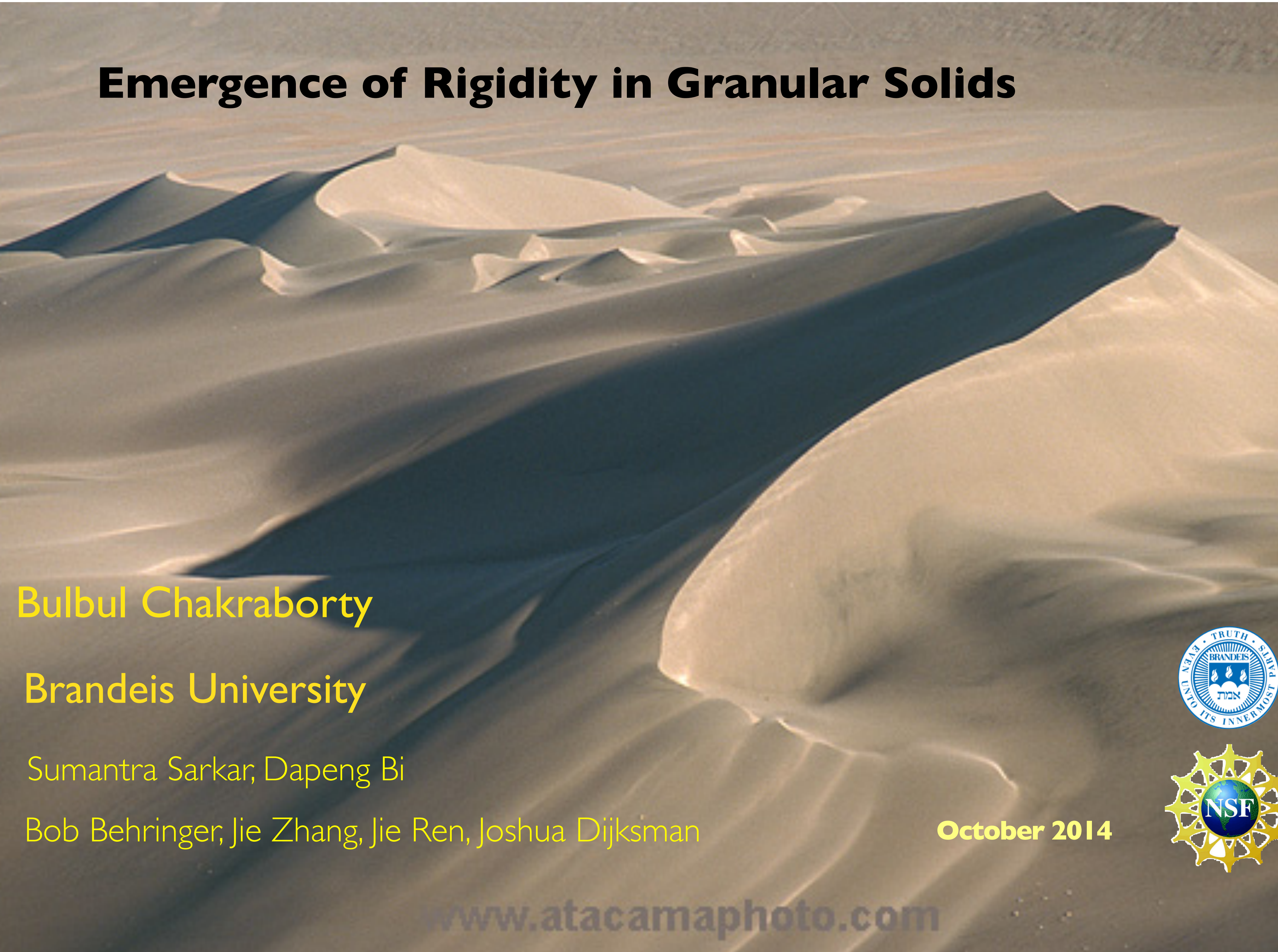
Emergence of Rigidity in Granular Solids



Bulbul Chakraborty

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October 2014

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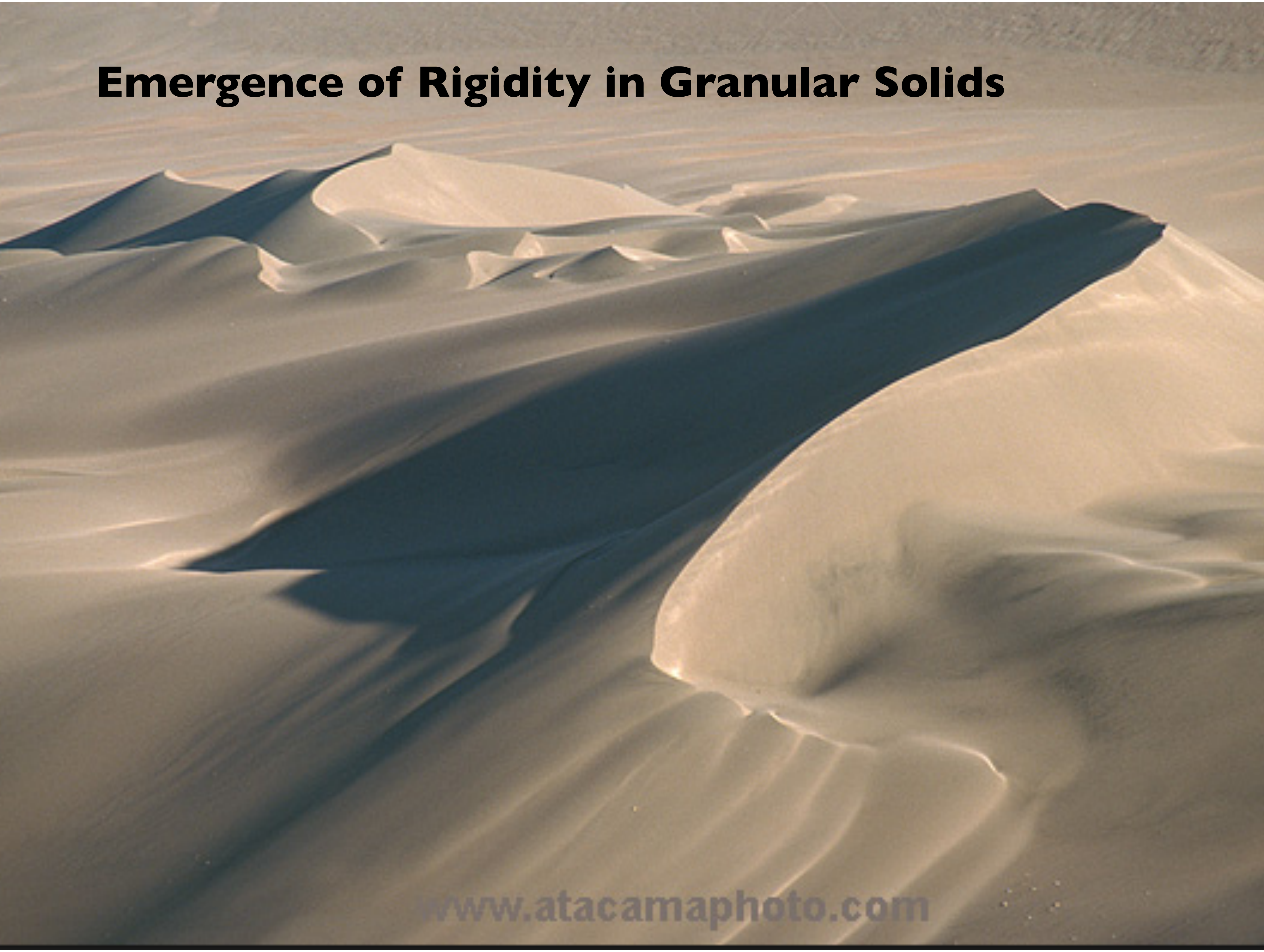

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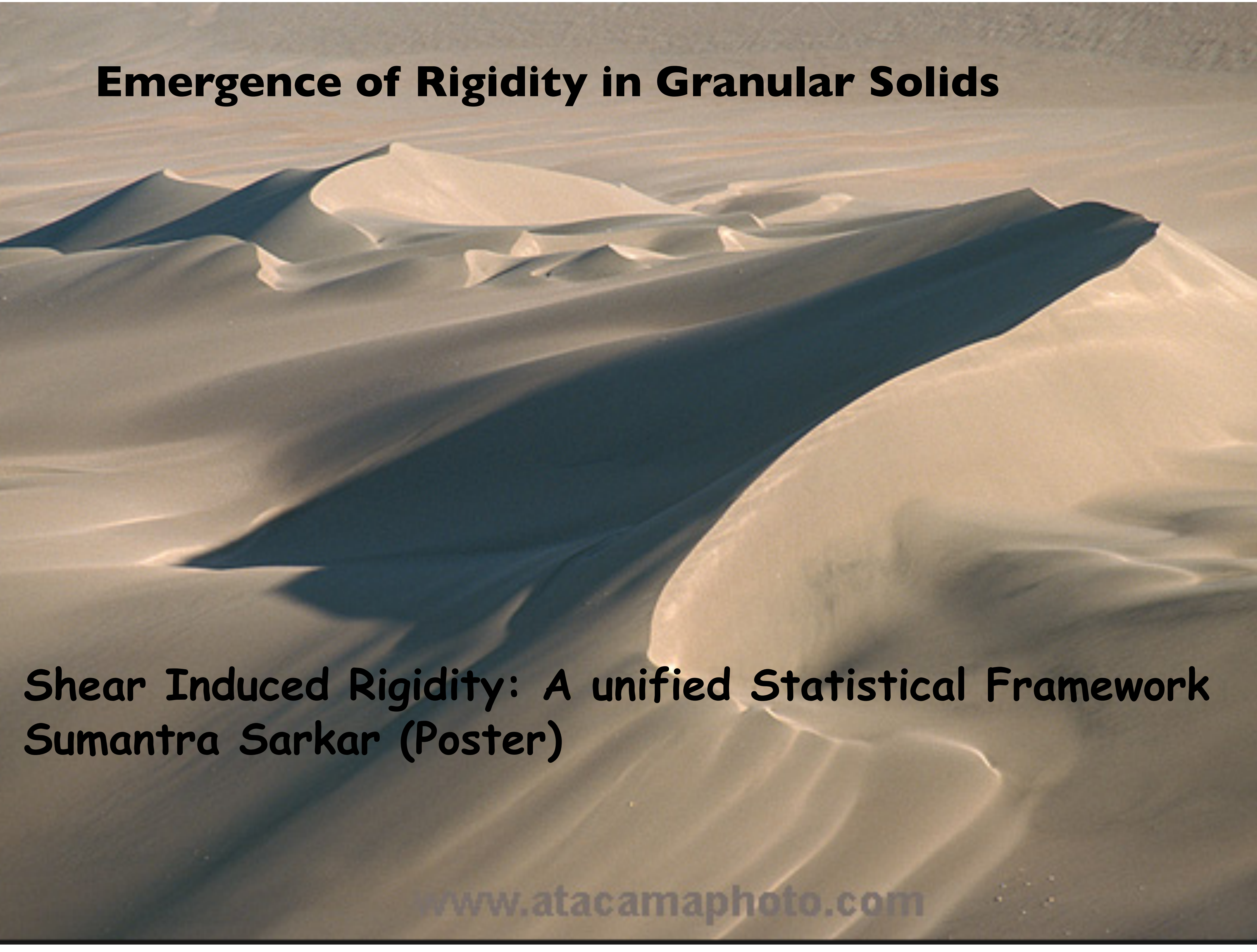
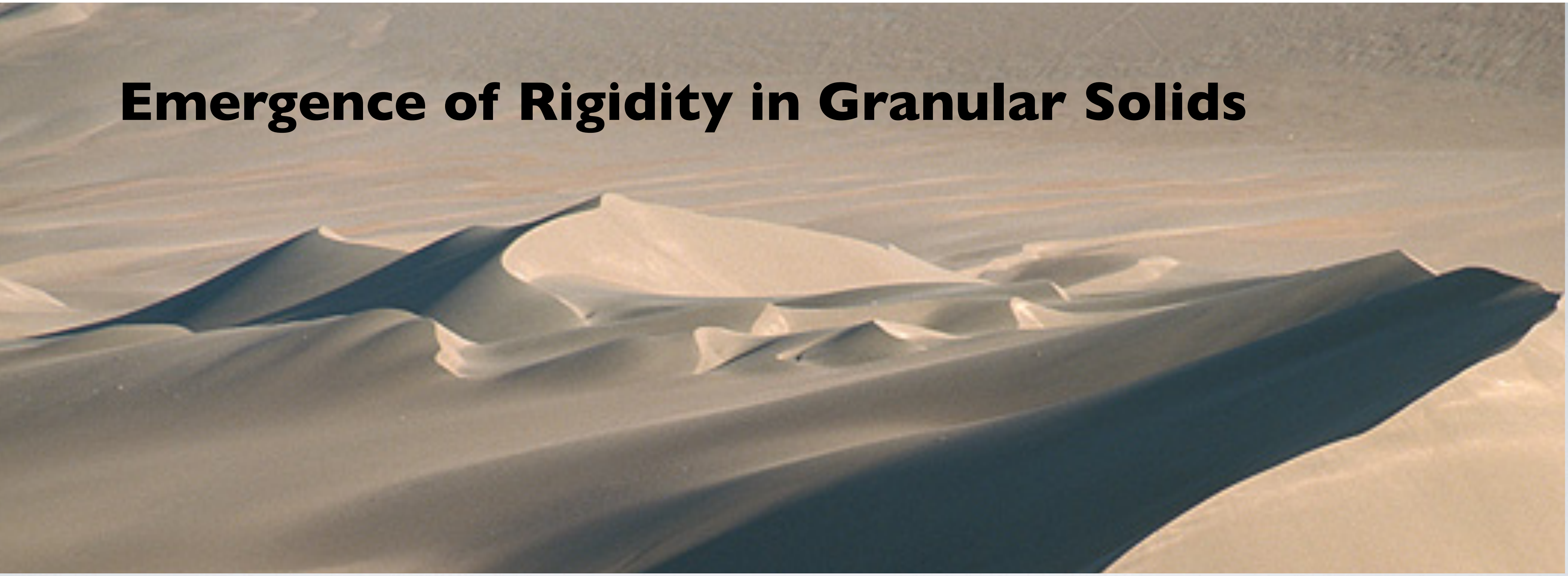


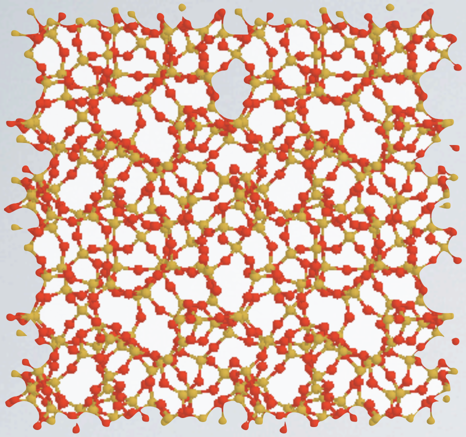
Shear Induced Rigidity: A unified Statistical Framework

Sumantra Sarkar (Poster)

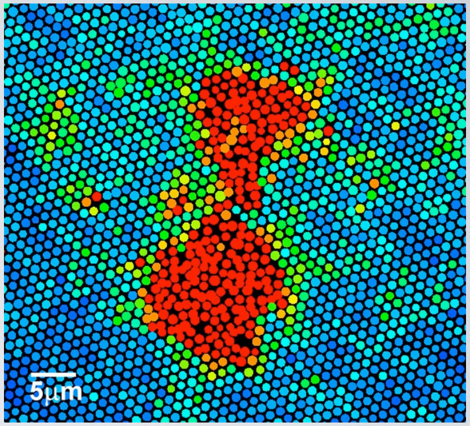
Emergence of Rigidity in Granular Solids



- Dry grains: Purely Repulsive Contact Interactions
- Thermal Fluctuations Irrelevant
- Frictional contacts
- Stay in a single microscopic configurations unless driven
- Limit of “just touching” where contact breaking/forming is important



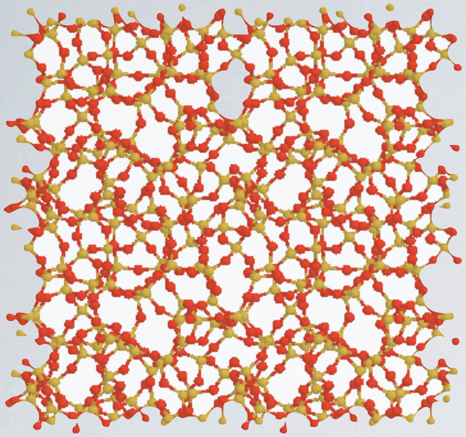
Rigidity of amorphous solids



- Broken translational symmetry (not obvious because structure is not crystalline) Patterns of particles : not destroyed by small thermal fluctuations, average survives, correlations survive, & Patterns persist under strain: shear rigidity

- Traditionally: energy or entropy gain leads to solidification

- Dry grains: no cohesive interactions and no thermal fluctuations



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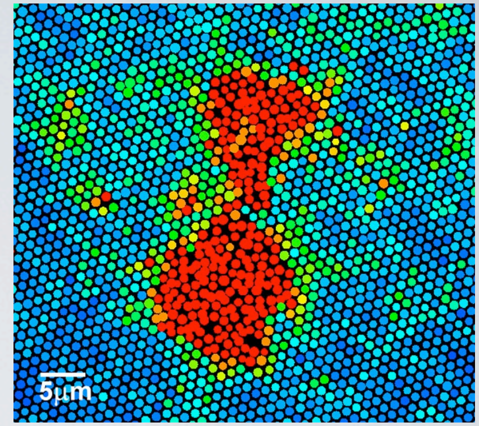

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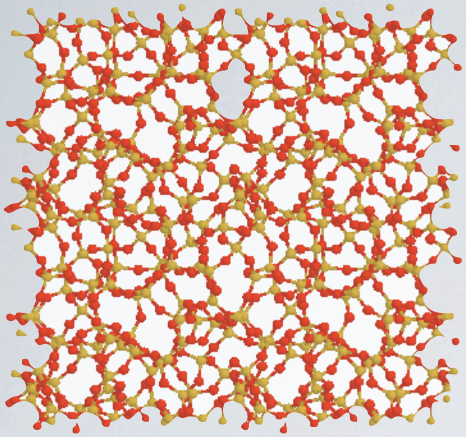
Broken Translational Symmetry in position space is a necessary but not a sufficient condition for rigidity



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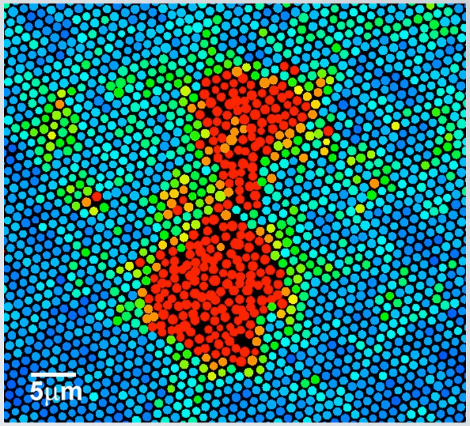

