Temporal correlations in avalanching processes

Lucilla de Arcangelis

Second University of Naples

CNISM, INFN

…or what power law distributions don't tell you….

- Power laws in natural phenomena
- Earthquakes and solar flares
- Temporal clustering
- Time-energy correlations
- Understanding the physical mechanisms
- … Also in brain activity…
- Homeostatic balance between excitation and inhibition

BRAIN TEAM

Hans J. Herrmann, ETH Zurich

Carla Perrone Capano, Univ. Naples

Dante Chialvo, Conicet

Dietmar Plenz, NIH Bethesda

Vittorio Capano, Univ. Naples

Fabrizio Lombardi, ETH Zurich

Gianluca Pellegrini, Univ. Naples

Roberta Russo, Univ. Naples

EARTHQUAKES

Eugenio Lippiello, SUN
Cataldo Godano, SUN
Warner Marzocchi, INGV

Alvaro Corral, CRM Barcelona

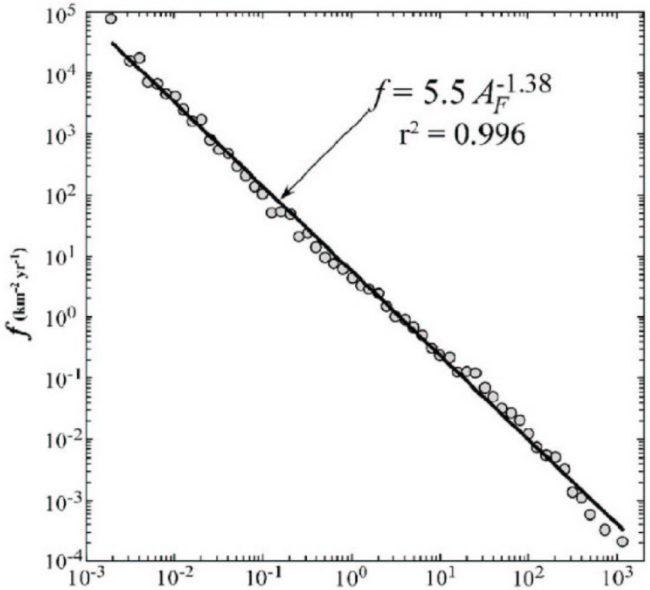
Milena Bottiglieri, SUN

SOLAR FLARES

Eugenio Lippiello, SUN
Cataldo Godano, SUN

Hans J. Herrmann, ETH Zurich
Miller Mendoza, ETH Zurich
José Soares de Andrade, Univ. Fortaleza

Power laws in natural hazards

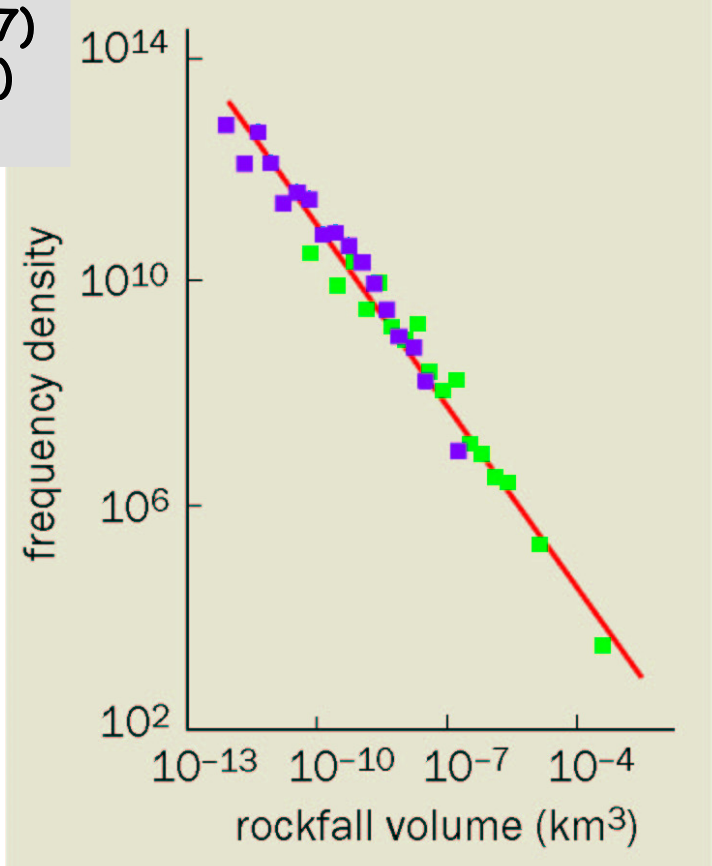


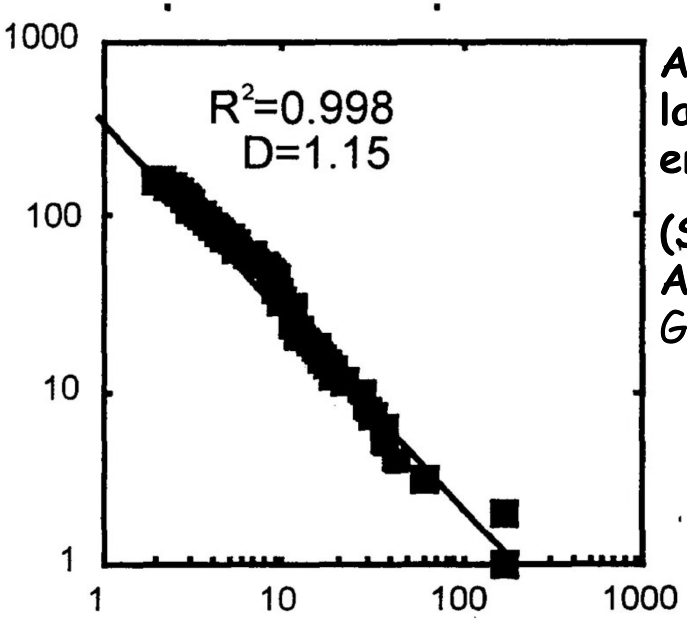
Forest fires in Ontario (Canada) 1976-1996

Turcotte & Malamud 2004

Exponent 1.1

Rockfall in Umbria (1997) & Yosemite (1980-2002)
Malamud 2004

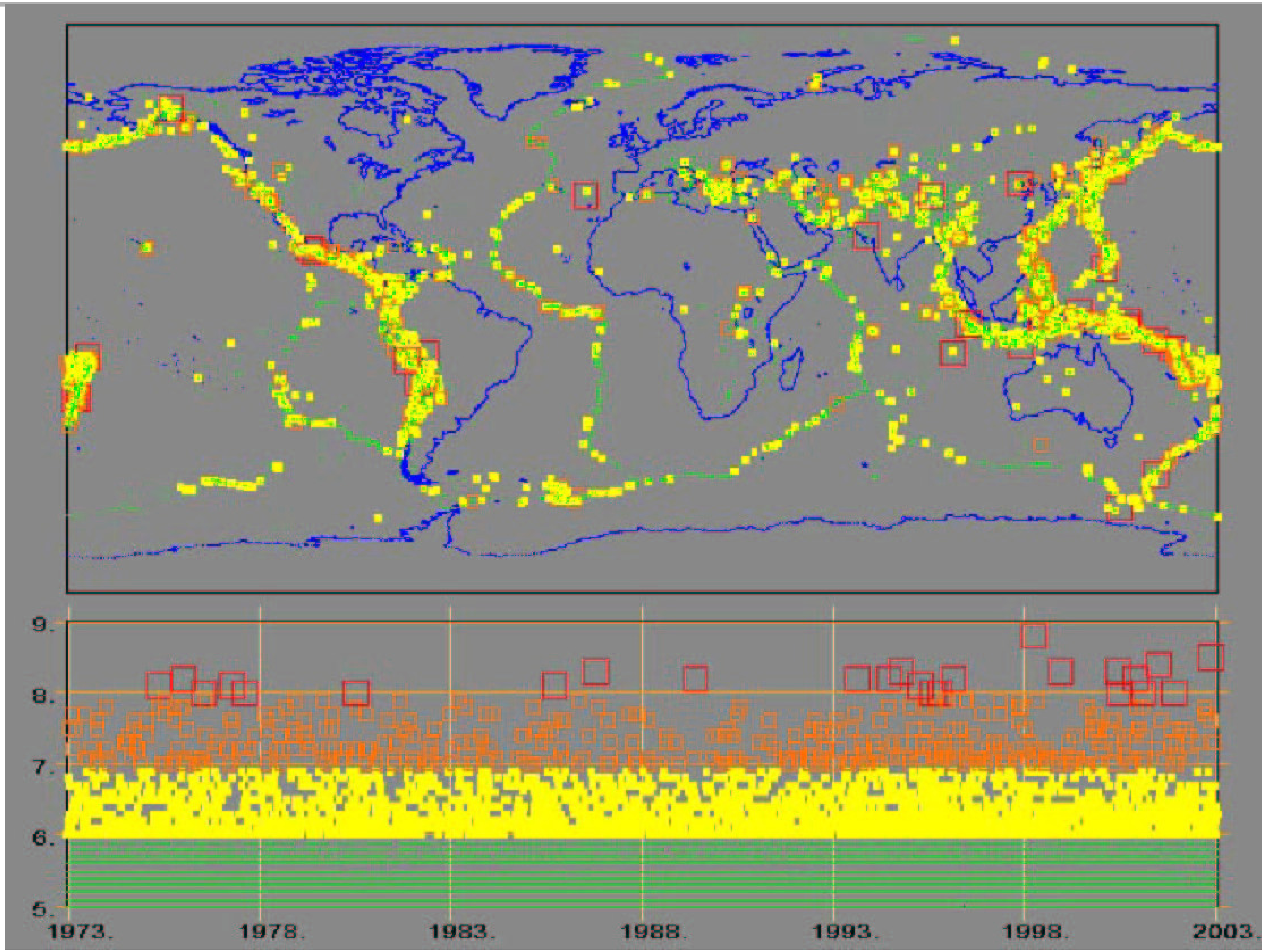




Areas covered by lava in volcanic eruptions

(Springerville, Arizona) Lahaie & Grasso 1998

Earthquakes in the world during 1973-2003



How big can an earthquake be?

Gutenberg-Richter Law (1954)

$$P(>M) \sim 10^{-b\,M} \quad (b \sim 1)$$

Seismic moment $M_0 = \mu A \Delta u$

$M = (2/3)\log(M_0) - 6$
(Kanamori,Anderson 1975)

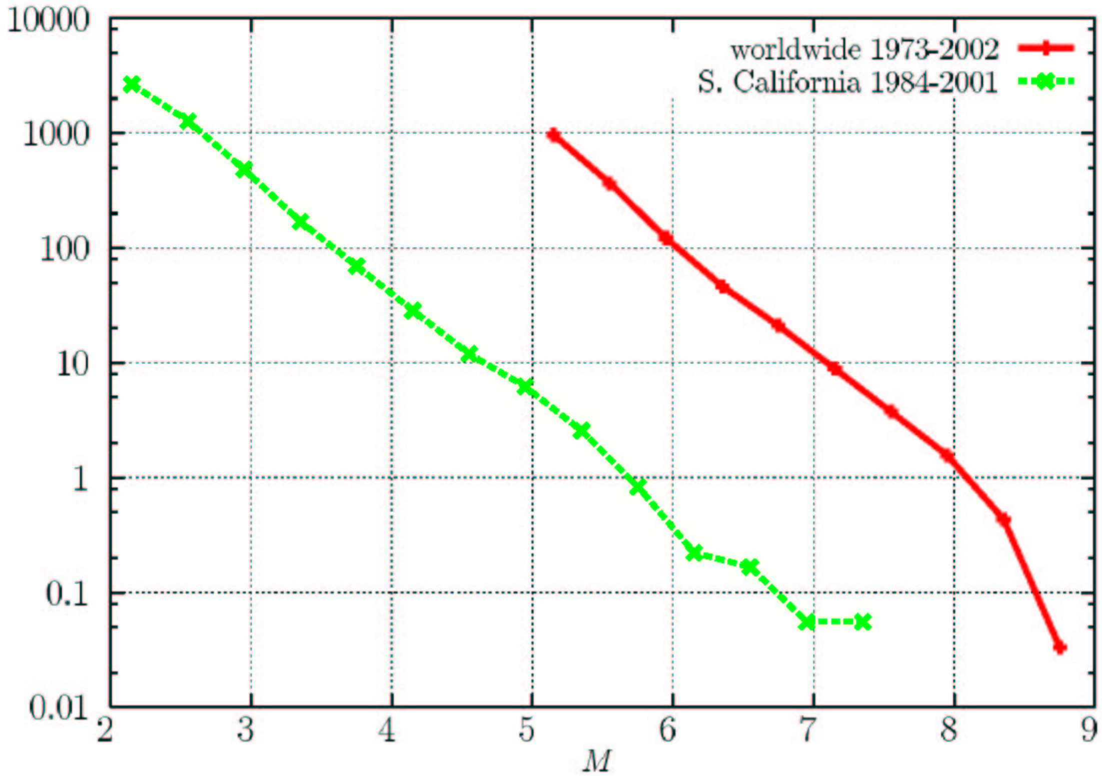
$$P(>M_0) \sim M_0^{-\alpha}$$

Energy

$M = (2/3)\log(E) + cost$

$$P(>E) \sim E^{-\alpha}$$

Universality of $\alpha \sim 0.7$



Temporal correlations: Sequences of aftershocks

At time t after a main shock at t=0

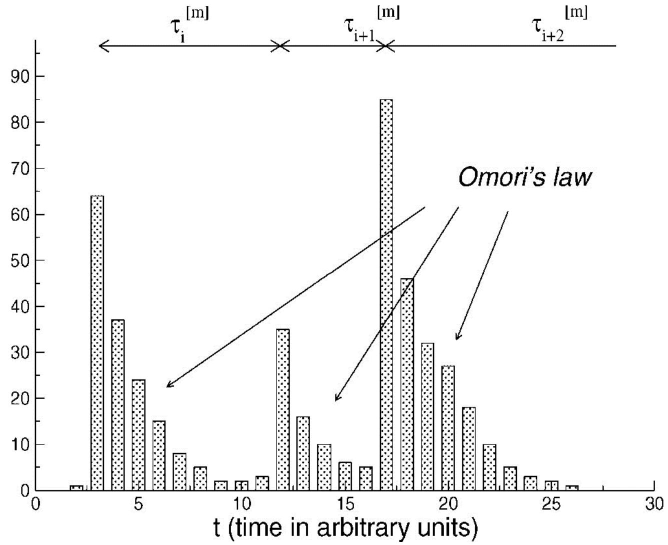
Omori law (JCSIUT,1894)

$$\boxed{n_{AS}(t) \sim (c + t)^{-p}} \quad p \sim 1$$

c depends on M main shock

and M lower cutoff

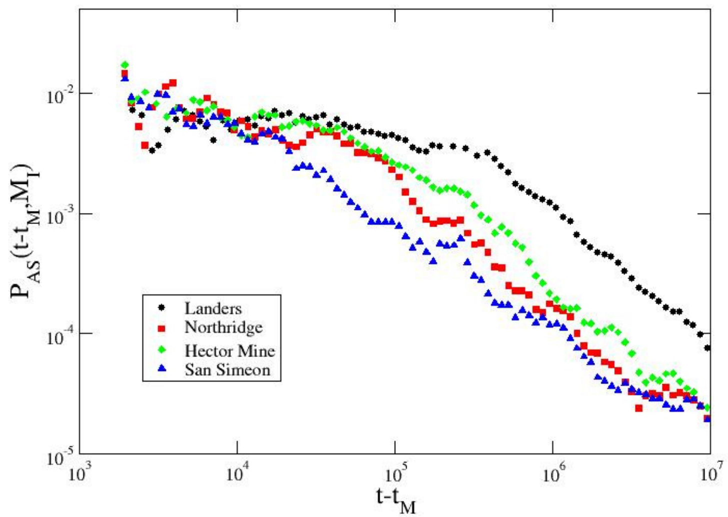
(Kagan 2004, Shcherbakov et al 2004, Lise et al 2004)



