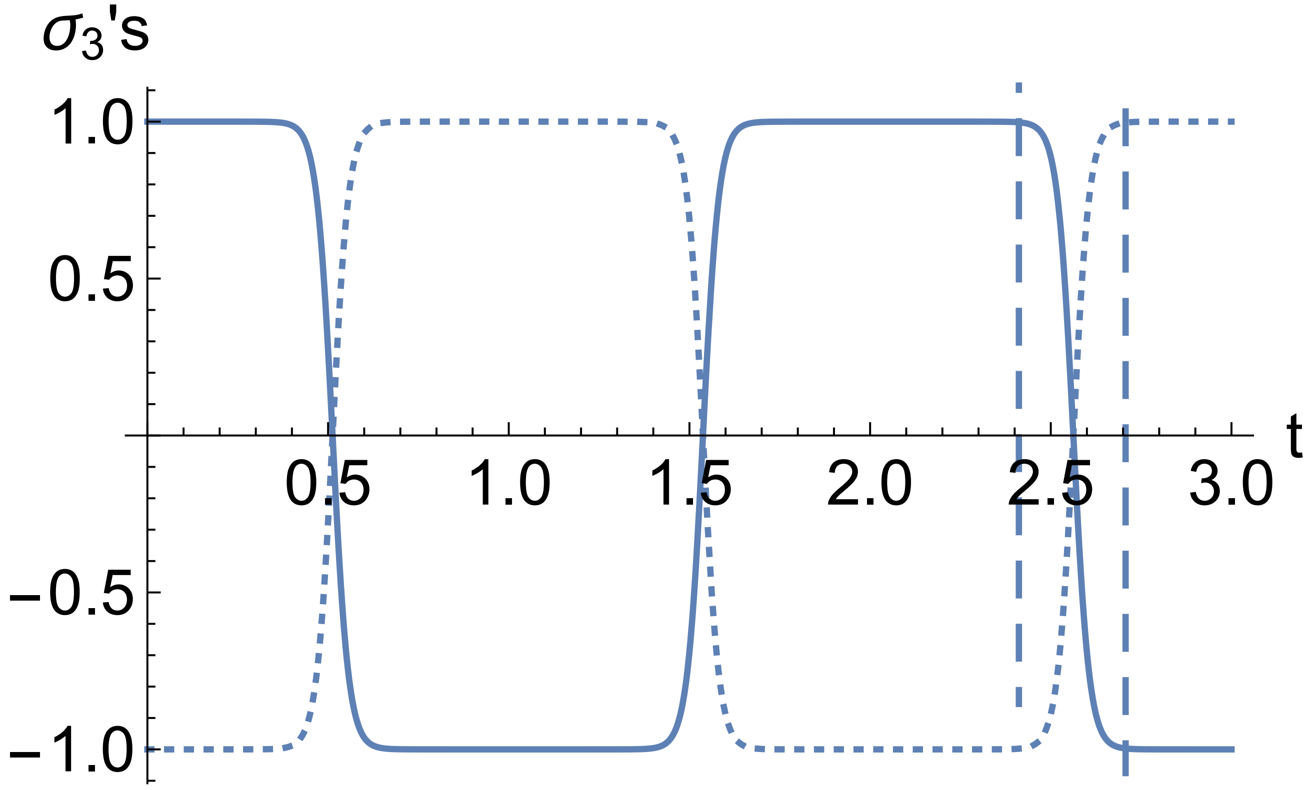
So nothing happened ?

Look at the expectation within one beam with an initial flavor of say, ν_e .

That is: following exactly the directional lines of any of the pictured 10^{10} beam constituents.

And it shows instanton-like breaks.

Single Beam Plots



Dives=instantons

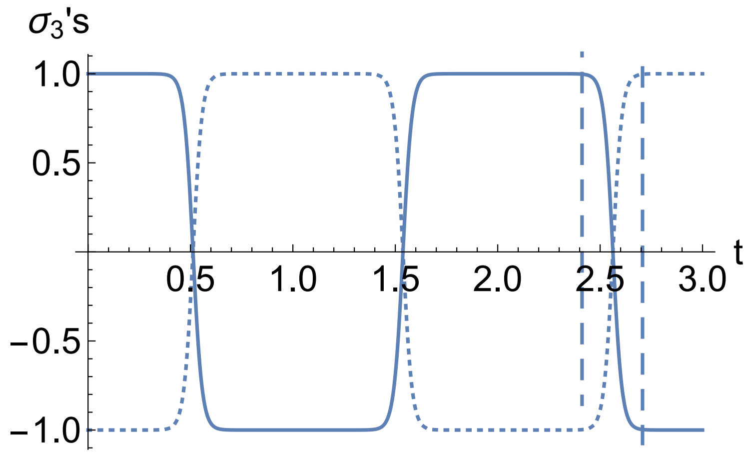
Shapes: Jacobean Elliptic

Solid line: <Flavor> of initial ν_e beam.