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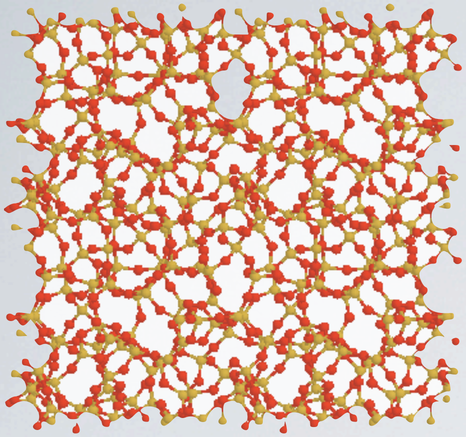
Need Broken Translational Symmetry in a Reciprocal Space



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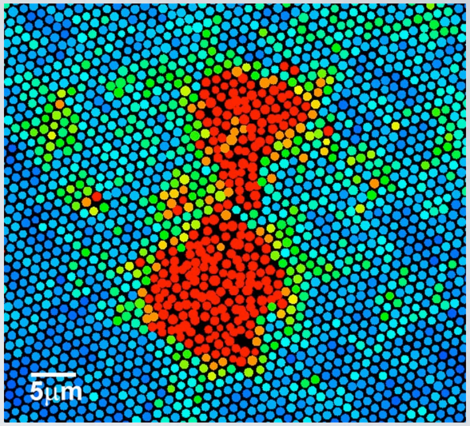

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Broken Translational Symmetry in position space is a necessary but not a sufficient condition for rigidity

Need Broken Translational Symmetry in a Reciprocal Space

Since the nature of these solids is different the way they fail is most likely different also

A Story of Constraints

Local force & torque balance satisfied for every grain

Friction law on each contact $f_t \leq \mu f_N$

Positivity of all forces $f_N \geq 0$

Imposed stresses determine sum of stresses over all grains

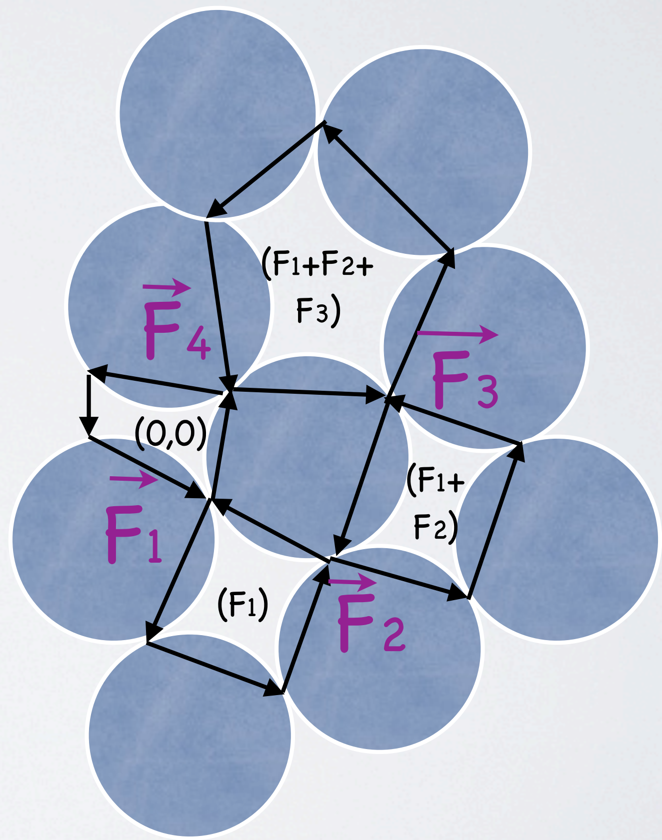
Imposing the conditions through gauge potentials (2D)

Ball & Blumenfeld (2002), Henkes, Bi, & BC (2007---), DeGuili (2011--)

- Vector fields enforce force balance constraint
- Additional scalar field enforces torque balance
- There is a relation between the two

Looking at the vector fields

We refer to them as heights: like a vector height field familiar in the context of groundstates of some frustrated magnet



Here the fields are continuous

Imposing the conditions through gauge potentials (2D)

Ball & Blumenfeld (2002), Henkes, Bi, & BC (2007---), DeGuili (2011--)

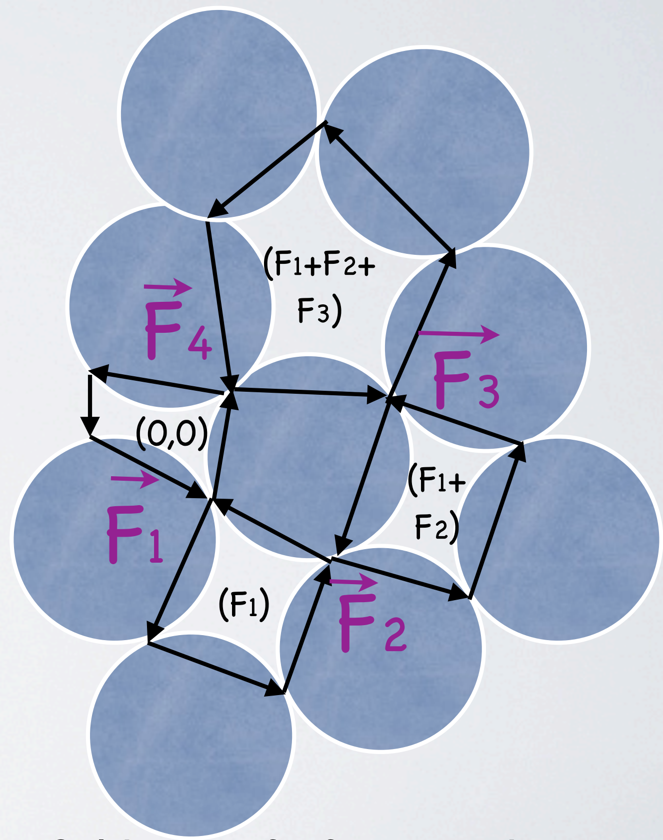
loops enclosing voids

$$\boldsymbol{f}_g^c = \boldsymbol{\rho}^{\ell'} - \boldsymbol{\rho}^{\ell},$$

$$\boldsymbol{r}^c \times \boldsymbol{f}_g^c = \varphi^{\ell'} - \varphi^{\ell} + \boldsymbol{r}^{\ell'} \times \boldsymbol{\rho}^{\ell'} - \boldsymbol{r}^{\ell} \times \boldsymbol{\rho}^{\ell}$$

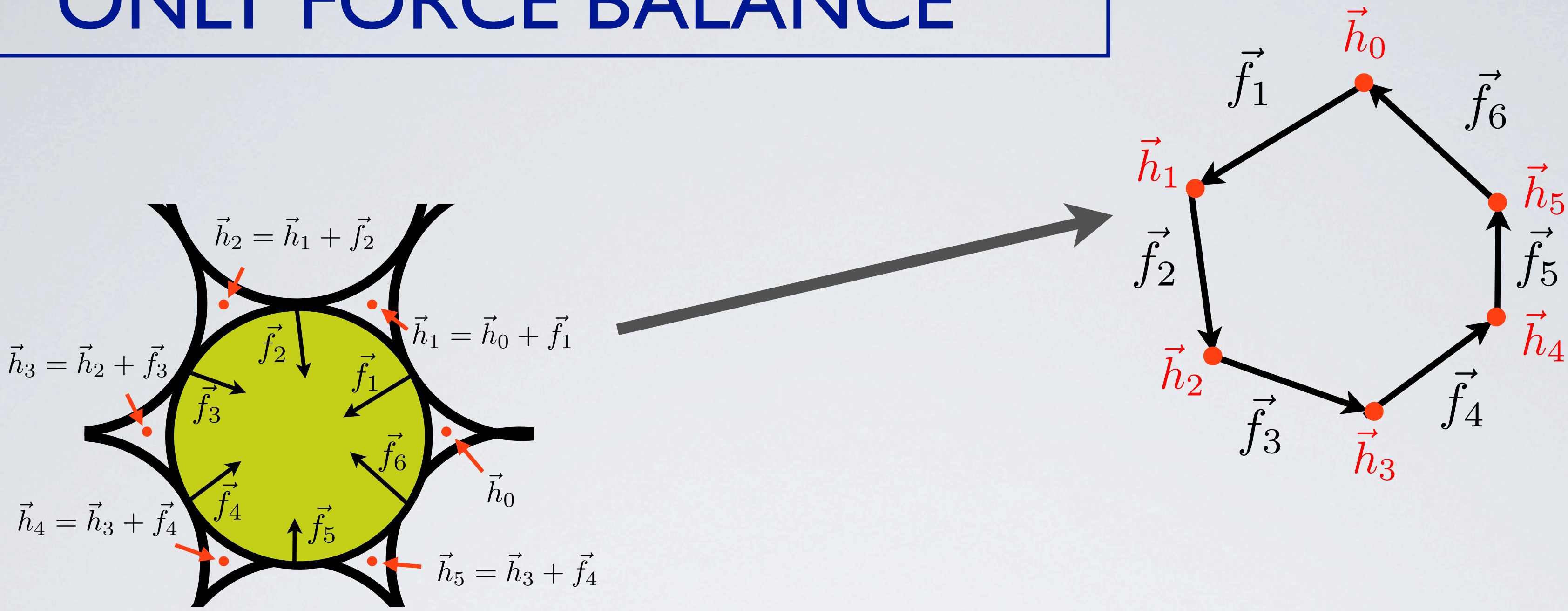
scalar potential

height vector



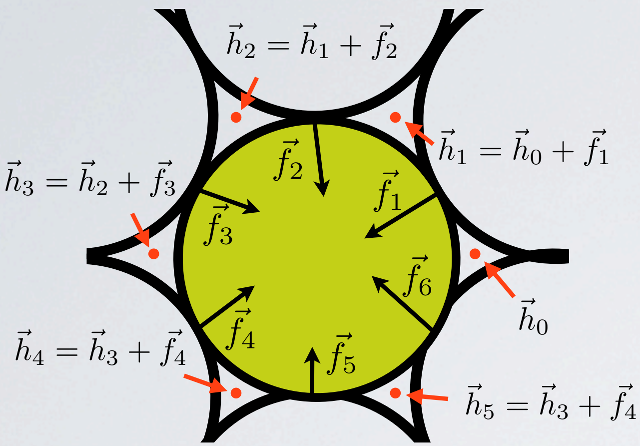
Gauge potentials: irrelevant additive constant. Any set of these fields satisfy force and torque balance. There are constraints relating the two potentials.

ONLY FORCE BALANCE



force tile

ONLY FORCE BALANCE



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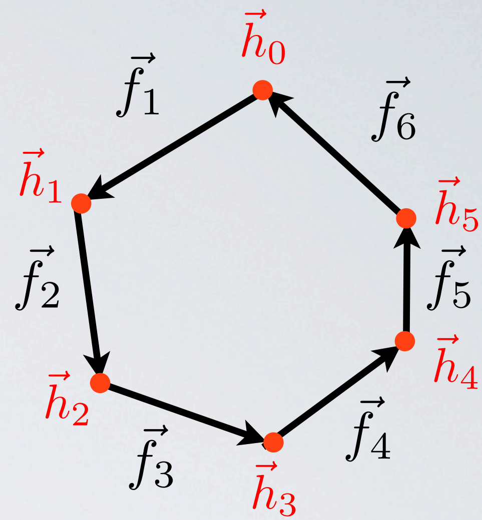
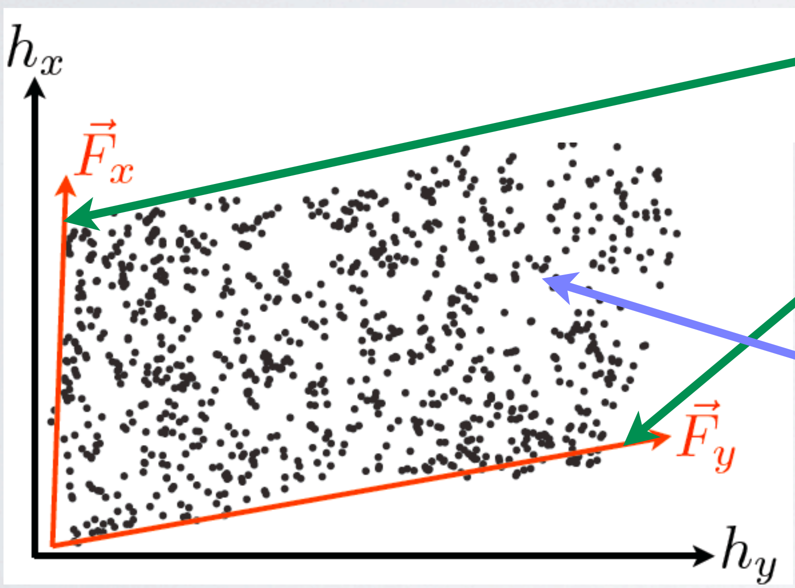
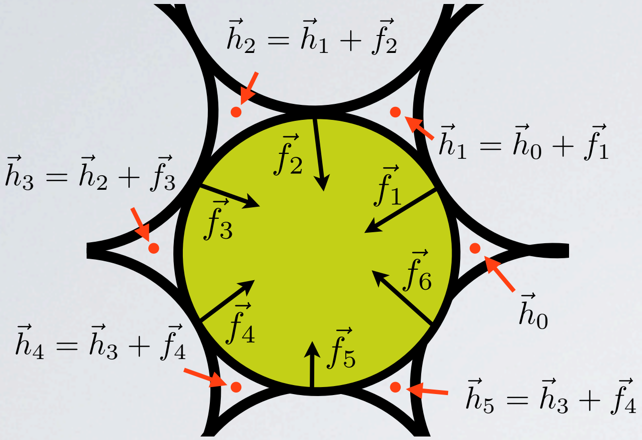


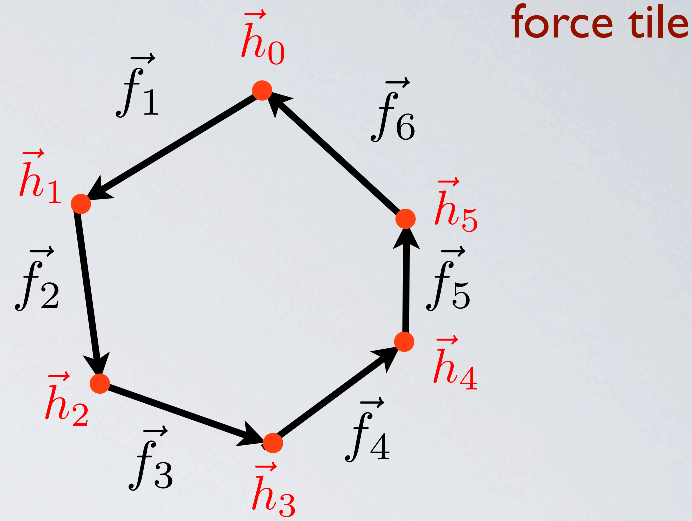


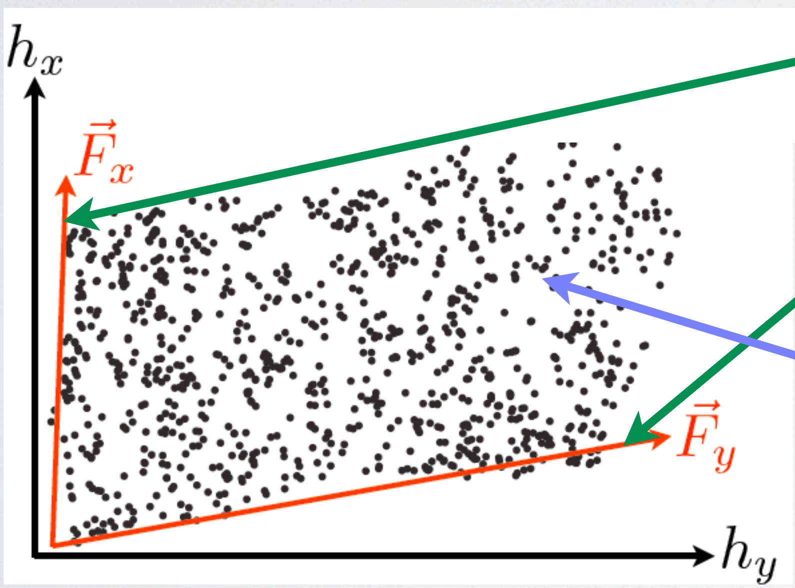
Height difference across the boundaries: determined by boundary stresses

Points: height vectors starting from some arbitrary origin

ONLY FORCE BALANCE







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for systems where forces are all repulsive, we have a single sheet

