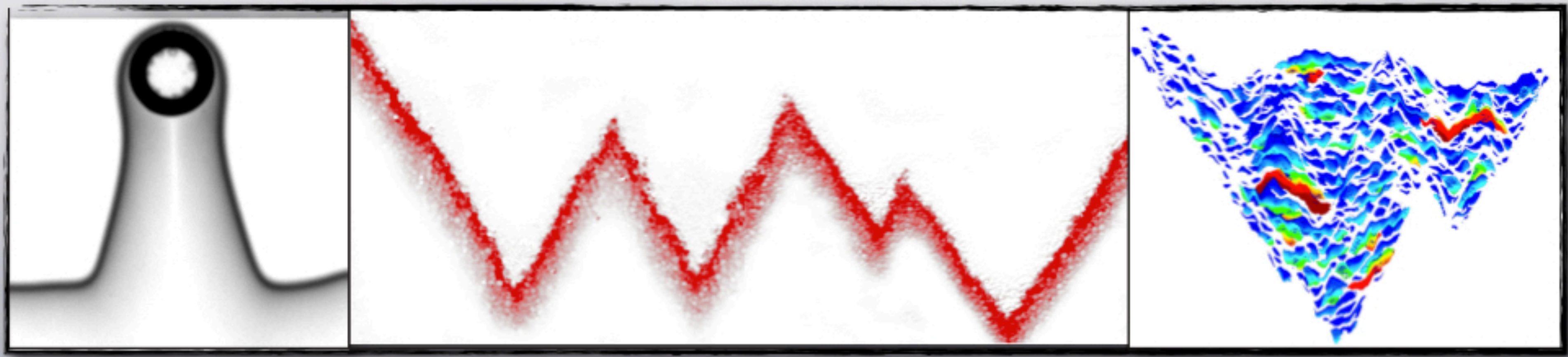
Avalanches and Dynamical Phase Transition of Reaction Waves in Adverse Flow



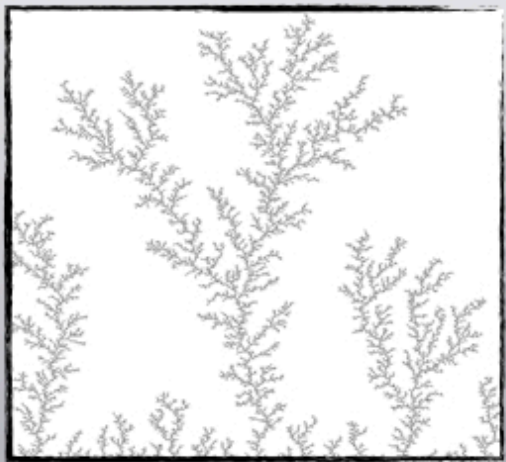
Séverine Atis
Massachusetts Institute of Technology

Dominique Salin and Laurent Talon
FAST - Université Paris Sud, Orsay

Kay Wiese and Pierre Le Doussal
LPT- Ecole Normale Supérieure, Paris

Growth phenomena and scale invariant structures

Diffusion limited aggregation

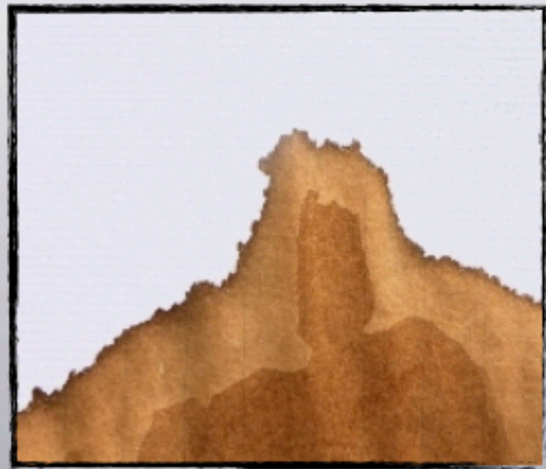


Numerical model

Vapor atom deposition

[Castro et al., 2012]

Imbibition fronts



Coffee stain on paper

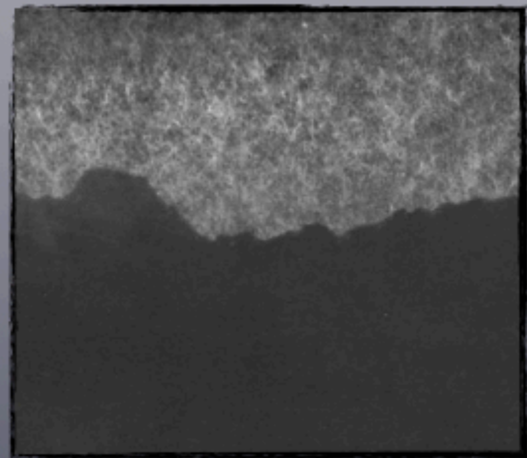
Solidification front

Crystal growth in supercooled liquids

Clouds

Boulder sky, summer 2011

Slow paper combustion



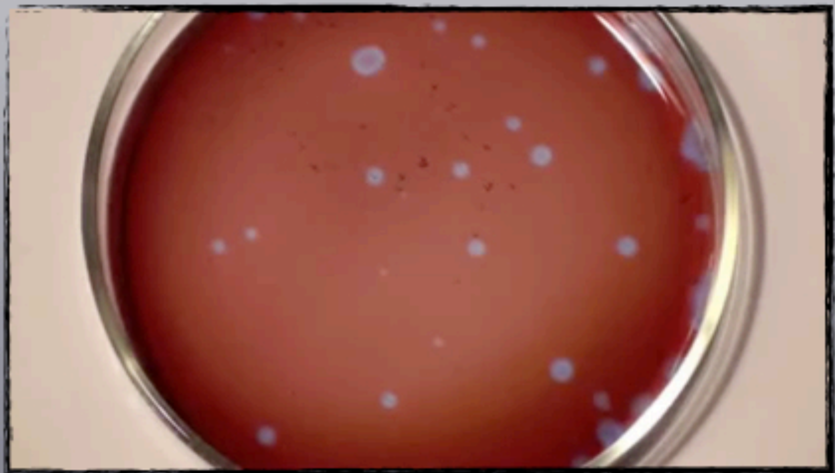
[Zhang et al., 1992]

Out of equilibrium system

- Autocatalytic reaction
- Fundamentally nonlinear
- Self-organization - living systems

feedback

$$A + B \longrightarrow 2B + C$$



Belousov Zhabotinski oscillations
[movie S. Morris]

Bacterial colonnies

[Benjacob et al., 1994]

Plants growth

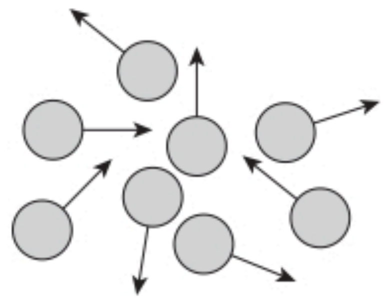
Lychen, New Hampshire

- Reaction Diffusion equation

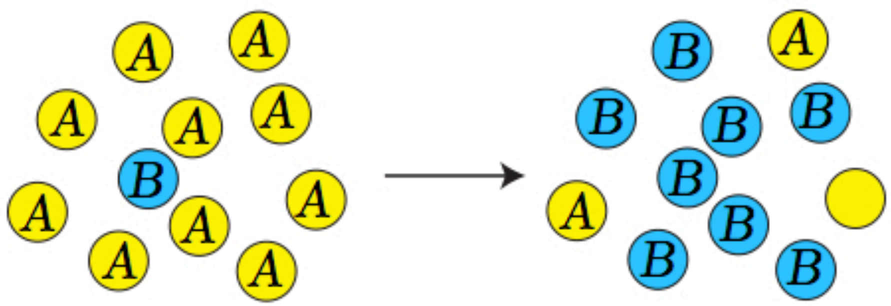
$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + f(u)$$

$$u = [B]$$

$\longrightarrow$ • $D\Delta u$ diffusion term



autocatalytic process $\longrightarrow$ nonlinearity $f(u) = r\,u(1-u)$



- Fisher-Kolmogorov equation (FKPP model)

[Kolmogorov et al. (1937), R. A. Fisher (1937)]

$$\frac{\partial u}{\partial t} = D\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + ru(1-u)$$

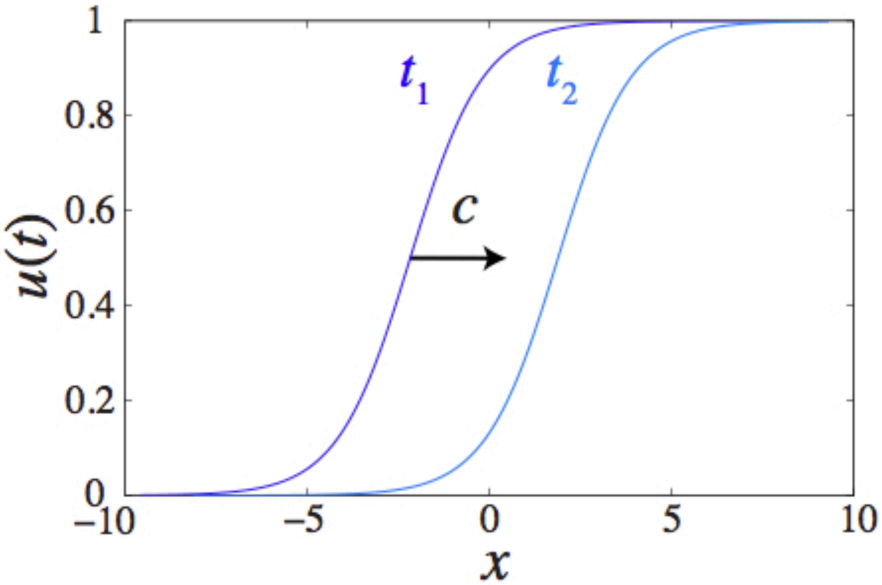
$$X = x \pm ct \longrightarrow c\frac{\partial u}{\partial X} = D\frac{\partial^2 u}{\partial X^2} + f(u)$$

- Progressive wave solutions

$$u(x,t) = u(x \pm ct)$$

FKPP

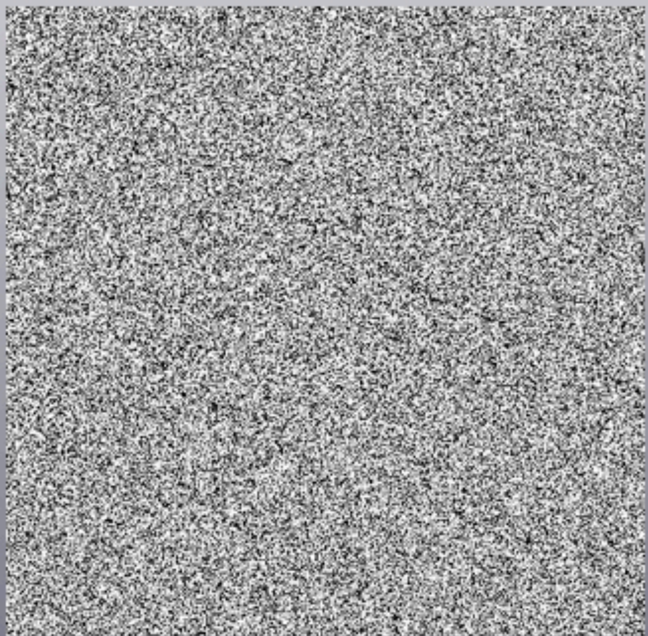
$$\longrightarrow c = 2\sqrt{rD}$$

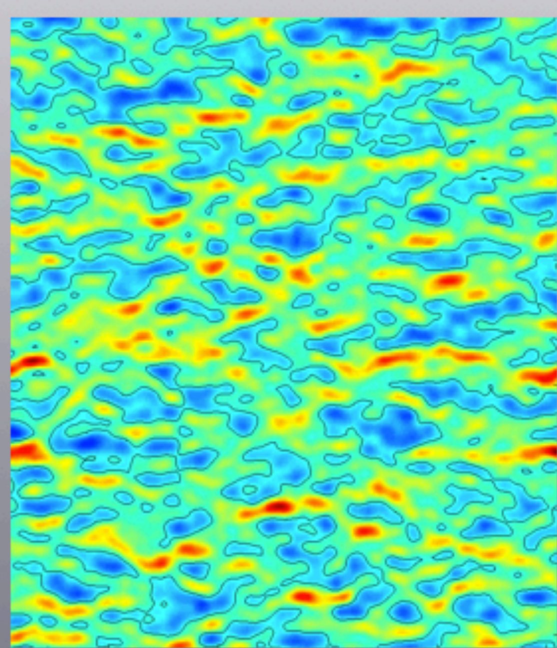


Growth phenomena and scale invariant structures

- What's happening in the presence of noise?

Disordered reactive flow flied





PLAN

1 - Experimental setup

2 - Front dynamics in high flow strength

3 - Frozen pattern formation

4 - Critical behavior

5 - Conclusion and perpectives

PLAN

1 - Experimental setup

2 - Front dynamics in high flow strength

3 - Frozen pattern formation

4 - Critical behavior

5 - Conclusion and perpectives

1 - Experimental setup

- Iodate acid arseneous reaction (IAA)

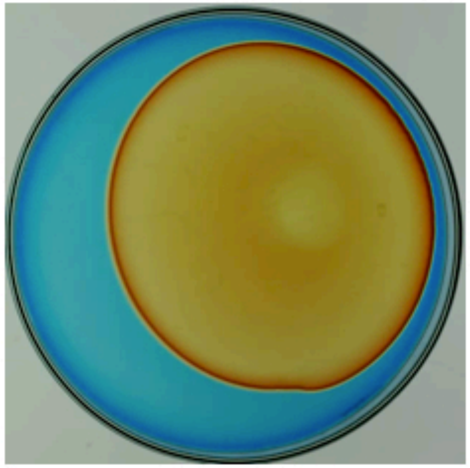
$$IO_3^- + 5I^- + 3H_3AsO_3 \rightarrow 6I^- + 3H_3AsO_4$$

- 3rd order chemical kinetics $f(C) = \alpha C^2(1-C)$

$$\frac{\partial C}{\partial t} = D_m\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \alpha C^2(1-C)$$

C : autocatalists concentration $[I^-]/[IO_3^-]_0$

α : reaction rate



IAA wave front
[movie D. Salin]

1 - Experimental setup

- **Stationary solution**

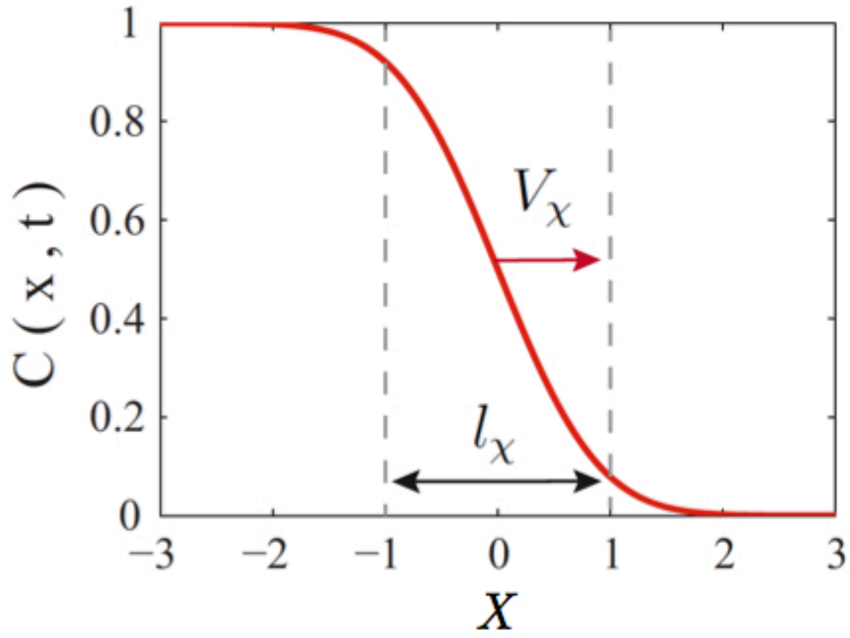
$$C(x,t) = \frac{1}{1+\exp[(x-V_\chi t)/l_\chi]}$$

⟶ resulting from the balance between diffusion and reaction

reaction front velocity and thickness remain constant

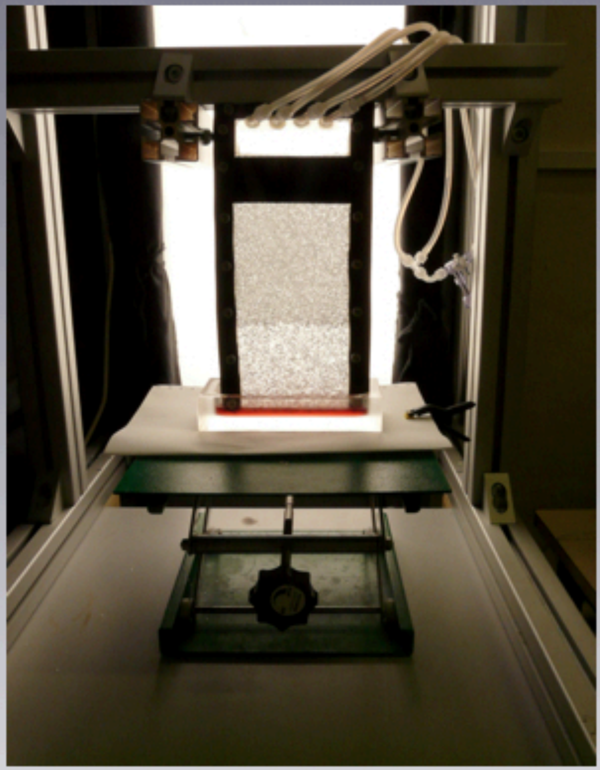
$$\begin{cases} V_\chi = \sqrt{\dfrac{\alpha D_m}{2}} \simeq 10 \mu m/s \\ l_\chi = \sqrt{\dfrac{2 D_m}{\alpha}} \simeq 100 \mu m \end{cases}$$