Dotted line: initial ν_x

See Coleman lecture:

“The double well done doubly well.”



The length of the plateaus between instantons is proportional to
Log N –where N is ν number in box.

The constant of proportionality is calculable, as in refs.
arXiv 2111.07204, and previously.

It is:

higher order in hbar but it is **not** “vacuum fluctuations”

Physical consequences for E.U.

A. Dodelson-Widrow –like mechanism for a sterile ν_s .

mass=1KeV……weakly coupled to active .

Nonlinear oscillation with 1 mm. period ……is magic
at freeing a tiny virtual component.
As compared with waiting 10^{10} cm. for a collision.

B. Reworking Dolgov's 1997 review article on small corrections to the neutrino physics in the period just before decoupling.

There is all kinds of fun to be had there.

But first some major misery.....

Bad superpositions: In the near-equilibrium EU problem I emphasized the necessity of using separate beams for each flavor in the initial state.

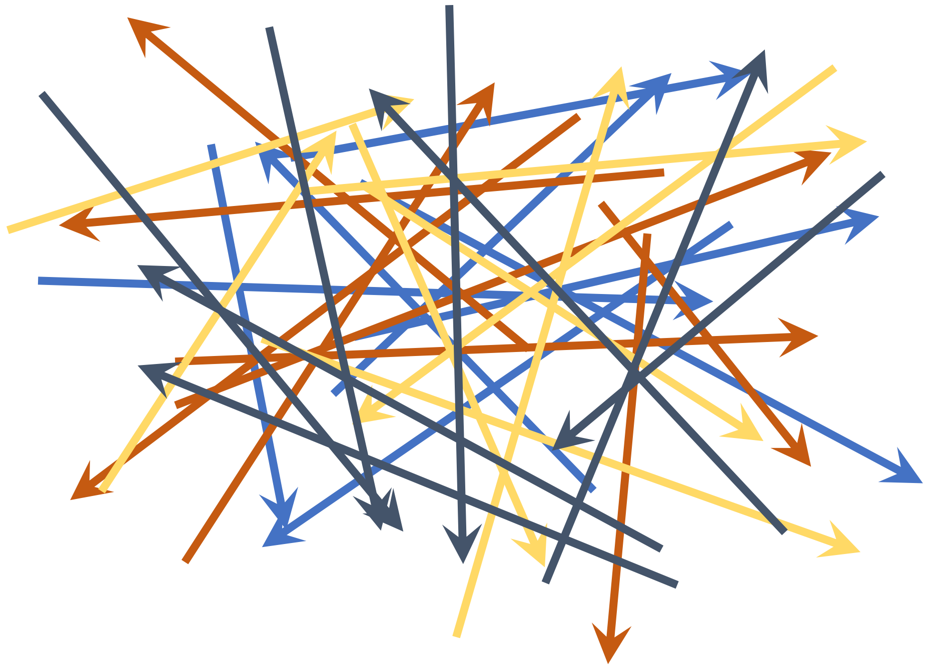
Formulating an initial condition as a set of angular moments violates that precept in supernova simulations. It is an additive superposition of beams that individually have random phase factors.

Plane waves are the required basis for ``fast" ν work precisely because neutrinos move in exact straight lines during these processes.... whatever their angular distribution might be.

How much will this matter in the S-N problem?