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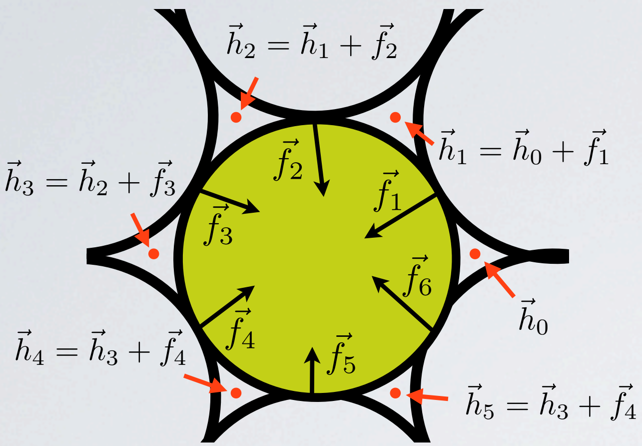
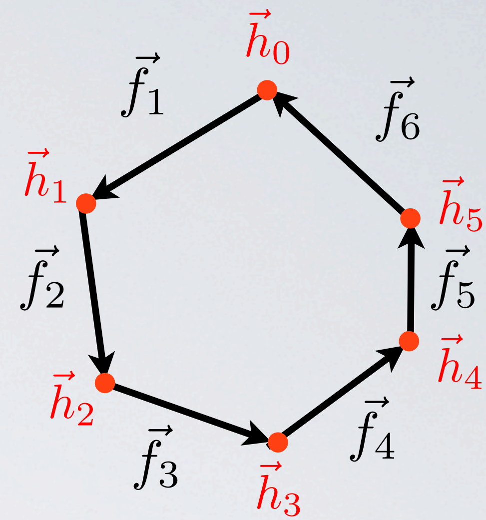
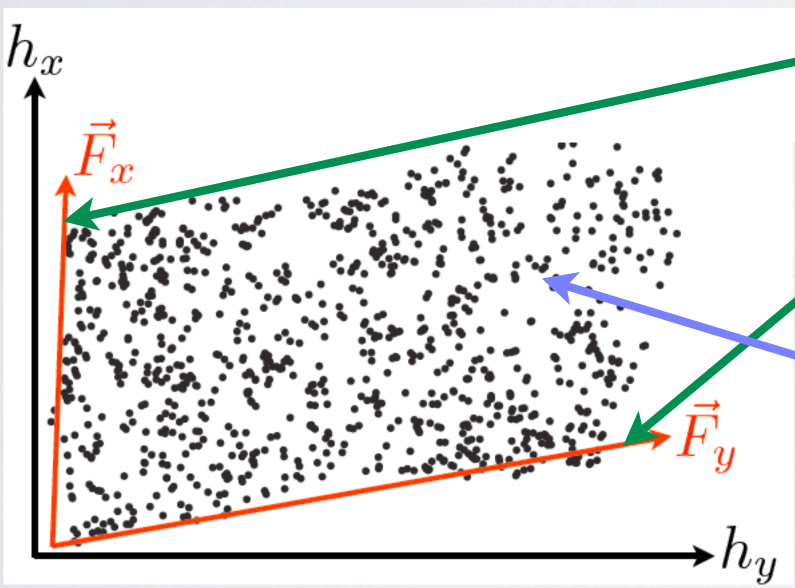


force tile





Height difference across the boundaries: determined by boundary stresses

Points: height vectors starting from some arbitrary origin

Reciprocal space: a tiling of polygons whose vertices are the heights

for systems where forces are all repulsive, we have a single sheet

Torque Balance

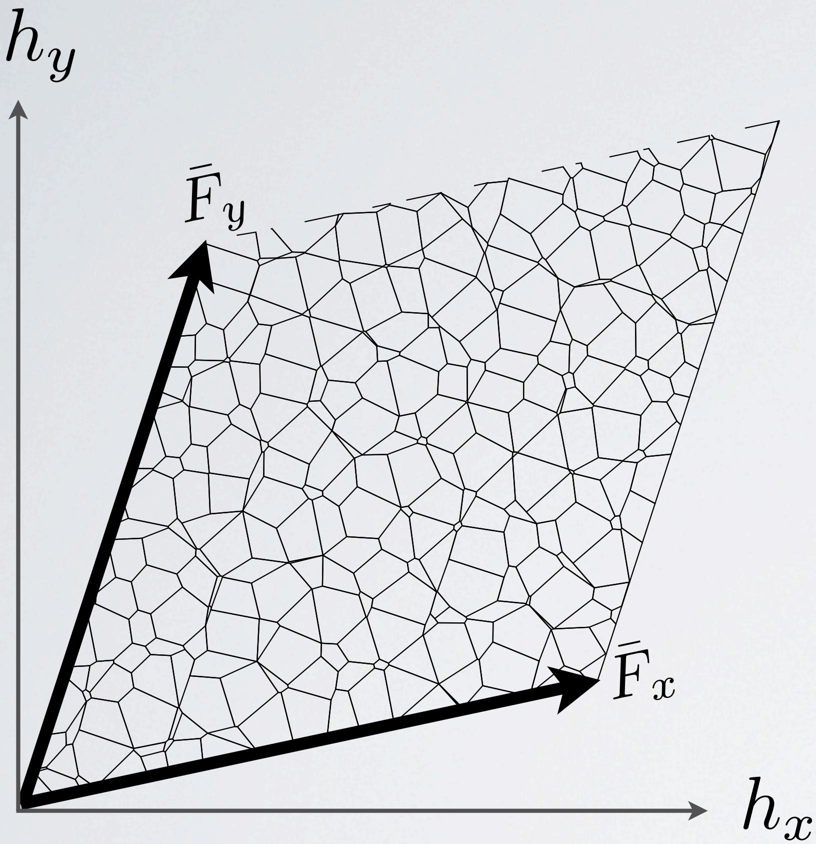
Friction law on each contact

Torque Balance | **Friction law on each contact**

★**Do these introduce correlations ?**

★**Example: Polygons have to be convex for frictionless, convex-shaped grains**

Torque Balance | **Friction law on each contact**



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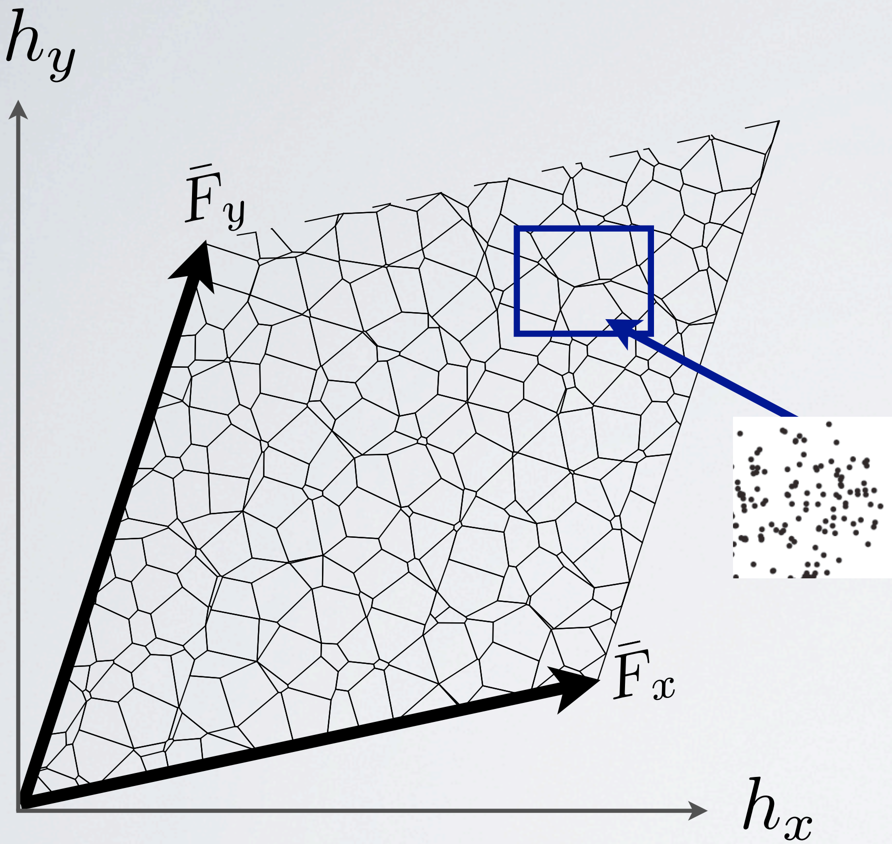
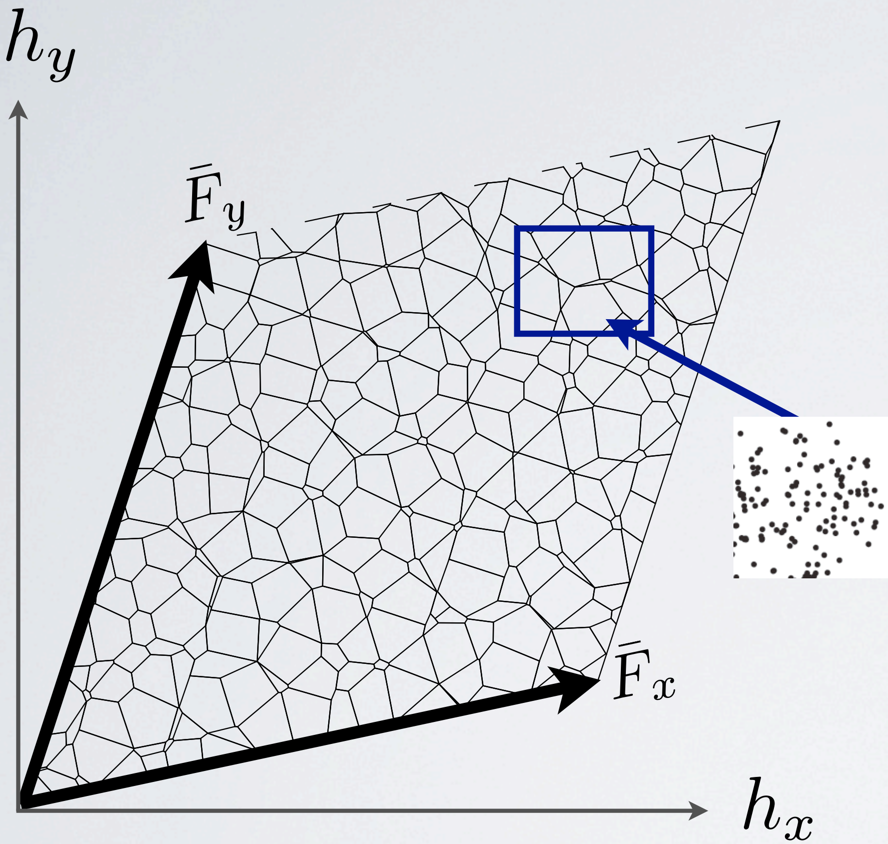


$$\langle \rho(\vec{h}) \rangle \neq const$$

Torque Balance | **Friction law on each contact**



★Do these introduce correlations ?
★Example: Polygons have to be convex for frictionless, convex-shaped grains

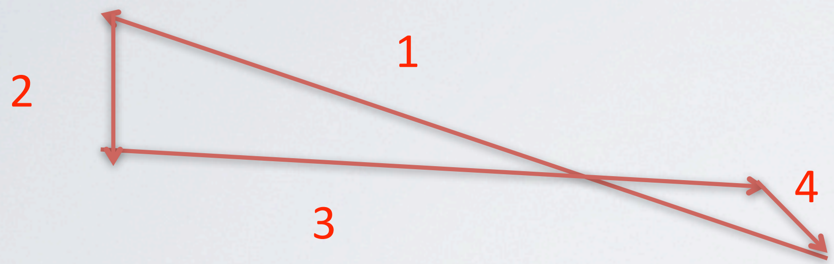
$$\langle \rho(\vec{h}) \rangle \neq const$$

RIGIDITY

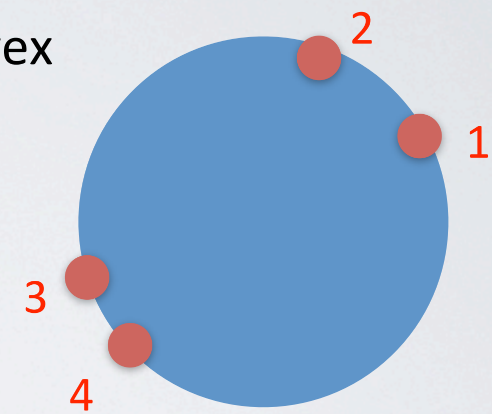
★Changing bounding box is the analog of straining
★Does the pattern persist ?

Torque Balance | **Friction law on each contact**

With friction: force tiles can be convex and non-convex



Torque balance from 2 large and two small forces



★**Do not have a theory of how the additional constraints affect the geometry of the tiles, and therefore correlations amongst heights**

★**Let's analyze experiments and see**

Shear-induced Solidification in a Model Granular System

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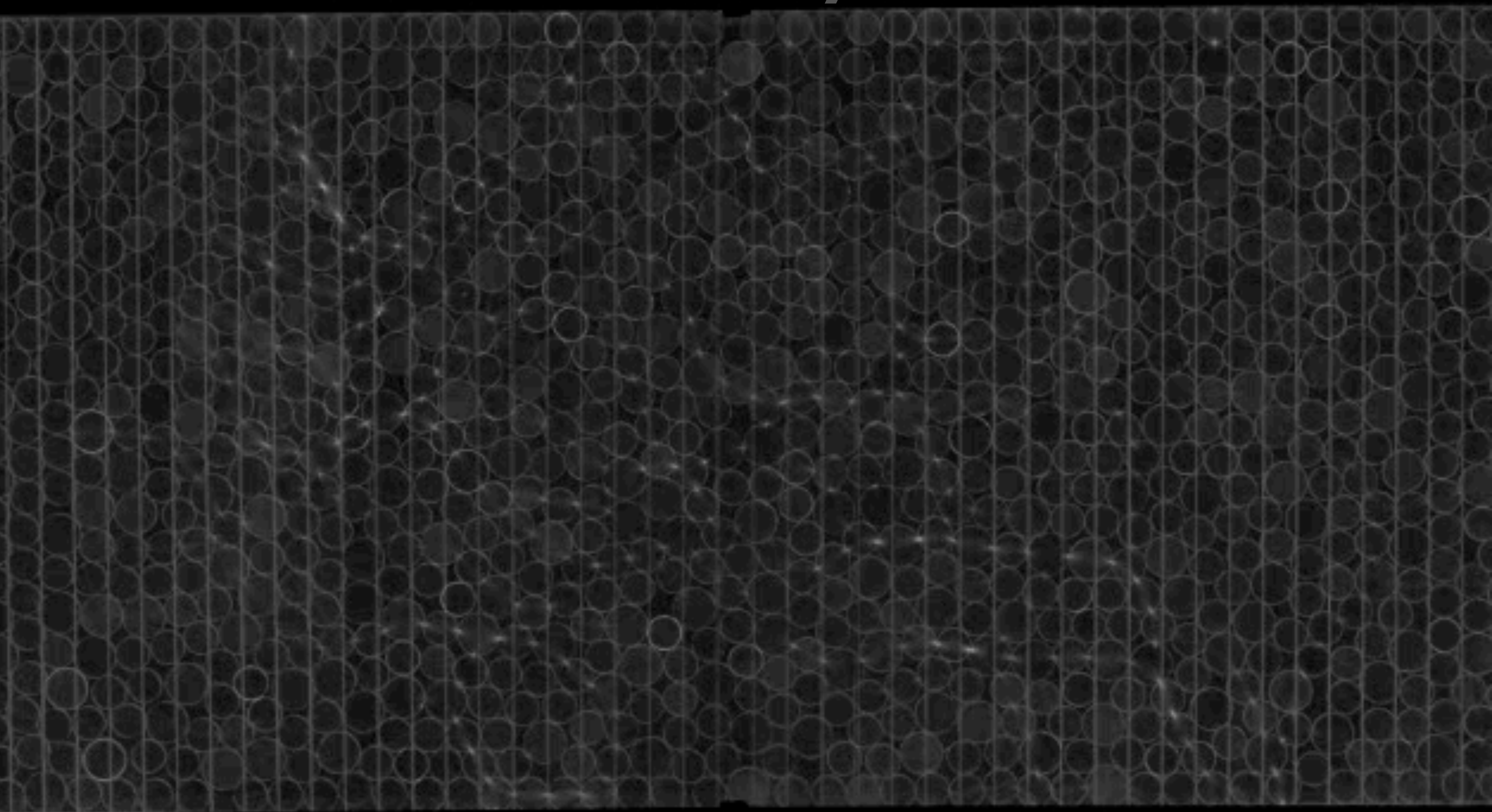