Productivity law

$$\boxed{N_{AS}(M) \sim 10^{\alpha M}} \quad \alpha \sim \mathrm{b}$$

(Helmstetter 2003, Felzer et al 2004, Helmstetter et al 2005, 2006)

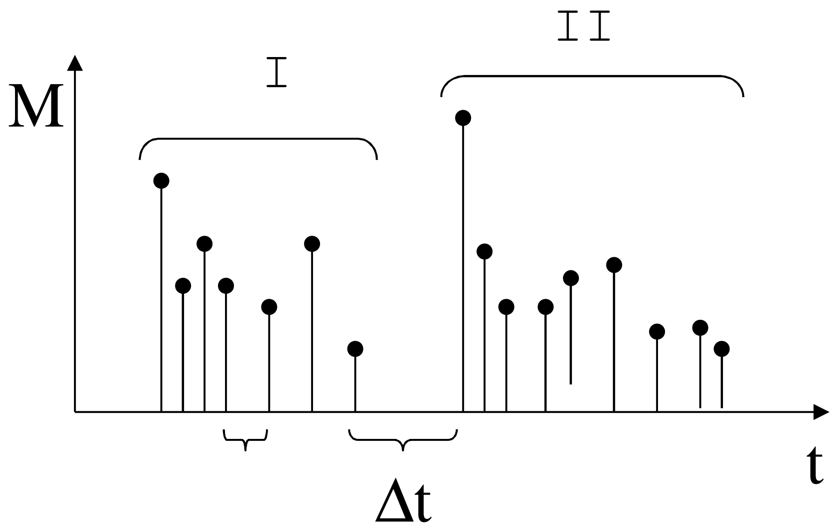


Intertime distribution

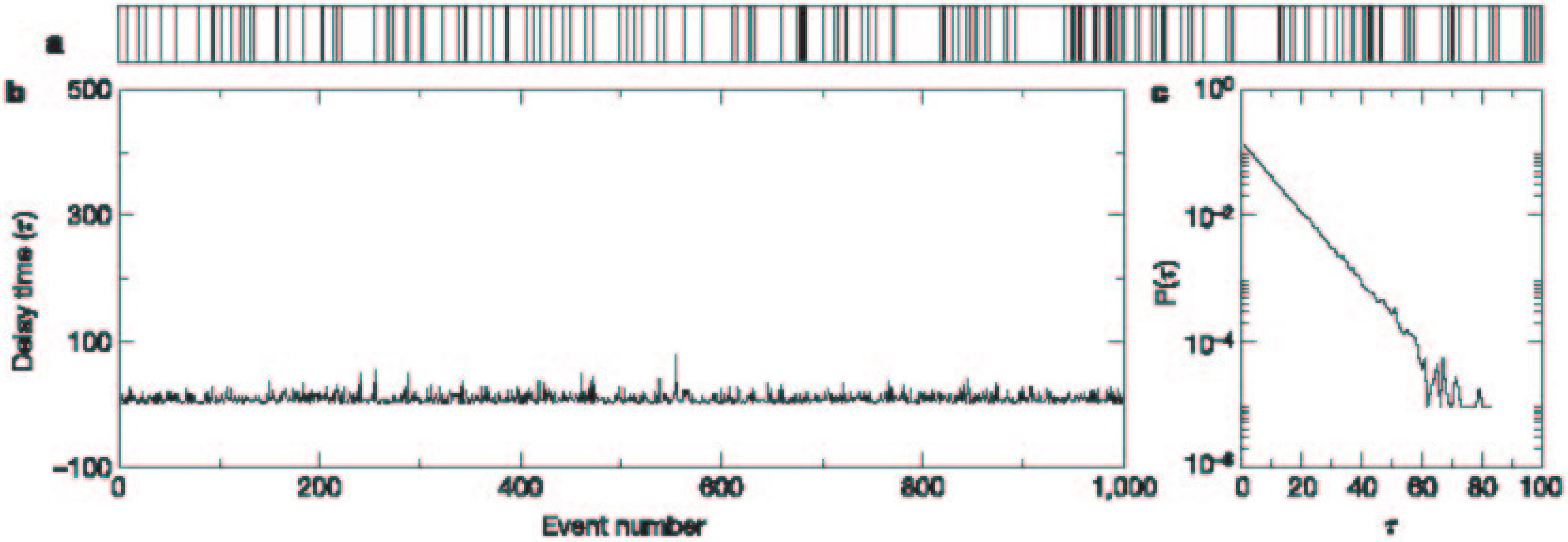
Probability distribution of intertimes

$$\Delta t$$

between consecutive events



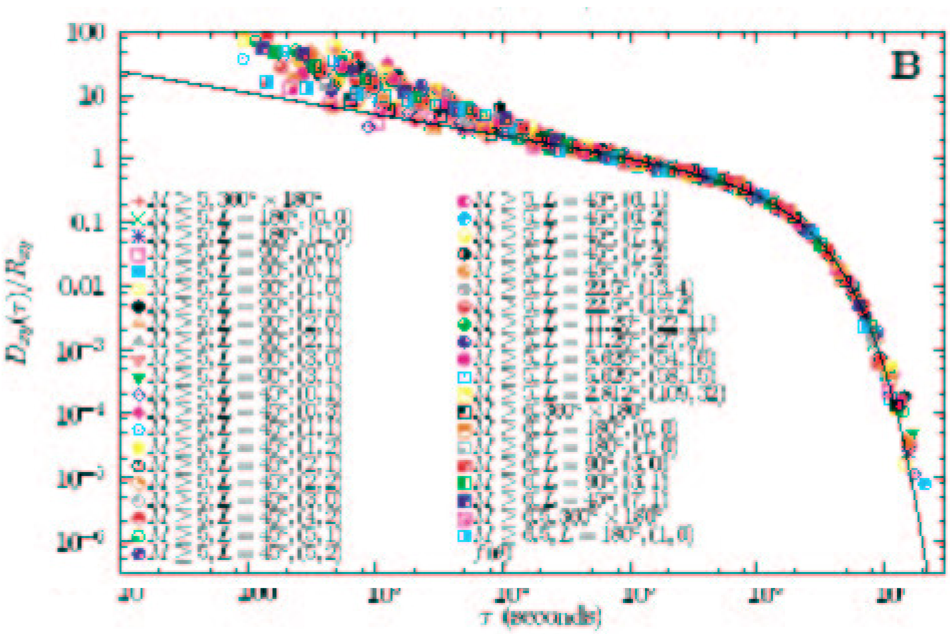
- $P(\Delta t)$ is an exponential for a Poisson process



- It exhibits a more complex structure as temporal correlations are present in the process

➡ Corral (PRL, 2004) rescaling Δt by the average rate in the area obtained a universal scaling law for the probability density

$$D(\Delta t, M_c) = R(M_c) f(R(M_c)\Delta t)$$

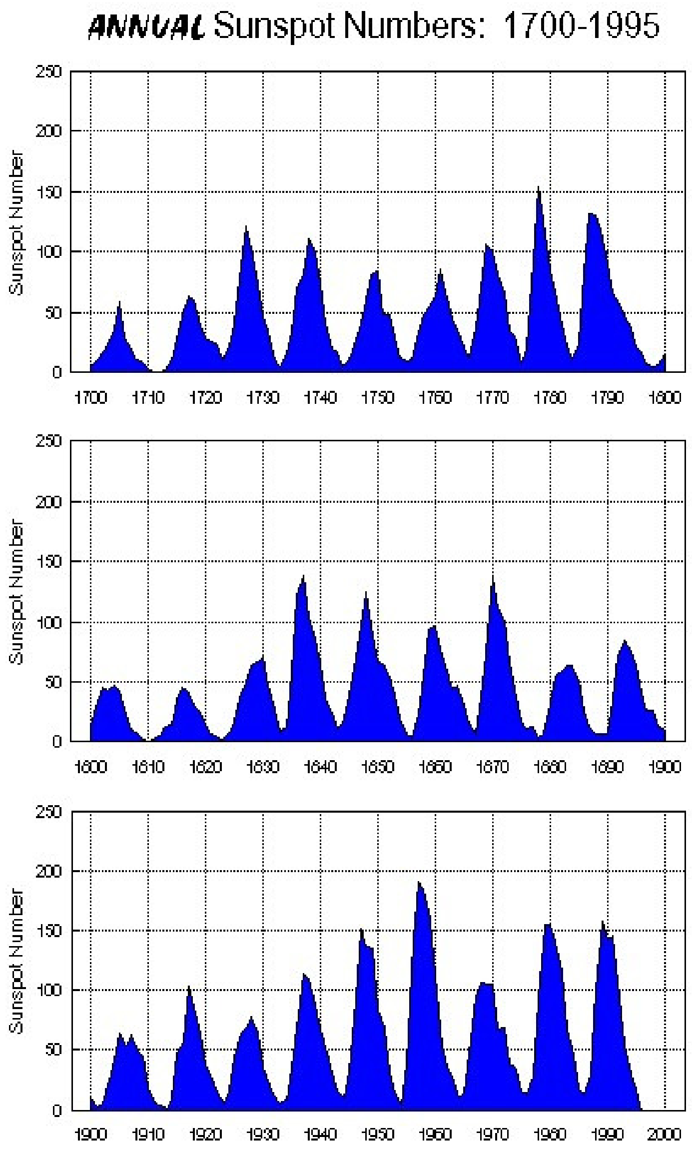


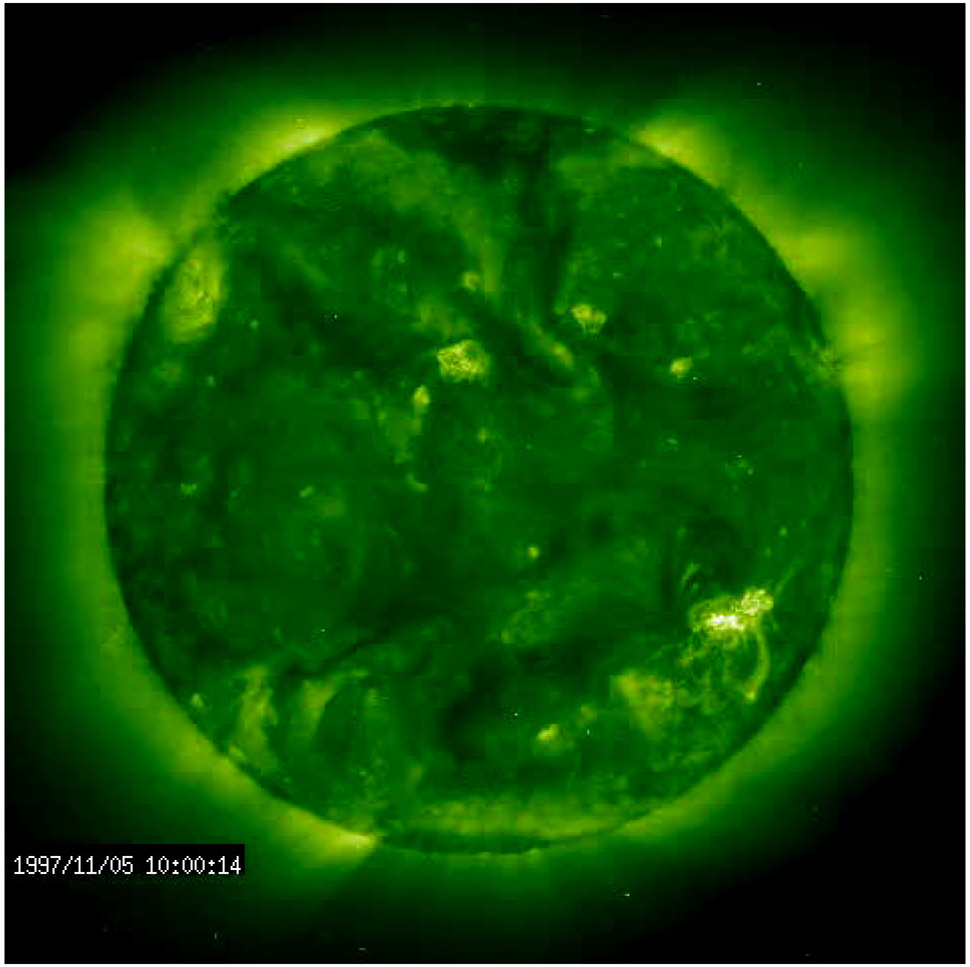
holds also for Japan, Spain, New Zeland…
scaling function not universal
(different areas are characterized by different rates)

Solar flares

Sudden rearrangement of stressed magnetic field lines gives rise to energetic bursts from solar corona.

These phenomena take place in active regions identified by sunspots, dark-looking due to the effects of intense magnetic field.