concentration profile of the front



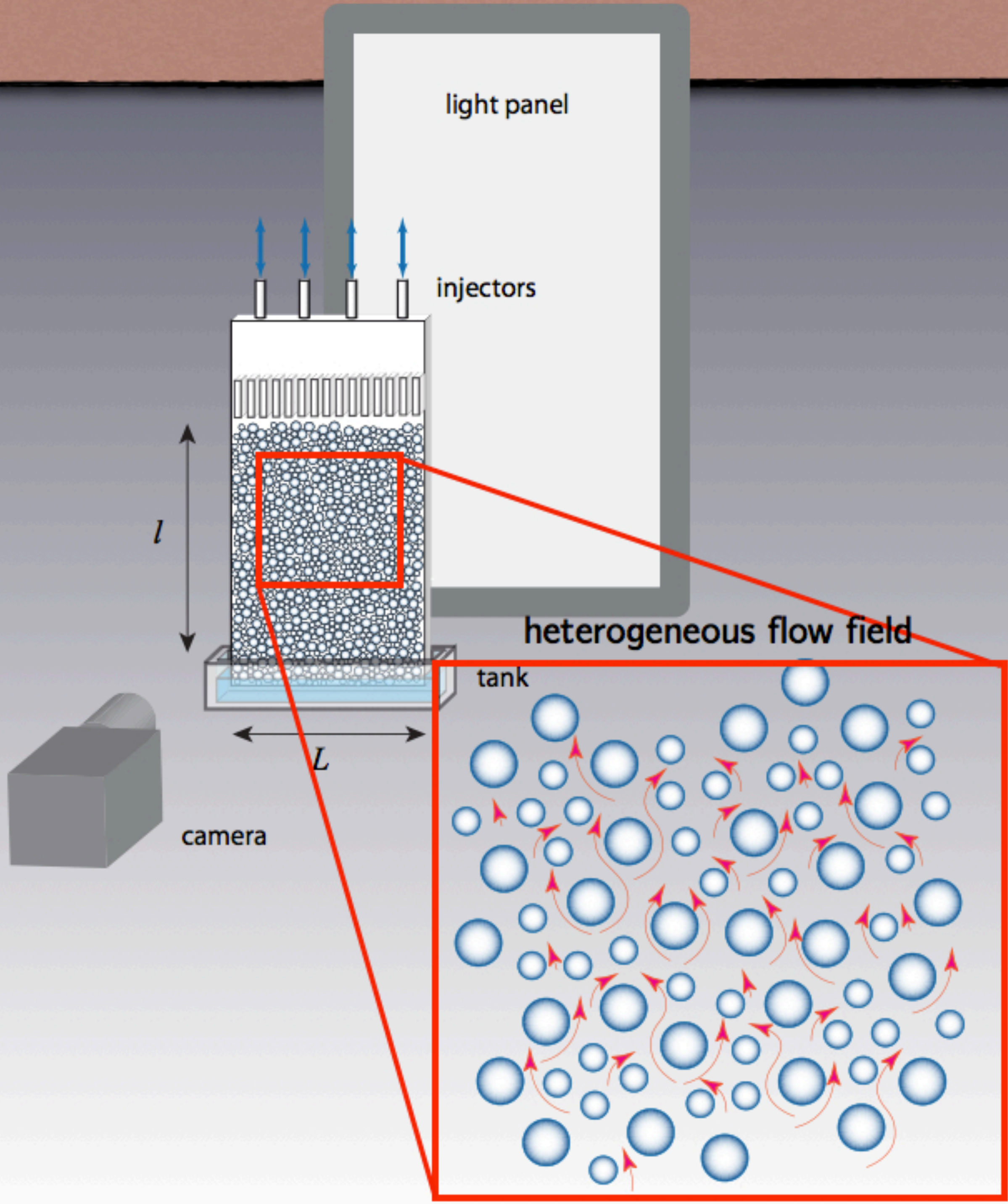
1 - Experimental setup

- Spatially disordered flow



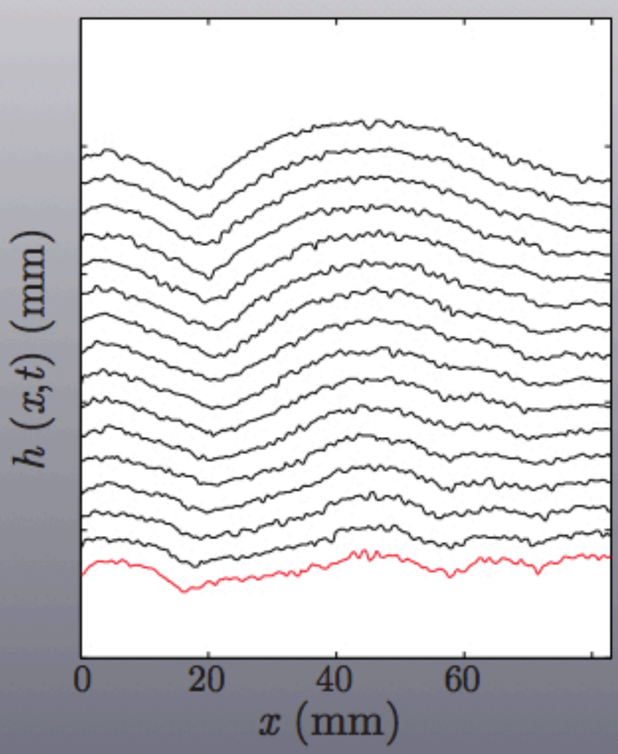
flow through a granular medium

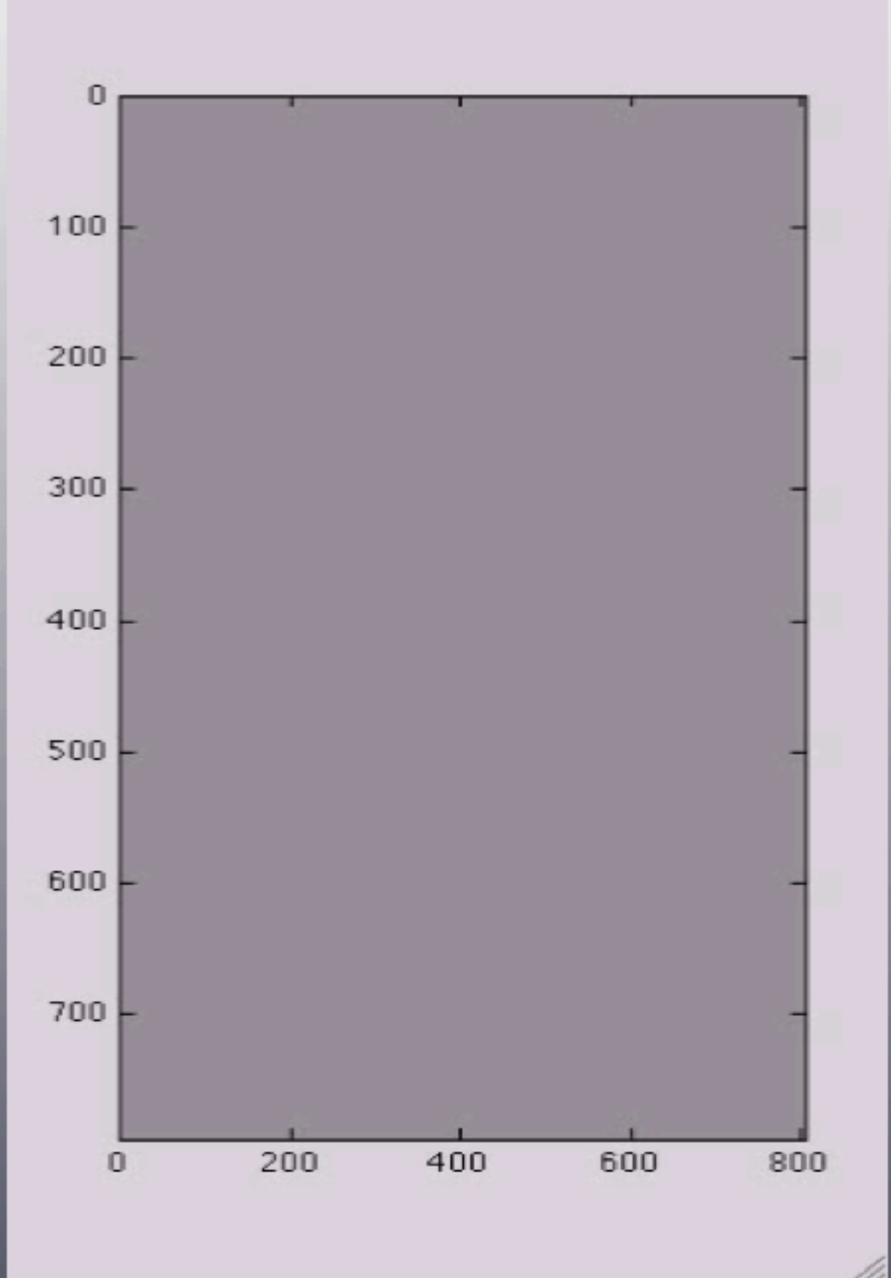
1.5 mm and 2 mm
diameter glass beads



1 - Experimental setup

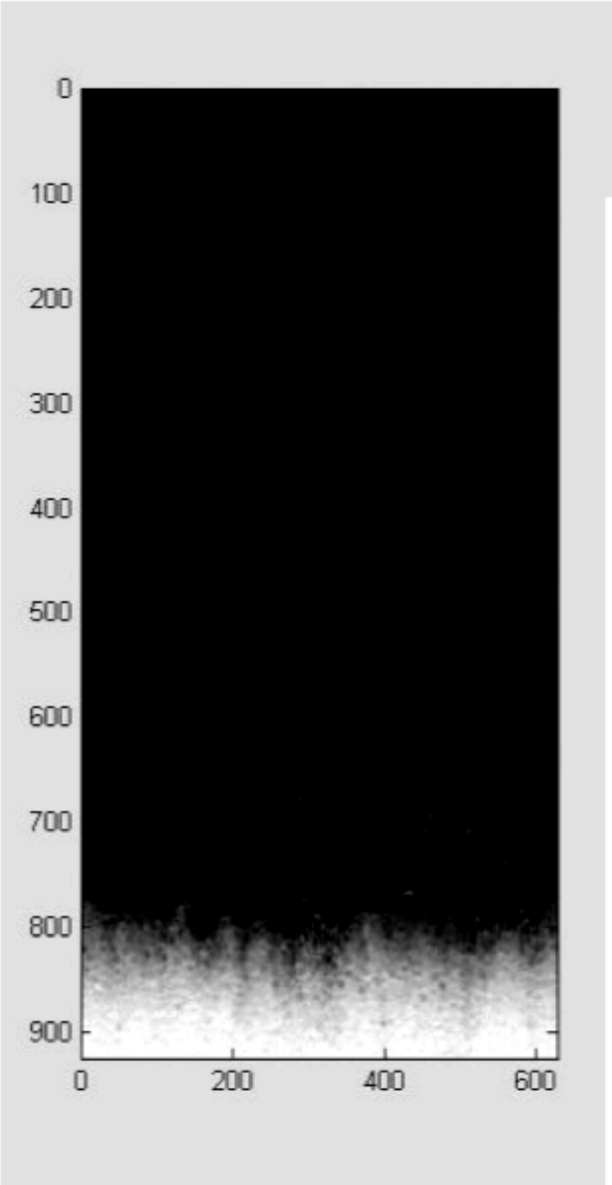
- Reaction front propagation without disordered flow





1 - Experimental setup

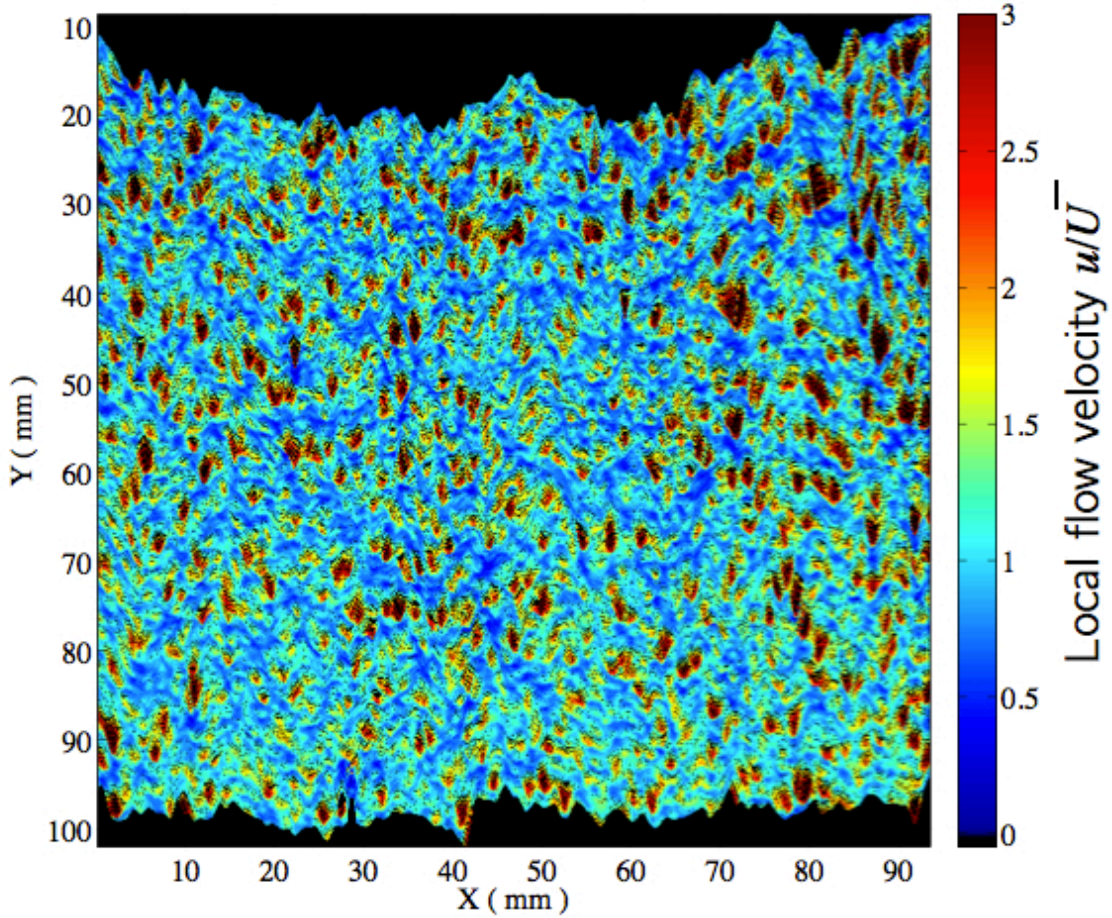
- **Tracers dispersion experiments: measurements of the local flow velocity**



Fluctuations correlation length:

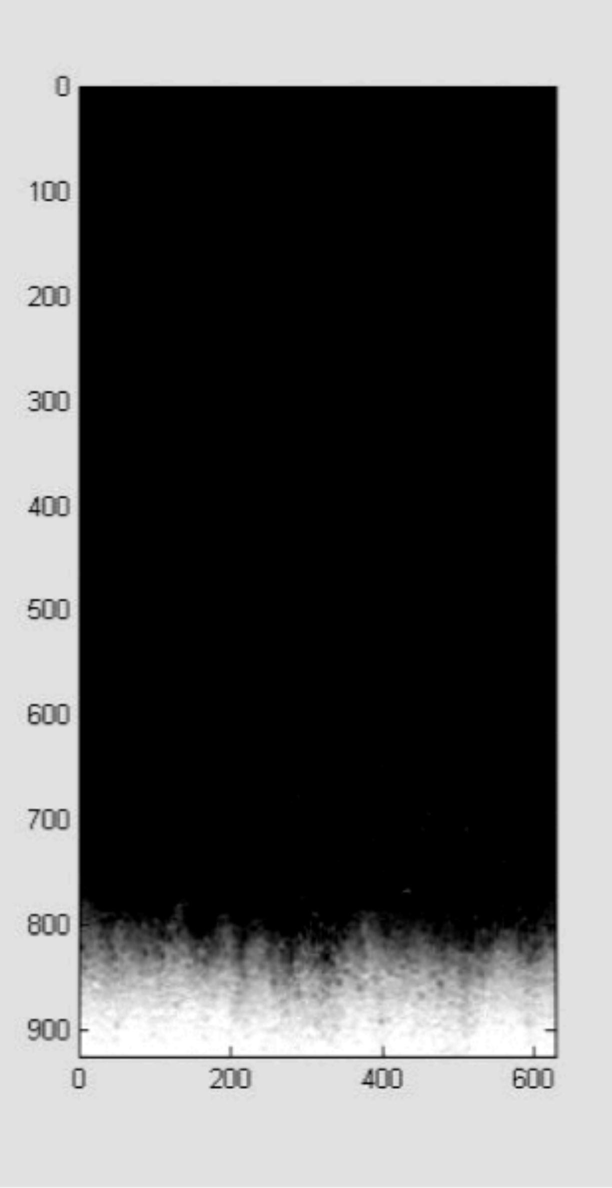
$$d_{\parallel} = 1.8 \pm 0.1\,\mathrm{mm}$$

Disordered flow of mean velocity $\bar{U}$



1 - Experimental setup

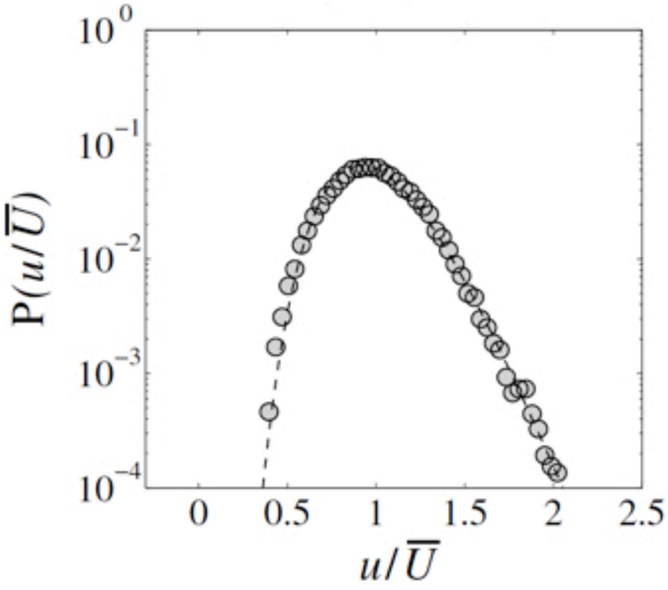
- **Tracers dispersion experiments: measurements of the local flow velocity**



Fluctuations correlation length:

$$d_{\parallel} = 1.8 \pm 0.1\,\mathrm{mm}$$

PDF of the local flow velocity fluctuations



[Lebond & al. (1997)]