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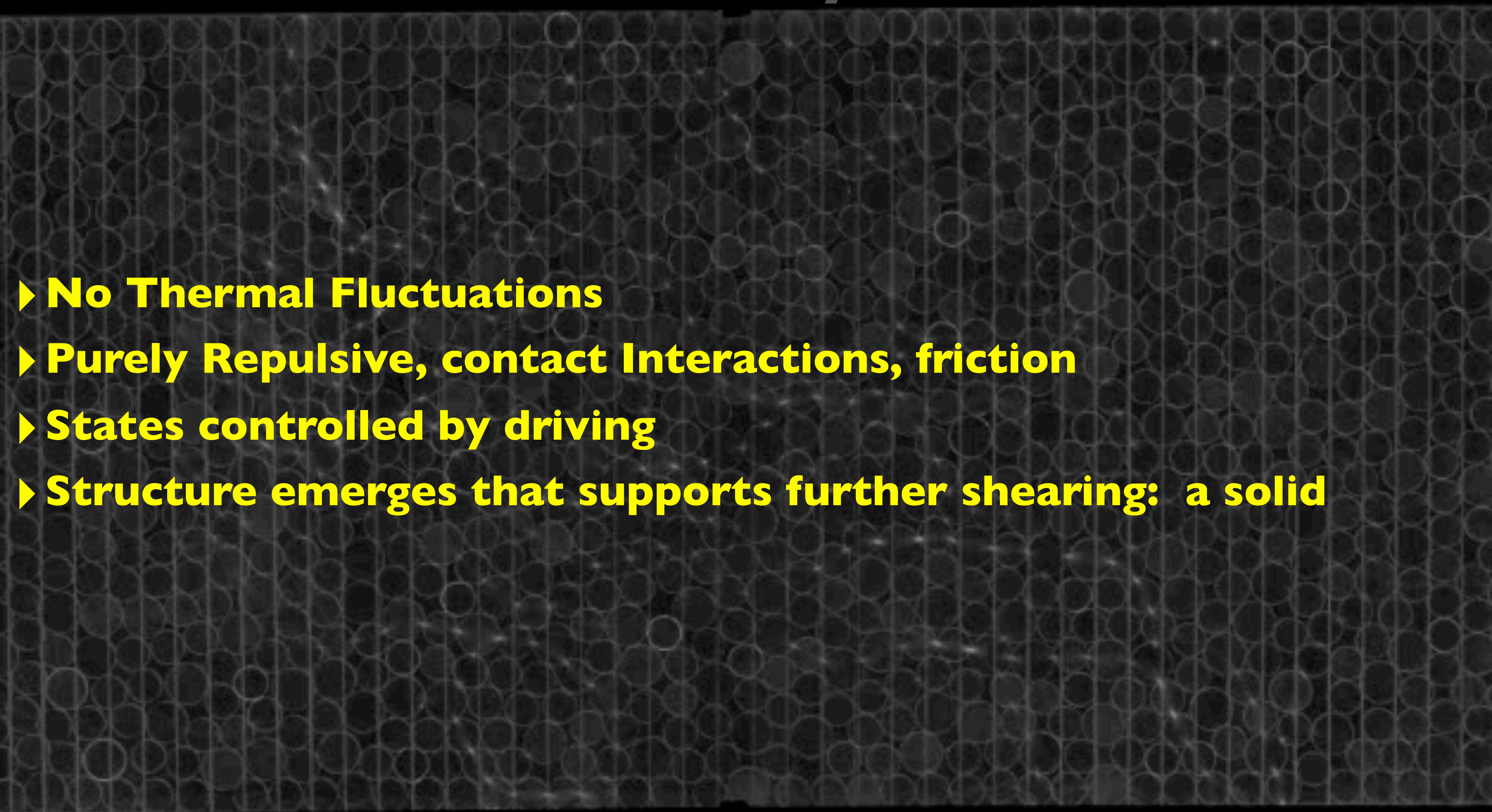
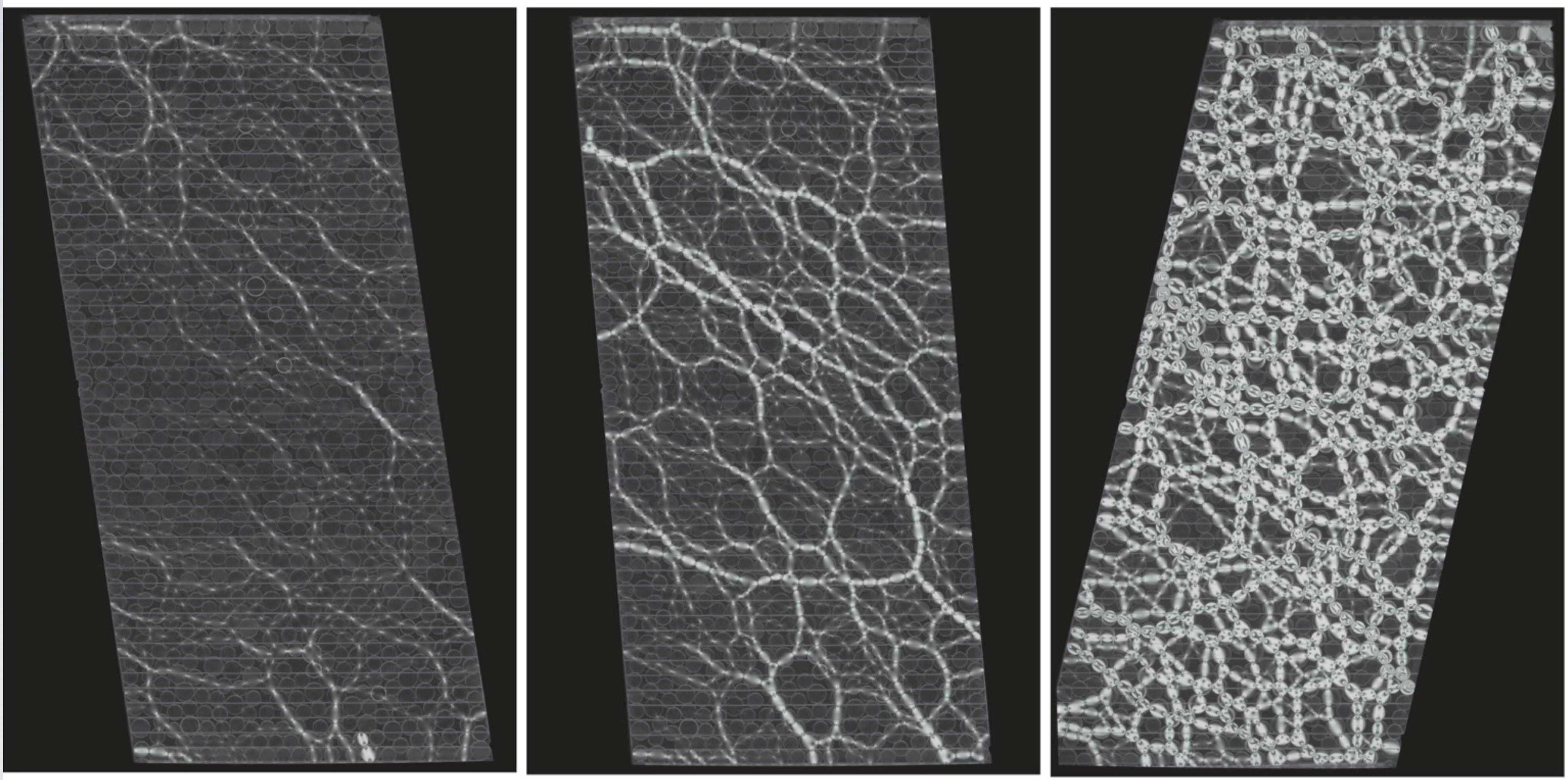
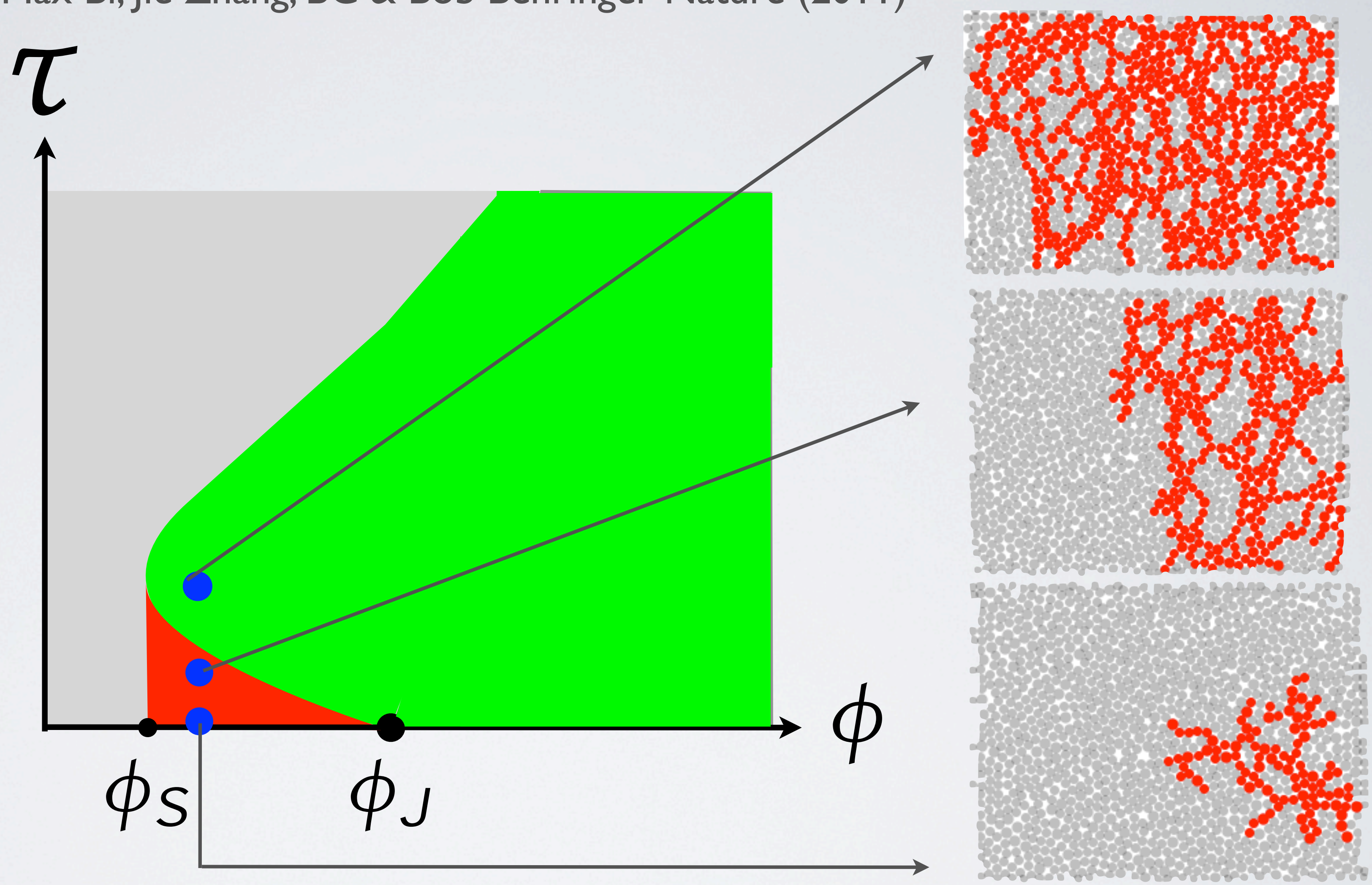


- **No Thermal Fluctuations**
- **Purely Repulsive, contact Interactions, friction**
- **States controlled by driving**
- **Structure emerges that supports further shearing: a solid**



Shear-Jamming Experiments (Quasistatic Forward + Cyclic Shear)

Max Bi, Jie Zhang, BC & Bob Behringer Nature (2011)



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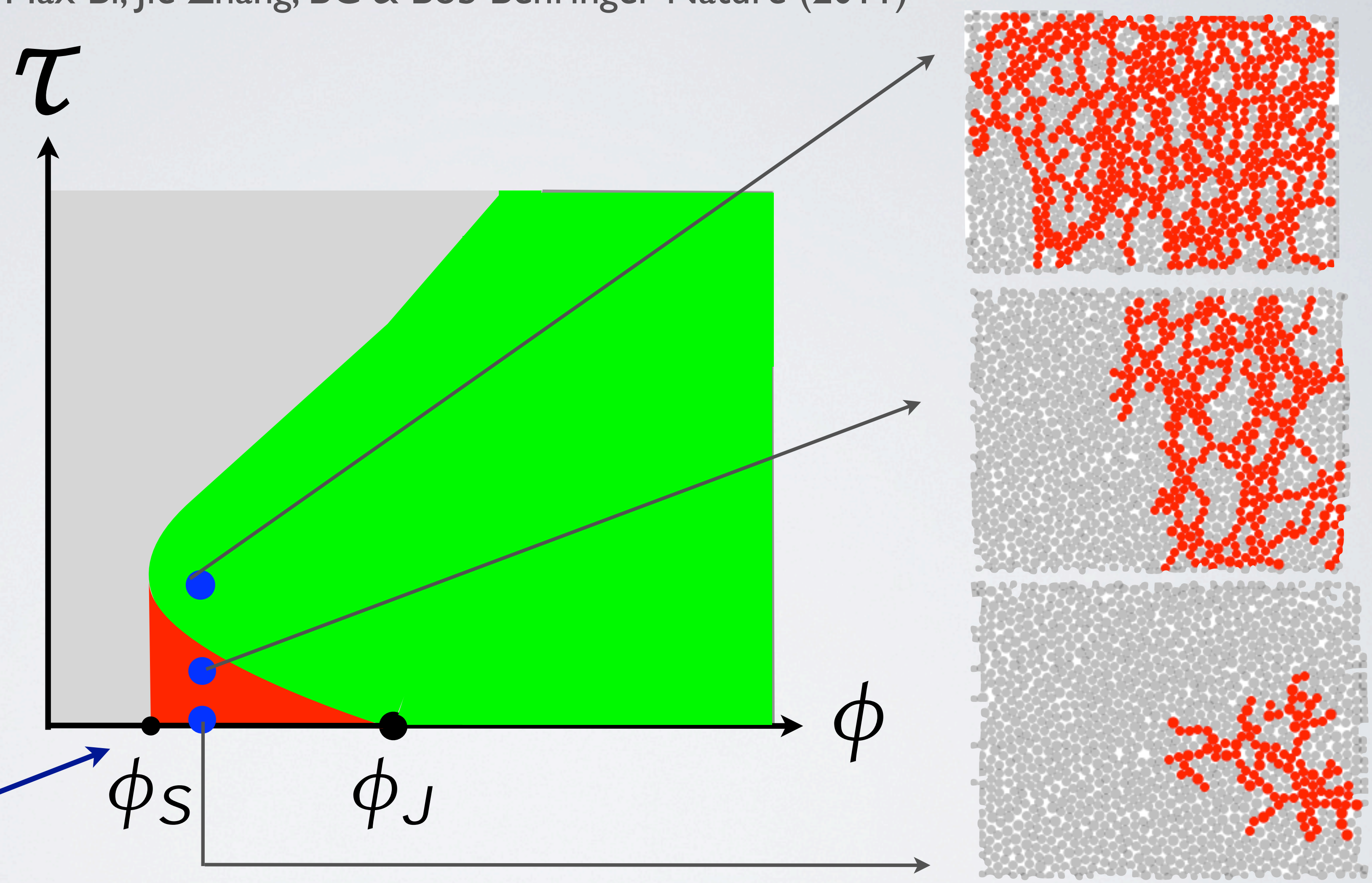

Onset density for broken translational symmetry in position space

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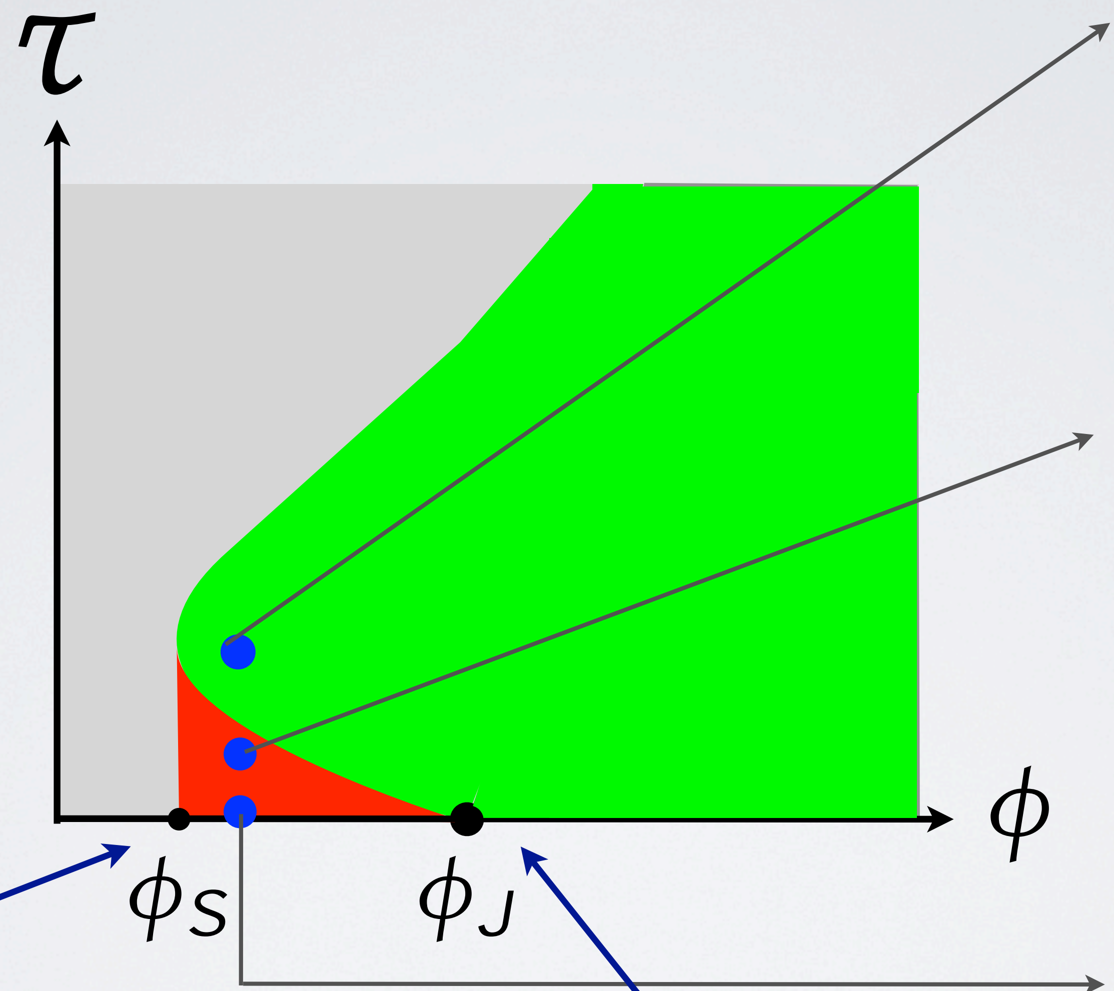
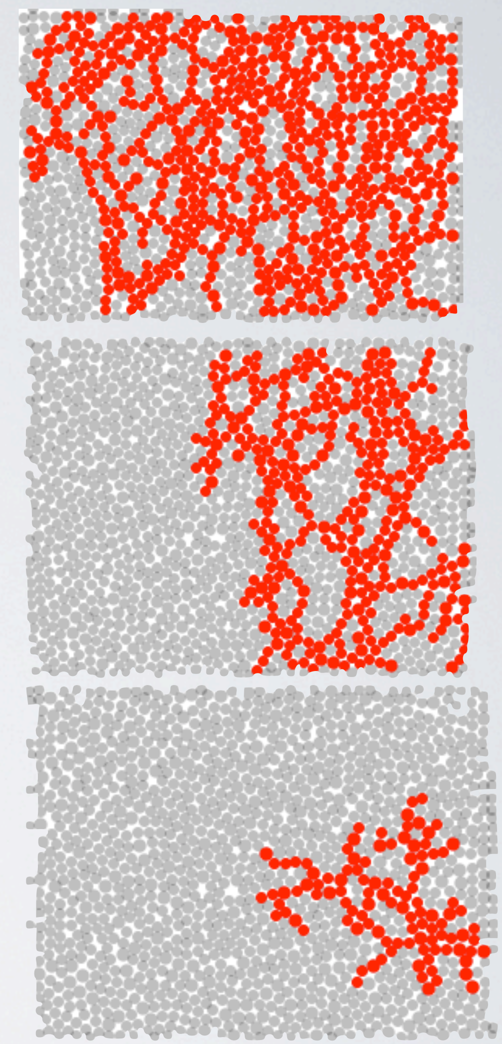
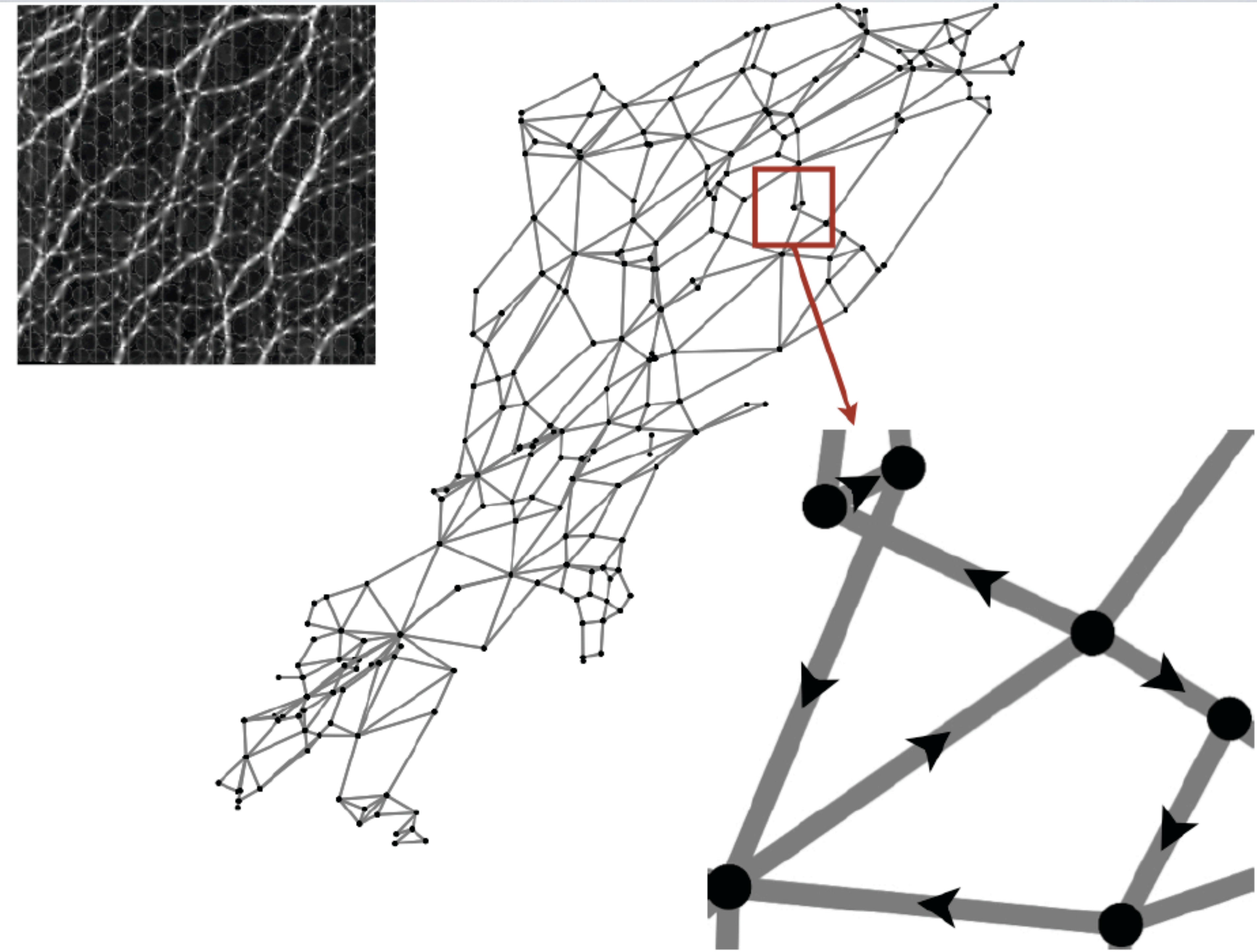