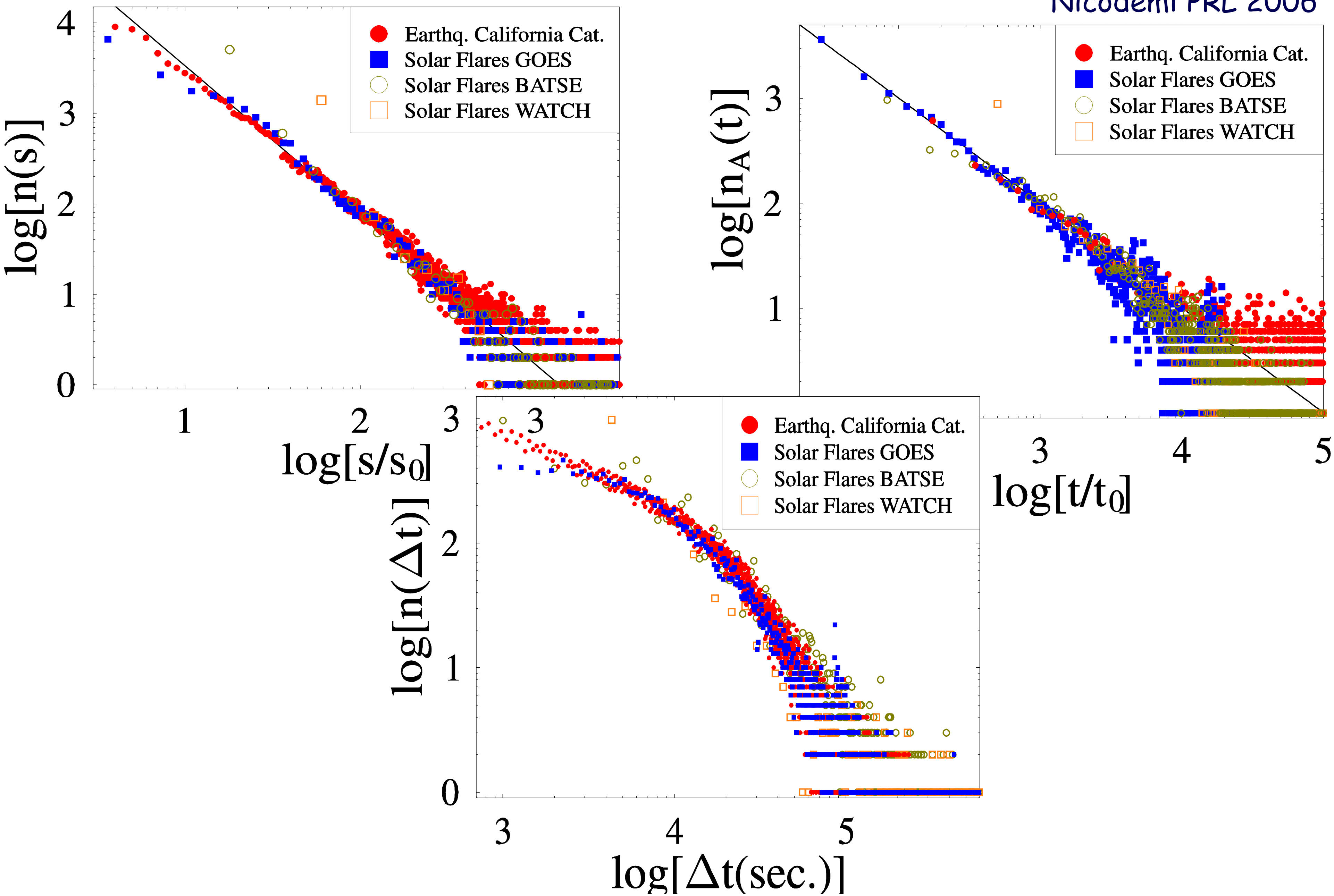




Solar flares and Earthquakes

LdA, Godano, Lippiello, Nicodemi PRL 2006

$\alpha = 1.65 \pm 0.1$



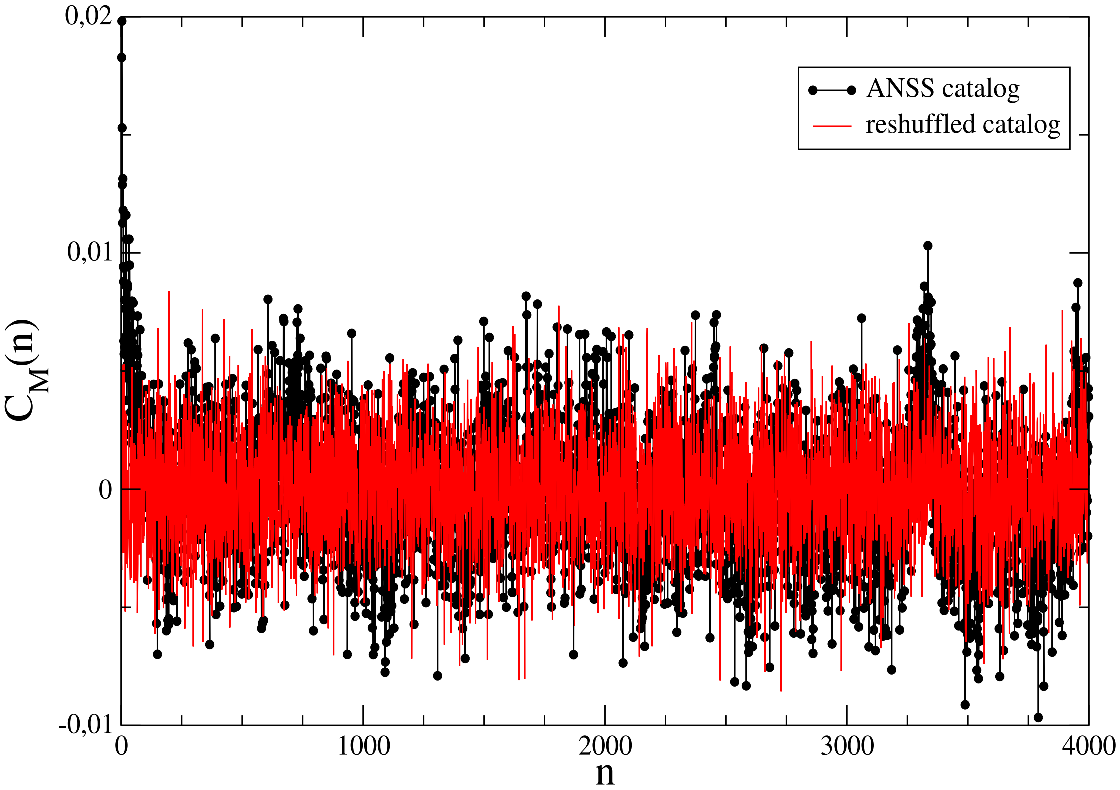
Is the occurrence of two phenomena
as different as
earthquakes and solar flares
driven by the
same physical mechanism?

Can statistical properties discriminate?

Magnitude correlations

Evaluating the $\langle M_i M_j \rangle - \langle M_i \rangle^2$ gives values comparable with statistical noise

red data represent the correlations evaluated in a catalog where magnitudes are reshuffled with respect to occurrence time → uncorrelat



Time-energy correlations

Lippiello, LdA, Godano, PRL 2008

We define for any couple of successive events of the catalog

The time distance $\boxed{\Delta t_i = t_{i+1} - t_i}$ and the magnitude difference

$$\boxed{\Delta m_i = m_{i+1} - m_i}$$

....and $\Delta m_i^* = m_{i+1} - m_{i^*}$

for a catalog where we reshuffle the previous magnitude $i^* \neq i$

We evaluate the conditional probability

$$P(\Delta m_i < m_0 \mid \Delta t_i < t_0) = \frac{N(m_0, t_0)}{N(t_0)}$$

couples of subsequent events with both $\Delta m_i < m_0, \Delta t_i < t_0$

couples of subsequent events with $\Delta t_i < t_0$

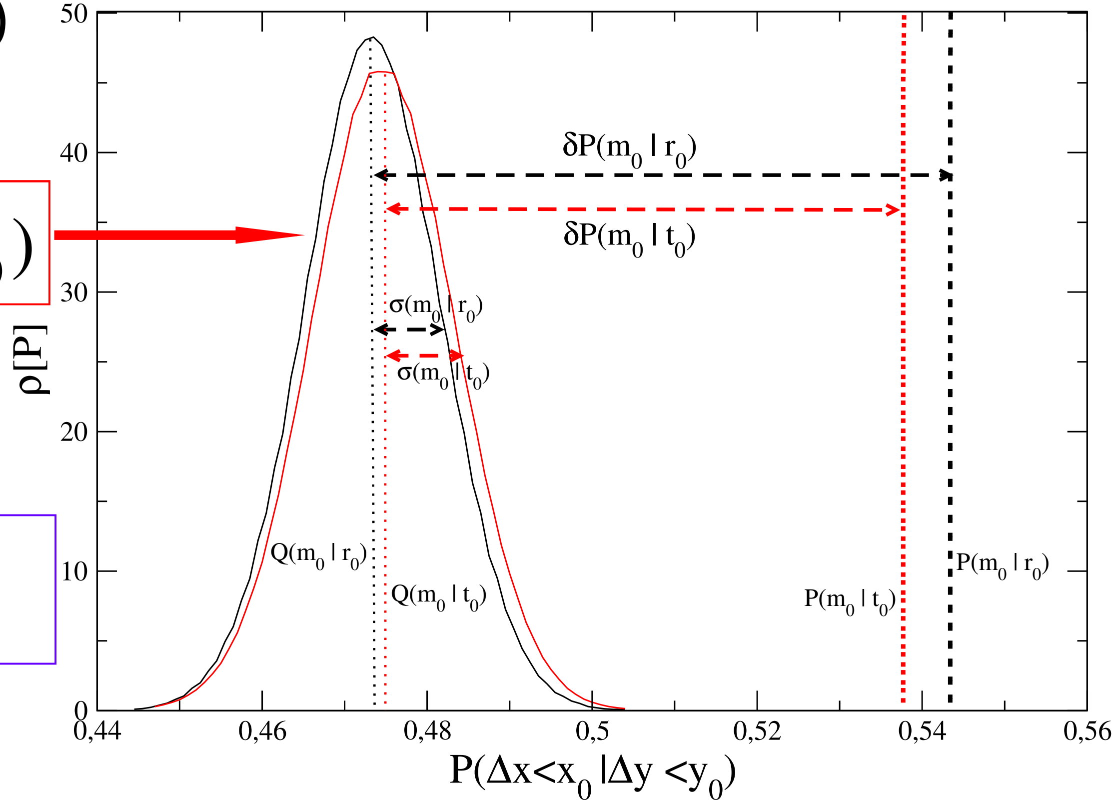
We calculate the conditional probabilities in the California catalog $P(\Delta m_i < m_0 \mid \Delta t_i < t_0)$

and for 10^4 realizations of the reshuffled catalog (Gaussian distributed)

$P(\Delta m_i^* < m_0 \mid \Delta t_i < t_0)$

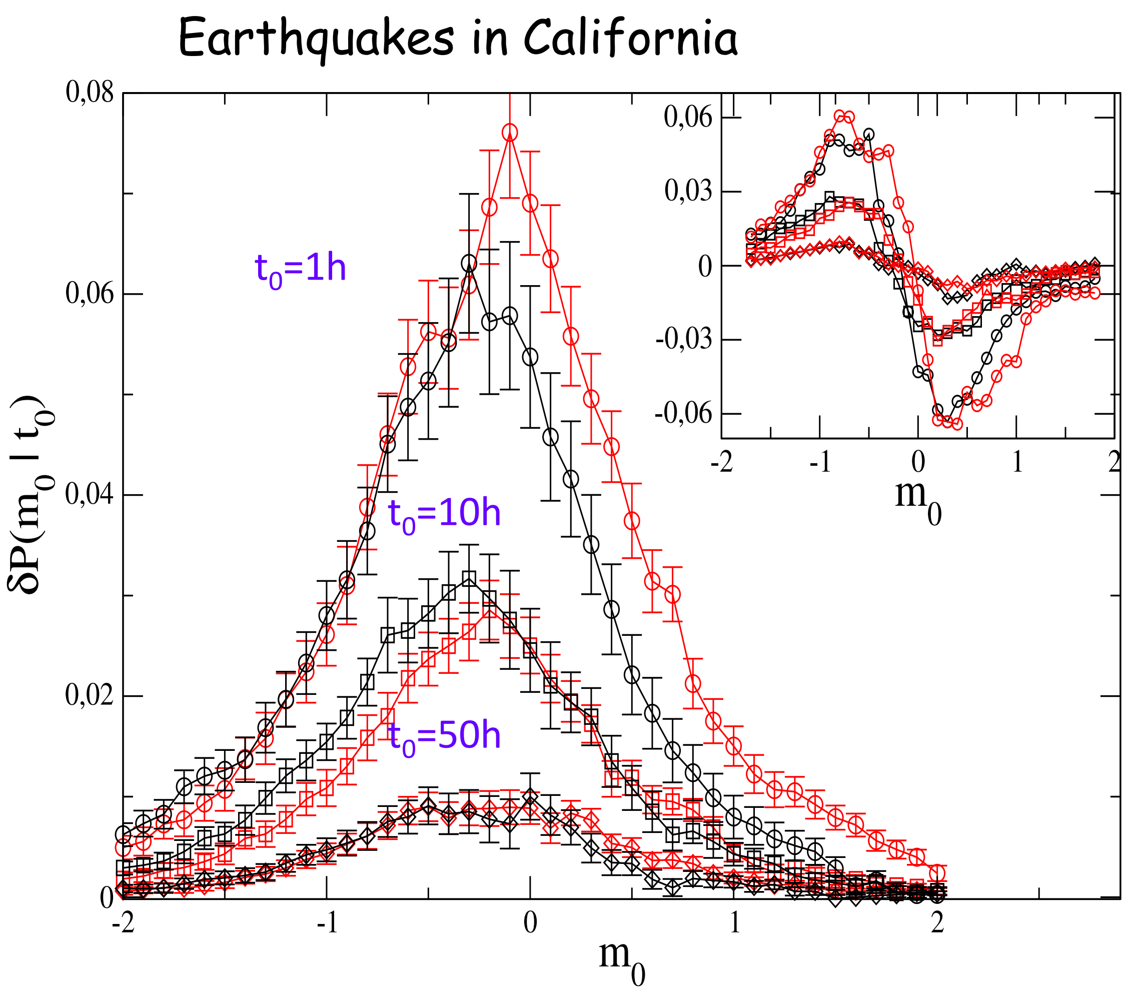
t_0=1h

m_0=0



IF $\delta P(m_0 \mid t_0) = P(\Delta m_i < m_0 \mid \Delta t_i < t_0) - Q(m_0 \mid t_0) > \sigma(m_0 \mid t_0)$