PLAN

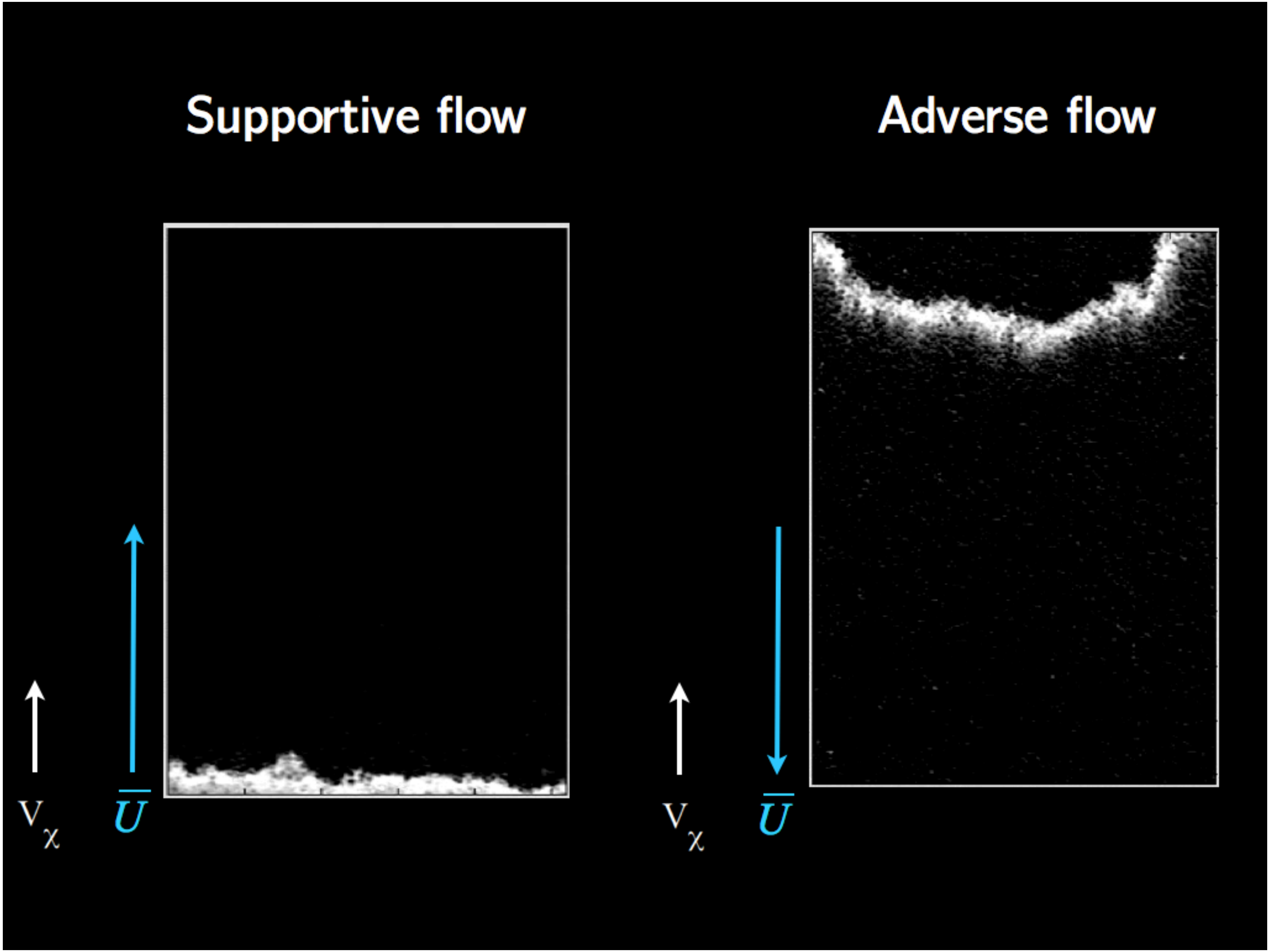
1 - Experimental setup

2 - Front dynamics in high flow strength

3 - Frozen pattern formation

4 - Critical behavior

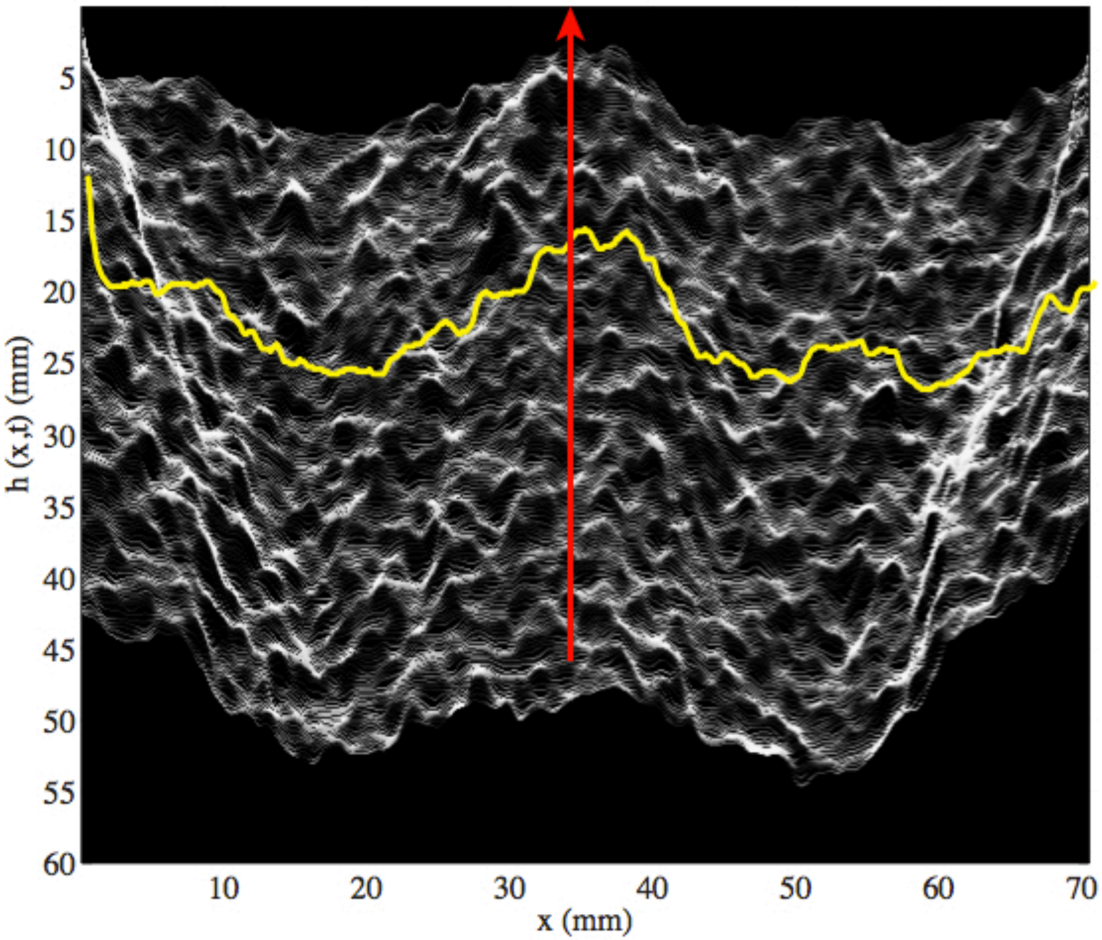
5 - Conclusion and perpectives

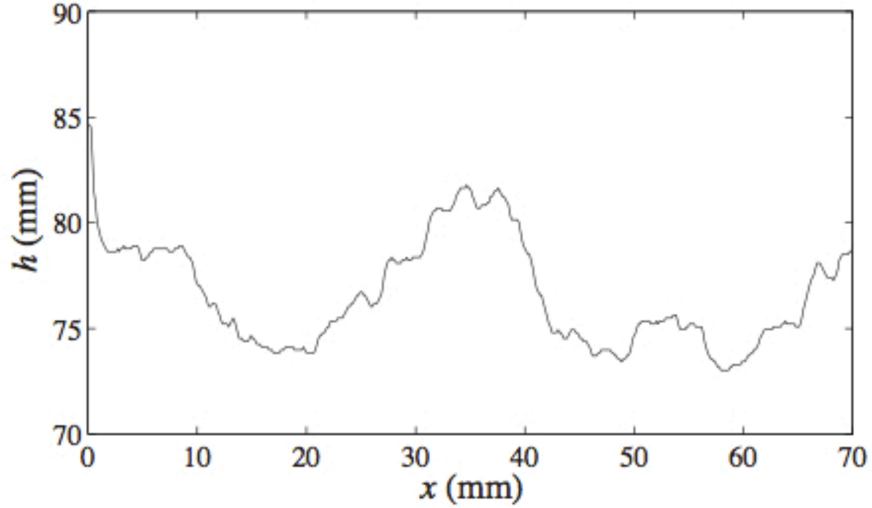


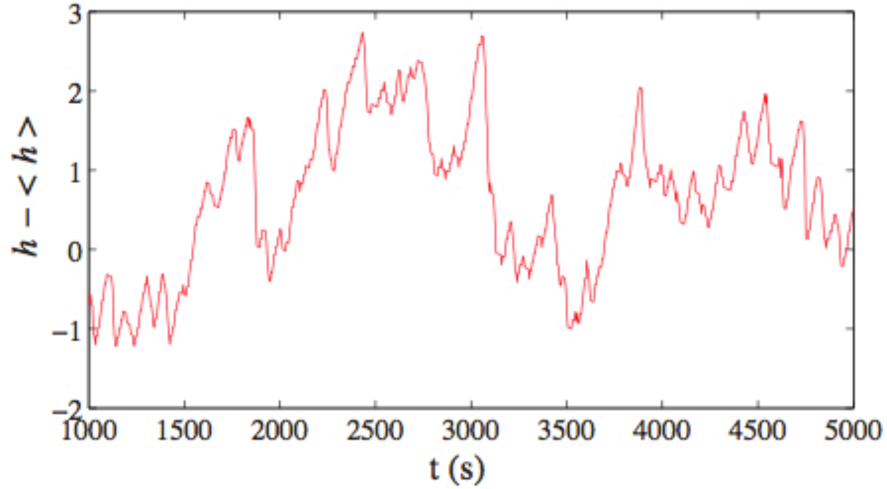
Supportive flow
Adverse flow
V_χ
$\overline{U}$
V_χ
$\overline{U}$

2 - Front dynamics in high flow strength

- **Front height spatiotemporal fluctuations measurements**





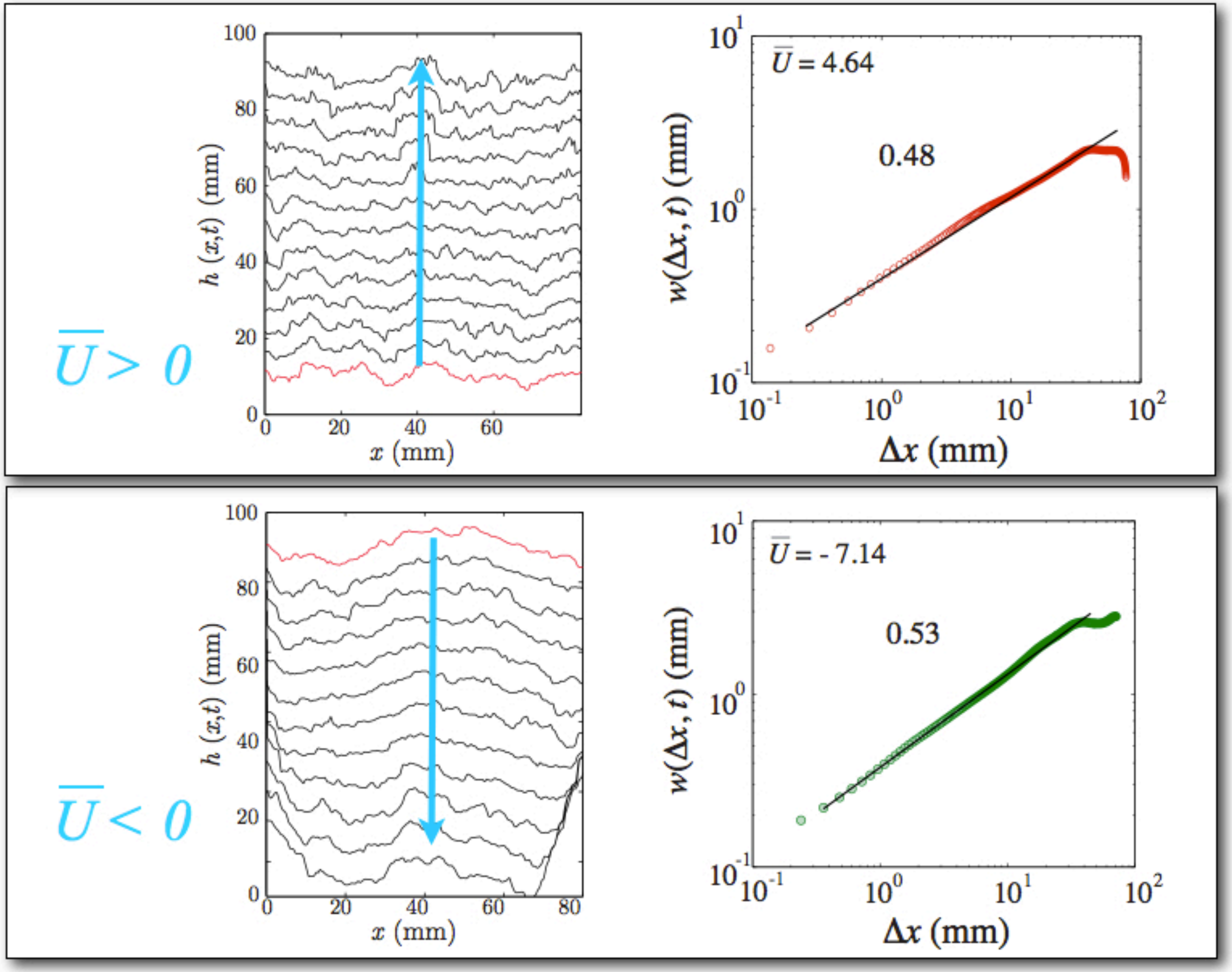


2 - Front dynamics in high flow strength

- **Roughness** $w(\Delta x, t) = \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_{\Delta x}]^2 \rangle_{\Delta x}} \right\rangle_L$

power law:

$$w(\Delta x, t) \sim \Delta x^{\alpha}$$

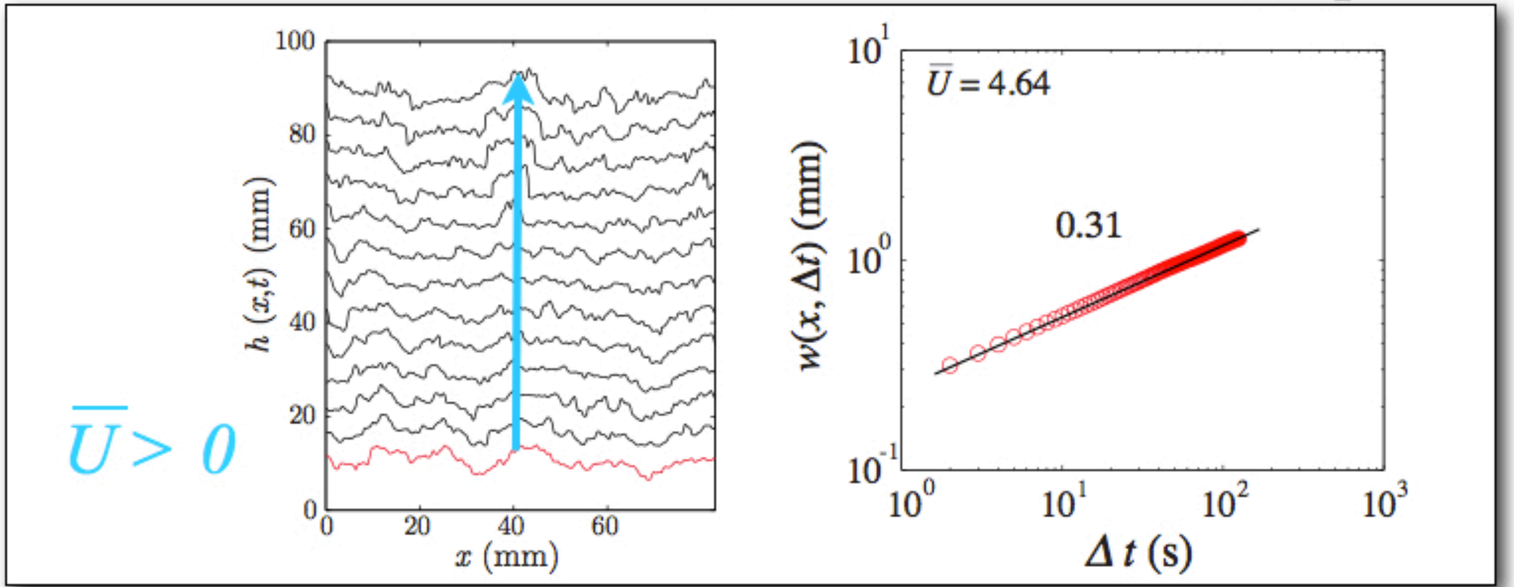


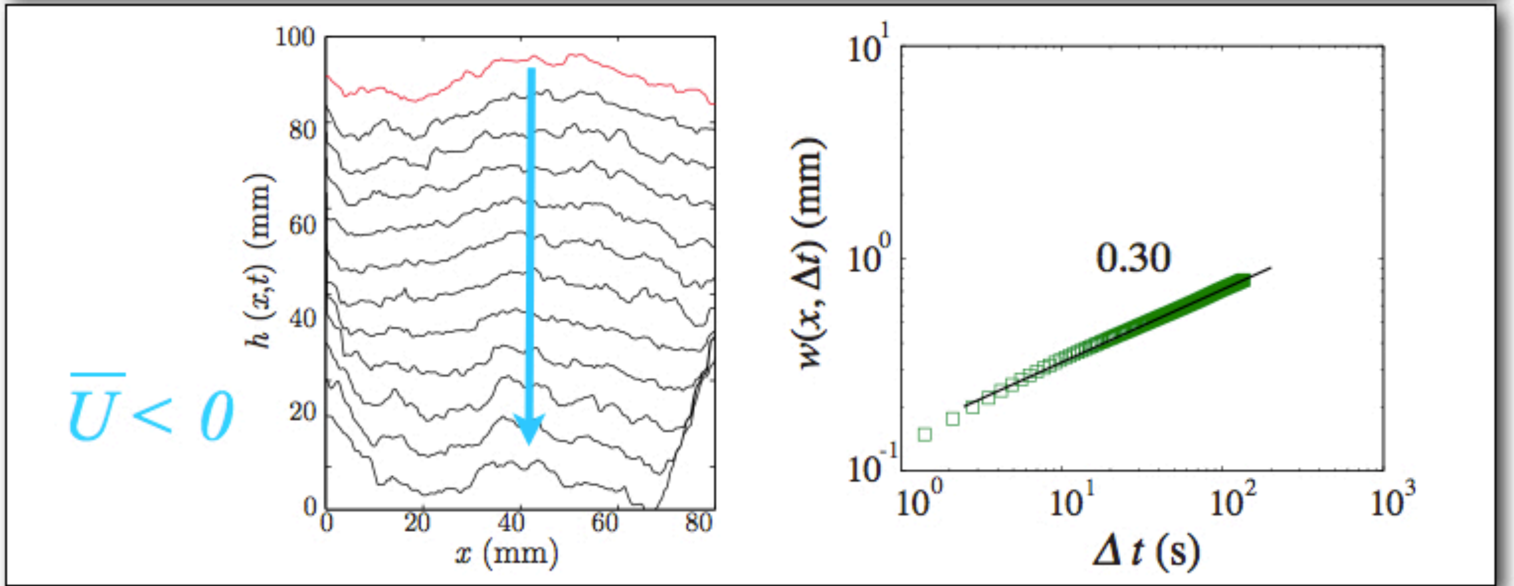
2 - Front dynamics in high flow strength

- **Temporal fluctuations** $w(x,\Delta t) = \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_{\Delta t}]^2 \rangle_{\Delta t}} \right\rangle_T$

power law:

$w(x,\Delta t) \sim \Delta t^{\beta}$





2 - Front dynamics in high flow strength

- **Theory**

Kardar-Parisi-Zhang (KPZ) model:

Nonlinear continum growth equation

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}\left[\nabla h(x,t)\right]^2 + \eta(x,t) + f$$

Predicted exponents: $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$

[Kardar & al. 1986]

random deposition

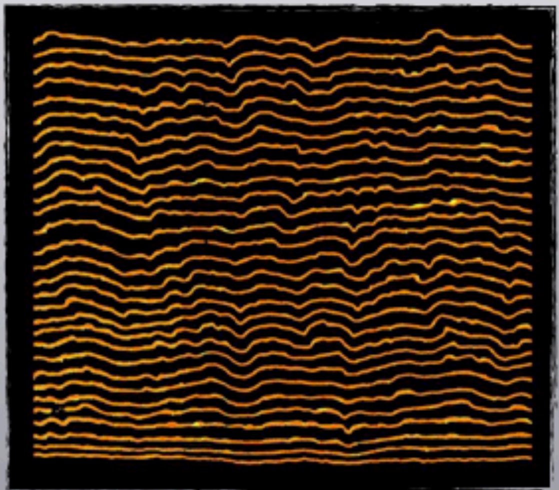
lateral growth

+ relaxation term
+ driving force

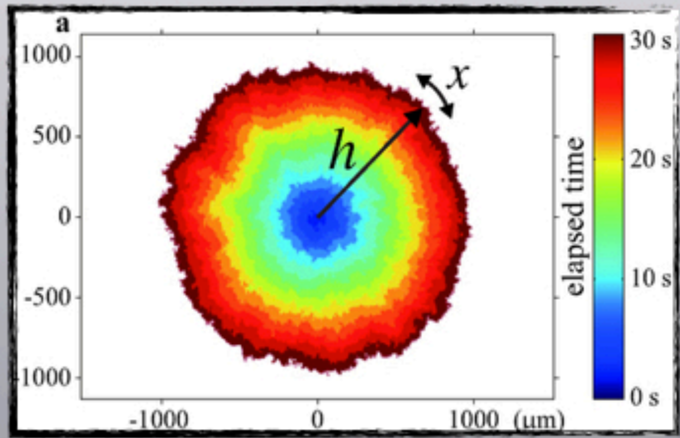
2 - Front dynamics in high flow strength

- Experimental observation

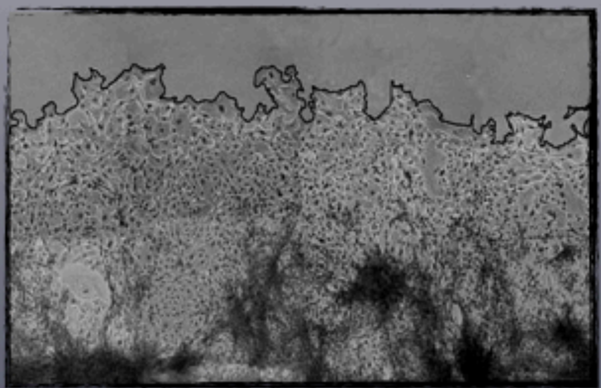
Myllys et al. 1993



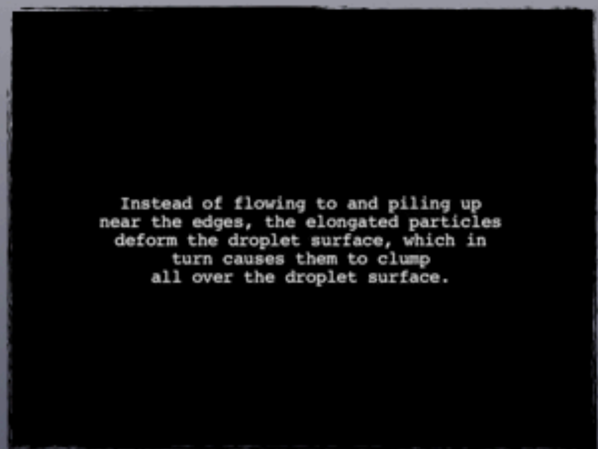
Takeuchi et al.2010



Huergo et al.2010



Yunker et al.2013

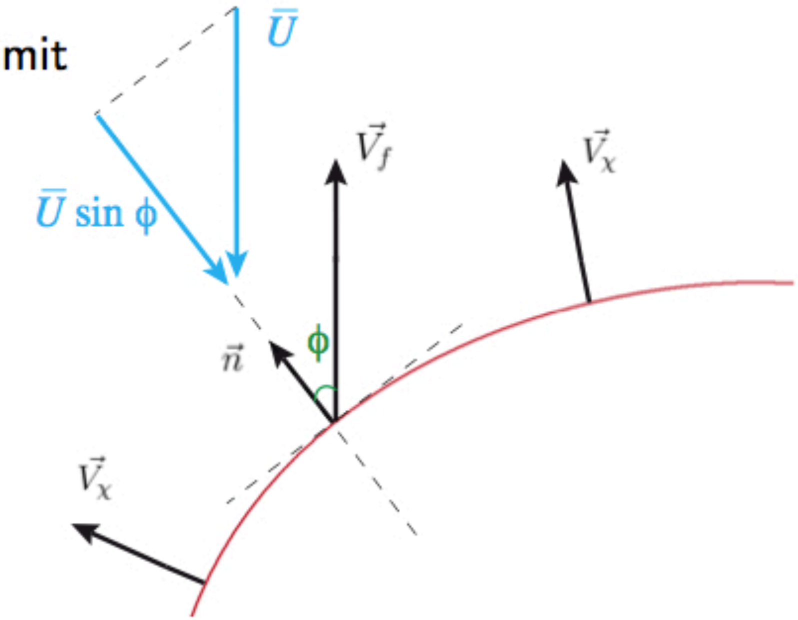


Advection - Reaction - Diffusion equation in thin front limit

- **Eikonal approximation** [Edwards (2002)]

$$\vec{V}_f \cdot \vec{n} = D_m \kappa + V_\chi + \vec{U}(x, h(x,t), t) \cdot \vec{n}$$

$$V_f \cos\phi = -U_x \sin\phi + U_y \cos\phi + V_\chi + D_m \kappa$$



Small gradients limite $|\nabla h| \ll 1$:

$$\frac{\partial h}{\partial t} = D_m \nabla^2 h + \frac{V_\chi}{2}(\nabla_x h)^2 + U_y + V_\chi - U_x \nabla_x h - D_m \nabla^2 h (\nabla_x h)^2$$

KPZ equation :