Onset density for broken translational symmetry in position space

Lowest density for jamming without shear



EVOLUTION IN RECIPROCAL SPACE

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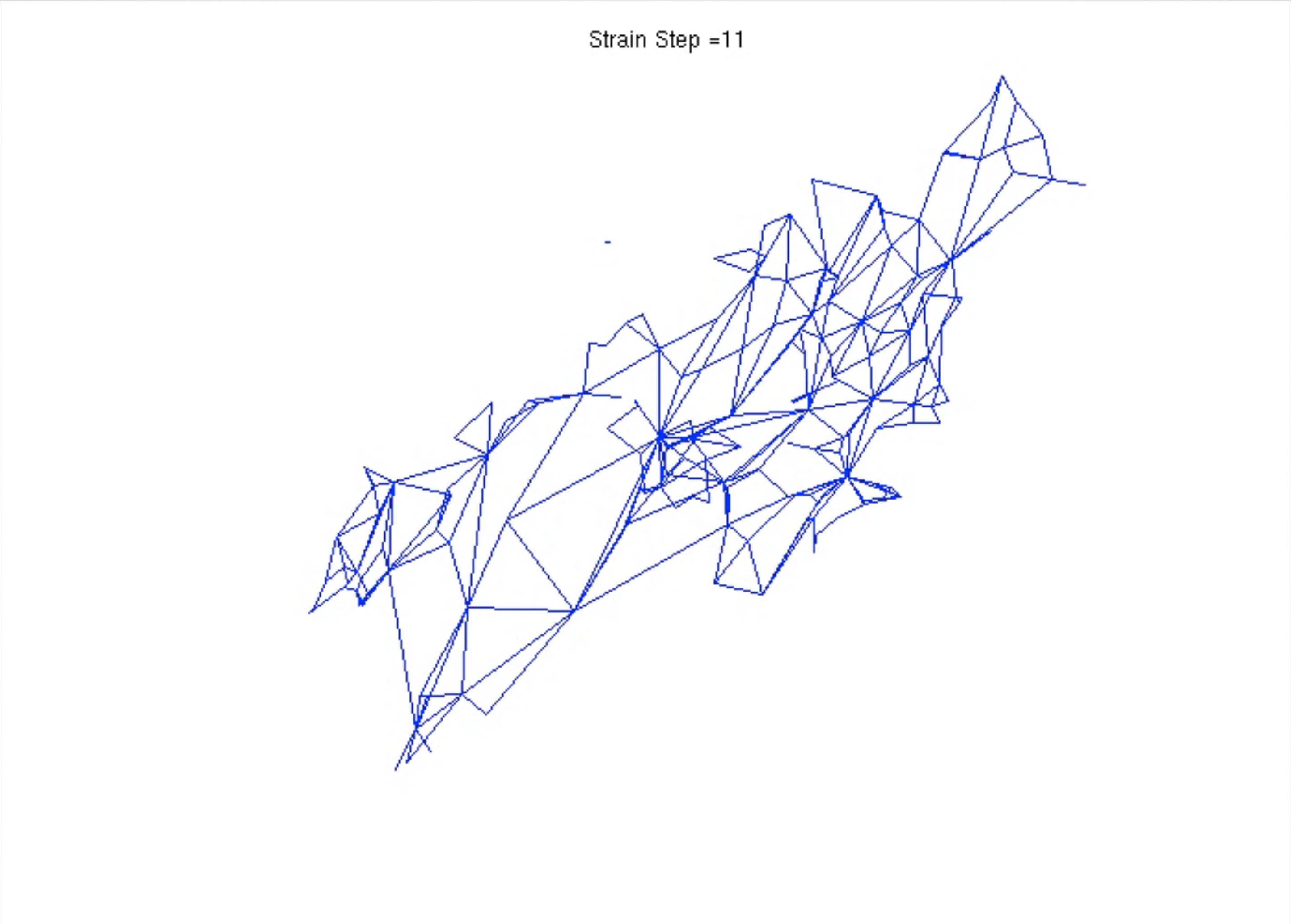


TEST OF PERSISTENT PATTERN IN HEIGHT SPACE

Overlap between two configurations

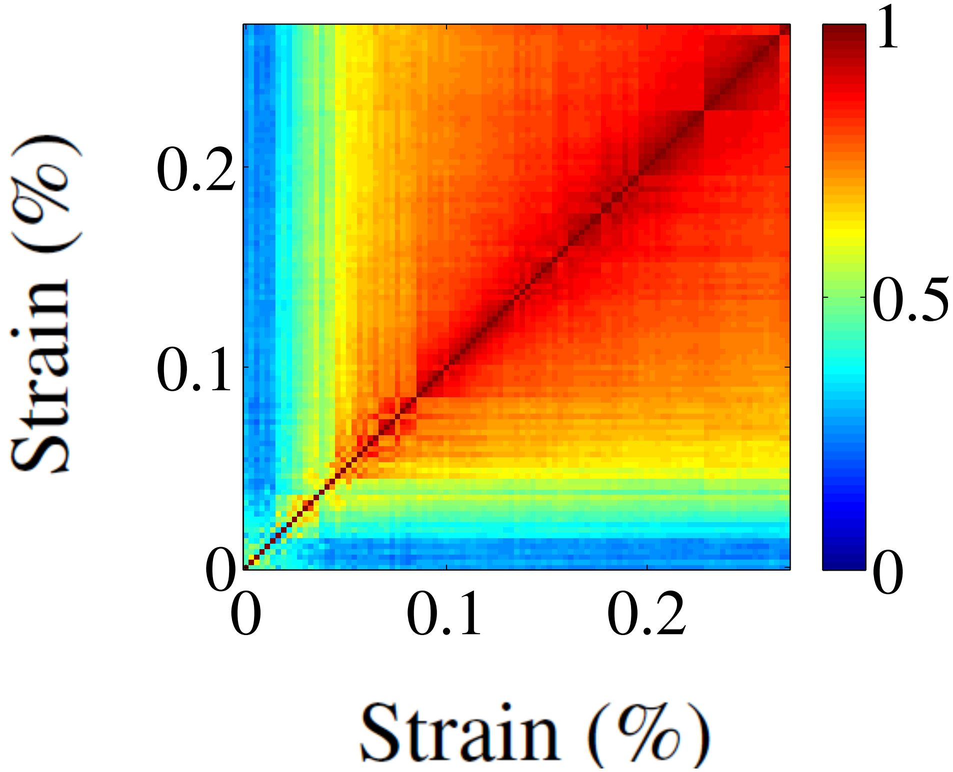
Grid stretched affinely with bounding box

$$d^{\alpha,\beta} = \sum_{m,n} \rho^{\alpha}_{m,n} \rho^{\beta}_{m,n}$$

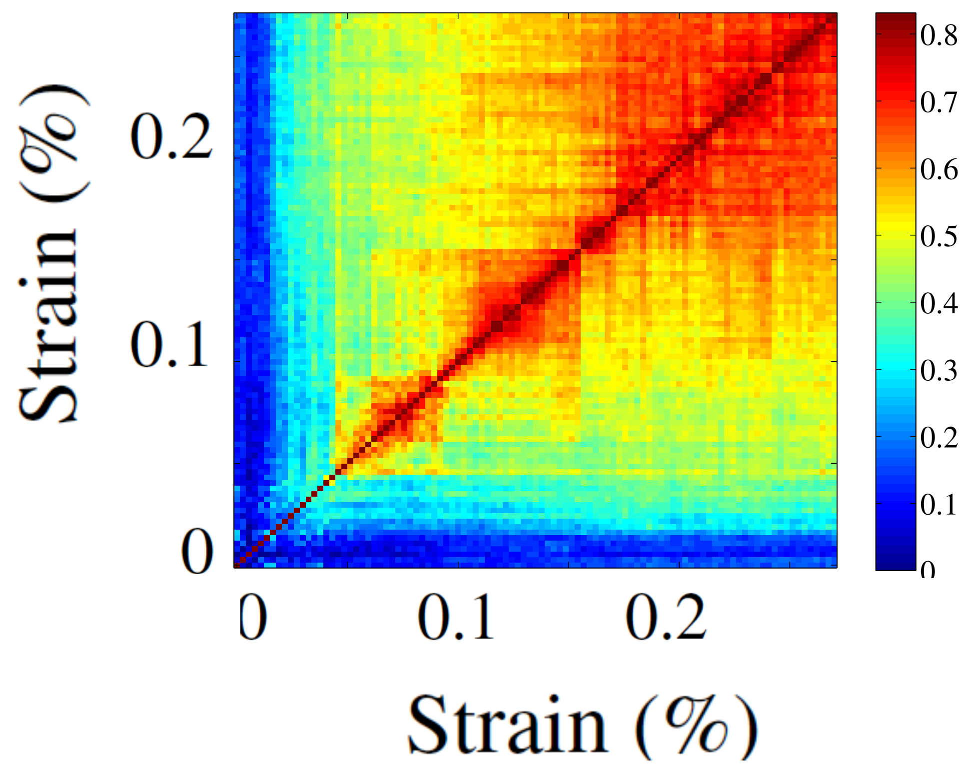
If height pattern evolves affinely, large overlap
The stress-generated pattern can sustain further loading