Evidence for time-energy correlations



$$\frac{d\delta P(m_0 \mid t_0)}{dm_0}$$ probability difference

For $m_0 < 0$ the probability is larger in the real than in the reshuffled catalog, where magnitudes are uncorrelated

Maximum for

m_0 in [-1,-0.5]

Experimental data
Numerical data

The next earthquake tends to have magnitude

close but smaller than the previous one

Branching model for seismicity

We treat seismicity as a **point process in time** , where $\{m_i(t_i)\}$ is the history of past events

Given the history, one assumes that each event can trigger future ones according to a two point conditional rate

the rate of events of magnitude *m* at time *t* is

$$\rho\big(m(t) \mid \{m_i(t_i)\}\big) = \sum_{i:t_i<t} \rho\big(m(t) \mid m_i(t_i)\big) + \mu P(m)$$

where μ is a constant rate of independent sources

P(m) their magnitude distribution

In the ETAS model (Ogata, JASA 1988) the magnitude *m* is independent of previous events

$$\rho\big(M_i(t) \mid M_i(t_i)\big) = P(m_i)\,g(t_i - t_j; m_j) \propto 10^{-bm_i} 10^{\alpha m_j} (t_i - t_j + c)^{-p}$$

Dynamical scaling

Lippiello, Godano, LdA, PRL 2007, 2008

We assume that the magnitude difference fixes a characteristic time

$$\boxed{\tau_{ij} = \tau_0 10^{b(m_j - m_i)}}$$

where τ_0 is a constant measured in seconds

and that $\rho\big(m_i(t_i) \mid m_j(t_j)\big)$ is invariant for $\Delta t \rightarrow \lambda \Delta t = \dfrac{\Delta t}{\tau}$

This time represents the temporal scale for correlations:

A m=2 earthquake is correlated to a previous m=6 event over a time scale of about 2 years

A m=5 earthquake is correlated to a previous m=6 event over a time scale of few days

Therefore the conditional rate becomes with time rescaled by τ_{ij} where $F(x)$ is a normalizable function

$$\rho\big(m_i(t_i) \mid m_j(t_j)\big) = F\left(\frac{t_i - t_j}{\tau_{ij}}\right)$$

On the basis of this scaling hypothesis we recover the GR law:

Total number of aftershocks

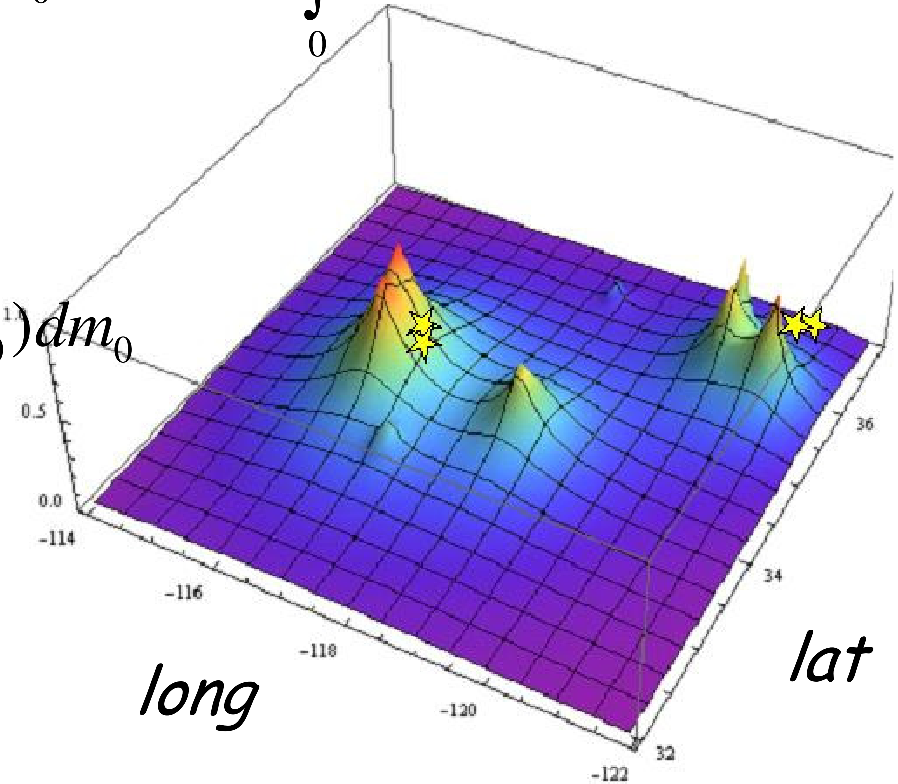
$$\int_{t_0}^{\infty} \rho(m(t) \mid m_0(t_0))dt = \tau_0 10^{-b(m-m_0)} \int_0^{\infty} F(x)dx$$

and the Omori law:

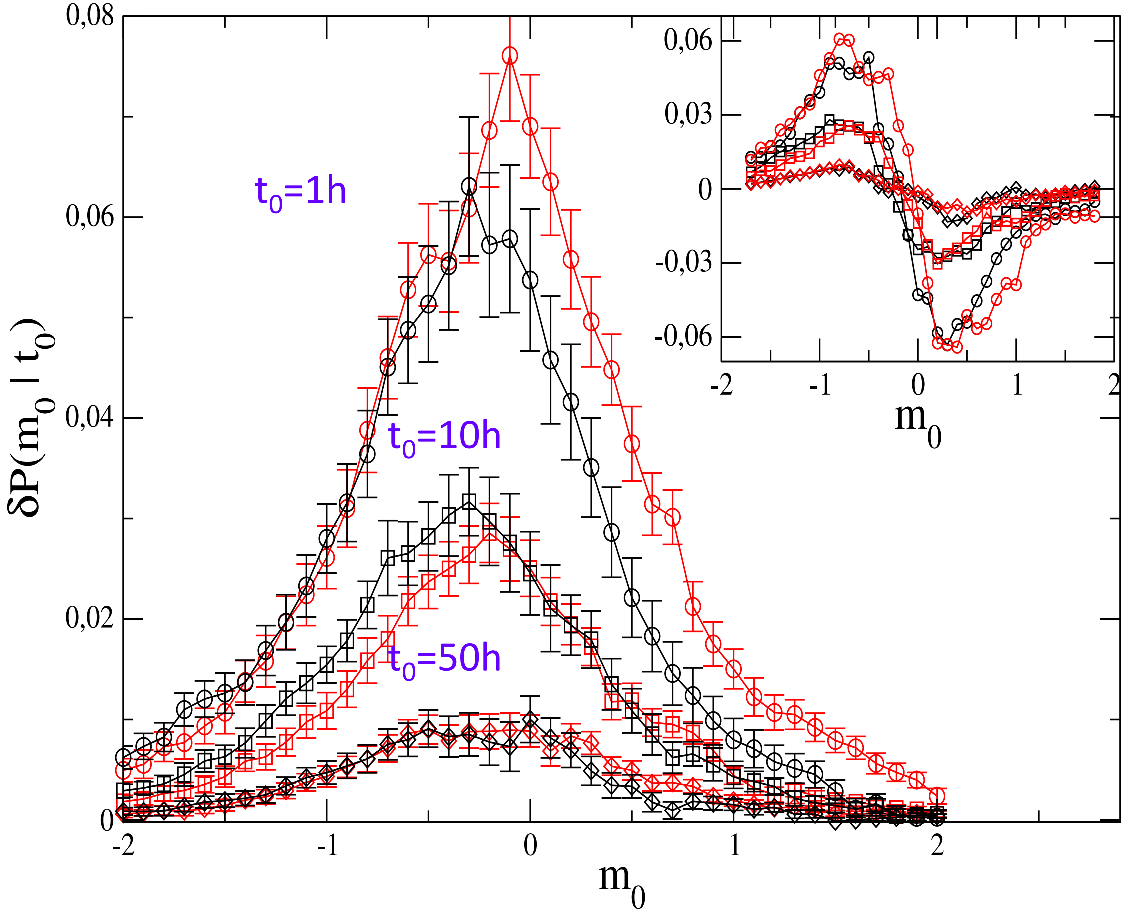
$$\rho(m, t-t_0) = \int_{-\infty}^{\infty} \rho(m(t) \mid (m_0(t_0))P(m_0)dm_0$$

$$\propto \frac{10^{-bm}}{t-t_0} \int_{-\infty}^{\infty} F(z)dz$$

Rate of m events at time t



Earthquakes in California



$$\frac{d\delta P(m_0 \mid t_0)}{dm_0}$$ probability difference

For $m_0 < 0$ the probability is larger in the real than in the reshuffled catalog, where magnitudes are uncorrelated

Maximum for

m_0 in [-1,-0.5]

Experimental data
Numerical data

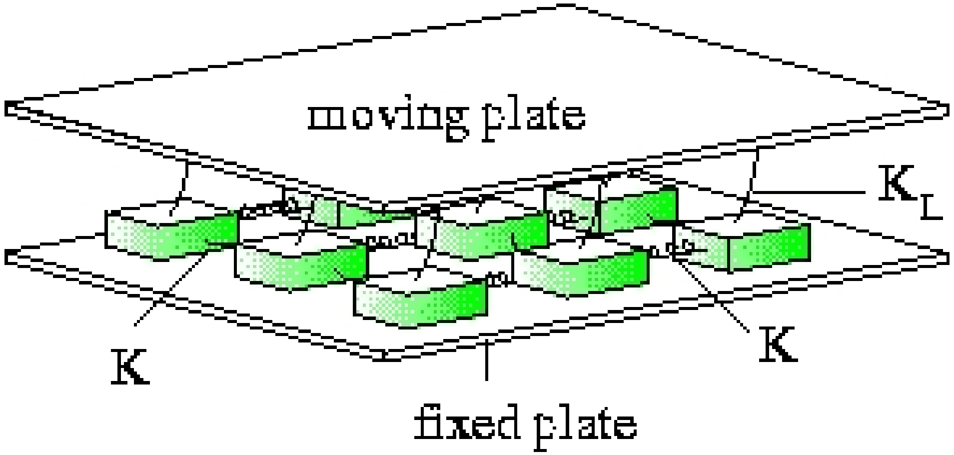
The next earthquake tends to have magnitude

close but smaller than the previous one

In good agreement with models implementing avalanching:

Olami-Feder-Christiensen model

Non conservative spring-block model, where $\alpha<0.25$ is the degree of dissipation



Successive instabilities generated by the stress redistribution

$\alpha \in [0.175 : 0.2]$

Lippiello et al EPL 2013

Spatio-temporal organization of foreshocks and aftershocks

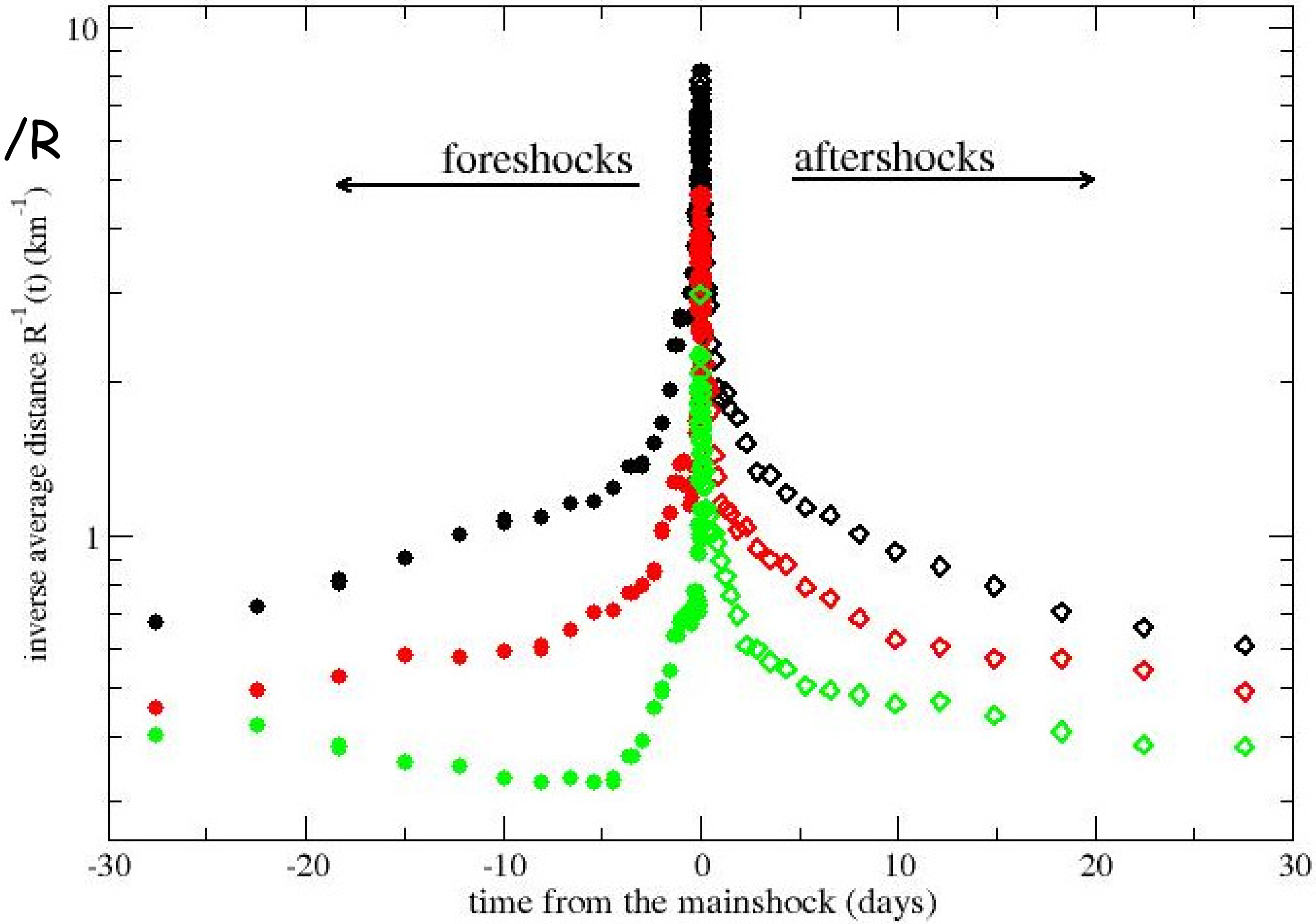
Lippiello et al, Nat Sci Rep 2012

Earthquakes tend to concentrate towards the future mainshock (foreshocks)
Then they spread out after mainshock occurrence (aftershocks)