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}(\nabla h(x,t))^2 + \eta(x,t) + F$$

$$U_y = \overline{U_y} + \delta U_y(x, h(x,t))$$

with

$$F = V_\chi + \overline{U_y}$$

[S. Atis, K. D. Awadhesh, D. Salin, L. Talon, P. Le Doussal, K. Wiese, *submitted* (ArXiv)]

$$\vec{V}_f = \begin{pmatrix} 0 \\ V_f \end{pmatrix}, \quad \vec{U} = \begin{pmatrix} U_x \\ U_y \end{pmatrix}, \quad \vec{n} = \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}$$

$$V_f = \frac{\partial h}{\partial t} \quad \text{et} \quad \kappa = \frac{\partial^2 h/\partial x^2}{(1+(\partial h/\partial x)^2)^{3/2}},$$

$$\tan\phi = \nabla_x h$$

$$\cos\phi = \frac{1}{\sqrt{1+(\nabla_x h)^2}}$$

PLAN

1 - Experimental setup

2 - Front dynamics in high flow strength

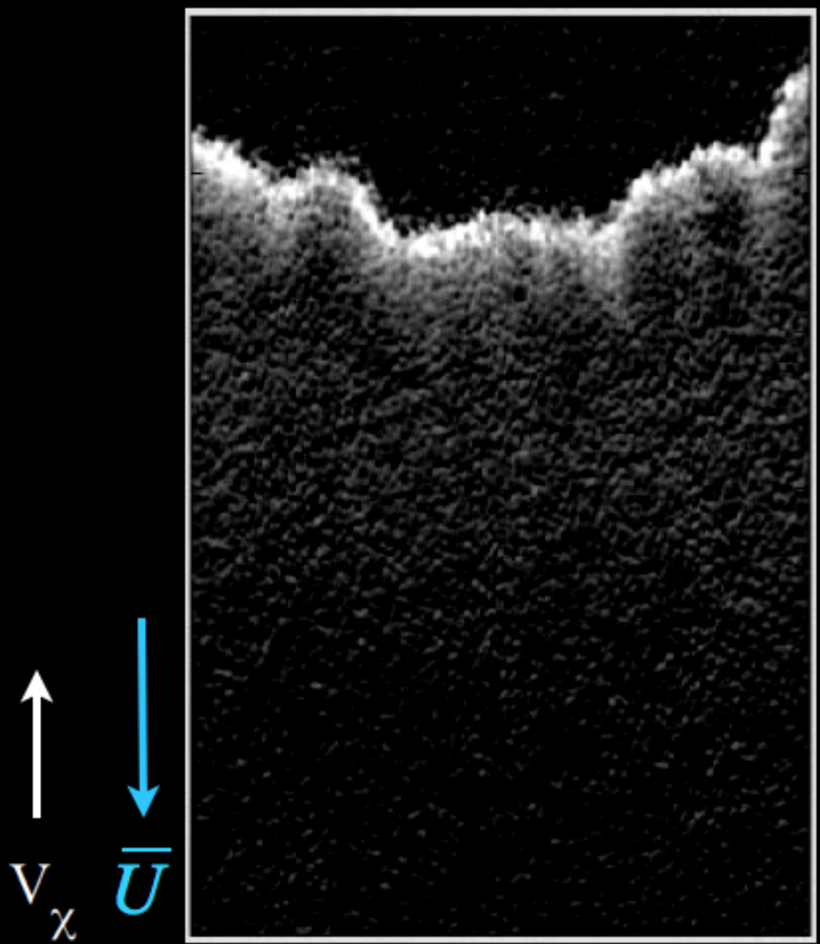
3 - Frozen pattern formation

4 - Critical behavior

5 - Conclusion and perpectives

Adverse flow

backward



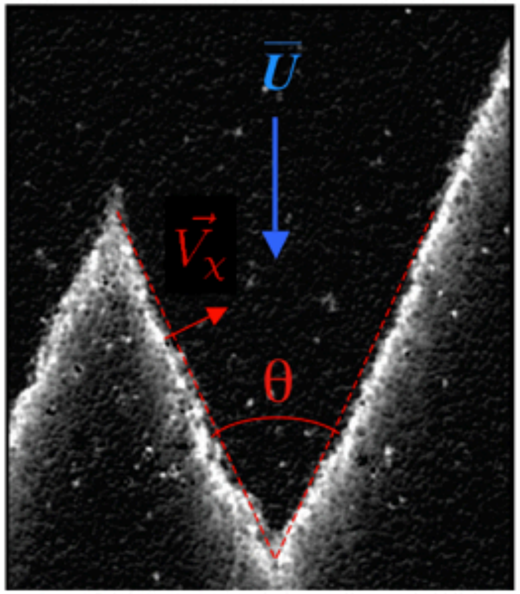
3 - Frozen pattern formation

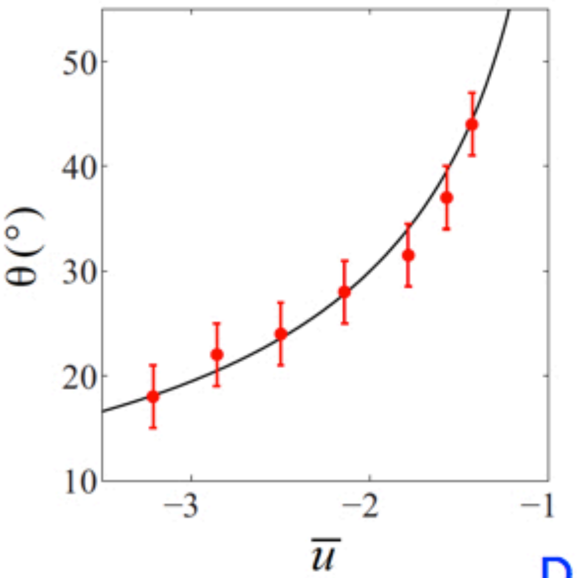
- **approximation eikonal** $l_\chi << l_d$

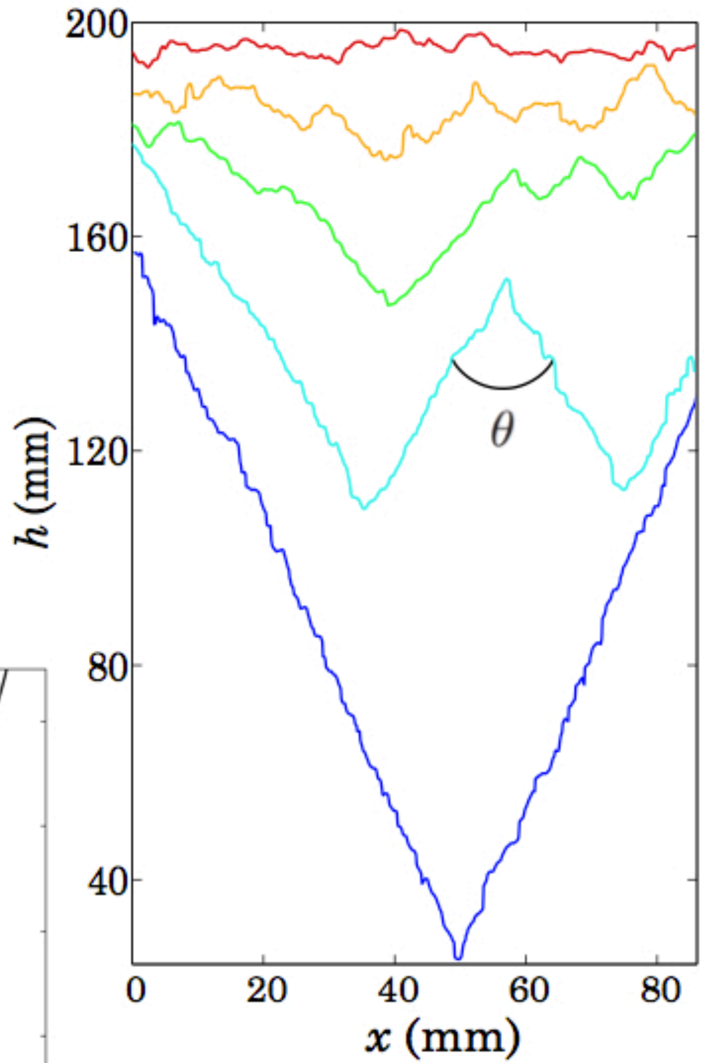
$$\vec{V_f}(\vec{r})\cdot\vec{n} = V_\chi + \vec{U}(\vec{r})\cdot\vec{n} + D_m\kappa$$

- **final static fronts**

$$V_\chi + \overline{U}\sin(\theta/2) = 0$$

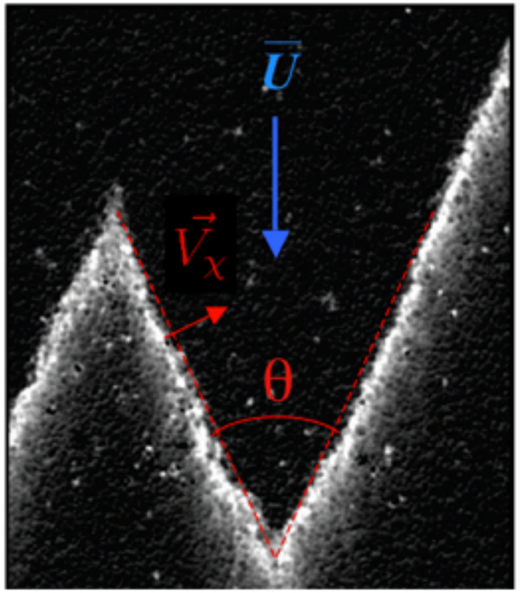






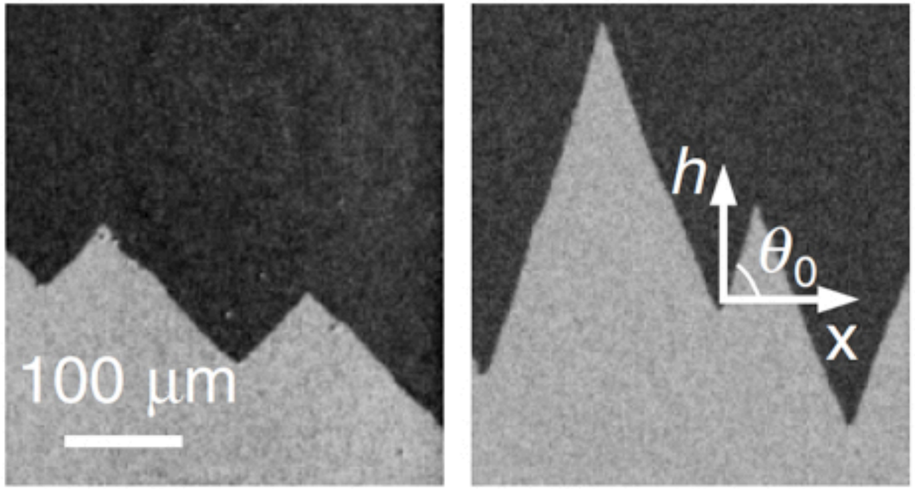
[S. Atis, S. Saha, H. Auradou, D. Salin, L. Talon, *PRL* **110** (2013)]

3 - Frozen pattern formation



$$V_\chi = \overline{U} \sin(\theta/2)$$

- **Magnetic domain wall frozen steady states**



[Moon et al. 2013]

$$H = \epsilon J \cos\theta_0$$

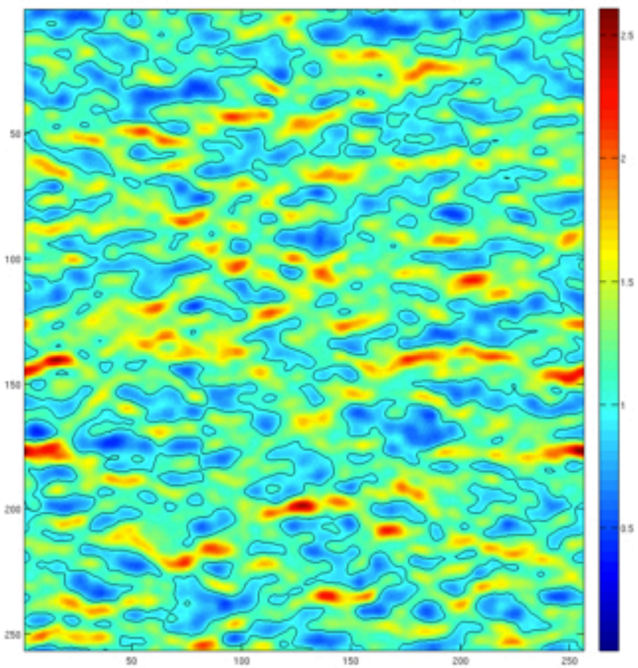
H : applied magnetical filed

J : applied electrical current

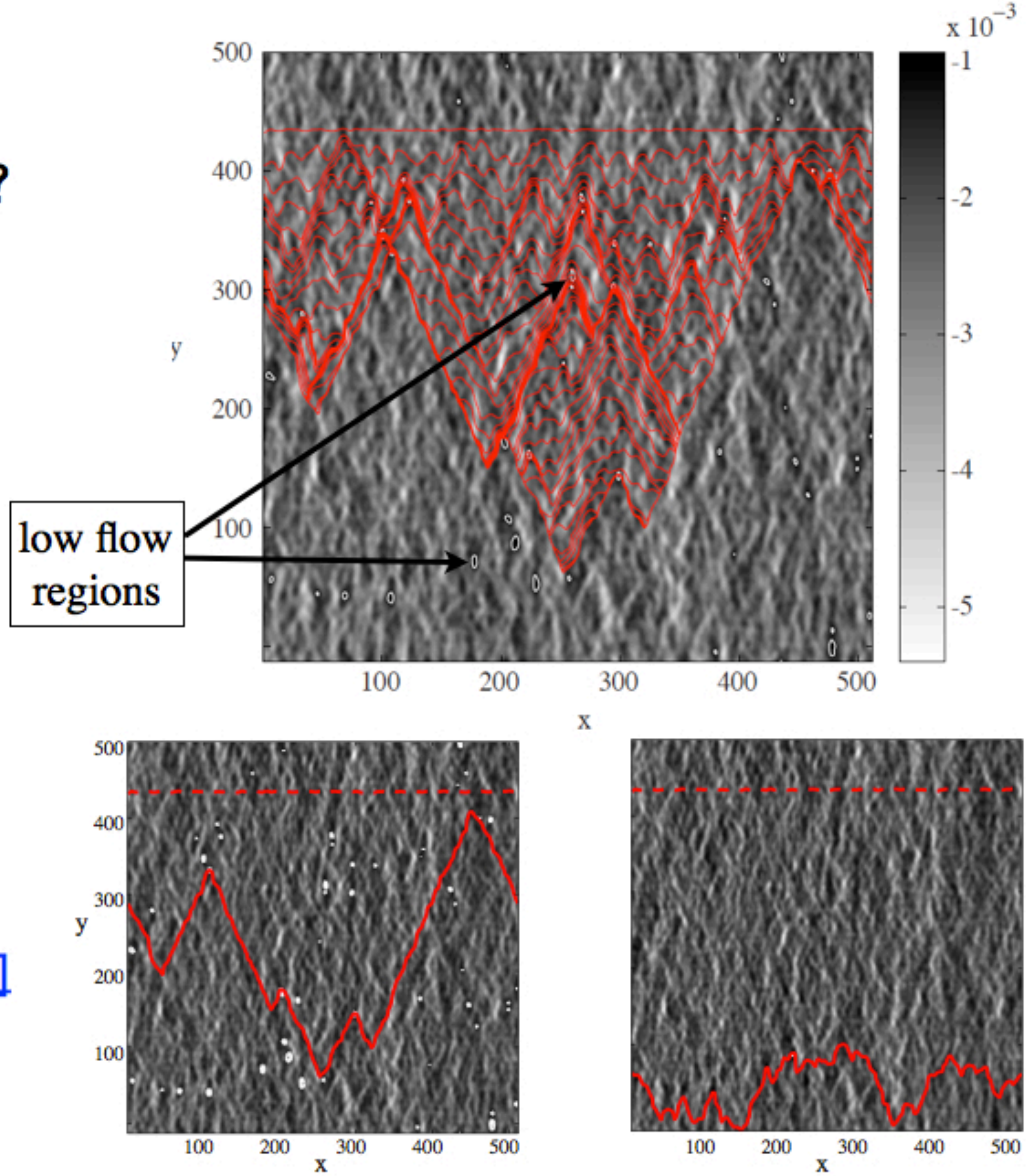
ϵ : nonadiabatic STT

- **Thermodynamic study of non-equilibrium steady states**

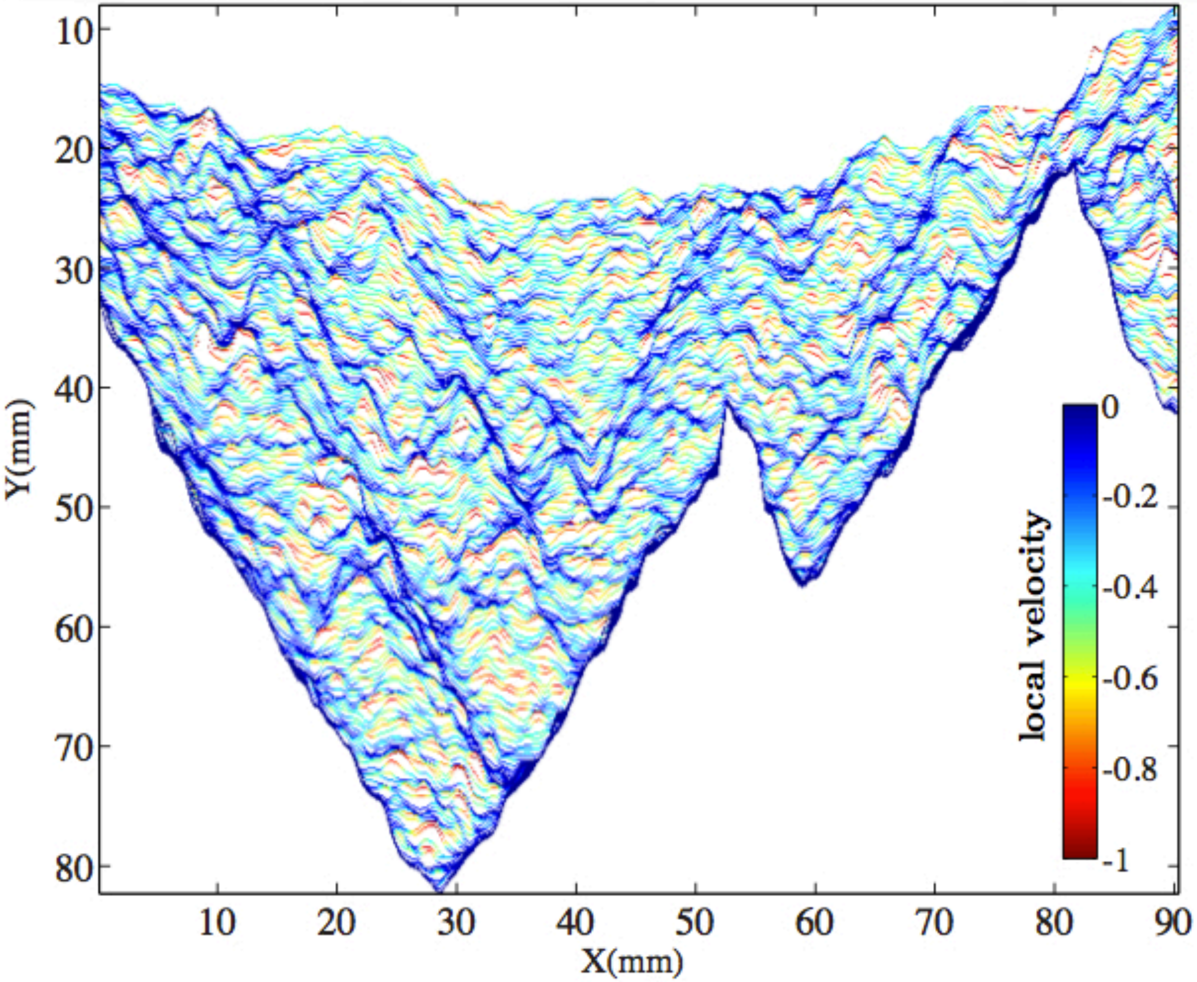
- **What is pinnig reaction fronts?**

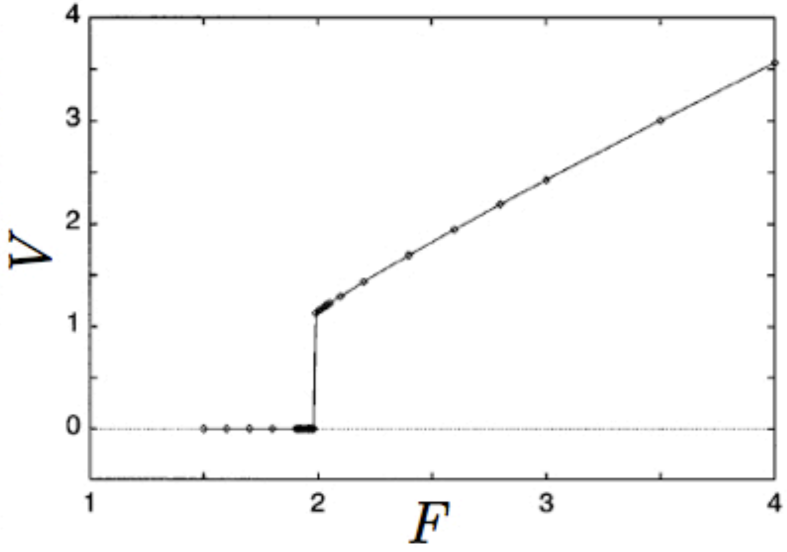


[S. Saha, S. Atis, D. Salin, L Talon (2013)]