Persistence of Structure: Overlap

Real Space

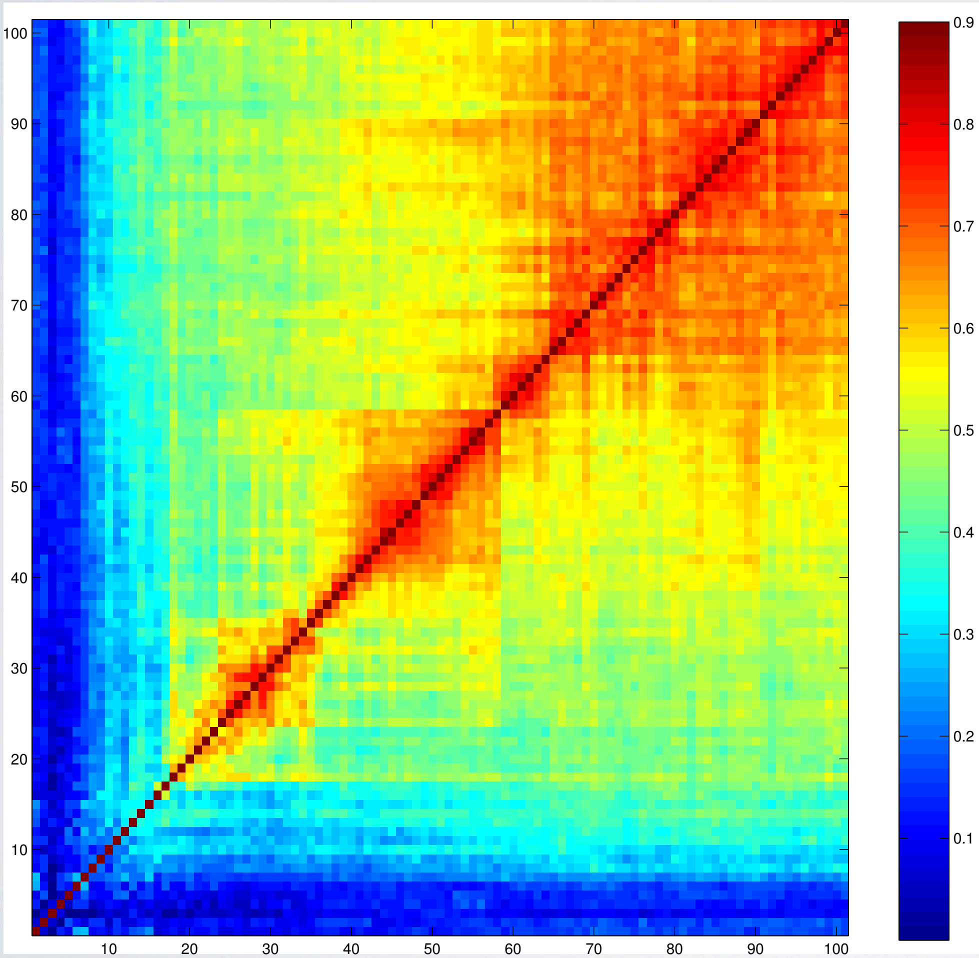


Reciprocal Space

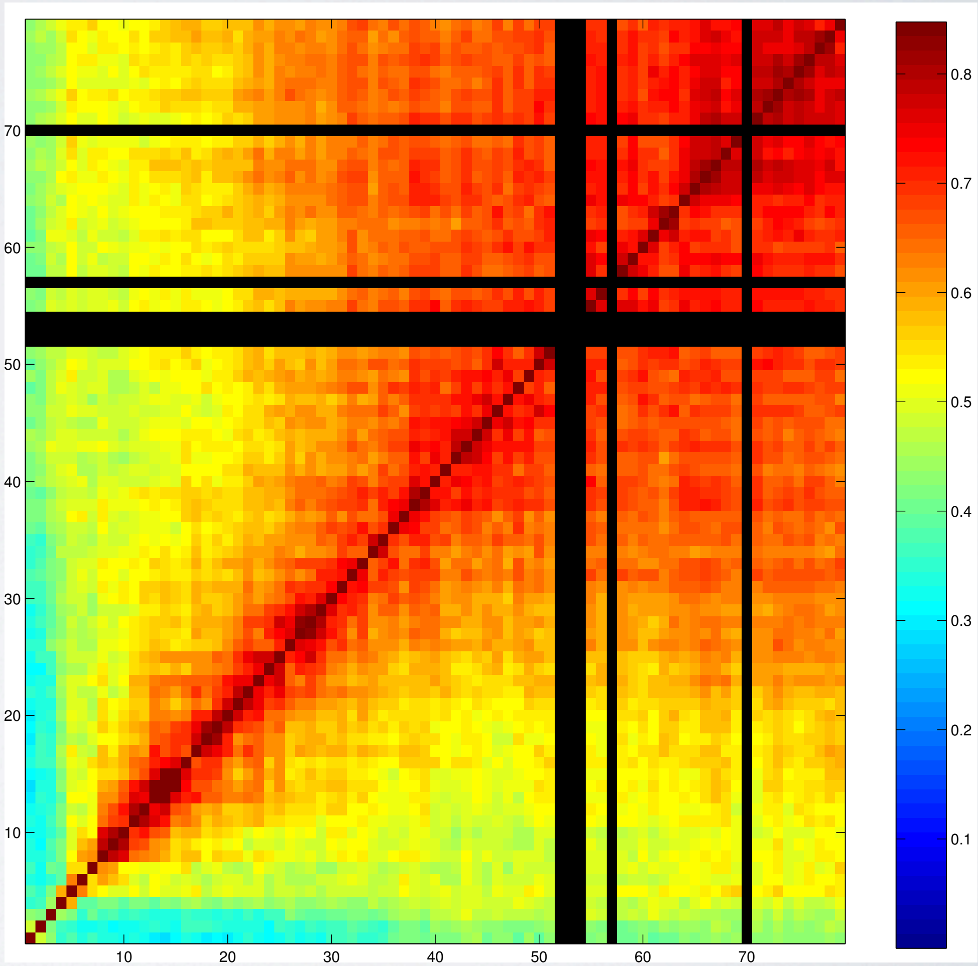


PERSISTENCE OF PATTERNS

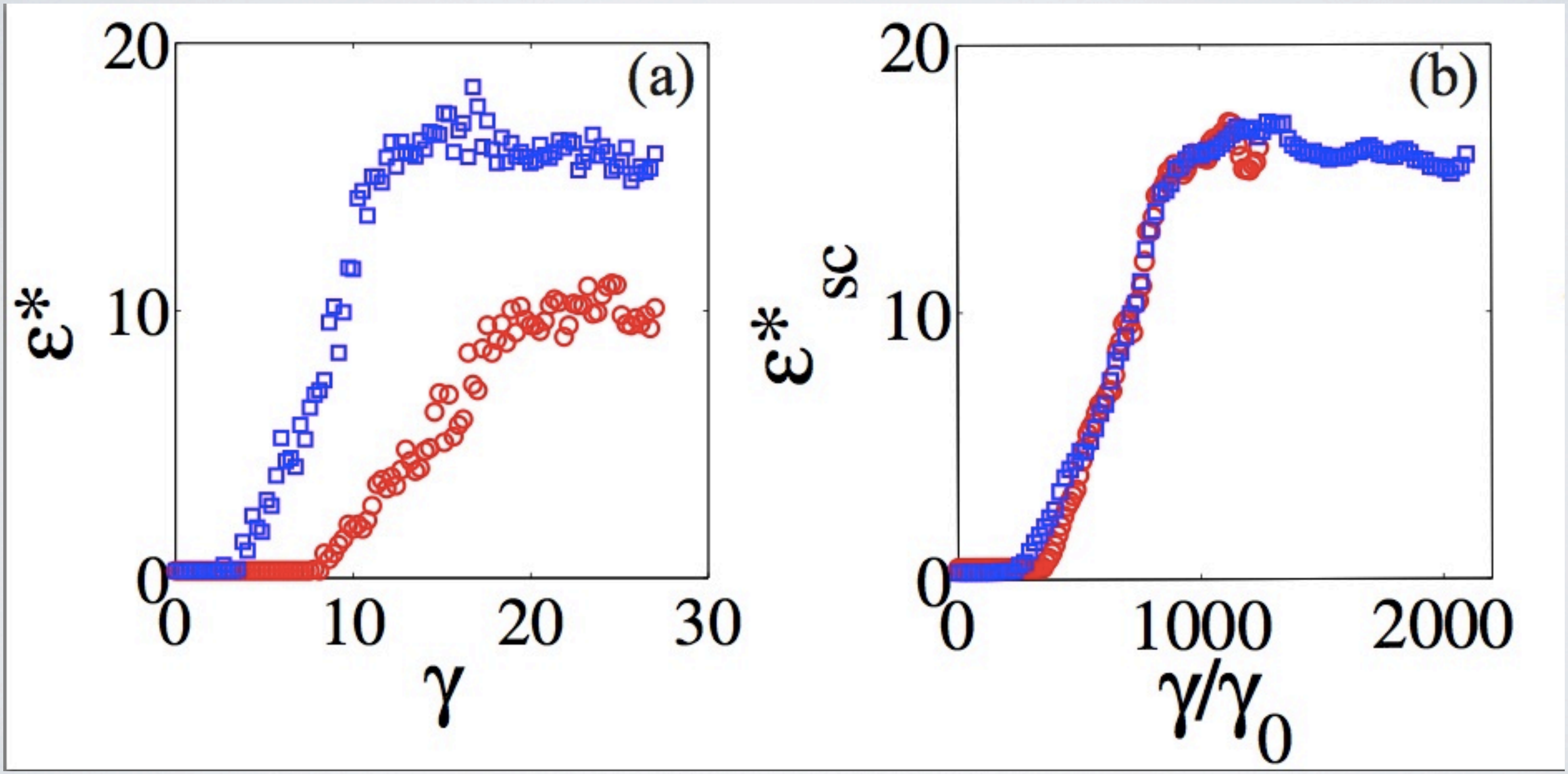
Low density



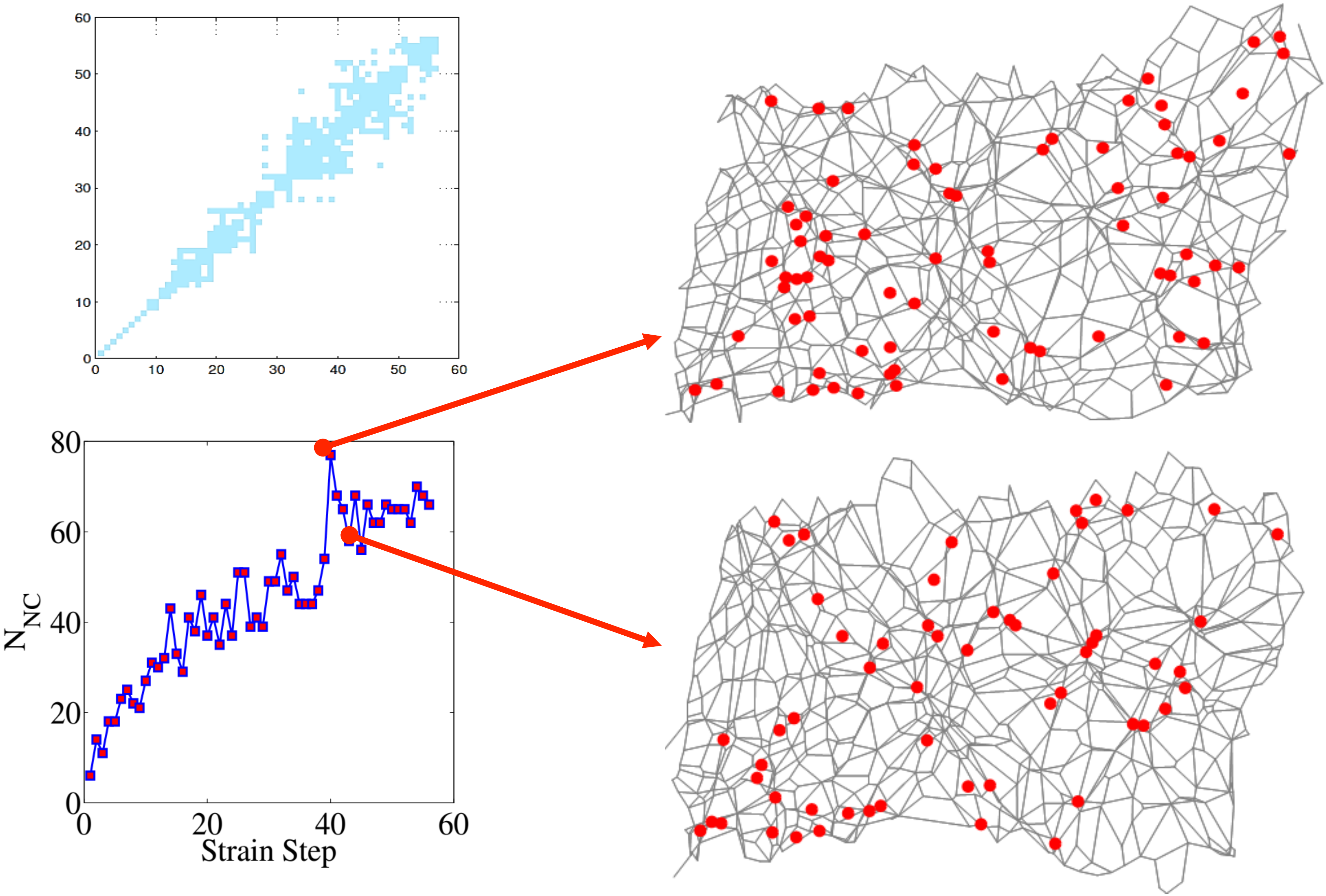
High density



ORDER PARAMETER



Failures: mini avalanches



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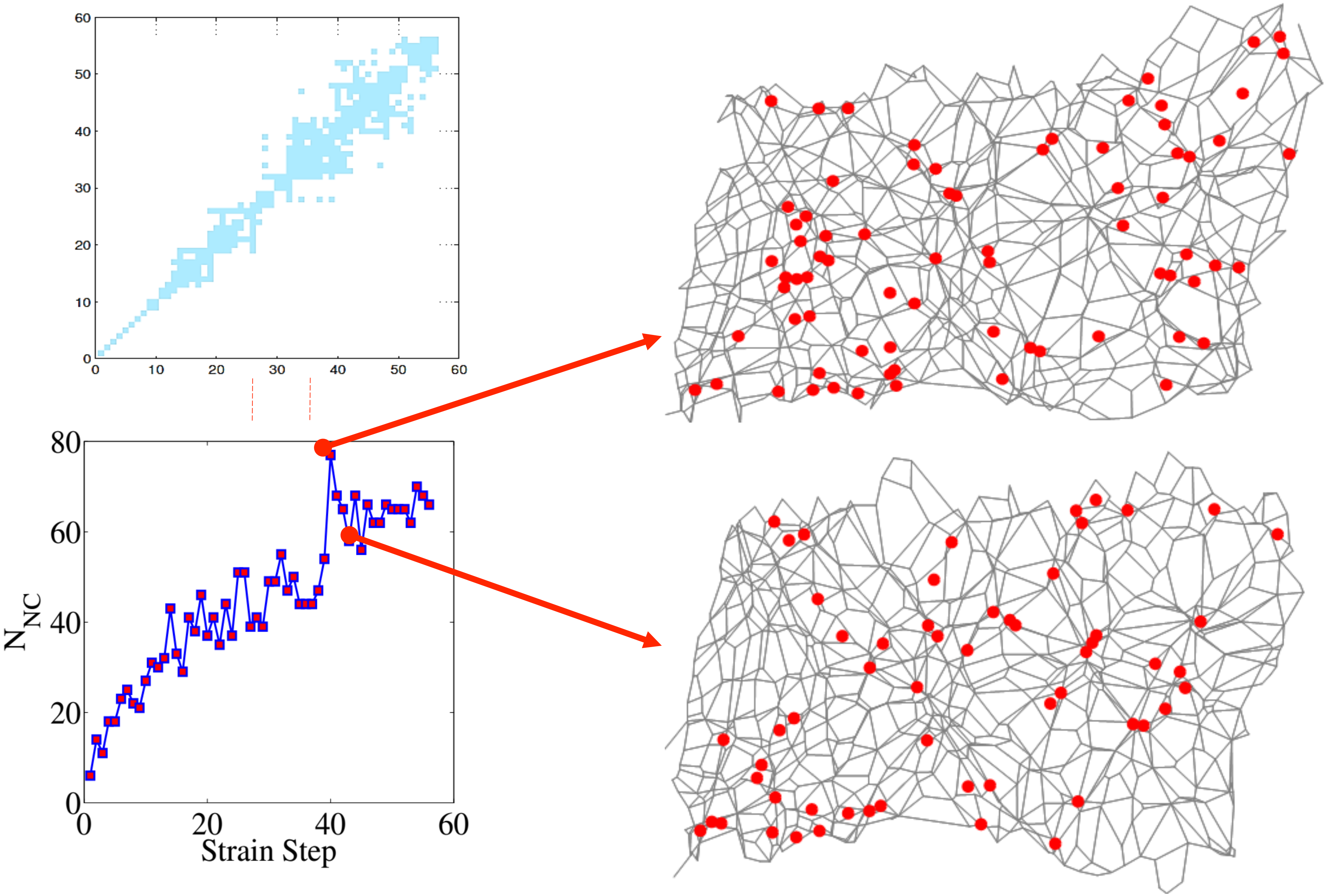
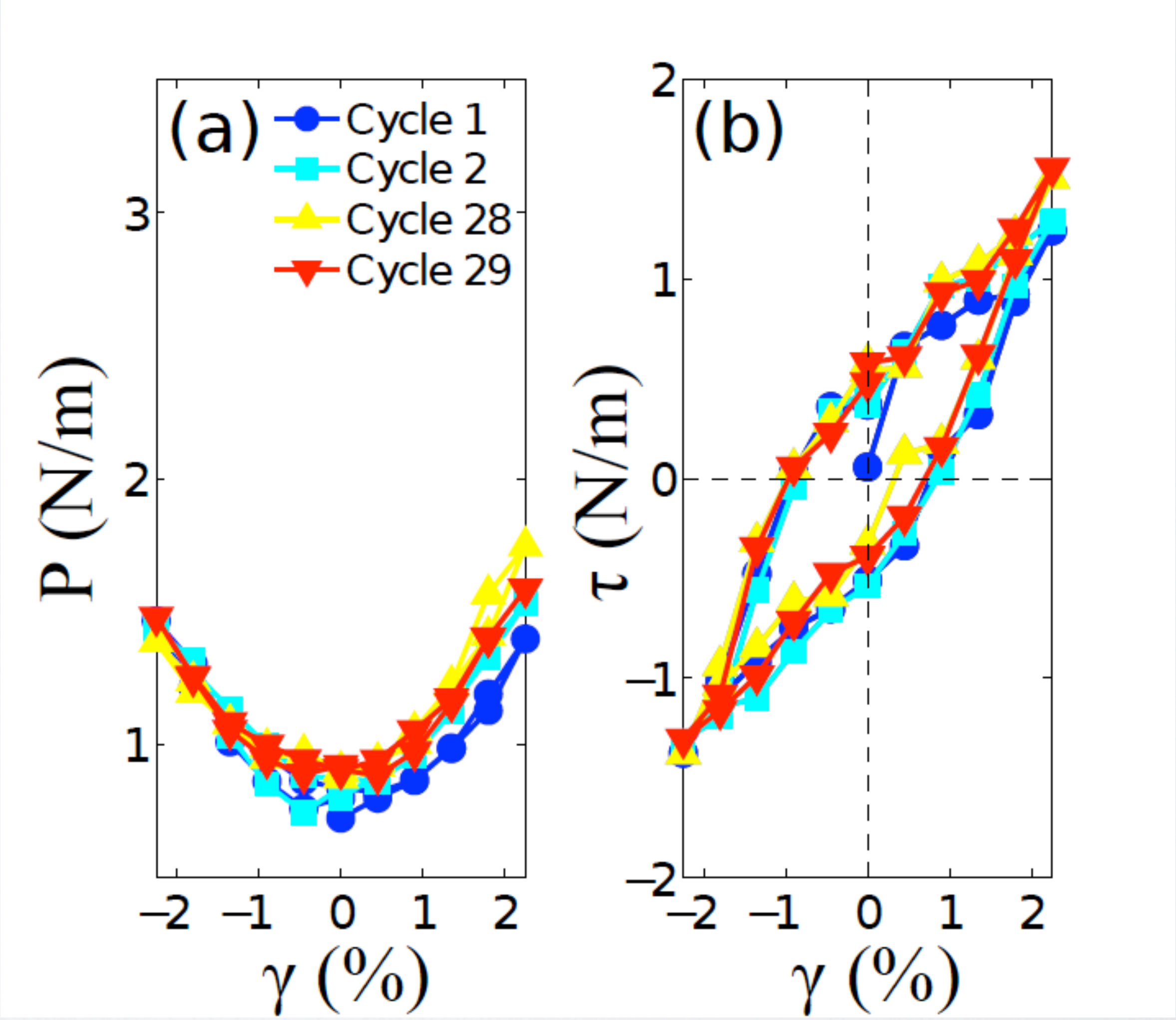


Hysteresis

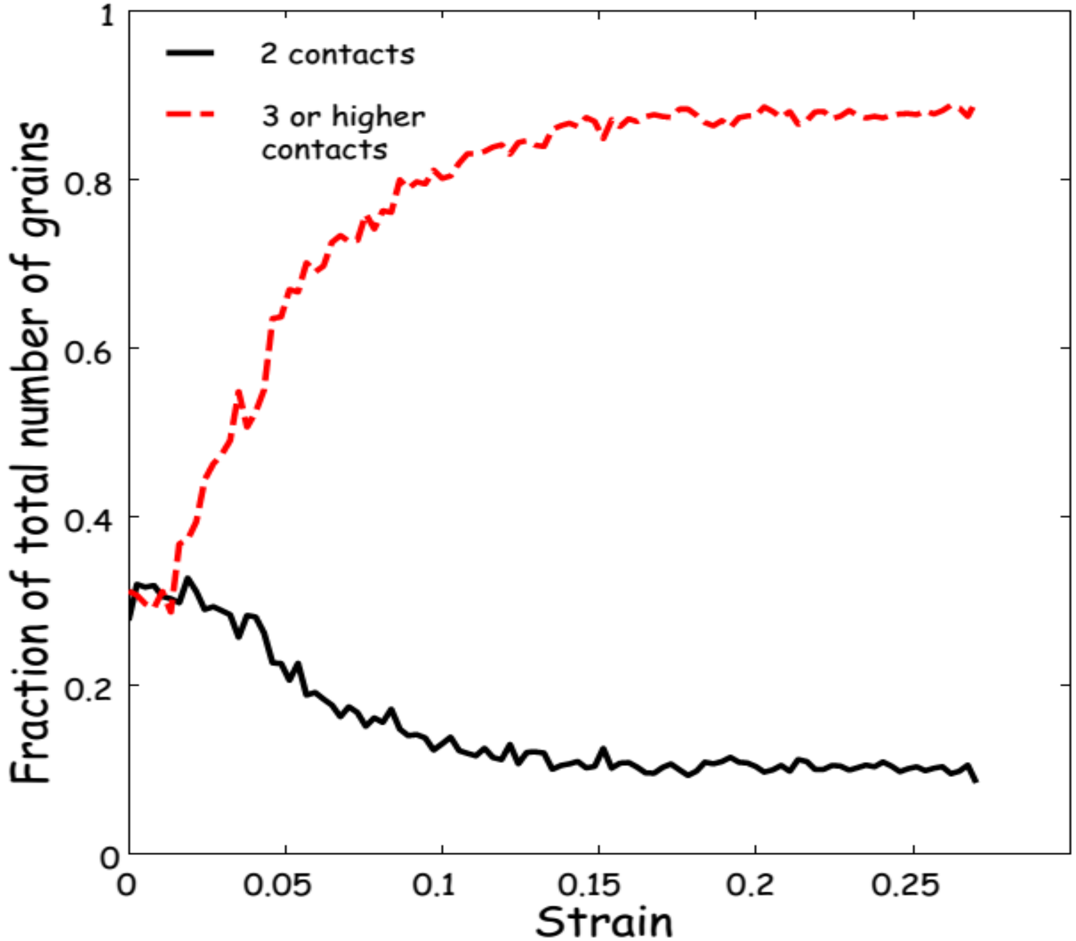


Shear Induced Rigidity: A unified Statistical Framework
Sumantra Sarkar (Poster)

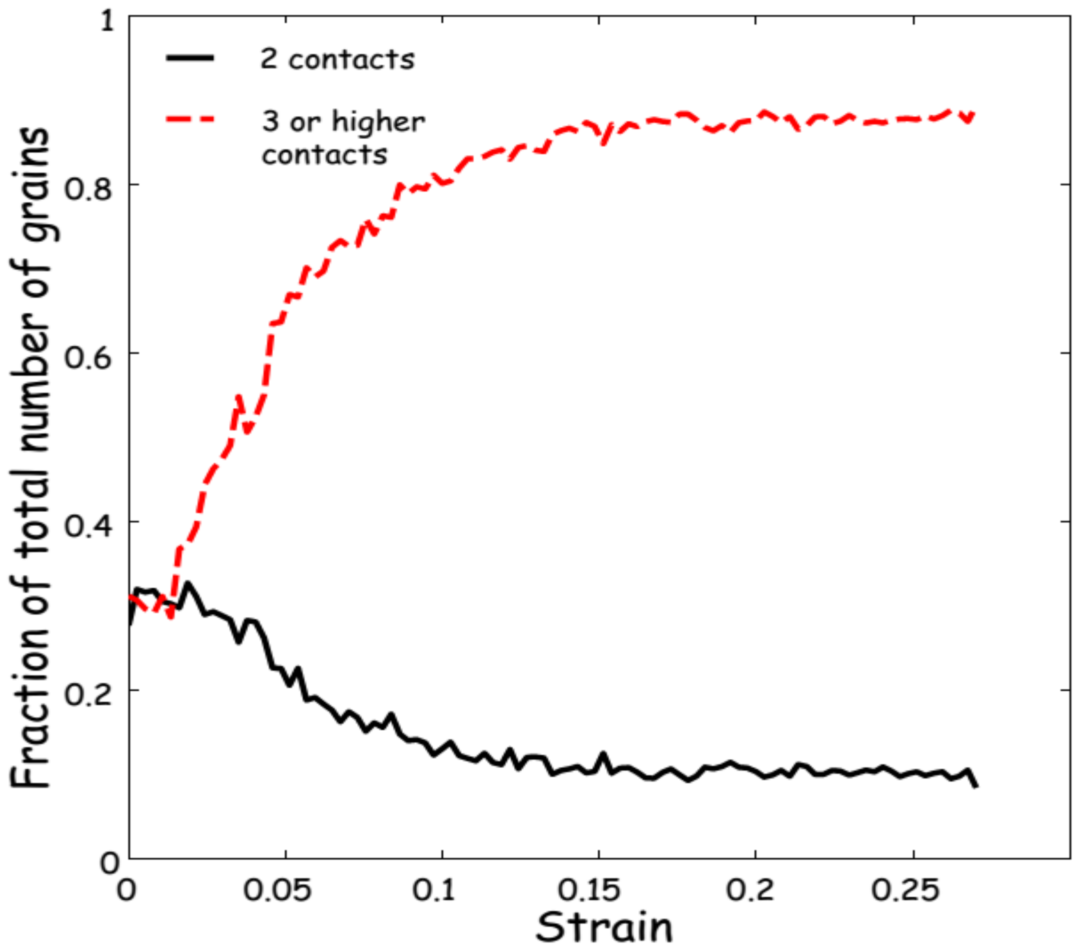
Stress state of grains maps to spins: 0, +1, -1
External magnetic field: Imposed strain

Summary

- Forces and positions have to be treated as distinct variables in granular systems
- Broken symmetry in position space does not guarantee rigidity because contacts can break
- Need a robust force network
- Signature of mechanical rigidity is broken translational symmetry in "force space"
- Theories of plasticity and failure for granular materials close to jamming need to incorporate the reciprocal space framework.