R=Average distance from the mainshock 1/R

main magnitude
2
3
4

Southern California catalog

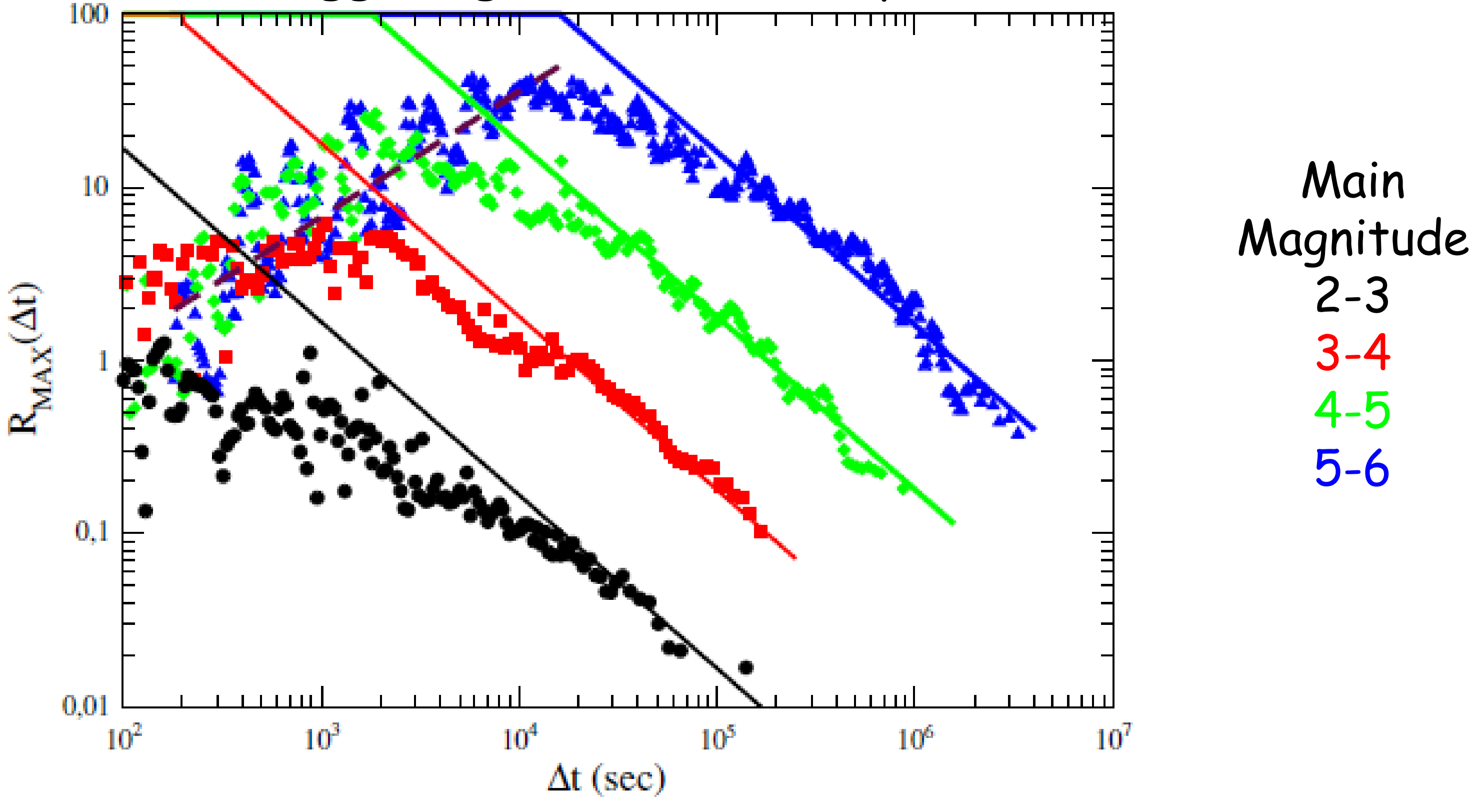


The maximum distance between the main and aftershocks occurring in

$$[\Delta t, \Delta t(1+\varepsilon)]$$

increases as a power law as long as events are 90% aftershocks

Aftershock triggering is controlled by stress diffusion



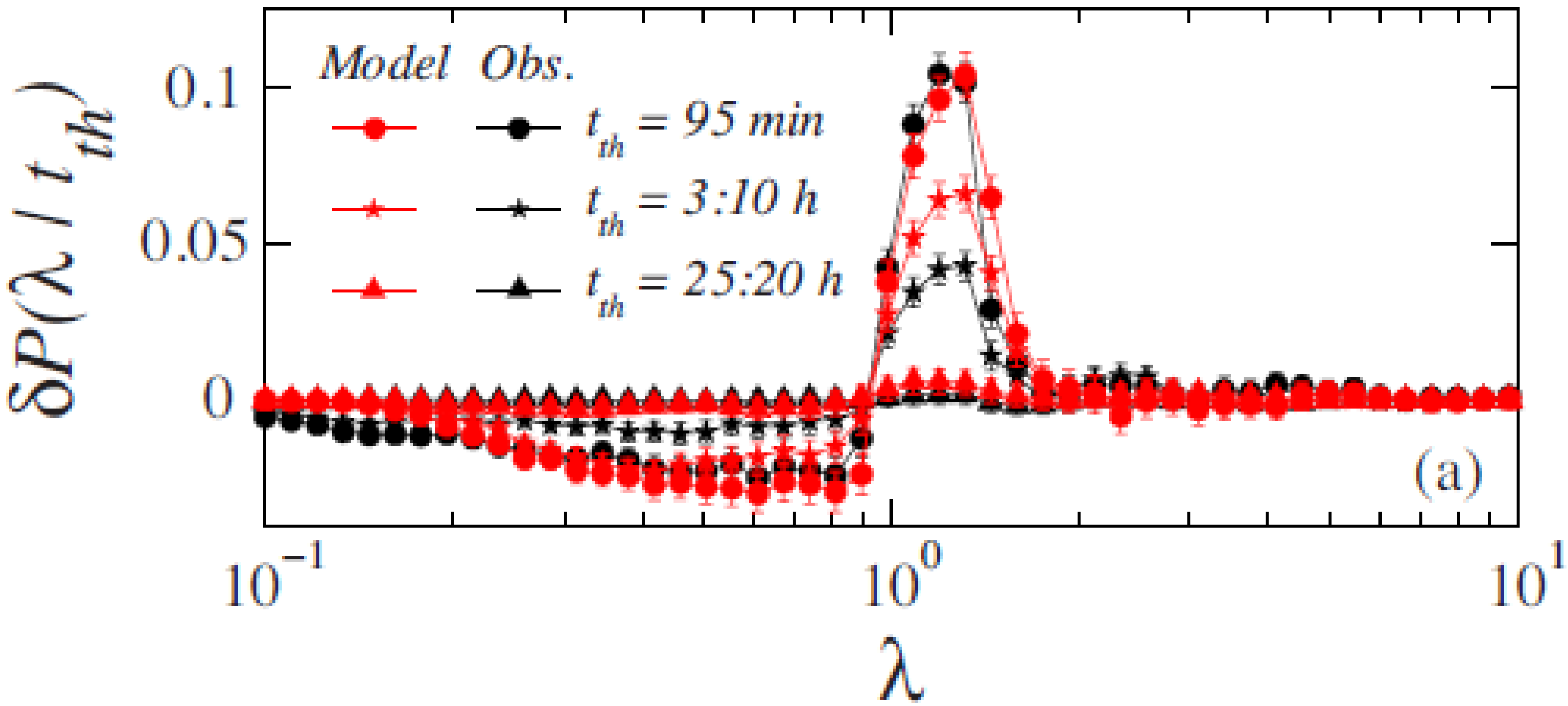
$$R_{MAX}(\Delta t) \propto \Delta t^{H} \qquad H = 0.54 \pm 0.05$$

Lippiello, Godano LdA, PRL 2009

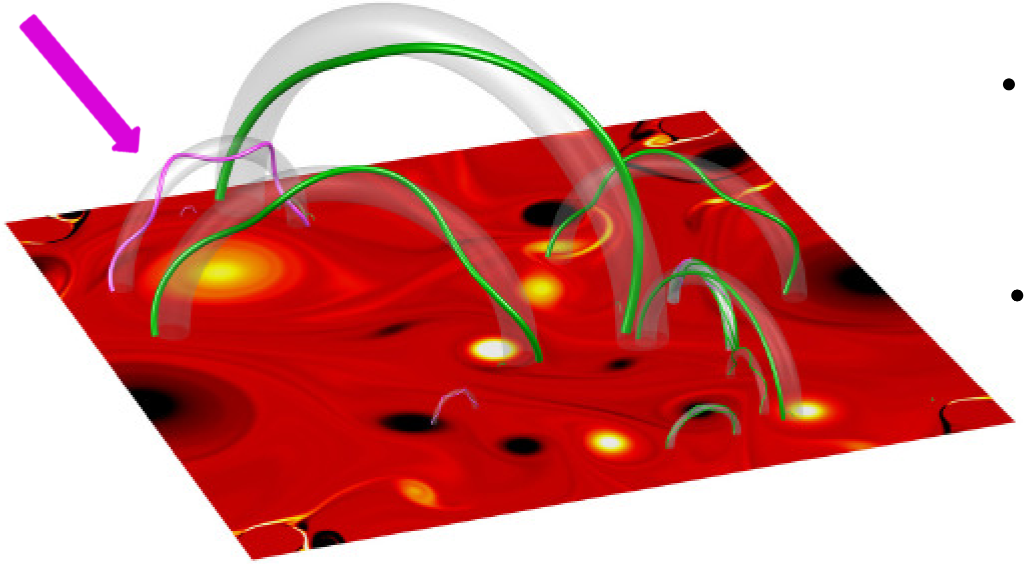
Correlations in solar flare occurrence

$$\delta P(\lambda | t_{th}) = P(E_{i+1}/E_i > \lambda | \Delta t_i < t_{th}) - Q(\lambda | t_{th}) > \sigma(\lambda | t_{th})$$

In consecutive flares (occurring within 3 hours)
the energy of the second flare
is close but larger than the energy of the previous one



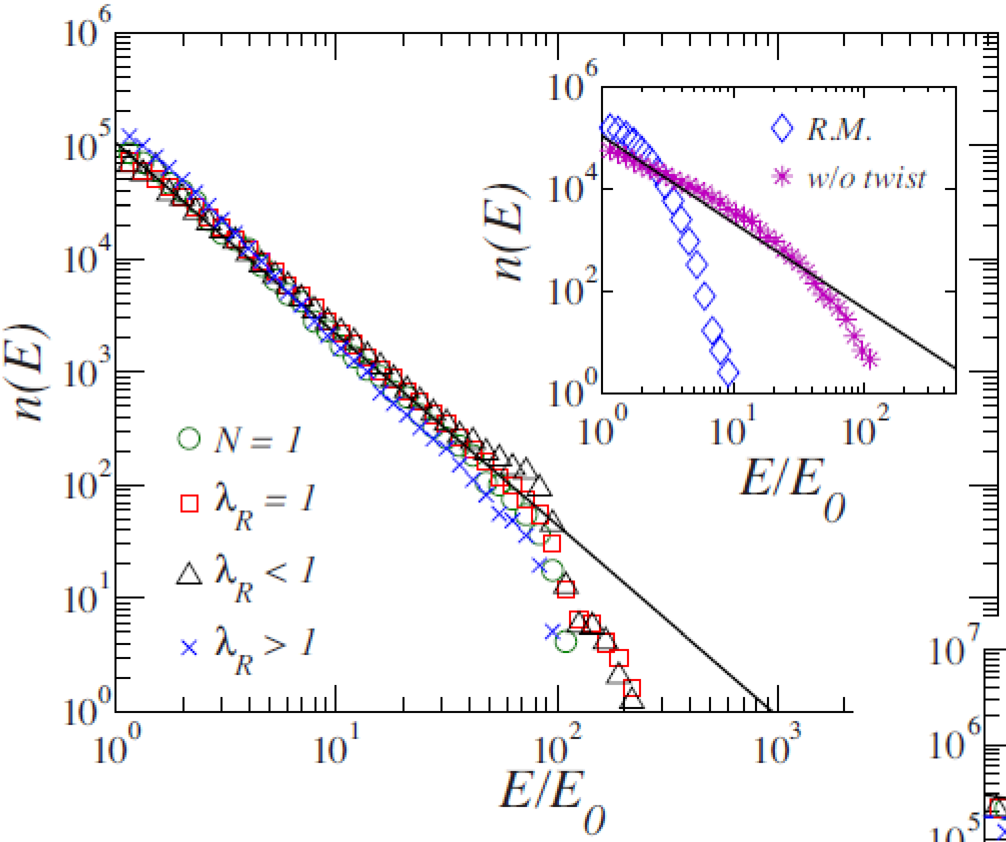
Understanding flare triggering



- A flare is due to the reconnection of magnetic flux tubes
  Parker Astro. J. 1988, Hughes et al PRL2003
- Footprints of magnetic flux tubes are anchored in the photosphere i.e. plasma in turbulent flow
- Magnetic flux tubes follow the local velocity field and are twisted by the vorticity
- A flare is released as soon as a tube reaches a critical twist (scale free energy distribution)

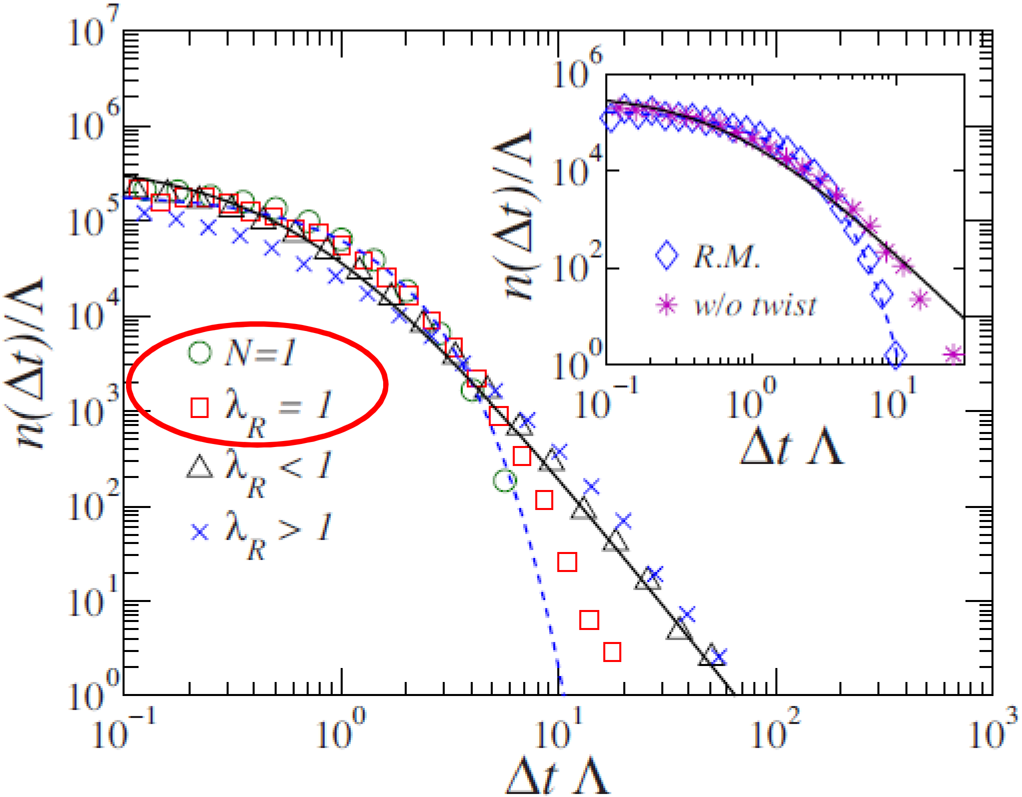
⟹ Tube-tube interactions:
Reconnection of one tube affects the surrounding magnetic flux tubes