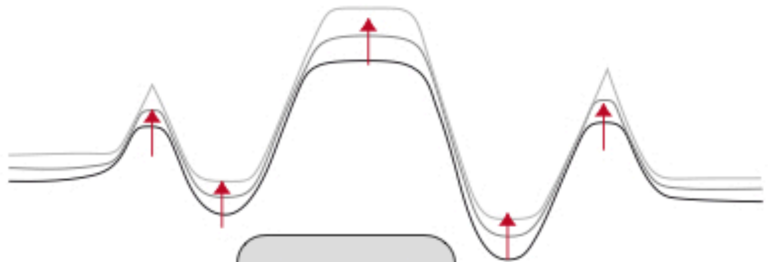


3 - Frozen pattern formation





[Jeong et al. (1996)]



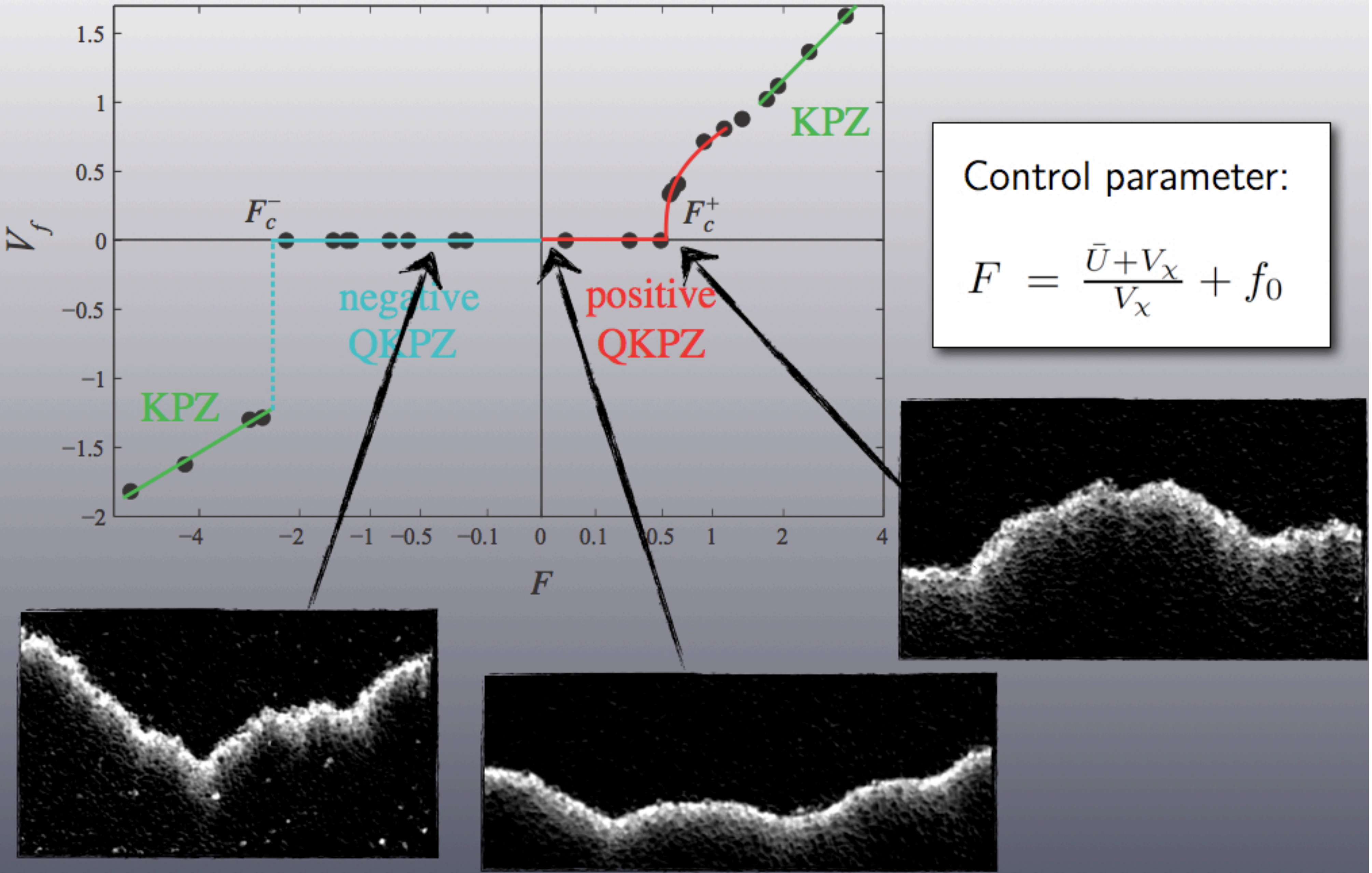
$V_f < 0$

$V_\chi > 0$

- **negative quenched KPZ model**

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) - \frac{\lambda}{2}\left[\nabla h(x,t)\right]^2 + \bar{\eta}\Big(x, h(x,t)\Big) + f$$

3 - Frozen pattern formation



Control parameter:

$$F = \frac{\bar{U} + V_x}{V_x} + f_0$$

PLAN

1 - Experimental setup

2 - Front dynamics in high flow strength

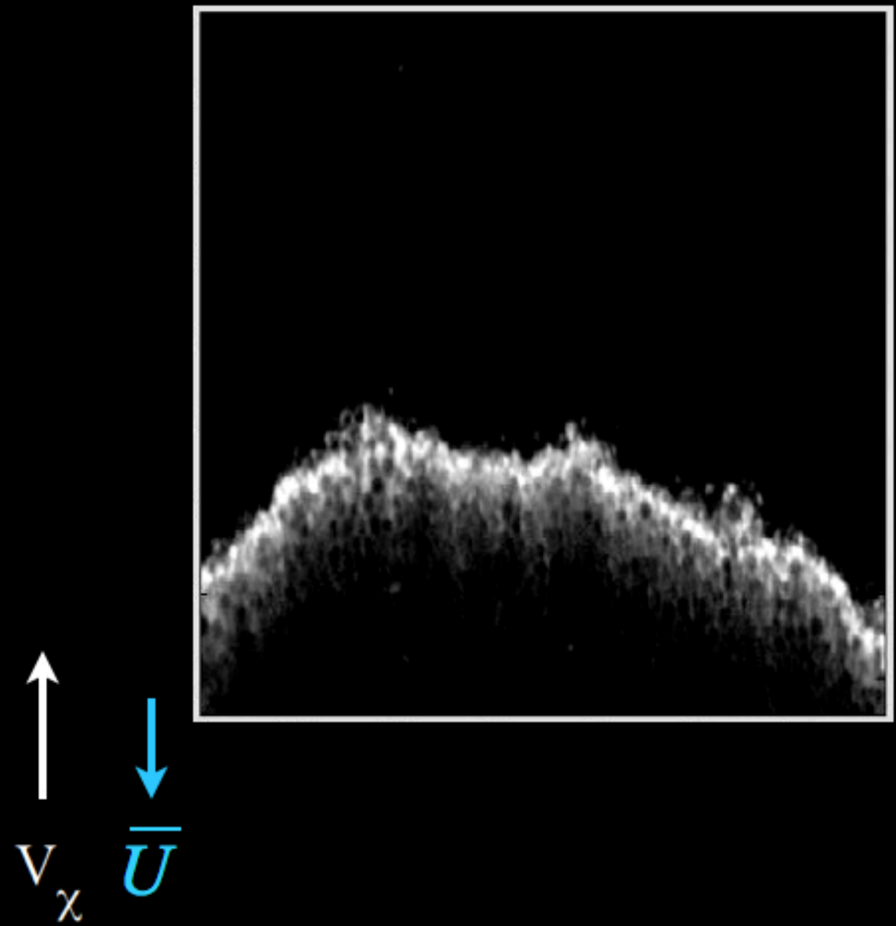
3 - Frozen pattern formation

4 - Critical behavior

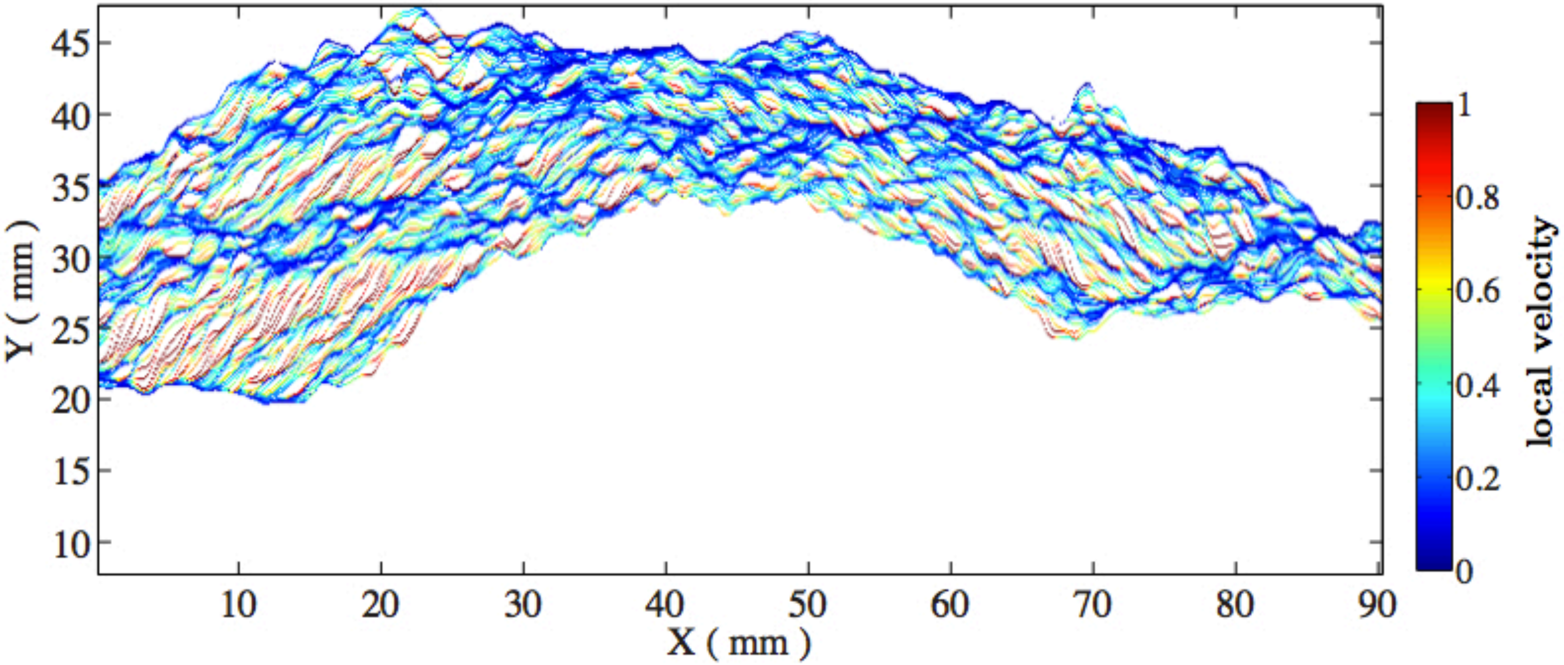
5 - Conclusion and perpectives

Adverse flow

upward

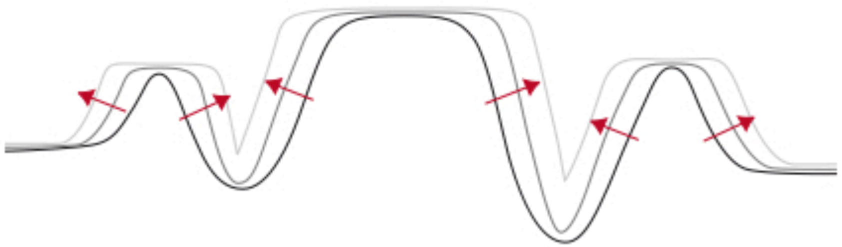


4 - Critical behavior



$V_f > 0$

$V_\chi > 0$



- **positive quenched KPZ model**

$$\frac{\partial h(x,t)}{\partial t} = \nu \nabla^2 h(x,t) + \frac{\lambda}{2}\left[\nabla h(x,t)\right]^2 + \bar{\eta}\Big(x, h(x,t)\Big) + f$$

$\alpha = \beta = 0.63$

4 - Transcient dynamics and universality

qKPZ exponents: $\alpha = \beta = 0.63$

Directed percolation class

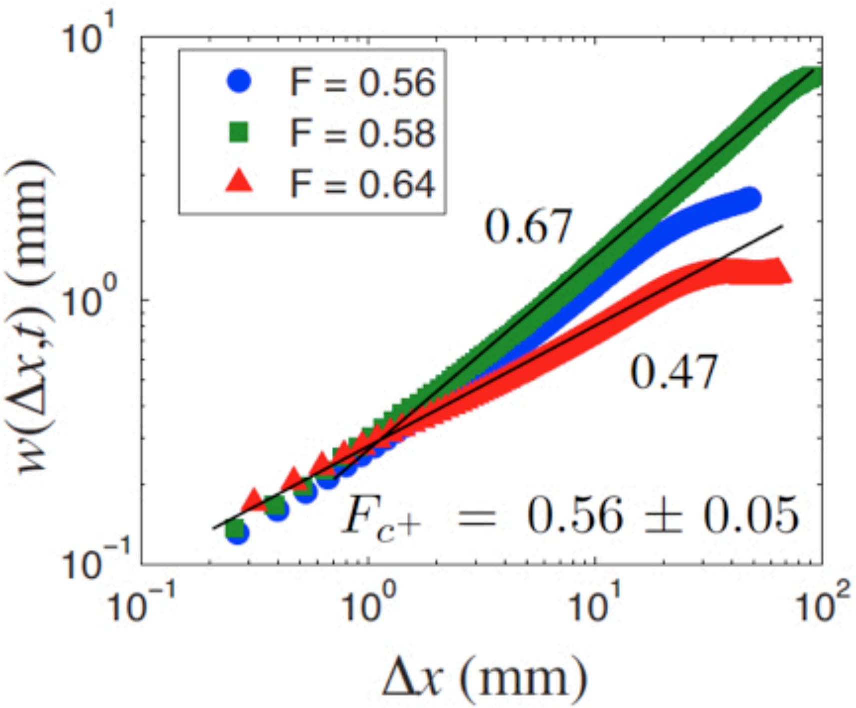
$$\left.\begin{array}{l} \nu_{\parallel} = 1.733 \pm 0.001 \\ \nu_{\perp} = 1.097 \pm 0.001 \end{array}\right\} \alpha_{\mathrm{DP}} = \frac{\nu_{\parallel}}{\nu_{\perp}} \simeq 0.63$$

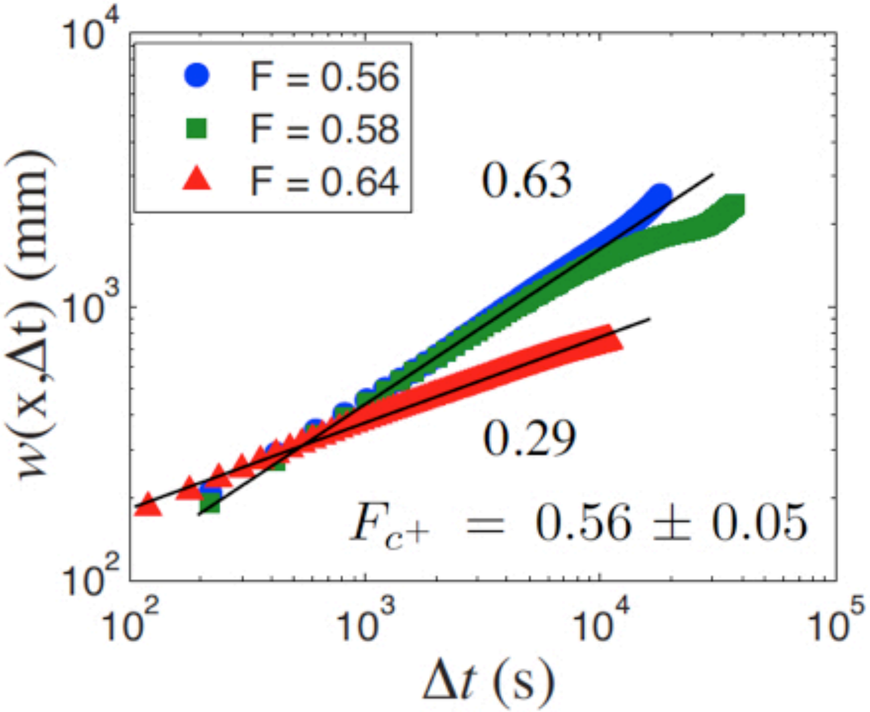
- **Roughness**

$$w(\Delta x, t) = \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_{\Delta x}]^2 \rangle_{\Delta x}} \right\rangle_L$$

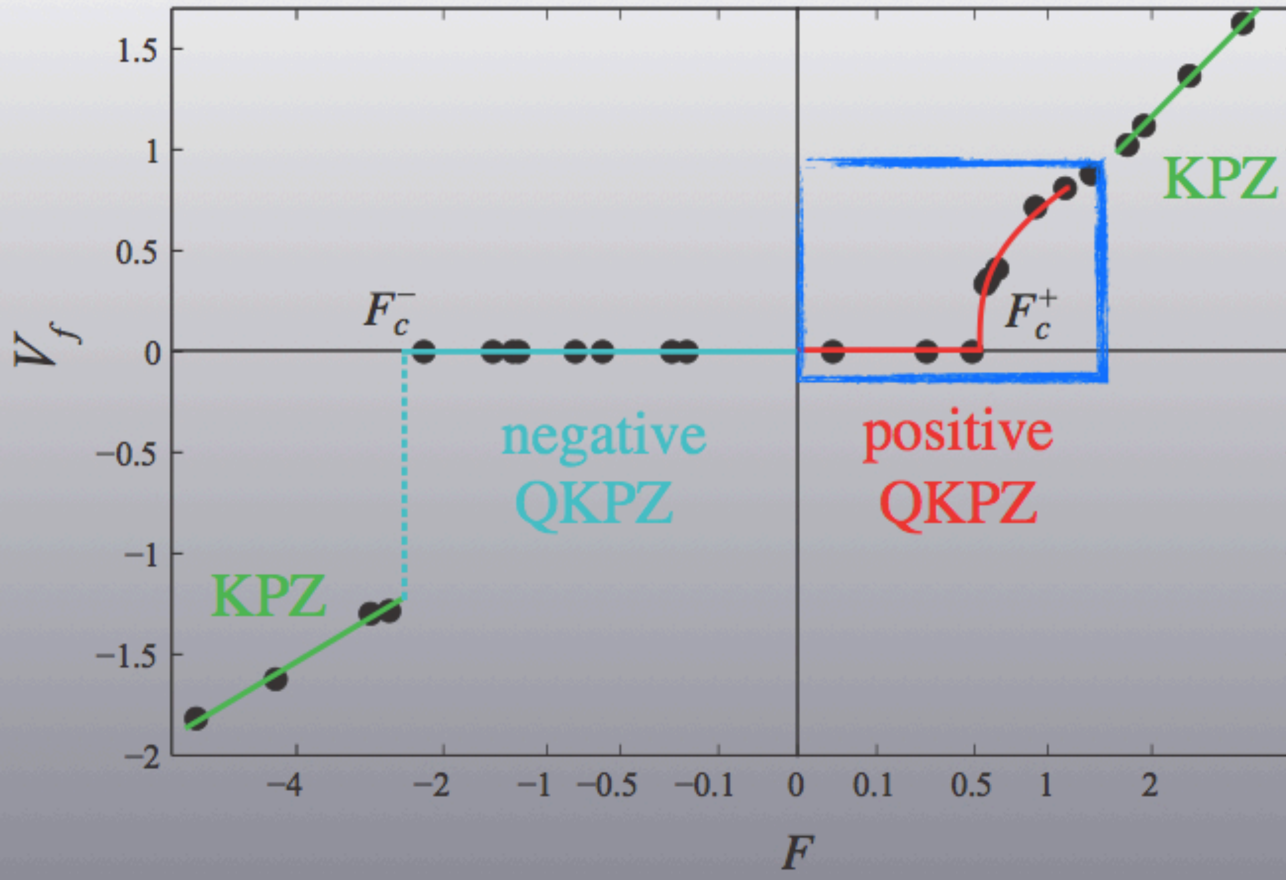
- **Temporal fluctuations**

$$w(x, \Delta t) = \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_{\Delta t}]^2 \rangle_{\Delta t}} \right\rangle_T$$