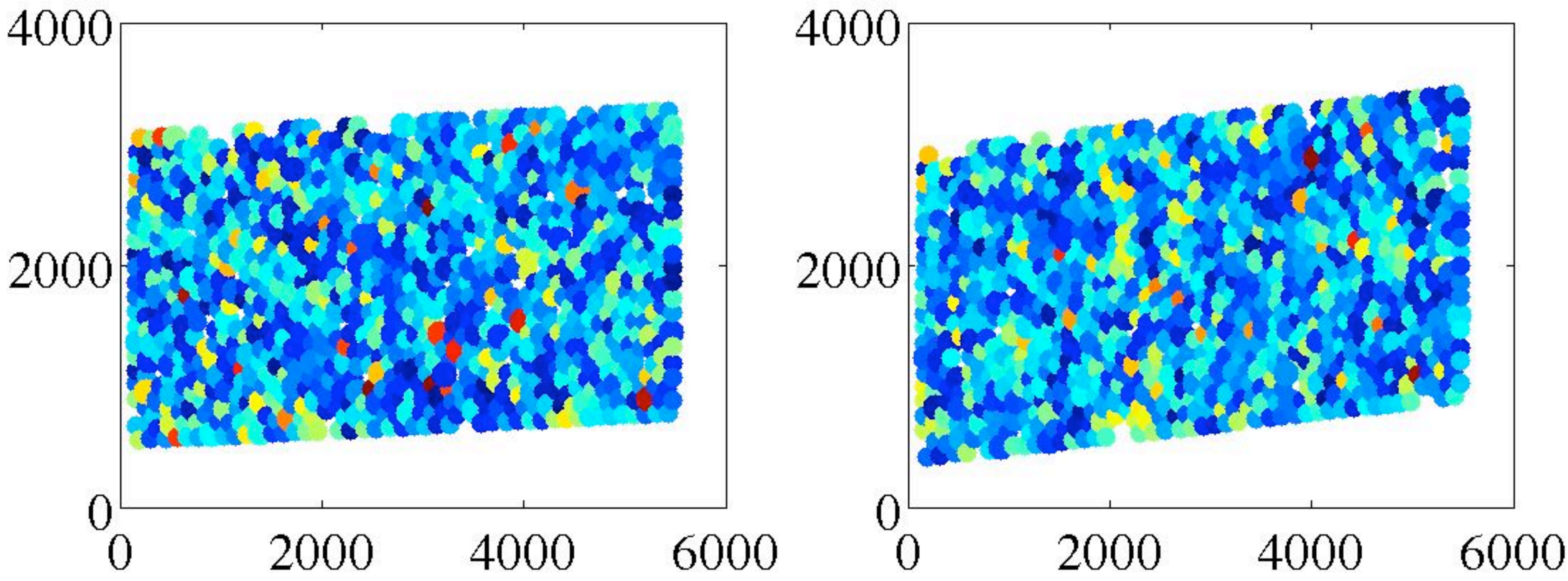


2 contacts
3 or higher contacts
Fraction of total number of grains
Strain
0
0.2
0.4
0.6
0.8
1
0
0.05
0.1
0.15
0.2
0.25



Shear increases number of grains with more than 2 contacts

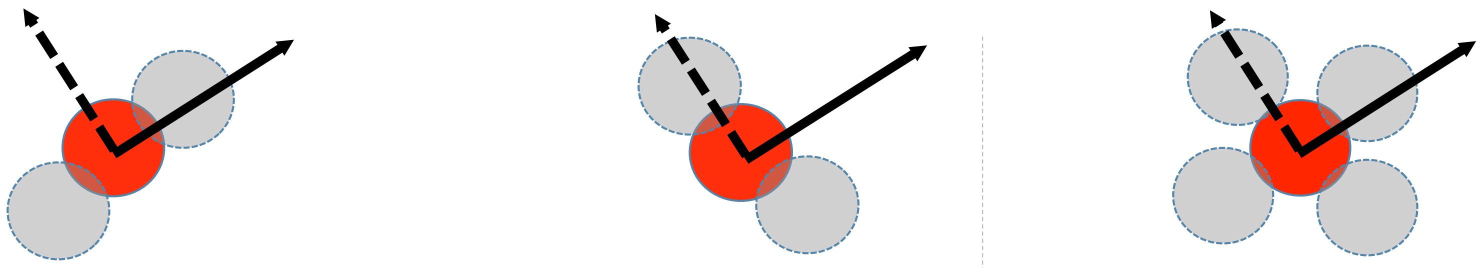
"Quenched" Non-Affine strain



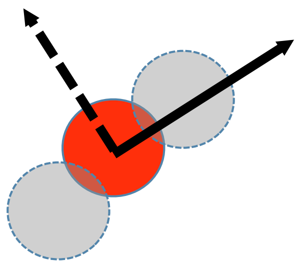
Random Field Model: Sumantra Sarkar (Poster)

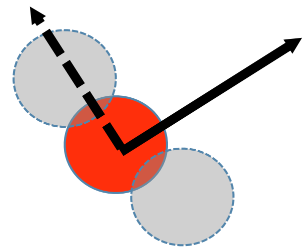
Stress state of grains maps to spins: 0, +1, -1
External magnetic field: Imposed strain

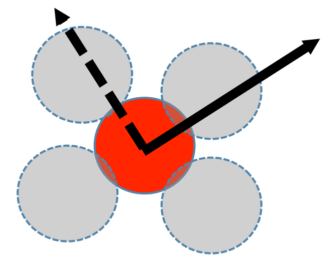
Model



Model

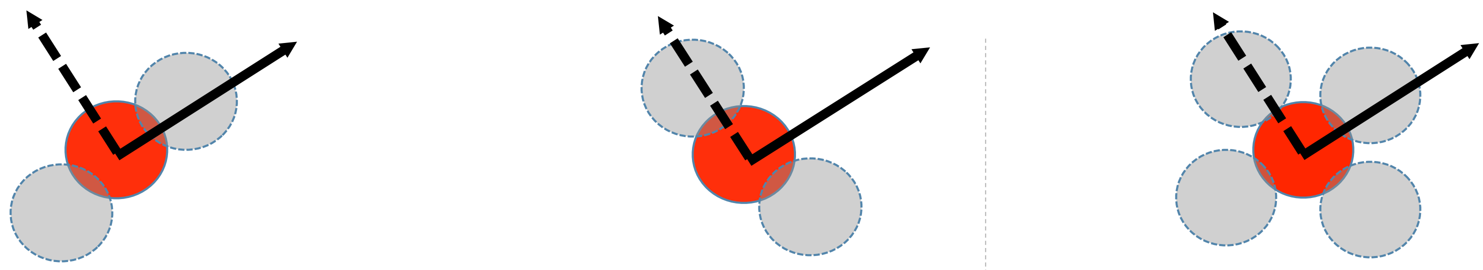






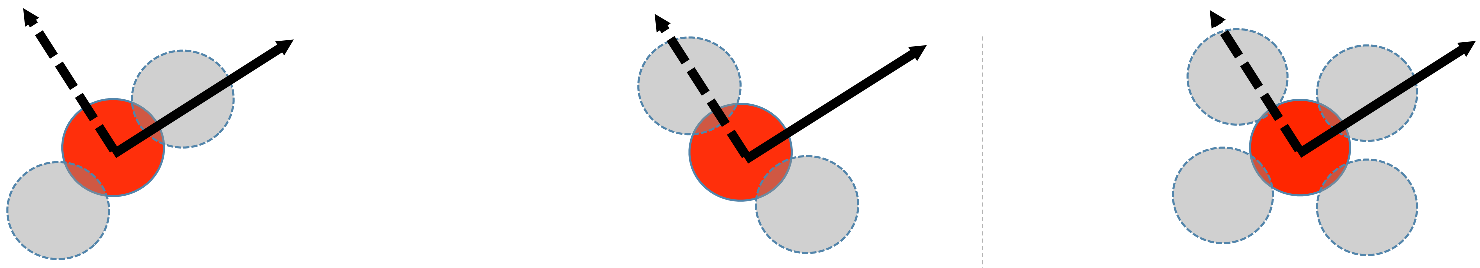
$$\mathcal{H} =$$

Model



$$\mathcal{H} = \boxed{-J \sum_{<i,j>} S_i S_j}$$

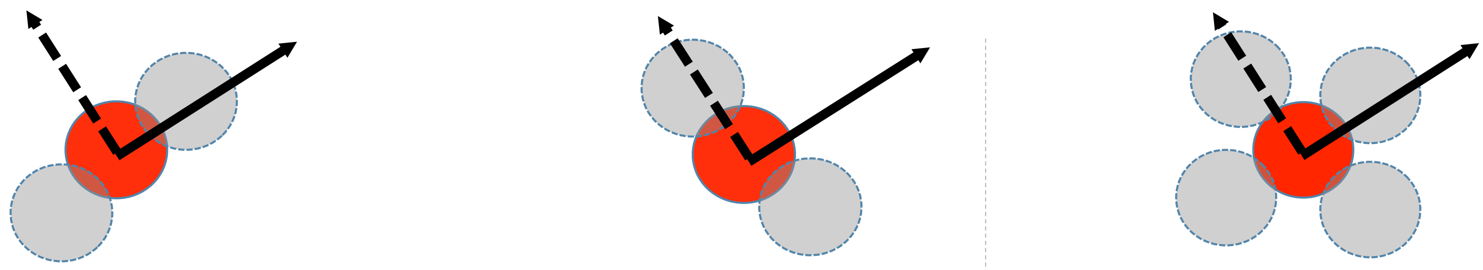
Model



$$\mathcal{H} = -J \sum_{<i,j>} S_i S_j - \sum_i h_i S_i$$

Non-affine strain.
Gaussian with zero
mean and disorder R

Model

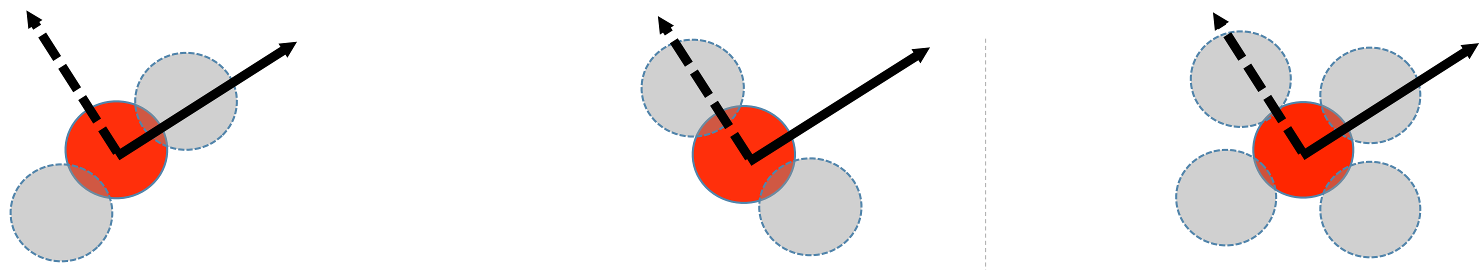


$$\mathcal{H} = -J \sum_{<i,j>} S_i S_j - \sum_i h_i S_i - H \sum_i S_i$$

Non-affine strain. Gaussian with zero mean and disorder R

Applied strain

Model

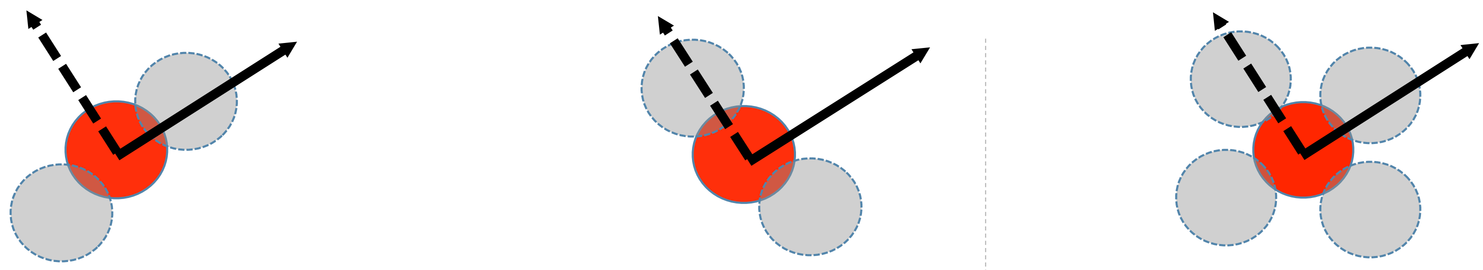


$$\mathcal{H} = -J \sum_{<i,j>} S_i S_j - \sum_i h_i S_i - H \sum_i S_i + \Delta(H) \sum_i S_i^2$$

Non-affine strain.
Gaussian with zero
mean and disorder R

Applied strain

Model



$$\mathcal{H} = -J \sum_{<i,j>} S_i S_j - \sum_i h_i S_i - H \sum_i S_i + \Delta(H) \sum_i S_i^2$$

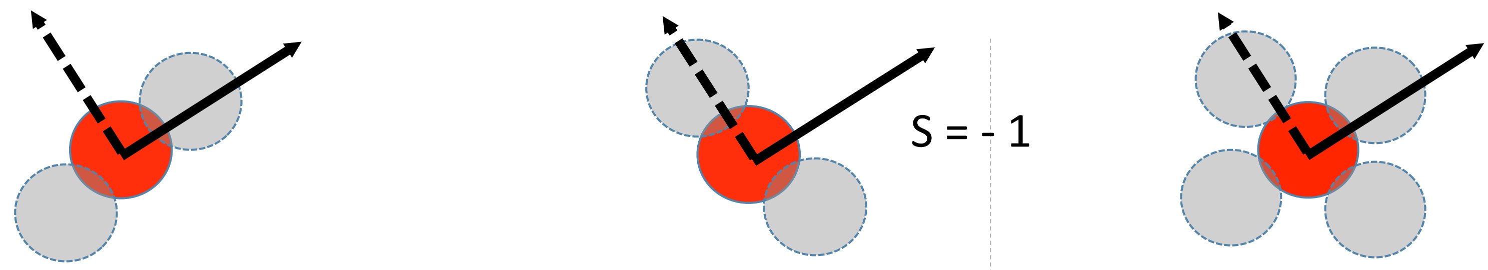
Non-affine strain. Gaussian with zero mean and disorder R

Applied strain

Magnetization ⟶ Stress Anisotropy

Fraction of zero spins ⟶ Fraction of grains with > 2 contacts

Model



$$\mathcal{H} = -J\sum_{<i,j>} S_i S_j - \sum_i h_i S_i - H\sum_i S_i + \Delta(H)\sum_i S_i^2$$

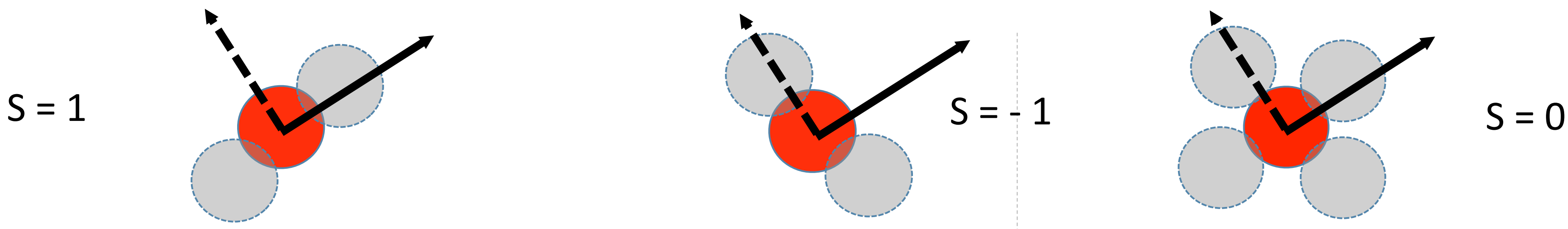
Non-affine strain. Gaussian with zero mean and disorder R

Applied strain

Magnetization ⟶ Stress Anisotropy

Fraction of zero spins ⟶ Fraction of grains with > 2 contacts

Model



$$\mathcal{H} = -J \sum_{<i,j>} S_i S_j - \sum_i h_i S_i - H \sum_i S_i + \Delta(H) \sum_i S_i^2$$

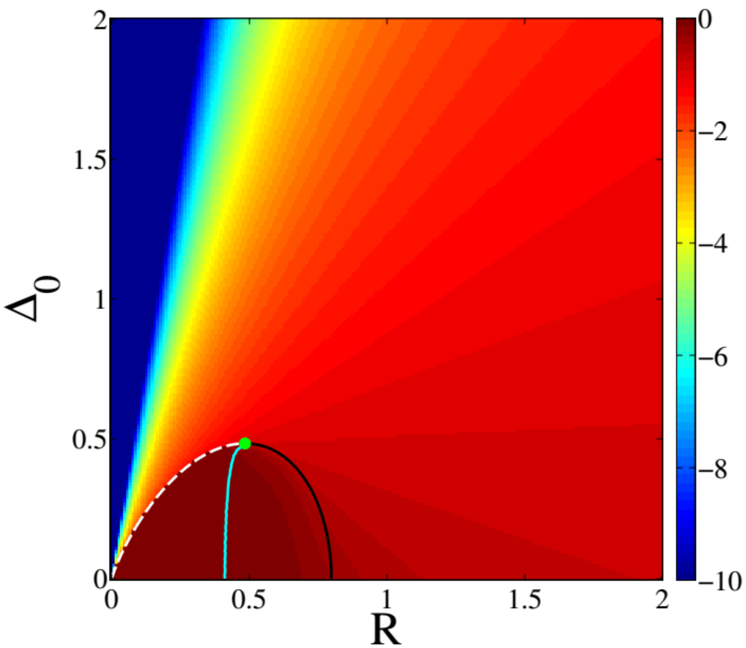
Non-affine strain. Gaussian with zero mean and disorder R