Mendoza, Kaydul, LdA, Andrade, Herrmann, Nat. Comm. 2014



For the size distribution
Evolution according to fluid velocity is sufficient (even without twisting)

For the intertime distribution
Tube interactions are necessary

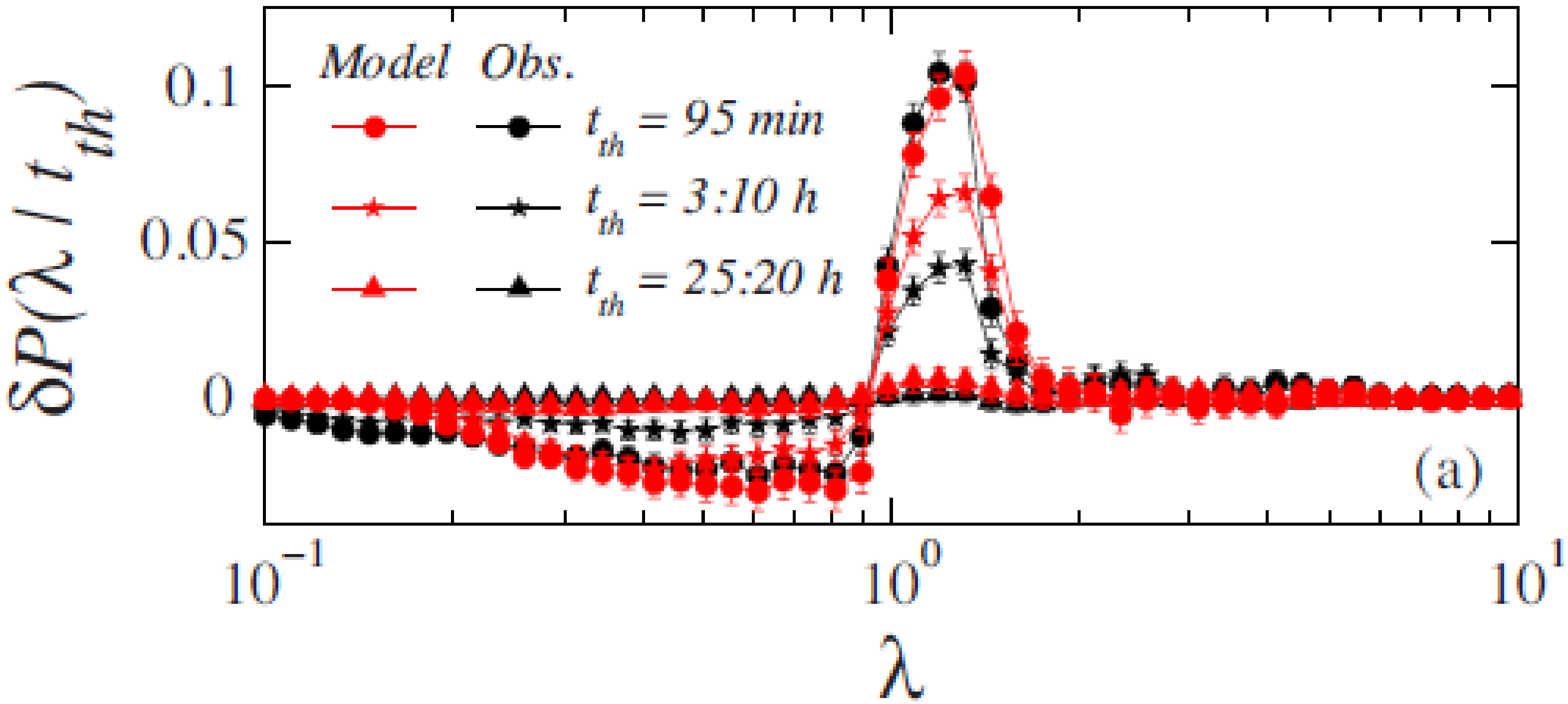


In order to observe that for close-in-time flares the energy of the second flare is close but larger than the energy of the previous one

⟹ Reconnection heats up the surrounded plasma increasing the local coronal pressure and the "critical" twist of the surrounding tubes

⟹ Rather than avalanching, this leads to a stabilizing effect
Following flare larger than the previous one!

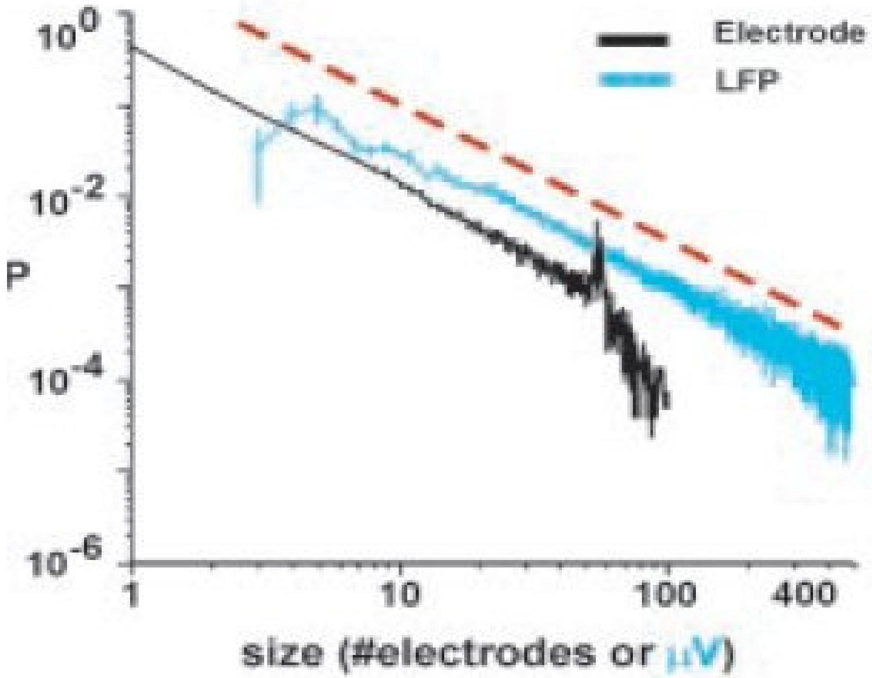
$$\delta P(\lambda | t_{th}) = P(E_{i+1}/E_i > \lambda | \Delta t_i < t_{th}) - Q(\lambda | t_{th}) > \sigma(\lambda | t_{th})$$



Neuronal avalanches

Beggs & Plenz (J. Neuroscience 2003, 2004) have measured spontaneous local field potentials continuously using a 60 channel multielectrode array in mature organotypic cultures of rat cortex *in vitro* and *in vivo* (rat & monkey) (PNAS 2008, 2009)

- dissociated neurons (V. Pasquale et al, Neurosci. 2008; A. Mazzoni et al PLoS ONE 2007)



- Avalanche size distribution is a power law with an exponent close to -3/2
- Avalanche duration distribution is a power law with an exponent close to -2.0

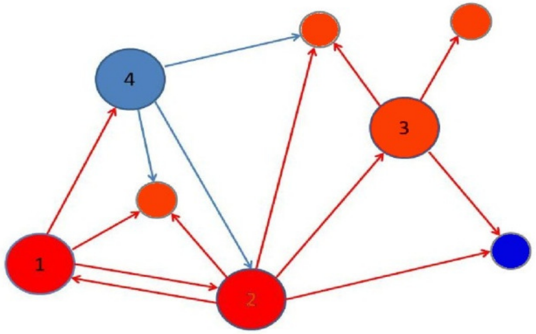
→ Critical state optimizes information transmission

ACTIVITY DEPENDENT MODEL

LdA,CPC, HJH, PRL 2006, PRE 2007

We introduce the main ingredients of neural activity:

Threshold firing, Neuron refractory period, Activity dependent synaptic plasticity



- We assign to each neuron a potential v_i

  and to each synapse a strength g_{ij}

$$\boxed{g_{ij} \neq g_{ji}}$$

- A neuron fires when the potential is at

  or above threshold v_{max} (-55mV)

$$v_j(t+1) = v_j(t) \pm \frac{k_{out}^i}{k_{in}^j} v_i \frac{g_{ij}(t)}{\sum_k g_{ik}(t)}$$

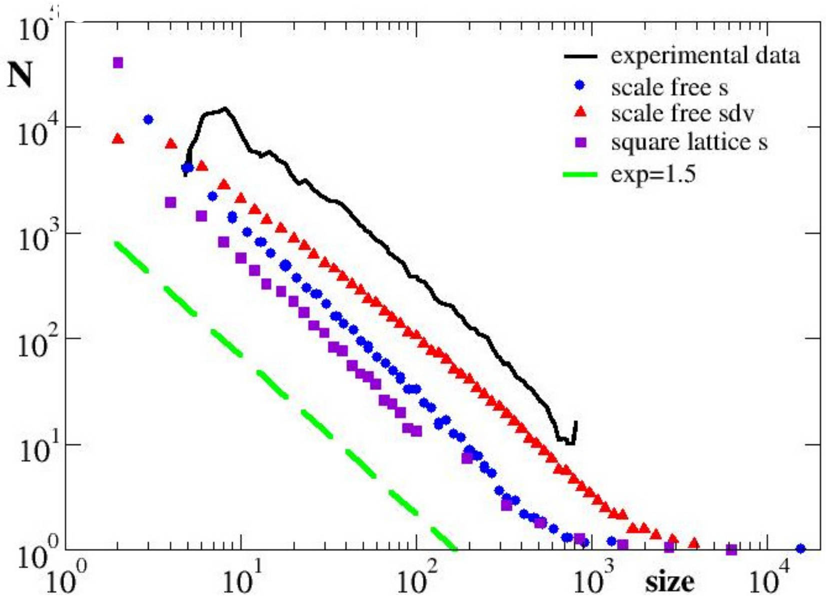
- Synapses can be excitatory or inhibitory
- After firing a neuron is set to zero resting potential (-70mV) and remains quiescent for one time step (refractory period)
- Activity dependent (Hebbian) plasticity and pruning

$$g_{ij}(t+1) = g_{ij}(t) + \alpha\left(v_j(t+1) - v_j(t)\right) / v_{max}$$

- Activity is triggered by random stimulation of a single neuron

AVALANCHE DISTRIBUTIONS

After training the network by plastic adaptation, we apply a sequence of stimuli at random to trigger avalanche activity

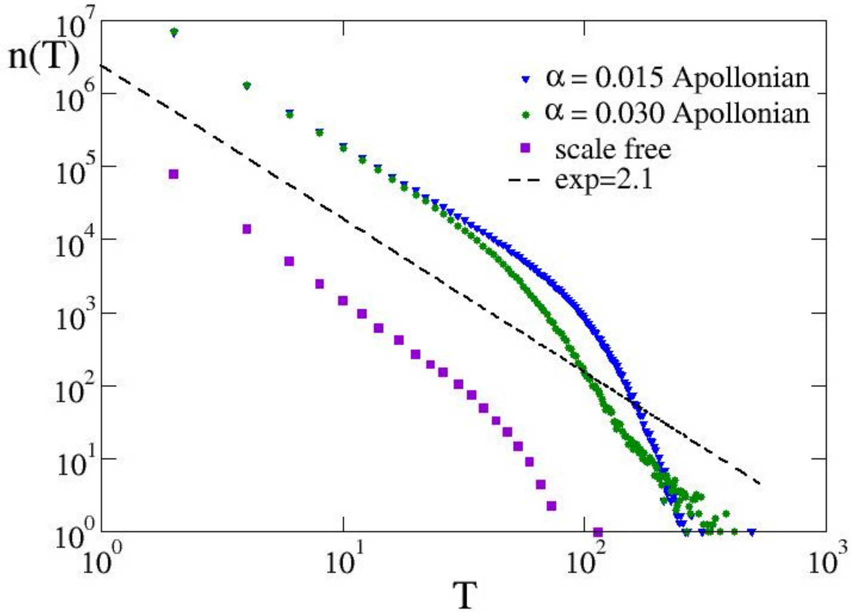


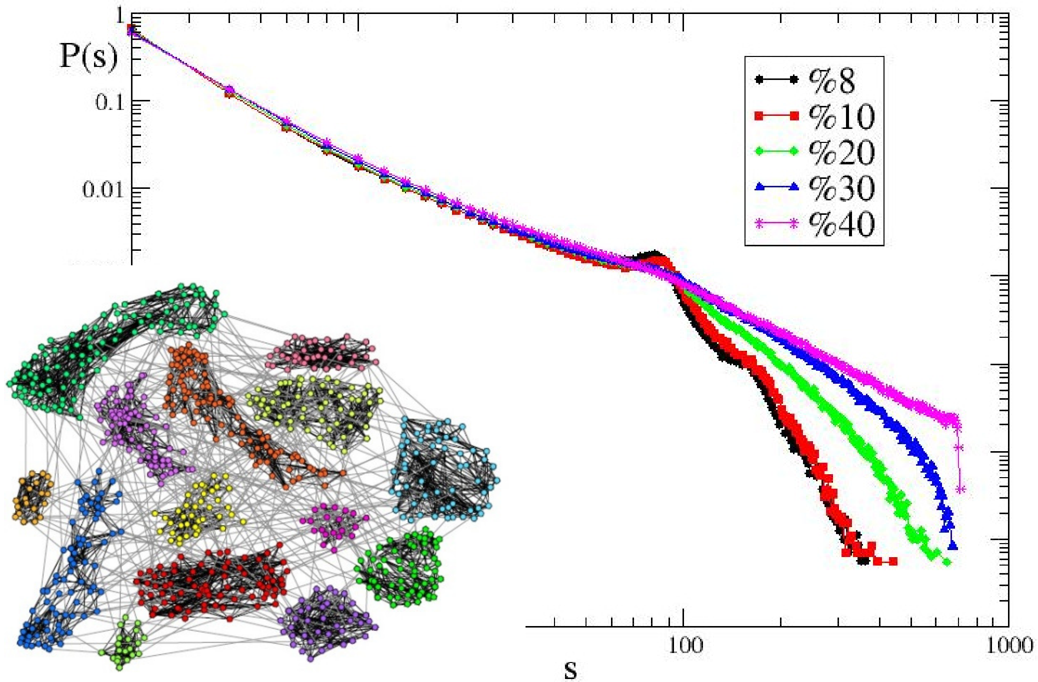
- different α
- regular, small world, scale-free networks
- excitatory and inhibitory synapses