4 - Critical behavior



Control parameter:

$$F = \frac{\bar{U}+V_x}{V_x} + f_0$$

- Depinning transition

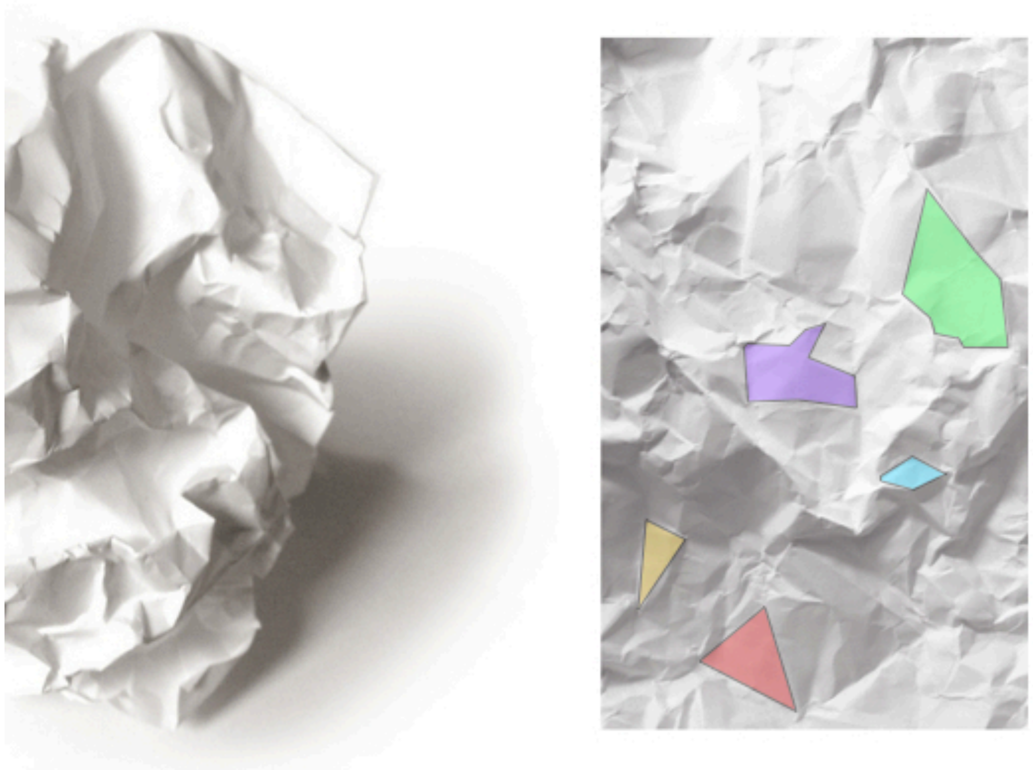
[S. Atis, K. D. Awadhesh, D. Salin, L. Talon, P. Le Doussal, K. Wiese, *submitted* (ArXiv)]

4 - Critical behavior

- **crackling noise**

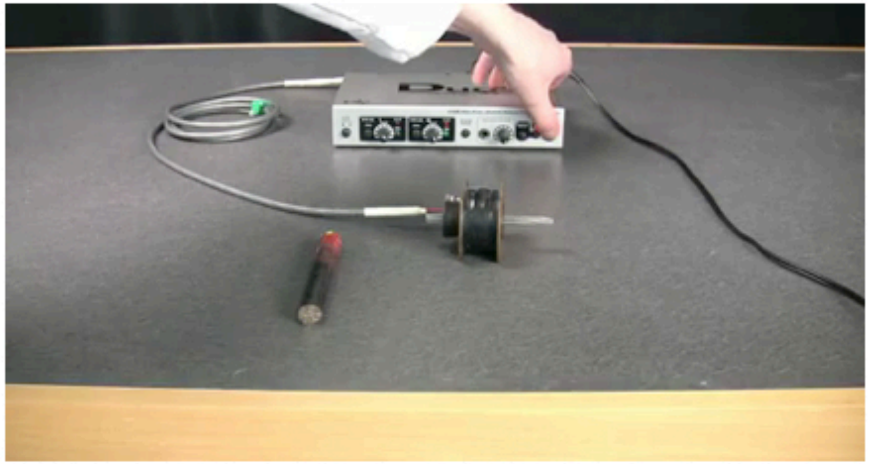
[J. P. Sethna et al. 2001]

paper crumpling



[P. Le Doussal, K. Wiese, *submitted* (ArXiv)]

Barkhausen noise

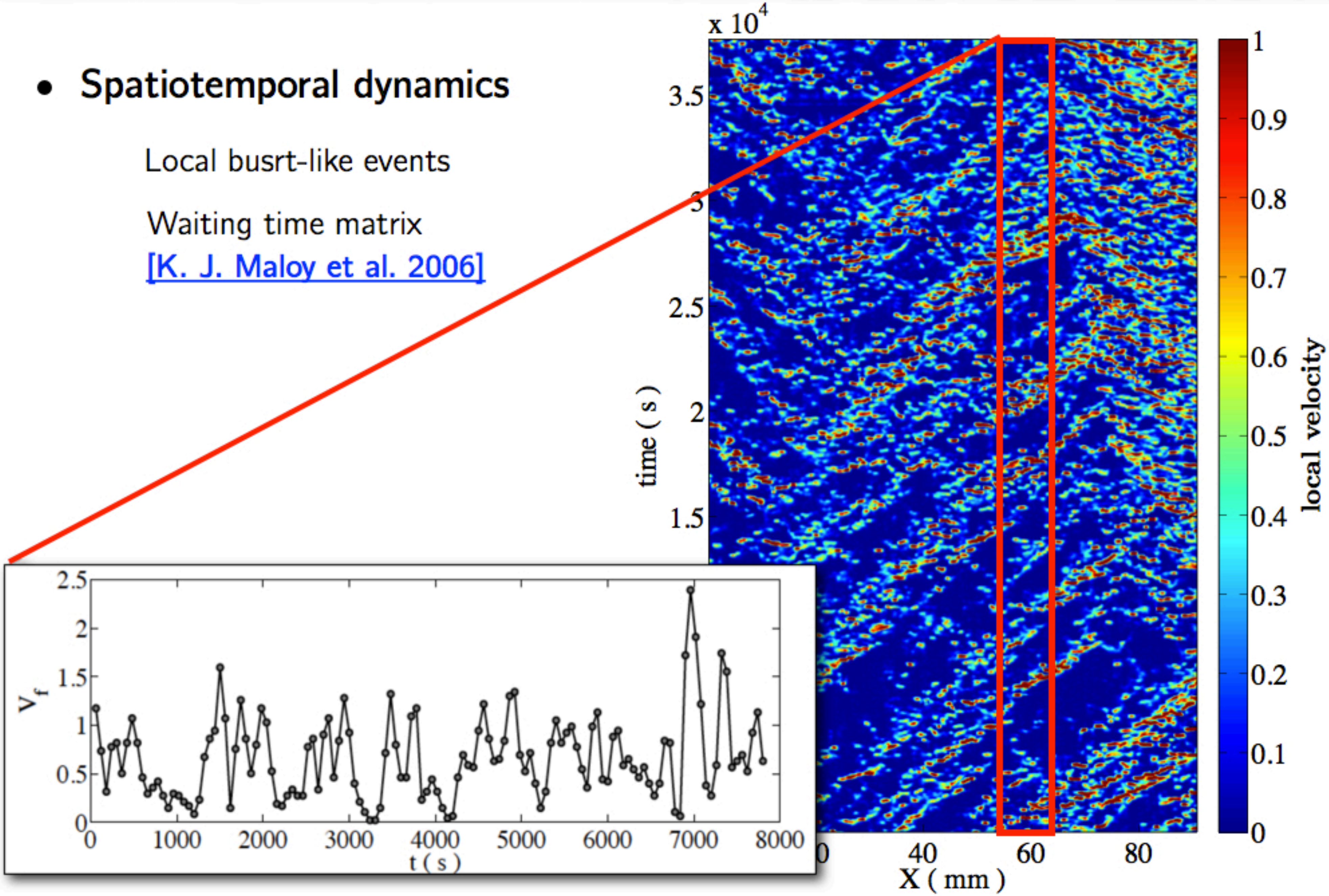


Dendritic flux avalanches in type II superconductors films

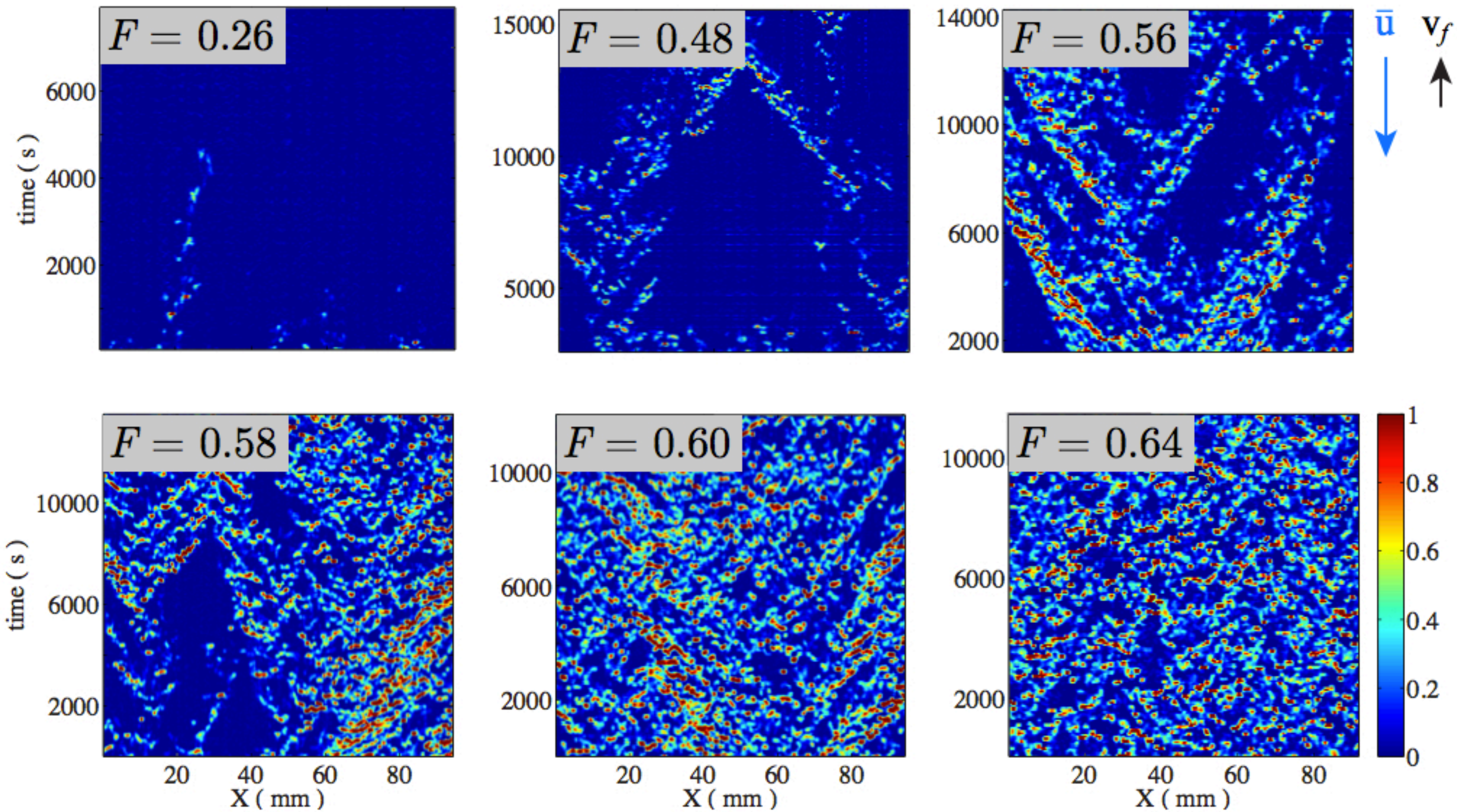
- **Spatiotemporal dynamics**

  Local busrt-like events

  Waiting time matrix
  [K. J. Maloy et al. 2006]

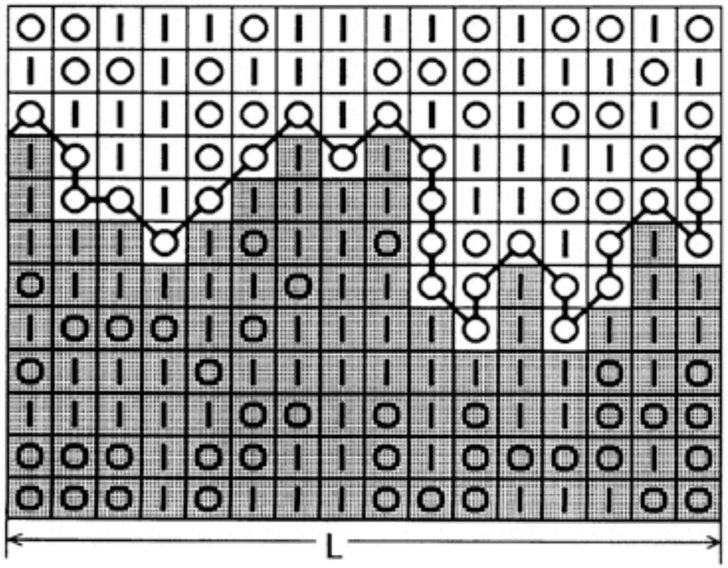


4 - Critical behavior



4 - Critical behavior

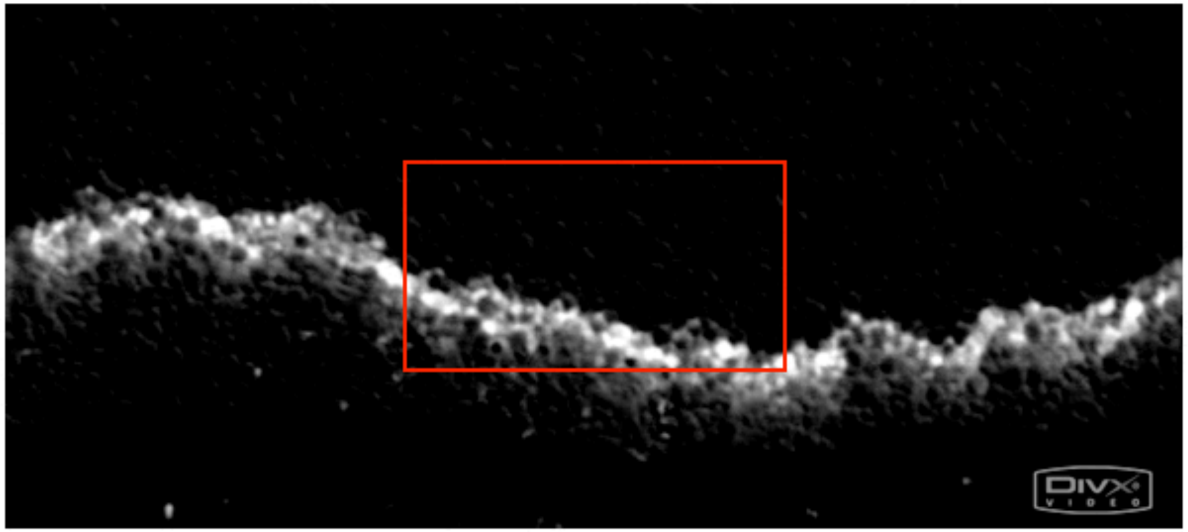
- percolation-like mechanism

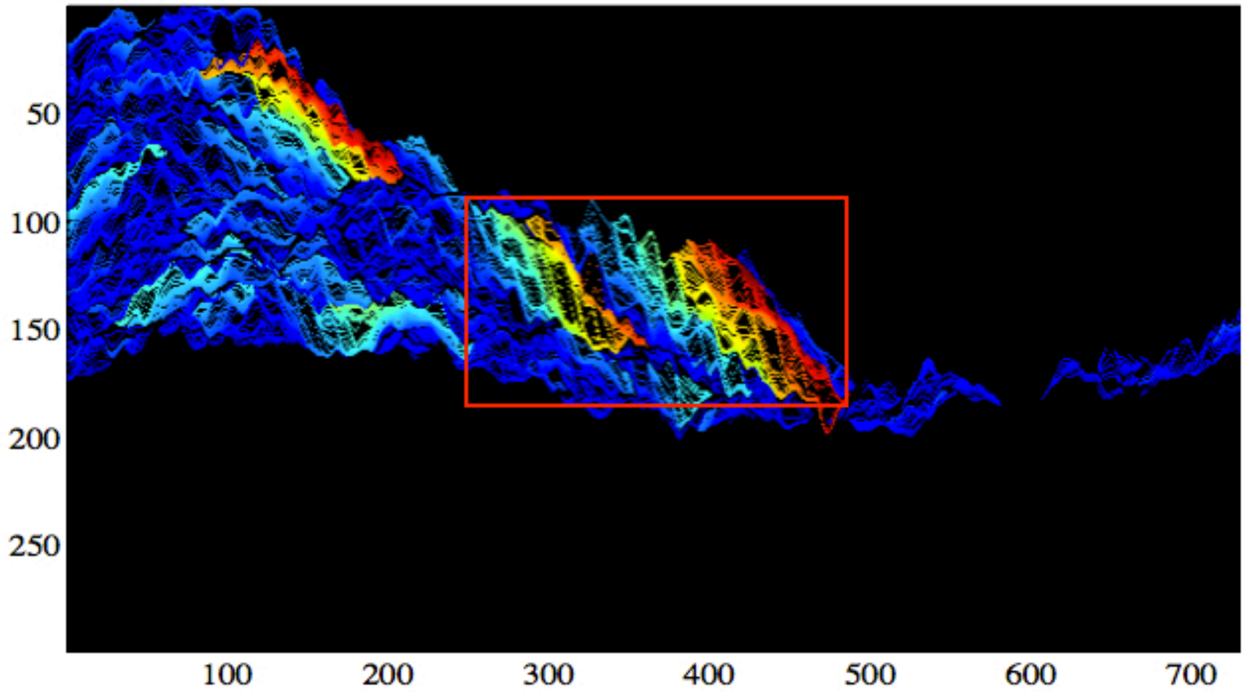


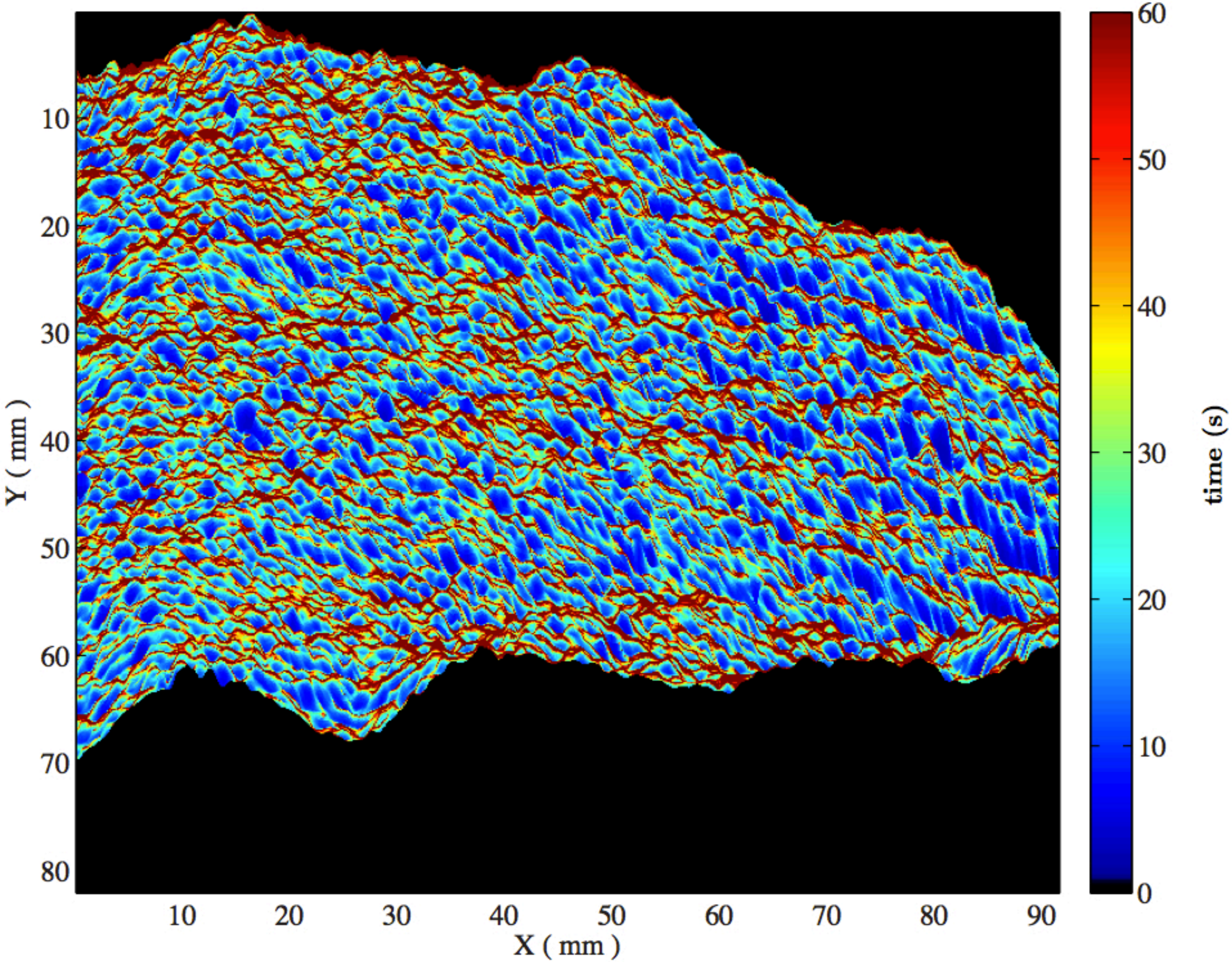
[Buldyrev et al. (1992)]

[Tang and Leschhorn (1992)]