Applied strain

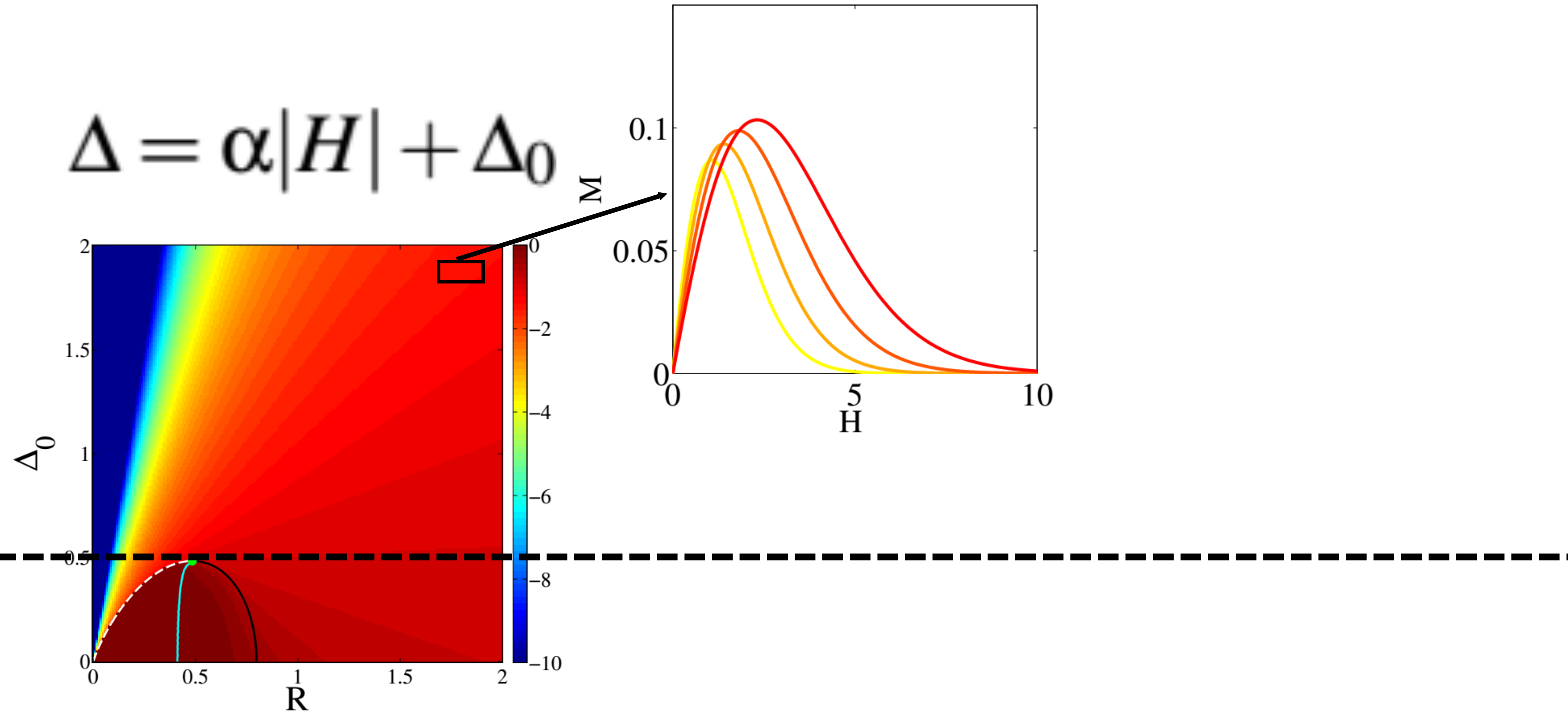
Magnetization ⟶ Stress Anisotropy

Fraction of zero spins ⟶ Fraction of grains with > 2 contacts

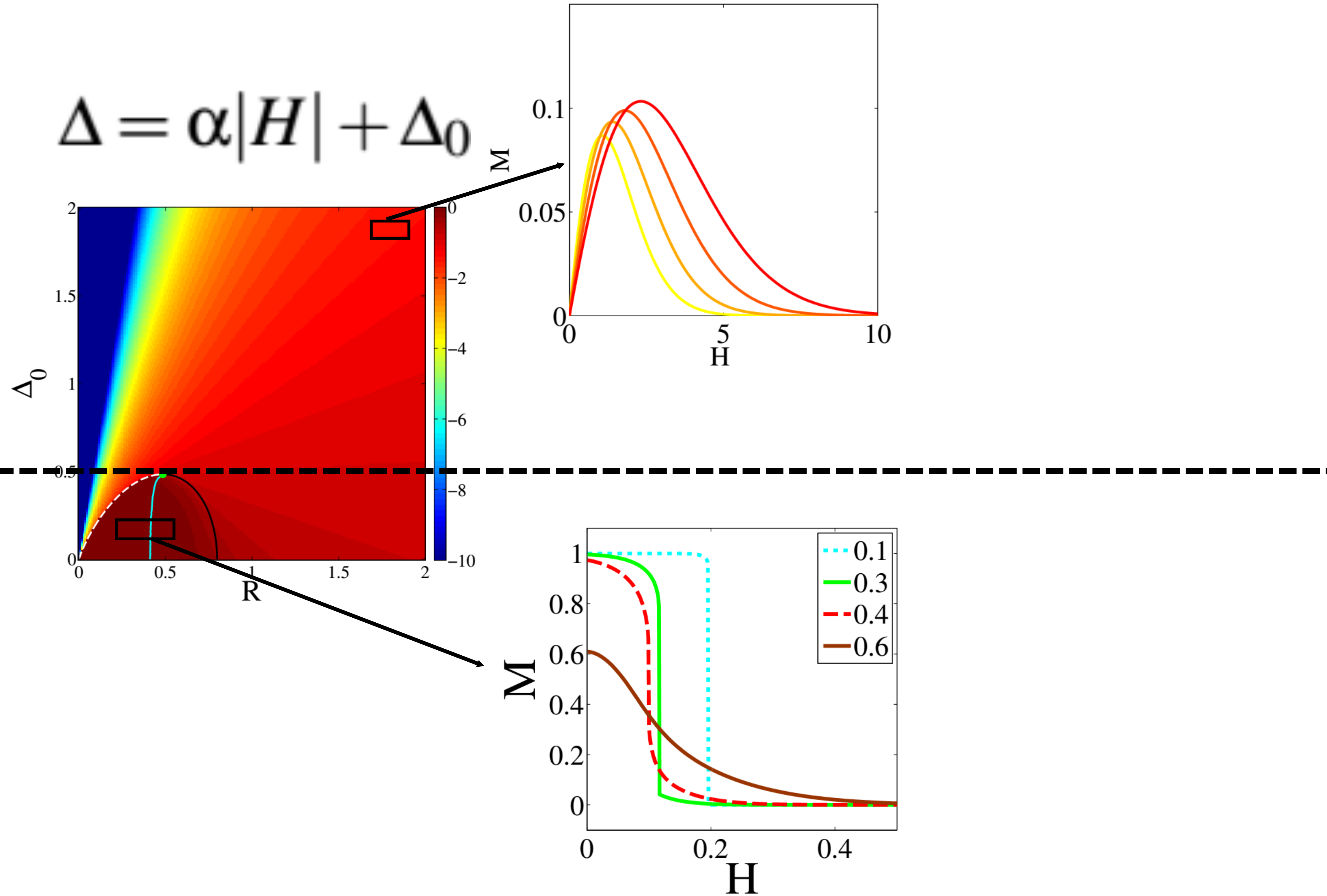
$$\Delta = \alpha |H| + \Delta_0$$



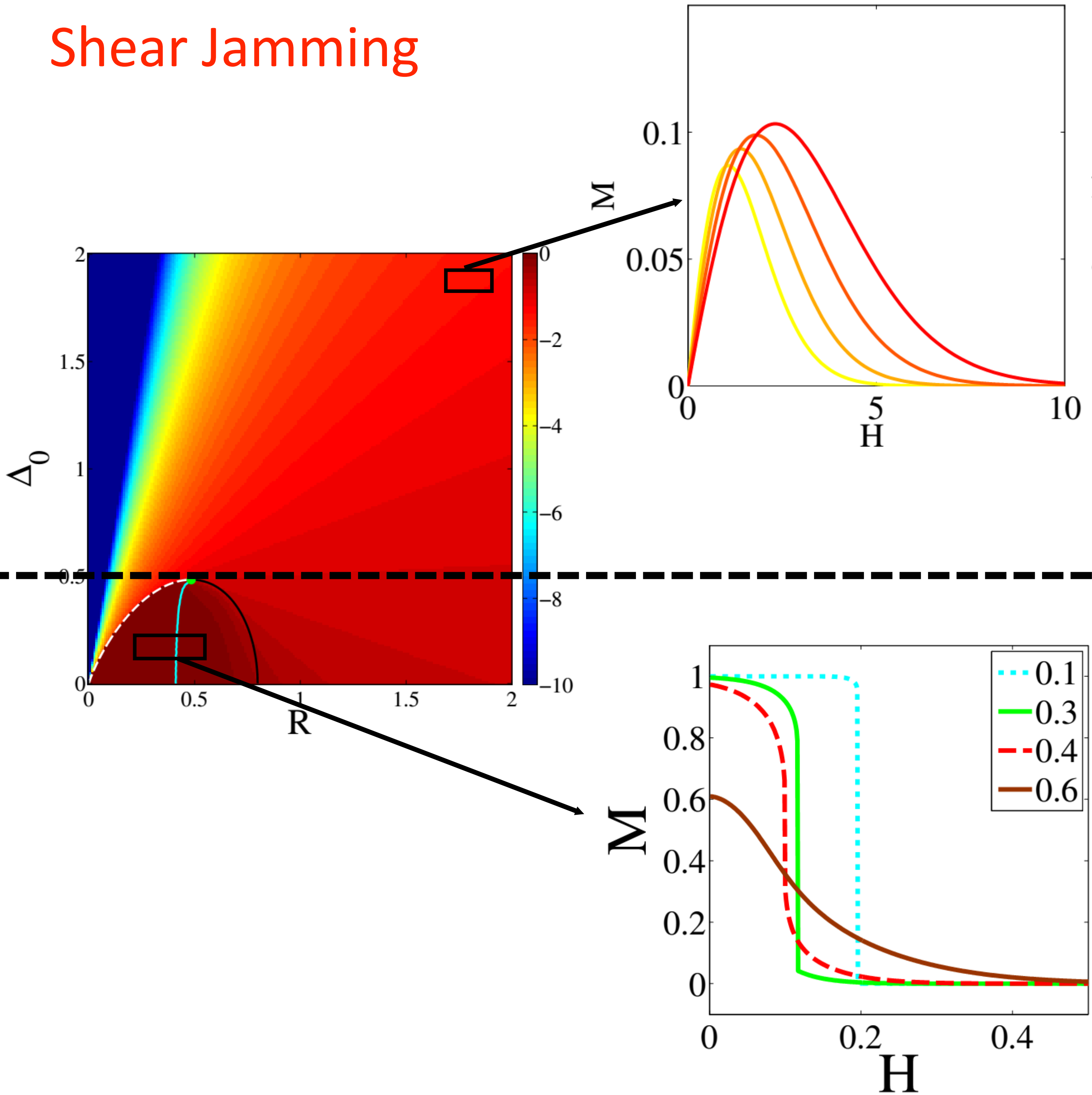
$$\Delta = \alpha|H| + \Delta_0$$

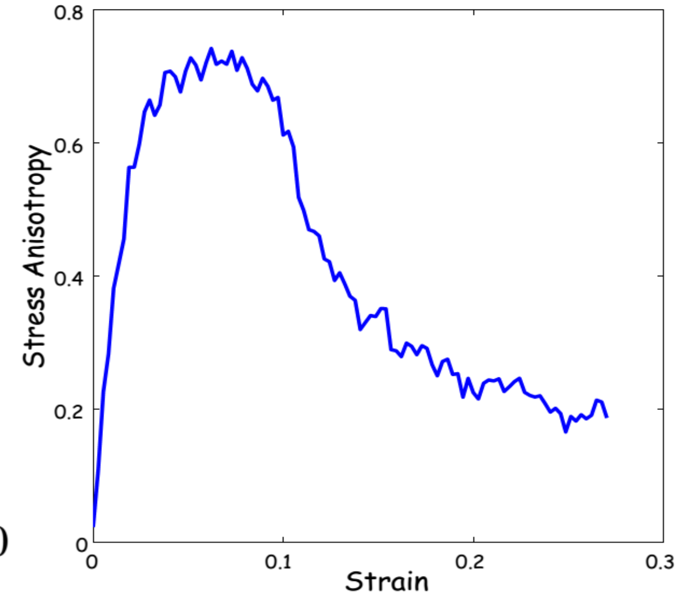


$$\Delta = \alpha|H| + \Delta_0$$



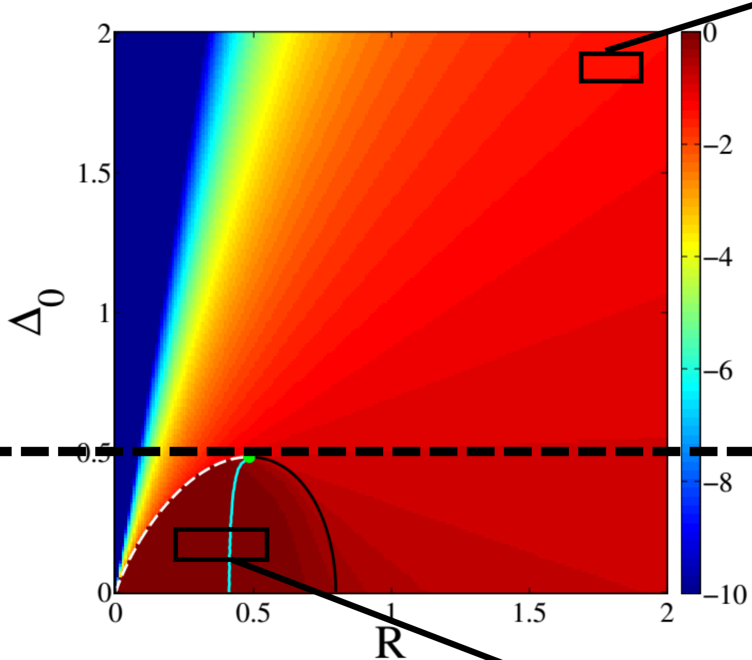
Shear Jamming

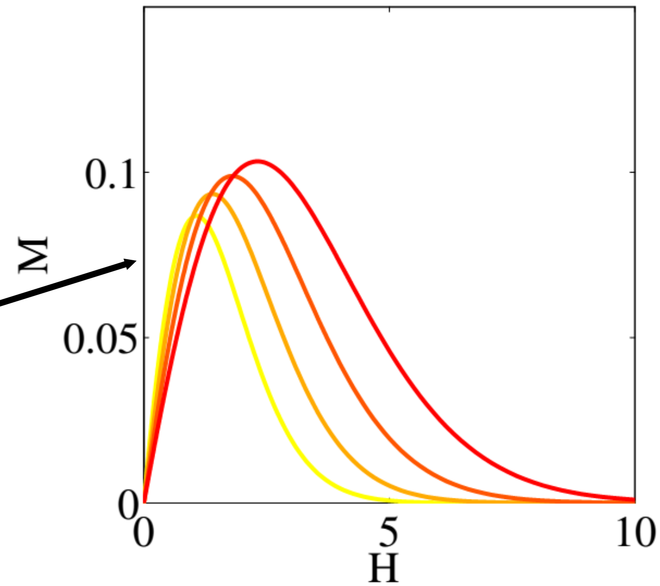


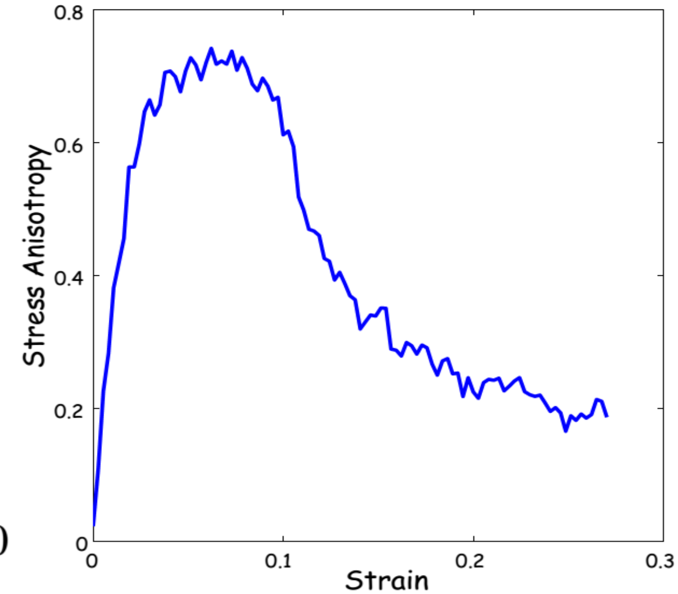


Data: Jie Ren

Shear Jamming

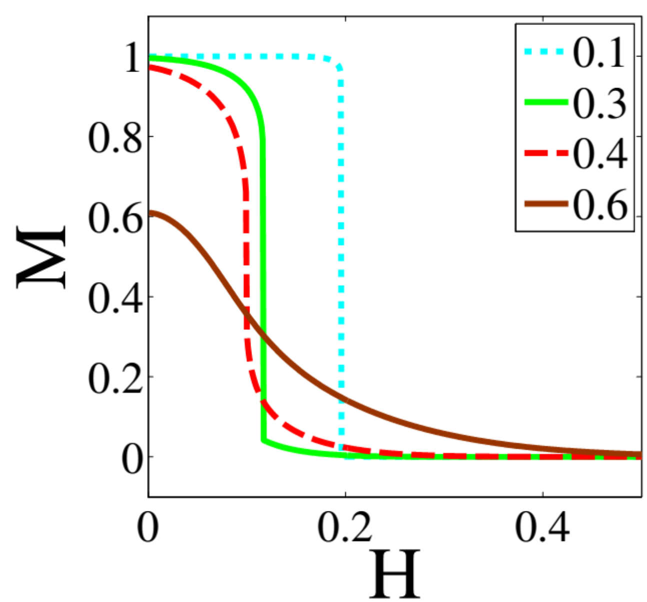


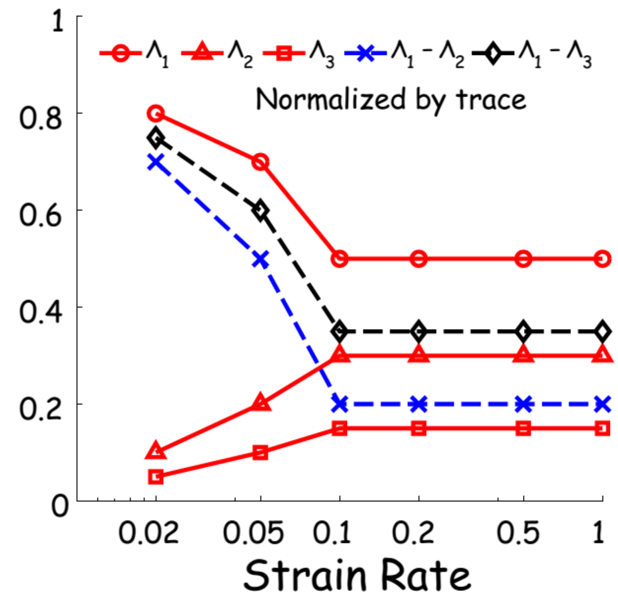




Data: Jie Ren

Discontinuous Shear Thickening





Constructed from: Seto et. al. PRL, 2013