1.5±0.1 & 2.1±0.1 for avalanche size & duration

Levina, Herrmann, Geisel, Nat Phys 2007

Millmann, Mihalas, Kirkwood, Niebur, Nat Phys 2010

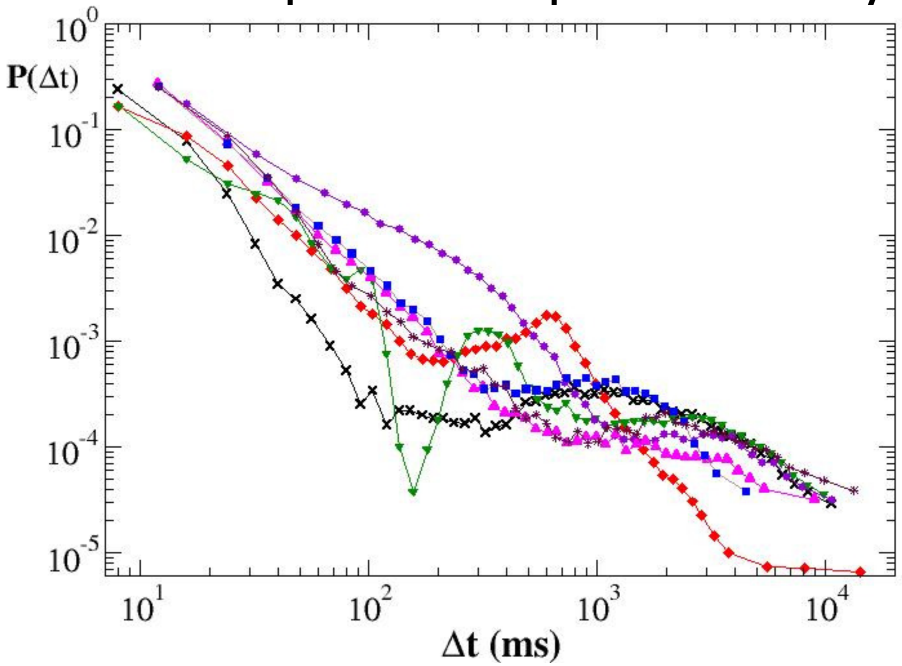


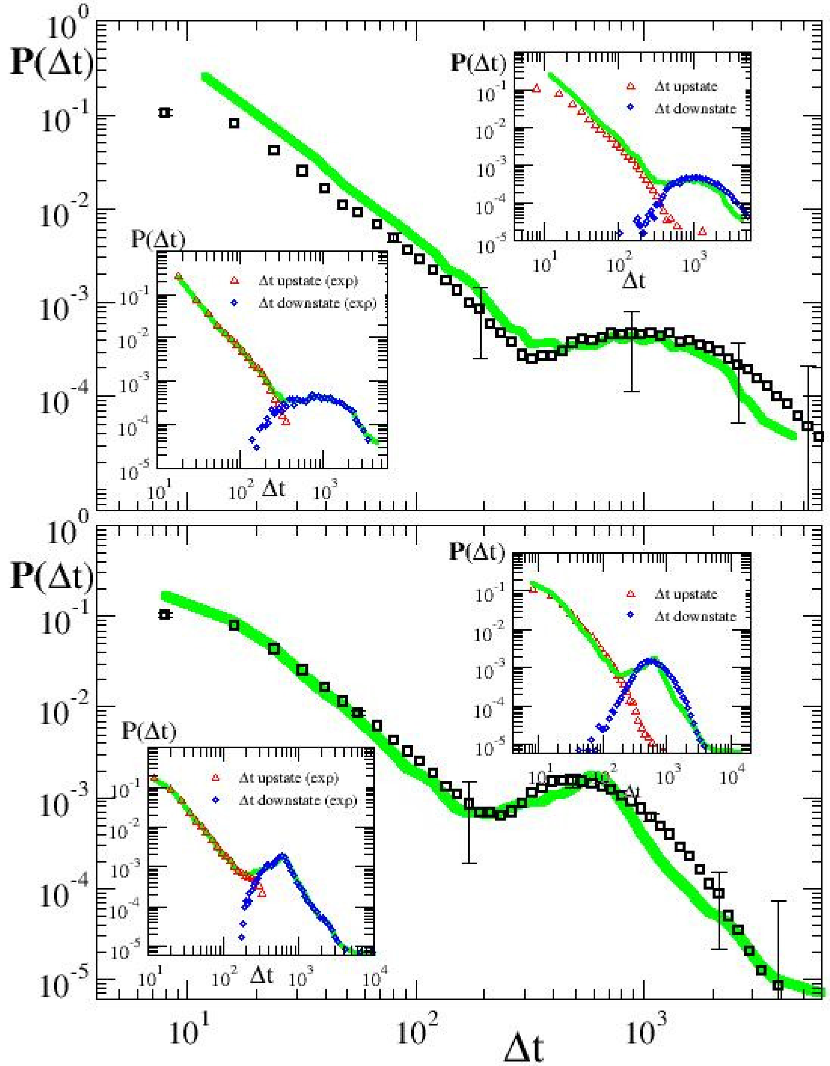


Russo et al Nat. Sci. Rep. 2013

Avalanche inter-time distribution

Experiments performed by D. Plenz (Lombardi et al PRL 2012)





Spontaneous neuronal activity can exhibit slow oscillations between bursty periods, up-states, and quiet periods, down-states

Small correlated avalanches,
neurons depolarized after firing

Disfacilitation period after large avalanche
Neurons hyperpolarized after firing

Implementation of up and down states

- Down-state → After an avalanche with $\boxed{s \geq s_{\min}}$

all neurons active in the last avalanche become hyperpolarized depending on their own activity

$$\boxed{v_i = v_i - h\delta v_i}$$

h > 0 is a hyper-polarization constant

short term memory at neuron level

System is stimulated by a small constant random drive

- Up-state → After an avalanche with $\boxed{s < s_{\min}}$

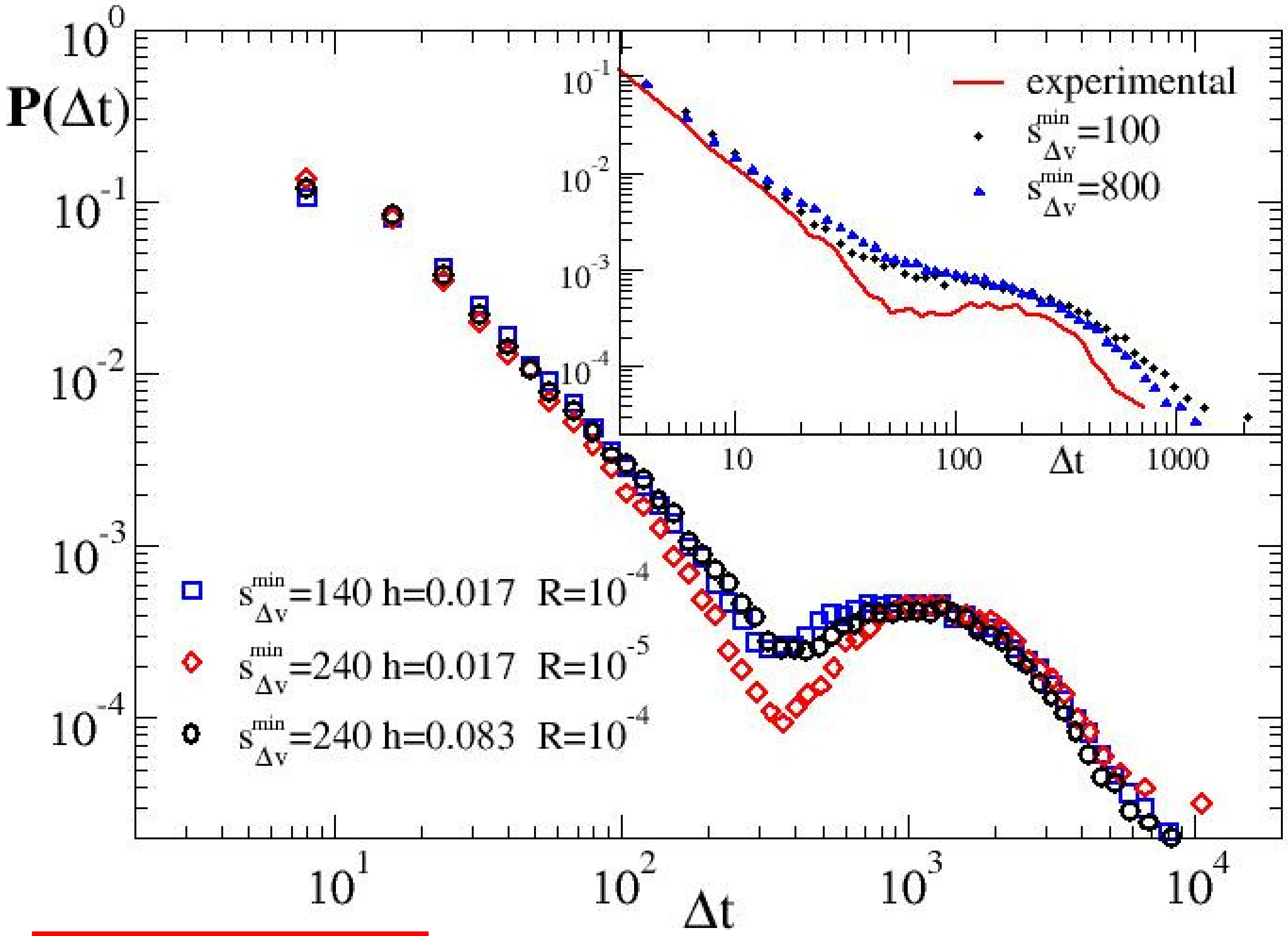
all neurons active in the last avalanche become depolarized depending on the last avalanche size

$$\boxed{v_i = v_{\max}(1 - s/s_{\min})}$$

the smaller the last avalanche
the closer the potential to the firing threshold

Memory at the network level

System is stimulated by a random drive
(network effect which sustains the up-state) $\boxed{\in]0, s_{\min}/s[}$



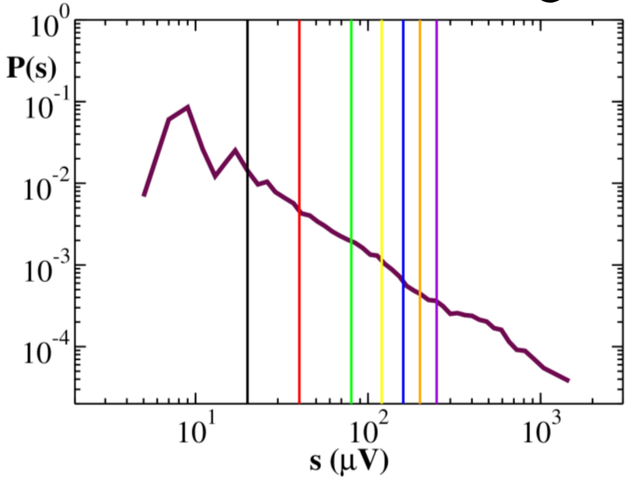
simple dual drive does not work!

$$\boxed{R = h / s_{\min}}$$

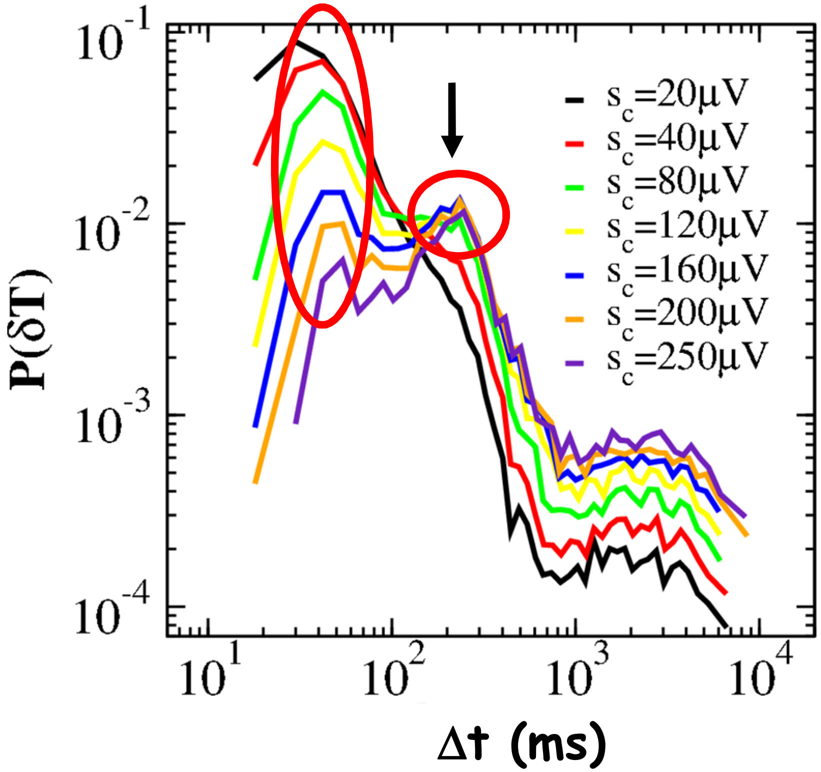
espressing the balance between excitation and inhibition is the unique parameter controlling the distribution