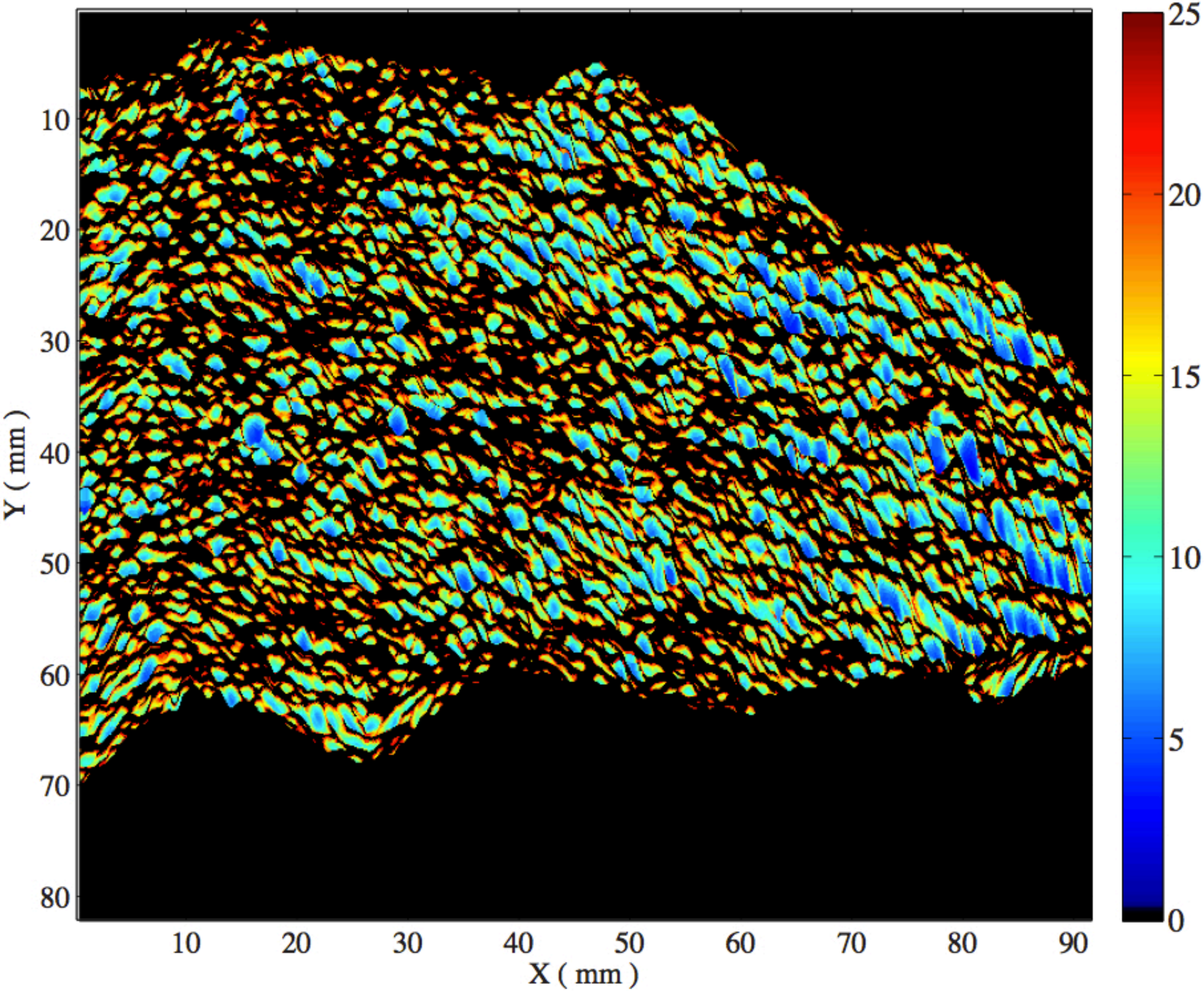
- local avalanches



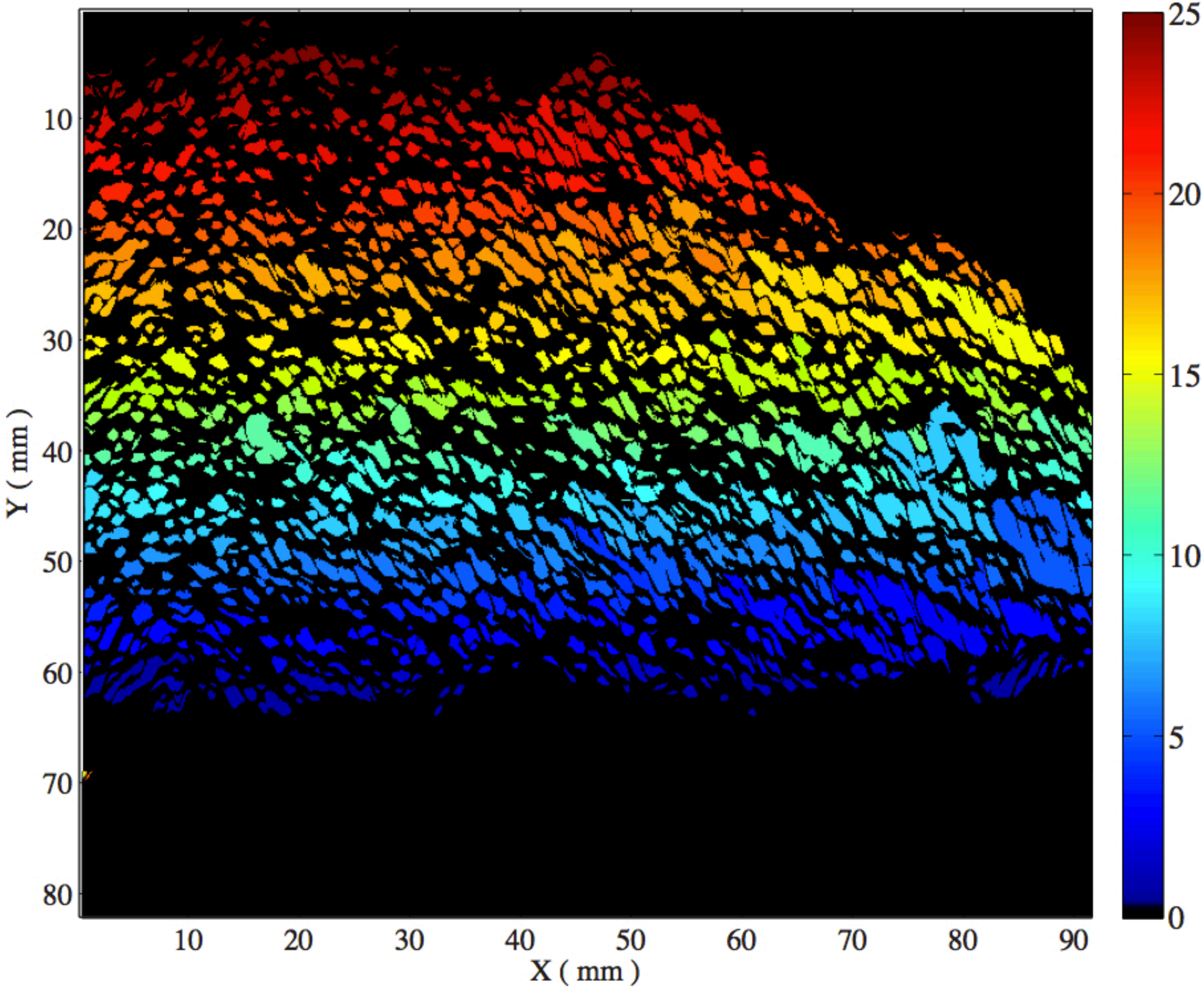




Y (mm)
10
20
30
40
50
60
70
80
X (mm)
10
20
30
40
50
60
70
80
90
time (s)
60
50
40
30
20
10
0

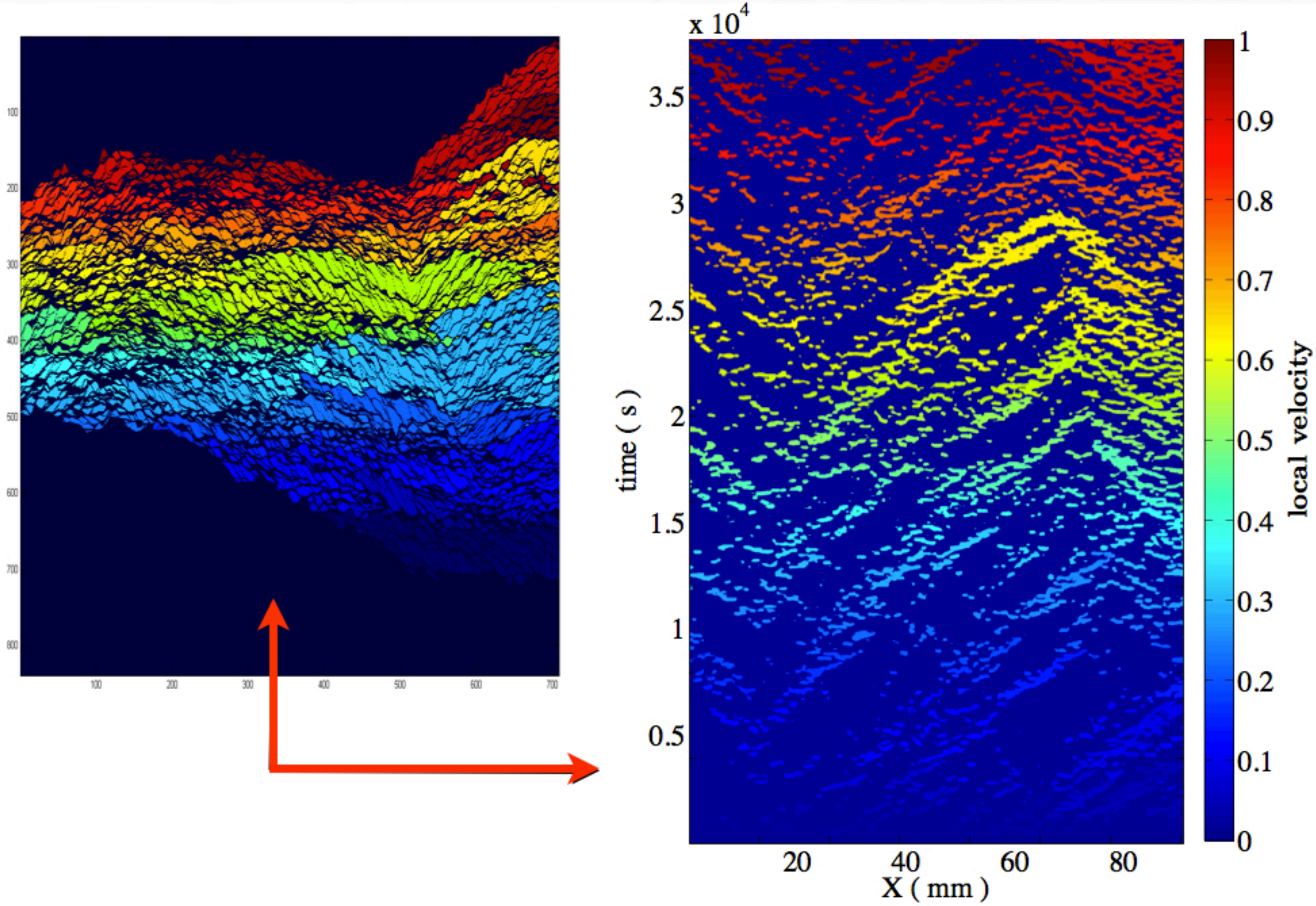


25
20
15
10
5
0
10
20
30
40
50
60
70
80
Y (mm)
10
20
30
40
50
60
70
80
90
X (mm)



Y (mm)
10
20
30
40
50
60
70
80
X (mm)
10
20
30
40
50
60
70
80
90
25
20
15
10
5
0

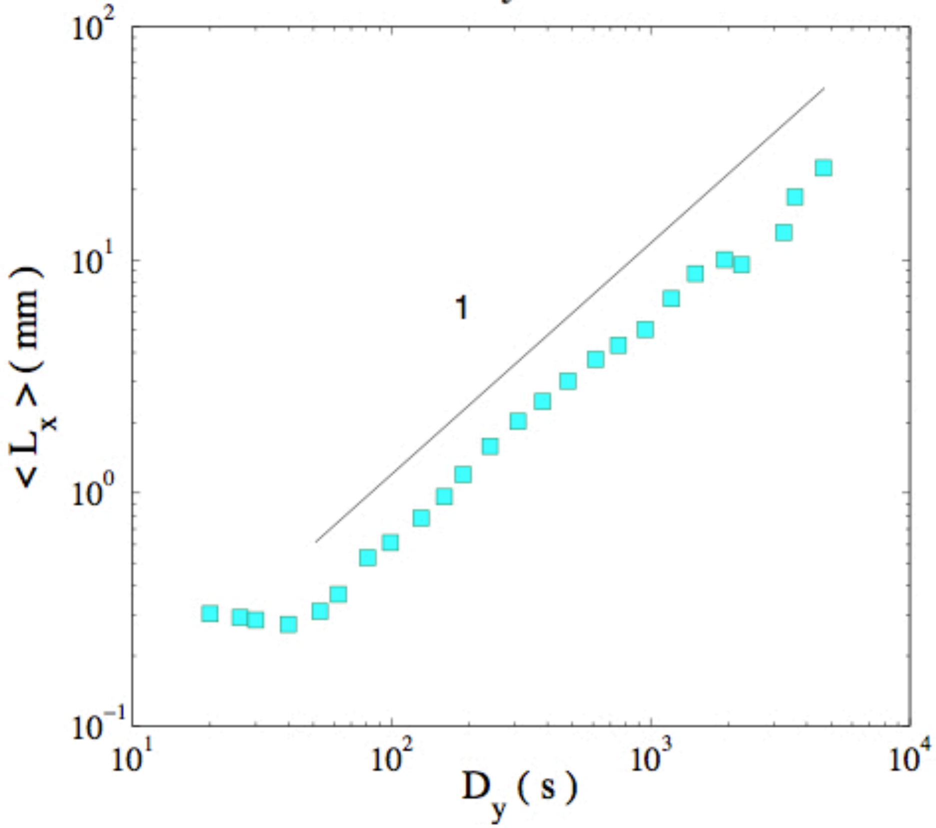
4 - Critical behavior



4 - Critical behavior

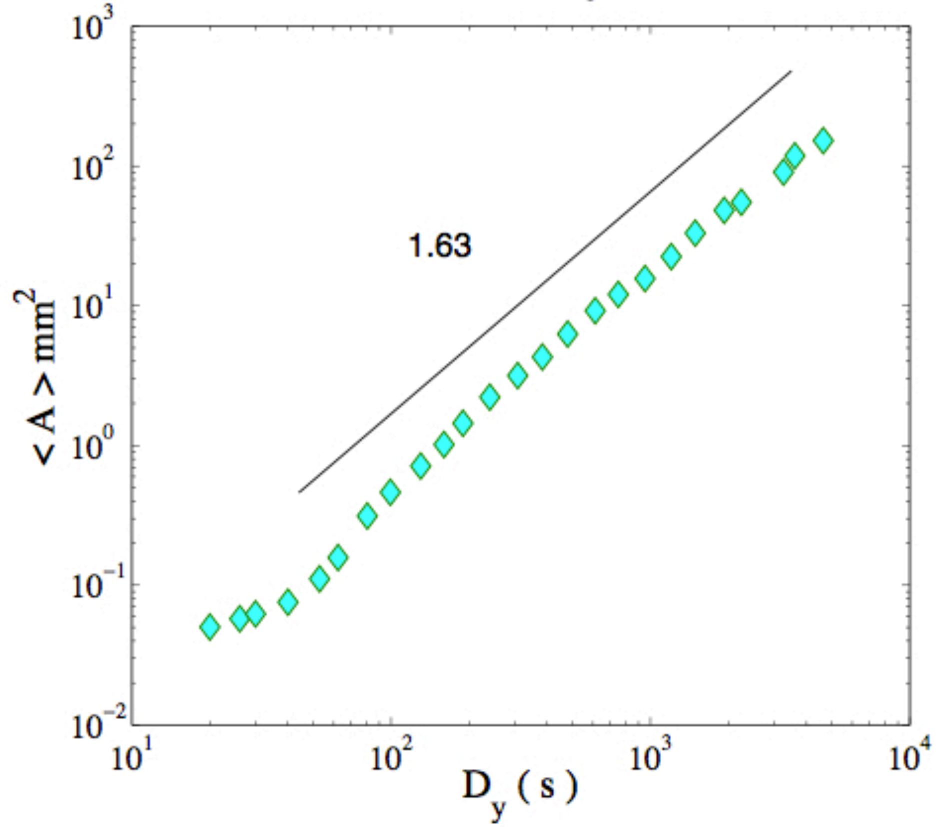
- **Dynamical exponent**

$$< L_x > \sim D_y^{1/z_A}$$



- **Avalanche size-duration**

$$< A > \sim D_y^{\gamma}$$



$$A \sim L_x L_y \sim L_x^{1+\alpha_A} \longrightarrow A \sim D_y^{\frac{1+\alpha}{z}}$$

[L. A. N. Amaral et al. (1995)]

[S. Santucci et al. (2011)]

$$\frac{1+\alpha}{z} = \gamma$$

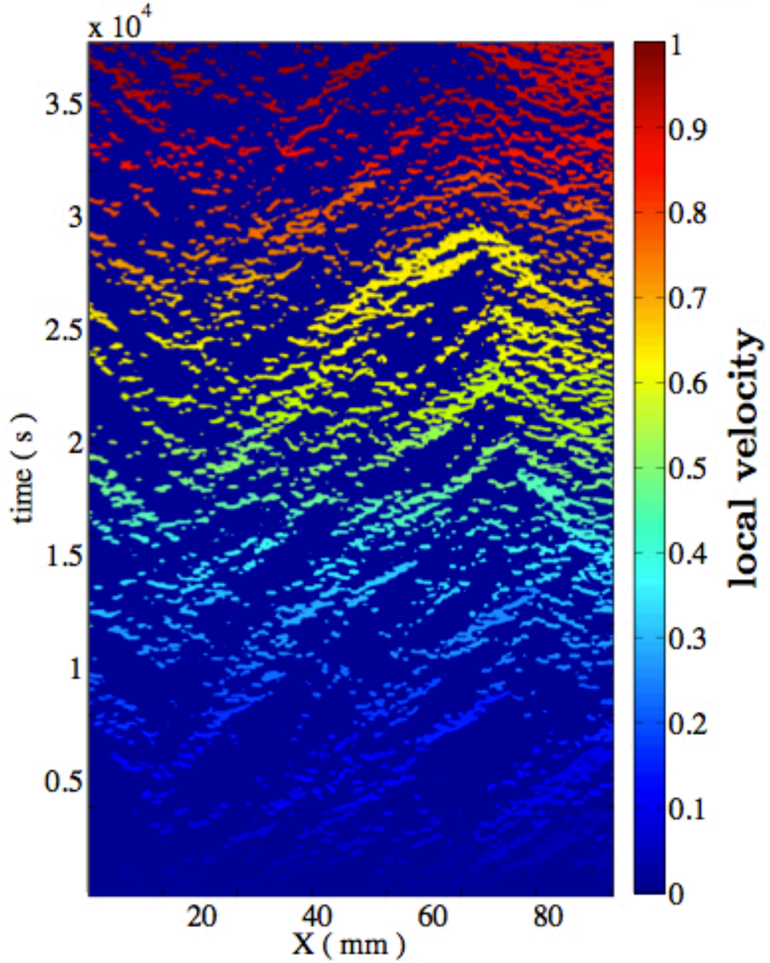
$$\alpha = 0.66 \pm 0.04$$
$$z = 1.06 \pm 0.05$$
$$\boxed{\gamma = 1.57 \pm 0.07}$$