Consider the dictum: “if angular distributions are independent of azimuthal angle, then we can average over ϕ and write equations with only polar angle.” Step 1 of rebuttal: “stay in 2-D. Calculate fate of an isotropic distribution of <u>initial</u>: ν_e , ν_x , $\bar{\nu}_e$, $\bar{\nu}_x$ “

Each beam contains 10^{10} neutrinos



Outcome:

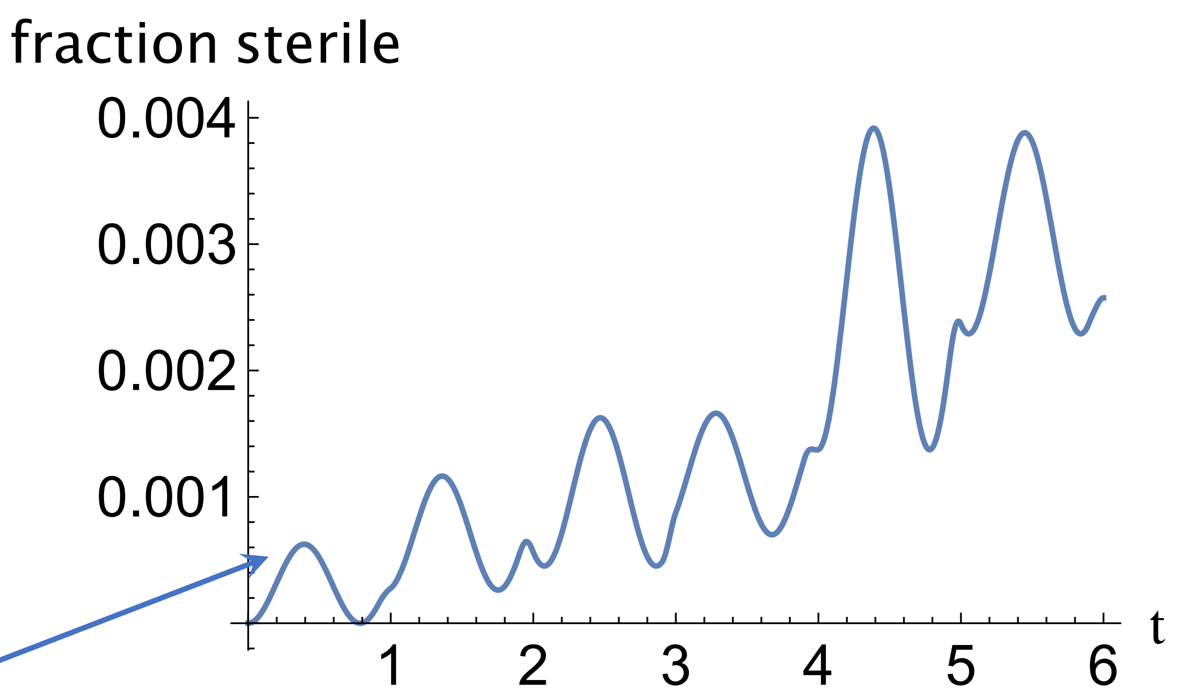
Just as in my quasi-1D case, the flavor of each beam will mix at the fast rate $n\ G_F$ as time progresses. If I take 200 beams these rates are proportional to $N\ G_F$/vol.

where $N = 200 \times 10^{10}$ ν

"So what? ", they will say. "It all averages to zero".

Go figure

Sterile production from a ν_e



First peak, more or less ---e-s oscillation in vacuum.

Later peaks : from the turmoil in the medium, as seen by single ν_e

$$
\begin{aligned}
H_{\text{eff}} =& \Big[\sigma_+\tau_- + \sigma_-\tau_+ + \tfrac{1}{2}\sigma_3\tau_3 + \bar{\sigma}_+\bar{\tau}_- + \bar{\sigma}_-\bar{\tau}_+ + \tfrac{1}{2}\bar{\sigma}_3\bar{\tau}_3 \\
& -\bar{\sigma}_+\tau_- - \bar{\sigma}_-\tau_+ - \tfrac{1}{2}\bar{\sigma}_3\tau_3 - \sigma_+\bar{\tau}_- - \sigma_-\bar{\tau}_+ - \tfrac{1}{2}\sigma_3\bar{\tau}_3)\Big] \\
& \times(1-\cos\theta)
\end{aligned}
$$

8 Equations of development

$$
-i\tfrac{d}{dt}\sigma_+ = [\sigma_+, H] \quad , \quad -i\tfrac{d}{dt}\sigma_3 = [\sigma_3, H] \ldots\ldots.
$$

$$
-i\tfrac{d}{dt}\bar{\tau}_-+ = [\bar{\tau}_+, H] \quad , \quad -i\tfrac{d}{dt}\bar{\tau}_3 = [\bar{\tau}_3, H]
$$

Etc.