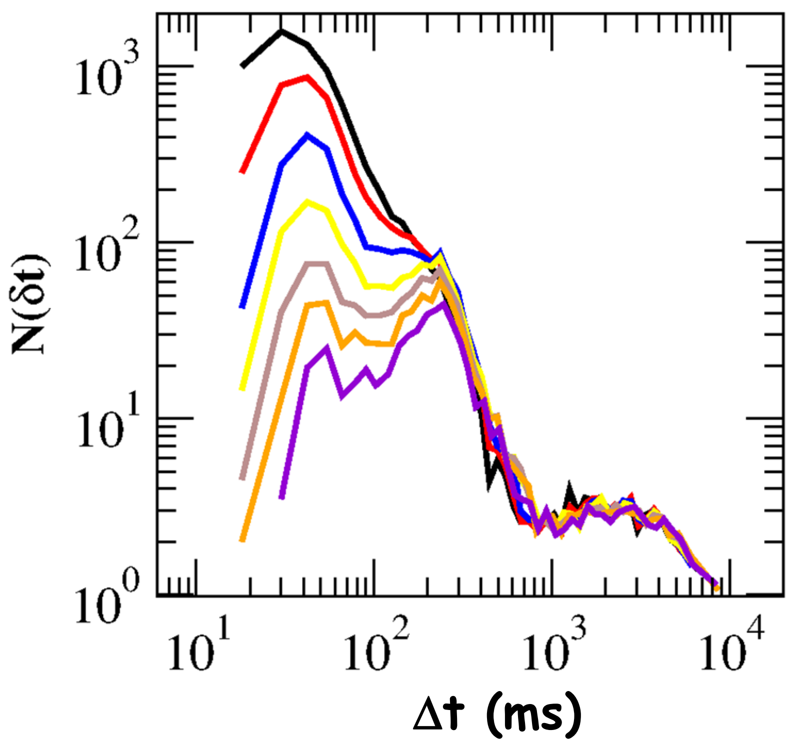
→ Homeostatic regulatory mechanism

$P(\Delta t;\ s_c)$ for avalanches with $s > s_c$

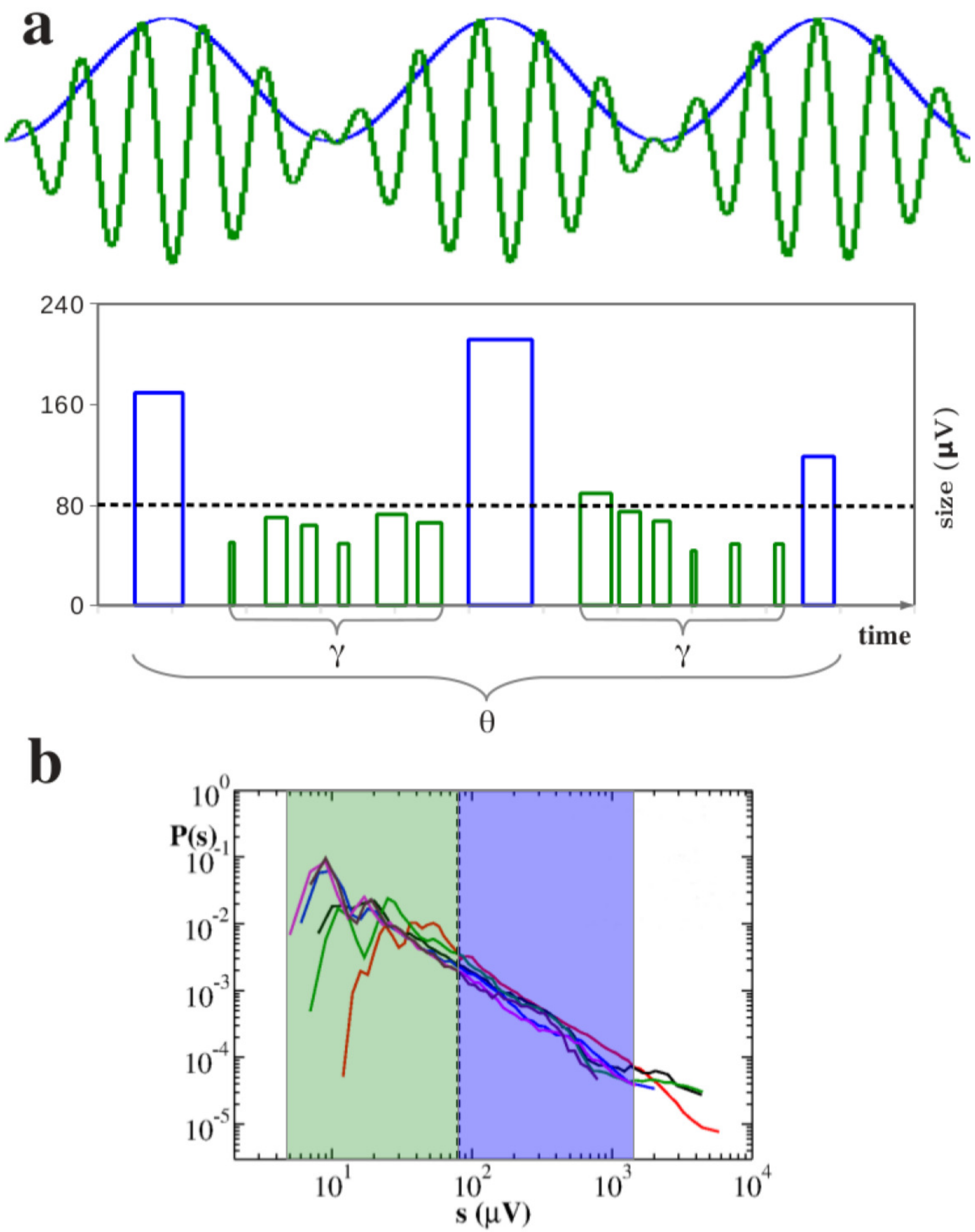


- Remove avalanches smaller than a given **threshold s_c**
- Evaluate new **$P(\Delta t;\ s_c)$**
- Fixed point at θ period





Avalanches and oscillations



- Hierarchical structure corresponding to nested θ-γ oscillations
- **Large avalanches** occur with **θ** frequency and trigger **smaller ones** related to **γ**
- Sizes related to **θ** cycles fall within the **blue region of P(s)**
- Sizes related to **γ** cycles fall within the ***green region of P(s)***
- The relationship between avalanches and **oscillations** does **not** imply a **characteristic size**

Lombardi F, Herrmann HJ, Plenz D, de Arcangelis L. *Front. Syst. Neurosci.* 8:204 (2014)

Correlations in the brain

Lombardi, Chialvo, Herrmann, LdA CSF 2013

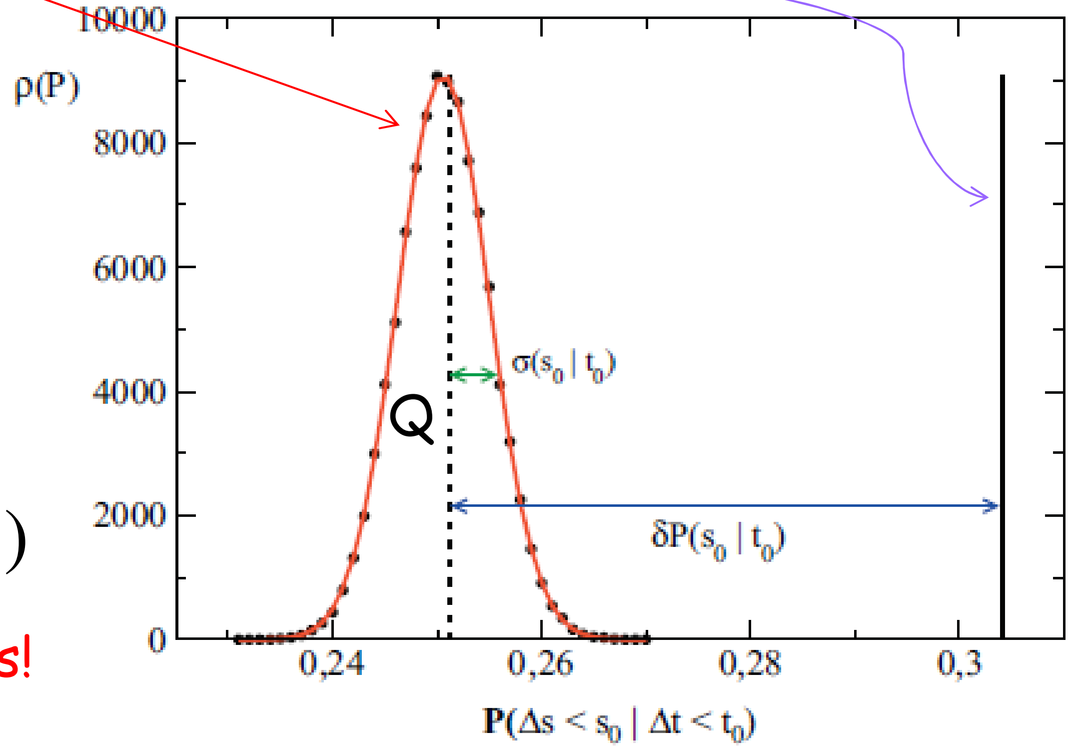
In fMRI data from 7 healthy humans we analyse extreme activity ($B>B_c$)

$$s_i(t) = B(\vec{r}_i, t+\delta t) - B(\vec{r}_i, t)$$ activity variation at each voxel *i*

We evaluate the conditional probability
with $\Delta t = t' - t$ and $\Delta s = s_l(t') - s_m(t)$

$$\boxed{P(\Delta s < s_0 \mid \Delta t < t_0)}$$

Both in the real and in a reshuffled catalog
where B are uncorrelated

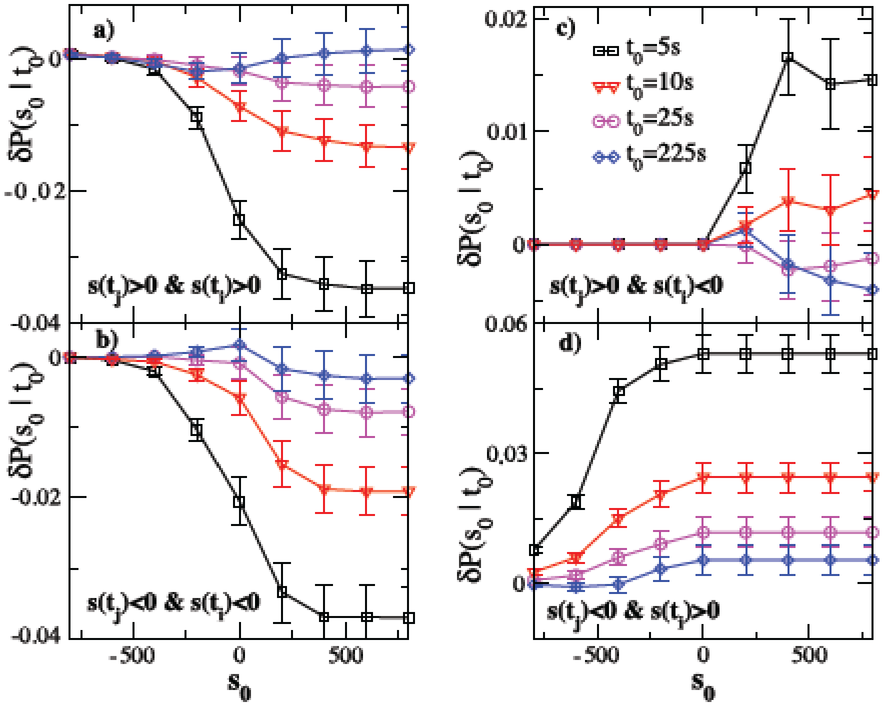


We monitor the conditional probability difference

$$\delta P(s_0 \mid t_0) =$$

$$P(\Delta s < s_0 \mid \Delta t_i < t_0) - Q(s_0 \mid t_0)$$

$$> \sigma(m_0 \mid t_0)$$ ➡ correlations!



$P(\Delta s < s_0 \mid \Delta t < t_0)$ is different than zero → Consecutive variations with opposite sign are correlated

↓

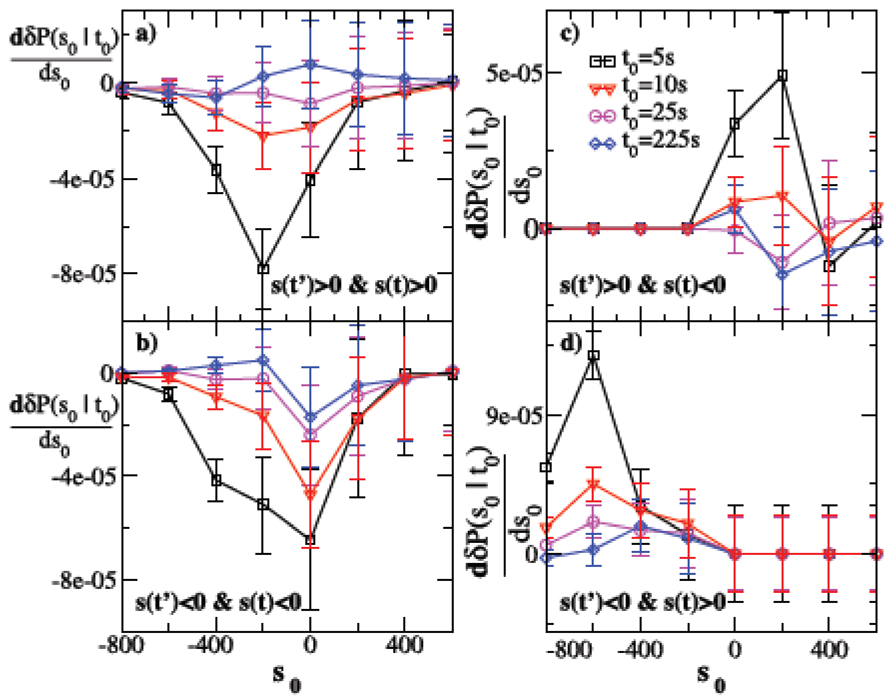
A local increase in activity induces a close-in-time activity depression

The derivative $\dfrac{d\delta P(s_0 \mid t_0)}{ds_0}$ represents the probability difference to observe $\Delta s = s_0$ with $\Delta t < t_0$

↓

Brain tends to realize activity balance

depressions are compensated by successive enhancements and vice versa



CONCLUSIONS

- Power law behaviour for event size distribution in many natural

  phenomena $\longrightarrow$ critical activity
- Complex temporal correlations
- Scaling properties of energy/time distributions unable to discriminate among different phenomena
- Conditional probability analysis $\longrightarrow$
- Aftershock triggering controlled by stress diffusion
- Solar flare occurrence driven by kink instability due to turbulent flow
- Balance between excitation and inhibition controls temporal organization in brain activity