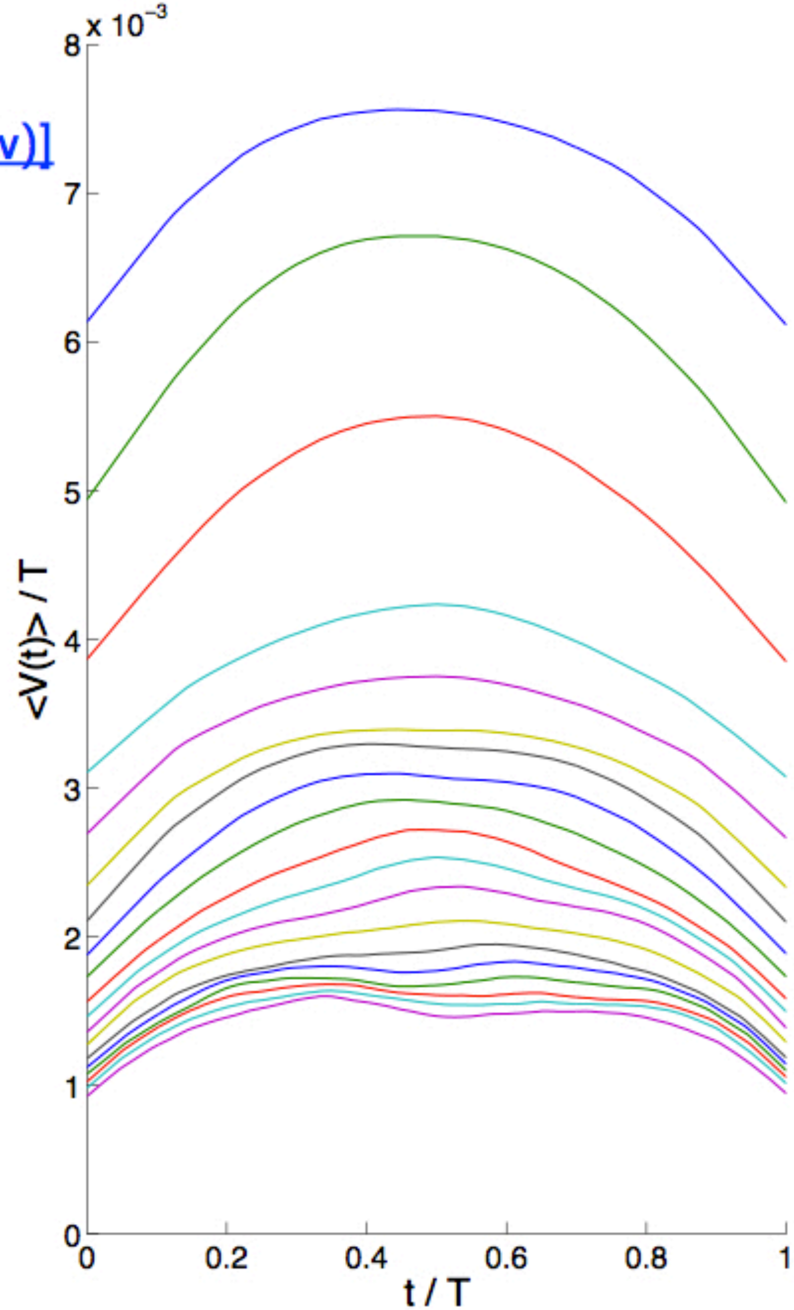
4 - Critical behavior

- **Avalanches shape**

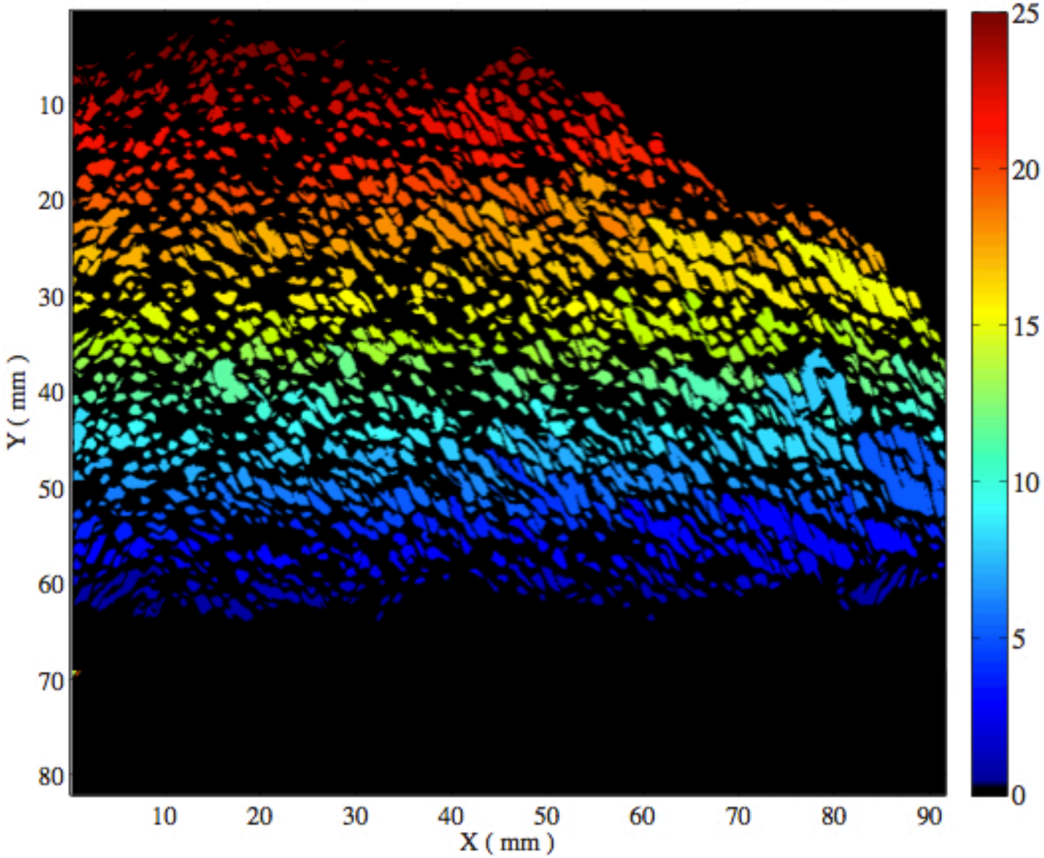
[A. Dobrinevski, P. Le Doussal, K. Wiese, *submitted* (ArXiv)]

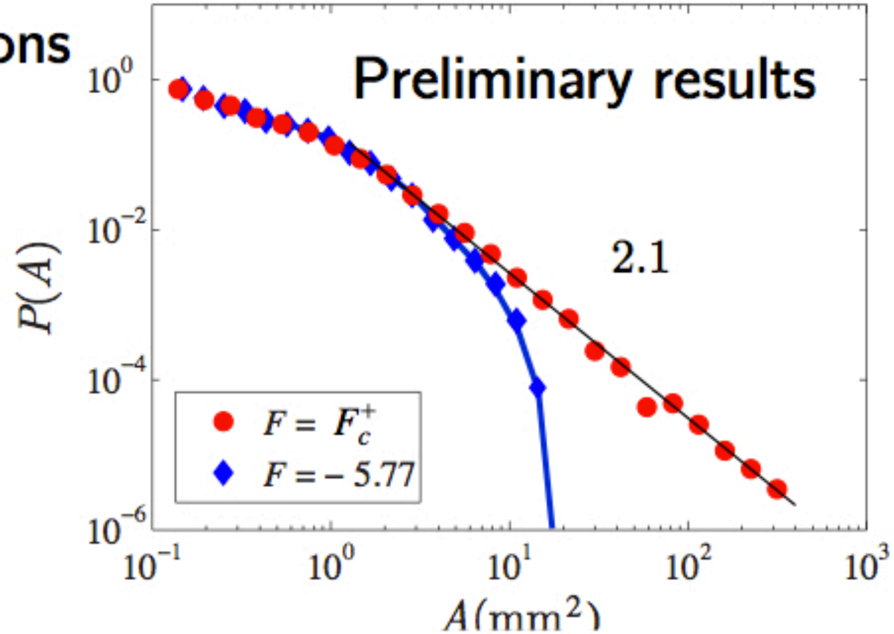
[L. Laurson et al. , *Nature Com.* (2013)]

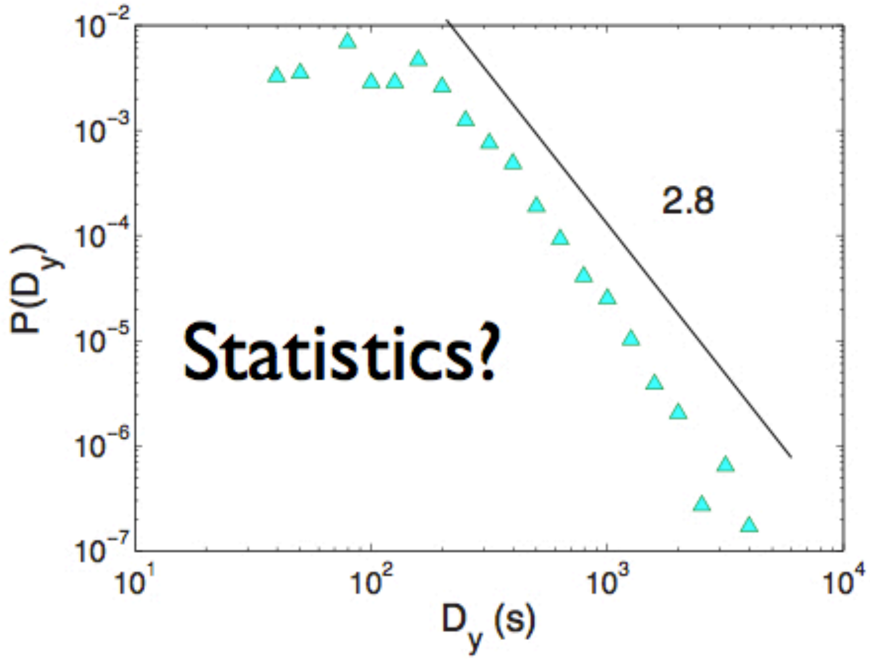




- avalanches size and duration distributions







$$\frac{\tau_D - 1}{\tau_S - 1} = \gamma$$

$$\boxed{\gamma = 1.68 \pm 0.08}$$

PLAN

1 - Experimental setup

2 - Front dynamics in high flow strength

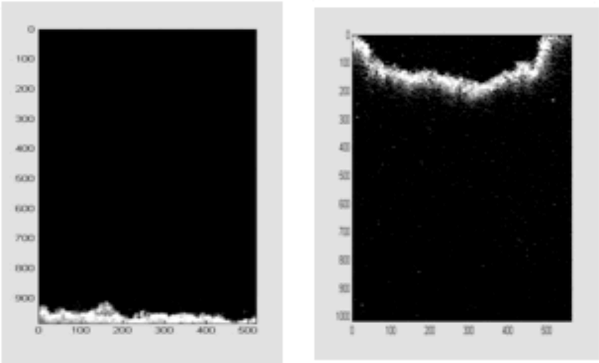
3 - Frozen pattern formation

4 - Critical behavior

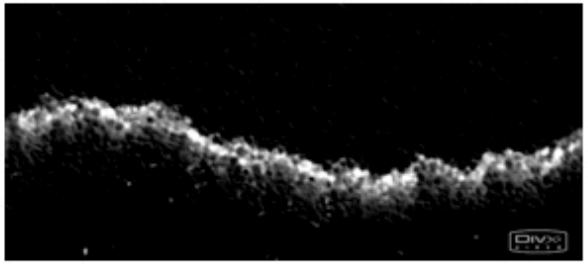
5 - Conclusion and perpectives

5 - Conclusion and perpectives

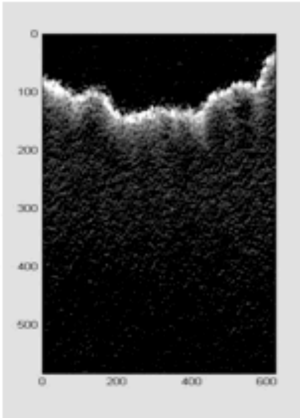
3 universality classes



- KPZ behavior for moving phase



- Positif qKPZ growth process for upward propagating fronts



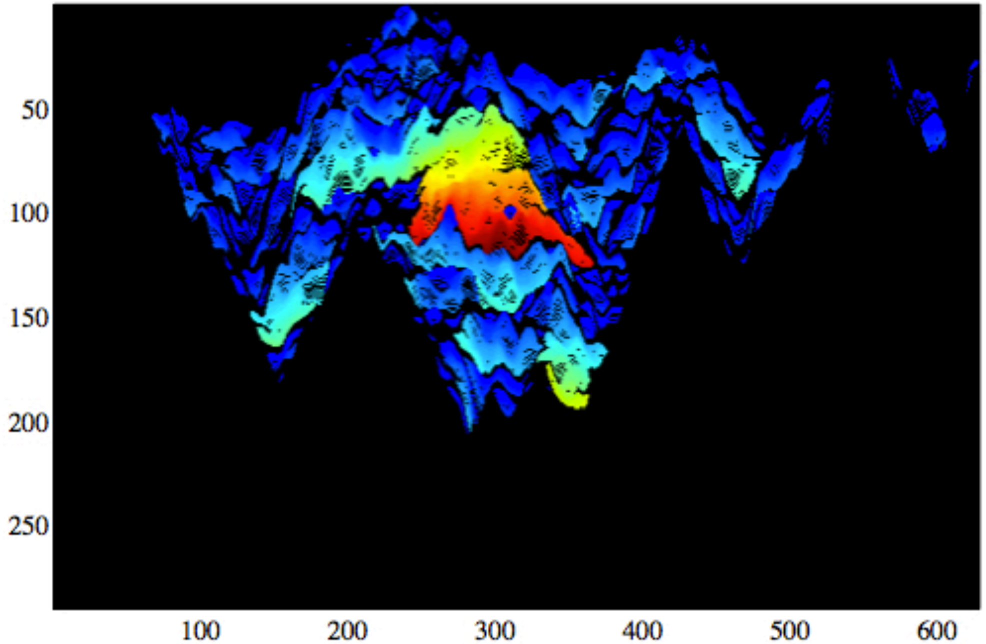
- Negatif qKPZ growth with static sawtooth pattern formation for bakward propagating fronts

- **Unique control parameter**

- **Avalanches statistical properties yet to be explored**

5 - Conclusion and perpectives

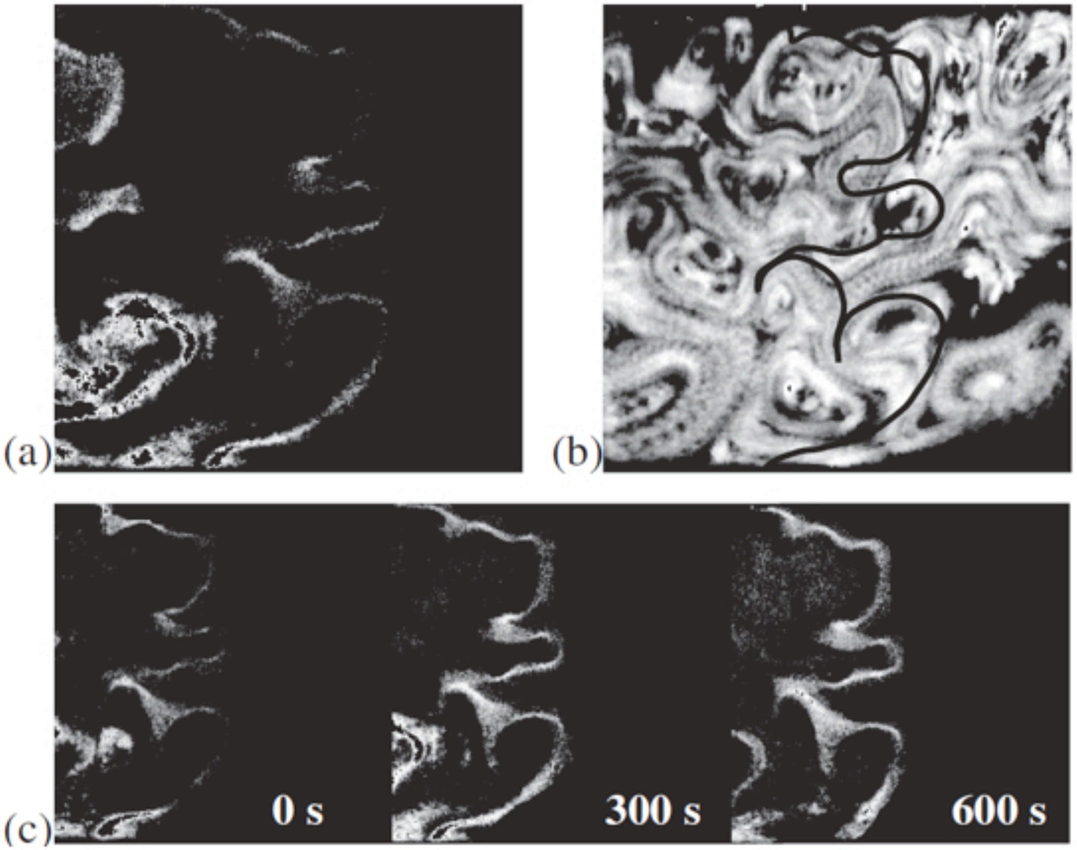
- **Avalanches statistics at the depinning transition with negative Force**



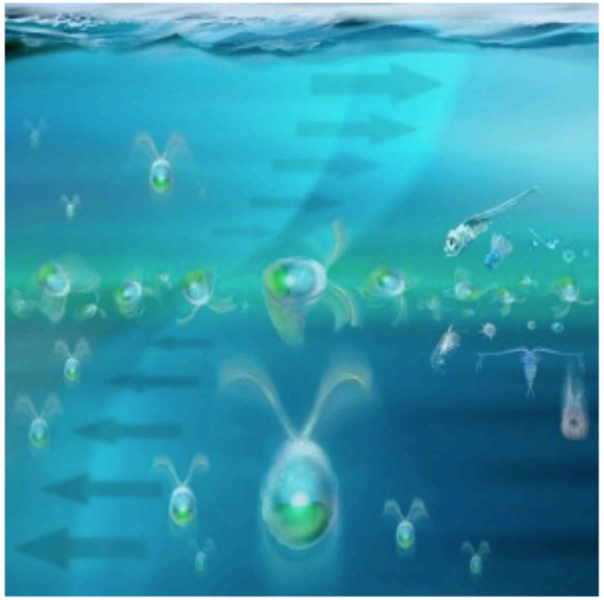
5 - Conclusion and perpectives

- **Bacterial dynamics in complex flows**

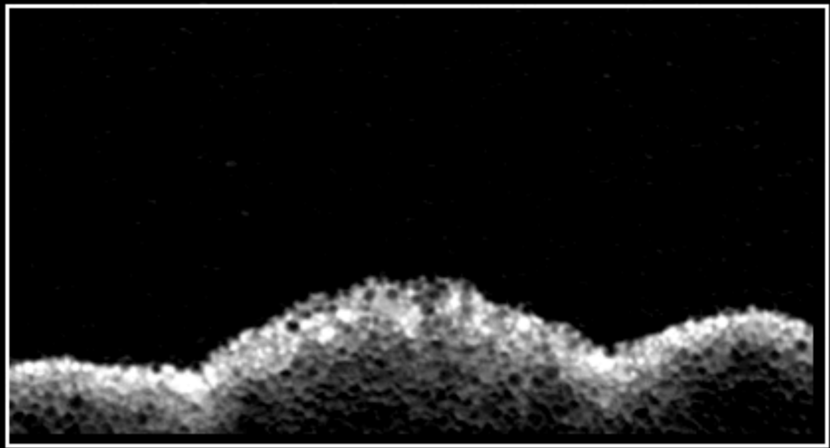
Disordered cellular flow

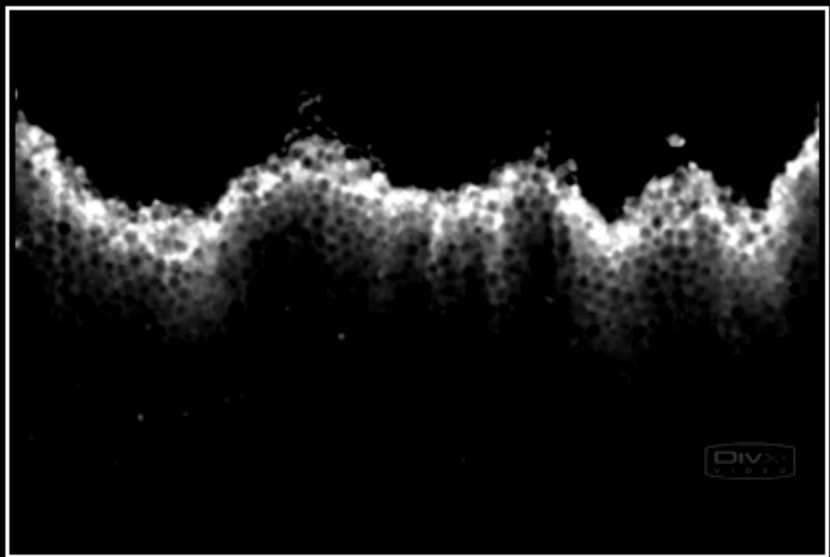


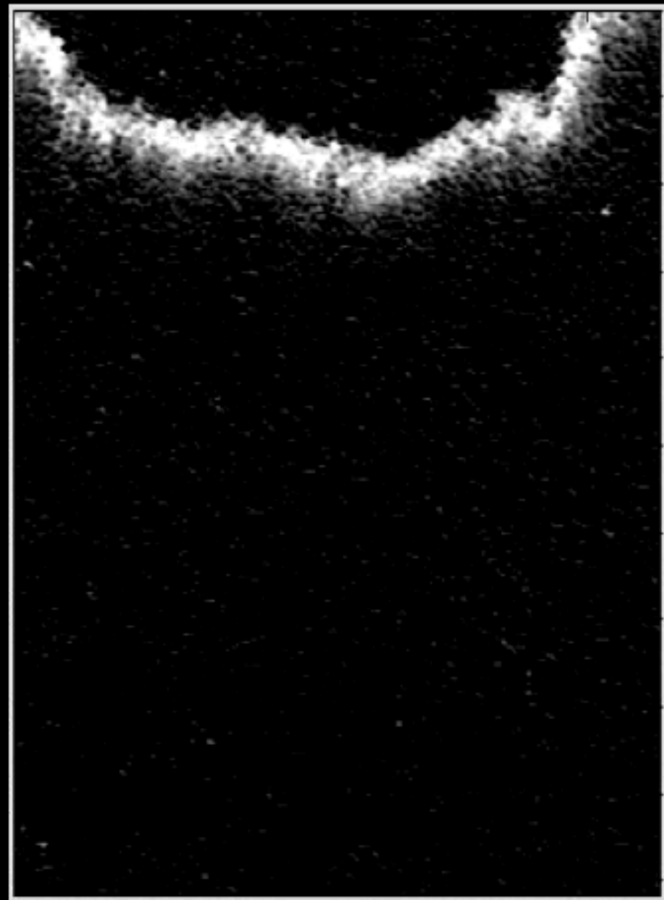
[Schwartz et Solomon, 2008]



[Stocker at al., 2010]







Thank you^^ !